

**ESE
GATE**

**MADE EASY
workbook 2024**



**Detailed Explanations of
Try Yourself Questions**

**Electrical Engineering
Electromagnetic Theory**



MADE EASY
Publications

1

Vector Analysis

T1. (d)

$$\bar{B} = -\rho \hat{a}_\phi + z \hat{a}_z$$

$$\rho = \sqrt{x^2 + y^2}$$

$$z = z$$

$$\begin{bmatrix} a_\phi \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

$$\Rightarrow \hat{a}_\phi = -\sin\phi \hat{a}_x + \cos\phi \hat{a}_y$$

$$\tan\phi = \frac{y}{x}$$

$$\hat{B} = -\sqrt{x^2 + y^2} \{-\sin\phi \hat{a}_x + \cos\phi \hat{a}_y\} + z \hat{a}_z$$

$$= -\sqrt{x^2 + y^2} \left\{ \frac{-y}{\sqrt{x^2 + y^2}} \hat{a}_x + \frac{x}{\sqrt{x^2 + y^2}} \hat{a}_y \right\} + z \hat{a}_z$$

$$= y \hat{a}_x - x \hat{a}_y + z \hat{a}_z$$

T2. (-3)

As

$$\int_C \bar{F} \cdot d\bar{l} \equiv \int y dx - \int x dy$$

So,

$$y = x^2 \Rightarrow dy = 2x dx$$

$$\int_C \bar{F} \cdot d\bar{l} = \int x^2 dx - \int x \cdot 2x dx$$

$$= \int (x^2 - 2x^2) dx \equiv - \int_{x=-1}^2 x^2 dx$$

$$\equiv - \left[\frac{x^3}{3} \right]_{-1}^2 \equiv - \left[\frac{8}{3} + \frac{1}{3} \right] \equiv -3$$

T3. (224)

$$\bar{A} = \nabla f = 4xyz\hat{a}_x + 2x^2z\hat{a}_y + 2x^2y\hat{a}_z$$

$$(0,0,0) \xrightarrow{dx\hat{a}_x} (2,0,0) \xrightarrow{dy\hat{a}_y} (2,7,0) \xrightarrow{dz\hat{a}_z} (2,7,4)$$

$$\begin{aligned}\therefore \int \bar{A} \cdot d\bar{l} &= \int 4xyz dx \text{ (at } y=0 \text{ and } z=0) + \int 2x^2z dy \text{ (at } z=0 \text{ and } x=2) + \int_{z=0}^4 2x^2y dz \text{ (at } x=2 \text{ and } y=7) \\ &= 224\end{aligned}$$

T4.

$$\oint \bar{D} \cdot d\bar{S} = \int \frac{5r^2}{4} \cdot r^2 \sin\theta d\theta d\phi \quad (\text{at } \theta=0, \frac{\pi}{4}, \phi=0, 2\pi) \\ = 589.1 \text{ C}$$

$$\int (\nabla \cdot \bar{D}) dV = \int (5r) \cdot r^2 \sin\theta d\theta d\phi dr = 589.1 \text{ C}$$

T5. (d)

$$\begin{aligned}\nabla \cdot F &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} [\rho(\sin^2 \phi)] + \frac{\partial}{\partial z} (-z) \\ &= 2 + 2\sin\phi\cos\phi - 1 = 1 + \sin 2\phi\end{aligned}$$

T6. Sol.

$$\nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{1}{r} \right) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (-r^2 \sin\theta \sin\theta) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} (10 \cos\phi)$$

$$= \frac{1}{r^2} - 2r \cos\theta - \frac{10 \sin\phi}{r \sin\theta}$$

$$\nabla \cdot \vec{A} \text{ at } \left(2, \frac{\pi}{4}, \frac{\pi}{2} \right) = \frac{1}{4} - 4 \times \frac{1}{\sqrt{2}} - \frac{10}{2 \times \frac{1}{\sqrt{2}}} = \frac{1}{4} - 7\sqrt{2} = -9.65$$

T7. (b)

$$\nabla \cdot A = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot Kr^n) = \frac{1}{r^2} K(n+2) r^{n+1}$$

$$\nabla \times A = 0$$

Hence, for $n = -2$ given vector is solenoidal and always irrotational.

T8. (d)

$$\oint \bar{A} \cdot d\bar{l} = \left[\int_A^B + \int_B^C + \int_C^A \right] \bar{A} \cdot d\bar{l}$$

$$\begin{aligned}\bar{A} &= 3x^2y^3\hat{a}_x - x^3y^2\hat{a}_y \\ d\bar{l} &= dx\hat{a}_x + dy\hat{a}_y\end{aligned}\left\{\begin{array}{l}\bar{A} \cdot d\bar{l} = 3x^2y^3dx - x^3y^2dy\end{array}\right.$$

$$\bar{A} \cdot d\bar{l} = 3x^2y^3dx - x^3y^2dy$$

Path AB :

$$y = x \Rightarrow dy = dx$$

$$\int \bar{A} \cdot d\bar{l} = \int 3x^2y^3dx - x^3y^2dy = \int 3x^5 - x^5dx = \int_1^2 2x^5dx = 2 \cdot \frac{x^6}{6} \Big|_1^2 = 21$$

Path CA :

$$x = 2 \Rightarrow dx = 0$$

Path CA :

$$y = 1 \Rightarrow dy = 0$$

$$\int \bar{A} \cdot d\bar{l} = \int 3x^2y^3dx = 3y^3 \int_{x=2}^1 x^2dx \text{ at } y = 1 = 3 \cdot \frac{x^3}{3} \Big|_2^1 = -7$$

Path BC :

$$x = 2 \Rightarrow dx = 0$$

$$\therefore \oint \bar{A} \cdot d\bar{l} = 21 + \frac{56}{3} - 7 \equiv \frac{98}{3}$$

$$\int \bar{A} \cdot d\bar{l} = - \int_2^1 x^3y^2dy \text{ at } x = 2$$

$$= \frac{56}{3}$$

$$\oint \bar{A} \cdot d\bar{l} = 21 + \frac{56}{3} - 7 = \frac{98}{3}$$

$\int (\nabla \times \bar{A}) \cdot d\bar{S} :$

$$\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3 & -x^3y^2 & 0 \end{vmatrix} = -12x^2y^2\hat{a}_z$$

$$d\bar{S} = dx dy (-\hat{a}_z)$$

(using RH curl)

$$\begin{aligned}\therefore \int \nabla \times \bar{A} \cdot d\bar{S} &= 12 \int_{x=1}^2 x^2 dx \int_{y=1}^x y^2 dy = 12 \int_{x=1}^2 x^2 dx \frac{y^3}{3} \Big|_{y=2}^x = \frac{12}{3} \int_{x=1}^2 x^2 dx (x^3 - 1) \\ &= 4 \left[\int_1^2 x^5 dx - \int_1^2 x^2 dx \right] = \frac{98}{3}\end{aligned}$$



2

Electrostatics

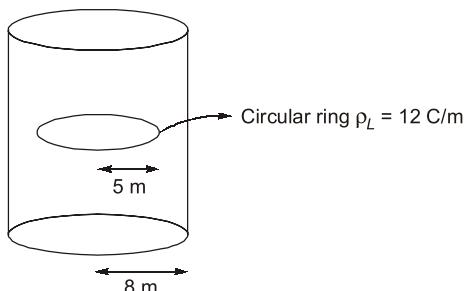
T1. Sol.

$$\begin{aligned}
 \psi &= Q_{\text{enc}} = \int P_V dV = \int (\nabla \cdot \bar{D}) dV \\
 \Rightarrow \quad \psi &= \int_V (y + x + z) dx dy dz \\
 &= \int_{y=-2}^2 y dy \int_{x=1}^4 dx \int_{z=-1}^2 dz + \int_{z=-1}^2 zdz \int_{x=1}^4 dx \int_{y=-2}^2 dy \\
 \Rightarrow \quad \psi &= \left. \frac{y^2}{2} \right|_{-2}^2 \cdot x|_1^4 \cdot z|_{-1}^2 + \left. \frac{x^2}{2} \right|_1^4 \cdot y|_{-2}^2 \cdot z|_{-1}^2 + \left. \frac{z^2}{2} \right|_{-1}^2 \cdot x|_1^4 \cdot y|_{-2}^2 \\
 \Rightarrow \quad \psi &= 0 + \frac{15}{2} \cdot 4 \cdot 3 + \frac{3}{2} \cdot 3 \cdot 4 = 90 + 18 = 108C
 \end{aligned}$$

T2. (b)

$$\nabla \cdot D = \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho 20\rho) + \frac{\partial}{\partial \rho} \left(\rho \frac{\rho^2}{3} \right) \right] = \frac{20 \times 2\rho}{\rho} + \frac{1}{3} \cdot \frac{3\rho^2}{\rho} = 40 + \rho$$

T3. Sol.



$$\text{Total charge enclosed} = \rho_L \times 2\pi R = 12 \times 2\pi \times 5 = 120\pi \text{ C}$$

T4. (c, d)

T5. (d)

Calculate the distance of the charges from the centre of the sphere and identify whether the charge is inside or outside the sphere.

For 2 C, $r_1 = \sqrt{4^2 + 8^2 + 3^2} = 9.43 \rightarrow$ Outside the sphere

For 8 C, $r_2 = \sqrt{2^2 + 1^2 + 3^2} = 3.74 \rightarrow$ Inside the sphere

For -12 C, $r_3 = \sqrt{4^2 + 0^2 + 1^2} = 4.123 \rightarrow$ Inside the sphere

Flux leaving the surface = 8 - 12 = -4 C

T6. (c)

1 nC and 3 nC.

T7. (a)

$$\begin{aligned}\text{Charge enclosed} &= \rho_L \times \text{length}_{\text{enclosed}} \\ &= 15 \times 10 = 150 \text{ nC}\end{aligned}$$

T8. (b)

$$\bar{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_p$$

$$d\hat{s} = dydz\hat{a}_x$$

$$\therefore \phi = \int \bar{D} \cdot d\hat{s} = \frac{\rho_L}{2\pi\rho} dydz \hat{a}_p \cdot \hat{a}_x$$

$$\because \rho = \sqrt{x^2 + y^2}; \hat{a}_p \cdot \hat{a}_x = \cos\phi$$

$$\therefore \phi = \int \frac{\rho_L}{2\pi\sqrt{x^2 + y^2}} dydz \cdot \cos\phi$$

$$\therefore \tan\phi = \frac{y}{x}$$

$$\therefore \cos\phi = \frac{x}{\sqrt{x^2 + y^2}}$$

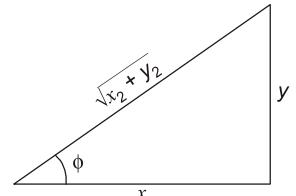
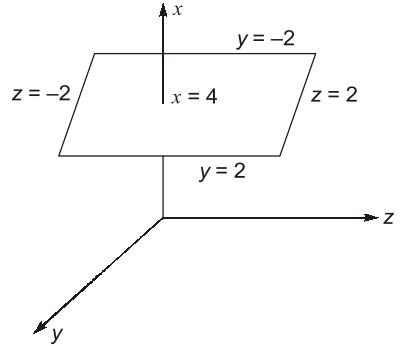
$$\begin{aligned}\text{Hence, } \phi &= \int \frac{\rho_L}{2\pi\sqrt{x^2 + y^2}} dy dz \cdot \frac{x}{\sqrt{x^2 + y^2}} \\ &= \int \frac{\rho_L x}{2\pi\sqrt{x^2 + y^2}} dy dz\end{aligned}$$

As $x = 4$, $y \in (-2, 2)$, $z \in (-2, 2)$

$$\therefore \phi = \int \frac{\rho_L \cdot 4}{2\pi(16 + y^2)} dy dz$$

$$= \frac{\rho_L}{2\pi} \int_{-2}^2 \frac{4dy}{16 + y^2} z \Big|_{-2}^2$$

$$= \frac{4\rho_L}{2\pi} \cdot \int_{-2}^2 \frac{4}{(4)^2 + y^2} dy$$



$$\begin{aligned}
 &= \frac{2\rho_L}{2\pi} \cdot 4 \cdot \int_{-2}^2 \frac{1}{(4)^2 + y^2} dy \\
 &= \frac{8\rho_L}{\pi} \cdot \frac{\tan^{-1}\left(\frac{1}{2}\right)}{4} \\
 &= \frac{2\rho_L}{\pi} \tan^{-1}\left(\frac{1}{2}\right) \cdot 2 \\
 &= \frac{4\rho_L}{\pi} \tan^{-1}\left(\frac{1}{2}\right) \\
 &= 5.903 \text{ nC}
 \end{aligned}$$

T9. Sol.

$$E \text{ due to line charge} = \frac{\rho_L}{2\pi\epsilon\sqrt{2^2+2^2}} \frac{(\hat{a}_x + \hat{a}_y)}{\sqrt{2}} = \frac{\rho_L}{8\pi\epsilon} (\hat{a}_x + \hat{a}_y)$$

$$E \text{ due to sheet charge} = \frac{\rho_s}{2\epsilon} (-\hat{a}_y)$$

$$\text{Equating the } y\text{-components, } \frac{\rho_L}{8\pi\epsilon} = \frac{\rho_s}{2\epsilon}$$

$$\rho_L = 4\pi\rho_s$$

T10. Sol.

$$Q = \int_1^1 \rho_s ds = \int_{x=-1}^1 \int_{y=0}^2 4x^2 y dx dy = 4 \cdot \frac{x^3}{3} \Big|_{-1}^1 \frac{y^2}{2} \Big|_0^2 = \frac{16}{3}$$

T11. Sol.

$$V = 100(x^2 - y^2)$$

At (2, 1, 1),

$$V = 100(4 - 1) = 300$$

So,

$$100(x^2 - y^2) = 300$$

\Rightarrow

$$x^2 - y^2 = 3$$

T12. (a, c, d)

(a) At (1, 0, -1);

$$V = 3(1)^2(0) - (0)(1) = 0$$

\therefore

$$\bar{E} = -\nabla V = -\left[\frac{\partial}{\partial x}(3x^2y - yz)\hat{a}_x + \frac{\partial}{\partial y}(3x^2y - yz)\hat{a}_y + \frac{\partial}{\partial z}(3x^2y - yz)\hat{a}_z \right] \hat{a}_z$$

\therefore

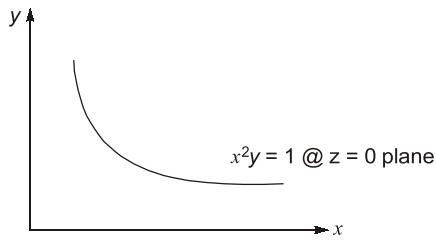
$$\bar{E} = -[6xy\hat{a}_x + (3x^2 - z)\hat{a}_y - (y)\hat{a}_z]$$

Hence, at (1, 0, -1);

$$\bar{E} \neq 0$$

(b) In xy-plane,

$$z = 0 = V = 3x^2y$$



So, at $x^2y = 1$; $V = 3$ is Constant

Hence, $x^2y = 1$ is an equipotential plane in $z = 0$ plane.

$$(c) \text{ At } P(2, -1, 4), V = 3(2)^2(-1) - (-1)(4) = -8$$

$\therefore V = -8$ is an equipotential surface passing through $(2, -1, 4)$.

$$(d) \text{ Given: } V = 3x^2y - yz$$

\therefore

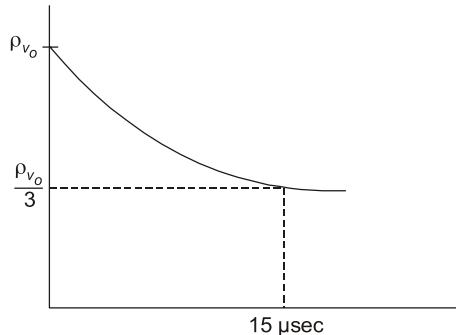
$$\hat{a}_n = \frac{\nabla V}{|\nabla V|}_{(2,-1,4)}$$

\Rightarrow

$$\hat{a}_n = \frac{6(2)(-1)\hat{a}_x + (3 \times 4 - 4) + 1\hat{a}_z}{\sqrt{12^2 + 8^2 + 1^2}}$$

\Rightarrow

$$\hat{a}_n = \frac{-12\hat{a}_x + 8\hat{a}_y + 1\hat{a}_z}{14.456} \equiv -0.83\hat{a}_x + 0.55\hat{a}_y + 0.07\hat{a}_z$$

T13. (b)


$$\rho_v = \rho_{v_0} e^{-t/T_r}; T_r = \frac{\epsilon}{\sigma}$$

\therefore

$$\frac{\rho_{v_0}}{3} = \rho_{v_0} e^{-t/T_r}$$

$$\frac{t}{T_r} = 1.0986$$

$$T_r = \frac{15 \times 10^{-6}}{1.0986} = 1.4 \times 10^{-5}$$

$$\epsilon_r = \frac{1.4 \times 10^{-5} \times 2 \times 10^{-4}}{8.85 \times 10^{-12}} = 308.56$$

T14. Sol.

$$3x + 4y = 12$$

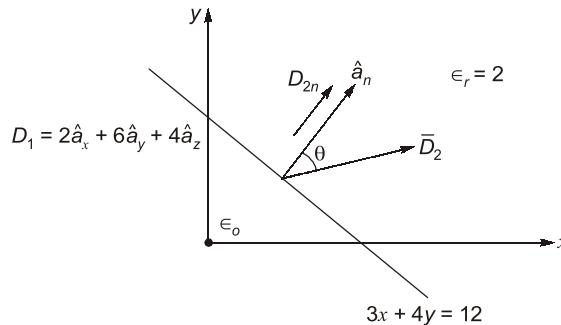
At $x = 0$,

$$y = 3$$

At $y = 0$,

$$x = 4$$

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{3\hat{a}_x + 4\hat{a}_y}{5}$$



Normal component :

$$\therefore \bar{D}_{in} = (\bar{D}_{in} \cdot \hat{a}_n) \hat{a}_n = \left(2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z \cdot \frac{3\hat{a}_x + 4\hat{a}_y}{5} \right) \left(\frac{3\hat{a}_x + 4\hat{a}_y}{5} \right)$$

$$\Rightarrow \bar{D}_{in} = 6 \left\{ \frac{3\hat{a}_x + 4\hat{a}_y}{5} \right\} \equiv 3.6\hat{a}_x + 4.8\hat{a}_y \equiv \bar{D}_{2n}$$

Tangential component :

$$\bar{E}_{1t} = \bar{E}_{2t}$$

$$\Rightarrow \frac{\bar{D}_{1t}}{\epsilon_1} = \frac{\bar{D}_{2t}}{\epsilon_2}$$

$$\Rightarrow \bar{D}_{2t} = \frac{\epsilon_2}{\epsilon_1} \bar{D}_{1t} \equiv \frac{2\epsilon_0}{\epsilon_0} \{ \bar{D}_1 - \bar{D}_{in} \}$$

$$\equiv 2\{(2, 6, 4) - (3.6, 4.8, 0)\}$$

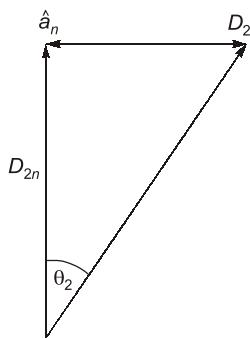
$$\equiv 2\{-1.6, 1.2, 4\}$$

$$\equiv -3.2, 2.4, 8$$

$$\therefore \bar{D}_{2t} = 3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z$$

$$\text{Hence, } \bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2x} \equiv -3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z + 3.6\hat{a}_x + 4.8\hat{a}_y$$

$$\Rightarrow \bar{D}_2 = 0.4\hat{a}_x + 7.2\hat{a}_y + 8\hat{a}_z \text{ V/m}$$



$$\cos \theta_2 = \frac{|D_{2n}|}{D_2} = \frac{6}{\sqrt{0.4^2 + 7.2^2 + 8^2}} \\ = 56.14^\circ$$

T15. Sol.

$$Q = C_o V_o = C_{eq} V' \\ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 1.6 \\ V' = \frac{24}{1.6} = 15 \text{ V}$$

T16. (c)

For air filled parallel plate capacitor

$$C_o = \frac{\epsilon_0 A}{d}$$

$$Q = C_o V$$

The equivalent arrangement is parallel capacitor

$$C_{eq} = \frac{4\epsilon_0 A}{2d} + \frac{\epsilon_0 A}{2d} = \frac{5\epsilon_0 A}{2d} = \frac{5}{2} C_o$$

$$Q_T = C_{eq} V'$$

$$2.5Q = 2.5C_o V'$$

$$Q = C_o V'$$

$$Q = C_o V$$

But

∴

$$V' = V$$

T17. Sol.

$$C_1 = \frac{\epsilon_1 A/2}{d}$$

$$C_2 = \frac{\epsilon_2 A/2}{d}$$

$$W_E = \frac{1}{2} C V^2$$

$$\frac{W_{E_1}}{W_{E_2}} = \frac{1}{2} = 0.5$$

T18. Sol.

$$\begin{aligned}\bar{E}_{\text{cyl.cap.}} &= \frac{\rho_L}{2\pi \epsilon_0 \rho} \hat{a}_\rho = \frac{Q}{2\pi \epsilon_0 l \rho} \hat{a}_\rho \\ \Rightarrow V &= - \int \bar{E} \cdot d\bar{l} = - \int \frac{Q}{2\pi \epsilon_0 l \rho} d\rho \\ &= - \frac{Q}{2\pi \epsilon_0 l} \int \frac{d\rho}{\rho \left(\frac{10+\rho}{\rho} \right)} = - \frac{Q}{2\pi \epsilon_0 l} \int_{\rho=a}^b \frac{d\rho}{10+\rho} \\ &= \frac{-Q}{2\pi \epsilon_0 l} \ln \left[\frac{10+b}{10+a} \right]\end{aligned}$$

Taking absolute value,

$$\begin{aligned}C &= \frac{Q}{|V|} = \frac{2\pi \epsilon_0 l}{\ln \left[\frac{10+b}{10+a} \right]} \\ \Rightarrow \frac{C}{l} &= \frac{2\pi \epsilon_0}{\ln \left[\frac{10+b}{10+a} \right]} = 434.6 \text{ PF/m}\end{aligned}$$

T19. (a, b)

Before touching,

$$F \propto q_1 q_2$$

After touching,

$$F \propto \left(\frac{q_1 + q_2}{2} \right)^2$$

∴

$$\begin{aligned}\frac{F'}{F} &= \frac{(q_1 + q_2)^2}{4q_1 q_2} \\ &= \frac{1}{4} \left[\frac{q_1}{q_2} + \frac{q_2}{q_1} + 2 \right]\end{aligned}$$

$$\text{Let } \frac{q_1}{q_2} = x$$

$$\frac{F'}{F} = \frac{1}{4} \left[x + \frac{1}{x} + 1 \right]$$

Let

$$f(x) = x + \frac{1}{x}$$

For minima,

$$f'(x) = 0$$

$$f'(x) = 1 - \frac{1}{x^2} = 0$$

$$\begin{aligned}x &= \pm 1 \\ \therefore f(x) &\geq 2 \text{ (minimum value)} \\ \frac{F'}{F} &\geq \frac{1}{4}[2+2] \geq 1 \\ F' &\geq F\end{aligned}$$

T20. Sol.

$$\begin{aligned}\tan 60^\circ &= \frac{OQ'}{\frac{d}{2}} \\ \sqrt{3} &= \frac{2OQ'}{d} \\ OQ' &= \frac{\sqrt{3}d}{2} \\ d' &= d - \frac{\sqrt{3}}{2}d = 0.134d \\ 2E \sin \theta + E' &= 0 \\ \frac{2Q \times 4}{4\pi\epsilon_0 d^2} \times \frac{d \times 2}{d} &= \frac{-Q'}{4\pi\epsilon_0 \times 0.018d^2} \\ \frac{16Q}{5\sqrt{5}} &= \frac{-Q'}{0.018} \\ Q' &= -\frac{16 \times 0.018}{5\sqrt{5}} = -0.025Q\end{aligned}$$

T21. (b)

Concept : Method of images

For \vec{F}_1 :

$$\begin{aligned}\bar{r} &= \bar{r}_f - \bar{r}_i = (1, 0, 1) - (1, 0, -1) = 2\hat{a}_z \\ \therefore \hat{a}_r &= \frac{\bar{r}}{|\bar{r}|} = \frac{2\hat{a}_z}{2} = \hat{a}_z \\ \therefore \bar{F}_1 &= \frac{-Q^2}{4\pi\epsilon_0(r)} \hat{a}_z = \frac{-Q^2}{16\pi\epsilon_0} \hat{a}_z\end{aligned}$$

For \bar{F}_2 :

$$\begin{aligned}\bar{r} &= \bar{r}_f - \bar{r}_i = (1, 0, 1) - (0, 0, -1) = \hat{a}_x + 2\hat{a}_z \\ |\bar{r}| &= \sqrt{5} \\ \therefore \hat{a}_r &= \frac{\bar{r}}{|\bar{r}|} = \frac{\hat{a}_x + 2\hat{a}_z}{\sqrt{5}} \\ \therefore \bar{F}_2 &= \frac{Q^2}{4\pi\epsilon_0(5)} \cdot \frac{\hat{a}_x + 2\hat{a}_z}{\sqrt{5}}\end{aligned}$$

For \bar{F}_3 :

$$\bar{r} = \bar{r}_f - \bar{r}_i = (1, 0, 1) - (0, 0, 1) = \hat{a}_x$$

$$|\bar{r}| = 1$$

$$\therefore \hat{a}_r = \frac{\hat{r}}{|\bar{r}|} = \hat{a}_x$$

$$\therefore \bar{F}_3 = \frac{-Q^2}{4\pi\epsilon_0(1)} \hat{a}_x$$

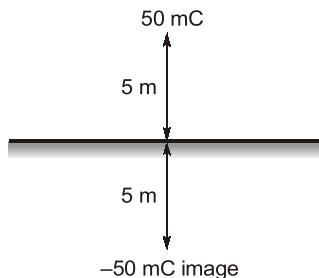
Hence,

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = -\frac{Q^2}{16\pi\epsilon} \hat{a}_z + \frac{Q^2}{4\pi\epsilon_0(5\sqrt{5})} (\hat{a}_x + 2\hat{a}_z) - \frac{Q^2}{4\pi\epsilon_0} \hat{a}_x$$

$$\bar{F} = \frac{Q^2}{4\pi\epsilon_0} \left[\frac{-\hat{a}_z}{4} + \frac{2\hat{a}_z}{5\sqrt{5}} + \frac{\hat{a}_x}{5\sqrt{5}} - \hat{a}_x \right]$$

$$\bar{F} = \frac{-Q^2}{4\pi\epsilon_0} [0.91\hat{a}_x + 0.071\hat{a}_z] \text{ N}$$

T22. Sol.



The induced surface charge density is the normal flux density at the point

$$D_{\text{normal}} = \rho_s$$

$$E \text{ due to charge} = \frac{50 \times 10^{-3} \times 9 \times 10^9}{25} = 18 \times 10^6 \text{ V/m}$$

$$E \text{ due to image} = 18 \times 10^6 \text{ V/m (same direction)}$$

$$\text{Total } E = 36 \times 10^6$$

$$D = 36 \times 10^6 \times \frac{1}{36\pi \times 10^9} = \frac{1}{\pi} \text{ mC}$$

■ ■ ■

3

Magnetostatics

T1. Sol.

$$H \propto \frac{I}{d} \text{ for a square loop}$$

$$\frac{H_1}{H_2} = \frac{I_1}{I_2} \frac{d_2}{d_1} = \frac{20}{5} \frac{d}{d/3} = 12$$

T2. (b)

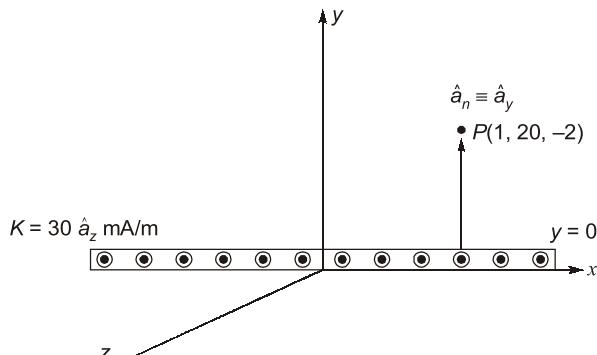
$$\begin{aligned} B \text{ at centre} &= \frac{\mu_o I}{2R} \left[1 - \frac{1}{2} + \frac{1}{4} \dots \right] \hat{a}_z \\ &= \frac{\mu_o I}{2R} \left[\frac{1}{1 - \left(-\frac{1}{2} \right)} \right] \hat{a}_z = \frac{\mu_o I}{3R} \hat{a}_z \end{aligned}$$

T3. (d)

$$\hat{a}_H = \hat{a}_L \times \hat{a}_r$$

1. For xy -plane wire : $\hat{a}_H = \hat{a}_x \times -\hat{a}_y = -\hat{a}_z$
2. For yz -plane wire : $\hat{a}_H = \hat{a}_y \times -\hat{a}_z = -\hat{a}_x$

T4. Sol.



$$\bar{K} = 30 \hat{a}_z \text{ mA/m at } y = 0, \text{ i.e., } xz\text{-plane.}$$

$P(1, 20, -2)$ has $y = 20$ which lies above $y = 0$.

So,

$$\hat{a}_x \equiv \hat{a}_y$$

Hence,

$$\begin{aligned} \bar{H} &= \frac{1}{2} \bar{K} \times \hat{a}_x \\ &= \frac{1}{2} \times 30 [\hat{a}_z \times \hat{a}_y] \\ &= -15 \hat{a}_x \end{aligned}$$

\therefore

$$\bar{H} = -15 \hat{i} \text{ mA/m}$$

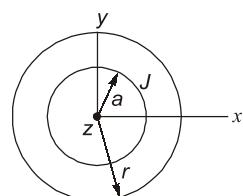
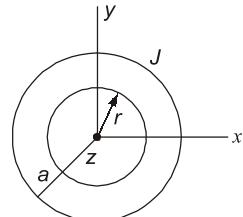
T5. (c)

Case (i) : $r < a$

$$\begin{aligned} I_{enc} &= \int J \cdot ds \equiv J\pi r^2 \\ H2\pi r &= J\pi r^2 \\ \Rightarrow H &= \frac{Jr}{2} \\ \therefore H &\propto r \end{aligned}$$

Case (ii) : $r > a$

$$\begin{aligned} I_{enc} &= \int \bar{J} \cdot d\bar{s} \equiv J \cdot \pi a^2 \\ H^2\pi r &= J\pi a^2 \\ \Rightarrow H &= \frac{Ja^2}{2r} \\ \therefore H &\propto \frac{1}{r} \end{aligned}$$



T6. Sol.

Net flux through the loop is zero considering opposite directions of flux as interpreted from arrow shown on the length of loop

$$\begin{aligned} \iint B \cdot ds &= \psi_1 - \psi_2 = 0 \\ \psi_1 - \psi_2 &= \int_0^b \int_a^b \frac{I}{2\pi\rho} dp dz = \int_0^c \int_b^c \frac{I}{2\pi\rho} dp dz \\ \ln\left(\frac{b}{a}\right) &= \ln\left(\frac{c}{b}\right) \\ b &= \sqrt{ac} \end{aligned}$$

T7. (a)

As

$$\bar{H} = -\nabla V_m$$

\therefore

$$\bar{H}|_{12A} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

$$-\nabla V_m = -\frac{1}{\rho} \cdot \frac{\partial V_m}{\partial \phi} \hat{a}_\phi$$

$$\therefore \frac{I}{2\pi r} = -\frac{1}{\rho} \cdot \frac{dV_m}{d\phi}$$

$$\int_0^{V_m} dV_m = -\frac{I}{2\pi} \int_{\pi/3}^{\pi/6} d\phi$$

$$\Rightarrow V_m = 1 \text{ A}$$

T8. Sol.

I flow direction = Vector potential direction

A direction at $\rho = \hat{a}_y$

B direction = $\hat{a}_y \times \hat{a}_x = -\hat{a}_z$

T9. Sol.

$$\nabla^2 A = -\mu J$$

$$\nabla^2 z^2 \hat{a}_\rho + \nabla^2 2\rho^2 \cos\phi \hat{a}_\phi = -\mu J$$

$$\nabla^2 z^2 = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial z^2}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\frac{1}{\rho} \frac{\partial z^2}{\partial \phi} \right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\rho \frac{\partial z^2}{\partial z} \right) = 2$$

$$\begin{aligned} \nabla^2 2\rho^2 \cos\phi &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} (2\rho^2 \cos\phi) \right) + \frac{1}{\rho} \frac{\partial}{\partial \phi} \left(\frac{1}{\rho} \frac{\partial}{\partial \phi} (2\rho^2 \cos\phi) \right) + \frac{1}{\rho} \frac{\partial}{\partial z} \left(\rho \frac{\partial}{\partial z} (2\rho^2 \cos\phi) \right) \\ &= \frac{1}{\rho} \cos\phi \cdot 4 \cdot 2 \cdot \rho - 2 \cos\phi = 6 \cos\phi \end{aligned}$$

At the origin,

$$\nabla^2 A = 2\hat{a}_\rho + 6\hat{a}_\phi = -\mu J$$

$$J = \frac{-1}{\mu} (2\hat{a}_\rho + 6\hat{a}_\phi)$$

T10. Sol.

$$\bar{B} = \nabla \times \bar{A} = \frac{-\partial A}{\partial \rho} \hat{a}_\phi = \frac{\rho}{2} \hat{a}_\phi, \quad d\bar{S} = d\rho dz \hat{a}_\phi$$

$$\therefore \phi = \int \bar{B} \cdot d\bar{S} = \frac{1}{2} \int_{z=0}^5 \int_{\rho=1}^2 \rho d\rho dz = \frac{\rho^2}{4} \Big|_0^1 (5) = \frac{15}{4} = 3.75 \text{ Wb}$$

T11. (a)

T12. Sol.

$$f = \mu N = \mu_{mg} = 0.1 \times 1 \times 10 = 1 \text{ N}$$

$$\bar{F} = i\bar{l} \times \bar{B} = 10(0.5)\hat{a}_y \times -1\hat{a}_z \equiv -5\hat{a}_x \Rightarrow |\bar{F}| = 5$$

$$F_{\text{net}} = F - f = 5 - 1 = 4$$

Also,

$$F_{\text{net}} = ma$$

$$\therefore ma = 4 \Rightarrow a = \frac{4}{m} = \frac{4}{1} = 4 \text{ m/s}^2$$

Now,

$$V^2 = u^2 + 2as$$

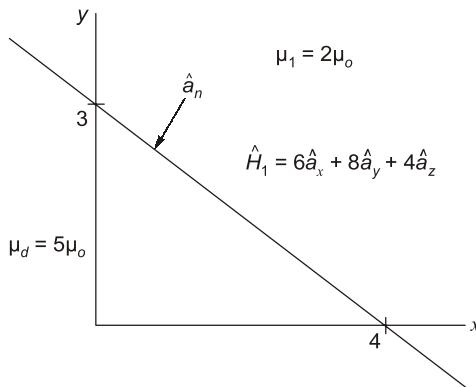
\Rightarrow

$$V^2 = 2as$$

\Rightarrow

$$V = \sqrt{2as} \equiv \sqrt{2 * 4 * 1} \equiv \sqrt{8} \equiv 2.8 \text{ m/s}$$

T13. (b)



$$3x + 4y = 12$$

At $x = 0$,

$$y = 3$$

At $y = 0$,

$$x = 4$$

As

$$\bar{H}_1 = 6\hat{a}_x + 8\hat{a}_y + 4\hat{a}_z$$

\Rightarrow

$$\begin{aligned} \bar{H}_{1n} &= (\bar{H}_1 \cdot \hat{a}_n) \hat{a}_n = \left\{ (6, 8, 4) \left(\frac{3, 4, 0}{5} \right) \right\} \left(\frac{3, 4, 0}{5} \right) \\ &= \left\{ \frac{18 + 32}{25} \right\} (3\hat{a}_x + 4\hat{a}_y) \equiv 6\hat{a}_x + 8\hat{a}_y \end{aligned}$$

\therefore

$$\bar{H}_{1t} = \bar{H}_1 - \bar{H}_{1n} = (6, 8, 4) - (6, 8, 0) \equiv 4\hat{a}_z$$

Now, at $\bar{K} = 0$,

$$\bar{H}_{1t} = \bar{H}_{2t} \Rightarrow \bar{H}_{2t} = 4\hat{a}_z$$

Also,

$$\bar{B}_{1n} = B_{2n}$$

\Rightarrow

$$\mu_1 \bar{H}_{1n} = \mu_2 \bar{H}_{2n}$$

\Rightarrow

$$\bar{H}_{2n} = \frac{\mu_1}{\mu_2} \bar{H}_{1n} = \frac{2}{5} \{ 6\hat{a}_x + 8\hat{a}_y \} \equiv 2.4\hat{a}_x + 0.8\hat{a}_y$$

\therefore

$$\bar{H}_2 = \bar{H}_{2t} + \bar{H}_{2n} = 2.4\hat{a}_x + 0.8\hat{a}_y + 4\hat{a}_z \text{ A/m}$$

T14. Sol.

$$\begin{aligned}
 H_{t_1} &= 4\hat{a}_x - 5\hat{a}_z \\
 H_{t_2} &= 8\hat{a}_x - 5\hat{a}_z \\
 H_{t_2} &= H_{t_1} + \bar{K} \times \hat{a}_y \\
 (H_{t_2} - H_{t_1}) &= \bar{K} \times \hat{a}_y \\
 \bar{K} &= \hat{a}_y \times (H_{t_2} - H_{t_1}) = \hat{a}_y \times (8\hat{a}_x - 5\hat{a}_z - 4\hat{a}_x + 5\hat{a}_z) = \hat{a}_y \times 4\hat{a}_x = -4\hat{a}_z \\
 \bar{K} &= -4\hat{a}_z
 \end{aligned}$$

T15. Sol.

$$\begin{aligned}
 \oint \bar{H} \cdot d\bar{l} &= I_{\text{enc}} \\
 I_{\text{enc}} &= NI \\
 \oint \bar{H} \cdot d\bar{l} &= H 2\pi r \\
 \therefore H &= \frac{NI}{2\pi r} \\
 \Rightarrow \bar{B} &= \frac{\mu NI}{2\pi r} \hat{a}_\phi \\
 \text{Now, } d\bar{s} &= d\rho dz \hat{a}_\phi \\
 \therefore \phi &= \int \bar{B} \cdot d\bar{s} = \frac{\mu NI}{2\pi} \ln \rho \Big|_a^b \cdot h \\
 \Rightarrow \phi &= \frac{\mu NI}{2\pi} \ln \left(\frac{b}{a} \right) h \\
 \Rightarrow \psi &= N\phi = \frac{\mu N^2 I}{2\pi} \ln \left(\frac{b}{a} \right) h \\
 \therefore L &= \frac{\psi}{2} = \frac{\mu N^2}{2\pi} \ln \left(\frac{b}{a} \right) h
 \end{aligned}$$

T16. (d)

$$\begin{aligned}
 \text{Magnetic energy, } W_m &= \frac{1}{2} \int \bar{B} \cdot \bar{H} dV \\
 W_m &= \frac{1}{2} \mu \int |H|^2 dV \\
 \oint \bar{H} \cdot d\bar{l} &= I_{\text{enc}} = \int \bar{J} \cdot d\bar{s} \\
 I_{\text{enc}} &= \frac{I}{\pi R^2} \pi r^2 = \frac{Ir^2}{R^2} \\
 \oint \bar{H} \cdot d\bar{l} &= H 2\pi r \\
 H 2\pi r &= \frac{Ir^2}{R^2}
 \end{aligned}$$

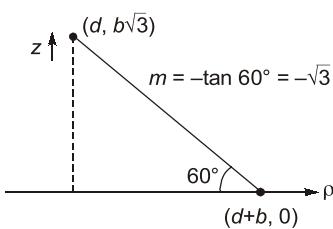
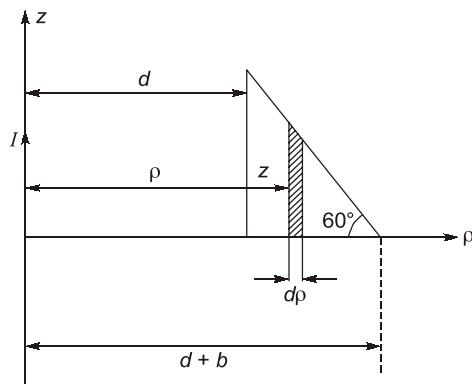
$$\Rightarrow H = \frac{Ir}{2\pi R^2}$$

$$\therefore W_{in} = \frac{\mu}{2} \int \frac{I^2 r^2}{4\pi^2 R^4} \cdot r dr d\theta dz \equiv \frac{\mu}{2} \cdot \frac{I^2}{4\pi^2 R^4} \int_{r=0}^R r^3 dr \int_{d=0}^{2\pi} d\phi \int_{z=0}^l dz$$

$$\Rightarrow W_{in} = \frac{\mu I^2}{8\pi^2 R^4} \cdot \frac{R^4}{4} \cdot 2\pi l \equiv \frac{\mu I^2 l}{16\pi}$$

$$\therefore W_{in} = \frac{\mu I^2 l}{16\pi}$$

T17. Sol.



$$z = -\sqrt{3}[\rho - (d+b)] \\ \Rightarrow z = \sqrt{3}[(d+b) - \rho]$$

$$M = \frac{\phi_{loop}}{I_{wire}} = \frac{\int \bar{B} \cdot \bar{ds}}{I_{wire}}$$

$$\bar{B} = \frac{\mu_0 I}{2\pi\rho} \hat{a}_\phi$$

$$\int \bar{B} \cdot \bar{ds} = \int \frac{\mu_0 I}{2\pi\rho} ds \cdot z = \frac{\mu_0 I}{2\pi} \int_d^{d+b} \frac{d\rho}{\rho} z$$

$$= \frac{\mu_0 I}{2\pi} \int_{\rho=d}^{d+b} \frac{dl}{\rho} (\sqrt{3}(d+b) - \rho) = \frac{\mu_0 I}{2\pi} \sqrt{3} \left[(d+b) \ln \rho \Big|_d^{d+b} - \rho \Big|_d^{d+b} \right]$$

$$\begin{aligned}&= \frac{\mu_0 I \sqrt{3}}{2\pi} \left[(d+b) \ln \left(1 + \frac{b}{d} \right) - b \right] \\&= \frac{\mu_0 \sqrt{3}}{2\pi} \left[(d+b) \ln \left(1 + \frac{b}{d} \right) - b \right] \\&= \frac{\mu_0 \sqrt{3}}{2\pi} [6 \ln(3) - 4] \\&= 897 \text{ nH}\end{aligned}$$

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4

Time Varying Fields

T1. (c)

T2. (200)

$$V_{\text{emf}} = \int (\bar{v} \times \bar{B}) d\bar{l}$$

$$\bar{v} = \rho \omega \hat{a}_\phi$$

$$\bar{B} = B_o(-\hat{a}_z)$$

$$\bar{v} \times \bar{B} = B_o \rho \omega (\hat{a}_\rho)$$

$$d\bar{l} = d\rho \hat{a}_\rho$$

$$\therefore V_{\text{emf}} = \int_0^R -B_o \rho \omega d\rho = \frac{-B_o \omega R^2}{2}; \text{ -ve sign } \because \text{ for } 0 \rightarrow R \downarrow$$

$$\therefore i_{\text{ind}} = \frac{B_o \rho^2 \omega}{2R} = 0.1 \text{ A}$$

$$\therefore I_{P-P} = 2 \times 0.1 = 0.2 \text{ A} = 200 \text{ mA}$$

T3. (d)

T4. (a, b, c)

Region has no charge, i.e., $\rho_v = 0$ and no current, i.e., $\bar{J} = 0$.

$$(a) \quad \nabla \cdot \bar{D} = \rho_v$$

$$\text{at } \rho_v = 0, \quad \nabla \cdot \bar{D} = 0 \Rightarrow \nabla \cdot \bar{E} = 0$$

$$(b) \quad \nabla \cdot \bar{B} = 0$$

$$(c) \quad \nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$(d) \quad \nabla \times \bar{H} = \bar{J}_c + \bar{J}_d = \bar{J}_d$$

$$\Rightarrow \nabla \times \frac{\bar{B}}{\mu} = \frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow \nabla \times \bar{B} = \mu \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \nabla \times \bar{B} = \frac{1}{C^2} \cdot \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \nabla \times \bar{B} - \frac{1}{C^2} \cdot \frac{\partial \bar{E}}{\partial t} = 0$$

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