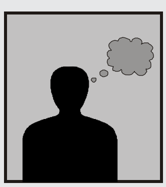
ESE GATE

State Engg. Exams

MADE EASY WORKBOOK 2027



Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Theory of Machines

MADE EASY PUBLICATIONS

1 Mechanism

Detailed Explanation of Try Yourself Questions

T1 : Solution

Pair Symbol	Constrained motion	Relative Motion	Degrees of Freedom
Revolute pair	1	Circular	5
Cylindrical pair	2	Cylindrical	4
Screw pair	1	Helical	5
Spherical pair	3	Spherical	3

■■■■

3 Gears and Gear Trains

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given data : $T_p = 36$, $T_g = 96$, $\phi = 20°$, m = 10 mm, a_m = 10 mm

Pitch circle radius,

$$R = \frac{mT_g}{2} = \frac{10 \times 96}{2} = 480 \text{ mm}$$

Gear Addendum radius,

$$R_a = R + 10 = 490 \text{ mm}$$

$$r = \frac{mT_p}{2} = \frac{10 \times 36}{2} = 180 \text{ mm, pinion}$$

$$r_a = r + 10 = 190 \text{ mm}$$

Path of contact $= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi + \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$

or $= \sqrt{490^2 - (480\cos 20°)^2} - 480\sin 20° + \sqrt{190^2 - (180\cos 20°)^2} - 180\sin 20°$

$= 191.446 - 164.17 + 86.54 - 61.56 =$ **52.256 mm**

Arc of contact $= \frac{\text{Path of contact}}{\cos 20°} = \frac{52.256}{\cos 20°} =$ **55.6 mm**

Contact ratio $= \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{55.6}{\pi \times 10} =$ **1.77**

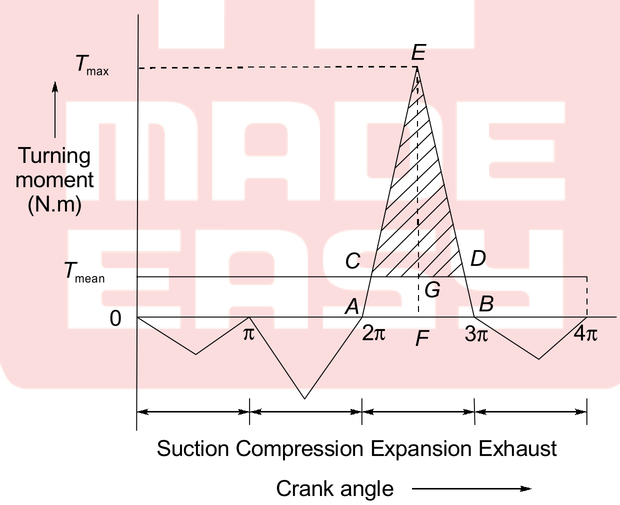
■■■■

5 Flywheel

Detailed Explanation of Try Yourself Questions

T1 : Solution

It is a four-stroke engine, thus, a cycle is completed in 4π radians. The turning moment diagram is shown in figure.



The energy is produced only in the expansion stroke whereas in the other three strokes, it is spent only.

Net energy produced in one cycle

$= [7200 - (440 + 1600 + 660)] \times 3 = 13500$ N.m

Also $\quad T_{mean} \times 4\pi = 13500$

or $\quad T_{mean} = \mathbf{1074\ N.m}$

Energy produced during expansion stroke

$$= \text{Area} \times \frac{\text{Energy}}{\text{mm}^2}$$

$= 7200 \times 3 = 21600$ N.m

As the area of the turning-moment diagram during the expansion stroke indicates the energy produced during the expansion stroke,

$$\therefore \quad \frac{T_{max} \times \pi}{2} = 21600$$

or $$T_{max} = \mathbf{13751\ N.m}$$

In triangle *ABE*,

$$\frac{CD}{AB} = \frac{EG}{EF} = \frac{13751 - 1074}{13751}$$

$$= \frac{12677}{13751} = 0.9219$$

or $$CD = 0.9219 \times \pi = 2.896 \text{ rad}$$

and maximum fluctuation of energy,

$$e = \text{Area}$$

$$CDE = \frac{CD \times EG}{2}$$

$$= \frac{2.896 \times 12677}{2} = 18356 \text{ N.m}$$

■■■■

6 Governor

Detailed Explanation of Try Yourself Questions

T1 : Solution

As per given information, $r_1 = 120$ mm, $r_2 = 80$ mm,
ball arm = sleeve arm ($a = b$), $m = 2$ kg

$N_2 = 400$ rpm, $\omega_1 = \dfrac{2\pi \times 400}{60}$, $N_1 = 420$ rpm, $\omega_2 = \dfrac{2\pi \times 420}{60}$

Sprint constant?

$$F_1 = mr_1\omega_1^2 = 2 \times 0.120 \times \left(\frac{2\pi \times 420}{60}\right)^2 = 464.266 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 2 \times 0.80 \times \left(\frac{2\pi \times 400}{60}\right)^2 = 280.735 \text{ N}$$

$$\text{Spring constant, } K = 2\left(\frac{a}{b}\right)^2\left(\frac{F_1 - F_2}{r_1 - r_2}\right)$$

$$= 2(1)^2\left(\frac{464.266 - 280.735}{0.040}\right) = 9.176 \times 10^3 \text{ N/m}$$

(ii) Spring constant, $K = 9.176$ N/mm

$$F_2 \times a = 0 + \frac{F_{s1}}{2} \cdot b$$

$$F_{s1} = 2F_2 = 2 \times 280.735 \text{ N}$$

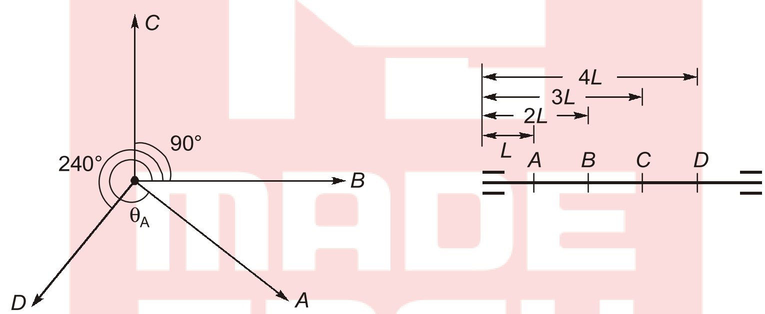
(i) Initial compression $= \dfrac{F_{s1}}{K} = \dfrac{2 \times 280.735}{9.176} \text{N} = 61.1889$ mm

■■■■

7 Balancing

Detailed Explanation of Try Yourself Questions

T1 : Solution



$\Sigma F_x = 0,\ m_A\, r \cos\theta + m_B\, r \cos 0° + m_C\, r \cos 90° + m_D\, r \cos 240° = 0$

$\Sigma F_y = 0,\ m_A\, r \sin\theta + m_B\, r \sin 0° + m_C\, r \sin 90° + m_D\, r \sin 240° = 0$

$$\Sigma F_x = mr\cos\theta + m_B r - \frac{m_D r}{2} = 0$$

$$m_A \cos\theta + m_B = \frac{m_D}{2}$$

$$m_A \cos\theta + 7 = \frac{m_D}{2}$$

$$\Sigma F_y = 0$$

$$m_A \sin\theta + m_C - \frac{\sqrt{3}}{2} m_D = 0$$

$$m_A \sin\theta + m_C = \frac{\sqrt{3}}{2} m_D$$

Dynamic

$\Sigma M_x = 0$

$m_A\, r\, l \cos\theta + m_B\, r\, 2\, l \cos 0° + m_C\, r\, 3\, l \cos 90° + m_D\, r\, 4\, l \cos 240° = 0$

$$m_A \cos\theta + 2\, m_B = 2\, m_D$$

$\Sigma M_y = 0$

$m_A\, r\, l \sin\theta + m_B\, r\, 2\, l \sin 0° + m_C\, r\, 3\, l \sin 90° + m_D\, r\, 4\, l \sin 240° = 0$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D$$

$$m_A \cos\theta + 7 = \frac{m_D}{2} \quad \text{... (i)}$$

$$m_A \sin\theta + m_C = \frac{\sqrt{3}}{2} m_D \quad \text{... (ii)}$$

$$m_A \cos\theta + 14 = 2\, m_D \quad \text{... (iii)}$$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D \quad \text{... (iv)}$$

From equation (iii) – (i)

$$m_D = 4.667 \text{ kg}$$

$$m_A \cos\theta + 7 = 2.33$$

$$m_A \sin\theta + m_C = 4.04$$

$$m_A \cos\theta + 14 = 9.332$$

$$m_A \sin\theta + 3\, m_C = 16.14$$

$$m_A \sin 0° + m_C = 4.04$$

$$m_C = 6.0667 \text{ kg}$$

$$m_A \sin\theta = -2$$

$$m_A \cos\theta = -4.66$$

$$m_A = 5.087 \text{ kg}$$

$$\theta = 203.456°$$

■■■■

10 Vibration

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given; d = 50 mm = 0.05 m; l = 300 mm = 0.03 m; m = 100 kg; E = 200 GN/m^2 = 200 × 109 N/m^2

We knowt hat cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6} \text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l.}{A.E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6} \text{ m}; \quad \omega_n = \sqrt{g/\delta}$$

∴ Frequency of longitudinal vibration,

$$f_n = \frac{\omega_n}{2\pi} = \left(\frac{\sqrt{g}}{2\pi}\right) \times \frac{1}{\sqrt{\delta}}$$

⇒
$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz}$$

Frequency of transverse vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l^3}{3E.I} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

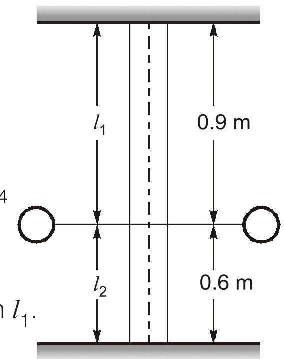
$$f_n = \frac{\omega_n}{2\pi} = \left(\sqrt{\frac{g}{2\pi}}\right) \times \frac{1}{\sqrt{\delta}}$$

⇒
$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{Hz}$$

T2 : Solution

Given: d = 50 mm = 0.05 m; m = 500 kg; E = 200 GN/m^2 = 200 × 10^9 N/m^2

We know that cross-sectional area of shaft,



$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3}\,\text{m}^2$$

and moment of inertia of shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\,\text{m}^4$$

Natural frequency of longitudinal vibration

Let m_1 = Mass of flywheel carried by the length l_1.

∴ $m - m_1$ = Mass of flywheel carried by length l_2.

We know that extension of length l_1

$$= \frac{W_1 \cdot l_1}{A.E} = \frac{m_1.g.l_1}{A.E} \quad \text{...(i)}$$

Similarly, compression of length l_2

$$= \frac{(W - W_1)l_2}{A.E} = \frac{(m - m_1)g.l_2}{A.E} \quad \text{...(ii)}$$

Since extenson of length l_1 must be equal to compression of length l_2, therefore equating equations (i) and (ii),

$$m_1.l_1 = (m - m_1)l_2$$

$$m_1 \times 0.9 = (500 - m_1)0.6 = 300 - 0.6\,m_1 \quad \text{or} \quad m_1 = 200 \text{ kg}$$

∴ Extension of length l_1,

$$\delta = \frac{m_1.g.l_1}{A.E} = \frac{200 \times 9.81 \times 0.9}{1.96 \times 10^{-3} \times 200 \times 10^9} = 4.5 \times 10^{-6}\,\text{m}$$

We know that natural frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{4.5 \times 10^{-6}}} = 235 \text{ Hz}$$

Alternate: Natural freuency of longitudinal vibration

$$\text{Axial stiffness} = \frac{AF}{l} \Rightarrow s_1 = \frac{AE}{l_1}; s_2 = \frac{AE}{l_2}$$

The 2-stiffness are in parallel

⇒ $$s = s_1 + s_2 = 1.0908 \times 10^9$$

$$w_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.0908 \times 10^9}{500}} = 1477.04$$

$$f = \frac{w_n}{2\pi} = 235 \text{ Hz}$$

Natural frequency of tranverse vibration

We know that the static deflection for a shaft fixed at both ends and carrying a point load is given by

$$\delta = \frac{Wa^3b^3}{3EIl^3} = \frac{500 \times 9.81(0.9)^3(0.6)^3}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6}(1.5)^3} = 1.24 \times 10^{-3}\,\text{m}$$

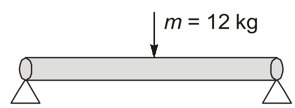
...(Substituting $W = m.g$; $a = l_1$, and $b = l_2$)

We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{1.24\times10^{-3}}} = 14.15 \text{ Hz}$$

T3 : Solution

Given:



Since it is a short bearing
∴ It is simply supported.

(i) Deflection at mid span, $\Delta = \frac{(mg)L^3}{48EI}$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{48EI}{L^3\times m}} = \sqrt{\frac{48\times200\times10^9\times\pi\times(.01)^4}{64\times0.4^3\times12}}$$

$$\omega_n = 78.332 \text{ rad/s} \Rightarrow 748 \text{ rpm}$$

(ii) $$\Delta = \frac{Fl^3}{48EI} + \frac{5\omega l^4}{384EI}$$

Where $$\omega = \frac{W}{L} = \frac{\rho\times V_g}{L} = \rho\times\frac{\pi}{4}d^2\times g$$

∴ $$\Delta = 1.5987\times10^{-3} + 1.962\times10^{-5}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = 77.85 \text{ rad/s}$$

$$N = \frac{\omega_n\times60}{2\pi} = 743.48 \text{ rpm} \approx 744 \text{ rpm}$$

T4 : Solution

Given: f_d = 90/min = 90/60 = 1.5 Hz

We know that time period,

$$t_p = 1/f_d = 1/1.5 = 0.67 \text{ s}$$

Let x_1 = Initial amplitude, and

x_2 = Final amplitude after one complete vibration

$= 20\%\ x_1 = 0.2\,x_1$ ...(Given)

We know that $\log_e\left(\frac{x_1}{x_2}\right) = a.t_p$ or $\log_e\left(\frac{x_1}{0.2x_1}\right) = a\times0.67$

∴ $\log_e 5 = 0.67\,a$ or $1.61 = 0.67\,a$ or $a = 2.4$ ...(∵ $\log_e 5 = 1.61$)

We also know that frequency of free damped vibration

$$f_d = \frac{1}{2\pi}\sqrt{(\omega_n)^2 - a^2}$$

or $(\omega_n)^2 = (2\pi\times f_d)^2 + a^2$ ...(By squaring and arranging)

$$= (2\pi\times1.5)^2 + (2.4)^2 = 94.6$$

$$\omega_n = 9.726 \text{ rad/s}$$

We know that frequency of undamped vibration,

$$f_n = \frac{\omega_n}{2\pi} = \frac{9.726}{2\pi} = 1.55 \text{ Hz}$$

Alternate

Damped frquency, $f_d = 1.5$ /s

Given $n_1 = 0.2\,x_0$

$$\Rightarrow \quad \frac{x_0}{x_1} = \frac{x_1}{x_2} = \ldots \frac{x_{n-1}}{x_n} = 5 = e^{\delta} \qquad \text{where } \delta = \text{logarithmic decrement}$$

$$\Rightarrow \quad d = \ln 5 = 1.609$$

$$\Rightarrow \quad \frac{2\pi_{\xi}}{\sqrt{1-\xi^2}} = 1.609$$

$$\Rightarrow \quad \frac{\xi^2}{1-\xi^2} = 0.066$$

$$\Rightarrow \quad \frac{1}{\xi^2} - 1 = 15.241$$

$$\Rightarrow \quad \xi = 0.248$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$\Rightarrow \quad \frac{\omega_d}{2\pi} = \sqrt{1-\xi^2}\,\frac{\omega_n}{2\pi}$$

$$\Rightarrow \quad f_d = \sqrt{1-\xi^2}\,f$$

$$\Rightarrow \quad f_n = \frac{f_d}{\sqrt{1-\xi^2}} = \frac{1.5}{\sqrt{1-0.248^2}} = 1.55 \text{ Hz}$$

■■■■