

**ESE**

**GATE**

**State Engg. Exams**

**MADE EASY**  
**WORKBOOK 2027**



**Detailed Explanations of  
Try Yourself *Questions***

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**Mechanical Engineering**  
**Industrial Engineering**



# 7

## LPP, Transportation and Assignment



### Detailed Explanation of Try Yourself Questions

#### T1 : Solution

By introducing slack variable  $s_1, s_2, s_3$  and  $s_4$  the problem may be written as follows

$$\text{Maximum } Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$$

$$x_1 + x_2 + s_1 = 6$$

$$2x_1 + x_2 + s_2 = 8$$

$$-x_1 + x_2 + s_3 = 1$$

$$x_2 + s_4 = 2$$

$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

The above information is tabulated

Table - 1

Basis	$C_j$	3	2	0	0	0	0		$\theta = \frac{b}{C_i}$
		$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$b$	
0 $s_1$		1	1	1	0	0	0	6	6
0 $s_2$		2*	1	0	1	0	0	8	4
0 $s_3$		-1	1	0	0	1	0	1	-1
0 $s_4$		0	1	0	0	0	1	2	$\infty$
$E_j$		0	0	0	0	0	0		
$E_j - C_j$		-3	-2	0	0	0	0		

$E_j$  and  $E_j - C_j$  are calculated

The entering column is identified and the  $\theta$  values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

$s_2$  leaves the basis,  $x_1$  enters the basis

The following row operations are performed

$$\begin{aligned} R_{2\text{new}} &= R_{2\text{old}}/2 \\ R_{1\text{new}} &= R_{1\text{old}} - R_{2\text{new}} \\ R_{3\text{new}} &= R_{3\text{old}} + R_{2\text{new}} \end{aligned}$$

Then the table 1 becomes

$E_j$  and  $E_j - C_j$  are calculated

The entering column is identified and the  $\theta$  values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

$s_4$  leaves the basis,  $x_2$  enters the basis

Table - 2

Basis	$C_j$	3	2	0	0	0	0	$b$	$\theta = \frac{b}{C_j}$
		$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$		
0 $s_1$		1	1/2	1	-1/2	0	0	2	4
0 $s_2$		1	1/2	0	1/2	0	0	4	8
0 $s_3$		0	3/2	0	1/2	1	0	5	10/3
0 $s_4$		0	1*	0	1	0	1	2	2
$E_j$		3	3/2	0	3/2	0	0		
$E_j - C_j$		0	-1/2	0	3/2	0	0		

The following row operations are performed

$$\begin{aligned} R_{1\text{new}} &= R_{1\text{old}} - (R_{4\text{new}})/2 \\ R_{2\text{new}} &= R_{2\text{old}} - (R_{4\text{new}})/2 \\ R_{3\text{new}} &= 2R_{3\text{old}} - 3(R_{4\text{new}})/2 \end{aligned}$$

Then the table 2 becomes table 3

$E_j$  and  $E_j - C_j$  are calculated

Table - 3

Basis	$C_j$	3	2	0	0	0	0	$b$
		$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	
0 $s_1$		0	0	1	-1/2	0	-1/2	1
0 $x_1$		1	0	0	1/2	0	-1/2	3
0 $s_3$		0	0	0	1/2	1	-3/2	2
0 $x_2$		0	1	0	0	0	1	2
$E_j$		3	2	0	3/2	0	1/2	
$E_j - C_j$		0	0	0	3/2	0	1/2	

It is an optimum matrix since all the values obtained in ' $E_j - C_j$ ' row are positive

Thus the optimum solution will change as

$$\Rightarrow \left. \begin{array}{l} x_1 = 3 \\ x_2 = 2 \\ S_1 = 1 \\ S_3 = 2 \end{array} \right\} \text{Basic variables} \quad \left. \begin{array}{l} S_2 = 0 \\ S_4 = 0 \end{array} \right\} \text{Non basic variables}$$

Hence,  $Z = 3 \times 3 + 2 \times 2 = 13$

### T2 : Solution

The simplex problem can be expressed in the form of matrices as follows

$$\text{Minimum } Z = (5 \ 2 \ 2)^A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^X$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 6 & 8 & 5 \\ 7 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}^B \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^X \geq \begin{pmatrix} 20 \\ 30 \\ 40 \\ 50 \end{pmatrix}^C$$

Note: If primal problem is  $\begin{pmatrix} \min Z = AX \\ BX \geq C \end{pmatrix}$  then Dual problem is  $\begin{pmatrix} \min Z = C^T Y \\ B^T Y \leq A^T \end{pmatrix}$

So the dual for the given problem becomes

$$\text{Maximum } Z = (20 \ 30 \ 40 \ 50) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 6 & 7 & 1 \\ 3 & 8 & 1 & 2 \\ 1 & 5 & 3 & 4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

### T3 : Solution

Let  $s_1, s_2, s_3$  be slack variables then the constraints become

$$x + y + s_1 = 90$$

$$x + 2y + s_2 = 80$$

$$x + y + s_3 = 50$$

$$x, y, s_1, s_2, s_3 \geq 0$$

$$\Rightarrow Z = 45x + 40y + 0s_1 + 0s_2 + 0s_3$$

The above information is tabulated,  $E_j$  and  $E_j - C_j$  are calculated.

The entering column is identified and the  $\theta$  values are calculated, then the leaving row are identified

Table - 1

Basis	$C_j$						$\theta = \frac{b}{C_i}$
	45	40	0	0	0		
	$x$	$y$	$s_1$	$s_2$	$s_3$	$b$	
$0 s_1$	1	1	1	0	0	90	90
$0 s_2$	1	2	0	1	0	80	80
$0 s_3$	1*	1	0	0	1	50	50 ←
$E_j$	0	0	0	0	0		
$E_j - C_j$	-45 ↑	-40	0	0	0		

$s_3$  leaves the basis and  $x$  enters into the basis, then the table 1 becomes table 2,  $E_j$  and  $E_j - C_j$  are calculated.

Table - 2

Basis	$C_j$						$\theta = \frac{b}{C_i}$
	45	40	0	0	0		
	$x$	$y$	$s_1$	$s_2$	$s_3$	$b$	
$0 s_1$	0	0	1	0	-1	40	
$0 s_2$	0	1	0	1	-1	30	
$45 x$	1	1	0	0	1	50	
$E_j$	45	45	0	0	45		
$E_j - C_j$	0	5	0	0	46		

As ' $E_j - C_j$ ' values present in the matrix are positive. So it is an optimum matrix.

So solution can be read out from matrix i.e.,

$x = 50, y = 0,$

∴ by putting  $(x, y) = (50, 0)$  in the objective function the maximum value of  $Z = 45x + 40y = 2250$ .

**T4 : Solution**

6	5	8	11	16
1	13	16	1	10
16	11	8	8	8
9	14	12	10	16
10	13	11	8	16

**Steps I** - If the number of lines is equal to the order of matrix then it is the optimal table. If the number of lines is less than the order of matrix, then select the minimum number which is not covered by the lines. Subtract this value from each and every element which is uncovered and add this value at the intersection point of lines. Here adding "3" minimum uncovered value at intersection and subtracting at non covered values.

1	0	3	6	11
<del>8</del>	12	15	<del>6</del>	9
8	3	0	<del>6</del>	<del>11</del>
0	5	3	1	7
2	5	3	0	8

We will continue to repeat step I, till number of lines are equal to the order of the matrix. Now number of lines are equal to the order of the matrix, so select zeros such that every column and row consists of one zero i.e., one allocation for one machine.

1	0	<del>3</del>	6	8
<del>8</del>	12	12	0	6
11	6	<del>3</del>	3	0
0	5	<del>3</del>	1	4
2	5	0	<del>6</del>	5

A-II, B-IV, C-V, D-I, E-III

The total processing time is 34.

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