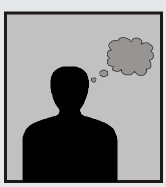
ESE GATE

State Engg. Exams

MADE EASY Workbook 2027



Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Heat Transfer

MADE EASY PUBLICATIONS

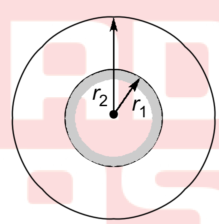
1 Conduction

Detailed Explanation of Try Yourself Questions

T1 : Solution

r_1 = 1.2 cm = 0.012 m, r_2 = 1.8 cm = 0.018 m, T_1 = 500°C

$\left(\frac{dT}{dr}\right)_{r=r_1} = 0$ (Insulated at inner surface)



Temperature profile in cylinder is given by

$$T = \frac{-\dot{q}r^2}{4k} + C_1 \ln r + C_2 \quad \text{...(i)}$$

At $\quad r = r_1,\ T = 500°C$

$$500 = \frac{-500 \times 10^3 \times (0.012)^2}{4 \times 0.55} + C_1 \ln(0.012) + C_2$$

$$C_2 - 4.42\, C_1 = 532.73 \quad \text{...(ii)}$$

At $\quad r = r_1,\ \frac{dT}{dr} = 0$

$$0 = \frac{-\dot{q}r}{2k} + \frac{c_1}{r} + 0$$

$$0 = \frac{-500 \times 10^3 \times 0.012}{2 \times 0.55} + \frac{C_1}{0.012}$$

$$C_1 = 65.45$$

From equation (ii)
$$C_2 = 822.019$$

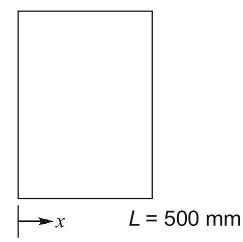
From equation (i)
$$T = \frac{-500 \times 10^3 r^2}{4 \times 0.55} + 65.45 \ln r + 822.019$$

$$T = -227272.72 \times r^2 + 65.45 \ln r + 822.019$$

at
$$r = r_2$$

$$T = -227272.72 \times (0.018)^2 + 65.45 \ln 0.018 + 822.019 = 485.44°C$$

T2 : Solution



$$T = 120 - 100x + 24x^2 + 40x^3 - 30x^4$$

$$\frac{\partial^2 T}{\partial x^2} + 0 + 0 + 0 = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Rate of heating or cooling,

$$\frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial \tau}$$

Location for maxima.

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial \tau}\right) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial \tau}\right) = \alpha \frac{\partial^3 T}{\partial x^3}$$

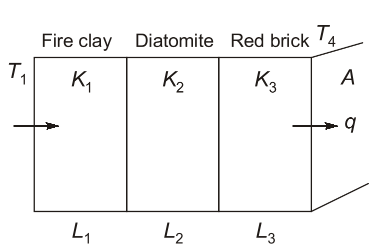
$$\alpha \frac{\partial^3 T}{\partial x^3} = 0$$

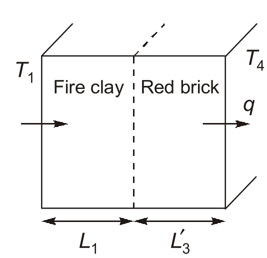
$$\frac{\partial^3 T}{\partial x^3} = 0 - 0 + 0 + 240 - 720x$$

$$240 - 720x = 0$$

$$x = \frac{240}{720} = 0.333 \text{ m}$$

T3 : Solution





$$q = \frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}}; \quad q = \frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L_3'}{k_2 A}}$$

$$\frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}} = \frac{T_1 - T_4}{\frac{L_1}{k_1 A} + \frac{L_3'}{k_2 A}}$$

$$\frac{1}{\frac{0.11}{0.94} + \frac{0.06}{0.13} + \frac{0.25}{0.7}} = \frac{1}{\frac{0.11}{0.94} + \frac{L_3'}{0.7}}$$

$$L_3' = 0.573 = 57.3 \text{ cm}$$

T4 : Solution

t = 150 mm, k = 15 W/mK

(i)

$$\dot{q} = \frac{h(T - T_\infty)}{0.150} = \frac{500 \times (100 - 20)}{0.150}$$

$$= 0.267 \times 10^6 \text{ W/m}^2$$

$$T(X) = a + bx + cx^2$$

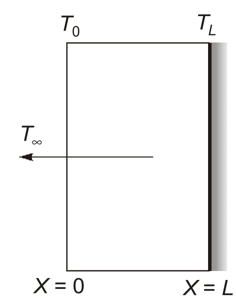
$X = 0$

$$T(0) = T_0 = 100°\text{C},\ T_\infty = 20°\text{C},\ h = 500 \text{ W/m}^2\text{-K}$$

at $X = 0$,

$$T_0 = 100°\text{C}$$

$$100 = a + 0 + 0, \qquad (a = 100)$$



$$\left.\frac{dT}{dX}\right|_{x = L} = 0$$

$$b + 2cx = 0$$

$$b + 2c(0.15) = 0$$

$$b + 0.3c = 0$$

(ii) For steady state,

$$-\left[-k \left.\frac{dT}{dX}\right|_{x=0}\right] = h(T - T_\infty)$$

$$15 \times b = 500 \times (100 - 20)$$

$$b = 266.67 \times 10$$

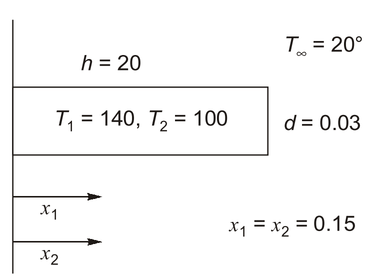
$$b = 2.67 \times 10^3 \text{ k/m}$$

(iii)

$$2.67 \times 10^3 + 0.3c = 0$$

$$c = -8.9 \times 10^3 \text{ k/m}$$

T5 : Solution



Since it is mentioned long rod, i.e., $L \to \infty$

$$\frac{T_1 - T_\infty}{T_s - T_\infty} = e^{-mx_1} \quad \text{...(i)}$$

$$\frac{T_2 - T_\infty}{T_s - T_\infty} = e^{-mx_2} \quad \text{...(ii)}$$

$$\frac{T_1 - T_\infty}{T_2 - T_\infty} = \frac{e^{-mx_1}}{e^{-mx_2}}$$

$$\frac{140 - 30}{100 - 30} = e^{m(x_2 - x_1)}$$

$$m \times 0.15 = \ln\left[\frac{110}{70}\right]$$

$$m = 3.01$$

$$\sqrt{\frac{h \times 4}{K \cdot d}} = 3.01$$

$$\frac{20 \times 4}{K \times 0.03} = 3.01^2$$

$$K = 294.3308 \text{ W/mK}$$

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2 Heat Exchanger

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a) LMTD method: The rate of heat transfer in the heat exchanger is found as

$$Q = [\dot{m} c_p (T_{out} - T_{in})]_{water}$$

$$= 1.2\,(4.18)\,(90 - 30) = 301 \text{ kW}$$

The outlet temperature of geothermal fluid is determined as

$$T_{out} = 160 - \frac{301}{2(4.31)} = 125.10°C$$

Therefore, $\Delta T_1 = 160 - 90 = 70°C$

$$\Delta T_2 = 125.10 - 30 = 95.10°C$$

and $LMTD = \dfrac{70 - 95.10}{\ln(70/95.10)} = 81.91°C$

Hence $A = \dfrac{Q}{U(LMTD)} = \dfrac{301 \times 10^3}{(600)(81.91)} = 6.12 \text{ m}^2$

To provide this surface area, the length of the tube required is found as

$$L = \frac{A}{\pi D} = \frac{6.12}{\pi(0.015)} = 129.87 \text{ m}$$

(b) *NTU Method:* We first determine the heat capacity rates of the hot and cold fluids to identify the smaller value of the two.

$$C_h = \dot{m}_h c_h = 2(4.31) = 8.62 \text{ kW/°C}$$

$$C_c = \dot{m}_c c_c = 1.2(4.18) = 5.02 \text{ kW/°C}$$

Therefore, $C_{min} = C_c = 5.02$ kW/°C

and $C = C_{min}/C_{max} = 5.02/8.62 = 0.583$

$$\varepsilon = \frac{Q}{C_{min}\left(T_{h,in} - T_{c,in}\right)} = \frac{301.00}{5.02(160 - 30)} = 0.461$$

Now we determine the value of *NTU* by making use of the expression of NTU for a counterflow heat exchanger from equation.

$$NTU = \frac{1}{C-1}\ln\left(\frac{\varepsilon-1}{\varepsilon C-1}\right) = \frac{1}{0.582-1}\ln\left(\frac{0.4615-1}{0.4615\times0.582-1}\right) = 0.7325$$

We know $$NTU = \frac{UA}{C_{min}}$$

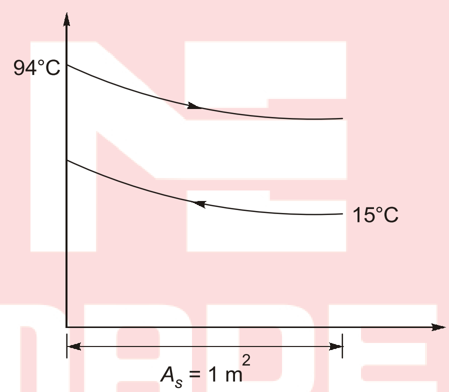
or $$A = \frac{NTU.C_{min}}{U} = \frac{0.7325\left(5.016\times10^3\right)}{600} = 6.1237\text{ m}^2$$

$$= \pi\times0.015\times L$$

Hence, $$L = 129.95\text{ m}$$

Therefore, we find that almost same result is obtained in both the methods.

T2 : Solution



$$\text{NTU} = \frac{UA_s}{C_{min}} = \frac{1075\times1}{305.4} = 3.52$$

$$U = 1075\text{ W/m}^2\text{-K}$$

$$C_h = \dot{m}_h c_h = 0.1527\times2000 = 305.4$$

$$C_{min} = 305.4$$

$$C_c = \dot{m}_c c_c = 0.361\times480 = 1508.98$$

$$C_{max} = 1508.98$$

$$C_h < C_c$$

$$R = \frac{C_{min}}{C_{max}} = 0.20$$

$$\dot{m}_h = \frac{550}{3600} = 0.1527\text{ kg/sec}$$

$$\dot{m}_c = \frac{1300}{3600} = 0.361\text{ kg/sec}$$

$$\frac{C_h\Delta T_h}{C_{min}\Delta T_{max}} = \varepsilon = \frac{1-\exp\left[-NTU(1-R)\right]}{1-R\exp\left[-NTU(1-R)\right]}$$

$$\frac{[T_{hi} - T_{ho}]}{[T_{hi} - T_{ci}]} = \frac{1 - \exp[-3.52(1-0.2)]}{1 - 0.2\exp[-3.52(1-0.2)]}$$

$$\frac{94 - T_{ho}}{94 - 15} = \frac{1 - 0.0598}{1 - 0.2 \times 0.598} = 0.932$$

$$T_{ho} = 20.372°C$$

$$q = \dot{m}_h c_h [T_{hi} - T_{ho}] = 305.4[94 - 20.372]$$

$$= 22.486 \text{ kW}$$

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3 Radiation

Detailed Explanation of Try Yourself Questions

T1 : Solution

We know that,

$$\frac{Q}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{R}$$

Here $A_1 = A_2 = A$, and R is the equivalent resistance of the thermal network

$$\varepsilon_1 = 0.75,\ \varepsilon_2 = 0.70$$

By summation rule of view factors

$$F_{33} + F_{31} + F_{32} = 1$$

$$F_{33} = 0 \text{ (in consideration of furnace surfaces to be plane)}$$

From symmetry, $F_{31} = F_{32}$

Hence, $F_{31} = F_{32} = 0.5$

Again from the reciprocity relation, $F_{13} = F_{31} = 0.5$

$$F_{23} = F_{32} = 0.5 \text{ (since } A_1 = A_2 = A_3 = A)$$

Again $F_{11} + F_{12} + F_{13} = 1$

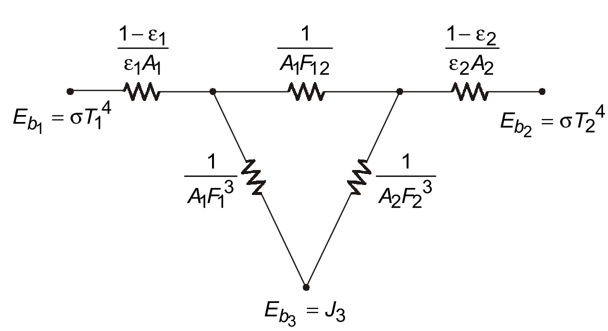
$$F_{11} = 0 \text{ and } F_{13} = 0.5$$

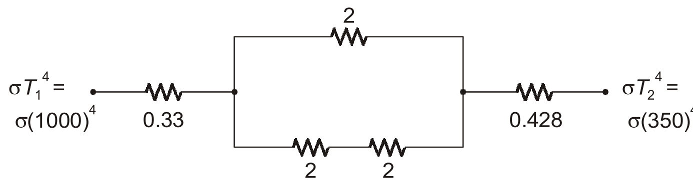
Hence, $F_{12} = 0.5$

As we know,

$$F_{12} = F_{13} = F_{23} = 0.5$$

Therefore



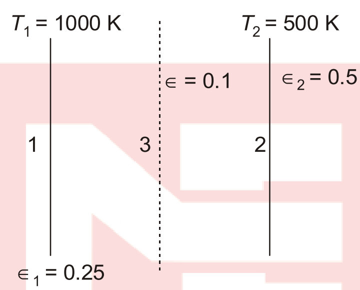


$$R = \frac{1-0.75}{0.75} + \frac{1}{0.5+(2+2)^{-1}} + \frac{0.3}{0.7} = 2.09$$

$$\frac{Q}{A} = \frac{5.67\times 10^{8}\left(1000^4 - 350^4\right)}{2.09}$$

$$= 26.72 \times 10^3 \text{ W/m}^2 = 26.72 \text{ kW/m}^2$$

T2 : Solution



(a)

$$\frac{\dot{q}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\in_1}{\in_1} + \frac{1}{F_{12}} + \frac{1-\in_2}{\in_2}} = \frac{5.67\times 10^{-8}\times\left(1000^4 - 500^4\right)}{3+1+1}$$

$$\frac{q}{A} = 10631.25 \text{ W/m}^2 = 10.631 \frac{\text{kW}}{\text{m}^2}$$

(b)

$$\frac{\dot{q}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\in_1}{\in_1} + \frac{1}{F_{13}} + \left(\frac{1-\in}{\in}\right) + \left(\frac{1-\in}{\in}\right) + \frac{1}{F_{23}} + \frac{1-\in_2}{\in_2}}$$

$$\left(\frac{\dot{q}}{A}\right) = \frac{5.67\times\left(1000^4 - 500^4\right)\times 10^{-8}}{3+1+9+9+1+1}$$

$$\frac{q}{A} = 2214.843 \text{ W/m}^2$$

or

$$\frac{q}{A} = 2.214 \frac{\text{kW}}{\text{m}^2}$$

(c)

$$\frac{\dot{q}}{A} = \frac{\sigma\left(T_1^4 - T^4\right)}{\frac{1-\in_1}{\in_1} + \frac{1}{F_{23}} + \frac{1-\in}{\in}}$$

$$2214.84 = \frac{5.67\times 10^{-8}\times\left(1000^4 - T^4\right)}{3+1+9}$$

$$T = 837.59 \text{ K}$$

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4 Convection

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{2.131\times 10^{-5}\times 1.01\times 10^{3}}{0.031} = 0.694$$

$$\text{Re} = \frac{\rho VL}{\mu} = \frac{0.962\times 12\times 2}{2.131\times 10^{-5}} = 1.0834\times 10^{6} > 5\times 10^{5}$$

So, flow is turbulent at the end of the plate. Distance upto which flow is laminar (x_{cr}):

$$\text{Re}_{cr} = 5\times 10^{5} = \frac{\rho V x_{cr}}{\mu}$$

$$5\times 10^{5} = \frac{0.9620\times 12\times x_{cr}}{2.131\times 10^{-5}}$$

$$x_{cr} = 0.923\text{ m}$$

For laminar region, $\text{Nu} = 0.332\,\text{Re}_x^{1/2}\,\text{Pr}^{1/3}$

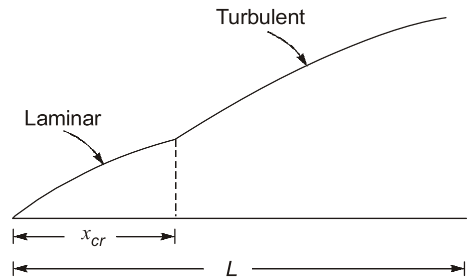
$$\frac{h_x\times x}{0.031} = 0.332\left(\frac{0.962\times 12\times x}{2.131\times 10^{-5}}\right)^{1/2}(0.694)^{1/3}$$

$$h_{x,L} = \frac{6.7067}{\sqrt{x}}$$

For turbulent region, $\text{Nu} = 0.0296\,\text{Re}_x^{4/5}\,\text{Pr}^{1/3}$

$$\frac{h_{x,T}\,x}{0.031} = 0.0296\left(\frac{0.962\times 12x}{2.131\times 10^{-5}}\right)^{4/5}(0.694)^{1/3}$$

$$h_{x,T} = \frac{31.39}{x^{1/5}}$$



Average heat transfer coefficient,

$$\bar{h} = \frac{1}{L}\left[\int_0^{x_{cr}} h_{x,L}\,dx + \int_{x_{cr}}^{L} h_{x,T}\,dx\right] = \frac{1}{2}\left[\int_0^{0.923}\frac{6.7067}{\sqrt{x}}dx + \int_{0.923}^{2}\frac{31.39}{x^{1/5}}dx\right]$$

$$= \frac{1}{2}\left[6.7067\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^{0.923} + 31.39\left[\frac{x^{\frac{-1}{5}+1}}{-\frac{1}{5}+1}\right]_{0.923}^{2}\right]$$

$$= \frac{1}{2}\left[2\times 6.7067\times 0.923^{1/2} + 31.39\times\frac{5}{4}\times\left(2^{4/5} - 0.923^{4/5}\right)\right]$$

$$= 22.20 \text{ W/m}^2\text{-k}$$

T2 : Solution

Given: Height, L = 1.5, Width, W = 1 m, Plate temp, t_s = 150°C, Surrounding temp, t_∞ = 30°C,
Average temp t_{avg} = 90°C

for

$$\rho = 0.946 \text{ kg/m}^3,\ k_a = 0.0313 \text{ W/mK}$$

$$\nu = 22.10 \times 10^{-6} \text{ m}^2/\text{s},\ C_p = 1.009 \text{ kJ/kg-K}$$

So,

$$G_r = \frac{g\beta\Delta t L^3}{\nu^2} = \frac{9.81\times\frac{1}{273+90}\times 120\times 1.5^3}{(22.10\times 10^{-6})^2} \qquad \left[\text{As } \beta = \frac{1}{273+t_{avg}}\right]$$

$$\text{Gr} = 2.24095 \times 10^{10}$$

Prandtl number,

$$\text{Pr} = \frac{\rho\gamma C_p}{k} = 0.67395$$

$$R_{a_L} = \text{Gr} \times \text{Pr} = 1.51 \times 10^{10}$$

As given

$$N_{u_L} = 0.59\,(R_{a_L})^{0.25}$$

$$\frac{hL}{k_a} = 50.562 \times 0.59 = 206.832$$

$$h = 4.316 \text{ W/m}^2\text{K}$$

So rate of heat transfer from both the surfaces,

$$\text{'}Q\text{'} = 2 \times h \times A \times (t_s - t_\infty) = 1553.76 \text{ W}$$

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