ESE **GATE** **PSUs**

State Engg. Exams

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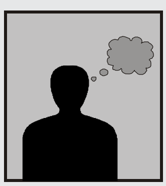


Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Fluid Mechanics and
Hydraulic Machines

MADE EASY PUBLICATIONS

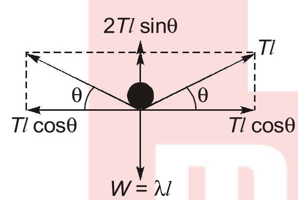
1 Fluid Properties

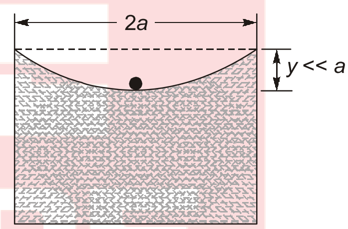
Detailed Explanation *of* Try Yourself Questions

T1 : Solution

Given: λ = Weight per unit length

FBD of the wire





Considering the equilibrium of wire in vertical direction, we have

$$2Tl\sin\theta = \lambda l; \qquad \because \quad \theta \text{ is very small}$$

$$2Tl \times \frac{y}{a} = \lambda l \qquad \sin\theta \simeq \tan\theta \simeq \theta = \frac{y}{a}$$

$$T = \frac{\lambda a}{2y}$$

So, option (b) is correct.

T2 : Solution

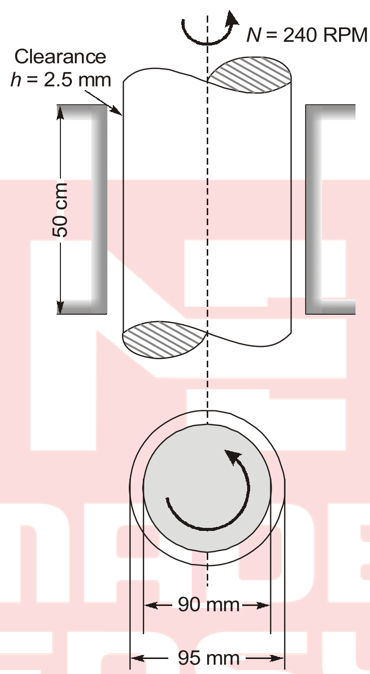
$$\text{Power} = T\omega$$

Calculating torque,

$$\text{Torque} = F \times \text{radius}$$

$$F = \frac{\mu A v}{y}$$

$$\mu = 2 \times 10^{-1}\ \text{Ns/m}^2$$



$$A = \pi D l = \pi \times \frac{90}{1000} \times \frac{50}{100} = 0.1414\ \text{m}^2$$

$$v = \frac{90}{2000} \times \frac{2\pi N}{60} = \frac{90}{2000} \times \frac{2 \times \pi \times 240}{60} = 1.131\ \text{m/s}$$

$$y = 2.5\ \text{mm} = 2.5 \times 10^{-3}\ \text{m}$$

$$F = \frac{2 \times 10^{-1} \times 0.1414 \times 1.131}{2.5 \times 10^{-3}} = 12.79\ \text{N}$$

$$\text{Torque} = F \times \text{radius} = 12.79 \times \frac{90}{2000} = 0.576\ \text{Nm}$$

$$\omega = \frac{2 \times \pi \times 240}{60} = 25.12\ \text{rad/s}$$

$$P = 0.576 \times 25.12 = 14.47\ \text{Watt} \simeq 14.5\ \text{Watt}$$

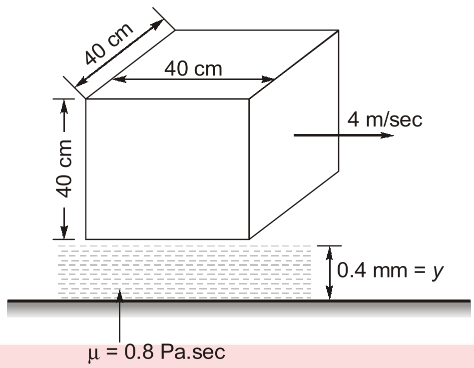
T3 : Solution

Given:

$$\text{Velocity of block, } V = 4 \text{ m/sec}$$

$$\text{Side of cube} = 40 \text{ cm} = 0.40 \text{ m}$$

$$\text{Viscosity, } \mu = 0.8 \text{ N.sec/m}^2$$



Force required,

$$F = \tau A = \mu\left(\frac{V}{y}\right)A$$

$$= 0.8 \times \frac{4}{0.4 \times 10^{-3}} \times (0.4 \times 0.4)$$

$$F = 1280 \text{ N}$$

So, option (a) is correct.

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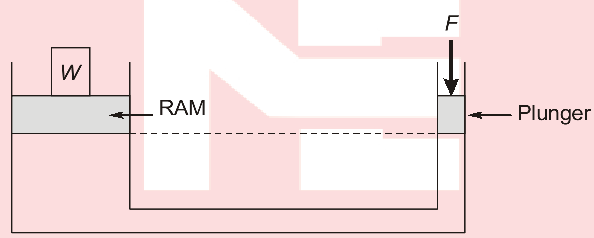
2 Fluid Statics

Detailed Explanation of Try Yourself Questions

T1 : Solution



Pressure intensity produced by force,

$$F = \frac{F}{a}$$

$$\text{Pressure intensity on RAM} = \frac{W}{A}$$

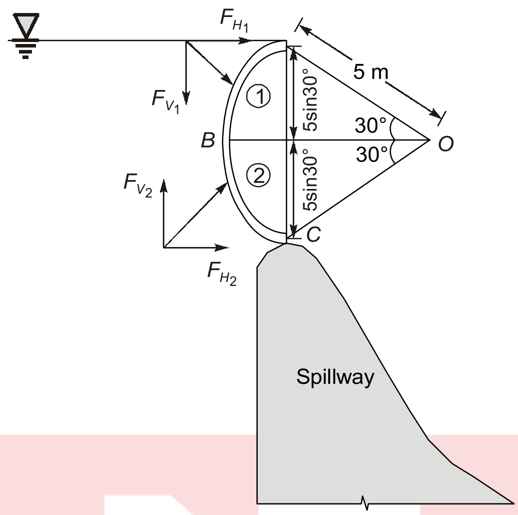
According to Pascal law,
$$\frac{W}{A} = \frac{F}{a} \quad A = \text{Area of Ram},\ a = \text{Area of plunger}$$

$$\frac{W}{\frac{\pi}{4}\times(0.3)^2} = \frac{50}{\frac{\pi}{4}\times(0.045)^2}$$

$$W = 2222.22 \text{ N} \simeq 2223 \text{ N}$$

So, option (b) is correct.

T2 : Solution



Horizontal force (F_H):

$$F_H = F_{H_1} + F_{H_2} \ (\rightarrow)$$

$$= \rho g \bar{h}_1 A_{v_1} + \rho g \bar{h}_2 A_{v_2}$$

$$A_{v_1} = A_{v_2} = 5\sin 30° \times 1 = 2.5 \text{ m}^2$$

$$\bar{h}_1 = \frac{5\sin 30°}{2} = 1.25 \text{ m}$$

$$\bar{h}_2 = 5\sin 30° + \frac{5\sin 30°}{2} = 3.75 \text{ m}$$

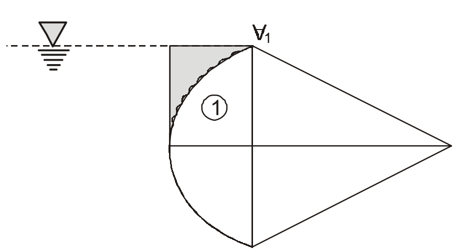
$$F_H = \rho g(2.5)(\bar{h}_1 + \bar{h}_2)$$

$$= (10^3)(10)(2.5)(1.25 + 3.75)$$

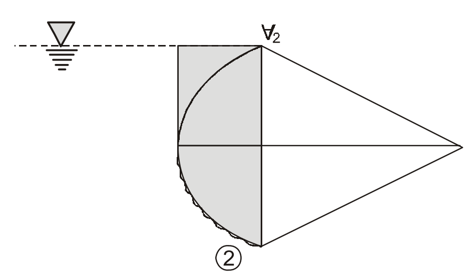
$$= 125 \text{ kN} \ (\rightarrow)$$

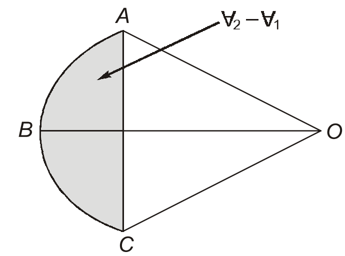
Vertical force (F_v):

Part-1



Part-2



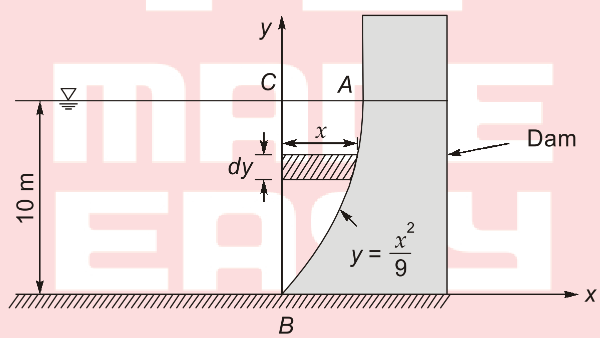


$$F_v = F_{v_2} - F_{v_1}\ (\uparrow)$$
$$= \rho g \forall_2 - \rho g \forall_1$$
$$= \rho g(\forall_2 - \forall_1)$$
$$= \rho g \times \text{volume of } ABCA$$
$$= \rho g\,(\text{Area of arc } OABCO - \text{Area of } \Delta OAC) \times \text{Width}$$

$$F_v = (10^3)(10)\left[\frac{\pi(5)^2}{6} - \left(\frac{1}{2} \times 5\cos 30° \times 5\sin 30° \times 2\right)\right] \times 1$$
$$= 22.6 \text{ kN } (\uparrow)$$

$$F_R = \sqrt{F_H^2 + F_V^2}$$
$$= \sqrt{(125)^2 + (22.6)^2}$$
$$= 127.03 \text{ kN/m}$$

T3 : Solution



Horizontal force (F_H):

$$F_H = \rho g \bar{h} A_V\ (\rightarrow)$$
$$= (10^3)(9.81)\left(\frac{10}{2}\right)(10 \times 1)$$
$$= 490.5 \text{ kN } (\rightarrow)$$

Vertical force (F_V):

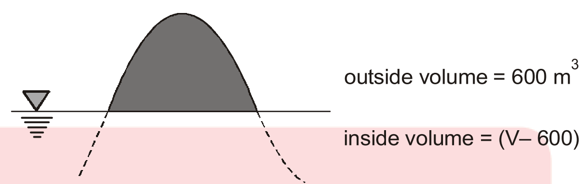
$$F_V = \rho g \forall$$
$$= (10^3)\ (9.81) \times (\text{Area of ABC}) \times \text{Width of dam}$$
$$= (10^3)(9.81)\left[\int_0^{10} x\,dy\right] \times 1 \qquad \left(x = \sqrt{9y}\right)$$
$$= (10^3)(9.81)\left[\int_0^{10} \sqrt{9y}\,dy\right] \times 1$$

$$= (100)(9.81)(63.246) \times 1$$
$$= 620.439 \text{ kN} (\downarrow)$$

Resultant force (F_R):

$$F_R = \sqrt{F_H^2 + F_V^2}$$
$$F_R = \sqrt{(490.5)^2 + (620.439)^2}$$
$$= 790.906 \text{ kN}$$

T4 : Solution



$$\rho_{\text{ice berg}} = 915 \text{ kg/m}^3$$
$$\rho_{\text{sea water}} = 1025 \text{ kg/m}^3$$

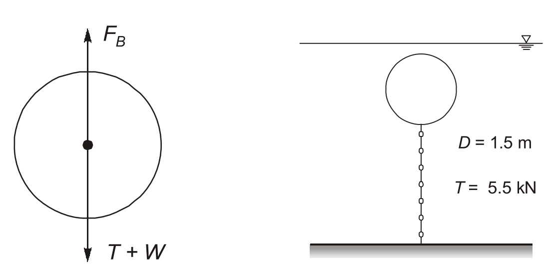
Let the total volume of iceberg be "V".

$$\text{Buoyancy force} = \text{Weight of iceberg}$$
$$\Rightarrow \quad \rho_{\text{sea water}} \times (V - 600) \times 9.81 = \rho_{\text{iceberg}} \times V \times 9.81$$
$$\Rightarrow \quad 1025\,(V - 600) = 915\,V$$
$$\Rightarrow \quad 1025\,V - 915\,V = 1025 \times 600$$
$$\Rightarrow \quad V = \frac{2025 \times 600}{1025 - 915} = 5590.9 \text{ m}^3$$

Weight of the iceberg

$$= \rho_{\text{iceberg}} \times V_{\text{iceberg}} \times 9.81$$
$$= 915 \times 5590.9 \times 9.81$$
$$= 50184757.04 \text{ N}$$
$$= 50.185 \text{ MN}$$

T5 : Solution



$$F_{\text{buoyancy}} = \text{Tension} + \text{Weight}$$
$$\rho_w \times \text{Volume } 5\ g = \text{Tension} + \text{Weight},$$

$$\text{Weight} = F_{\text{buoyancy}} - \text{Tension}$$

$$= \left[\rho_w \times \frac{4}{3} \times \pi \times r^3 \times g\right] - [5.5 \times 10^3]$$

$$= \left[1000 \times \frac{4}{3} \times \pi \times \left(\frac{1.5}{2}\right)^3 \times 9.81\right] - [5.5 \times 10^3]$$

$$= 17335.7 - 5500 = 11835.7 \text{ N} \simeq 12 \text{ kN}$$

T6 : Solution

For the gate to be in equilibrium and not have any rotation, summation of moment of all the forces about the hinge must be zero.

Depth of water (H) = 2 m.

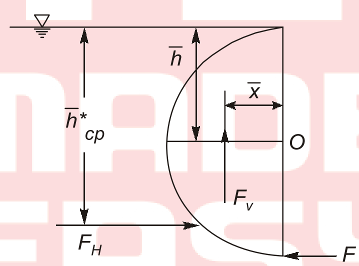
Consider unit width, of cylinder,

$$F_H = \rho g \bar{h} A_V$$

$$= 1000 \times 9.81 \times \left(\frac{2}{2}\right) \times (2 \times 1) = 19.62 \text{ kN/m width}$$

Vertical component, $F_v = \rho g\,(\text{volume})$

$$= 1000 \times 9.81 \left(\frac{\pi R^2}{2} \times 1\right) = 15.41 \text{ kN/m width}$$



location of center of pressure of F_H,

$$\bar{h}^*_{cp} = \frac{2}{3}H = \frac{4}{3} \text{ m}$$

location of center of pressure of F_V,

$$\therefore \quad \bar{x} = \frac{4R}{3\pi} = \frac{4 \times 1}{3\pi} = 0.424 \text{ m}$$

Moment about hinge,

$$\therefore \quad F_H \times (\bar{h}^*_{cp} - \bar{h}) = (F_V \times \bar{x}) + (F \times 1)$$

$$\therefore \quad 19.62 \times \left(\frac{4}{3} - 1\right) = 15.41 \times 0.424 + F \times 1$$

$$\therefore \quad F = 0 \text{ kN}$$

So, option (b) is correct.

T7 : Solution

Given: Density of water, ρ_w = 1000 kg/m^3, Density of oil, ρ_{oil} = 800 kg/m^3, Acceleration due to gravity, g = 10 m/sec^2.

Pressure exetend on the bottom wall inside the vessel.

$$P_{bottom} = \text{Gas pressure + Pressure by weight of fluids (oil + water)}$$

$$= 2\text{ bar} + \frac{(800\times10\times1+1000\times10\times3)}{10^5}\text{ bar}$$

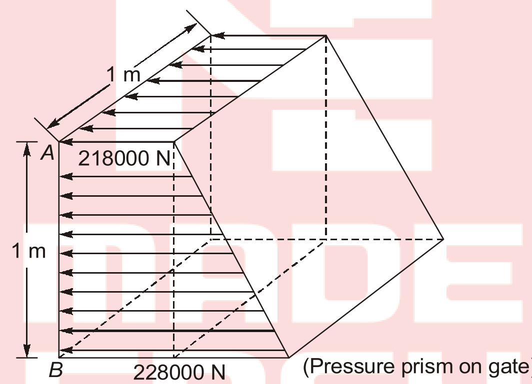
$$= 2\text{ bar} + 0.38\text{ bar}$$

$$P_{bottom} = 2.38\text{ bar}$$

So, option (b) is correct.

T8 : Solution

Now, pressure prism on gate (1m × 1m)



$$\text{Pressure at point 'A' } (P_A) = 2\times10^5 + 800\times10\times1 + 1000\times10\times1$$

$$= 218000\text{ N}$$

$$\text{Pressure at point 'B' } (P_B) = P_A + 1000\times10\times1 = 218000 + 1000\times10\times1$$

$$= 228000\text{ N}$$

$$\text{Force exerted on the gate, } F_{gate} = \text{Volume of pressure prism}$$

$$= \frac{1}{2}(218000+228000)\times1\times1$$

$$F_{gate} = 2.23\times10^5\text{ N}$$

So, option (c) is correct.

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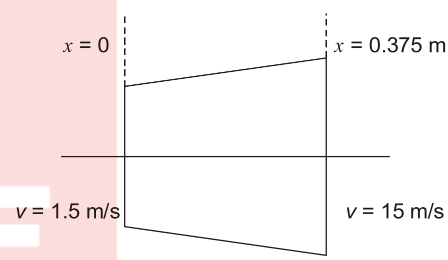
3 Fluid Kinematics

Detailed Explanation of Try Yourself Questions

T1 : Solution

Let the velocity by given by



$$u = a + bx$$

$\therefore$ At $x = 0, \quad u = 1.5$

$\therefore$ $a = 1.5$

At $x = 0.375, u = 15$

$\therefore$ $$b = \frac{15 - 1.5}{0.375} = 36$$

Hence $u = 1.5 + 36x$

$$a_x = \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} + \frac{w\partial u}{\partial z}$$

$\therefore$ $$\frac{v\partial u}{\partial y} = \frac{w\partial u}{\partial z} = 0$$

$$a_x = (1.5 + 36x)\frac{\partial}{\partial x}(1.5 + 36x)$$

$$= (1.5 + 36x)(36)$$

$$a_x\Big|_{x = 0.375} = 36 \times \{1.5 + 36 \times 0.375\} = 540 \text{ m/s}^2$$

T2 : Solution

(i) $\psi = y^2 - x^2$

Flow to be irrotational it must satisfy the Laplace equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

checking $\psi = y^2 - x^2$

$$\therefore \qquad \frac{\partial \psi}{\partial x} = -2$$

$$\psi = y^2 - x^2$$

$$\therefore \qquad \frac{\partial \psi}{\partial y} = 2y$$

$$\frac{\partial^2 \psi}{\partial y^2} = +2$$

Hence $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = +2 - 2 = 0$$

Hence flow is irrotational.

(ii) $$\psi = Ax^2y^2$$

For flow to be irrotational stream function should satisfy the Laplace equation.

$$\therefore \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Checking $$\psi = Ax^2y^2$$

$$\frac{\partial \psi}{\partial x} = 2Ay^2x$$

$$\frac{\partial^2 \psi}{\partial x^2} = 2Ay^2$$

Checking $$\psi = Ax^2y^2$$

$$\frac{\partial \psi}{\partial y} = Ax^2 2y$$

$$\therefore \qquad \frac{\partial^2 \psi}{\partial y^2} = 2Ax^2$$

$$\therefore \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 2A(x^2 + y^2)$$

Flow is not irrotational.

(iii) $$\psi = Ax - By^2$$

For flow to be irrotational stream function should satisfy the Laplace equation.

∴ Checking $$\psi = Ax - By^2$$

$$\frac{\partial \psi}{\partial x} = A$$

$$\frac{\partial^2 \psi}{\partial x^2} = 0$$

Checking $$\psi = Ax - By^2$$

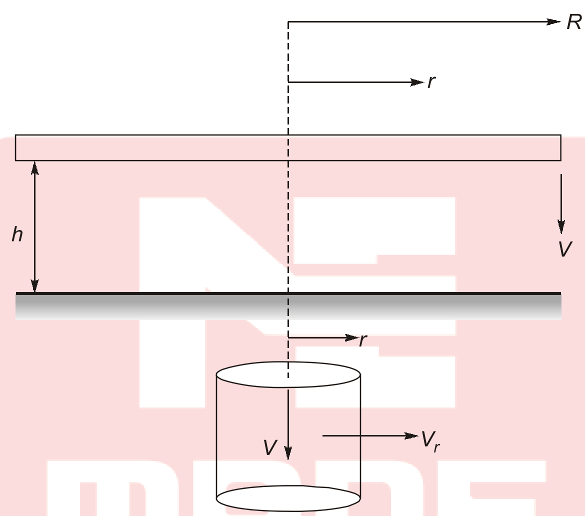
$$\frac{\partial \psi}{\partial y} = -2By$$

$$\frac{\partial^2 \psi}{\partial y^2} = -2B$$

Hence
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 - 2B \neq 0$$

Hence flow is not irrotational.

T3 : Solution



Apply continuity

$$\dot{m}_{inlet} = \dot{m}_{exit}$$

$$\rho\left(\pi r^2\right)V = \rho(2\pi rh)V_r$$

$$rV = 2hV_r$$

$$V_r = \frac{Vr}{2h}$$

So, option (a) is correct.

T4 : Solution

Given: Temperature field $T = (60 - 0.2\,xy)$°C, Velocity field, $\vec{V} = \left(2xy\hat{i} + ty\hat{j}\right)$ m/sec

Rate of change of temperature $\left(\frac{DT}{Dt}\right)_{\text{at }(2,-4),\, t=40\text{ sec}} = ?$

The rate of change of temperature with time in vector field is given by

$$\frac{DT}{Dt} = u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}+\frac{\partial T}{\partial t}\times\frac{dt}{dt}$$

where, T = Temperature, t = Time

$$u = \frac{dx}{dt} = \text{Velocity in } x\text{-direction} = 2xy$$

$$v = \frac{dy}{dt} = \text{Velocity in } y\text{-direction} = ty$$

$$w = \frac{dz}{dt} = \text{Velocity in } z\text{-direction} = 0$$

$$\frac{DT}{Dt} = 2xy(-0.2y) + ty(-0.2x) + 0 + 0$$

$$\frac{DT}{Dt} = -0.4\,xy^2 - 0.2xyt$$

$$\left(\frac{DT}{Dt}\right)_{\text{at }(2,-4), t=4\,\text{sec}} = -0.4\times 2\,(-4)^2 - 0.2\times 2(-4)\times 4 = -6.4°\text{C}$$

So, option (c) is correct.

T5 : Solution

Given: Velocity vector, $\vec{V} = \left(x^2+y^2+z^2\right)\hat{i}+\left(xy+yz+y^2\right)\hat{j}+\left(xz-z^2\right)\hat{k}$

$$\text{Volume dilation rate, } \dot{\epsilon}_V = \dot{\epsilon}_x+\dot{\epsilon}_y+\dot{\epsilon}_y = \nabla\cdot\vec{V} = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

$$= 2x + (x + z + 2y) + (x - 2z)$$

$$(\dot{\epsilon}_V)_{\text{at}(1,2,3)} = 2\times 1 + (1 + 3 + 2\times 2) + (1 - 2\times 3)$$

$$= 5$$

So, option (b) is correct.

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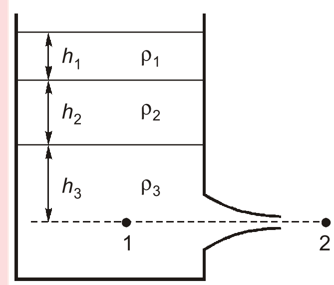
4 Fluid Dynamics & Flow Measurement

Detailed Explanation of Try Yourself Questions

T1 : Solution



Applying Bernoullis between points 1 and 2

$$\therefore \quad \frac{P_1}{\rho_3 g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_3 g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$P_1 = (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)g$$

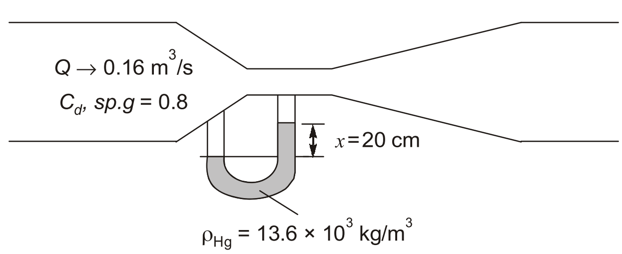
$$P_2 = 0 \qquad \text{(gauge pressure)}$$

$$V_1 = 0$$

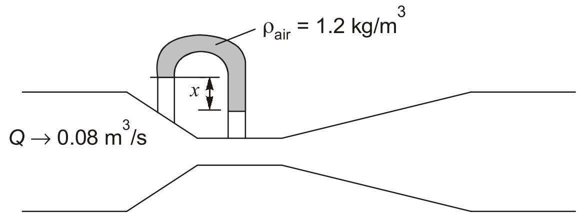
$$\therefore \quad \frac{V_2^2}{2g} = \frac{(\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)g}{\rho_3 g}$$

$$V_2 = \sqrt{2gh_3\left\{\frac{\rho_1 h_1}{\rho_3 h_3} + \frac{\rho_2 h_2}{\rho_3 h_3} + 1\right\}}$$

T2 : Solution



$$\Delta h_1 = \left[\frac{s_m}{s_p} - 1\right]x = \left[\frac{13.6}{0.8} - 1\right]20 = 320 \text{ cm}$$



$$\Delta h_2 = \left[1 - \frac{s_m}{s_P}\right]x = \left[1 - \frac{\frac{\rho_{air}}{1000}}{0.8}\right]x$$

$$\rho_{air} = \frac{1.013 \times 10^5}{287 \times 298} = 1.184 \text{ kg/m}^3$$

$$\Delta h_2 = \left[1 - \frac{1.184 \times 10^{-3}}{0.8}\right]x = 0.9952x \text{ m}$$

$$Q_{ac.} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g\Delta h}$$

$$\frac{Q_1}{Q_2} = \sqrt{\frac{\Delta h_1}{\Delta h_2}}$$

$$\frac{0.16}{0.08} = \sqrt{\frac{320}{0.99852x}}$$

$$2 = \sqrt{\frac{320}{0.99852x}}$$

$$4 = \frac{320}{0.99852x}$$

$$x = \frac{320}{4 \times 0.99852} = 80.12 \text{ cm}$$

T3 : Solution

Given: Inflow rate = 0.02 m^3/sec.

Cross-section area of tank A = 1 m^2

Inner diameter of outlet pipe, d = 60 mm = 0.06 m

Rate of water level increase = 5 mm/sec = 0.005 m/sec

Volumetric rate of increase = 0.005 m/sec × 1 m^2 = 0.005 m^3/sec.

Now, Out flow rate, Q_{out} = (0.02 – 0.005) m^3/sec = 0.015 m^3/sec

Now, Average velocity in the outlet pipe.

$$V_{outlet} = \frac{Q_{out}}{\text{Area of outlet pipe}} = \frac{0.015}{\frac{\pi}{4} \times (0.06)^2}$$

$$V_{outlet} = 5.3 \text{ m/sec}$$

So, option (c) is correct.

T4 : Solution

As we know that,

$$\int \frac{dP}{\rho g} + \frac{V^2}{2g} + z = \text{constant} \quad \text{...(i)}$$

For a compressible flow, undergoing an adiabatic process

$$\frac{P}{\rho^k} = c \text{ (constant)}$$

$$dP = K \,.\, C \,.\, \rho^{k-1} \,.d\rho$$

By equation (i)

$$\int \frac{K.C.\rho^{k-1}.d\rho}{\rho g} + \frac{V^2}{2g} + z = \text{constant}$$

$$\frac{KC}{g}\int \rho^{k-2}\, d\rho + \frac{V^2}{2g} + z = \text{constant}$$

$$\frac{K.C.\rho^{k-1}}{g(k-1)} + \frac{V^2}{2g} + z = \text{constant}$$

$$\frac{K.C.\rho^{k-1}}{g(k-1)}.\frac{\rho}{\rho} + \frac{V^2}{2g} + z = \text{constant}$$

$$\frac{K}{K-1}.\frac{C.\rho^k}{\rho.g} + \frac{V^2}{2g} + z = \text{constant} \quad (\because P = C.\rho^k)$$

$$\frac{K}{K-1}.\frac{\rho}{\rho.g} + \frac{V^2}{2g} + z = \text{constant}$$

So, option (b) is correct.

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5 Dimensional Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

As per Reynold's model law

$$\frac{\rho_r V_r l_r}{\mu_r} = 1$$

$$\Rightarrow \quad \frac{V_r l_r}{\nu_r} = 1$$

Viscosity scale ratio, $V_r = \frac{\nu_r}{l_r}$

Discharge scale ratio, $Q_r = V_r \times A_r = V_r \times l_r^2$

$$= \frac{\nu_r}{l_r} \times l_r^2 = \nu_r \times l_r$$

T2 : Solution

$$\left[\frac{\rho VL}{\mu}\right]_{model} = \left[\frac{\rho VL}{\mu}\right]_P$$

Given $\frac{L_m}{L_P} = \frac{1}{6}$

$$[VL]_m = [VL]_P$$

$$V_m \times L_m = 60 \times \frac{L_P}{L_m} = 60 \times 6 = 360 \text{ km/hr}$$

$$F_D = C_D \frac{1}{2} \rho A V^2$$

or $F_D \propto (LV)^2$

$\therefore \quad (F_D)_P = k[L_P v_P]^2$

$$(F_D)_m = k[L_m V_m]^2$$

$$\frac{(F_D)_P}{(F_D)_m} = \frac{L_P^2 \, V_P^2}{L_m^2 \, V_m^2}$$

$$= 6^2 \times \left(\frac{60}{360}\right)^2$$

$$\frac{(F_D)_P}{250} = 1$$

$$\therefore \quad (F_D)_P = 250 \text{ N}$$

Power required to overcome the drag in prototype

$$= (F_D)_P \times V_P$$

$$= 250 \times \frac{60 \times 1000}{3600}$$

$$= 4167.67 \text{ W} = 4.167 \text{ kW}$$

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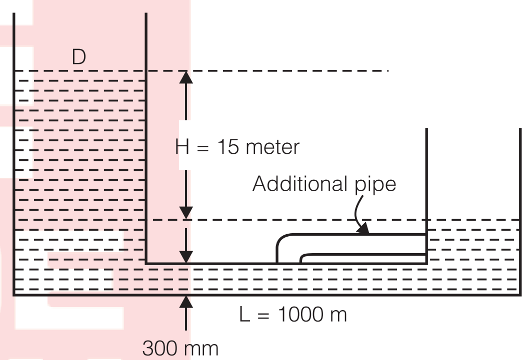
6 Flow Through Pipes

Detailed Explanation of Try Yourself Questions

T1 : Solution

All the losses are negligible except friction.



$$\therefore \quad H = \frac{4fL}{d}\cdot\frac{V^2}{2g}$$

$$15 = \frac{0.02\times 1000\times V^2}{0.3\times 2\times 9.81}$$

$\because$ f = 0.02 which is very high.
So it will be friction factor and 4f = 0.02

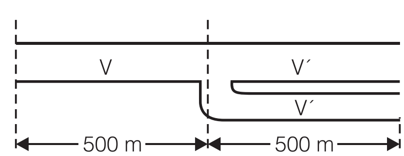
$$V^2 = \frac{15\times 0.3\times 2\times 9.81}{0.02\times 1000}$$

$$V = 2.101 \text{ m/sec}$$

$$\therefore \quad \text{Flow rate, } \dot{Q} = AV = \frac{\pi}{4}(0.3)^2\times 2.101$$

$$\dot{Q} = 0.1485 \text{ m}^3\text{/sec}$$

Now addition same pipe of length is added in later half of pipe as



$$\therefore \quad Q_1 = Q_2 + Q_3$$

$$AV = AV' + AV'$$

$$V' = \frac{V}{2}$$

$\therefore$ Friction head is same

$$h_f = 15 = \frac{4fL'}{d}\cdot\frac{V^2}{2g} + \frac{4fL'}{d}\cdot\frac{V'^2}{2g}$$

$$15 = \frac{0.02\times 500}{0.3}\frac{V^2}{2g} + \frac{0.02\times 500}{0.3}\times\frac{1}{4}\cdot\frac{V^2}{2g}$$

$$15 = 2.124\ V^2$$
$$V = 2.657 \text{ m/sec}$$
$$V' = \frac{V}{2} = 1.329 \text{ m/sec}$$

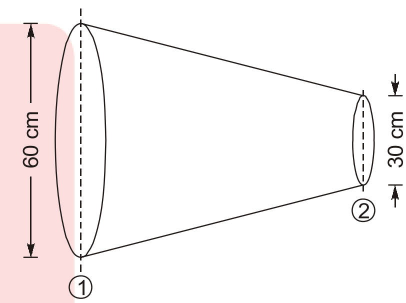
Discharge rate $\quad Q' = A.V = \frac{\pi}{4}.(0.3)^2 \times 2.657 = 0.18781 \text{ m}^3\text{/sec}$

$$\text{Increase in discharge} = \frac{Q' - Q}{Q} = 26.47\%.$$

T2 : Solution

Using the Bernaulli's equation, at points 1 and 2

∴ Let p_1, V_1, Z_1 be the pressure, velocity and head at point 1, and p_2 V_2, Z_2, be the corresponding values at point 2.



$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$h_L = \left(1 - \frac{1}{C_c}\right)^2 \frac{V_2^2}{2g}$$

$$\therefore \quad h_L = \left(1 - \frac{1}{0.65}\right)^2 \frac{V_2^2}{2g}$$

$$\therefore \quad h_L = 0.2899 \frac{V_2^2}{2g}$$

Also, $\quad Q = A_1 V_1 = A_2 V_2$

$$\Rightarrow \quad \frac{\pi}{4} \times (60)^2 V_1 = \frac{\pi}{4}(30)^2 \times V_2$$

$$\therefore \quad V_1 = \frac{V_2}{4}$$

Using the Bernaulli's equation

$$\therefore \quad \frac{100 \times 10^3}{1000 \times 9.81} + \frac{1}{2g}\left(\frac{V_2}{4}\right)^2 + Z_1 = \frac{80 \times 10^3}{1000 \times 9.81} + \frac{V_2^2}{2g} + Z_2 + 0.2899 \frac{V_2^2}{2g}$$

$$\therefore \quad 10.1936 + \frac{V_2^2}{32g} = 8.1549 + 1.2899 \frac{V_2^2}{2g} \qquad [\because Z_1 = Z_2]$$

$$\therefore \quad 10.1936 - 8.1549 = 1.2899 \frac{V_2^2}{2g} - \frac{V_2^2}{32g}$$

$$2.0387 = 0.06255\ V_2^2$$
$$\Rightarrow \quad V_2^2 = 32.5886$$
$$\therefore \quad V_2 = 5.7086 \text{ m/s}$$

∴ Flow rate, $\quad Q = A_2 V_2 = \frac{\pi}{4} \times (0.3)^2 \times 5.7086$

$$Q = 0.4035 \text{ m}^3\text{/s}$$

Also,
$$h_L = \left(1 - \frac{1}{C_c}\right)^2 \frac{V_2^2}{2g}$$

$$h_L = \left(1 - \frac{1}{0.65}\right)^2 \times \frac{(5.7086)^2}{2 \times 9.81}$$

$$h_L = 0.482 \text{ m}$$

T3 : Solution

L_1 = 1800 m $\quad$ L_2 = 1200 m $\quad$ L_3 = 600 m

D_1 = 50 cm = 0.5 m $\quad$ D_2 = 40 cm = 0.4 m $\quad$ D_3 = 30 cm = 0.3 m

(i) We know for the pipe connected in series

$$\frac{L_{eq}}{D_{eq}^{\,5}} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

$$\frac{L_{eq}}{(0.4)^5} = \frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}$$

$$L_{eq} = 4318.22 \text{ m}$$

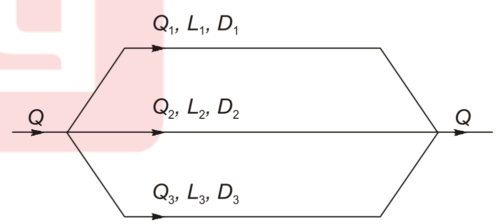
(ii)
$$\frac{L_{eq}}{D_{eq}^{\,5}} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$$

∴
$$\left(\frac{3600}{D_{eq}^{\,5}}\right) = \frac{1800}{(0.5)^5} + \frac{1200}{(0.4)^5} + \frac{600}{(0.3)^5}$$

On solving, D_{eq} = 0.38570 m

∴ D_{eq} = 38.57 cm

(iii)
$$Q = Q_1 + Q_2 + Q_3$$



Since,
$$h_f \propto \frac{LQ^2}{D^5}$$

So,
$$Q \propto \left(\frac{D^5}{L}\right)^{1/2}$$

[h_f is same for parallel connections]

Thus,
$$\left(\frac{D_{eq}^{\,5}}{L_{eq}}\right)^{1/2} = \left(\frac{D_1^{\,5}}{L_1}\right)^{1/2} + \left(\frac{D_2^{\,5}}{L_2}\right)^{1/2} + \left(\frac{D_3^{\,5}}{L_3}\right)^{1/2}$$

⇒
$$\left(\frac{0.5^5}{L_{eq}}\right)^{1/2} = \left(\frac{0.5^5}{1800}\right)^{1/2} + \left(\frac{0.4^5}{1200}\right)^{1/2} + \left(\frac{0.3^5}{600}\right)^{1/2}$$

⇒ On solving, L_{eq} = 377.345 m

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7 Laminar and Turbulent Flow

Detailed Explanation of Try Yourself Questions

T1 : Solution

Reynolds number, $$Re = \frac{\rho VD}{\mu} = \frac{1260 \times 5.0 \times 0.10}{1.50} = 420$$

(i) As this value is less than 2000, the flow is laminar. In laminar flow in a conduit

$$\tau_0 = \frac{8\mu V}{D} = \frac{8 \times 1.50 \times 5.0}{0.10} = 600 \text{ Pa}$$

(ii) In laminar flow the head loss

$$h_f = \frac{32\mu VL}{\gamma D^2} = \frac{32 \times 1.50 \times 5.0 \times 12}{(1260 \times 9.81)(0.1)^2} = 23.3 \text{ m}$$

(iii) Power expended $P = \gamma Q h_f$

Discharge $$Q = AV = \frac{\pi \times (0.1)^2}{4} \times 5.0 = 0.03927 \text{ m}^3/\text{s}$$

Power, $$P = (1260 \times 9.81) \times 0.03927 \times 23.3$$
$$= 11309.8 \text{ W} = 11.31 \text{ kW}$$

T2 : Solution

(i) For two-dimensional laminar flow between parallel plates

$$u_m = \text{Maximum velocity} = \frac{3}{2}V$$

$$= \frac{3}{2} \times 1.40 = 2.10 \text{ m/s}$$

(ii) Since $$V = \left(-\frac{dp}{dx}\right)\frac{B^2}{12\mu}$$

$$\left(-\frac{dp}{dx}\right) = \frac{12\mu V}{B^2} = \frac{12\times0.105\times1.40}{(0.012)^2} = 12250$$

Boundary shear stress $\tau_0 = \left(-\frac{dp}{dx}\right)\frac{B}{2} = 12250\times\frac{0.012}{2} = 73.5 \text{ Pa}$

(iii) Shear stress τ at any y from the boundary

$$\tau = \left(-\frac{dp}{dx}\right)\left(\frac{B}{2}-y\right)$$

At $y = 0.002$ m

1. $\tau = (12250)\left(\frac{0.012}{2}-0.002\right) = 49 \text{ Pa}$

Velocity, $v = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)(By-y^2)$

$$= \frac{1}{2\times0.105}\times12250\left[0.012\times0.002-(0.002)^2\right]$$

$$v = 1.167 \text{ m/s}$$

T3 : Solution

Given:

At R: $\bar{u} = 1.5$ m/s

At $\frac{R}{2}$ $\bar{u} = 1.35$ m/s

Flow is turbulent

We know $\frac{u-\bar{u}}{U^*} = 5.75\log_{10}\left(\frac{y}{R}\right)+3.75$

Given, at $y = R$, $u = 1.5$ m/s

$\therefore$ $\frac{1.5-\bar{u}}{U^*} = 3.75$ ...(i)

Also at, $y = \frac{R}{2} = \frac{0.1}{2} \Rightarrow 0.05$ m, $u = 1.35$

$$\frac{1.35-\bar{u}}{U^*} = 5.75\log_{10}\left(\frac{1}{2}\right)+3.75$$

$\therefore$ $\frac{1.35-\bar{u}}{U^*} = 2.0190$ ...(ii)

Dividing eq. (i) by eq. (ii)

$$\frac{1.5-\bar{u}}{1.35-\bar{u}} = 1.857$$

$$1.5-\bar{u} = 1.857(1.35-\bar{u})$$

$$1.5-\bar{u} = 2.507-1.857\bar{u}$$

$$1.857\,\bar{u} - \bar{u} = 1.007$$

$$0.857\,\bar{u} = 1.007$$

$$\bar{u} = 1.175 \text{ m/s}$$

$\therefore$ $$Q = \bar{u} \times \pi R^2$$

$$Q = 1.175 \times \pi \times (0.1)^2$$

$$Q = 0.0369 \text{ m}^3\text{/s}$$

$$\frac{\bar{u}}{U^*} = 5.75\log_{10}\left(\frac{R}{k}\right) + 4.75$$

Also, from eq. (i)

$$\frac{15 - \bar{u}}{U^*} = 3.75$$

$\therefore$ $$\frac{1.5 - 1.175}{U^*} = 3.75$$

$\Rightarrow$ $$U^* = 0.0866 \text{ m/s}$$

$\therefore$ $$\frac{1.175}{0.0866} = 5.75\log_{10}\left(\frac{0.1}{k}\right) + 4.75$$

$\therefore$ $$k = 2.9 \times 10^{-3} \text{ m}$$

$\therefore$ $$k = 2.9 \text{ mm}$$

Also, $$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.1}{2.9 \times 10^{-3}}\right) + 1.74$$

$\therefore$ $$f = 0.043$$

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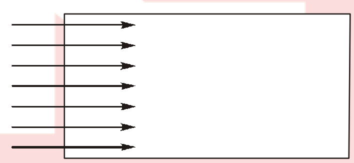
8 Boundary Layer Theory, Drag and Lift

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$F_{D1} = C_{fx}\rho\frac{1}{2}AV_\infty^2 \quad \text{[For first half]}$$

$$C_{fx} = \frac{k}{\sqrt{Re_x}}$$

$$= \frac{k}{\sqrt{Re_x}}\times\rho\times\frac{1}{2}\times b\times\frac{L}{2}\times U_\infty^2$$

$$= \frac{k\sqrt{2\mu}}{\sqrt{\rho VL}}\times\frac{\rho\times bU_\infty^2\times L}{4} \quad \text{....(1)}$$

$$F_{D2} = C_{fx}\rho\frac{1}{2}AV_\infty^2 \quad \text{[for full plate]}$$

$$C_{fx} = \frac{k}{\sqrt{Re_L}}$$

$$= \frac{k\times\rho\times b\times L\times U_\infty^2\sqrt{\mu}}{\sqrt{\rho VL}\times 2}$$

$$\frac{F_{D_1}}{F_{D_2}} = \frac{\sqrt{2}/4}{1/2}$$

$$= \frac{\sqrt{2}}{4}\times 2 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

T2 : Solution

Given:

I[st] velocity profile

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

or
$$u = \frac{3U}{2}\left(\frac{y}{\delta}\right) - \frac{U}{2}\left(\frac{y}{\delta}\right)^3$$

Differentiating w.r.t y, the above equation becomes,

$$\frac{\partial u}{\partial y} = \frac{3U}{2}\times\frac{1}{\delta} - \frac{U}{2}\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

At $y = 0$,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{3U}{2\delta} - \frac{3U}{2}\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta} = \frac{3U}{2\delta}$$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is positive. Hence flow will not separate or flow will remain attached with the surface.

2[nd] Velocity profile

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3$$

∴
$$u = 2U\left(\frac{y}{\delta}\right)^2 - U\left(\frac{y}{\delta}\right)^3$$

∴
$$\frac{\partial u}{\partial y} = 2U\times 2\left(\frac{y}{\delta}\right)\times\frac{1}{\delta} - U\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

at $y = 0$,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 2U\times 2\left(\frac{0}{\delta}\right)\times\frac{1}{\delta} - U\times 3\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta} = 0$$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$, the flow is on the verge of separage.

3[rd] velocity profile

$$\frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2$$

∴
$$u = -2U\left(\frac{y}{\delta}\right) + U\left(\frac{y}{\delta}\right)^2$$

$\therefore \quad \dfrac{\partial u}{\partial y} = -2U\left(\dfrac{1}{\delta}\right) + 2U\left(\dfrac{y}{\delta}\right) \times \dfrac{1}{\delta}$

At $y = 0$, $\quad \left(\dfrac{\partial u}{\partial y}\right)_{y=0} = -\dfrac{2U}{\delta} + 2U\left(\dfrac{0}{\delta}\right) \times \dfrac{1}{\delta} = -\dfrac{2U}{\delta}$

As $\left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ is negative the flow has separated.

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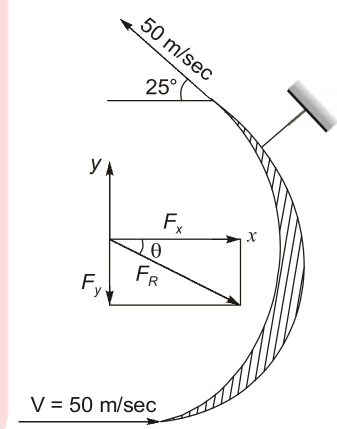
9 Hydraulic Machines

Detailed Explanation of Try Yourself Questions

T1 : Solution



Given: **(a)** Velocity of jet, V = 50 m/s

Angle at outlet = 25°

For the stationary vane, the force in the direction of jet is given as

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where,

$$V_{1x} = 50 \text{ m/s}$$

$$V_{2x} = -50 \cos 25° = -45.315$$

∴ Force in direction of jet per unit weight of water

$$= \frac{\text{Mass/sec}[50-(-45.315)]}{\text{Weight of water/sec}}$$

or

$$F_x = \frac{(Mass/\text{sec})[50+45.315]}{(\text{Mass/sec})\times g}$$

$$= \frac{1}{g}[50+45.315]\text{ N} = \frac{95.315}{9.81} = 9.716 \text{ N}$$

Force exerted by jet in the direction perpendicular to the direction of the jet per unit weight of the flow,

$$F_y = \frac{(\text{Mass per sec})\left[V_{1y} - V_{2y}\right]}{g \times \text{ Mas per sec}}$$

$$= \frac{1}{g}\left[V_{1y} - V_{2y}\right] = \frac{1}{g}[O - 50\sin 25°] \qquad (\because V_{1y} = 0,\ V_{2y} = 50 \sin 25°)$$

$$= \frac{-50\sin 25°}{9.81} = -2.154 \text{ N}$$

–ve sign means the force F_y is acting in the downward direction.

∴ Resultant force per unit weight of water = $\sqrt{F_x^2 + F_y^2}$

or $$F_R = \sqrt{(9.716)^2 + (2.154)^2} = 9.952 \text{ N}$$

The angle made by the resultant with the x-axis.

$$\tan\theta = \frac{F_y}{F_x} = \frac{2.154}{9.716} = 0.2217$$

∴ $$\theta = \tan^{-1} 0.2217 = 12.50°$$

(b) Velocity of the vane = 20 m/s

When the vane is moving in the direction of the jet, the force exerted by the jet on the plate in the direction of jet,

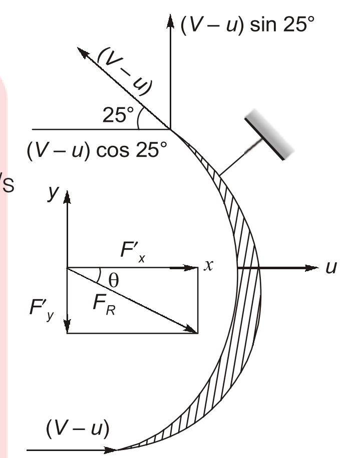
$$F'_x = [\text{Mass of water striking/sec}] \times [V_{1x} - V_{2x}]$$

where, V_{1x} = Initial velocity of the striking water

$= (V - u) = 50 - 20 = 30$ m/s

V_{2x} = Final velocity in the direction of x

$= -(V - u) \cos 25° = 30 \times \cos 25° = -27.189$ m/s



∴ F_x = Mass per sec [30 + 27.189]

Force in the direction of jet per unit weight,

$$F'_x = \frac{\text{Mass per sec } [30 + 27.189]}{\text{Mass per sec} \times g}$$

$$= \frac{(30 + 27.189)}{9.81} = 5.829 \text{ N}$$

Force exerted by the jet in the direction perpendicular to direction of jet, per unit weight

$$F'_y = \frac{1}{g}\left[V_{1y} - V_{2y}\right]$$

where, $V_{1y} = 0$; $V_{2y} = (V - u) \sin 25° = (50 - 20) \sin 25° = 30 \sin 25°$

$$F'_y = \frac{1}{9.81}\left[0 - 30 \sin 25°\right] = -1.292 \text{ N}$$

∴ Resultant force $= \sqrt{(5.829)^2 + (1.292)^2} = 5.917$ N

The angle made by the resultant with x-axis,

$$\tan\theta = \frac{1.292}{5.829} = 0.2217$$

∴ $$\theta = \tan^{-1} 0.2217 = 12.30°$$

∴ Work done per second per unit weight of flow

$= F'_x \times u = 5.829 \times 20 = 116.58$ N m/s

∴ Power developed $= \dfrac{\text{Work done per second}}{1000} = \dfrac{116.58}{1000} = 0.116$ kW

T2 : Solution

Given:

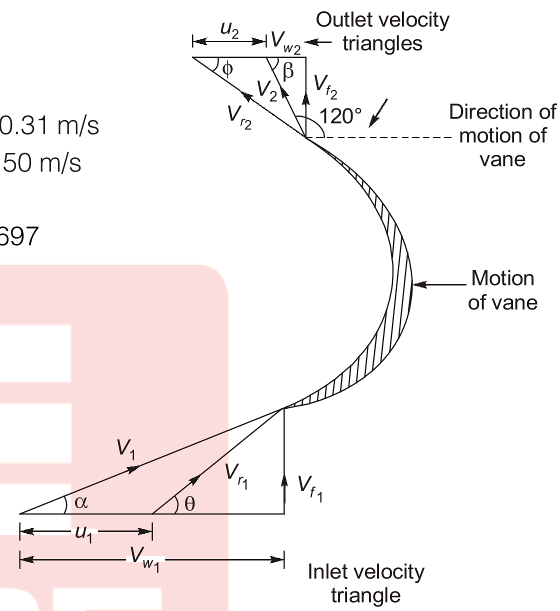
Velocity of jet, $V_1 = 35$ m/s

Velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes = 120°

∴ Angle $\beta = 180° - 120° = 60°$



(a) Angle of vanes tips.

From inlet velocity triangle,

$$V_{w1} = V_1 \cos\alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f1} = V_1 \sin\alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

∴ $\theta = \tan^{-1} 1.697 = 59.49°$

By sine rule, $$\frac{V_{r1}}{\sin 90°} = \frac{V_{f1}}{\sin\theta}$$

or $$\frac{V_{r1}}{1} = \frac{17.50}{\sin 59.49°}$$

∴ $$V_{r1} = \frac{17.50}{0.866} = 20.31 \text{ m/s}$$

Now, $V_{r2} = V_{r1} = 20.31$ m/s

From outlet velocity triangle, by sine rule

$$\frac{V_{r2}}{\sin 120°} = \frac{u_2}{\sin(60° - \phi)}$$

or $$\frac{20.25}{0.866} = \frac{20}{\sin(60° - \phi)}$$

∴ $$\sin(60° - \phi) = \frac{20 \times 0.866}{20.31} = 0.852 = \sin(58.50°)$$

∴ $\phi = 60° - 58.50° = 1.5°$

(b) Work done per unit weight of water entering $= \frac{1}{g}(V_{w1} + V_{w2}) \times u_1$ ...(i)

$$V_{w1} = 30.31 \text{ m/s and } u_1 = 20 \text{ m/s}$$

The value of V_{w2} is obtained from outlet velocity triangle

$$V_{w2} = V_{r2} \cos\phi - u_2 = 20.31 \cos 1.5° - 20.0 = 0.30 \text{ m/s}$$

∴ Work done/unit weight $$= \frac{1}{9.81}[30.31 + 0.30] \times 20 = 62.41 \text{ Nm/N}$$

(c) Efficiency $= \dfrac{\text{Work done per kg}}{\text{Energy supplied per kg}}$

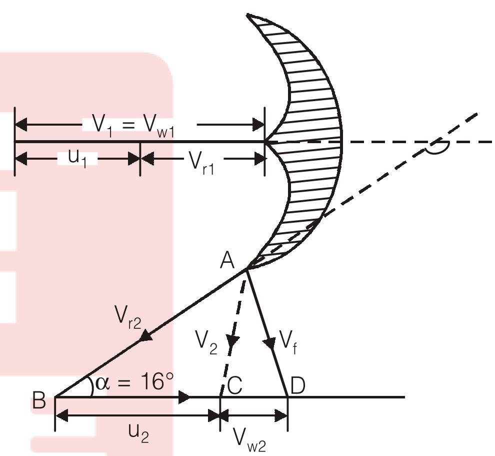
$$= \frac{62.41}{\frac{V_1^2}{2g}} = \frac{62.41\times 2\times 9.81}{35\times 35} = 99.96\%$$

T3 : Solution

Gross head, $H_g = 220$ m, Net head, $H = 200$ m, $C_V = 0.98$, $N = 200$ rpm, power = 3.7 MW, $u_1 = u_2 = u$

Given: $\dfrac{u}{V_1} = 0.46,\ D = ?$

Speed of jet at vena contracta i.e. max. speed of jet



$$V_1 = C_V\sqrt{2gH}$$
$$= 0.98\sqrt{2\times 9.81\times 200}$$
$$= 61.4 \text{ m/sec}$$

Speed of wheel

$$u = 0.46\times V_1$$
$$= 0.46\times 61.4 = 28.24 \text{ m/sec}$$

$$u = \frac{\pi DN}{60} = 28.24 \quad [u = u_1 = u_2]$$

$$D = \frac{28.24\times 60}{\pi\times 200}$$

$$D = 2.697 \text{ m}$$

$\therefore$ $V_{r2} = V_{r1} = V_1 - u$
$$= 61.4 - 28.24$$
$$= 33.16 \text{ m/sec}$$
$$V_{w2} = V_{r2}\cos 16 - u$$
$$= 33.16\times\cos 16 - 28.24$$
$$V_{w2} = 3.635 \text{ m/sec}$$

Blade efficiency,

$$\eta_b = \frac{2u(V_{w1}+V_{w2})}{V_1^2} = \frac{2\times 28.24\,(61.4+3.635)}{61.4^2}$$

$$\eta_b = 97.5\%$$

Hydraulic efficiency

$$= \frac{u(V_{w1}+V_{w2})}{gH} = \frac{28.24(61.4+3.635)}{9.81\times 200} = 0.936 = 93.6\%$$

T4 : Solution

Given: Gross head, $H_g = 500\text{ m}$

Head lost in friction, $h_f = \dfrac{H_g}{3} = \dfrac{500}{3} = 166.7\text{ m}$

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.3\text{ m}$

Discharge, $Q = 2.0\text{ m}^3/\text{s}$

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio, $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86\text{ m/s}$

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

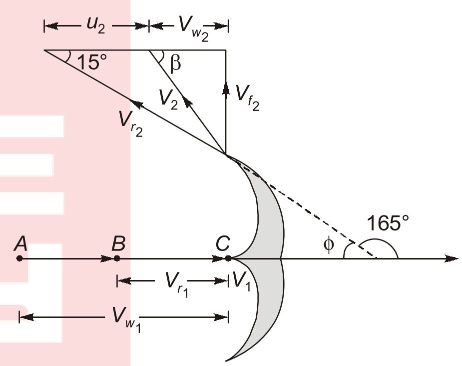
or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387\text{ m/s}$

∴ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473\text{ m/s}$

Also $V_{w_1} = V_1 = 80.86\text{ m/s}$

From outlet velocity tringle, we have



$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2}\cos\phi = u_2 + V_{w_2}$$

or $44.473 \cos 15° = 36.387 + V_{w_2}$

or $V_{w_2} = 44.473 \cos 15° - 36.387$

$= 6.57\text{ m/s}$

Work done by the jet on the runner per second is given by equation as

$$\rho a V_1\left[V_{w_1} + V_{w_2}\right] \times u = \rho Q\left[V_{w_1} + V_{w_2}\right] \times u \qquad (\because aV_1 = Q)$$

$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630\text{ Nm/s}$

∴ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = 6362.63\text{ kW} = 6.36\text{ MW}$$

Hydraulic efficiency of the turbine is given by equation as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$= 0.9731$ or 97.31%

T5 : Solution

Inlet diameter, $D_1 = 1.0$ m

Rotational speed, $N = 400$ rpm

Area of flow, $A = 0.25\ m^2$

Net available head, $H = 65$ m

Velocity of flow at inlet, $V_{f1} = 8.0$ m/s

Velocity of whirl at inlet, $V_{w1} = 25.0$ m/s

Flow is radial at outlet i.e. velocity of whirl at outlet, $V_{w2} = 0$

Let the peripheral velocity at inlet and outlet be u_1 and u_2 respectively

$$\therefore \quad u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1 \times 400}{60} = 20.94 \text{ m/s}$$

Discharge, $Q = A \times V_{f1} = 0.25 \times 8 = 2\ m^3/s$

Power developed by the wheel is expressed as

$$P = \rho Q(u_1 V_{w1} - u_2 V_{w2})$$

$$= 1000 \times 2 \times (20.94 \times 25 - u_2 \times 0) \times 10^{-3} = 1047 \text{ kW}$$

Hydraulic efficiency,

$$\eta_h = \left[\frac{u_1 V_{w1} - u_2 V_{w2}}{gH}\right] \times 100$$

$$= \left[\frac{20.94 \times 25 - u_2 \times 0}{9.81 \times 65}\right] \times 100 = 82.1\%$$

T6 : Solution

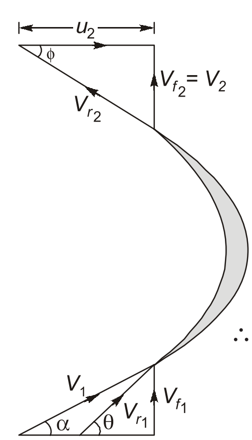
Given:

Head, $H = 12$ m

Hub diameter, $D_b = 0.35 \times D_0$ Where D_0 = Dia. of runner

Speed, $N = 100$ rpm

Vane angle at outlet, $\phi = 15°$



Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.6$

$$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 12} = 9.2 \text{ m/s}$$

From the outlet velocity triangle, $V_{w_2} = 0$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2} \quad \left(\because V_{f_2} = V_{f_1} = 9.2\right)$$

$$\tan 15° = \frac{9.2}{u_2}$$

$$\therefore \quad u_2 = \frac{9.2}{\tan 15°} = 34.33 \text{ m/s}$$

∴

But for Kaplan turbine, $u_1 = u_2 = 34.33$

Now, using the relation, $u_1 = \frac{\pi D_0 \times N}{60}$ or $34.33 = \frac{\pi \times D_0 \times 100}{60}$

$$D_0 = \frac{60 \times 34.33}{\pi \times 100} = 6.56 \text{ m}$$

∴ $D_b = 0.35 \times D_0 = 0.35 \times 6.35 = 2.23$ m

Discharge through turbine is given by eq. as

$$Q = \frac{\pi}{4}\left[D_0^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}\left[6.55^2 - 2.3^2\right] \times 9.2$$

$$= \frac{\pi}{4}(42.9026 - 5.29) \times 9.2 = 271.77 \text{ m}^3/\text{s}$$

T7 : Solution

Given:

Head, $H = 25$ m

Speed, $N = 200$ rpm

Discharge, $Q = 9$ cumec $= 9 \text{ m}^3/\text{s}$

Efficiency, $\eta_0 = 90\% = 0.90$ (Take the efficiency as overall η)

Now using relation, $\eta_0 = \frac{\text{Work developed}}{\text{Water power}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$

∴ $P = \eta_0 \times \frac{\rho \times g \times Q \times H}{1000} = \frac{0.90 \times 9.81 \times 1000 \times 9 \times 25}{1000} = 1986.5$ kW

(a) Specific speed of the machine (N_s)

Using equation $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{1986.5}}{25^{5/4}} = 159.46$ rpm

(b) Power generated $P = 1986.5$ kW

(c) As the specific speed lies between 51 and 255, the turbine is a Francis turbine.

T8 : Solution

Given: $Q = 0.04 \text{ m}^3/\text{s}$

$H_g = 20$ m

$$\eta_0 = \frac{\rho g Q H}{P}$$

$f = 0.015$

$l = 100$ m

$D = 0.15$ m

$\eta_0 = 70\%$, $\eta_0 = 0.7$

$$h_f = \frac{4flQ^2}{12D^5} = \frac{4 \times 0.015 \times 100 \times (0.04)^2}{12 \times (0.15)^5} = 10.534 \text{ m}$$

∴ $H_{net} = H_g + h_f = 20 \text{ m} + 10.534$

$$\Rightarrow \qquad H_{net} = 30.534 \text{ m}$$

$$\eta_0 = \frac{\frac{\rho g Q H_{net}}{1000}}{P}$$

$$0.70 = \frac{\frac{1000 \times 9.81 \times 0.04 \times 30.534 \text{ kW}}{1000}}{P}$$

$$\therefore \qquad P = \frac{9.81 \times 0.04 \times 30.534}{0.7} \text{ kW}$$

$$P = 17.116 \text{ kW}$$

Hence power required to derive the pump is 17.116 kW.

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