ESE GATE

State Engg. Exams

MADE EASY WORKBOOK 2027

Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

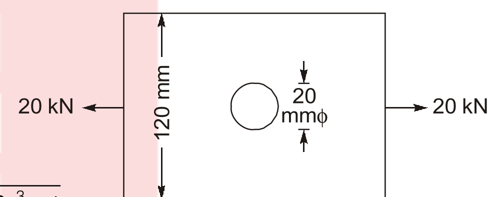
Design of Machine Elements

MADE EASY PUBLICATIONS

1 Static & Fluctuating Stresses

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$\sigma = \frac{P}{A}$$

$$\Rightarrow \quad \frac{600\times10^6}{3\times5} = \frac{20\times10^3}{(120-20)\times10^{-3}\times t}$$

$$\Rightarrow \quad t = \frac{2\times10^5}{4\times10^7} = 5\times10^{-3}\text{ m} = 5\text{ mm}$$

T2 : Solution

Mean stress,
$$\sigma_m = \frac{200+(-100)}{2} = 50\text{ MPa}$$

Variable stress,
$$\sigma_v = \frac{200-(-100)}{2} = 150\text{ MPa}$$

Soderberg's formula,
$$\frac{1}{N} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \frac{50}{0.55\times\sigma_u} + \frac{150}{0.5\times\sigma_u} = \frac{1}{\sigma_u}\left[\frac{50}{0.55}+\frac{150}{0.5}\right]$$

$$\Rightarrow \quad \sigma_u = 781.818\text{ MPa} \approx 781.82\text{ MPa}$$

T3 : Solution

$$\sigma_{max} = \frac{32M_{max}}{\pi d^3} = \frac{32\times500}{\pi d^3} = \frac{5092.96\times10^3}{d^3}\text{MPa}$$

$$\sigma_{min} = \frac{32M_{min}}{\pi d^3} = \frac{32\times-200}{\pi d^3} = \frac{-2037.18\times10^3}{d^3}\text{MPa}$$

$$\sigma_{mean} = \frac{\sigma_{max}+\sigma_{min}}{2} = \frac{1527.89\times10^3}{d^3}\text{MPa}$$

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{3565.07 \times 10^3}{d^3} \text{ MPa}$$

For ductile material-using Soderberg equation

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} \qquad [\sigma_e = 0.5\,\sigma_u]$$

$$\frac{1}{2.5} = \frac{1527.89 \times 10^3}{400 d^3} + \frac{3565.07 \times 10^3}{\frac{540}{2} \times d^3}$$

$$\Rightarrow \quad d^3 = 2.5 \times 10^3 \times \left[\frac{1527.89}{400} + \frac{3565.07 \times 2}{540}\right] = 34.91 \text{ mm}$$

$$\Rightarrow \quad d \simeq 35 \text{ mm}$$

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2 Welded, Riveted and Bolted Joint

Detailed Explanation of Try Yourself Questions

T1 : Solution

Pitch of rivets, $$p = \frac{\pi(D+t)}{\text{No. of rivets/row}} = \frac{\pi(1600+30)}{45}$$
$$= 113.8 \text{ mm} \simeq 115 \text{ mm}$$

Efficiency, $$\eta_C = 1 - \frac{d}{p} = 1 - \frac{35}{115} = 69.56\ \% \simeq 70\%$$

T2 : Solution

Area of two welds,
$$A = 2(100 \times t) = (200\ t) \text{ mm}^2$$

Primary shear stress $$\tau_1 = \frac{P}{A} = \frac{20\times10^3}{200t} = \left(\frac{100}{t}\right)\text{MPa}$$

Moment of inertia of two welds, $$I = 2\,I_{xx}$$

$$I_{xx} = \frac{100t^3}{12} + Ay_1^2 = \frac{100t^3}{12} + (100t)(100)^2 \text{ mm}^4$$

Since dimension t is very small compared with 100, so term t^3 can benegelcted.

Therefore, $$I_{xx} = (100^3\ t) \text{ mm}^4$$

$$I = 2I_{xx} = t\times\left(\frac{bd^2}{2}\right) = t\times\left(\frac{100\times200^2}{2}\right) = 2\times10^6\ t \text{ mm}^4$$

Bending stress in the top weld $$= 2\times(10^6\ t) \text{ mm}^4$$

$$\sigma_b = \frac{M_b y}{I} = \frac{\left(20\times10^3\times200\right)\times100}{2\times10^6\ t} = \frac{200}{t}\text{MPa}$$

Maximum shear stress $$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_1^2} = \sqrt{\left(\frac{200}{2t}\right)^2 + \left(\frac{100}{t}\right)^2}$$

$\Rightarrow$ $$\tau = \frac{141.42}{t}\text{MPa}$$

$\Rightarrow$ $$100 = \frac{141.42}{t}$$

$\Rightarrow$ $$t = 1.4142 \text{ mm}$$

$\Rightarrow$ $$h = \frac{t}{0.707} = 2 \text{ mm}$$

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3 Bearings

Detailed Explanation of Try Yourself Questions

T1 : Solution

Life of bearing in millions of revolutions

$$= \left(\frac{C}{W}\right)^3 = \left(\frac{35}{45}\right)^3 = 0.4705$$

$$L = 60\,NL_H$$

$$\Rightarrow \quad \text{Life in hours, } L_H = \frac{L}{60N} = \frac{0.4705\times10^6}{60\times1800} = 4.356 \text{ hrs}$$

This the life expected for 90% of the bearings.
The average life expectancy is 5 times of the above life.
i.e. Average life = 5 × 4.356 = 21.78 hrs

T2 : Solution

$$L = 60\,NL_H = 60\times900\times2000 = 108\times10^6 \text{ revolutions}$$

$$L = \left(\frac{C}{W}\right)^3 \times 10^6$$

$$108\times10^6 = \left(\frac{C}{2.0}\right)^3 \times 10^6$$

$$C = 9.5 \text{ kN} = 9500 \text{ N}$$

∴ 6204 bearing is suitable

T3 : Solution

Given, $W = 6$ kN = 6000 N, $N = 1500$ rpm, $D = 0.05$ m, $L = 0.05$ m, $S = 0.121$, $C = 50 \times 10^{-6}$ m

$$P = \frac{W}{LD} = \frac{6000}{50\times10^{-3}\times50\times10^{-3}} = 2.4 \times 10^{6} \text{ Pa}$$

Sommerfeld number,
$$S = \left(\frac{D}{C}\right)^2 \times \frac{ZN}{P}$$

$\Rightarrow$
$$0.121 = \left(\frac{50\times10^{-3}}{50\times10^{-6}}\right)^2 \times \frac{Z\times1500}{60\times2.4\times10^{6}}$$

$\Rightarrow$
$$Z = 0.011616 \text{ Ns/m}^2 = 0.11616 \text{ Poise} = 11.62 \text{ cP}$$

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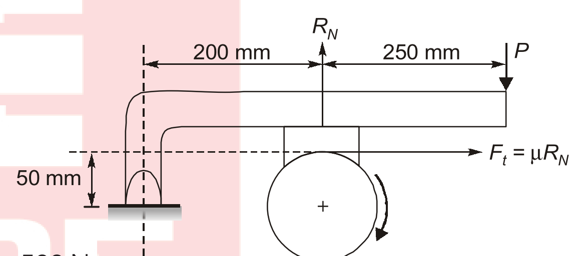
4 Brakes

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$D = 300 \text{ mm} = 0.3 \text{ m}$$
$$T = 75 \text{ N-m}$$
$$T = F_t \times r = \mu R_N \times r$$
$$\Rightarrow \quad 75 = 0.35 \times R_N \times 0.15$$
$$\Rightarrow \quad R_N = 1428.57 \text{ N}$$
$$F_T = \mu R_N = 0.35 \times 1428.57 = 500 \text{ N}$$
$$P \times (250 + 200) + F_T \times 50 = R_N \times 200$$
$$\Rightarrow \quad P \times 450 + 500 \times 50 = 1428.57 \times 200$$
$$\Rightarrow \quad P = 579.365 \text{ N}$$

T2 : Solution

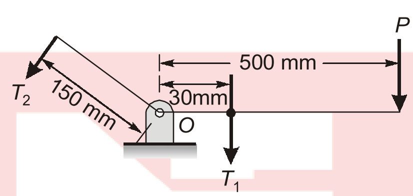
Tension ratio,
$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= e^{\frac{\pi \times 270}{180} \times 0.2} = 2.566 \qquad \text{...(i)}$$

Torque equation
$$T = (T_1 - T_2)$$
$$\Rightarrow \quad \frac{P \times 60}{2\pi N} = T_1 - T_2$$
$$\Rightarrow \quad \frac{30 \times 10^3 \times 60}{2\pi \times 400} = (T_1 - T_2) \times r \qquad \Rightarrow \quad r = 1 \text{ m}$$
$$\Rightarrow \quad (T_1 - T_2) = 716.197 \text{ N} \qquad \text{...(ii)}$$

From Eq (i) and (ii) we get
$$T_1 = 1173.54 \text{ N}$$
$$T_2 = 457.34$$
$$\text{So, maximum tension} = T_1 = 1173.54 \text{ N}$$

T3 : Solution

$$\text{Drum diameter} = 850 \text{ mm}$$
$$\text{Thickness of block} = 75 \text{ mm}$$
$$\mu = 0.4$$
$$\frac{T_1}{T_2} = \left(\frac{1+\mu\tan 7.5^\circ}{1-\mu\tan 7.5^\circ}\right)^{12}$$
$$\frac{T_1}{T_2} = \left(\frac{1+0.4\tan 7.5^\circ}{1-0.4\tan 7.5^\circ}\right)^{12}$$
$$\frac{T_1}{T_2} = 3.5432$$



By taking moment about 'O'

$$P \times 500 + T_1 \times 30 = T_2 \times 150$$
$$P \times 500 + 3.5432 \times 30 T_2 = T_2 \times 150$$
$$P \times 500 = 43.704\, T_2$$

Power absorbed by blocks

$$P = (T_1 - T_2)\omega \times \frac{d}{2}$$
$$d = 850 + 2 \times 75 = 1000$$
$$225 \times 10^3 \times 10^3 = (T_2 \times 3.5432 - T_2) \times \frac{2\pi \times 240}{60} \times \frac{1000}{2}$$
$$T_2 = 7040.27 \text{ N}$$
$$P \times 500 = 43.704 \times 7040.27$$
$$P = 615.35 \text{ N}$$

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5 Friction Clutches

Detailed Explanation of Try Yourself Questions

T1 : Solution

Pressure variation for uniform wear is given by

$$P = \frac{W}{2\pi(r_0 - r_i)r}$$

Maximum pressure occurs at smallest radius,

$$P_{max} = \frac{W}{2\pi(r_0 - r_i)r_i} = \frac{8000}{2\pi(0.2 - 0.1)0.1} = 127323.9\,\text{N/m}^2$$

$$= 127.3\,\text{kN/m}^2$$

Minimum pressure occurs at largest radius,

$$P_{min} = \frac{W}{2\pi(r_0 - r_i)r_0} = \frac{8000}{2\pi(0.2 - 0.1)0.2} = 63.66\,\text{kN/m}^2$$

T2 : Solution

Design torque, $T = 2 \times 50 = 100$ N-m

For uniform wear conditions, pressure variation is given by

$$P = \frac{C}{r} = \frac{W}{2\pi(r_0 - r_i)r}$$

Maximum pressure occurs at inner radius i.e. at $r = r_i$

So, $$P_{max} = \frac{W}{2\pi(r_0 - r_i)r_i}$$

$$\Rightarrow \quad 1 \times 10^6 = \frac{W}{2\pi\left(\frac{0.1}{2} - \frac{0.065}{2}\right) \times \frac{0.065}{2}}$$

$$\Rightarrow \quad W = 3573.56\,\text{N}$$

$$T = \mu W R_m \cdot n$$

$$100 = 0.08 \times 3573.56 \times \left(\frac{\frac{0.1}{2} + \frac{0.065}{2}}{2}\right) \times n$$

$\Rightarrow$ $n = 8.479 \simeq 9$ pair of contact surface

Generally we take friction disk in such a way so that total number of contact pair are even, so answer should be 10.

T3 : Solution

$$P = T \cdot \omega$$

$$873.8 = T \times \frac{2\pi \times 900}{60}$$

$$T = 9.272$$

Also, $T = I\alpha$

$$T = (mk^2)\,\alpha$$

$$9.272 = 14 \times (160)^2 \times \alpha$$

$$\alpha = 25.87 \text{ rad/se c}^2$$

Also, $\omega_F = \omega_i + \alpha t$

$$t = \frac{2\pi \times 900}{60 \times 25.87} = 3.64 \text{ sec}$$

T4 : Solution

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 4000} = 47.746 = \mu W R_m$$

$\Rightarrow$ $$R_m = \frac{T}{\mu W}$$

$\Rightarrow$ $$\frac{2}{3}\left(\frac{r_0^3 - r_i^3}{r_0^2 - r_i^2}\right) = \frac{T}{\mu \times P \times \pi\left(r_0^2 - r_i^2\right)}$$

$\Rightarrow$ $$\left(r_0^3 - r_i^3\right) = \frac{3T}{2\mu P\pi} \Rightarrow r_o^3 = r_i^3 + \frac{3T}{2\mu P\pi}$$

$$= \left(50 \times 10^{-3}\right)^3 - \frac{3 \times 47.746}{2 \times 0.3 \times 1.2 \times 10^6 \times \pi}$$

$$r_i = 0.0395 \text{ m} = 39.5 \text{ mm}$$

T5 : Solution

$$r_i = r_o - b \sin\alpha = 350 - 130 \sin 12.5 = 321.863 \text{ mm}$$

$$r_m = \frac{350 + 321.863}{2} = 335.93 \text{mm}$$

Torque, $$T = \frac{\mu W}{\sin\alpha} \times r_m$$

$$W = \frac{T \sin\alpha}{\mu r_m} = \frac{400 \times 1000 \times 0.21644}{0.4 \times 335.93}$$

$$= 644.298 \text{ N} \simeq 644.3 \text{ N}$$

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6 Gears

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$\frac{N_p}{N_g} = 2 = \frac{T_g}{T_p}$$

Wear strength, $S_w \text{ (or) } P_w = kQwd_p$

$k = 1.5 \text{ N/mm}^2$

$$Q = \frac{2T_g}{T_p + T_g} = \frac{2}{1+\frac{T_p}{T_g}} = \frac{2}{1+\frac{1}{2}} = \frac{4}{3}$$

$w = 100 \text{ mm}$

$d_p = 400 \text{ mm}$

Now, $P_w = kQwd_p$

$$= 1.5 \times \frac{4}{3} \times 100 \times 400 = 80000 \text{ N} = 80 \text{ kN}$$

T2 : Solution

$\phi = 20°$

Power $= 20 \text{ kW} = 20 \times 10^3 \text{ W}$

$N_P = 300 \text{ rpm}$

$$\text{Velocity ratio} = \frac{T_g}{T_P} = 3$$

$\sigma_{OG} = 100 \text{ MPa} = 100 \times 10^6 \text{ N/m}^2$

$= 100 \text{ N/mm}^2$

$$\sigma_{OP} = 120 \text{ MPa} = 120 \times 10^6 \text{ N/m}^2$$
$$T_P = 15$$
$$T_G = 3\,T_P = 3 \times 15 = 45$$
$$b = 14 \text{ m}$$
$$\sigma_{es} = 600 \text{ MPa} = 600 \text{ N/mm}^2$$
$$E_P = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$
$$E_s = 100 \text{ GPa} = 100 \times 10^3 \text{ N/mm}^2$$

Module:

let m = Module in mm and

D_P = Pitch circle diameter of the pinion in mm

Pitch line velocity,
$$V = \frac{\pi D_P N_P}{60} = \frac{\pi m T_P N_P}{60} = \frac{\pi \times m \times 15 \times 300}{60}$$
$$= 0.235\, m \text{ m/s}$$

Assuming steady load condition and 8-10 hours of service per day, the service factor C_s is taken as 1, i.e. $C_s = 1$.

We know that the design tangential tooth load,
$$W_T = \frac{P}{V} \times C_s = \frac{20 \times 10^3}{0.235m} \times 1 = \frac{85.10 \times 10^3}{m} \text{ N}$$

and Velocity factor,
$$C_v = \frac{3}{3+v} = \frac{3}{3+0.235m}$$

Tooth from factor for pinion
$$y_p = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{15} = 0.0932$$

And tooth from factor for gear
$$y_g = 0.154 - \frac{0.912}{T_g} = 0.154 - \frac{0.912}{45} = 0.133$$

$\therefore$
$$\sigma_{PO} \times y_p = 120 \times 0.0932 = 11.184$$
$$\sigma_{OG} \times y_G = 100 \times 0.133 = 13.3$$

Since $(\sigma_{OP} \times y_P)$ is less than $(\sigma_{OG} \times y_G)$ therefore the pinion is weaker. Now using the Lewis equation to the pinion, we have
$$W_T = \sigma_{wp} \times b\pi m y_p = \sigma_{OP} \times C_v\, b\pi m y_p \qquad [\because \sigma_{wp} = \sigma_{OP} \times C_v]$$
$$\frac{85.1 \times 10^3}{m} = 120 \times \left(\frac{3}{3+0.235m}\right) \times (14m \times \pi m \times 0.0932)$$
$$\frac{85.1 \times 10^3}{m} = \frac{1475.69}{(3+0.235m)}$$
$$\Rightarrow \quad \frac{57.66}{m} = \frac{m^2}{3+0.235m} = 172.98 + 13.55\, m = m^3$$
$$\Rightarrow \quad m^3 - 13.55 \text{ m} - 172.98 = 0$$
$$m = 6.37 \text{ mm} \simeq 7 \text{ mm}$$

Face width:
$$b = 14\, m = 14 \times 7 = 98 \text{ mm}$$

Pitch diameter of gears,
$$D_P = mT_P = 7 \times 15 = 105 \text{ mm}$$
$$D_g = mT_g = 7 \times 45 = 315 \text{ mm}$$

Checking the gears for wear:–

Ratio factor, $$Q = \frac{2 \times VR}{VR+1} = \frac{2 \times 3}{3+1} = 1.5$$

$$\text{Load stress factor} = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_g}\right]$$

$$= \frac{600^2 \times \sin 20°}{1.4}\left[\frac{1}{200 \times 10^3} + \frac{1}{100 \times 10^3}\right]$$

$$= 1.3192 \text{ N/mm}^2$$

The maximum or limiting load for wear

$$W_w = D_P \times bQ\,K$$

$$= 105 \times 98 \times 1.5 \times 1.319$$

$$= 20358.765 \text{ N} = 20.358 \text{ kN}$$

Tangential load on tooth, $$W_T = \frac{85.1 \times 10^3}{m} = \frac{85.1 \times 10^3}{7}$$

$$= 12157 \text{ N} = 12.157 \text{ kN}$$

Since maximum wear load (20.358 kN) is more than the tangential load (12.157 kN) on the tooth, the design is satisfactory from the wear point of view.

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