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UPSC ESE 2020

Main Exam Detailed Solutions

Electrical Engineering

PAPER-I

EXAM DATE : 18-10-2020 | 9:00 AM to 12:00 PM

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Electrical Engineering Paper Analysis
ESE 2020 Main Examination

Sl.	Subjects	Total Marks
1.	Electric Circuits	52
2.	Electromagnetic Fields	44
3.	Electrical Materials	62
4.	Engineering Mathematics	72
5.	Basic Electronics Engineering	94
6.	Computer Fundamental	72
7.	Electrical and Electronic Measurements	84
	Total	**480**

Scroll down for detailed solutions

1. (a) (i) Obtain the partial differential equation governing the equations

$$f(u, v) = 0, \quad u = x + yz, \quad v = x + y + z$$

(ii) Construct a partial differential equation of all surfaces of revolution having z-axis as the axis of rotation.

[12 Marks]

Solution:

(i) Given equations $\phi(u, v) = 0$

where $u = x + yz,\ v = x + y + z$

The required partial differential equation of these equation is of the form

$$P\frac{\partial z}{\partial k} + Q\frac{\partial z}{\partial y} = R$$

where P, Q, R are functions of x, y, z.

These are given by

$$P = \begin{vmatrix} \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \end{vmatrix} = \begin{vmatrix} z & y \\ 1 & 1 \end{vmatrix} = z - y$$

$$Q = \begin{vmatrix} \frac{\partial u}{\partial z} & \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \end{vmatrix} = \begin{vmatrix} y & 1 \\ 1 & 1 \end{vmatrix} = y - 1$$

$$R = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & z \\ 1 & 1 \end{vmatrix} = 1 - z$$

$\therefore$ Required P.D.E. is

$$Pp + Qq = R$$

$$\Rightarrow \quad (z-y)\frac{\partial z}{\partial x} + (y-1)\frac{\partial z}{\partial y} = (1 - z)$$

(ii) Any surface rotated about z-axis it form circular planes along z-axis. Then equation of surface can be taken as

$$z = \phi\left(\sqrt{x^2 + y^2}\right) \quad \text{...(1)}$$

ϕ is an arbitrary function.

We need to form the corresponding partial differential equation.

We need to eliminate arbitrary function 'ϕ' by differentiating partially w.r.t. $x.y$.

Partially diff. (1) w.r.t. x

$$\frac{\partial z}{\partial x} = \phi'\left(\sqrt{x^2 + y^2}\right) \times \frac{1}{2\sqrt{x^2 + y^2}} \times 2x \quad \text{...(2)}$$

Diff. (1) w.r.t. y :

$$\frac{\partial z}{\partial y} = \phi'\left(\sqrt{x^2+y^2}\right) \times \frac{1}{2\sqrt{x^2+y^2}} \times 2y \qquad \text{...(3)}$$

Eliminating ϕ' from (2), (3)

$$(2) \div (3) \Rightarrow \quad \frac{\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}} = \frac{\phi'\left(\sqrt{x^2+y^2}\right) \times \frac{x}{\sqrt{x^2+y^2}}}{\phi'\left(\sqrt{x^2+y^2}\right) \times \frac{y}{\sqrt{x^2+y^2}}}$$

$$\frac{p}{q} = \frac{x}{y} \Rightarrow yp - xq = 0$$

$yp - xq = 0$ is P.D.E. which represents all the surfaces of revolutions having z-axis as the axis of rotation.

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- **MADE EASY Classnotes**

End of Solution

1. (b) Discuss the phenomena of Hysteresis in ferromagnetic materials. On the B-H curve show the retentivity, coercivity and saturation points. What is coercive force and the energy dissipated per unit volume of the ferromagnetic substance during the hysteresis cycle?

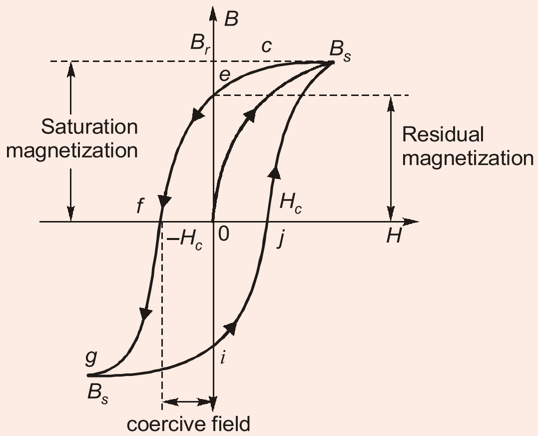
[12 Marks]

Solution:

When a fresh ferromagnetic material is subjected to an external field, dipoles are going to be aligned in the direction of applied H. Size of corresponding domain (D) increases. If the field is further increased, size further increases and for a particular value of $H = H_m$ no further expansion will be possible, that means saturation has been achieved

$$M_{sat} = N\beta$$

If direction of H is reversed, dipoles start dealigning and magnetization start decreasing when H is removed, there is no complete dealignment of the dipoles and have some residual magnetization remains. The magnetic flux density corresponding to this is known as **residual** or **remanent flux density** or **retentivity.**



The amount of magnetic field required to be applied in reverse direction to reduce the residual magnetization to zero is known as coercive field. Thus characteristic never return to origin if once ferromagnetic material is magnetized below ferromagnetic Curie temperature. In this region the susceptibility increases with increase in temperature.

Consider small change in flux (dB), induced emf,

$$e = N\frac{d\phi}{dt} = NA\frac{dB}{dt}$$

$$dE = N\frac{d\phi}{dt} \times i \times dt$$

$$dE = NAi\frac{dB}{dt}dt$$

$$dE = NAi\ dB$$

and
$$H = N\frac{i}{l}$$

$$i = \frac{Hl}{N}$$

∴
$$dE = NA\frac{Hl}{N}dB$$

$$\frac{dE}{Al} = H \cdot dB$$

Energy loss per unit volume per cycle

$$E = \oint H \cdot dB$$

The integral gives the area enclosed by the curve.

Thus, power loss per unit volume in a hysteresis cycle equals area under the hysteresis loop.

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- **Theory Book:** Material science **(Click Here for Reference)**

End of Solution

1. (c) Explain and derive continuity of current equation using the principle of conservation of charge.

[12 Marks]

Solution:

From the principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume.

$$\text{Mathematically, } I_{out} = \frac{-dQ_{in}}{dt}$$

$$I = \frac{-dQ}{dt} \quad ...(i)$$

Where Q is the total charge enclosed by the closed surface

I is current going out of the volume, which is enclosed by the closed surface,

$$Q = \iiint \rho_v dv \qquad \text{...(ii)}$$

$$I = \oiint \vec{J} \cdot d\vec{s} \qquad \text{...(iii)}$$

Put (ii), (iii) in (i),

$$\oiint \vec{J} \cdot d\vec{s} = \frac{-d}{dt}\left(\iiint \rho_v dv\right) \qquad \text{...(iv)}$$

From the divergence theorem,

$$\oiint \vec{J} \cdot d\vec{s} = \iiint (\vec{\nabla} \cdot \vec{J}) dv \qquad \text{...(v)}$$

Put (v) in equation (iv),

$$\iiint (\vec{\nabla} \cdot \vec{J}) dv = \iiint \left(-\frac{\partial \rho_v}{\partial t}\right) dv \qquad \text{...(vi)}$$

Compare both sides

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \qquad \text{...(vii)}$$

equation (vii) is called the continuity of current equation.

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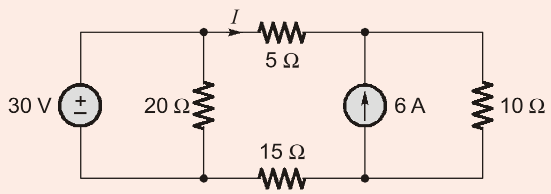
- **MADE EASY Conventional Classnotes**

End of Solution

1. (d) Using the principle of superposition, determine the current I in the 5 Ω resistor in the circuit shown in the figure.



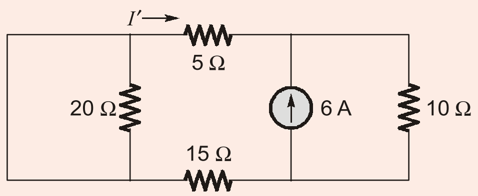
[12 Marks]

Solution:

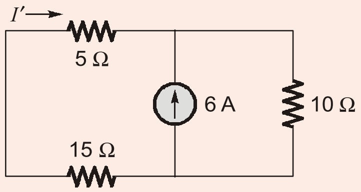
We let, $I = I' + I''$

Where I' and I'' are due to 6 A current source and 30 V voltage source respectively.

To obtain I', we turn off the 30 V voltage source so that we have the circuit in figure,



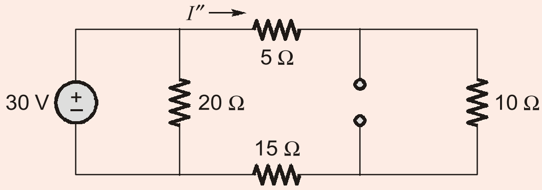
Circuit can be redrawn as



Applying current division rule,

$$I' = -6 \times \frac{10}{20+10} = -2 \text{ A}$$

To obtain I'', we turn off the 6 A current source so that we have the circuit in figure



Write KVL equation,

$$30 - 5I'' - 10I'' - 15I'' = 0$$

$$I'' = \frac{30}{30} = 1 \text{ A}$$

So, $I = I + I'' = 1 - 2 = -1$ A

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- **MADE EASY Classnotes (Click Here for Reference)**
- **ESE 2020 Mains Test Series:** Q.8(b) (ii) of Test-3
- **Mains Work Book:** (Q.21, Page 26)

End of Solution

1. (e) State Ampere's circuital law. A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find $\vec{H}$ everywhere.

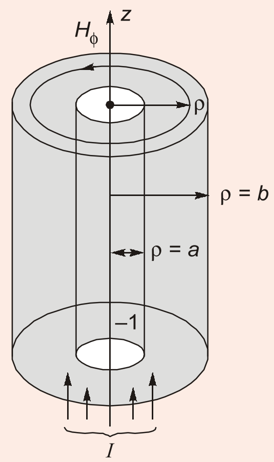
[12 Marks]

Solution:

Ampere's circuital law states that the closed line integral of magnetic field intensity is equal to the total current enclosed by that closed surface.

Mathematically $\oint \vec{H} \cdot d\vec{l} = I_{enclosed}$ ...(i)

Derivation of $\vec{H}$ for hollow conducting cylinder :



Case-1: $\vec{H}$ in the region $0 < \rho < a$

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$H_\phi (2\pi\rho) = 0$$
$$H_\phi = 0 \qquad \text{...(ii)}$$

Case-2: $\vec{H}$ in the region $a < \rho < b$

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$H_\phi (2\pi\rho) = I\frac{(\rho^2 - a^2)}{(b^2 - a^2)}$$
$$H_\phi = \frac{I}{2\pi\rho}\left[\frac{\rho^2 - a^2}{b^2 - a^2}\right] \qquad \text{...(iii)}$$

Case-3: $\vec{H}$ in the region $b < \rho < \infty$

$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed}$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$H_\phi (2\pi\rho) = I$$
$$H_\phi = \frac{I}{2\pi\rho} \qquad \text{...(iv)}$$

$$\vec{H} = \begin{cases} 0 \text{ for } 0 < \rho < a \\ \dfrac{I}{2\pi P}\left[\dfrac{\rho^2 - a^2}{b^2 - a^2}\right]\hat{a}_\phi \text{ for } a < \rho < \beta \\ \dfrac{I}{2\pi\rho}\hat{a}_\phi \text{ for } b < \rho < \infty \end{cases}$$

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- **Mains Workbook:** (Q.20)
- **Mains Classroom Video Lecture**

End of Solution

2. (a) (i) If a square matrix A of order n with entries in field F has n distinct eigenvalues, then prove that matrix A is similar to a diagonal matrix.

[10 Marks]

Solution:

Let A be matrix order $n \times n$ and let A has n distinct eigen values, i.e., all its eigen values are distinct.

$\Rightarrow$ All the eigen vectors of $A_{n \times n}$ are linearly independent.

($\because$ No. of distinct eigen values $\Rightarrow$ No. of linearly independent eigen vectors)

Then the modal matrix P = $[X_1 \quad X_2 \, ... \, X_n]$

(where X_1, X_2, ... X_n are eigenvectors of A) is a non-singular matrix.

$\Rightarrow \qquad |P| \neq 0$

$\Rightarrow$ P^{-1} exists.

$\therefore$ By diagonalization :

$$D = P^{-1}AP \text{ is possible.}$$

Hence, A and diagonal matrix (D) are similar matrices whose determinants and eigen values are same.

Hence Proved.

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- **Mains Classroom Video Lecture :** Introduction of diagonalization (lecture-5)
- **Theory Book:** Engg. Maths (Page 37)
- **Mains Work Book:** (Q.9, Page 92) **(Click Here for Reference)**

End of Solution

2. (a) (ii) Find the matrix P which diagonalizes the matrix associated with the quadratic form

$$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$$

[10 Marks]

Solution:

Given quadrant form $3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy$.

The symmetric matrix can be written as

$$A = \begin{array}{c} \\ x \\ y \\ z \end{array}\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \end{array}$$

For Eigen Values (λ) :

By solving characteristic equation $|A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$= [3-\lambda][(5-\lambda)(3-\lambda) - 1] + 1[(\lambda - 3) + 1] + 1[(1-(5-\lambda)] = 0$$

$$= (3-\lambda)[15 - 8\lambda + \lambda^2 - 1] + \lambda - 2 + \lambda - 4 = 0$$

$$= -\lambda^3 + 11\lambda^2 - 36\lambda + 36 = 0$$

Solving we get $\lambda = 2, 3, 6$

Find eigen vectors X_1, X_2, X_3

1.Sub $\lambda_1 = 2$ in $(A - \lambda_1 I)X_1 = 0$

We get equations,

$$x - y + z = 0 \quad \text{...(1)}$$

$$-x + 3y - z = 0 \quad \text{...(2)}$$

$$x - y + z = 0 \quad \text{...(3)}$$

Solving (1) and (2)

x	y	z	
–1	1	1	–1
3	–1	–1	3

$$\Rightarrow \quad \frac{x}{1-3} = \frac{y}{-1+7} = \frac{z}{3-1}$$

$$\Rightarrow \quad \frac{x}{-2} = \frac{y}{0} = \frac{z}{2} = K \text{ (say)}$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2. Sub $\lambda = 3$ in $(A - \lambda_2 I)X_2 = 0$, we get

$$0x - y + z = 0 \quad \text{...(4)}$$

$$-x + 2y - z = 0 \quad \text{...(5)}$$

$$x - y + 3z = 0 \quad \text{...(6)}$$

Solving (4), (5)

x	y	z	
–1	1	0	–1
2	–1	–1	2

$$\Rightarrow \quad \frac{x}{-1} = \frac{y}{-1} = \frac{z}{-1} = K \text{ (say)}$$

$$\Rightarrow \quad X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Sub $\lambda = 6$ in $(A - \lambda_3 I)X_3 = 0$

We get equations

$$-3x - y + z = 0 \quad \text{...(7)}$$

$$-x - y - z = 0 \quad \text{...(8)}$$

$$x - y - 3z = 0 \quad \text{...(9)}$$

Solving (7), (8)

$$\begin{matrix} x & y & z & \\ -1 & 1 & -3 & -1 \\ -1 & -1 & -1 & -1 \end{matrix}$$

$$\frac{x}{2} = \frac{y}{-4} = \frac{z}{2} = K \text{ (say)}$$

$$X_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Hence, the required modal matrix for the given quadratic form is

$$P = \begin{bmatrix} \frac{X_1}{\|X_1\|} & \frac{X_2}{\|X_2\|} & \frac{X_3}{\|X_3\|} \end{bmatrix}_{3\times 3} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

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- **Mains Work Book:** (Q.9)

End of Solution

2. (b) (i) On the basis of specific resistance ρ, discuss the difference between conductors, semi-conductors and insulators.

[8 Marks]

Solution:

Electric specific resistance also known as electrical resistivity is fundamental property of material that quantifies how strongly it resists or conducts electric current.

It is commonly represented by $\rho(\Omega - m)$

$$\rho = \frac{RA}{l}$$

where, R = Electrical resistance of uniform specimen

l = Length of specimen

A = Area of cross-section

Metals: A metal consists of a lattice of atoms, each with an outer shall of electrons that freely dissociate from their parent atoms and travel through the lattice and hence value of specific resistance of metal is comparatively lowest.

Semiconductors: The resistivity of semiconductor is generally intermediate but varies widely under different condition such as exposure of material to electric field and specific frequencies of light, degree of semiconductor doping etc.

Insulators: The insulators have generally very high value of resistivity due to non availability of free electron like in conductors.

The below table mentions the order of values of specific resistance for different electrical materials.

Material	Resistivity (specific resistance)
Super conductors	0
Metals	10^{-8}
Semiconductor	Variable
Electrolytes	Variable
Insulators	10^{16}
Super insulators	∞

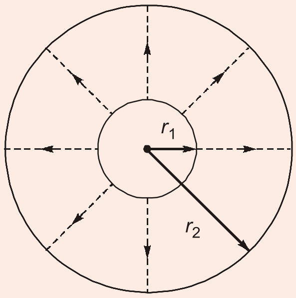
End of Solution

2. (b) (ii) Prove that insulation resistance of a cable is inversely proportional to its length. Define insulation resistance.

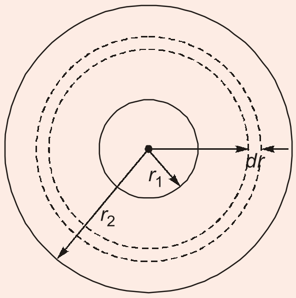
[12 Marks]

Solution:

The cable conductor is provided with an insulation of suitable thickness so as to avoid leakage of current. The path for leakage current is radial, as shown in figure (a), through the insulating material. The opposition offered by the insulation to the leakage current is called the insulation resistance of the cable.



(a)



(b)

Consider a single core cable of conductor radius r_1, internal sheath radial r_2, length l and insulation material resistivity ρ.

Let an elementary cylindrical section of the insulation of radius r and thickness dr be considered. Now the length through which the leaking current will flow is dr and area of cross-section is $2\pi rl$.

The insulation resistance offered to the leakage current by elementary cylindrical section of the insulation under consideration is $\frac{\rho dr}{2\pi rl}$.

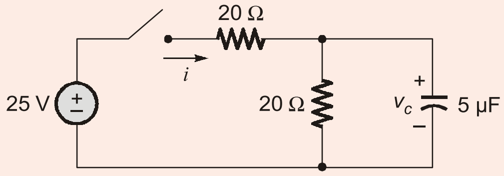
Insulation resistance of the cable,

$$R_{INS} = \int_{r_1}^{r_2} \frac{\rho dr}{2\pi rl} = \frac{\rho}{2\pi l}[\log_e r]_{r_1}^{r_2} = \frac{\rho}{2\pi l}\log_e \frac{r_2}{r_1}$$

i.e., the insulation resistance of the cable varies inversely as the length of the cable $\left(R_{INS} \alpha \frac{1}{l}\right)$.

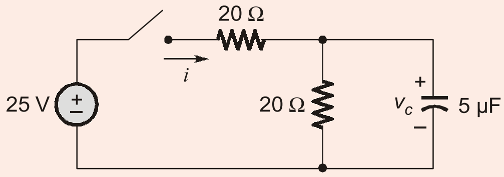
End of Solution

2. (c) In the circuit shown in the figure given below, the switch is closed at $t = 0$. Obtain the current i and capacitor voltage v_c, for $t > 0$.



[20 Marks]

Solution:



At $t = 0^-$

$$V_C = 0 \text{ V} \qquad \text{(As no sources is connected with capacitor)}$$

$$\therefore \qquad v_c(0^+) = v_c(0^-) = 0 \text{ V}$$

For $t > 0$

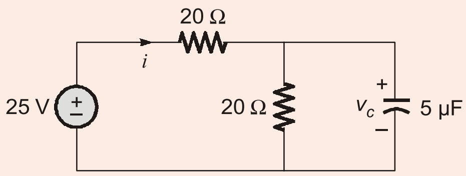
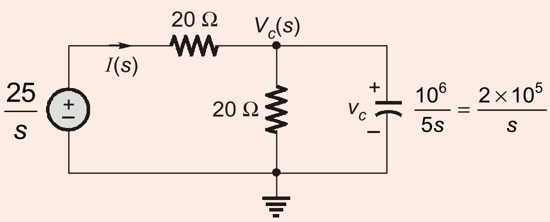


figure (i)

Transforming the circuit from time domain to the Laplace domain,



Apply KCL,

$$\frac{V_c(s) - \frac{25}{s}}{20} + \frac{V_c(s)}{20} + \frac{V_c(s)\cdot s}{20\times 10^4} = 0$$

$$V_c(s)\left[10^4 + 10^4 + s\right] - \frac{25}{s}\times 10^4 = 0$$

$$V_c(s) = \frac{\frac{25\times 10^4}{s}}{2\times 10^4 + s} = \frac{25\times 10^4}{s(s+2\times 10^4)}$$

$$V_c(s) = \frac{a_0}{s} + \frac{a_1}{s+2\times 10^4}$$

$$V_c(s) = \frac{12.5}{s} + \frac{-12.5}{s+2\times 10^4}$$

Taking inverse laplace transform,

$$v_c(t) = 12.5u(t) - 12.5e^{-2\times 10^4 t}u(t)$$

$$v_c(t) = 12.5\left[1 - e^{-2\times 10^4 t}\right]u(t)\ \text{V}$$

From figure (i)

$$i = \frac{25 - v_c(t)}{20}$$

$$= \frac{25 - 12.5 + 12.5e^{-2\times 10^4 t}}{20}$$

$$\text{i}(t) = \frac{12.5}{20}\left[1 + e^{-2\times 10^4 t}\right]$$

$$i(t) = \frac{5}{8}\left[1 + e^{-2\times 10^4 t}\right]\text{A} \qquad t > 0$$

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- **Work Book:** (Q.29, Page 21) **(Click Here for Reference)**
- **ESE 2020 Mains Test Series:** Q.4(c) (iii) of Test-10
- **MADE EASY Classnotes**

End of Solution

3. (a) (i) If the density function of a continuous random variable is given by

$$f(x) = \begin{cases} 0, & x < 0 \\ ax, & 0 \le x \le 2 \\ (4-x)a, & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

(p) Find value of *a*.
(q) Find the cumulative distribution function (cdf).
(r) Find P(X > 2.5).

[10 Marks]

Solution:

Given p.d.f. of a continuous random variable

$$f(x) = \begin{cases} 0, & x < 0 \\ ax, & 0 < x \le 2 \\ (4-x)a, & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$$

(p) Find value of *a*?

p.d.f. is a valid only if $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\therefore \quad \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} 0.dx + \int_{0}^{2} ax dx + \int_{2}^{4} a(4-x)dx + \int_{4}^{\infty} 0dx$$

$$= \left(0 + a\frac{x^2}{2}\right)_0^2 + a\left(4x - \frac{x^2}{2}\right)_0^4 + 0 = 1$$

$$\Rightarrow \quad 2a + 8a - 6a = 1$$

$$\Rightarrow \quad 4a = 1$$

$$\Rightarrow \quad a = \frac{1}{4}$$

$$\therefore \text{ Then} \quad f(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \le x \le 2 \\ 1 - \frac{x}{4} & 2 \le x \le 4 \\ 0, & x > 4 \end{cases}$$

(q) W.K.T. C.D.F. $F(x_K) = \int_{-\infty}^{x_K} f(x)dx$

C.D.F. is obtained by integrating $f(x)$ within the interval.

$$\therefore \quad F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{8}, & 0 \le x \le 2 \\ x - \frac{x^2}{8}, & 2 \le x \le 4 \\ 0, & x > 4 \end{cases} \quad \text{is C.D.F.}$$

(r) $$P(X > 2.5) = \int_{2.5}^{\infty} f(X)dx$$

$$= \int_{2.5}^{4}\left(1 - \frac{x}{4}\right)dx + \int_{4}^{\infty} 0.dx = \left. x - \frac{x^2}{8} \right]_{2.5}^{4}$$

$$= \left(4 - \frac{16}{8}\right) - \left(2.5 - \frac{(2.5)^2}{8}\right)$$

$$= 0.281$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.3(a) (i) of Test-8
- **Mains Work Book:** (Q.50, page 95-96) **(Click Here for Reference)**

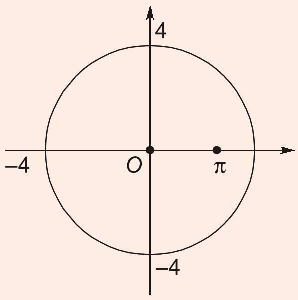
End of Solution

3. (a) (ii) Compute $\oint_C \frac{\cos z}{z^2(z-\pi)^3} dz$, , where $c : |z| = 4$.

[10 Marks]

Solution:

$$\oint_C \frac{\cos z}{z^2(z-\pi)^3} dz, \quad C : |z| = 4$$



Singularities of $f(z)$ are given by

$$z^2(z-\pi)^3 = 0$$

$\Rightarrow z = 0, \pi$ both lies inside $C : |z| = 4$

Here, $z = 0$ is pole of order $m = 2$.

$z = \pi$ is pole of order $m = 3$.

By C.R. T. :

$$\oint_C f(z)dz = 2\pi i[\text{Res}_0 + \text{Res}_\pi]$$

Residue at $z = 0$:

$$\text{Res}_0 = \frac{1}{(2-1)!} \lim_{z\to 0} \frac{d}{dz}\left[(z-0)^2 \frac{\cos z}{z^2(z-\pi)^3}\right]$$

$$= \lim_{z\to 0} \frac{d}{dz}\left[\frac{\cos z}{(z-\pi)^3}\right]$$

$$= \lim_{z\to 0} \frac{(z-\pi)^3(-\sin z) - \cos z(3(z-\pi)^2}{(z-\pi)^6}$$

$$\text{Res}_0 = \frac{-3\pi^2}{\pi^6} = \frac{-3}{\pi^4}$$

Residue at $z = \pi$:

$$\text{Res}_\pi = \frac{1}{(3-1)!} \lim_{z\to \pi} \frac{d^2}{dz^2}\left[(z-\pi)^3 \frac{\cos z}{z^2(z-\pi)^3}\right]$$

$$= \frac{1}{2}\lim_{z\to\pi}\frac{d}{dz}\left[\frac{z^2(-\sin z)-2z\cos z}{z^4}\right]$$

$$\text{Res}_\pi = \frac{1}{2}\left[\frac{\pi^2-6}{\pi^4}\right]$$

∴ by C.R.T.

$$\oint_C f(z)dz = 2\pi i[\text{Res}_0 + \text{Res}_\pi]$$

$$= 2\pi i\left[\frac{-3}{\pi^4}+\frac{1}{2}\cdot\frac{\pi^2-6}{\pi^4}\right]$$

$$= 2\pi i\left[\frac{-6}{\pi^4}+\frac{1}{2\pi^2}\right]$$

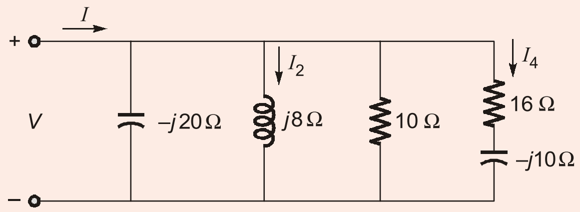
$$= 2i\left[\frac{-6}{\pi^3}+\frac{1}{2\pi}\right]$$

MADE EASY Source

- **Mains Classroom Video Lecture :** First problem on residue theorem (lecture-13)
- **Mains Work Book:** (Q.58, Page 141) **(Click Here for Reference)**

End of Solution

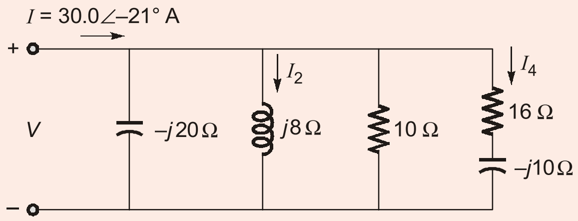
3. (b) For the circuit shown in the figure given below, the total current I entering the circuit is 30.0 ∠–21° A. Determine the voltage V and the branch current I_2 and I_4.



[20 Marks]

Solution:

Given that,



$$\frac{1}{Z_{eq}} = \frac{1}{-j20}+\frac{1}{j8}+\frac{1}{10}+\frac{1}{16-j10}$$

$$Z_{eq} = 6.245 + j2.02\ \Omega$$
$$V = Z_{eq}\, I$$
$$= 196.9\angle{-3.06}\text{ V}$$
$$I_2 = \frac{V}{j8} = 24.6\angle -93.066\text{ A}$$
$$I_4 = \frac{V}{16 - j10} = 10.43\angle 28.945\text{ A}$$

MADE EASY Source

- **Work Book:** (Q.20, Page 15) **(Click Here for Reference)**
- **ESE 2020 Mains Test Series:** Q.1(b) of Test-2
- **Mains Work Book:** (Q.7, Page 4)

End of Solution

3. (c) (i) Discuss the factors affecting electrical resistance of conductors.

[10 Marks]

Solution:

Temperature

Average collision time of electron in conducting material reduces with increase in temperature. Since the conductivity of the material is directly proportional to average collision time. Therefore conductivity reduces with increasing temperature. Since resistivity is the reciprocal of the conductivity, so resistivity of conducting material increases with increase in temperature. The resistivity of conducting material as a function of temperature is given below:

$$\rho_{T_2} = \rho_{T_1}(1 + \alpha \Delta T)$$

Where, $\Delta T = T_2 - T_1$ = difference of temperature

α = temperature coefficient

Since the electrical conductivity is proportional to I, and resistivity (ρ) to I^{-1}.

Hence, $\rho_{Total} = \rho_T + \rho_i + \rho_d = \rho_T + \rho_r$

Equation is statement of **"Matthiessen's Rule"** and it indicates the various contribution to the resistivity of metals are independently additive.

Also, $\rho_T \rightarrow$ ideal resistivity varies with temperature.

$\rho_r \rightarrow$ residual resistivity (depends on impurity).

NOTE: The temperature coefficient of conducting material is positive.

Alloying:

The resistivity of alloy has two components called (a) thermal component and (b) residual component. **Thermal component** of conducting material is due to lattice vibration caused by thermal energy. This component reduces with decrease in temperature and becomes zero at absolute zero temperature. The **residual component** of resistivity arise due to impurity of material and lattice imperfection. This component is independent of temperature and will be existing even at absolute zero temperature. Therefore, alloying can induce the resistivity. Alloys generally have a less regular structure than pure metals. Consequently, the electrical conductivity of a solid solution alloy, drops off rapidly with the increase alloy content.

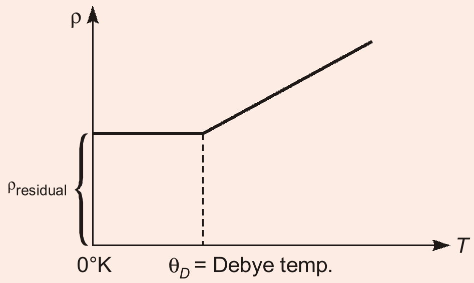
Resistivity of alloy (ρ_{alloy}) = $\rho_{puremetal} + s\rho_i$

where, s = atomic percentage of added impurity

ρ_i = increase in resistivity per atomic%

Unit of "ρ_{alloy}" is μ-ohm-cm.

⇒ When a copper is added to silver (Ag), the resistivity of alloy is more than resistivity of copper (Cu), and conductivity of alloy is less than conductivity of copper (Cu).



⇒ When a small amount of Ni i.e. 1% is a added to Cu, the resistivity of Cu goes up by 1.3 μ-ohm-cm.

Cold Working:

The mechanical treatments such as cold working produces localized strain in the material which results in the increase in resistivity of material. **A hard drawn copper wire has lower conductivity than annealed copper.**

Age Hardening:

If material gets hardened with age then its resistivity increases and hence conductivity decreases.

MADE EASY Source

- **Theory Book:** Material science **(Click Here for Reference)**

End of Solution

3. (c) (ii) Find the diffusion coefficients of electrons and holes of a single silicon crystal at 27° C, if the mobilities of electrons and holes are 0.17 and 0.025 m^2/volt-sec respectively at 27° C.
(Boltzmann's constant $k = 1.38 \times 10^{-23}$ Joule/degree)

[10 Marks]

Solution:

Given : T = 27°C, i.e., T = 300 K

Electron mobility, μ_n = 0.17 m^2/ V-sec

Hole mobility, μ_P = 0.025 m^2/ V-sec

Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J/degree

From Einstein relation,

$$\frac{D_n}{\mu_n} = V_T = \frac{kT}{q} = \frac{D_P}{\mu_P}$$

For electrons

Diffusion coefficient, $D_n = \mu_n V_T$

$$\Rightarrow \quad D_n = \mu_n \frac{kT}{q}$$

$$= \frac{0.17 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

$$\Rightarrow \quad D_n = 4.39 \times 10^{-3} \text{ m}^2/\text{sec}$$

$$\Rightarrow \quad D_n = 43.9 \text{ cm}^2/\text{sec}$$

For holes,

Diffusion coefficient, $D_p = \mu_P V_T = \mu_P \frac{kT}{q}$

$$\Rightarrow \quad D_p = \frac{0.025 \times 1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \text{ m}^2/\text{sec}$$

$$\Rightarrow \quad D_p = 6.47 \text{ cm}^2/\text{sec}$$

End of Solution

4. (a) (i) State Dirichlet's conditions for existence of Fourier series of a function. Determine the half range Fourier cosine series of

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x < \pi \end{cases}$$

[10 Marks]

Solution:

Dirichlet Conditions

If a signal $x(t)$ satisfies certain conditions, its Fourier series is guaranteed to converge point-wise at all points where $x(t)$ is continuous. These conditions are known as Dirichlet conditions.

(i) The signal x(t) must be absolutely integrable over th range of time-period,

$$\int_0^T |x(t)|\,dt < \infty$$

(ii) It should have finite number of finite size discontinuities in one period.

(iii) It should have finite number of maxima and minima in one period.

Comment: Condition (i) is necessary whereas (ii) and (iii) are sufficient conditions.

$$C = \pi$$

$$a_0 = \frac{2}{C}\int_0^C f(x)\cdot dx = \frac{2}{\pi}\int_0^C f(x)\cdot dx$$

$$= \frac{2}{\pi}\int_0^{\pi} f(x)\cdot dx = \frac{2}{\pi}\left[\int_0^{\pi/2} x\cdot dx + \int_{\pi/2}^{\pi}(\pi - x)dx\right]$$

$$= \frac{2}{\pi}\left[\left(\frac{x^2}{2}\right)_0^{\pi/2} + \left(\pi x - \frac{x^2}{2}\right)_{\pi/2}^{\pi}\right]$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{8} + \left(\pi^2 - \frac{\pi^2}{2}\right) - \left(\frac{\pi^2}{2} - \frac{\pi^2}{8}\right)\right]$$

$$a_0 = \frac{2}{\pi}\left[\frac{\pi^2}{8} + \pi - \frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^2}{8}\right]$$

$$a_0 = \frac{2}{\pi}\left[\frac{2\pi^2}{8}\right] = \frac{\pi}{2}$$

$$a_0 = \frac{\pi}{2}$$

$$a_n = \frac{2}{c}\int_0^C f(x)\cdot \cos\frac{n\pi x}{c}\cdot dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \cos nx\cdot dx$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi/2} x\cdot\cos nx\cdot dx + \int_{\pi/2}^{\pi}(\pi - x)\cdot\cos nx\cdot dx\right]$$

$$\begin{array}{ccc|ccc} + & x & \dfrac{\sin nx}{n} & + & (\pi - x) & \dfrac{\sin nx}{n} \\ - & 1 & \dfrac{-\cos nx}{n^2} & - & -1 & \dfrac{-\cos nx}{n^2} \end{array}$$

$$a_n = \frac{2}{\pi}\left[\left[\left(x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right)\right]_0^{\pi/2} + \left[(\pi - x)\frac{\sin nx}{n} - \frac{\cos nx}{n^2}\right]_{\pi/2}^{\pi}\right]$$

$$a_n = \frac{2}{\pi}\left[\left[\frac{\pi}{2}\cdot\frac{\sin n\frac{\pi}{2}}{n} + \frac{\cos n\frac{\pi}{2}}{n^2} - \frac{1}{n^2}\right] + \left[-\frac{\cos n\pi}{n^2} - \frac{\pi}{2}\frac{\sin\frac{n\pi}{2}}{n} + \frac{\cos\frac{n\pi}{2}}{n^2}\right]\right]$$

$$a_n = \frac{2}{\pi}\left[\frac{1}{n^2}\left(2\cos\frac{n\pi}{2} - \cos n\pi - 1\right)\right]$$

∴ Fourier half range cosine series is

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{c}\right)$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left(2\cos\frac{n\pi}{2} - \cos n\pi - 1\right) \cdot \cos nx$$

MADE EASY Source

- **Mains Classroom Video Lecture:** Last problem in fourier series (Lecture-17)
- **Theory Book:** Engg. Maths (Page 534) **(Click Here for Reference)**

End of Solution

4. (a) (ii) By converting into a line integral, evaluate $\iint_S (\nabla \times \bar{F}) \cdot \hat{n}\, dS$, where $\bar{F} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xy + z^2)\hat{k}$ and S is the surface of paraboloid $x^2 + y^2 + z = 4$ above xy-plane.

[10 Marks]

Solution:

$$\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xy + z^2)\hat{k}$$

$\int_c \vec{F} \cdot d\vec{r}$ where c is the boundary of paraboloid $x^2 + y^2 + z = 4$

For surface of paraboloid over xy-plane

$$\partial\vec{r} = i\ dx + j\ dy$$

$$\int_c \left[(x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xy + z^2)\hat{k}\right]\left[idx + \hat{j}dy\right]$$

$$\int_C (x^2 + y - 4)dx + 3xy\,dy$$

Now put

$$x = 2\cos\theta$$
$$y = 2\sin\theta$$
$$dx = -2\sin\theta\ d\theta$$
$$dy = 2\cos\theta$$

$$\int_0^{2\pi} \left(-4\cos^2\theta + 2\sin\theta - 4\right)(-2\sin\theta)d\theta + 24\sin\theta\cos^2 d\theta$$

$$I = 4\int_0^{2\pi} (4\sin\theta\cos^2\theta - \sin^2\theta + \sin\theta)d\theta$$

$$\int_0^{2\pi} \sin\theta \, d\theta = 0$$

$$\int_0^{2\pi} 4\sin\theta\cos^2\theta = 0 \qquad \because \int_0^{2\pi} \cos^n\theta\sin\theta = 0$$

$$4\int_0^{2\pi} -\sin^2\theta \, d\theta = -16\int_0^{\pi/2} \sin^2\theta \, d\theta = -16 \times \frac{1}{2} \times \frac{\pi}{2} = -4\,\pi$$

MADE EASY Source

- **Mains Classroom Video Lecture:** First question on strokes theorem (Lecture-19)

End of Solution

4. (b) Discuss photoelectricity and photoemissive effect along with laws of photoemissive effect.

[20 Marks]

Solution:

Photo electric effect, phenomenon in which electrically charged particles are released from or within a material when it absorbs electromagnetic radiation. A material that can exhibit the photo electric effect is said to be photo emissive.

The photons must have sufficient energy to over come the work function of the material. If this condition is not met, increased irradiance and longer exposure time will not induce emission of free electrons from the surface.

- The electrons were emitted immediately without any time lag.
- On increasing the intensity of light, there was an increase in the number of photo electrons, but not to their maximum kinetic energy.
- Red light did not cause the emission of electrons, no matter what the intensity is.
- A weak violet light ejected only a few electrons, but their maximum kinetic energies were greater than those for intense light of longer wavelengths.
- The energy of the emitted electrons is proportional to the frequency of illuminating light.
- The emission energy is independent of the total energy of illumination.

Laws of photo electricity:

1. Photo electric effect is an instantaneous process.
2. Photo electric current is directly proportional to the intensity of incident light and is independent of frequency.
3. The stopping potential and hence the maximum velocity of the electrons depends upon the frequency of incident light and is independent of its intensity.
4. The emission of electrons stops below a certain minimum frequency known as threshold frequency.

End of Solution

4. (c) Show that the ratio of the amplitudes of conduction current density and displacement current density is $\frac{\sigma}{\omega \in}$ for the applied field $E = E_m \sin \omega t$. Assume $\mu = \mu_0$.
What is the amplitude ratio if the applied field is $E = E_m e^{-t/\tau}$, where t is real?
[20 Marks]

Solution:

Conduction current density is $J_C = \sigma E$ ...(i)

$J_C \rightarrow$ Conduction current density (A/m^2)

σ is conductivity (1/Ω–m)

E is applied electric field intensity (volt/m)

displacement current density,

$$J_d = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\in E) = \in \frac{\partial E}{\partial t} \quad ...(ii)$$

J_d is displacement flux density (A/m^2)

$D = \in E$ = displacement flux density (c/m^2)

$\in = \in_0 \in_r$ = permittivity (F/m)

Given, $E = E_m \sin\omega t$ ...(iii)

Put (iii) in (i),

$$J_C = \sigma E_m \sin \omega t$$

amplitude of J_C is σE_m

Put (iii) in (ii) then $J_D = \in \frac{d}{dt}(E_m \sin \omega t) = \in E_m \cos(\omega t)\, \omega$

amplitude of J_D is $\omega \in E_M$.

$$\frac{\text{amplitude of } J_C}{\text{amplitude of } J_d} = \frac{\sigma E_m}{\omega \in E_m} = \frac{\sigma}{\omega \in} \quad ...(iv)$$

Given, $E = E_m e^{-t/\tau}$ ...(v)

$$J_C = \sigma E = \sigma E_m e^{-t/\tau} \quad ...(vi)$$

amplitude of J_C is σE_m

$$J_d = \in \frac{\partial E}{\partial t} = \in \frac{\partial}{\partial t}\left(E_m e^{-t/\tau}\right) = \in E_m e^{-t/\tau}\left(-\frac{1}{\tau}\right) \quad ...(vii)$$

$$J_d = \frac{-\in E_m}{\tau} e^{-t/\tau}$$

amplitude of J_d is $\frac{\in E_m}{\tau}$

$$\frac{\text{amplitude } J_c}{\text{amplitude of } J_d} = \frac{\sigma E_m}{\left(\frac{\in E_m}{\tau}\right)} = \frac{\sigma \tau}{\in} \quad ...(viii)$$

where τ is time constant.

MADE EASY Source

- **MADE EASY Classnotes**

End of Solution

5. (a) (i) Using an iterative method, write *C* program segment to generate first $n (n \geq 8)$ Fibonacci numbers.

[6 Marks]

Solution:

In the fibonacci series, except for the first two terms of the sequence every other is the sum of the previous two.

shown below terms in the series w.r.t. (n = 8) is

0 (First) 1 (Second) 1 2 3 5 8 13

Code:

```
#include <stdio.h>
int main ()
{
int n, first = 0, second = 1, next, i ;
printf ("enter the number of terms\n") ;
scanf ("%d", %n);
printf ("first %d terms of Fibonacci series are :\n", n);
for (i = 0; i < n; i++)
{
if (i < = 1)
next = i;
else
{
next = first + second;
first = second;
second = next;
}
printf ("%d\n", next);
}
return 0;
}
```

End of Solution

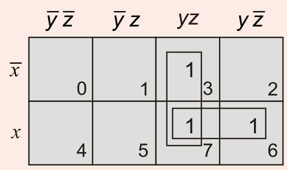
5. (a) (ii) Find minimal disjunctive normal form (DNF) for the expressions $\bar{x}yz + xyz + xy\bar{z}$ and $x + yz + z\bar{x}y + \bar{y}xz$.

[6 Marks]

Solution:

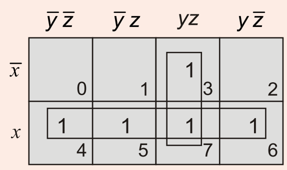
Given, $f = \bar{x}yz + xyz + xy\bar{z}$

Corresponding K-map



$f = xy + yz = (x \cap y) \vee (y \cap z)$

$f = x + yz + z\bar{x}y + y\bar{x}z$



$f = x + yz$

$= x \cup (y \cap z)$

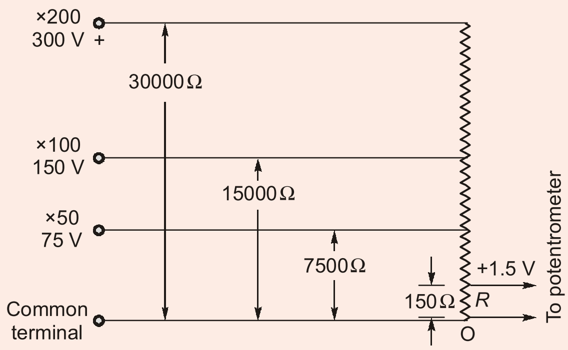
End of Solution

5. (b) Why and how are volt-ratio boxes utilized along with d.c. potentiometers? How should the value of the volt-ratio box resistance be chosen?

[12 Marks]

Solution:

Volt-Ratio Box: General purpose potentiometers cover a low voltage range. (approximately 0 - 1.6 V in case of American Instruments and 0 - 1.8 V in case of British Instruments). If higher voltage ranges have to be measured a precision potential divider, called a volt-ratio box, is used. The volt-ratio box provides multiple ranges as shown in figure.



The voltage to be measured is connected to the appropriate binding post. Sometimes the volt-ratio box is provided with selector switch for voltage ranges. Supposing the voltage to be measured is connected between common terminal and 300 V switch. If the reading of the potentiometer is 0.826, then the value of unknown voltage is 0.826 × (300/1.5) = 165.2 V.

It was stated earlier that the potentiometer consumes no power from the source under balance conditions. However, when a volt-ratio box is used, this statement is no longer true. An examination of figure above indicates that a current must flow through resistance *R*, in order to produce a voltage drop so that a voltage measurement be made by the potentiometer in order that the unknown voltage be determined. Therefore, the resistance of the volt-ratio box represents a load on the unknown voltage source and thus consumes power. The value of volt-ratio box resistance should be as high as possible in order that the power consumption is low. However, low values of resistance are preferred since they have a greater degree of stability. Also low values of resistance give greater galvanometer sensitivity and also minimize the effects of high resistance leakage paths around the terminals.

Thus the choice of volt-ratio box resistance involves a compromise between low and high values. Volt-ratio boxes are usually constructed with residences of 100 Ω/V or 200 Ω/V of the nominal voltage range, allowing a maximum current of 10 mA or 5 mA respectively from the measured source. Figure shows a volt-ratio box with a resistance of 100 Ω/V and a maximum current of 10 mA.

MADE EASY Source

- **Theory Book:** Measurement & Instrumentation (Page No. 265) **(Click Here for Reference)**

End of Solution

5. (c) The self-capacitance or distributed capacitance of a coil is measured using *Q* meter. The first measurement is carried out at 2.5 MHz, when the tuning capacitor is set at 425 pF. The second measurement is carried out by increasing the frequency to 6 MHz, when the tuning capacitor is set at 60 pF. Determine the distributed capacitance of the coil.

[12 Marks]

Solution:

In case of first measurement,

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

$$2.5 \times 10^6 = \frac{1}{2\pi\sqrt{L(425\times 10^{-12} + C_d)}} \quad \text{...(i)}$$

In case of second measurement,

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

$$6 \times 10^6 = \frac{1}{2\pi\sqrt{L(60\times 10^{-12} + C_d)}} \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we get

$$\frac{6}{2.5} = \frac{\sqrt{L(425+C_d)}}{\sqrt{L(60+C_d)}}$$

$$5.76 = \frac{425+C_d}{60+C_d}$$

$$345.6 + 5.76\ C_d = 425 + C_d$$

$$4.76\ C_d = 79.4$$

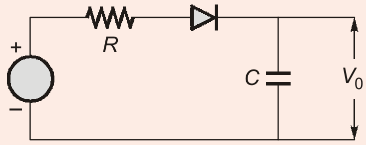
$$C_d = 16.68 \text{ pF}$$

MADE EASY Source

- **MADE EASY Classnotes (Click Here for Reference)**

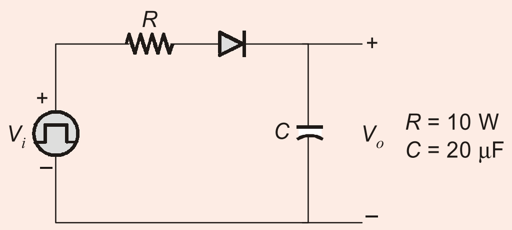
End of Solution

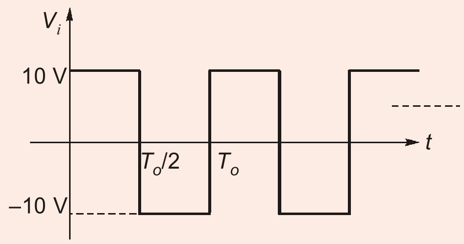
5. (d) Calculate the output voltage V_0 at 7 ms in the figure shown below if *a*, ±10 V square wave of 250 Hz source is applied to R = 10 Ω, C = 20 μF. The diode is ideal and capacitor is initially uncharged.



[12 Marks]

Solution:

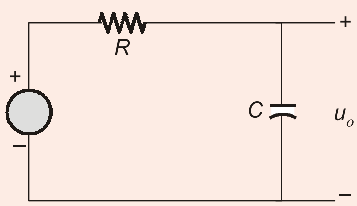




$$T_o = \frac{1}{f_o} = \frac{1}{250} = 4 \text{ m sec}$$

$$\frac{T_o}{2} = 2 \text{ m sec}$$

For the positive half cycle of V_i, diode is forward bias and capacitor will start to change through resistance R.



Charging time constant (τ) = RC

$$\tau = 10 \times 10^{-6} \text{ sec} = 0.01 \text{ m sec}$$

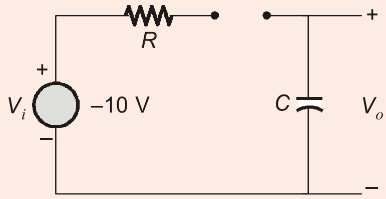
So, $5\tau = 5 \times 0.01 \text{ m sec} = 0.05 \text{ m sec}$

Hence, $5\tau < \frac{T_o}{2}$

So, capacitor will fully charge upto peak value of V_i, during first positive cycle only.

So, $V_o = 10$ V

For next negative half cycle of V_i, diode is reverse biased. So, circuit will be open.

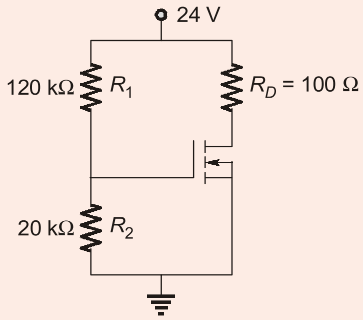


Here in this case, there is no discharge path available for capacitor to discharge. So, capacitor will always retain its voltage at constant 10 V for $t > 2$ m sec.

$$V_0\big|_{t=7\text{ msec}} = 10 \text{ volts}$$

End of Solution

5. (e) Determine V_{GS} and V_{DS} for the E-MOSFET circuit shown in the figure below. The minimum values of $I_{D(ON)}$ = 200 mA at V_{GS} = 4 V and Gate to Source threshold voltage $V_{GS(Th)}$ = 2 V.



[12 Marks]

Solution:

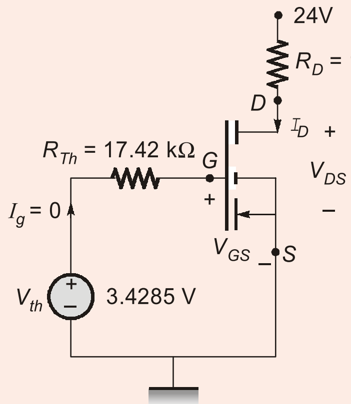
Taking Thevenin equivalent on gate terminal

$$V_{Th} = \frac{R_2}{R_1 + R_2} \times V_{DD}$$

$$V_{Th} = \frac{20}{120 + 20} \times 24 = 3.4285 \text{ volts}$$

$$R_{Th} = R_1 \,//\, R_2 = (120 \,//\, 20) = 17.42 \text{ k}\Omega$$

Equivalent circuit for figure given in problem is



Since $I_g = 0$, so

$$V_G = 3.4285 \text{ volts}$$
$$V_S = 0 \text{ Volts}$$
$$V_{GS} = 3.4285 \text{ Volts}$$

Since given MOSFET is N-channel. So, drain current is given by

$$I_{DS} = \text{Kn}(V_{GS} - V_{Th})^2$$

Assuming MOSFET operating saturation in region.

Given that $V_{GS} = 4$ V, $\quad V_{GS(Th)} = 2$ and $I_{D(ON)} = 200$ mA

$$0.2 = K_n(4-2)^2$$

$$K_n = \frac{0.2}{4} = 0.05 \text{ A/V}^2$$

Now, $\quad V_{GS} = 3.4285$ V and $V_{GS(Th)} = 2$

$$I_{DS} = K_n(V_{GS} - V_{Th})^2$$
$$I_{DS} = 0.05(3.4285 - 2)^2$$
$$I_{DS} = 0.10203 \text{ A}$$

and $\quad V_{DS} \geq V_{GS} - V_{th}$ [TRUE],

so, MOSFET is operating in saturation region.

$$I_{DS} = V_{DD} - I_{DS} \cdot R_D$$
$$V_{DS} = 24 - 100 \times 0.10203 = 13.80 \text{ Volts}$$

For saturation region

$$V_{DS} \geq V_{GS} - V_{Th}$$
$$13.80 \geq 1.4285 \text{ (True)}$$

So, $\quad V_{GS} = 3.4285$ Volts

$$V_{DS} = 13.80 \text{ Volts}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.6(c) of Test-6
- **Mains Work Book:** (Q.27, Page 50) **(Click Here for Reference)**
- **MADE EASY Classnotes**

End of Solution

6. (a) (i) Explain in brief the following and differentiate between
I. Stack and Queue
II. Sort and Search

[12 Marks]

Solution:

I.

Stack	Queue
1. It is an ordered list where the new item is added and existed element is deleted from only one end called as the top of the stack (TOS). All the insertions and deletions in a stack is done from the TOS. So, the last element added will be the first to be removed from the stack. ∴ Stack is called as last in first out (LIFO) type of list.	1. It is a collection of similar type of elements. It contains. It contain two ends named as real and front. The addition of new elements takes place at one end called rear end. Deletion of he existing elements takes place at the other end called front end. ∴ Queue is called as first in first out (FIFO) type of list.
2. Stack follows LIFO mechanism.	2. Queue follows FIFO mechanism.
3. In a stack, same end is used to insert and delete the elements.	3. In a Queue two different ends are used to insert and delete the elements.
4. Stack is using only one pointer to refer the TOS.	4. Queue uses two pointers to refer the front and the rear ends of a queue.
5. It performs two operations known as push and POP.	5. It performs two operations known as enqueue and dequeue.
6. Stack implation is easy.	6. Queue implation is tricky.
7. Stack does not have variants.	7. Queue has variants like circular queue, priority queue, etc.

II.

Sort	Search
1. Searching is method for finding a target value within a list. It sequentially checks each element of the list for the target value until a match is found (or) until all the elements have be on searched.	1. Sorting is the method of arranging the element in particular order mostly either in ascending (or) descending order.
2. Popular types of searching are: · linear search · Binary search.	2. Types of sorting techniques are: · Bubble sort · Insertion sort · Selection sort · Quick sort · Merge sort · Heap sort

MADE EASY Source

- **MADE EASY Mains Classnotes**

End of Solution

6. (a) (ii) Write a pseudo code or in any standard programming language for interchanging the values of two variables :
I. Using a third variable.
II. Not using any extra variable.

[8 Marks]

Solution:

Swap two numbers with third variable

Assign variable-1 value to a temp variable

Assign variable-2 value to variable-1

Assign temp variable value to variable-2

Code :

step - 1 : Start
step - 2 : enter x, y, temp
step - 3 ; temp = x
step - 4 : $x = y$
step - 5 : y = temp
step - 6 : print x, y
step - 7 : stop

Swap two numbers without third variable

Code :

step - 1 : Start
step - 2 : Enter x, y
step - 3 : print x, y
step - 4 : $x = x + y$
step - 5 : $y = x - y$
step - 6 : $x = x - y$
step - 7 : print x, y
step - 8 : stop

End of Solution

---

6. (b) A 230 V, single phase, watt hour meter has a constant load of 5 A passing through it for 8 hours at unity power factor. If the meter constant is 460 revolutions per kWh, how many revolutions does the meter disc make during this period ? If the same meter makes 1638 revolutions when operating at 230 V and a constant load of 6 A passing through it for a certain duration at a power factor of 0.86, determine the duration of operation of the meter in hours.

[20 Marks]

Solution:

Given, $V = 230$ V, $I = 5$ A

Power factor = 1

Watt hour meter reading = $230 \times 5 \times 1 \times 8 = 9200$ Wh

Given meter constant is 460 revolutions per kWh

No. of revolutions during this period is

$$= \frac{9200 \times 4600}{1000} = 4232 \text{ revolutions}$$

Given meter constant is 460 revolutions per kWh

∴ Meter reading in 1638 revolution is

$$= \frac{1000}{460} \times 1638 = 3.56 \text{ kW}$$

Given, $V = 230$ V,

$I = 6$ A

Meter reading = 230 × 6 × 0.866 × time

230 × 6 × 0.866 × time = 3.56 kW

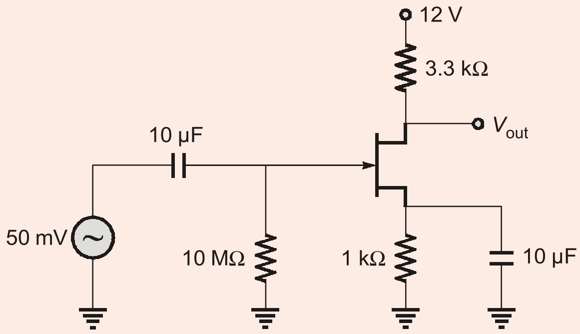
$$\text{time} = \frac{3.56 \times 10^3}{230 \times 6 \times 0.866} = 3 \text{ Hours}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.7(c) (ii) of Test-7 **(Click Here for Reference)**

End of Solution

6. (c) (i) Find the voltage gain of JFET amplifier shown in the figure below for the drain to source current with gate shorted, I_{DSS} = 10 mA, cut-off voltage $V_{GS(OFF)}$ = –4 V and I_D = 2 mA. If a load resistance of 4.7 kΩ is a.c. coupled to the output of this amplifier, calculate the percentage change in voltage gain.



[10 Marks]

Solution:

Given :

$$I_{DSS} = 10 \text{ mA}$$
$$I_D = 2 \text{ mA}$$
$$V_{GS(OFF)} = -4 \text{ V}$$

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_{GS(OFF)}}\right]^2 \quad ...(1) \quad \text{Shockley equation}$$

Transconductance g_m is given by

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

On performing partial differentiation of eqn. (1)

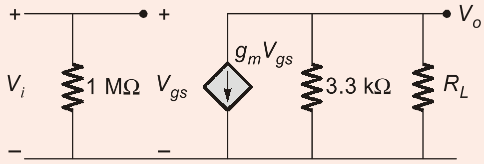
$$\frac{\partial I_D}{\partial V_{GS}} = \frac{-2I_{DSS}}{V_{GS(OFF)}}\left[1 - \frac{V_{GS}}{V_{GS(OFF)}}\right]$$

$$g_m = \frac{-2I_{DSS}}{V_{GS(OFF)}} \times \sqrt{\frac{I_D}{I_{DSS}}}$$

On putting values

$$g_m = \frac{-2\times 10}{-4} \times \sqrt{\frac{2}{10}} = 5\sqrt{\frac{2}{10}} = 2.33 \text{ mA/V} = 2.23 \text{ mS}$$

⇒ Equivalent small signal modal of given problem figure.



Case 1 : When $R_L = \infty$ (absent)

$$A_V = \frac{V_o}{V_i} = \frac{V_o}{V_{gs}} = \frac{-g_m \cdot V_{gs}(3.3\ \text{k}\Omega)}{V_{gs}} = -g_m\,(3.3\ \text{k}\Omega)$$

$$A_v = -2.23 \times 3.3 = -7.359$$

Case 2 : When $R_L = 4.7$ kΩ

$$A_V' = \frac{V_o}{V_i} = \frac{V_o}{V_{gs}} = \frac{-g_m V_{gs}(3.3\ \text{k}\Omega \parallel 4.7\text{k}\Omega)}{V_{gs}}$$

$$= -2.23 \text{ ms} \times 1.9387\ \text{k}\Omega$$

$$A_V' = -4.3234$$

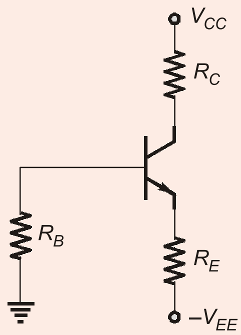
$$\%\text{ change in voltage gain} = \frac{|A_V| - |A_V'|}{|A_v|} \times 100 = 41.25\%$$

MADE EASY Source

- **Mains Work Book:** (Q.24, Page 47) **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classroom Video Lecture**

End of Solution

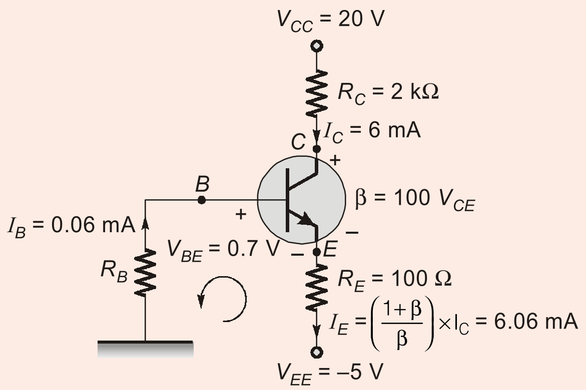
6. (c) (ii) Find R_B in the figure shown below for silicon transistor with β = 100 and negligible leakage current, if V_{CC} = 20 V, V_{EE} = 5 V, R_E = 100 Ω, R_C = 2 kΩ and I_C = 6 mA.



[10 Marks]

Solution:

Given BJT Circuit



$$I_B = \frac{I_C}{\beta} = \frac{6}{100} = 0.6 \text{ mA}$$

$$I_E = I_B + I_C = 6.06 \text{ mA}$$

Since given transistor is nPn.

So, $I_B + I_C = I_E$

$$I_B = I_E - I_C$$

$$= 6.06 - 6 = 0.06 \text{ mA}$$

Apply KVL in input loop

$$I_B R_B + V_{BE} + I_E R_E - V_{EE} = 0$$

$$I_B R_B = V_{EE} - I_E R_E - V_{BE}$$

$$= 5 - 0.1 \times 6.06 - 0.7 = 3.694 \text{ Volts}$$

$$R_B = \frac{3.694}{0.06} = 61.56 \text{ k}\Omega$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.5(d) of Test-6
- **Mains Workbook:** (Q.7, Page 4) **(Click Here for Reference)**
- **MADE EASY Classnotes**

End of Solution

7. (a) (i) Execution of a sequence of instructions of a program involves 200 instruction fetch operations, 100 memory operand read operations, and 80 memory operand write operations. Find the average memory access time in executing this sequence of instructions if the memory access time is 2 ns for a read operation with a hit in cache, 5 ns for a read operation with a miss in cache, 3 ns for a write operation with a hit in cache and 10 ns for a write operation with a miss in cache. The cache hit ratio is 0.9. Consider the time taken for fetch operation to be equal to that of read operation.

[10 Marks]

Solution:

$$\begin{aligned}
\#\text{instruction fetch} &= 200 \\
\#\text{memory read operation} &= 100 \\
\#\text{memory write operation} &= 80 \\
HC &= 0.9 \\
T_{avg\ r} &= H_r T_c + (1 - H_r)(T_M + T_L) \text{ (given in question)} \\
&= (0.9 * 2ns) + (1 - 0.9)\ 5\ ns = 2.3\ nsec \\
T_{avg\ w} &= H_w T_C + (1 - H_w)(T_M + T_C) \\
&= (0.9 * 3\ ns) + (1 - 0.9)\ 10\ ns \\
&= 3.7\ ns \\
T_{avg} &= \frac{(f_r * T_{avg\ r}) + (f_\omega * T_{avg\omega})}{(f_r + f_\omega)} \\
&= \frac{300 \times 2.3 + 80 \times 3.7}{380} = 2.594\ ns
\end{aligned}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Similar question mains mock-1 test series.
- **MADE EASY Mains Classnotes**

End of Solution

7. (a) (ii) Derive the expression for the number of comparisons required in the worst case for sorting an array of '*n*' elements using Bubble sort. Calculate it for an array of 100 elements. When will such maximum number comparisons occur in Bubble sort ?

[10 Marks]

Solution:

Bubble sort is a simple algorithm which is used to sort a given set of '*n*' elements provided in form of an array with '*n*' number of elements.

Bubble sort compares all the elements one by one and sort them based on their values.

```
#include <stdio.h>
void bubblesort (int arr[ ], int n]
{
int i, j, temp;
for (i = 0; i < n; i ++)
{
for (j = 0; j < n – (i – 1); j ++)
{
if (arr [ j ] > arr [j + 1];
{
        temp = arr [ j ];
        arr [ j ] = arr [j + 1];
```

```
            arr [j + 1] = temp;
            }
        }
    }
    printf ("sorted array: ")
    for (i = 0; i < n; i++)
    {
            printf ("sorted array:");
            for (i = 0; i < n; i++)
            {
            printf ("%d", arr[i]) ;
            }
    }
    int main ()
    {
    int arr [100], i, n, step, temp;
    printf ("enter the number of elements to be sorted:");
    scanf ("%d", % n);
    for (i = 0; i < n; i++)
    {
    printf ("enter element no %d,", i + 1);
    scan f("%d", & arr [i]);
    }
    bubble sort (arr, n);
    return 0;
    }
```

In bubble sort, (n – 1) comparisons will be done in the 1st pass, (n – 2) in 2nd pass, (n – 3) in 3rd pass and so on.

So the total # of comparisons will be

(n – 1) + (n – 2) + (n – 3) + ... + 3 + 2 + 1

$$\text{Sum} = \frac{n(n-1)}{2}$$

i.e., $O(n^2)$

For the arr[100] elements

$$\text{Sum} = \frac{100(100-1)}{2} = 4950 \text{ comparisons required in the worst case.}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Similar question Q.2(c) (i) of Test-7
- **MADE EASY Mains Classnotes**

End of Solution

7. (b) A spring controlled, electrodynamic voltmeter has a range of 100 V, has a square low scale response, and it takes 0.08 A on d.c. for full scale deflection of 120°. The control constant is 1 x 10^{-6} N-m/degree. The true potential difference across the instrument is 100.42 V, when it reads 100 V at 50 Hz. Determine the initial mutual inductance of the instrument.

[20 Marks]

Solution:

Actual voltage across the instrument

$$= 100.42 \text{ V}$$

Current through instrument when reading 100 V = 0.08 A

Impedance of the instrument

$$= \frac{100.42}{0.08} = 1255.25\ \Omega$$

$$\text{Resistance of instrument} = \frac{100}{0.08} = 1250\,\Omega$$

$$\text{Reactance of instrument} = \sqrt{(1255.25)^2 - (1250)^2} = 114.68\ \Omega$$

Mutual inductance at $\theta = 120°$ is

$$M = \frac{114.68}{2\pi \times 50} = 0.365 \text{ H}$$

The expression for deflection for a square low response voltmeter is

$$\theta = \frac{V^2}{KZ^2}\frac{dM}{d\theta}$$

For d.c. impedance, Z = resistance R and therefore,

$$\theta = \frac{V^2}{KR^2}\frac{dM}{d\theta}$$

$$\frac{V}{R} = 0.08 \text{ A},$$

$$K = 1 \times 10^{-6},$$

$$\theta = 120°$$

$$120° = \frac{(0.08)^2}{1\times 10^{-6}}\frac{dM}{d\theta}$$

$$\frac{120° \times 1 \times 10^{-6}}{0.08 \times 0.08} = \frac{dM}{d\theta}$$

$$\frac{dM}{d\theta} = 0.01875 \text{ H}$$

$$M = \frac{dM}{d\theta}(\theta_2 - \theta_1) = 0.01875 \times \left(\frac{2\pi}{3} - 0\right) = 0.03927$$

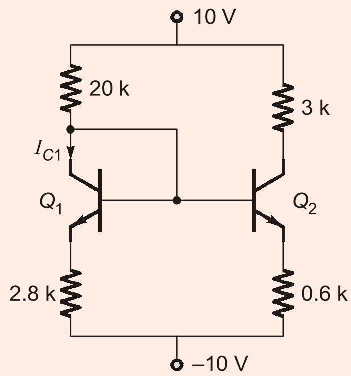
Initial mutual inductance = 0.365 – 0.03927 = 0.32573 H

MADE EASY Source

- **Mains Work Book:** (Q.18, Page 111) **(Click Here for Reference)**

End of Solution

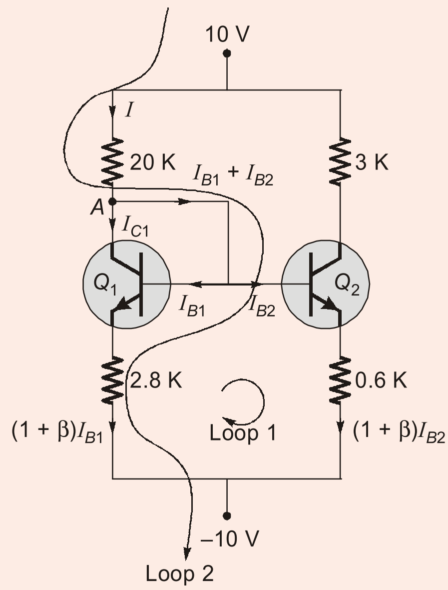
7. (c) (i) Fin I_{C1} in the circuit shown in the figure below. Assume that the two transistors are matched and V_{BE} = 0.7 V, β = 100.



[12 Marks]

Solution:

Given : $V_{BE} = 0.7\text{ V}, \beta = 100$



KVL in loop (1)

$$V_{BE2} + (1+\beta)I_{B2} \times 0.6K - 2.8K(1+\beta)I_{B1} - V_{BE1} = 0$$

$$0.7 + 101I_{B2} \times 0.6K - 2.8K \times 101I_{B1} - 0.7 = 0$$

$$101I_{B2} \times 0.6K = 2.8K \times 101I_{B1}$$

$$I_{B2} = 4.67I_{B1}$$

On applying KCL on node (A)

$$I = I_{C1} + I_{B1} + I_{B2}$$

$$I_R = I_{C1} + 5.67I_{B1}$$

$$I_R = 100I_{B1} + 5.67I_{B1} = 105.67I_{B1}$$

On applying KVL in loop (2)

$$-10 + 20KI_R + V_{BE1} + 2.8K(1+B)I_{B1} - 10 = 0$$

$$20K \times 105.67 I_{B1} + 101 I_{B1} \times 2.8K = 20 - 0.7$$

$$I_{B1} = \frac{19.3}{2396.2K}$$

$$I_{B1} = 8.05\ \mu A$$

and $$I_{C1} = \beta I_{B1}$$

$$I_{C1} = 0.8054\ mA$$

MADE EASY Source

- **Mains Workbook:** (Q.17, Page 7) **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classroom Video Lecture**

End of Solution

7. (c) (ii) Find the feedback factor β of the negative feedback network required for an amplifier with open loop gain $A_0 = 2000 \pm 200$ to reduce the variation to less than ± 0.2%. Find the overall gain of the system with feedback.

[8 Marks]

Solution:

Given : $$A_o = 2000 \pm 200$$

$$\frac{dA_f}{A_f} = \pm 0.2\% = \pm\frac{0.2}{100}$$

We have $$\frac{dA_f}{A_f} = \frac{dA_o}{A_o} \times \frac{1}{(1+A_o\beta)}$$

where A_f = Gain with feedback

A_o = Open loop gain

$$\pm\frac{0.2}{100} = \frac{\pm 200}{2000} \times \frac{1}{(1+A_o\beta)}$$

$$(1+A_o\beta) = \frac{200}{20 \times 0.2} = 50$$

$$A_o\beta = 49 \Rightarrow \beta = \frac{49}{2000} = 0.0245$$

Gain with feedback

$$A_f = \frac{A_1}{1+A_o\beta} = \frac{2000}{50} = 40$$

$$A_f = 40$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.8(a) (i) of Mock Test-3
- **Mains Workbook:** (Q.45, Page 17) **(Click Here for Reference)**
- **MADE EASY Classnotes**
- **Mains Classroom Video Lecture**

End of Solution

8. (a) Name the layers of Open Systems Interconnection (OSI) model created by the International Organization for Standardization for different computer systems to communicate with each other using standard protocols. Mention the important functions of each of these layers in brief.

[20 Marks]

Solution:

OSI reference model describes how information from a s/w application in one computer moves through a physical medium to the software application in another computer.

OSI consists of 7 layers and each layer performs a particular network function, named as

Responsibility of the host:
- Application layer — layers-7
- Presentation layer — layers-6
- Session layer
- Transport layer

Responsibility of the network:
- Network layer
- Datalink layer
- Physical layer — layers-1

OSI model is divided into two layers upper layers and lower layers.

Upper layers of the OSI model mainly deals with the application related issues and lower layers of the OSI model delays with the data transport issues.

Functions of physical layer:

- line configuration
- data transmission
- Topology
- Signals

Functions of a data link layer:

- Framing
- physical addressing
- Flow control
- Error control
- Access control

Functions of a Network layer :

- Internetworking
- Addressing
- Routing
- Packetizing

Functions of a transport layer :

Two protocols are used in this layer

1. Transmission control protocol (TCP)
2. User datagram protocol (UDP)
   - Service point addressing
   - Segmentation and reassembly
   - Connection control

- Flow control
- Error control

Functions of a session layer :

- Dialog control
- Synchronization

Functions of presentation layer :

- Translation
- Encryption
- Compression

Functions of application layer :

- File transfer, across and management
- Mail services
- Directly services

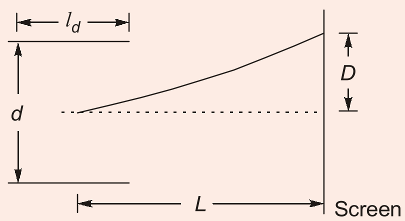
MADE EASY Source

- **ESE 2020 Mains Test Series:** Q.7(a) of Test-9
- **MADE EASY Classnotes**
- **Theory Book:** Computer fundamental **(Click Here for Reference)**

End of Solution

8. (b) In an oscilloscope, the deflection factor of CRT is 80 V/cm and the accelerating voltage is 2500 V. What is the minimum distance required from center of deflection plates to screen that allows full deflection of 4 cm on the oscilloscope screen ? [20 Marks]

Solution:



$$D.F. = 80 \text{ Volt/cm}$$

$$V_a = 2500 \text{ volts}, D = 4 \text{ cm}$$

$$D = \frac{L \cdot l_d \cdot V_d}{2V_a \cdot d}$$

$$\text{Sensitivity} = S' = \frac{D}{V_d} = \frac{L \cdot l_d}{2V_a \cdot d}$$

$$\text{Deflection factor} = D.F = \frac{1}{S'} = \frac{V_d}{D}$$

$$\Rightarrow \quad V_d = 80 \text{ Volt/cm} \times 4 \text{ cm} = 320 \text{ Volt}$$

$$L = \frac{2V_a \cdot dD}{I_d \cdot V_d} = \frac{2D \cdot V_a}{V_d} \times \left(\frac{d}{I_d}\right) = \frac{2 \times 4 \text{ cm} \times 2500 \text{ Volt}}{320 \text{ Volt}} \times K$$

where, $K = \frac{d}{I_d}$ = Constant

$L = 62.5 \text{ cm} \times K$

Minimum value of L depends on value of K

Taking $K = 1$

$L = 62.5 \text{ cm}$

MADE EASY Source

- **Mains Work Book:** (Q.51, Page 134) **(Click Here for Reference)**

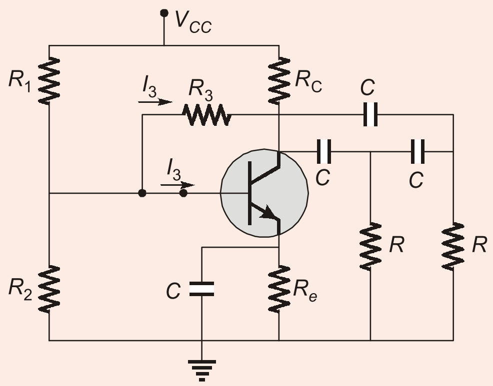
End of Solution

8. (c) Derive the expression for the frequency and the condition for starting of sustained oscillation in a transistorised R-C phase shift oscillator. Neglect h_{0e} and h_{re}.. Assume $R >> h_{ie}$ and a load resistance R_L is a.c. coupled to the oscillator.

[20 Marks]

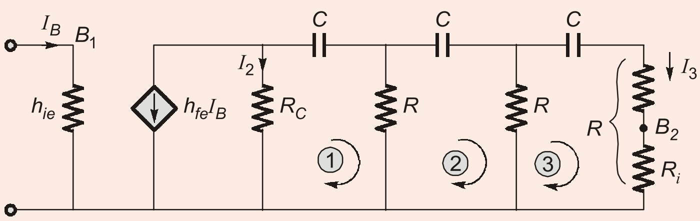
Solution:

RC Phase shift oscillator using BJT:



(a) Transistor phase shift oscillator

- The resistor $R_3 = R - R_i$, where $R_i \simeq h_{ie}$ is the input resistance of the transistor. This choice makes the three Rc sections of the phase shifting network alike and simplifies the calculations.
- We assume that the biasing resistors R_1, R_2 and R_e have no effect on the signal operation and neglect these in the following analysis.



(b) Equivalent circuit for this oscillator

- Hence, we imagine the loop broken at the base between B_1 and B_2, but in order not to change the loading on the feedback network, we place R_i from B_2 to ground. If we assume a current I_B to enter the base at B_1, the loop current gain equals $\frac{I_3}{I_B}$ and is found by writing KVL for three meshes.

KVL in Loop 1 :

$I_1X_C + R(I_1 - I_2) + R_C(I_1 + h_{fe}I_B) = 0$

$\Rightarrow \quad (R_C + R + X_C)I_1 - RI_2 + R_Ch_{fe}I_B = 0$

$\Rightarrow \quad (R + R_C + X_C)I_1 - RI_2 = -R_Ch_{fe}I_B \qquad$ (i)

KVL in Loop 2 :

$I_2X_C + R(I_2 - I_3) + R(I_2 + I_1) = 0$

$\Rightarrow \quad -RI_1 + (2R + X_C)I_2 - RI_3 = 0 \qquad$ (ii)

KVL in Loop 3 :

$I_3(R + X_C) + R(I_3 - I_2) = 0$

$\Rightarrow \quad -RI_2 + (2R + X_C)I_3 = 0 \qquad$ (iii)

Writing equations in matrix form

$$\begin{bmatrix} (R+R_C+X_C) & -R & 0 \\ -R & (2R+X_C) & -R \\ 0 & -R & (2R+X_C) \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -R_Ch_{fe}I_B \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
\Delta &= (R + R_C + X_C)[(2R + X_C)2 - R^2] - (-R)[(-R)(2R + X_C)] + 0 \\
&= (R + R_C + X_C)[4R^2 + X_C^2 + 4RX_C - R^2] + R(-2R^2 - RX_C) + 0 \\
&= (R + R_C + X_C)[3R^2 + X_C^2 + 4RX_C] - 2R^3 - R^2X_C \\
&= 3R^3 + RX_C^2 + 4R^2XC + 3R^2R_C + R_CX_C^2 + 4RR_CX_C + 3R^2X_C + X_C^3 \\
&\quad + 4RX_C^2 - 2R^3 - R^2X_C \\
\Delta &= R^3 + X_C^3 + 3R^2R_C + 6R^2X_C + 5RX_C^2 + R_CX_C^2 + 4RR_CX_C
\end{aligned}$$

$$\Delta_3 = \begin{vmatrix} R+R_C+X_C & -R & -R_Ch_{fe}I_B \\ -R & 2R+X_C & 0 \\ 0 & -R & 0 \end{vmatrix}$$

$$= -R_Ch_{fe}I_B[R^2] = -R_Ch_{fe}R^2I_B$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-R_CR^2h_{fe}I_B}{R^3 + X_C^3 + 3R^2R_C + 6R^2X_C + 5RX_C^2 + R_CX_C^2 + 4RR_CX_C}$$

$$\Rightarrow \quad \frac{I_3}{I_B} = \frac{-\frac{R_C}{R}h_{fe}}{1+\left(\frac{X_C}{R}\right)^3 + 3\frac{R_C}{R} + 6\frac{X_C}{R} + 5\left(\frac{X_C}{R}\right)^2 + \frac{R_C}{R}\left(\frac{X_C}{R}\right)^2 + 4\frac{R_C}{R}\cdot\frac{X_C}{R}}$$

Put $\frac{R_C}{R} = K$

$$\Rightarrow \quad \frac{I_3}{I_B} = \frac{-Kh_{fe}}{1+\left(\frac{X_C}{R}\right)^3 + 3K + \frac{6X_C}{R} + 5\left(\frac{X_C}{R}\right)^2 + K\left(\frac{X_C}{R}\right)^2 + 4K\frac{X_C}{R}}$$

Put $X_C = \frac{1}{j\omega C}$

$$\Rightarrow \quad \frac{I_3}{I_6} = \frac{-Kh_{fe}}{1+\left(\frac{1}{j\omega RC}\right)^3 + 3K + 6\frac{1}{j\omega RC} + 5\left(\frac{1}{j\omega RC}\right)^2 + K\left(\frac{1}{j\omega RC}\right)^2 + \frac{4K}{j\omega RC}}$$

So, $4 - \frac{29}{K^2} = 0$

$$\Rightarrow \quad \frac{29}{K^2} = 4$$

$$\Rightarrow \quad K = \sqrt{\frac{29}{4}}$$

$$\Rightarrow \quad K = 2.7$$

So, minimum h_{fe} required to sustain oscillations is

$$h_{fe} \geq (4 \times 2.7) + 23 + \frac{29}{2.7}$$

$$\Rightarrow \quad h_{fe} \geq 44.5$$

A transistor with a small-signal common emitter short-circuit current gain less than 44.5 cannot be used in this phase shift oscillator.

MADE EASY Source

- **Mains Workbook:** (Q.35, Page 57)
- **MADE EASY Classnotes (Click Here for Reference)**
- **Mains Classroom Video Lecture**

End of Solution

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