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UPSC ESE 2020

Main Exam Detailed Solutions

Electronics and Telecom. Engineering

PAPER-II

EXAM DATE : 18-10-2020 | 2:00 PM to 5:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

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Electronics and Telecom. Engineering Paper Analysis
ESE 2020 Main Examination

Sl.	Subjects	Marks
1.	Control Systems	80
2.	Electromagnetics	110
3.	Communication Systems	80
4.	Advanced Communication	60
5.	Advanced Electronics	110
6.	Computer Organization and Architecture	40
	Total	**480**

Scroll down for detailed solutions

1. (a) A certain speech signal is sampled at 8 kHz and coded with DPCM, the output of which belongs to a set of 8 symbols $s_1 - s_8$.
The probabilities of these symbols are $p(s_1) = 0.4$, $p(s_2) = p(s_3) = 0.2$, $p(s_4) = 0.1$, $p(s_5) = 0.05$, $p(s_6) = p(s_7) = 0.02$ and $p(s_8) = 0.01$. Calculate the entropy in bits/sec. If all the symbols are equiprobable, what will be the entropy?

[10 Marks]

Solution:

Given set of symbols is $s = \{s_1, s_2, \ldots\ldots s_8\}$

The entropy is given as : $H(s) = \sum_{K=1}^{8} p(s_k)\log_2\left[\frac{1}{p(s_k)}\right]$

$\therefore$ $H(s) = -[0.4 \log_2 0.4 + 0.2 \times 2 \log_2 0.2 + 0.1 \log_2 0.1 + 0.05 \log_2 0.05 + 2(0.02)\log_2(0.02) + 0.01 \log_2 0.01]$

$= 2.298$ bits/sample

f_s = sampling frequency = 8 kHz

$\therefore$ Entropy (in bits/sec) $= H(s) \times r$

$= 2.298 \times 8$

$= 18384$ bits/sec

If all the symbols are equiprobable, then the entropy is maximum, which is given by

$H(s) = \log_2 n$

where, $p(s_k) = \frac{1}{n}, k = 1, 2, \ldots..8$

and n = total no. of symbols.

$\therefore$ $H(s) = \log_2 8 = 3$ bits/sample

$\therefore$ $R = H \times r = 3 \times 8000 = 24000$ bits/sec

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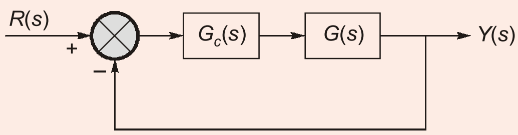
- **Theory Book:** Communication Systems (Page No. 336) **(Click here for reference)**
- **MADE EASY Classnotes**

End of Solution

1. (b) In the figure shown below, $G(s) = \frac{K}{(\tau s + 1)}$ has a time constant of 0.5 seconds, and has unity DC gain. An integral controller is placed in forward path as $G_c(s) = \frac{K_1}{s}$ such that the open loop transfer function $G(s)G_c(s)$ has a velocity error constant $K_V = 1$. Find the sensitivity of the closed loop system transfer function with respect to K_1 at $\omega = 1$ rad/sec.



[10 Marks]

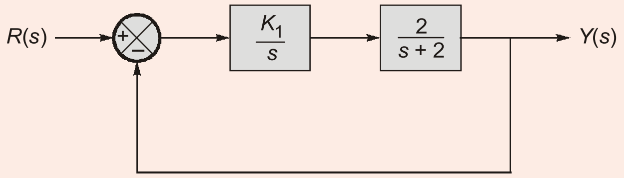
Solution:

Given, $G(s) = \dfrac{K}{(\tau s+1)}$

Time constant, $\tau = 0.5$ sec

DC gain = 1, i.e. $G(j0) = 1$

$\therefore \quad 1 = \dfrac{K}{0.5j(0)+1} \Rightarrow K = 1$



Let closed loop transfer function,

$$G_{CL}(s) = \frac{\dfrac{2K_1}{s(s+2)}}{1+\dfrac{2K_1}{s(s+2)}} = \frac{2K_1}{s(s+2)+2K_1}$$

$$\therefore \quad G_{CL}(s) = \frac{2K_1}{s^2+2s+2K_1}$$

Sensitivity of closed loop transfer function with respect to K_1 is

$$S_{K_1}^{G_{CL}(s)} = \frac{K_1}{G_{CL}(s)}\frac{\partial G_{CL}(s)}{\partial K_1}$$

$$= \frac{K_1}{\dfrac{2K_1}{s^2+2s+2K_1}} \times \frac{\partial}{\partial K_1}\left[\frac{2K_1}{s^2+2s+2K_1}\right]$$

$$= \frac{s^2+2s+2K_1}{2} \frac{(s^2+2s+2K_1)(2)-(2K_1)(2)}{(s^2+2s+2K_1)^2}$$

$$= \frac{s^2+2s+2K_1}{2} \cdot \frac{2s^2+4s+4K_1-4K_1}{(s^2+2s+2K_1)^2}$$

$$= \frac{s^2+2s}{(s^2+2s+2K_1)}$$

It is also given that, velocity error constant $K_V = 1$

i.e., $\lim\limits_{s\to 0} sG_{OL}(s) = 1$

$$\lim_{s\to 0} s\cdot\frac{2K_1}{s(s+2)} = 1 \qquad \left(\because\ G_{OL}(s) = \frac{2K_1}{s(s+2)}\right)$$

$$\therefore \quad K_1 = 1$$

$$\therefore \quad S_{K_1}^{G_{CL}(s)} = \frac{s(s+2)}{s^2+2s+2}$$

$$S_{K_1}^{G_{CL}(j\omega)} = \frac{j\omega(2+j\omega)}{-\omega^2+2j\omega+2}$$

At $\omega = 1$ rad/sec

$$\left|S_{K_1}^{G_{CL}(s)}\right| = \left|\frac{j(2+j)}{1+2j}\right| = 1$$

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- **ESE 2020 Mains Test Series:** Mock Test-4 (Q.6(d)) **(Click here for reference)**

End of Solution

1. (c) List and define various scheduling performance criteria used for comparing various CPU-scheduling algorithms. Compute and compare the average process waiting time of First come First serve, Shortest task first and Priority scheduling algorithms for the processes with their details as listed in the table.

Process	Arrival Time	Burst Time	Priority
P_0	0	3	1
P_1	2	2	2
P_2	3	4	3
P_3	4	7	1

[10 Marks]

Solution:

Different CPU scheduling algorithms have different properties and choice of a particular algorithm depends on the various factors. Many criteria have been suggested for comparing CPU scheduling algorithms.

The criteria include the following:

(i) **CPU utilisation:** The main objective of any CPU scheduling algorithm is to keep the CPU as busy as possible to increase the CPU utilisation.

(ii) **Turnaround time:** The time elapsed from the time of submission of a process to the time of completion is known as turnaround time. An important criteria is how long it takes to execute that process.

(iii) **Waiting time:** The time spent by a process in a ready queue is called waiting time. To increase the performance of a scheduling algorithm waiting time should be minimal.

(iv) **Response time:** The time taken from submission of process of request until the first response is produced is called response time.

(v) **Throughput:** The number of processes being executed and completed per unit time is called throughput. The throughput varies depending upon the length or duration of processes.

First Come First Serve

Gantt Chart

P_0	P_1	P_2	P_3

0 3 5 9 16

$$\text{Avg. waiting time} = \frac{(0-0)+(3-2)+(5-3)+(9-4)}{4} = \frac{8}{4} = 2$$

Shortest Task First

Gantt Chart

P_0	P_1	P_2	P_3

0 3 5 9 16

$$\text{Avg. waiting time} = \frac{(0-0)+(3-2)+(5-3)+(9-4)}{4} = \frac{8}{4} = 2$$

Priority scheduling algorithm

Gantt Chart

P_0	P_1	P_3	P_2

0 3 5 12 16

$$\text{Avg. waiting time} = \frac{(0-0)+(3-2)+(5-4)+(12-3)}{4} = \frac{11}{4} = 2.75$$

Comparison of Avg. waiting time =

(First Come First Serve = Shortest Task First < Priority Scheduling Algorithm)

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- **MADE EASY Classnotes : (Click here for reference)**

End of Solution

1. (d) A uniform plane wave is propagating in z-direction with velocity 1.4×10^8 m/s in a perfect dielectric medium of intrinsic impedance 474 Ω. If $E_x(z, t) = 1750 \cos(10^6 \pi t - \beta z)$ V/m represents instantaneous electric field, what will be the magnetic field? Determine the wavelength and average power of the wave.

[10 Marks]

Solution:

(i) Given $E_x(z, t) = 1750 \cos(10^6 \pi t - \beta z)$ V/m

The wave is EM wave

$$\therefore \quad H_y(z, t) = \frac{E_0}{\eta_0}\cos(10^6 \pi t - \beta z)$$

$$= \frac{1750}{474}\cos(10^6 \pi t - \beta z)$$

$$= 3.692 \cos(10^6 \pi t - \beta z) \text{A/m}$$

(ii) Wavelength $\lambda = \dfrac{2\pi}{\beta}$

where, $\beta = \dfrac{\omega}{v_p} = \dfrac{10^6\pi}{1.4\times10^8} = 0.0224$ rad/m

$\therefore$ $\lambda \approx 280$ m

(iii) Average power $= P_{avg} = \dfrac{E_0^2}{2|\eta|} = 3.23$ kW/m^2

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- **Theory Book:** Electromagnetics (Page No. 145) **(Click here for reference)**

End of Solution

1. (e) Processor technology deals with computation architectures whereas IC technology deals with implementation style for a given functionality. What are the different processor and IC technologies? Is processor technology orthogonal to IC technology or interdependent with IC technology? Justify your answer.

[10 Marks]

Solution:

IC technology deals with the process of implementing a transistor on a single chip. The aim of IC technology is to design a transistor in such a way what it performs ideally and take minimum amount of space in an integrated circuit. Different type of IC technology include :

1. Large scale integration technology
2. Very large scale integration technology
3. Silicon on insulator technology

Processing technology deals with the designing and architecture of the circuit which have to be implemented. It includes the design of multiple cores for a single processor, design of an efficient ALU system, cache sharing systems, multithread processors. It mainly exploits the transistor made by an integrated circuit to optimise the system performance.

Example of processor technology include :

1. RISC processors
2. ARM processors
3. CISC processors

Different comprise can name their processors on different parameters.

A processor technology can be independent of the IC technology as both Intel and ARM processors can be built on the same 14 nm technology and will have very different performance and core structure.

The IC technology can only delete the physical parameters of a processor like the power requirement and power dissipation but different processors within a some company (family) can also be built on the same IC technology e.g. the 10th generation intel i3, i5, i7 are built on the same IC technology.

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- **Theory Book:** Computer Org. and Architecture (Page No. 3) **(Click here for reference)**

End of Solution

1. (f) Explain the following terms:
(i) Model Birefringence
(ii) Coherence Length
(iii) Beat Length
The difference between the propagation constants for the two orthogonal modes in the single mode fiber is 250. It is illuminated with light of peak wavelength 1.55 μm from an injection laser source with a spectral width of 0.8 nm. Calculate Model Birefringence, Coherence Length and Beat Length.

[10 Marks]

Solution:

(i) **Modal Birefringence:** A single-mode fiber with normal circular symmetric about the core axis allow the propagation of two nearly degenerate modes with orthogonal polarization. For an ideal optically circular symmetric core both polarization mode propagate with identical velocities. However, manufactured optical fiber has difference in core geometry resulting from internal and external stresses and fiber bending. The modes therefore have different propagation constant brand by which are directed by anisotropy of fiber cross-section, where β_x and β_y are the propagation constant for slow and fast mode respectively. The value of modal birefringence B_F can be calculated as

$$B_F = \frac{(\beta_x - \beta_y)}{2\pi/\lambda}$$

(ii) **Coherence length:** It is the length of optical fiber over which the birefringence coherence is maintained i.e., the phase coherence $\phi(z)$ between the two modes is maintained. Thus,

$$L_{bc} \approx \frac{\lambda^2}{\beta_F(\delta\lambda)}$$

where $\delta\lambda$ is the source line width.

(iii) **Beat length:** If the phase coherence is maintained it leads to a polarization state which is generally elliptical but which varies periodically along the fiber. The characteristic length L_B for this process corresponding to the propagation distance for which a 2π phase difference accumulates between two modes is known as beat length.

i.e., $$L_B = \frac{\lambda}{B_F} = \frac{2\pi}{(\beta_x - \beta_y)}$$

Now, $$B_F = \frac{(\beta_x - \beta_y)}{2\pi} \cdot \lambda$$

$$= \frac{250}{2\pi} \times 1.55 \times 10^{-6} = 61.67\ \mu\text{m}$$

$$L_{bc} \approx \frac{\lambda^2}{B_F \times \delta\lambda} = \frac{(1.55 \times 10^{-6})^2}{61.67 \times 10^{-6} \times 0.8 \times 10^{-9}}$$

$$L_{bc} \approx 48.69 \text{ m}$$

$$L_B = \frac{\lambda}{B_F} = 25.13 \text{ mm}$$

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- **MADE EASY Classnotes : (Click here for reference)**

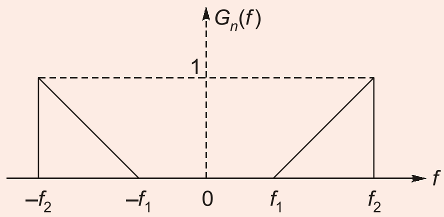
End of Solution

2. (a) Narrow band noise $n(t)$ having bandwidth 2B centered at f_0 is expressed as $n(t) = n_1(t)\cos(2\pi f_0 t) - n_Q(t)\sin(2\pi f_0 t)$, where $n_I(t)$ and $n_Q(t)$ are inphase and quadrature components respectively.

(i) Draw the block diagram of the scheme and show the extraction of $n_I(t)$ and $n_Q(t)$ from $n(t)$.

(ii) If $G_n(f)$ is power spectral density (PSD) of $n(t)$, derive expressions in terms of $G_n(f)$ for PSD of $n_I(t)$ and $n_Q(t)$.

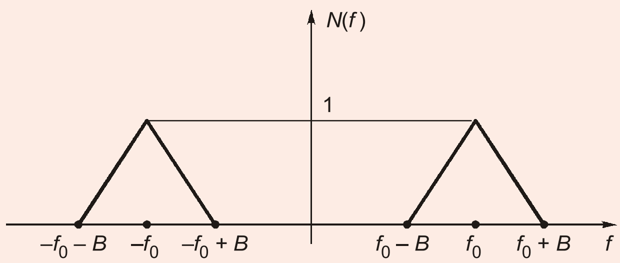
(iii) If $G_n(f)$ is as shown, sketch PSD of $n_I(t)$ assuming $f_0 = f_1$.



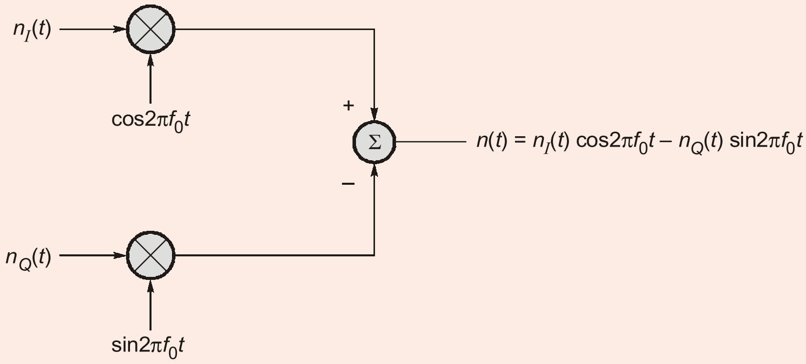
[6 + 8 + 6 Marks]

Solution:

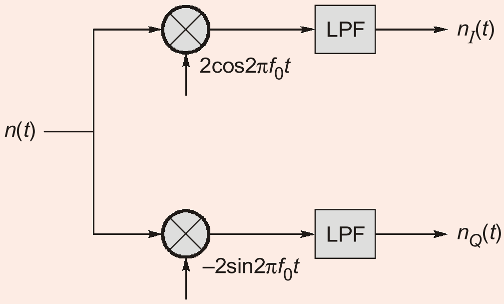
$n(t)$ is narrowband noise having bandwidth $2B$, centered at f_o.



(i) Given $n(t) = n_I(t)\cos 2\pi f_0 t - n_Q(t)\sin\pi f_0 t$



Extraction of $n_I(t)$ and $n_Q(t)$ from $n(t)$:



(ii) If $X(t) \xrightarrow{PSD} S_X(f)$

Then $X(t)\cos 2\pi f_0 t \xrightarrow{PSD} \dfrac{S_X(f - f_0) + S_X(f + f_0)}{4}$

Given, $n(t) \xrightarrow{PSD} G_n(f)$

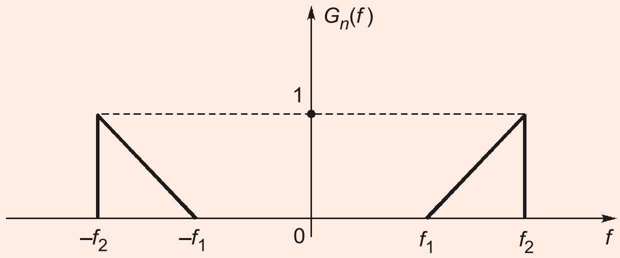
Then, $2n(t)\cos 2\pi f_0 t \xrightarrow{PSD} G_n(f - f_0) + G_n(f + f_0)$

$-2n(t)\sin\pi f_0 t \xrightarrow{PSD} G_n(f - f_0) + G_n(f + f_0)$

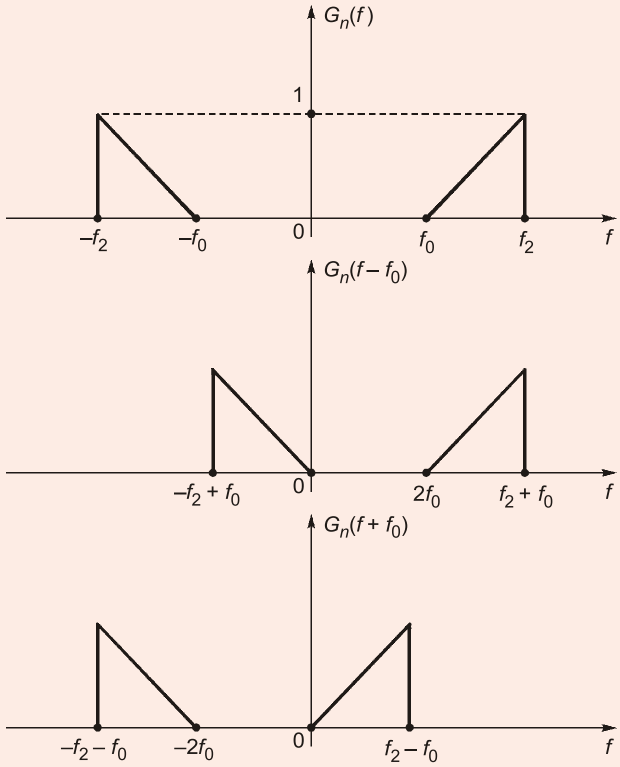
So, that $n_I(t)$ and $n_Q(t)$ will have same PSD.

$$G_{NI}(f) = G_{nQ}(f) = G_n(f - f_0) + G_n(f - f_0)\ ;\ |f| \le B$$

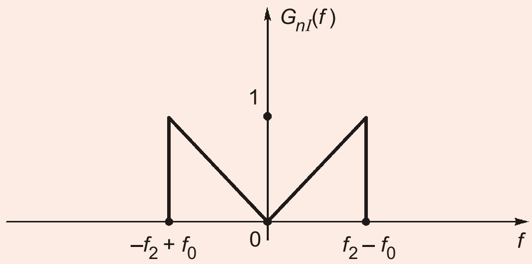
(iii)



Given $f_0 = f_1$



PSD of $n_I(t) \rightarrow G_{nI}(f) = G_n(f - f_o) + G_n(f + f_o)$
where $n_I(t)$ is low pass signal.



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- **Theory Book:** Communication Systems (Page No. 102-103) **(Click here for reference)**

End of Solution

2. (b) For a unity feedback system with $G(s) = \dfrac{3s+\alpha}{s(s+1)(s+5)}$, draw the root locus plot as parameter α varies from 0 to ∞. Also find the value of parameter α for which the closed loop system becomes unstable. From the root locus plot, obtain approximate location of the system poles with $\xi = 0.707$.

[20 Marks]

Solution:

$$1 + G(s) = 0$$

$$1+\frac{3s+\alpha}{s(s+1)(s+5)} = 0$$

$$s(s^2 + 6s + 5) + 3s + \alpha = 0$$

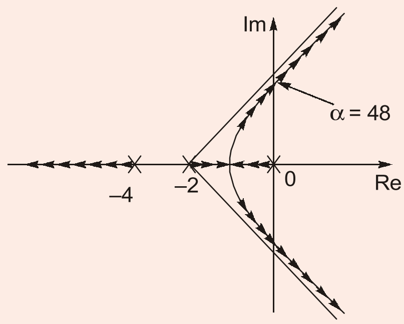
$$s^3 + 6s^2 + 5s + 3s + \alpha = 0$$

$$s^3 + 6s^2 + 8s + \alpha = 0$$

$$s(s + 2)(s + 4) + \alpha = 0$$

$$1+\frac{\alpha}{s(s+4)(s+2)} = 0$$

We need to draw root locus for $\dfrac{\alpha}{s(s+4)(s+2)}$



Root locus on real axis

Root locus exist between $s = 0$ and -2, $s = -4$ and ∞

$$\text{Centroid } \sigma = \frac{-2-4}{3} = \frac{-6}{3} = -2$$

Angle of asymptotes:

As $P - Z = 3$, Angle of Asymptotes are $\pm 60°$, $180°$.

$$s^3 + 6s^2 + 8s + \alpha = 0$$

s^3	1	8
s^2	6	α
s^1	$\dfrac{48-\alpha}{6}$	
s^0	α	

$$\frac{48-\alpha}{6} = 0$$

$$\alpha = 48$$

Root locus crosses imaginary axis at $\alpha = 48$

$\therefore$ for $\alpha > 48$, system becomes unstable.

Break away point:

$$\alpha = -s(s^2 + 6s + 8)$$

$$\alpha = -s^3 - 6s^2 - 8s$$

$$\frac{d\alpha}{ds} = 0$$

$$\frac{d}{ds}(-s^3 - 6s^2 - 8s) = 0$$

$$3s^2 + 12s + 8 = 0$$

$$s = -3.154, \; s = -0.845$$

Break away point is at $s = -0.845$

As seen from root locus, the third order system is combination of first and dominant second order system hence,

$$(s+p)(s^2 + 2\xi\omega_n s + \omega_n^2) = 0$$

$$s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + ps^2 + 2\xi\omega_n ps + p\omega_n^2 = 0$$

$$s^3 + (2\xi\omega_n + p)s^2 + (\omega_n^2 + 2\xi\omega_n p)s + p\omega_n^2 = 0$$

from characteristic equation

$$s^3 + 6s^2 + 8s + \alpha = 0$$

$$2\xi\omega_n + p = 6$$

$$\sqrt{2}\omega_n + p = 6$$

$$p = 6 - \sqrt{2}\omega_n$$

$$\omega_n^2 + 2\xi\omega_n p = 8$$

$$\omega_n^2 + \sqrt{2}\omega_n p = 8$$

$$\omega_n^2 + \sqrt{2}\omega_n(6 - \sqrt{2}\omega_n) = 8$$

$$\omega_n^2 + 6\sqrt{2}\omega_n - 2\omega_n^2 = 8$$

$$6\sqrt{2}\omega_n - \omega_n^2 = 8$$

$$\omega_n^2 - 6\sqrt{2}\omega_n + 8 = 0$$

$$\omega_n = 1.08 \quad \Rightarrow p = 4.47$$

$$\omega_n = 7.40 \quad \Rightarrow p = -4.47 \quad \text{(not possible)}$$

$\therefore$ $$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$s = -0.763 \pm j0.763 \text{ and } s = -4.47$$

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- **ESE 2020 Mains Test Series:** Test-15 Q.4(b) **(Click here for reference)**

End of Solution

2. (c) Memory sub-system for a product has been designed with 3-level memory hierarchy within a budget of ₹ 22,000. The known and unknown parameters for the design are tabulated below:

Memory Type	Access Time	Capacity	Cost per kilobyte in ₹
Cache	5 ns	1 MB	1
Main Memory	–	127 MB	0.1
Solid State Drive (SSD)	5 μs	–	0.001

The design achieved an effective memory access time of 20 ns with cache hit ratio 0.95 and main memory hit ratio 0.99. The SSD can be only in integer power of 2 in GB.

Find out the missing parameters in the above table.

[20 Marks]

Solution:

Given data:

Cache hit ratio, $(H_C) = 0.95$

Main memory hit ratio, $(H_M) = 0.99$

Cache access time, $(T_C) = 5$ ns

SSD access time $= 5\ \mu s$

$T_{SSD} = 5000$ ns

Let Main memory access time $(T_M) = x$

Formula for effective memory access time

$= H_C T_C + (1 - H_C)H_M(T_C + T_M) + (1 - H_C)(1 - H_M)H_{SSD}(T_C + T_M + T_{SSD})$

Since given effective memory access time is = 20 ns

So,

$20 \text{ ns} = 0.95 \times 5 + 0.05 \times 0.99 \times (5 + x) + 0.05 \times 0.01 \times (5 + x + 5000)$

$20 \text{ ns} = 4.75 + 0.0495(5 + x) + 0.0005(5005 + x)$

$$20 - 4.75 = 0.0495x + 0.2475 + 2.5025 + 0.0005x$$

$$15.25 = 2.75 + 0.05x$$

$$15.25 - 2.75 = 0.05x$$

$$12.5 = 0.05x$$

$$x = \frac{12.5}{0.05} = 250 \text{ ns}$$

Capacity of SSD

Given cast here is cast per KB.

So,
$$22000 = \frac{2^{20}}{2^{10}} \times 1 + \frac{2^{27}}{2^{10}} \times 0.1 + \frac{2^{30+x}}{2^{10}} \times 0.001$$
$$22000 = 1024 + 13107.2 + 1048.576 + 2^x \times 0.001$$
$$6820.224 = 2^x \times 0.001$$
$$2^x = \frac{6820.224}{0.001}$$
$$2^x = 6820224$$

On taking $\log_2$ both sides we get
$$x = 23$$

So,
$$\text{size of SSD} = 2^{23} \times 2^{10}$$
$$= 2^{33}B$$
$$= 8\text{ GB}$$

MADE EASY Source

- **Theory Book:** Computer Org. and Architecture (Page No. 17) **(Click here for reference)**

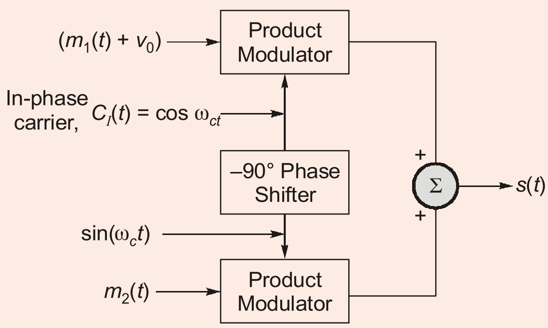
End of Solution

3. (a) In a particular AM system, quadrature modulation is used where the inphase carrier modulates $(m_1(t) + V_0)$ and quadrature carrier modulates $m_2(t)$, where $m_1(t)$ and $m_2(t)$ are low pass band-limited message signals and V_0 is constant.
   (i) Write the expression for quadrature AM signal.
   (ii) Assuming V_0 is large, show that $m_1(t)$ can be recovered using envelope detector.
   (iii) Propose a coherent demodulation scheme and show the recovery of $m_2(t)$.

[4 + 8 + 8 Marks]

Solution:

(i)



Expression for Quadrature AM signal is;
$$s_q(t) = m_2(t) \sin 2\pi f_c t$$

(ii) Recovering $m_1(t)$ by using Envelope Detector:
$$\left\{A\cos\omega_0 t + B\sin\omega_0 t \xrightarrow[\text{Detector}]{\text{Envelope}} \sqrt{A^2 + B^2}\right\}$$

$$\therefore \quad (ED)_{output} = \sqrt{(m_1(t)+V_0)^2 + m_2^2(t)}$$

$$= \sqrt{m_1^2(t) + V_0^2 + 2m_1(t)V_0 + m_2^2(t)}$$

$$= V_0\sqrt{\frac{m_1^2(t)}{V_0^2} + 1 + \frac{2m_1(t)}{V_0} + \frac{m_2^2(t)}{V_0^2}}$$

$\because$ V_0 is large $\Rightarrow$ V_0^2 will be very large.

$\therefore$ Neglect $\frac{m_1^2(t)}{V_0^2}$ and $\frac{m_2^2(t)}{V_0^2}$ terms.

$$\Rightarrow \quad (ED)_{o/p} = V_0\left[1 + \frac{2m_1(t)}{V_0}\right]^{1/2}$$

$$\left\{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...\right\}$$

$$\therefore \Rightarrow \quad (ED)_{o/p} = V_0\left[1 + \frac{1}{2}\times\frac{2m_1(t)}{V_0} - \frac{1}{8}\left(\frac{2m_1(t)}{V_0}\right)^2 + ...\right]$$

$\because$ Since V_0 is large, neglect higher order terms.

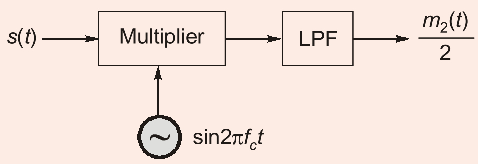
$$= V_0\left[1 + \frac{m_1(t)}{V_0}\right]$$

$$= V_0 + m_1(t)$$

DC term 'V_0' can be blocked by 'DC Blocker' $(ED)_{o/p} = m_1(t)$.

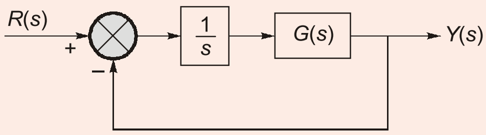
(iii) Recovering $m_2(t)$ by using synchronous detector:

$S(t) = [m_1(t) + V_0]\cos 2\pi f_c t + m_2(t)\sin 2\pi f_c t$



End of Solution

3. (b) For the unity feedback system shown in the figure, the plant $G(s)$ has a step response of $(3 - 6e^{-2t} + 3e^{-4t})u(t)$ and it is placed in cascade with a block of gain $\frac{1}{s}$. Sketch the Nyquist plot of the system and find its gain and phase margins. Also comment whether the closed loop system is stable or not.



[20 Marks]

Solution:

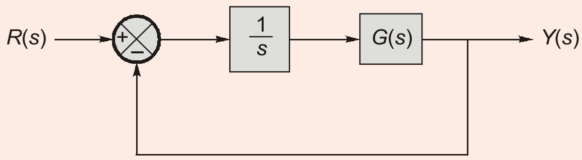
Given, step response of plant $G(s)$, i.e. $(3 - 6e^{-2t} + 3e^{-4t})\ u(t)$

The transfer function of the plant i.e., impulse response is $(12e^{-2t} - 12e^{-4t})\ u(t)$

$$\therefore \qquad G(s) = \frac{12}{s+2} - \frac{12}{s+4} = \frac{12s + 48 - 12s - 24}{(s+2)(s+4)}$$

$$= \frac{24}{(s+2)(s+4)}$$

Given closed loop system is



$\therefore$ Open loop transfer function,

$$G(s)H(s) = \frac{24}{s(s+2)(s+4)}$$

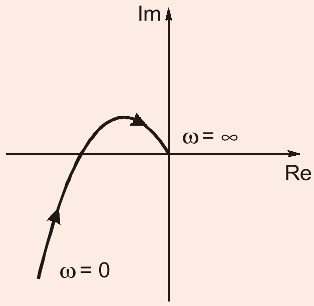
$$G(j\omega)H(j\omega) = \frac{24}{(j\omega)(2+j\omega)(4+j\omega)}$$

$$|G(j\omega)H(j\omega)| = \frac{24}{\omega\sqrt{4+\omega^2}\ \sqrt{16+\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

At $\omega = 0$ $\qquad |G(j\omega)H(j\omega)| = \infty\ ;\ \angle G(j\omega)H(j\omega) = -90^\circ$

At $\omega = \infty$ $\qquad |G(j\omega)H(j\omega)| = 0\ ;\ \angle G(j\omega)H(j\omega) = -270^\circ$



The intersection point with negative real axis,

$$\angle G(j\omega)H(j\omega) = -180^\circ$$

$$-90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right) = -180^\circ$$

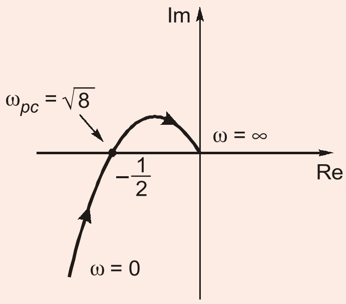
$$\tan^{-1}\left(\frac{\frac{\omega}{2}+\frac{\omega}{4}}{1-\frac{\omega^2}{8}}\right) = 90^\circ$$

$\therefore$ $$\omega_{pc} = \sqrt{8}$$

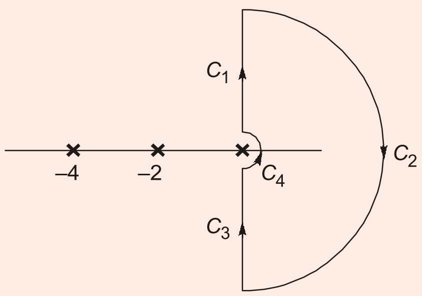
At $\omega_{pc} = \sqrt{8}$, $$M = \left|G(j\omega_{pc})H(j\omega_{pc})\right| = \frac{24}{\omega_{pc}\sqrt{4+\omega_{pc}^2}\sqrt{16+\omega_{pc}^2}}$$

$\therefore$ $$M = \frac{24}{\sqrt{8}\sqrt{4+8}\sqrt{16+8}} = \frac{1}{2}$$

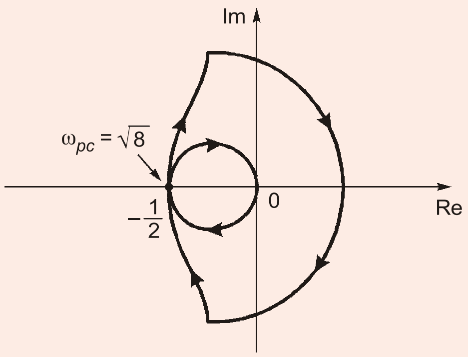
$\therefore$ Gain margin, $$\text{GM} = \frac{1}{M} = 2$$



Nyquist contour :



Nyquist plot :



Gain cross-over frequency,

ω_{gc} : $\left|G(j\omega_{gc})H(j\omega_{gc})\right| = 1$

$$\frac{24}{\omega_{gc}\sqrt{4+\omega_{gc}^2}\sqrt{16+\omega_{gc}^2}} = 1$$

$\therefore$ $\omega_{gc} = 1.938$

Phase margin, $PM = 180° + \angle G(j\omega)H(j\omega)\big|_{\omega=\omega_{gc}} = 20.05°$

Since, phase margin and gain margin both are positive, hence system is stable.

MADE EASY Source

- **ESE 2020 Mains Test Series:** Test-13 (Q.3(a)) **(Click here for reference)**

End of Solution

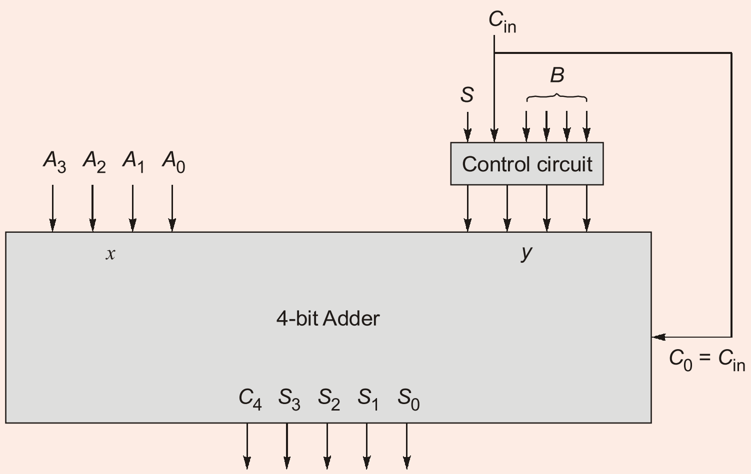
3. (c) Design a 4-bit arithmetic circuit with one selection variables and two four-bit data inputs A and B. The circuit generates the following four arithmetic operations in conjunction with the input carry C_{in}. Draw the logic diagram for the following:

S	$C_{in} = 0$	$C_{in} = 1$
0	$D = A + B$	$D = A - B$
1	$D = A + 1$	$D = A - 1$

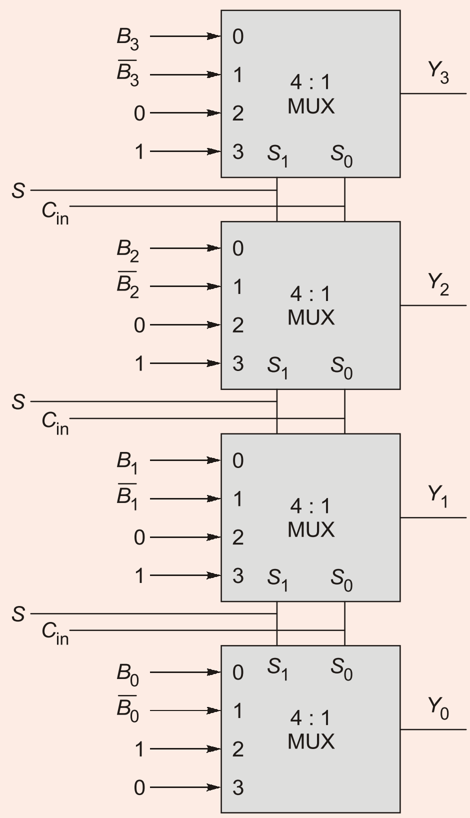
[20 Marks]

Solution:

S	C_{in}	Condition
0	0	$A + B$
0	1	$A - B$
1	0	$A + 1$
1	1	$A - 1$



Design of control circuit :



MADE EASY Source

- **MADE EASY Classnotes : (Click here for reference)**

End of Solution

4. (a) Twelve different audio signals each band-limited to 10 kHz are to be multiplexed and transmitted.

(i) TDM is used with flat top samples of 1 μsec duration and with provision of one extra pulse of 1 μsec duration for synchronisation. If sampling is at Nyquist rate, calculate the spacing between successive samples of TDM signal. What is the bandwidth of this TDM signal?

(ii) If the audio signals are multiplexed using FDM and transmitted using AM-SSB, what is the minimum bandwidth required?

[12 + 8 Marks]

Solution:

(i) Given : $N = 12$, $f_m = 10$ kHz,

Given that each flat top sample is 1 μsec.

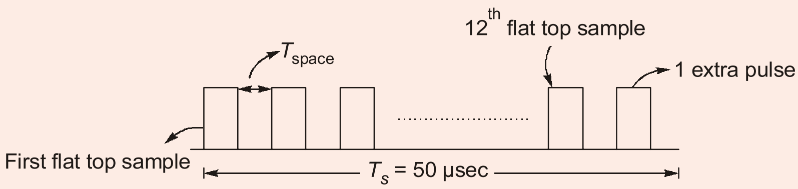
$$T_{sample} = T_b = 1 \text{ μsec}$$

$$f_s = 2\, f_m = 20 \text{ kHz}$$

$$T_s = \frac{1}{f_s} = \frac{1}{20k} = 50 \text{ μsec}$$

Frame time (T_s) = 50 μsec

In one frame, N = 12 samples are present



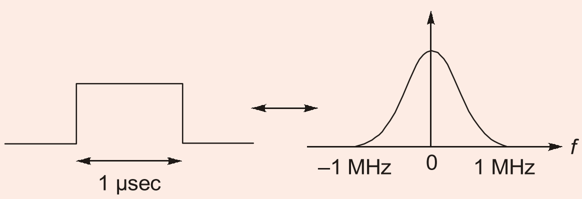
∴ $13\,[T_{sample} + T_{space}] = 50$ μsec

∴ $13\,[1 \text{ μsec} + T_{space}] = 50$ μsec

⇒ $T_{space} = 2.84$ μsec

Bandwidth:

In flat top sampling, pulse by pulse transmission will occur. Each flat top pulse duration = 1 μsec.



∴ Bandwidth = 1 MHz

(ii) For AM - SSB signal;

SSB B.W. = Message B.W. = f_m

CB ≥ SB

CB ≥ 12 [10 kHz]

CB ≥ 120 kHz

∴ Minimum bandwidth = 120 kHz

End of Solution

4. (b) Given a system with transfer function $G(s) = \frac{10}{(s+1)(s+4)}$, find its equivalent state space phase variable canonical representation in the form $\dot{x} = Ax + Bu$, $y = Cx + Du$. Also design a state feedback controller $u = Kx$ such that the system admits a peak response $M_{pw} = 1.25$ in frequency domain and a peak time $t_p = 3.53$ seconds in time step response.

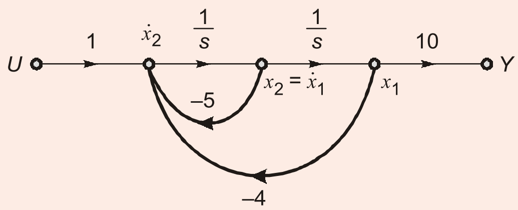
[20 Marks]

Solution:

Given, $$G(s) = \frac{10}{(s+1)(s+4)} = \frac{10}{s^2+5s+4} \times \frac{s^{-2}}{s^{-2}}$$

$$G(s) = \frac{10s^{-2}}{1-[-5s^{-1}-4s^{-2}]}$$

$\therefore$ Number of modes = 3 + 2 = 5



$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -5x_2 - 4x_1 + U$$
$$Y = 10x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \ \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] U$$

Characteristic equation, $|sI - (A - BK)| = 0$

$\Rightarrow$ $s^2 + s(5 - K_2) + (4 - K_1) = 0$ ...(i)

Given, $$M_r = 1.25 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$\therefore$ $$\xi = 0.447 \simeq 0.5$$

Peak time, $$t_p = 3.53 = \frac{\pi}{\omega_d}$$

$$3.53 = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$\therefore$ $$3.53 = \frac{\pi}{\omega_n\sqrt{1-(0.5)^2}}$$

$\therefore$ $$\omega_n \simeq 1$$

$\therefore$ Characteristic equation of the controller,

$$q(s) = s^2 + 2\xi\omega_n s + \omega_n^2$$

$$q(s) = s^2 + 2\times0.5\times1\times s + 1$$

$$q(s) = s^2 + s + 1 \quad \text{...(ii)}$$

By comparing equations (i) and (ii),

$$\therefore \quad K = [K_1 \;\; K_2] = [3 \;\; 4]$$

MADE EASY Source

- **MADE EASY Classnotes : (Click here for reference)**

End of Solution

4. (c) Following Register Transfer statements provide the operations to be performed with flip-flop F:

$X_1T_1 : F \leftarrow 0$

$X_2T_2 : F \leftarrow 1$

$X_3T_3 : F \leftarrow G$

$X_4T_4 : F \leftarrow \bar{F}$

In all other conditions, the contents of F do not change. Using J-K flip-flops, draw the logic diagram showing connections of the gates that implement control function for F.

[20 Marks]

Solution:

The input to the flip-flop F is

$X_1T_1 \rightarrow$ always $0 \rightarrow F$

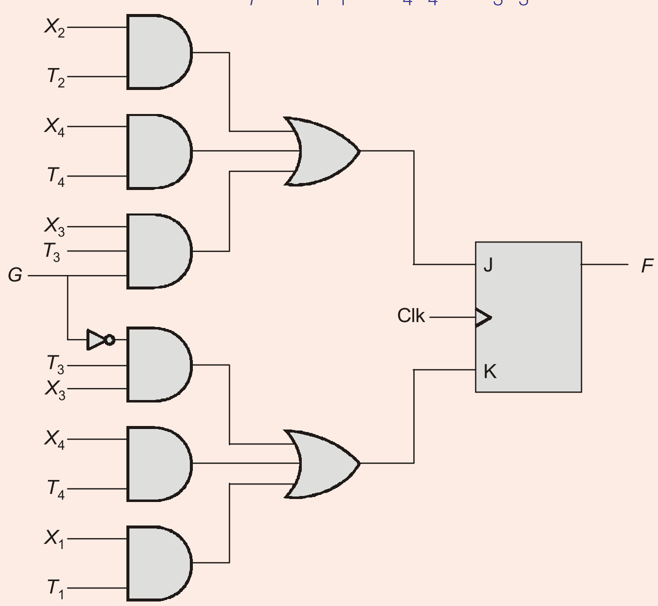
$X_2T_2 \rightarrow 1 \rightarrow F$

$X_3T_3 \rightarrow G \rightarrow F$

$X_4T_4 \rightarrow F \rightarrow F$

$$J_F = X_2T_2 + X_4T_4 + X_3T_3G$$

$$K_F = X_1T_1 + X_4T_4 + X_3T_3\bar{G}$$



End of Solution

5. (a) Band-limited message signal $m(t)$ is encoded using PCM system which uses uniform quantizer and 8-bit binary encoding. If the bit rate is 56 Mb/sec, what is the maximum bandwidth of $m(t)$ for satisfactory operation?
Calculate signal to quantization noise ratio if $m(t)$ is full load single tone sinusoidal signal of frequency 1 MHz.

[10 Marks]

Solution:

Given: Bit rate = 56 Mb/sec = R_b

$$\text{no. of bits} = 8 = n$$

$$R_b = nf_s$$

$$56 \times 10^6 = 8 \times f_s$$

$$f_s = 7 \text{ MHz}$$

According to sampling theorem,

$$f_s \geq 2f_m$$

$$7 \text{ MHz} \geq 2f_m$$

$$f_m \leq 3.5 \text{ MHz}$$

$$f_m = 3.5 \text{ MHz}$$

$$\text{Signal Power, } S = \frac{A_m^2}{2}$$

$$\text{Noise power, } N_Q = \frac{\Delta^2}{12} \qquad \left\{\text{Where } \Delta = \text{Step size} = \frac{2A_m}{2^n}\right\}$$

$$\therefore \qquad \frac{S}{N_Q} = \frac{A_m^2}{2}\frac{(2^n)^2 \times 12}{(2A_m)^2} = \frac{3}{2}(2^{2n})$$

$$= \frac{3}{2}(2^{16}) = 98304$$

$$\frac{S}{N_Q}(\text{in } dB) = 49.9 \text{ dB} \simeq 50 \text{ dB}$$

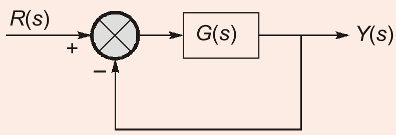
MADE EASY Source

- **Theory Book:** Communication Systems (Page No. 249) **(Click here for reference)**

End of Solution

5. (b) For a unity feedback system shown in the figure, $G(s) = \frac{K}{s(s+\alpha)}$ has resonant frequency 'ω_r' which is $\frac{1}{\sqrt{2}}$ times the damped frequency 'ω_d'. G(s) also has a setting time of $2\sqrt{3}$ seconds, for a 2% tolerance band in its time step response. Calculate the following:

(i) Undamped natural frequency
(ii) Decay rate
(iii) Peak overshoot
(iv) Steady state error for the input $r(t) = t{\cdot}u(t)$



[10 Marks]

Solution:

Given, $$G(s) = \frac{K}{s(s+\alpha)}$$

Resonant frequency, $$\omega_r = \frac{1}{\sqrt{2}}\times\omega_d$$

Settling time, $$t_s = 2\sqrt{3}\text{ sec} \quad \text{for 2\% tolerance}$$

But resonant frequency, $$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$\therefore$ $$\omega_n\sqrt{1-2\xi^2} = \frac{1}{\sqrt{2}}\,\omega_n\sqrt{1-\xi^2}$$

$$1 - 2\xi^2 = \frac{1-\xi^2}{2}$$

$$2 - 4\xi^2 = 1 - \xi^2$$

$$3\xi^2 = 1$$

$\Rightarrow$ $$\xi = \frac{1}{\sqrt{3}}$$

Settling time, $$t_s = \frac{4}{\xi\omega_n} \quad \text{for 2\% tolerance band}$$

i.e., $$\frac{4}{\frac{1}{\sqrt{3}}\omega_n} = 2\sqrt{3}$$

$\therefore$ $$\omega_n = 2\text{ rad/sec}$$

(i) Undamped natural frequency, ω_n = 2 rad/sec

(ii) Decay rate $= \xi\omega_n = \frac{1}{\sqrt{3}} \times 2 = \frac{2}{\sqrt{3}}$

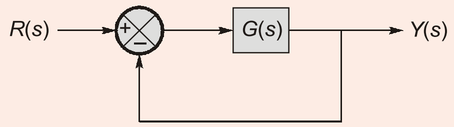
$\therefore$ Decay rate = 1.155 per sec

(iii) Peak overshoot, $\% M_p = e^{-\xi\omega_n/\sqrt{1-\xi^2}} \times 100\%$

$$\% M_p = e^{-\frac{1}{\sqrt{3}}\times 2 \Big/ \sqrt{1-\frac{1}{3}}} \times 100\%$$

$$\% M_p = 24.31\%$$

(iv)



Open loop transfer function,

$$G(s) = \frac{K}{s(s+\alpha)}$$

Input, $r(t) = tu(t)$

$\therefore$ Steady state error (e_{ss}) due to ramp input for type-1 system is

$$e_{ss} = \frac{1}{K_v}$$

where, K_v is velocity error constant,

i.e., $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{K}{s(s+\alpha)}$

$\therefore$ $K_v = \frac{K}{\alpha}$

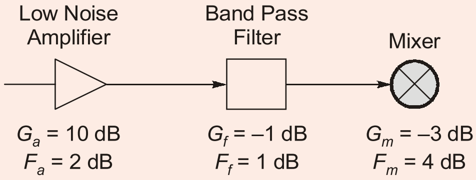
$\therefore$ $e_{ss} = \frac{\alpha}{K}$

MADE EASY Source

- **MADE EASY Classnotes : (Click here for reference)**

End of Solution

5. (c) The block diagram of a wireless receiver front end is shown below:



(i) Compute the overall Noise Figure of the sub-system.
(ii) Compute equivalent noise temperature (overall) assuming system temperature T_0 = 290 K.
(iii) Compute overall gain.
(iv) Compute output noise power assuming input noise power from the feeding antenna at 150 K temperature and 1 F.
(v) Bandwidth of 10 MHz.
(vi) Compute input power if we require minimum signal to noise ratio of 20 dB.
(vii) Compute minimum signal voltage assuming characteristic impedance of 150 Ω.

[10 Marks]

Solution:

(i) Overall Noise Figure (F_{eq}) is given as:

$$F_{eq} = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f}$$

Given: G_a = 10 dB = 10; G_f = –1 dB = 0.79; G_m = –3 dB = 0.5

F_a = 2 dB = 1.58; F_f = 1 dB = 1.25; F_m = 4 dB = 2.51

$$\therefore \quad F_{eq} = 1.58 + \frac{1.25 - 1}{10} + \frac{2.51 - 1}{10 \times 0.79} = 1.79$$

(ii) Equivalent Noise Temperature (T_{eq}) will be;

$$T_{eq} = T_0(F_a - 1) + \frac{T_0(F_f - 1)}{G_a} + \frac{T_0(F_m - 1)}{G_a \cdot G_f}$$

$$= 290(1.58 - 1) + \frac{290(1.25 - 1)}{10} + \frac{290(2.51 - 1)}{10 \times 0.79}$$

$$T_{eq} = 230.88 \text{ K}$$

(iii) Overall Gain = $G_a G_f G_m$

= 10 × 0.79 × 0.5

= 3.95

(iv) Output noise power assuming noise power from the feeding antenna at 150 K temperature and IF bandwidth of 10 MHz can be calculated as:

$$\text{Output noise power} = KT_e' BGF = P_0$$

where, $T_e' = T_{Ant} + T_e = 150\text{ K} + 230.8\text{ K} = 380.8\text{ K}$

$\therefore$ $P_0 = 1.38 \times 10^{-23} \times 380.8 \times 10 \times 10^6 \times 3.95 \times 1.79$

$P_0 = 0.37 \times 10^{-12}$ Watt

$P_0 = 0.37$ pWatt

(vi) We have taken bandwidth as 10 MHz from point 5 of the question.

Input power if we require minimum signal to noise ratio of 20 dB.

$(S/N)_i = 20\text{ dB}$

$(S/N)_i = 100$

$S_i = N_i \times 100 = \text{KTB} \times 100$

$S_i = 1.38 \times 10^{-23} \times 290 \times 10 \times 10^6 \times 100$

$S_i = 4.002$ Pico Watt

(vii) Given, characteristic Impedance = 150 Ω = R

$$P = \frac{V^2}{2R}$$

$$V = \sqrt{2P \times R}$$

$$V = \sqrt{2 \times 4.002 \times 10^{-12} \times 150}$$

$$V = 34.64 \times 10^{-6}\text{ Volt}$$

Minimum Signal Voltage is 34.64 μ volt.

MADE EASY Source

- **Theory Book:** Communication Systems (Page No. 99) **(Click here for reference)**

End of Solution

5. (d) Normalised radiation intensity of an antenna is given by

$U_n(\theta) = 1,\ 0 \le \theta < 30°$

$= \dfrac{\cos\theta}{0.866};\ 30° \le \theta < 90°$

$= 0;\ 90° \le \theta \le 180°$

It is independent of ϕ.

Determine exact directivity and maximum aperture area at operating frequency of 900 MHz.

[10 Marks]

Solution:

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} u(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= 2\pi\left[\int_0^{30^\circ} \sin\theta\, d\theta + \int_{30^\circ}^{90^\circ} \frac{\cos\theta\sin\theta}{0.866} d\theta\right]$$

$$= 2\pi\left[\int_0^{\pi/6} \sin\theta\, d\theta + \int_{\pi/6}^{\pi/2} \frac{\cos\theta\sin\theta}{0.866} d\theta\right]$$

$$= 2\pi\left[[-\cos\theta]_0^{\pi/6} + \frac{1}{0.866}\left(\frac{-\cos^2\theta}{2}\right)\Bigg|_{\pi/6}^{\pi/2}\right]$$

$$= 2\pi[-0.866 + 1 + 0.433]$$

$$= 3.5626$$

$\therefore$
$$D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \times 1}{3.5626}$$

$$= 3.5273$$

$$= 5.4745 \text{ dB}$$

Now,
$$A_e = \left(\frac{\lambda^2}{4\pi}\right)D$$

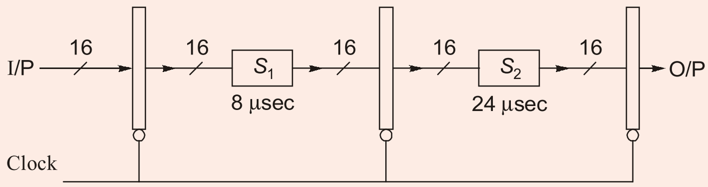
$$A_e = 0.0311$$

MADE EASY Source

- **Theory Book:** Electromagnetics (Page No. 194)
- **MADE EASY Classnotes :** **(Click here for reference)**

End of Solution

5. (e) The figure shown below indicates a two-state pipeline with stage delays indicated below the stages. Latch delays are to be ignored.



(i) Calculate throughput and latency of the pipeline shown above.

(ii) The pipeline stage 2 is not split in three equal sub-stages. Find out the new throughput and latency for the complete pipeline.

[5 + 5 Marks]

Solution:

(i) In pipelining, we take cycle time as maximum stage delay plus the register delay if there is any. Here is no register delay.

So, $t_p = \max(8\ \mu\text{sec}, 24\ \mu\text{sec})$

$= 24\ \mu s$

Now, latency = No. of stages in pipeline × Cycle time of pipeline

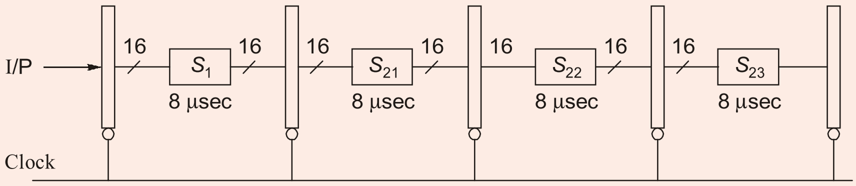
$= 2 \times 24\ \mu\text{sec}$

$= 48\ \mu\text{sec}$

Thus, throughput is $= \dfrac{1}{\text{Cycle time of pipeline}}$

$= \dfrac{1}{24\ \mu\text{sec}} = 0.0416$ instructions/μsec

(ii) When stage 2 with split up in three equal substages each will be of 8 μsec. Let these substages are S_{21}, S_{22}, S_{23} respectively.



Now, $t_p = 8\ \mu\text{sec}$; where t_p = cycle time in pipeline

Now, latency = No. of stages in pipeline × t_p

$= 4 \times 8\ \mu\text{sec}$

$= 32\ \mu\text{sec}$

Now, throughput $= \dfrac{1}{t_p} = \dfrac{1}{8\ \mu\text{sec}} = 0.125$ instructions/μsec

End of Solution

5. (f) An isolator has an insertion loss of 0.5 dB and an isolation of 30 dB. Determine the scattering matrix of the isolator if the isolated ports are perfectly matched to the junction.

[10 Marks]

Solution:

Since the isolator is perfectly matched

$$[s] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

The value of $s_{11} = s_{22} = 0$

Now, given the insertion loss $= -20\log_{10}|s_{21}|$

$$0.5 = -20\log_{10}|s_{21}|$$

$$s_{21} = 0.944$$

$$\text{Isolation} = 30 \text{ dB}$$

$$30 = -20\log_{10}|s_{12}|$$

$$\log_{10}|s_{12}| = \left(-\frac{30}{20}\right)$$

$$s_{12} = 0.0316$$

$$\therefore \quad [s] = \begin{bmatrix} 0 & 0.0316 \\ 0.944 & 0 \end{bmatrix}$$

End of Solution

6. (a) Lossless transmission line operating at 30 MHz has inductance L = 1 μH/m and capacitance C = 100 pF/m. Quarter wave transformer line is used to couple this transmission line to different loads for impedance matching.

(i) Calculate the characteristic resistance of the quarter wave line if load is an antenna offering pure resistance of 70 Ω.

(ii) If load is Z_L = 150 + j100 Ω, determine the characteristic resistance of the quarter wave line.

[12 Marks]

Solution:

(i) For a lossless transmission line

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1\times 10^{-6}}{100\times 10^{-12}}} = 100\ \Omega$$

Now, the characteristic impedance of the quarter wave transformer is equal to

$$Z_T = \sqrt{Z_0 Z_L}$$

$$= \sqrt{100\times 70} = 83.67\ \Omega$$

(ii) Now, Z_L = 150 + j100 Ω and Z_T be the impedance of the transformer

Now,
$$Z_{in} = Z_T\left[\frac{Z_L\cos(\beta l) + jZ_T\sin(\beta l)}{Z_T\cos(\beta l) + jZ_L\sin(\beta l)}\right]$$

$$Z_{in} = \frac{(Z_T)^2}{Z_L} \qquad (\because \beta l = \pi/2)$$

$$Z_T = \sqrt{Z_{in} Z_L}$$

$$= \sqrt{(100)\times(150+j100)}$$

$$= 100\sqrt{(1.5+j)}$$

$$= 100(1.284 + j0.389)$$

$$Z_T = 128.4 + j38.9$$

MADE EASY Source

- **Theory Book:** Electromagnetics (Page No. 174-175) **(Click here for reference)**

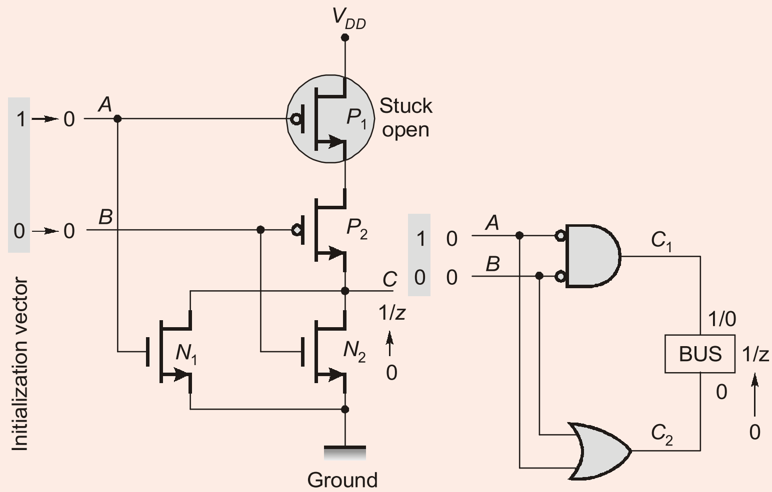
End of Solution

6. (b) Consider a CMOS schematic for 2-input NOR gate.
Design appropriate test scheme to check the following faults through control/observation of voltage/current levels at Input/Output/supply.
(i) One pMOS transistor stuck open
(ii) One nMOS transistor stuck short

[10 + 10 Marks]

Solution:

Stuck open and stuck-short faults are generally referred to as transistor faults. Faults at the physical level are called defects. The electrical or logic-level faults that can be produced by physical defects are classified as defect-oriented faults. Examples of physical defects are broken wires, bridges, improper semiconductor doping, and improperly formed devices. To understand the operation of purely digital MOS circuits, the simple model of the transistor is useful. A MOS transistor as a switch, a defect is modeled as the switch being permanently in either the open or the shorted state.



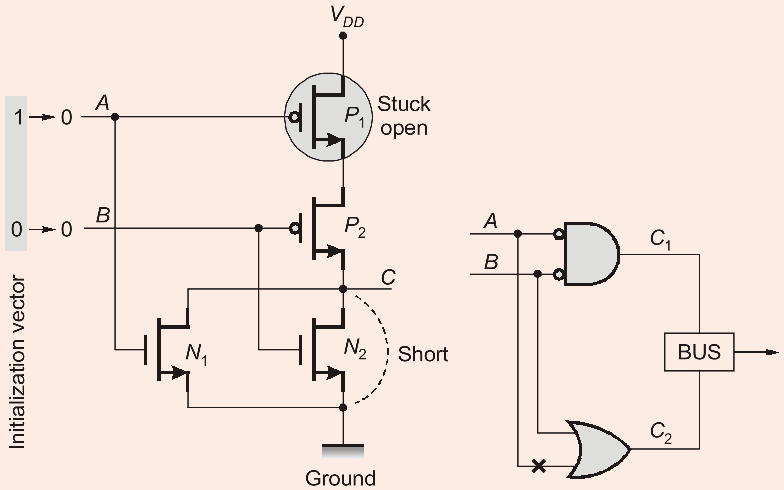
C_1	C_2	C
0	0	z
0	1	1
1	0	0
1	1	S(short)

Stuck-Open Fault:

Figure shows a NOR gate implemented using CMOS technology. P_1 and P_2 are PMOS transistors when the gate terminal inputs A and B are 0. Further, the inputs A and B also applied at the gate of NMOS transistors, N_1 and N_2, If $A = B = 0$ then P_1 and P_2 are shorted in the fault-free circuit and only P_2 is shorted in the faulty circuit. N_1

and N_2 are open in both circuits. In CMOS circuit output C has some parasitic capacitance with the charge from the previous operation of the circuit. In order to detect the fault, Z assumes value 0. The test vectors are 10 → 00 which produces an output 0 → 1 in the good circuit and 0 → 0 in the faulty circuit. Figure also shows gate level model of the CMOS NOR circuit. Here every series interconnection between a supply node to output is replaced by AND gate. Further, a parallel interconnection replaced by an OR gate. The output is produced by a BUS network whose truth table is shown in figure. Furthermore, as unknown three-state simulation, the state in this model indicates the short circuit between the supply nodes. Stuck-open fault of a PMOS transistor is modeled as a stuck-at-1 fault at the corresponding input signal and that of an NMOS transistor as a stuck-at-0 fault.

NMOS stuck short fault at NMOS



Stuck short at N_2 can be modeled as stuck at 1 at Q

if $A = 0$, $B = 0$ then $C_1 = 1$ and $C_2 = 1$

For stuck of 1 (for stuck short at N_2)

If $C_2 = 1$, $C_1 = 1$

Short circuit

The excess current flow in circuit and that can be detected.

End of Solution

6. (c) Write the expression for signal to noise ratio for PIN diode. A silicon PIN photodiode incorporate into the optical receiver has a quantum efficiency of 65% when operating at wavelength of 0.9 μM. The dark current at this point is 3 nA and load resistance is 4 kΩ. The post detection bandwidth of the receiver is 5 MHz and the thermal noise temperature is 20°C. If the overall signal to noise ratio is 5 dB, calculate the incident power.

[20 Marks]

Solution:

There are two main noise inside a PIN diode withut internal gain and they are dark current noise and quantum noise, both of which can be regarded as short noise on photo current. The expression of these noise source can be written as

$$i^2 = 2qB\,(I_p + I_d)$$

To this, the noise due to the background radiation can also be included

Thus, $$i_{TS}^2 = 2qB(\text{Ip} + I_d + I_s)$$

The thermal noise can be given as

$$i_t^2 = \frac{4KTB}{R_L}$$

Thus, the total noise power is given as

$$i_{\text{total}}^2 = i_{TS}^2 + i_t^2$$

The incident optical power can be given as I_p^2

$$\therefore \quad \frac{S}{N} = \frac{I_p^2}{2qB(I_p + I_d + I_b) + \dfrac{4KTB}{R_L}}$$

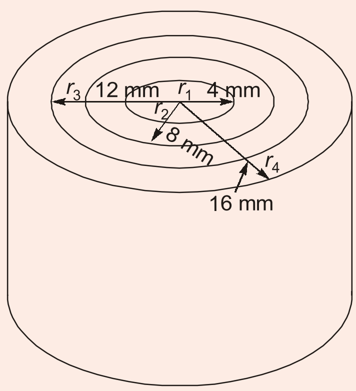
Data insufficient for numerical part.

End of Solution

7. (a) A coaxial capacitor of length 1 m is formed using two concentric cylindrical conductors. The inner conductor has radius 4 mm and the outer conductor radius is 16 mm. The space between them is filled with 3 layers of perfect dielectric materials with different dielectric constant such that $\varepsilon_{r_1} = 5$, 4 mm < ρ < 8 mm; $\varepsilon_{r_2} = 3$, 8 mm < ρ < 12 mm and $\varepsilon_{r_3} = 1$, 12 mm < ρ < 16 mm. If the potential difference between the inner and outer conductor is 100 V, determine the capacitance and charge on the inner conductor. ($\varepsilon_0 = 8.854 \times 10^{-12}$ F/m)

[20 Marks]

Solution:



Let Q be charge per unit length on the outer surface of the inner most conductor of radius r_1.

$\therefore \quad 2\pi rLD = \lambda L$ (By Gauss law)

$$D = \frac{\lambda}{2\pi r}$$

now $\quad D = \varepsilon_0\varepsilon_r E$

$$\therefore \quad E_1 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1}r}, E_2 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r2}r} \text{ and } E_3 = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r3}r}$$

$$\therefore \quad V = \int_{r_1}^{r_2} E_1 dr + \int_{r_2}^{r_3} E_2 dr + \int_{r_3}^{r_4} E_3 dr$$

$$= \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1}r} dr + \int_{r_2}^{r_3} \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r2}r} dr + \int_{r_3}^{r_4} \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r3}r} dr$$

$$V = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1}} \ln\left[\frac{r_2}{r_1}\right] + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r2}} \ln\left[\frac{r_3}{r_2}\right] + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r3}} \ln\left[\frac{r_4}{r_3}\right]$$

$$C = \frac{Q}{V} = \frac{\lambda L}{V}$$

$$C = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r1}} \ln\left[\frac{r_2}{r_1}\right] + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r2}} \ln\left[\frac{r_3}{r_2}\right] + \frac{\lambda}{2\pi\varepsilon_0\varepsilon_{r3}} \ln\left[\frac{r_4}{r_3}\right]}$$

$$= \frac{2\pi\varepsilon_0 L}{\frac{1}{\varepsilon_{r1}} \ln\left[\frac{r_2}{r_1}\right] + \frac{1}{\varepsilon_{r2}} \ln\left[\frac{r_3}{r_2}\right] + \frac{1}{\varepsilon_{r3}} \ln\left[\frac{r_4}{r_3}\right]}$$

$$= \frac{2\pi\varepsilon_0 \times 1000}{\frac{1}{5}\ln\left(\frac{8}{4}\right) + \frac{1}{3}\ln\left(\frac{12}{8}\right) + \ln\left(\frac{16}{12}\right)}$$

$$= 9.9082 \times 10^{-11} \text{ F}$$

$$C = 99.08 \text{ pF}$$

Now, charge on the inner most plate

$$Q = CV$$

$$= 99.08 \times 100 \times 10^{-12}$$

$$Q = 9.908 \text{ C}$$

End of Solution

7. (b) (i) The impulse response of an LTI system is given by

$$h(n) = \left[\left(\frac{1}{4}\right)^n \cos\left(\frac{\pi}{4}n\right)\right]u(n)$$

Realize this system using finite number of adders, multipliers and minimum possible unit delays.

(ii) Consider an initially relaxed system whose output $y(n)$ for $n \geq 0$ is the Fibonacci series. Describe this system in the form of difference equation relating input and output. Obtain impulse response of this system.

[10 + 10 Marks]

Solution:

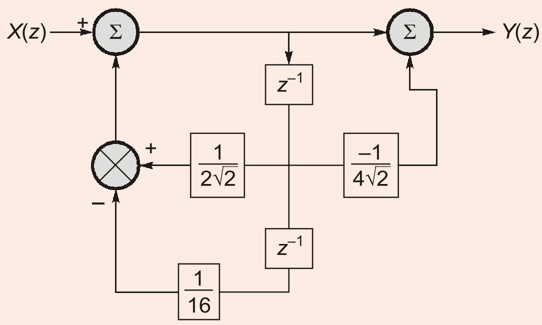
(i) Given impulse response of an LTI system;

$$h[n] = \left[\left(\frac{1}{4}\right)^n \cos\left(\frac{\pi n}{4}\right)\right]u[n]$$

$$\therefore \quad H(z) = \frac{1-\left[\frac{1}{4}\cos\left(\frac{\pi}{4}\right)\right]z^{-1}}{1-\left[2\left(\frac{1}{4}\right)\cos\left(\frac{\pi}{4}\right)\right]z^{-1}+\left(\frac{1}{4}\right)^2 z^{-2}}; \quad |z| > \frac{1}{4}$$

$$H(z) = \frac{1-\frac{z^{-1}}{4\sqrt{2}}}{1-\frac{z^{-1}}{2\sqrt{2}}+\frac{z^{-2}}{16}}; \quad |z| > \frac{1}{4}$$

The above system can be realized as:



(ii) The Fibonacci number is created by adding previous attained two values to attain the new value.

$$y[n] = y[n-1] + y[n-2]$$

Thus, the equation so obtained is a homogeneous equation.

$\therefore$ the solution of second order homogeneous equation can be given as

$$y[n] = C_1\alpha_1^n + C_2\alpha_2^n$$

where α_1 and α_2 are the roots of the equation and C_1 and C_2 are constants.

thus, the delay equation will become

$$z^2 - z - 1 = 0$$

$$\therefore \quad \alpha = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore \quad y[n] = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Now, since the series is Fibonacci.

$$y[1] = 1$$

and

$$y[0] = 0$$

Thus, for $y[0] = 0$, we have

$$C_1 + C_2 = 0$$

$$C_1 = -C_2$$

and for $y[1] = 1$, we have

$$C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$C_1\left(\frac{1+\sqrt{5}}{2}\right) - C_1\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$C_1\left(2\sqrt{5}\right) = 2$$

$$C_1 = \frac{1}{\sqrt{5}} \quad \therefore \quad C_2 = -\frac{1}{\sqrt{5}}$$

thus, the impulse response will become

$$y[n] = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

MADE EASY Source

- **MADE EASY Mains Workbook :** Signals & System (Page No. 697) **(Click here for reference)**

End of Solution

7. (c) A hexagonal cell within a four cell system has a radius of 1.387 km. A total of 60 channels are used in the entire system. If the load per user is 0.029 Erlangs and λ = 1 call/hour, compute the following for an Erlang C system that has 5% probability of a delayed call:

(i) How many users per square km will this system support?
(ii) What is the probability that a delayed call will have to wait for more than 10 s?
(iii) What is the probability that a call will be delayed for more than 10 s?

Erlang *C* Traffic Table
Maximum offered versus *B* and *N*

N \ B	1	2	5	10	15
14	6.70	7.31	8.27	9.15	9.76
15	7.39	8.03	9.04	9.97	10.60
16	8.09	8.76	9.82	10.79	11.44

[20 Marks]

Solution:

$\because$ Radius (r) = 1.387 km

$\therefore$ Area (A) = $2.598 \times (1.387)^2$

$= 4.997 \text{ km}^2$

Number of cells per cluster = 4

$\therefore$ Number, of channels per cluster = $\dfrac{60}{4} = 15$

(i) C = 15 and P_r[delay > 0] = 0.05 = 5%

From the tube it is clear the

Traffic Intensity = 9.04 Erlangs

$\therefore$ Number of users = $\dfrac{\text{total traffic intensity}}{\text{traffic per user}}$

$= \dfrac{9.04}{0.029} = 312$ users

thus we have $\dfrac{312}{4.997} \approx 62$ user/km^2

(iii) For λ = 1

Hold time (H) = $\dfrac{Au}{\lambda} = 0.029$ hr = 104.4 sec

$P_r[\text{delay} > t \mid \text{delay}] = \exp(-(C-A)t \,/\, H)$

$= \exp\left[\dfrac{-(15-9)\times 10}{104.4}\right] = 56.29\%$

(iii) P_r[delay > 0] = 5% = 0.05

$\therefore$ $P_r[\text{delay} > 10] = P_r[\text{delay} > 0] \times P_r[\text{delay} > t \mid \text{delay}]$

$= 0.05 \times 0.5629 = 2.81\%$

End of Solution

8. (a) Consider an air filled rectangular waveguide with inner dimension of width and height a and b respectively ($a > b$).

(i) With clear reasoning describe why propagation is not possible of both electric and magnetic fields in the direction of propagation are zero.

(ii) The propagation constant γ for TE and TM mode is given by

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon$$

where m and n are integers.

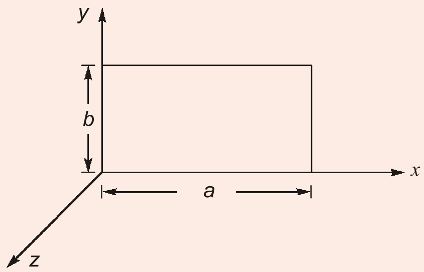
Obtain an expression for minimum frequency below which propagation is not possible.

(iii) If a = 2 cm and b = 1 cm, determine the range of frequency at which only one mode propagates. ($\varepsilon = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m)

[6 + 6 + 8 Marks]

Solution:

(i) For a rectangular waveguide.



1. According to boundary condition the tangential component of the electric field must be zero,

$$E_t = 0$$

2. The normal component of magnetic field must be zero

$$H_n = 0$$

at boundary $E_x = E_2 = 0$ at $y = 0$ and $y = b$

$E_y = E_2 = 0$ at $x = 0$ and $x = b$

Applying Maxwells equation we have,

$$\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) = -j\omega\mu\, H_x$$

$$\left(\gamma E_x + \frac{\partial E_z}{\partial x}\right) = j\omega\mu H_y$$

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\, \mu H_z$$

$$\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) = j\omega\varepsilon\, E_x$$

$$\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right) = -j\omega\, \varepsilon E_y$$

$$\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_y}{\partial y}\right) = j\omega\varepsilon E_z$$

From wave equation,

$$\nabla^2 \bar{E} + \gamma^2 \bar{E} = 0$$

$$\nabla^2 \bar{H} + \gamma^2 \bar{H} = 0$$

where, $\gamma^2 = -\omega^2\mu\varepsilon$ for nonconducting medium on solving, we have

$$\therefore \quad E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - j\frac{\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + j\frac{\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + j\frac{\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - j\frac{\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Now, from the above equation if both E_z and H_z are zero, thus all the fields inside the rectangular waveguide will be zero, hence both the fields can not be zero in the direction of propagation of the wave.

(ii)

$$\gamma^2 = \left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2 - \omega^2\mu\varepsilon$$

$$= \sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2 - \omega^2\mu\varepsilon}$$

Now, if $\omega^2\mu\varepsilon < \left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2$

then the wave will not propagate thus

$$\omega^2\mu\varepsilon = \left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2$$

$$\omega = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi m}{a}\right)^2 + \left(\frac{\pi n}{b}\right)^2}$$

$$f_c = \frac{v_0}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

(iii) For a = 2 cm and b = 1 cm

For TE_{10} $$f_c = \frac{v_0}{2a} = \frac{3\times10^8}{2\times2\times10^{-2}} = 7.5 \text{ GHz}$$

For TE_{01} $$f_c = \frac{v_0}{2b} = \frac{3\times10^8}{2\times10^{-2}} = 15 \text{ GHz}$$

Thus, $7.5 \text{ GHz} < f_c < 15 \text{ GHz}$

MADE EASY Source

- **Theory Book:** Electromagnetics (Page No. 229) **(Click here for reference)**

End of Solution

8. (b) A display is connected to port P1 of 8051 microcontroller. A sequence of 7-bit-patterns are to be displayed in cyclic manner continuously. Write a program is 8051 assembly to display the bit-patterns (8-bit each) with a delay of 1 second between each pair of bit-patterns. The bit-patterns are stored in program memory space at the start at location 400 H. Assume that sub-routing for delay is available directly. Comment on your program appropriately and mention any necessary assumptions explicitly.

[20 Marks]

Solution:

- Assuming that the 7 bit patterns are stored in program memory space from 400 H.
- Assuming the delay subrouting is available directly for 1 second.
- Considering display in connected to part 1 (P1) of 8051 as per given data.
- The program starts at 0000H.
- Register R_1 is considered as count register to count '7' patterns.
- Data pointer DPTH is used to point the starting address bit patterns @ 400 H.
- Assume patterns displayed are from 0-9.
-

	Memory
400H	00
401H	01
402H	02
403H	03
404H	04
405H	05
406H	06
407H	07
408H	08
409H	09

Assume bit patterns codes in memory are 00, 01, 02, 03, 04, 05, 06, 07, 08, 09 for 0 – 9 values.

```
            ORG 0000#
START :     MOV R1, #10           ;  R1 ® no. of patterns n
            MOV DPTR, #400#       ;  Initialize ROM pointer
BACK  :     CLRA                  ;  Accumulator = 0
            MOVC A, @ A + DPTR    ;  data from 400 H to accumulator
            MOV P1, A             ;  output data to port 1 i.e., display
        CALL Delay                ;  delay of 1 second (Absolute call)
        INC DPTR                  ;  Increment Memory Pointer
        DJN2 R1, BACK             ;  decrement R1 Register and check if non-zero
                                     Repeat loop until R1 → 0
        SJMP start                ;  Repeat continuously
        ORG 0400H                 ;  Store 7-bit patterns from 400 H.
        DB  00H, 01H, 02H, 03H, 04H, 05H, 06H, 07H, 08H, 09H
        END                       ;  Direct line for end of file
```

End of Solution

8. (c) The dominant mode TE_{10} is propagated in a rectangular waveguide of dimensions a = 6 cm and b = 4 cm. The distance between maximum and minimum is found to be equal to 4.47 cm with the help of travelling wave detector. Determine the signal frequency.

[20 Marks]

Solution:

a = 6 cm; b = 4 cm

Distance between maxima and minima = $\lambda_g/4$, where λ_g is the wavelength in waveguide.

$$\frac{\lambda_g}{4} = 4.47 \text{ cm} \quad \text{(Given)}$$

$$\lambda_g = 4 \times 4.47 = 17.88 \text{ cm}$$

Also,

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\lambda_c = 2a = 2 \times 6 = 12 \text{ cm}$$

$$\frac{1}{\lambda^2} = \frac{1}{(17.88)^2} + \frac{1}{(12)^2}$$

$$\lambda = 9.9639 \text{ cm}$$

Also, operating frequency, $f = \frac{c}{\lambda} = \frac{3\times10^8}{9.9639} \times 10^2 \text{ Hz}$

$$f = 3.0108 \text{ GHz}$$

MADE EASY Source

- **ESE 2020 Mains Test Series:** Test-11 (Q.5(b)) **(Click here for reference)**

End of Solution

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