2019

RANK Improvement WORKBOOK

Answer key and Hint of
Objective & Conventional *Questions*

Civil Engineering

Structural Analysis

MADE EASY Publications

1 Determinacy and Indeterminacy

LEVEL 1 Objective Questions

1.	(a)
2.	(d)
3.	(a)
4.	(9)
5.	(2.6)
6.	(a)
7.	(3)
8.	(3)
9.	(c)
10.	(b)
11.	(d)
12.	(8)
13.	(b)
14.	(a)

LEVEL 2 Objective Questions

15.	(d)
16.	(b)
17.	(a)
18.	(a)
19.	(c)
20.	(a)
21.	(a)
22.	(d)
23.	(b)
24.	(11)
25.	(a)
26.	(d)

2 Influence Line Diagram and Rolling Loads

LEVEL 1 Objective Questions

1. (d)
2. (c)
3. (b)
4. (b)
5. (d)
6. (240)
7. (22.5)
8. (c)
9. (1)
10. (a)
11. (b)
12. (c)
13. (a)

LEVEL 2 Objective Questions

14. (1.783)
15. (a)
16. (c)
17. (c)
18. (b)
19. (b)
20. (d)
21. (a)
22. (d)
23. (195.488)

LEVEL 3 Conventional Questions

Solution : 1

T = 60 kN, C = 135 kN

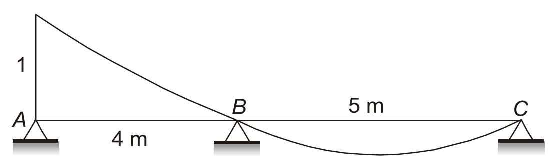
Solution : 2

$250\sqrt{3}$ (tension)

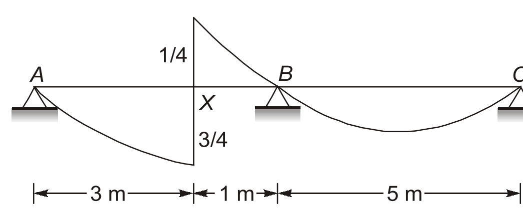
Solution : 3

Using Muller-Breslau Principle,

1. Influence line diagram for reaction at A, R_A



2. Influence line diagram for shear force at X



Solution : 4

26.44 t

Solution : 5

207.9 kNm

Solution : 6

SF = (–)79.37 kN

BM = 293.76 kN

Solution : 7

H = 587.5 kN and M = 750 kNm

Solution : 8

SF = + 62.5 kN, 303.75 kNm

Solution : 9

SF = +60 kN, M_{max} = 590.7 kNm

Solution : 10

60 kN and 160 kNm

■■■■

3 Arches and Suspended Cables

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(a)
4.	(12.02)
5.	(45.76)
6.	(a)
7.	(b)
8.	(35.35)
9.	(a)
10.	(c)
11.	(c)
12.	(c)
13.	(b)
14.	(c)
15.	(c)
16.	(b)
17.	(d)
18.	(a)
19.	(a)
20.	(b)
21.	(b)

LEVEL 3 Conventional Questions

Solution : 1

$H = 0, \quad M = 0$

Solution : 2

$H_B = 9.77$ t

Solution : 3

$$H = \frac{WL}{8r}$$

Solution : 4

$F = 2.017$ kN, $N = 3.188$ kN,

$M_{max}(+) = 7.68$ kNm, $M_{max}(-) = -2$ kNm

Solution : 5

$H = 26.94$ kN, $M = -12.68$ kNm, $N = 28.89$ kN, $F = 7.12$ kN

Solution : 6

$T_{AC} = 26.4$ N

Solution : 7

$T_{max} = 950.3$ kN

$(SF)_{max} = 75$ kN

$(M)_{max} = 774$ kNm

Solution : 8

BM = –281.25 kNm

N = 201.95 kN

Radial shear = 0 kN

Solution : 9

N = 11.86 kN

F = 10.49 kN

M = 31.7 kNm

■■■■

4 Methods of Structural Analysis

LEVEL 1 Objective Questions

1.	(5)
2.	(a)
3.	(b)
4.	(d)
5.	(5)
6.	(a)
7.	(17.8)
8.	(0.67)
9.	(c)
10.	(a)
11.	(d)
12.	(b)
13.	(d)
14.	(4)
15.	(0.87)
16.	(50)
17.	(0)
18.	(b)

LEVEL 2 Objective Questions

19.	(1.0027°)
20.	(60)
21.	(c)
22.	(c)
23.	(1.4)
24.	(b)
25.	(0)
26.	(a)
27.	(c)
28.	(c)
29.	(c)
30.	(b)
31.	(c)

LEVEL 3 Conventional Questions

Solution : 1

$$\theta_0 - \frac{Pa^2}{372EI}$$

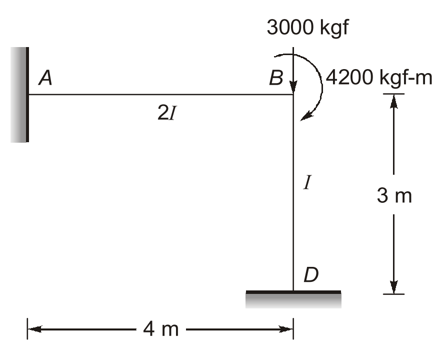
Solution : 2

$M_{12} = 17.97$ kNm $M_{21} = -52.94$ kNm
$M_{23} = 52.94$ kNm $M_{32} = -41.41$ kNm
$M_{34} = 41.42$ kNm $M_{43} = -9.29$ kNm

Solution : 3

For given frame,

Shifting 3000 kgf load at *C* to *B* as $M = 3000 \times 1.4 = 4200$ kgf-m



Using slope deflection method, fixed end moment

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$= \frac{2E(2I)}{4}(\theta_B) = EI\theta_B \qquad \{\theta_A = 0;\ \delta = 0\}$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= \frac{2E(2I)}{4}(2\theta_B) = 2EI\theta_B$$

$$M_{BD} = \frac{2EI}{L}\left(2\theta_B + \theta_D - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{3}(2\theta_B) = \frac{4EI\theta_B}{3} \qquad \{\theta_D = 0;\ \delta = 0\}$$

$$M_{DB} = \frac{2EI}{L}\left(2\theta_D + \theta_B - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{3}(\theta_B) = \frac{2EI}{3}\theta_B$$

Applying moment equilibrium equation at 'B'

$$M_{BA} + M_{BD} - 4200 = 0$$

$$\Rightarrow \qquad 2EI\theta_B + \frac{4}{3}EI\theta_B - 4200 = 0$$

$$\Rightarrow \qquad EI\theta_B = 1260 \text{ kgf-m}$$

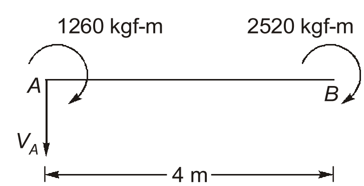
$$M_{AB} = EI\theta_B = 1260 \text{ kgf-m}$$

$$M_{BA} = 2EI\theta_B = 2 \times (1260) = 2520 \text{ kgf-m}$$

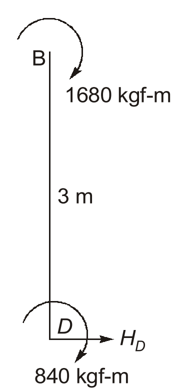
$$M_{BD} = \frac{4EI\theta_B}{3} = \frac{4}{3} \times (1260) = 1680 \text{ kgf-m}$$

$$M_{DB} = \frac{2EI\theta_B}{3} = \frac{2}{3} \times (1260) = 840 \text{ kgf-m}$$

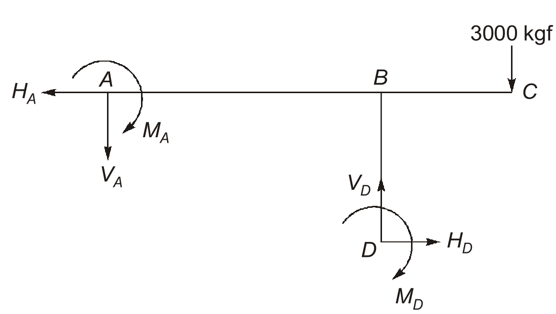
For member *AB*



Using $M_B = 0 \quad \Rightarrow \quad V_A \times 4 = 1260 + 2520 \quad \Rightarrow \quad V_A = 945$ kgf (↓)
For member *BD*



Using $M_B = 0 \quad \Rightarrow \quad 1680 + 840 = H_D \times 3 \quad \Rightarrow \quad H_D = 840$ kgf (→)
Applying equilibrium equations on frame



$$\Sigma F_y = 0 \qquad \Rightarrow \qquad V_A + V_D = 3000$$

$$\Rightarrow \quad -945 + V_D = 3000 \quad \Rightarrow \quad V_D = 3945 \text{ kgf-m } (\uparrow)$$
$$\Rightarrow \quad \Sigma F_X = 0 \quad \Rightarrow \quad H_A = H_D \quad \Rightarrow \quad H_A = 840 \text{ kgf } (\leftarrow)$$

So support reactions

at A, $\quad H_A = 840 \text{ kgf } (\leftarrow); \quad V_A = 945 \text{ kgf } (\downarrow); \quad M_A = 1260 \text{ kgf-m } (\circlearrowright)$

at D, $\quad H_D = 840 \text{ kgf } (\rightarrow); \quad V_D = 3945 \text{ kgf } (\uparrow); \quad M_D = 840 \text{ kgf-m } (\circlearrowright)$

Solution : 4

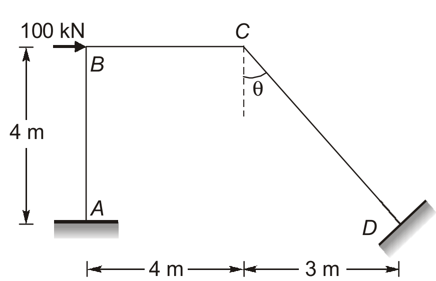
$H_A = 840$ kgf $(\leftarrow)$
$H_D = 840$ kgf $(\rightarrow)$
$V_A = 945$ kgf $(\downarrow)$
$V_D = 3945$ kfg $(\uparrow)$
$M_A = 1260$ kgfm
$M_D = 840$ kgfm

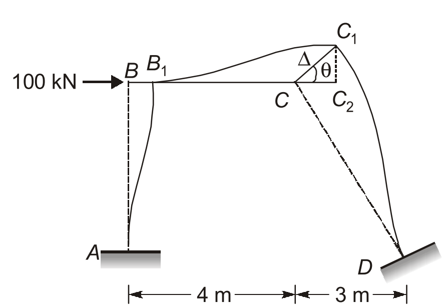
Solution : 5

There are no external loads acting on the portal frame except the sway force of 100 kN from left to right. So only the sway analysis will be carried out.

Distribution Factors:

Joint	Member	Relative Stiffness	Total Relative Stiffness	Distribution Factor
B	BA	$\frac{I}{4} = \frac{I}{4}$	$\frac{7I}{12}$	$\frac{3}{7}$
	BC	$\frac{4}{3}I \times \frac{1}{4} = \frac{I}{3}$		$\frac{4}{7}$
C	CB	$\frac{4}{3}I \times \frac{1}{4} = \frac{I}{3}$	$\frac{7I}{12}$	$\frac{4}{7}$
	CD	$\frac{5}{4}I \times \frac{1}{5} = \frac{I}{4}$		$\frac{3}{7}$





Let the frame $ABCD$ deflect to the position $AB_1\ C_1\ D$ due to the sway force.

Let, $\quad CC_1 = \Delta$

In ΔCC_1C_2, $\quad \cos\theta = \dfrac{CC_2}{CC_1}$

$$BB_1 = CC_2 = \Delta \cos\theta = \frac{4}{5}\Delta$$

$$C_1C_2 = \Delta \sin\theta = \frac{3}{5}\Delta$$

$$M_{FAB} = -\frac{6EI\Delta_{BB_1}}{l^2} = \frac{-6EI\frac{4}{5}\Delta}{16} = M_{FBA}$$

$$M_{FBC} = \frac{6E\frac{4}{3}I\Delta_{C_1C_2}}{l^2} = \frac{6E\frac{4}{3}I\times\frac{3}{5}\Delta}{16} = M_{FBC} = \frac{24}{80}EI\Delta$$

$$M_{FCD} = M_{FDC} = \frac{-6E\times\frac{5I\Delta}{4}}{25} = -\frac{6EI\Delta}{20} = -\frac{24EI\Delta}{80}$$

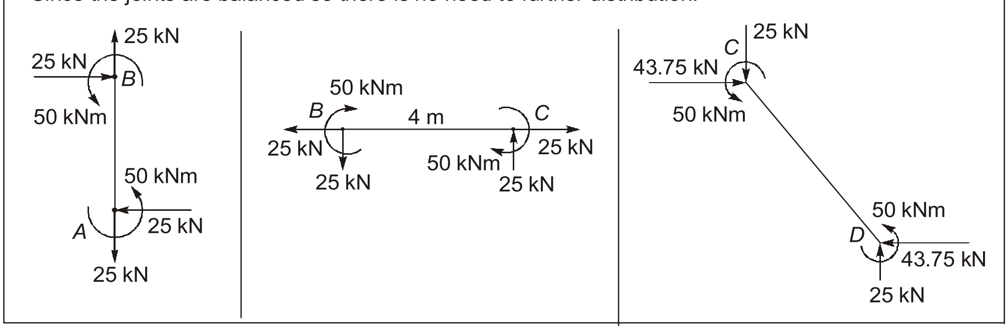
Ratio of fixed end moments for *AB*, *BA*, *BC*, *CB*, *CD* and *DC*

$$-\frac{24EI\Delta}{80} : -\frac{24EI\Delta}{80} :: \frac{24EI\Delta}{80} : \frac{24EI\Delta}{80} :: -\frac{24EI\Delta}{80} : -\frac{24EI\Delta}{80}$$

–50 : –50 :: 50 : 50 :: –50 : –50

	A	B		C		D
		3/7	4/7	3/7	4/7	
Col.(a)	–50	–50	50	50	–50	–50

Since the joints are balanced so there is no need to further distribution.



$$\text{Vertical reaction at } D = \frac{M_{BC}+M_{CB}}{L_{BC}} = \frac{50+50}{4} = 25 \text{ kN}(\uparrow)$$

$\therefore$ $$\text{Horizontal reaction at } A = \frac{M_{AB}+M_{BA}}{L_{AB}} = \frac{-50-50}{4} = -25 \text{ kN}(\rightarrow) = 25 \text{ kN}(\leftarrow)$$

$$\text{Horizontal reaction at } D = \frac{M_{CD}+M_{DC}-V_D\times 3}{4} = \frac{-50-50-25\times 3}{4}$$

$$= -43.75(\rightarrow) = 43.75 \text{ kN }(\leftarrow)$$

Let the sway force S be acting from left to right and then

$$H_A + H_D + S = 0$$

$$-25 - 43.75 + S = 0$$

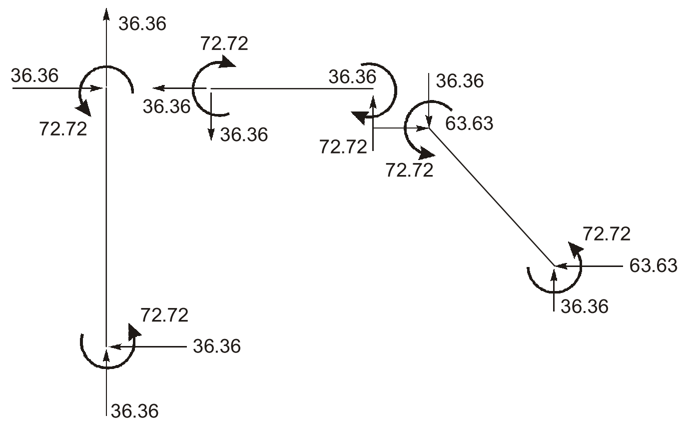
$$\Rightarrow \quad S = 68.75 \text{ kN }(\rightarrow)$$

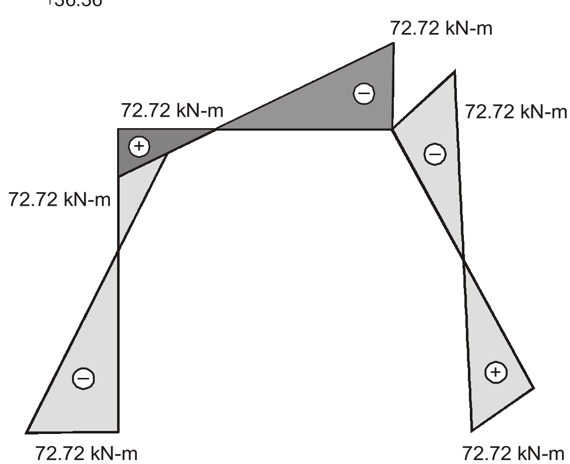
For a sway force of 68.75 kN, the sway moment are as per Col. (a)

For the actual sway force of 100 kN, the corresponding sway moments will be

$\frac{100}{68.75} \times$ Col. (a) moments

	A		B		C	D
col. (a)	–50	–50	50	50	–50	–50
Actual sway moments $\frac{100}{68.75}$ × col. (a)	–72.72	–72.72	72.72	72.72	–72.72	–72.72





Bending Moment Diagram

Solution : 6

Step 1 : To find the fixed end moments

First we write the fixed end moments

$$FEM_{12} = \frac{80 \times 1 \times 2^2}{3^2} = -35.55 \text{ kN.m}$$

$$FEM_{21} = -80 \times 2 \times 12 = 17.78$$

$$FEM_{23} = \frac{50 \times 4^2}{12} = -66.67 \text{ kN.m}$$

$$FEM_{32} = \frac{-50 \times 4^2}{12} = 66.67 \text{ kN.m}$$

$$FEM_{34} = \frac{40 \times 4}{8} = -20.00 \text{ kN.m}$$

$$FEM_{43} = -\frac{40 \times 4}{8} = 20.00 \text{ kN.m}$$

Step 2 : To write end moments

The continuous beam under goes rotation of joints at supports 2 and 3. We may designate them as θ_2 and θ_3 respectively. As the supports are rigid no translation of joints is possible.

Now we can write down slope deflection equations for moments.

$$M_{12} = -35.55 + 2\left(\frac{1.5EI}{3}\right)(0 + \theta_2)$$

$$M_{21} = 17.78 + 2\left(\frac{1.5EI}{3}\right)(0 + 2\theta_2)$$

$$M_{23} = -66.67 + 2\left(\frac{2EI}{4}\right)(2\theta_2 + \theta_3)$$

$$M_{32} = 66.67 + 2\left(\frac{2EI}{4}\right)(\theta_2 + 2\theta_3)$$

$$M_{34} = -20.00 + \frac{2EI}{4}(2\theta_3 + 0)$$

and
$$M_{43} = 20.00 + \frac{2EI}{4}(\theta_3 + 0)$$

Step 3 : To write equilibrium conditions

In these equations the unknowns are θ_2 and θ_3 and the two equilibrium equations are:

$$M_{21} + M_{23} = 0 \quad \text{...(a)}$$

$$M_{32} + M_{34} = 0 \quad \text{...(b)}$$

Substituting the values from above we have

$$4EI\theta_2 + EI\theta_3 = 48.89$$

$$EI\theta_2 + 3EI\theta_3 = -46.67$$

Solving the equations simultaneously, we get

$$EI\theta_2 = 17.58$$

and
$$EI\theta_3 = -21.42$$

Step 4 : To write the end moments

Substituting these values in moments equations, we get

$$M_{12} = 35.55 - 17.58 = 17.97 \text{ kN.m}$$

$$M_{21} = -17.75 + 2(-17.58) = -52.94 \text{ kN.m}$$

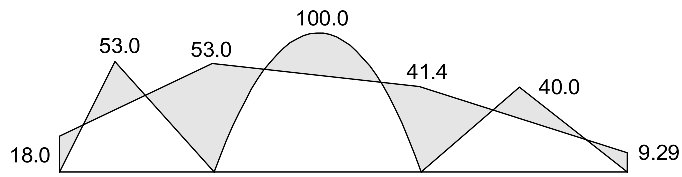
$$M_{23} = 66.67 + 2(-17.58 + 21.42) = 52.93 \text{ kN.m}$$

$$M_{32} = -66.67 - 17.58 + 2(21.42) = -41.41 \text{ kN.m}$$

$$M_{34} = 20.00 + 21.42 = 41.42 \text{ kN.m}$$

$$M_{43} = -20.00 + \left(\frac{-21.42}{2}\right) = -9.29 \text{ kN.m}$$

The moment diagram is shown in figure below.



Moment diagram

Solution : 7

It is seen from inspection that the beam can rotate at joints 2 and 3. There is no possibility of translation of any of the joints. Writing down the fixed end moments, we have

$$FEM_{12} = \frac{20(4)(4)}{12} = 26.67 \text{ kN.m}$$

$$FEM_{21} = -26.67 \text{ kN.m}$$

$$FEM_{23} = \frac{40(4)}{8} = 20.0 \text{ kN.m}$$

$$FEM_{32} = -20.0 \text{ kN.m}$$

$$FEM_{24} = -\frac{20(4)}{8} = -10.0 \text{ kN.m}$$

$$FEM_{42} = +10 \text{ kN.m}$$

Note that for the column, the bottom end is the left end and the top the right end.

Designating the rotations at joints 2 and 3 as θ_2 and θ_3 respectively, the end moments can be written as

$$M_{12} = 26.67 + 2EK\theta_2$$

Taking $EK = 1$, since only relative values are needed,

Similarly,
$$\left.\begin{aligned} M_{21} &= -26.67 + 4\theta_2 \\ M_{23} &= 20 + 4\theta_2 + 2\theta_3 \\ M_{32} &= -20 + 2\theta_2 + 4\theta_3 \\ M_{24} &= -10 + 4\theta_2 \\ M_{42} &= 10 + 2\theta_2 \end{aligned}\right\} \quad \text{...(i)}$$

Now to solve for the two unknown, θ_2 and θ_3, two conditons are used. They are

$$M_{21} + M_{23} + M_{24} = 0 \quad \text{...(ii)}$$

and
$$M_{32} = 0 \text{ (support 3 is a roller support)} \quad \text{...(iii)}$$

Substituting for moment terms from the expression in Eq. (i), we get

$$12\theta_2 + 2\theta_3 = 16.67 \quad \text{...(iv)}$$

$$2\theta_2 + 4\theta_3 = 20.00 \quad \text{...(v)}$$

Solving Eqs. (iv) and (v) simultaneously, we get

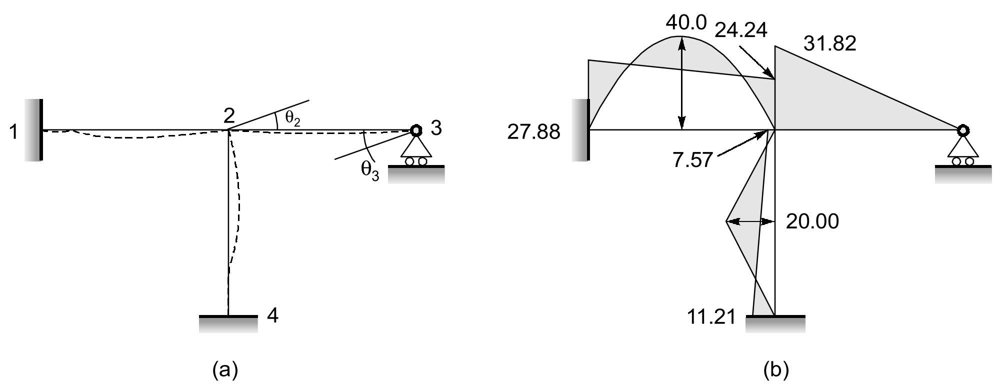
$$\theta_2 = 0.606$$

and
$$\theta_3 = 4.6968$$

Substituting back in Eq. (i), we have

$$M_{12} = 26.67 + 2(0.606) = 27.88 \text{ kN.m}$$
$$M_{21} = -26.67 + 4(0.606) = -24.24 \text{ kN.m}$$
$$M_{23} = 20.0 + 4(0.606) + 2(4.6968) = 31.82 \text{ kN.m}$$
$$M_{24} = -10 + 4(0.606) = -7.57 \text{ kN.m}$$
$$M_{32} = -20 + 2(0.606) + 4(4.6968) = 0$$
$$M_{42} = 10.0 + 2(0.606) = 11.21 \text{ kN.m}$$

The deflected shape and the moment diagram are shown in figure below.



(a) Deflected shape, (b) Moment diagram

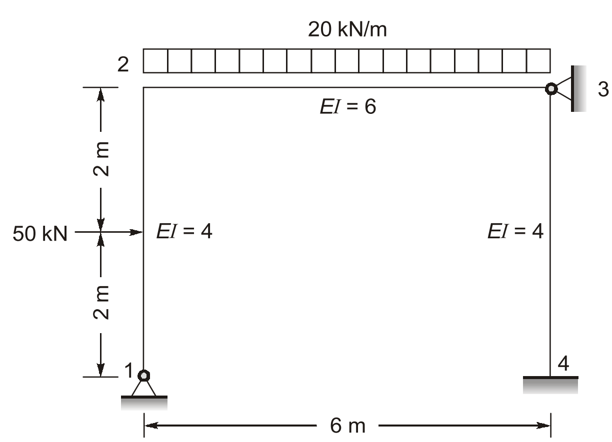
Solution : 8

The support condition at end 1 prevents the frame from undergoing lateral translation. As pointed out, the procedure for the analysis of this frame is the same as that for continuous beams without translation of supports. The fixed end moments due to external loading in the temporarily restrained joints are

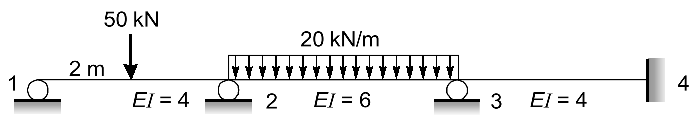
$$FEM_{12} = \frac{30(4)^2}{12} = 40.00 \text{ kN.m}, \quad FEM_{21} = -40.00 \text{ kN.m}$$

$$FEM_{23} = \frac{(100)(4)}{8} = 50.00 \text{ kN.m}, \quad FEM_{32} = -50.00 \text{ kN.m}$$

$$FEM_{34} = \frac{20(3)^2}{12} = 15.00 \text{ kN/m}, \quad FEM_{43} = -15.00 \text{ kN.m}$$



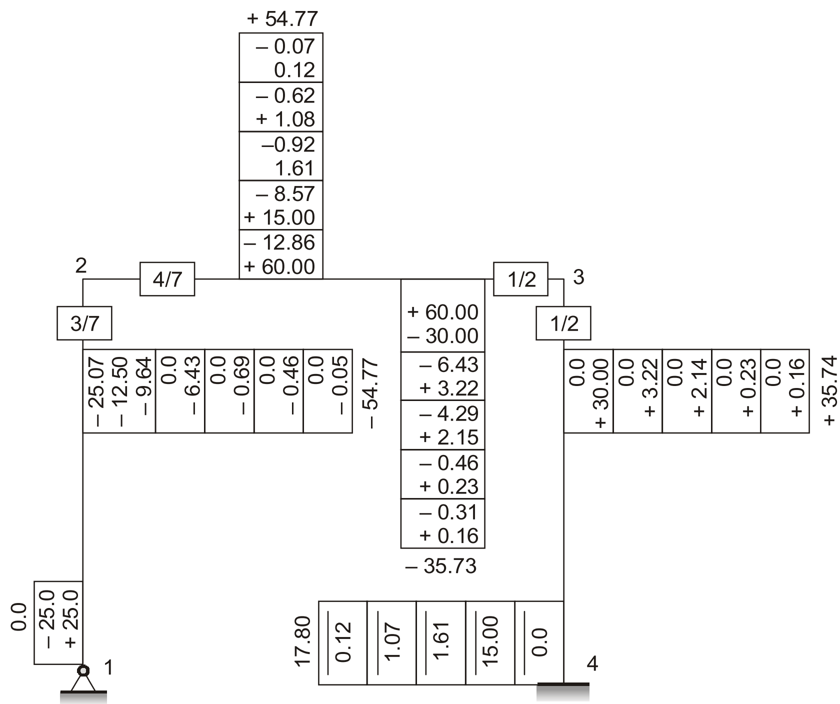
(a) Frame and loading



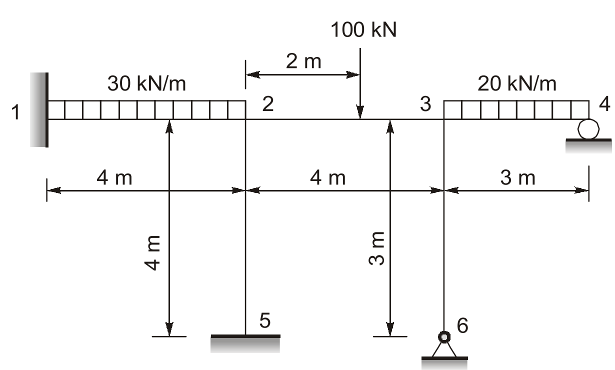
	1	2	2	3	3	4	
Span	4 m		6 m		4 m		
	$\frac{3}{4}\frac{EI}{L} = \frac{3}{4}$		$\frac{EI}{L} = 1$		$\frac{EI}{L} = 1$		Relative stiffness
	1.0	3/7	4/7	1/2	1/2	0	D.F
	+25.00	+25.00	+60.00	–60.00	0.0	0.0	FEM
	–25.00 →	–12.50					Bal. 1 and C.O
		–9.64	–12.86	+30.00	+30.00		Dist.
			+15.00	–6.43		+15.00	C.O
		–6.43	–8.57	+3.22	+3.21		Dist.
			+1.61	–4.29		+1.61	C.O
		–0.69	–0.92	+2.15	+2.14		Dist.
			+1.08	–0.46		+1.07	C.O.
		–0.46	–0.62	+0.23	+0.23		Dist.
			+0.12	–0.31		+0.12	C.O.
		–0.05	–0.07	+0.16	+0.16		Dist.
	0	–54.77	+54.77	–35.73	+35.74	+17.80	Final moments

(b) Frame opened out and distribution carried out

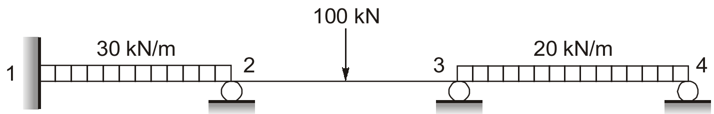
The calculations are performed on a sketch of the frame as shown in Fig. (e). For the girder of the frame, the calculations are entered as in a continuous beam. For columns, however, the values are entered below on a column line as shown in Fig. (e). This arrangement is convenient for single storey frames of any number of bays. For frames of more than one storey, the arrangement which is slightly different but more convenient is shown later.



(c) An alternative way of recording value



(d) Frame and loading



	1	2	2	3	3	4	
		$\frac{l}{4}$		$\frac{l}{4}$		$\left(\frac{3}{4}\right)\frac{l}{3}=\frac{l}{4}$	Re. stiff
	0	1/3	1/3	1/3	1/3	1.0	D.F
	+40.00	–40.00	50.00	50.00	+15.00	15.00	FEM
					+7.50	← +15.00	Dist.4 and C.O
		–3.33	–3.33	–9.17	+9.17		C.O
	–1.67		+4.59	–1.67			C.O
		–1.53	–1.53	+0.56	+0.56		Dist.
	–0.77		+0.28	–0.77			C.O
		–0.09	–0.09	+0.26	+0.26		Dist.
	–0.05		+0.13	–0.05			C.O.
		–0.04	–0.04	–0.02	+0.02		Dist.
	+37.51	–44.99	+50.01	–42.48	+32.51	0	Final moments

Column 2-5 Top	Column 2-5 Bottom	Column 3-6 Top	Column 3-6 Bottom	
$\frac{l}{4}$	$\left(\frac{3}{4}\right)\frac{l}{3}=\frac{l}{4}$			Re. stiff
1/3		1/3		D.F
0	0	0	0	FEM
				Dist.4 and C.O
–3.34	–	+9.16	–	C.O
	–1.67			C.O
–1.53	–	+0.55	–	Dist.
	–0.77	–	–	C.O
–0.10		+0.25		Dist.
	–0.05			C.O.
–0.05		+0.01		Dist.
–5.02	–2.49	+9.97	0	Final moments

Fig. (e)

The fixed end moments are entered in one row. It may be noted that there are no fixed end moments for the columns as there is no load transverse to them. Next the simply supported end 4 is balanced and half of it is carried to support 3 as the carry over moment. Joints 2 and 3 are balanced and the moments are distributed to the three member ends meeting at these joints according to their distribution factors. The distributed moment for the column tops are recorded below under 'column top'. Next the moments are carried over to the farther ends as carry over moments. For columns the carry over is from 'column top' to 'column bottom'. The moments are summed up as usual. As a check it can be verified that the sum of the moments at any joint must be zero. It may be noted that the moments at the bottom of columns are either equal to zero as in the case of hinged supports or equal to half the moment at the top as in the case of fixed bases. The moment diagram is shown in fig. (f) below drawn on the tension face of the members.

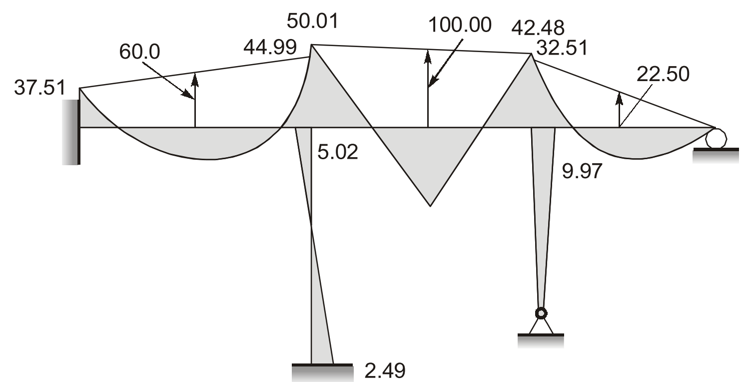
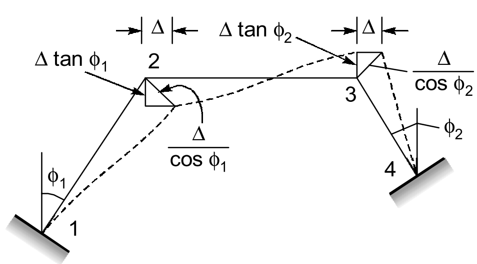


Fig. (f) Moment diagram

Solution : 9

The lateral translation of joints 2 and 3 under external force is shown in figure. If axial deformations are neglected, lateral translation Δ is the same for both joints. The amount of translation transverse to the columns and the beam is indicated in figure. The fixed end moments caused by joints translations only restraining joint rotations are

$$FEM_{12} = FEM_{21} = \frac{6EI}{L_{12}^2}\frac{\Delta}{\cos\phi_1} = \frac{6}{5\times 5}\frac{EI\Delta}{0.8}$$



Lateral translation of joints

1	2		3		4	
$\frac{I}{5}$		$\frac{I}{5}$		$\frac{I}{3.75}$		Re. stiff
0	1/2	1/2	3/7	4/7	0	D.F
+30.00	+30.00	–36.00	–36.00	+53.33	+53.33	FEM
	+3.00	+3.00	–7.43	–9.90		Dist
+1.50		–3.72	+1.50		–4.95	C.O
	+1.86	+1.86	–0.64	–0.88		Dist.
+0.93		–0.32	+0.93		–0.43	C.O
	+0.16	+0.16	–0.40	–0.53		Dist.
+0.08		–0.20	–0.08		–0.05	C.O.
	+0.10	+0.10	–0.03	–0.05		Dist.
+32.51	+35.12	–35.12	–41.99	+41.99	–47.68	Final moments

(c)

Letting $EI\Delta = 100$

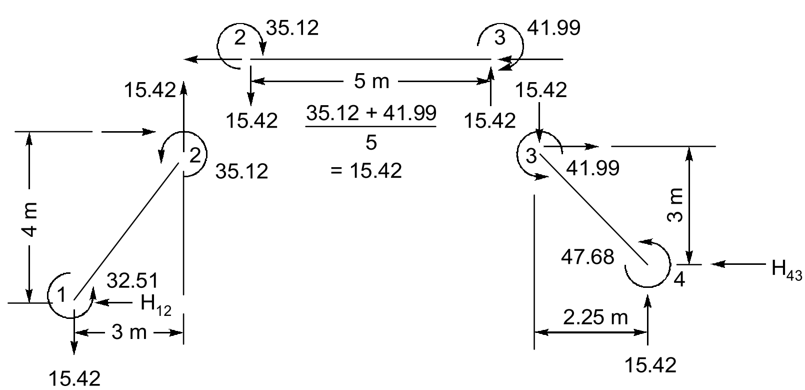
$$FEM_{12} = FEM_{21} = \frac{6 \times 100}{20} = +30.0 \text{ kN.m}$$

$$FEM_{23} = FEM_{32} = -\frac{6EI}{L_{23}^2}(\Delta \tan\phi_1 + \Delta \tan\phi_2)$$

$$= -\frac{6}{5^2}(1.5)EI\Delta = -36.00 \text{ kNm}$$

$$FEM_{34} = FEM_{43} = \frac{6EI\Delta}{L_{34}^2 \cos\phi_2} = \frac{6EI\Delta}{3.75^2 \times 0.8} = 53.33 \text{ kN.m}$$

The moment distribution is carried out using the general procedure and the calculations are recorded on the opened up frame shown in fig. (c). We shall now evaluate the external horizontal force which caused the final moments in the table of fig. (c). To obtain the horizontal force we shall consider the freebody diagram of the beam and column shown ni fig. (d). The summation of moments about 2 on the left inclined column gives



(d) Free-body diagram of frame members

$$\Sigma M_2 = -H_{12}(4) + 15.42(3) + 32.51 + 35.12 = 0$$

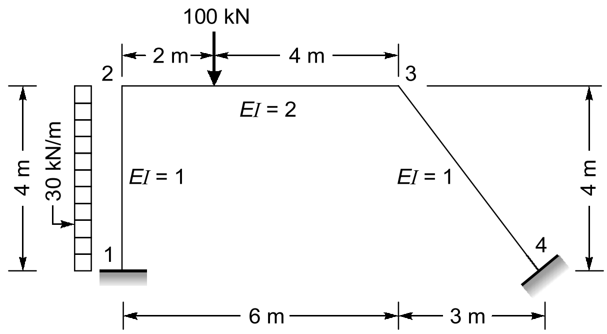
or $\quad H_{12} = 28.47$ kN

Similarly, for the right hand column taking moments about 3 and equating $\Sigma M_3 = 0$, we have

$$-H_{43}(3) + 15.42\,(2.25) + 47.68 + 41.99 = 0$$

or $\quad H_{43} = 41.46$ kN

The resultant external lateral force is $X' = 69.93$ kN from left to right. But the frame was actually subjected to an external force of 50.0 kN only. Therefore, the final moments in the table of Fig. (e) are to be multiplied by a factor $\frac{50}{69.93} = 0.715$. The true moments are



(e) Frame and loading

$$M_{12} = 32.51 \times 0.715 = 23.24 \text{ kN.m}$$
$$M_{21} = -M_{23} = 35.12(0.715) = 25.11 \text{ kN.m}$$
$$M_{34} = -M_{32} = 41.99(0.715) = 30.92 \text{ kN.m}$$
$$M_{43} = 47.68(0.715) = 34.09 \text{ kN.m}$$

Solution : 10

$M_{12} = +73.88$ kNm

$M_{21} = 12.77$ kNm

$M_{23} = -12.77$ kNm

$M_{32} = -40$ kNm

■■■■

5 Trusses

LEVEL 1 Objective Questions

1. (a)
2. (c)
3. (c)
4. (a)
5. (0.56)
6. (3.33)
7. (c)
8. (d)
9. (0.13)
10. (b)
11. (b)

LEVEL 2 Objective Questions

12. (3.33)
13. (c)
14. (d)
15. (d)
16. (2.25)
17. (12)
18. (b)
19. (c)
20. (b)
21. (d)
22. (b)
23. (c)
24. (0.084)
25. (33.33)
26. (a)
27. (b)

LEVEL 3 Conventional Questions

Solution : 1

Member	AO	AB	OB	BC	OC	OD	OE
Forces	20.71 kN	–54.11 kN	41.41 kN	–54.11 kN	–58.58 kN	41.41 kN	20.71 kN

Solution : 2

$F_{lk} = 142.16$ kN (C)

Solution : 3

$F_{5\text{-}6} = 75$ kN (T) $\quad F_{5\text{-}8} = \dfrac{25}{\sqrt{2}}$ kN (T)

$F_{5\text{-}7} = \dfrac{25}{\sqrt{2}}$ kN (C) $\quad F_{5\text{-}4} = 100$ kN (T)

Solution : 4

$F_{CB} = 10$ kN (T), $\quad F_{BE} = 10$ kN (C), $\quad F_{EF} = 30\sqrt{2}$ kN (C)

Solution : 5

$\delta_C = 0.335$ mm

Solution : 6

$F_{2\text{-}3} = 66.67$ kN (C) $\quad F_{7\text{-}3} = 41.67$ kN (C)

$F_{7\text{-}12} = 41.67$ kN (T) $\quad F_{11\text{-}12} = 66.67$ kN (T)

$F_{2\text{-}7} = 50$ kN (T) $\quad F_{7\text{-}11} = 0$ kN

Solution : 7

Memebrs	AC	CD	DE	EB	BD	BA	AD	BC
Forces (kN)	–8.75	–6.56	–15	0	11.25	–6.56	–14.06	10.94

Solution : 8

Members	BC	CE	ED	DA	AC	CD	BD
Forces (kN)	26.66	0	0	26.66	–33.33	20	–33.33

Solution : 9

$H_D = 400$ kN, $\quad V_b = 100$ kN

(a) $x = -43.4$m (b) $x = 0$ (c) $x = 44.3$ kN

Solution : 10

$\Delta = 15.168$ mm, $\quad \theta = 3.326°$

■■■■

6 Matrix Method of Structural Analysis

LEVEL 1 Objective Questions

1.	(d)
2.	(a)
3.	(a)
4.	(d)
5.	(c)
6.	(d)
7.	(b)
8.	(d)
9.	(b)
10.	(a)

LEVEL 2 Objective Questions

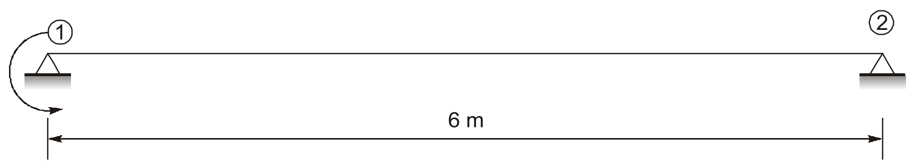
11.	(b)
12.	(c)
13.	(a)
14.	(906.48)
15.	(a)
16.	(a)
17.	(a)
18.	(a)
19.	(c)
20.	(c)
21.	(c)
22.	(133.93/EI)
23.	(b)
24.	(c)

LEVEL 3 Conventional Questions

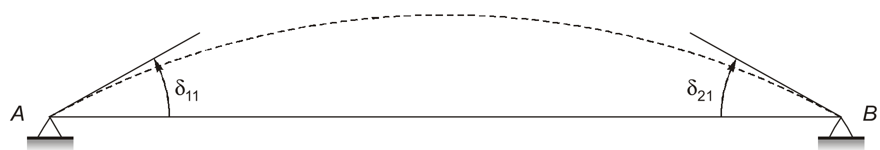
Solution : 1

The degree of static indeterminacy of the beam is two. Let the basic determinate structure be obtained by releasing bending moments at A and B. This can be done by inserting by hinges at A and B. Like simply supported beams.

The released structure and coordinates 1 and 2 assigned to redundant bending momets at A and B as shown below.



To develop flexibilty matrix, a unit force should be applied successively at coordinate 1 and 2. Thus to generate the first column of flexibility matrix, apply a unit force at coordinate 1. The deflector curve is shown due to unit force at 1.



$$\delta_{11} = \frac{L}{3EI} = \frac{6}{3EI} = \frac{2}{EI}$$

$$\delta_{21} = \frac{-6}{6EI} = \frac{-1}{EI}$$

To generate second column of flexibility matrix, apply unit force at coordinate 2.

$$\delta_{12} = \frac{-1}{EI}$$

$$\delta_{22} = \frac{6}{3EI} = \frac{+2}{EI}$$

Hence, flexibility matrix [δ] is given by;

$$[\delta] = \frac{1}{EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Deflection due to applied loads at nodes 1 and 2 are given as:

$$\Delta_{1L} = \frac{PL^2}{16EI} + \frac{wL^3}{24EI} + \frac{M_oL}{24EI} = \frac{12\times 6^2}{16EI} + \frac{2\times 6^3}{24EI} + \frac{24\times 6}{24EI} = \frac{51}{EI}$$

$$\Delta_{2L} = \frac{-PL^2}{16EI} - \frac{2\times 6^3}{24EI} + \frac{24\times 6}{24EI} = \frac{-39}{EI}$$

We know, $[P] = [\delta]^{-1}\{[\Delta]-[\Delta_L]\}$

If net displacements at redundants are zero,

$$\Delta_1 = \Delta_2 = 0$$

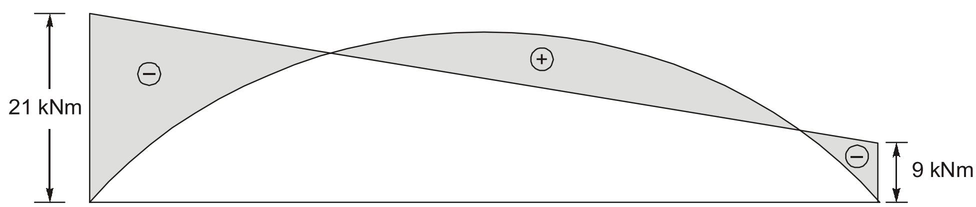
$\therefore$ $$[P] = [\delta^{-1}][\Delta_L]$$

$\Rightarrow$ $$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = -\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 51 \\ -39 \end{bmatrix} \times \frac{1}{EI}$$

$\Rightarrow$ $$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{-EI}{(4-1)} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 51 \\ -39 \end{bmatrix} \times \frac{1}{EI}$$

$\Rightarrow$ $$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{-EI}{3} \begin{bmatrix} 63 \\ -27 \end{bmatrix} \times \frac{1}{EI} = \begin{bmatrix} -21 \\ 9 \end{bmatrix}$$

$\Rightarrow$ $$M_A = -21 \text{ kNm} \qquad M_B = 9 \text{ kNm}$$

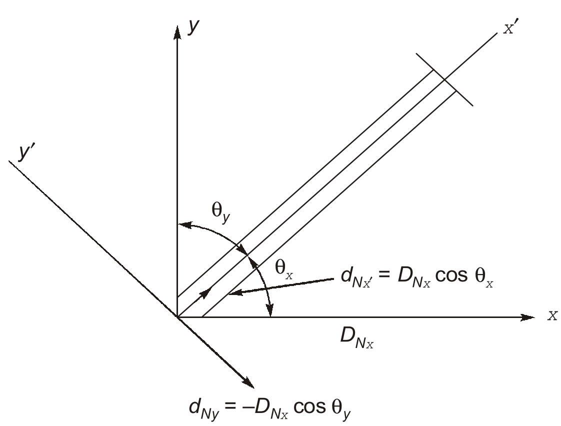


Bending Moment Diagram

Solution : 2

In order to transform the internal frame loads q and deformation d from local x', y', z' coordinates to global x, y, z coordinates. For this two approaches (transformation matrices) are explained below.

Displacement Transformation Matrix: Consider the frame member as shown below.



$$d_{Nx'} = D_{Nx} \cos \theta_x \qquad\qquad d_{Ny'} = -D_{Nx} \cos \theta_y$$

Likewise, a global coordinate displacement D_{Ny}, creates local coordinate displacement of,

$$d_{Nx'} = D_{Ny} \cos \theta_y \qquad\qquad d_{Ny} = D_{Ny} \cos \theta_x$$

Finally since the z and z' axes are coincident, that is directed out of page, causes a rotation D_{Nz} about z causes a corresponding rotation $D_{Nz'}$ about z'. Thus,

$$d_{Nz'} = D_{Nz}$$

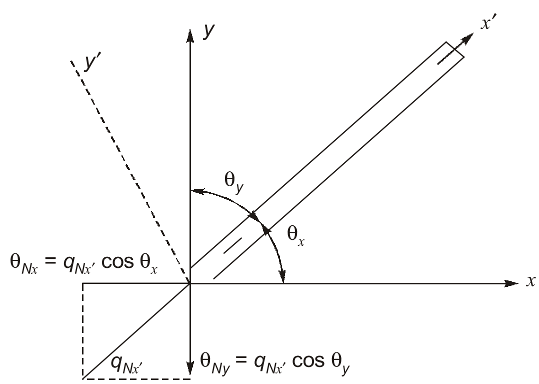
Let $\lambda_x = \cos\theta$, $\lambda_y = \cos\theta_y$ represents direction cosines of members,

$$\begin{bmatrix} d_{Nx'} \\ d_{Ny'} \\ d_{Nz'} \\ d_{Fx'} \\ d_{Fy'} \\ d_{Fz'} \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 & 0 & 0 \\ -\lambda_y & \lambda_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & 0 \\ 0 & 0 & 0 & -\lambda_y & \lambda_x & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Nz} \\ D_{Fx} \\ D_{Fy} \\ D_{Fz} \end{bmatrix}$$

Or, $$d = TD$$

By inspection, T transforms the dix global x, y, z displacements into the six local x', y', z' displacements d. Hence T is referred to as the displacement transformation matrix.

Force transformation matrix. If we now apply each component of load to near end of the member, to transform from gloabal to local coordinates. Applying $q_{Nx'}$, it can be seen that,



$$Q_{Nx} = -q_{Nx'}\cos\theta_x, \qquad Q_{Ny} = -q_{Nx'}\cos\theta_y$$

If $q_{Ny'}$ is applied, then its components are,

$$Q_{Nx} = -q_{Ny}\cos\theta_y \qquad Q_{Ny} = -q_{Ny'}\cos\theta_x$$

Finally, since $q_{Nz'}$ is collinear with Q_{NZ}, we have,

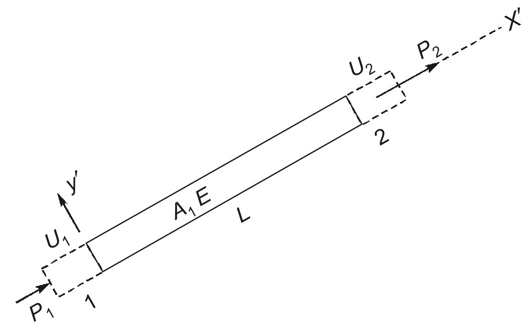
$$Q_{Ny} = q_{Nz'}$$

In a similar manner, end loads of $q_{Fx'}$, $q_{Fy'}$, $q_{Fz'}$ will yield the following respective components.

$$\begin{bmatrix} Q_{Nx} \\ Q_{Ny} \\ Q_{Nz} \\ Q_{Fx} \\ Q_{Fy} \\ Q_{Fz} \end{bmatrix} = \begin{bmatrix} \lambda_x & -\lambda_y & 0 & 0 & 0 & 0 \\ \lambda_y & \lambda_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & -\lambda_y & 0 \\ 0 & 0 & 0 & \lambda_y & \lambda_x & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{Nx'} \\ q_{Ny'} \\ q_{Nz'} \\ q_{Fx'} \\ q_{Fy'} \\ q_{Fz'} \end{bmatrix} \Rightarrow Q = T^T q$$

Solution : 3

For member oriented axis:



$$P_1 = \frac{AE}{L}U_1 - \frac{AE}{L}U_2$$

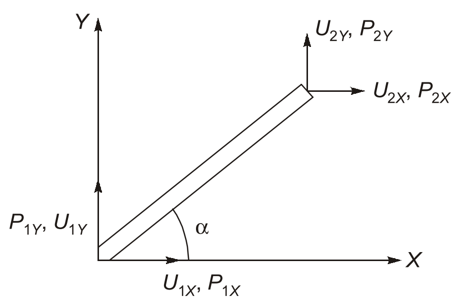
$$P_2 = -\frac{AE}{L}U_1 + \frac{AE}{L}U_2$$

i.e.,
$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Using, $P = k\Delta$

Stiffness matrix, $k = \dfrac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

For structure oriented axis



Comparing with member oriented axis

$$P_{1y} = P_1 \sin\alpha, \quad P_{1x} = P_1 \cos\alpha$$
$$P_{2y} = P_2 \sin\alpha, \quad P_{2x} = P_2 \cos\alpha$$

Also
$$U_1 = U_{1y}\sin\alpha + U_{1x}\cos\alpha$$
$$U_2 = U_{2y}\sin\alpha + U_{2x}\cos\alpha$$

i.e.,
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

and
$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{2x} \\ P_{2y} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \\ 0 & \cos\alpha \\ 0 & \sin\alpha \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Comparing with member oriented stiffness matrix

$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{2x} \\ P_{2y} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \\ 0 & \cos\alpha \\ 0 & \sin\alpha \end{bmatrix}\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \end{bmatrix}$$

$$\begin{bmatrix} P_{1x} \\ P_{1y} \\ P_{2x} \\ P_{2y} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha & -\cos^2\alpha & -\cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha & -\cos\alpha\sin\alpha & -\sin^2\alpha \\ -\cos^2\alpha & -\cos\alpha\sin\alpha & \cos^2\alpha & \cos\alpha\sin\alpha \\ -\cos\alpha\sin\alpha & -\sin^2\alpha & \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

So stiffness matrix

$$k' = \frac{EA}{L}\begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha & -\cos^2\alpha & -\cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha & -\cos\alpha\sin\alpha & -\sin^2\alpha \\ -\cos^2\alpha & -\cos\alpha\sin\alpha & \cos^2\alpha & \cos\alpha\sin\alpha \\ -\cos\alpha\sin\alpha & -\sin^2\alpha & \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$$

Solution : 4

Members	*OA*	*OB*	*OC*	*OD*
Forces	42.33 kN (T)	36.10 kN (T)	3.78 kN (T)	38.08 kN (T)

Solution : 5

$$K = \begin{bmatrix} \dfrac{12EI_1}{l_1^3} & \dfrac{-6EI_1}{l_1^2} \\ \dfrac{-6EI_1}{l_1^2} & \dfrac{4EI_1}{l_1} + \dfrac{3EI_2}{l_2} \end{bmatrix}$$

Solution : 6

$M_A = -34.5$ kNm　　$M_D = 28.5$ kNm
$M_B = -27.1$ kNm　　$M_E = 26.4$ kNm
$M_C = 0$

Solution : 7

$M_1 = -170.3$ kNm　　$M_2 = -73.4$ kNm

Solution : 8

$M_{AB} = -58.65$ kNm　　$M_{BA} = 85.20$ kNm
$M_{BC} = -85.20$ kNm　　$M_{CB} = +158.04$ kNm
$M_{CD} = +158.04$ kNm　　$M_{DC} = 0$

Solution : 9

$M_{AB} = -111.5$ kNm　　$M_{BA} = +72$ kNm
$M_{BC} = -72$ kNm　　$M_{CB} = 0$

Solution : 10

$M_{AB} = 0$　　$M_{BA} = 660$ kNm
$M_{BC} = -660$ kNm　　$M_{CB} = -330$ kNm

Solution : 11

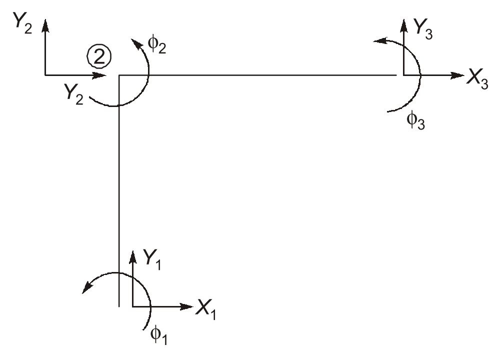
$M_{AB} = 3.28$ kNm $M_{BA} = 9.93$ kNm

$M_{BC} = -9.93$ kNm $M_{CB} = 30.23$ kNm

$M_{CD} = -7.7$ kNm $M_{CE} = -22.5$ kNm

$M_{DC} = -5.55$ kNm

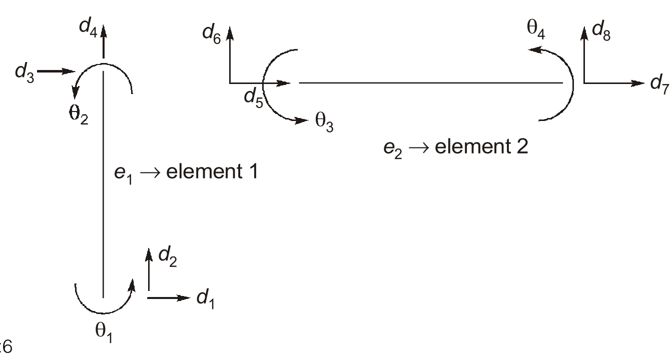
Solution : 12



$D' \rightarrow$ Displacement matrix for frame X, Y, ϕ, are degree of freedom for frame.

$d' \rightarrow$ Displacement matrix for unconnected element

Q, D are DOF of elements e_1 and e_2



Member 1, e_1, K_{e_1}, $K_{6\times 6} = \begin{bmatrix} K_{11_{3\times3}} & K_{12_{3\times3}} \\ K_{21_{3\times3}} & K_{22_{3\times3}} \end{bmatrix}_{6\times6}$

Member 2, e_2, K_{e_2}, $K_{6\times 6} = \begin{bmatrix} K'_{11_{3\times3}} & K'_{12_{3\times3}} \\ K'_{21_{3\times3}} & K'_{22_{3\times3}} \end{bmatrix}_{6\times6}$

To define relationship between d' and D'. Use matrix B where $B \rightarrow$ transformation Matrix.

At each joint displacement of element (1) and element (2) should be equal to the displacement of frame at the corresponding joint of frame.

Ex. $\theta_2 = \theta_3 = \phi_2$, $X_3 = d_7$, $\theta_1 = \phi_1$.

$$d' = BD'$$

Similarly all such relationships between the element and the frame are developed and the relationship is developed based on above relations.

$$\begin{bmatrix} \theta_1 \\ d_1 \\ d_2 \\ \theta_2 \\ d_3 \\ d_4 \\ \theta_3 \\ d_5 \\ d_6 \\ \theta_4 \\ d_7 \\ d_8 \end{bmatrix}_{12\times1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{12\times9} \begin{bmatrix} \phi_1 \\ X_1 \\ Y_1 \\ \phi_2 \\ X_2 \\ Y_2 \\ \phi_3 \\ X_3 \\ Y_3 \end{bmatrix}_{9\times1}$$

To form stiffness transformation matrix, use stiffness matrix of frame.

$$K = B^T kB$$

where, $k = \begin{bmatrix} k_{e1} & 0 \\ 0 & k_{e2} \end{bmatrix}_{12\times12}$

So, $K_{9\times9} = [B^T]_{9\times12} \times [k]_{12\times12} \times [B]_{12\times9}$

Using the relationship, global stiffness matrix for frame can be assembled.

■■■■