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ESE 2020 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-12: Full Syllabus Test

Paper-I

Name :

Roll No :

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	
Q.2	
Q.3	
Q.4	
Section-B	
Q.5	
Q.6	
Q.7	
Q.8	
Total Marks Obtained	

Signature of Evaluator

Cross Checked by

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Section-A

Q.1 (a)

A and B are two matrices with same number of rows.

(i) Compare the rank of A and the block matrix $[A \ B]$.(ii) If $B = A^2$, how do these ranks compare? Explain with reasoning.(iii) If A is $m \times n$ of rank r , what are the dimensions of null-spaces of A and $[A \ A]$?

[3 × 4 marks]

Ans (i) (i) Given $[A]$ and $[B]$ have same number of rows.

If rank of $[A] = \rho[A] = r$

So rank of $[A \ B] = \rho[A \ B] \geq r$

So, $\rho[A] \leq \rho[A \ B]$

\therefore both have same no. of rows.
and More columns in $[A \ B]$ than $[A]$

(ii) Given $B = A^2$

Rank of $[B] = \rho[B] = \rho[A]$

\therefore each element of $B = A^2$ has a linear combination of element of $[A]$ and no new column are there for matrix.

(iii) Given $[A]_{m \times n}$ and $\rho[A] = r$

Null space $N(A)$ dimension = $n - r$

For $[A \ A]$ matrix has dimension of $m \times 2n$


Hence ~~Null space~~ $[A \ A]$ has same column as that of A

So $\rho[A \ A] = r$

So Null space $[A \ A] = 2n - r$

- Q.1 (b) An infinite cylinder has a surface current \vec{K} , find the magnetic field \vec{B} , inside and outside the cylinder when \vec{K} is
- flowing vertically along length.
 - flowing around the cylinder.

[6 + 6 marks]

Ans (i) 

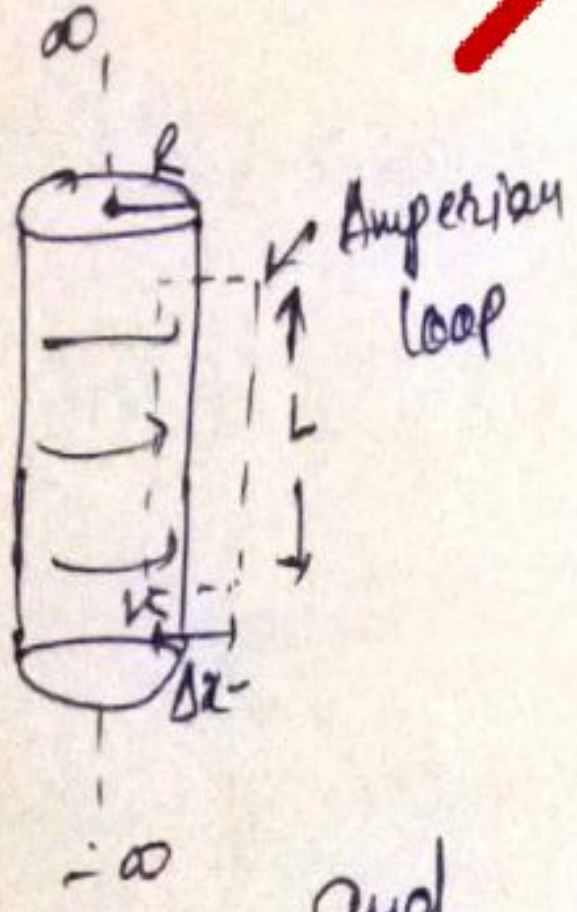
Using Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

for $r < R$, $I_{\text{enclosed}} = 0$, so $B = 0$

for $r > R$, $B \times 2\pi r = \mu_0 (K \times 2\pi R)$

$\Rightarrow \vec{B} = \frac{\mu_0 K R}{r} \hat{\phi}$

(ii) 

Amperian loop

$$\lim_{\Delta x \rightarrow 0} \oint \vec{B} \cdot d\vec{l} \Rightarrow B \cdot L = \mu_0 K \cdot L$$

$\Rightarrow \vec{B} = \mu_0 K \hat{\phi}$ for $r \leq R$

and No magnetic field outside conductor.

i.e. $B = 0$, $r > R$

- Q.1 (c) The input to a wattmeter is 12 A, 250 V at unity power factor. The current coil resistance is 0.1Ω and pressure coil resistance is $6.5 \text{ k}\Omega$. Calculate the percentage error due to resistance only in each of the methods of connection. Also determine at what load current, the errors can be equal for both the connections.

[12 marks]

Given, Ans (c) Case 1 When voltmeter connected across/near the load

Given, $R_A = 0.1 \Omega$, $R_V = 6.5 \text{ k}\Omega$

Now, True power = $V \cdot I \cdot \cos \phi$
 $P_T = 250 \times 12 \times 1 = 3000 \text{ watts}$ — (1)

Error $\Delta P = \frac{V^2}{R_V} = \frac{250 \times 250}{6.5 \times 1000} = 9.615 \text{ watt}$
 due to pressure coil resistance

So % Error = $\frac{\Delta P}{P_T} \times 100 = \frac{9.615}{3000} \times 100 = 0.32\%$

Case 2 When current coil connected near the load.

So, $\therefore P_T = V \cdot I \cdot \cos \phi = 3000 \text{ watt}$ (From (1))

Error due to C.C. resistance $\Delta P = I^2 \times R_A = 12^2 \times 0.1 = 14.4 \text{ watts}$

$$\therefore \% \text{ Error} = \frac{\Delta P}{P_t} \times 100 = \frac{14.4}{3000} \times 100 = 0.48\%$$

Now, let load current be I_0 when error can be both Equal

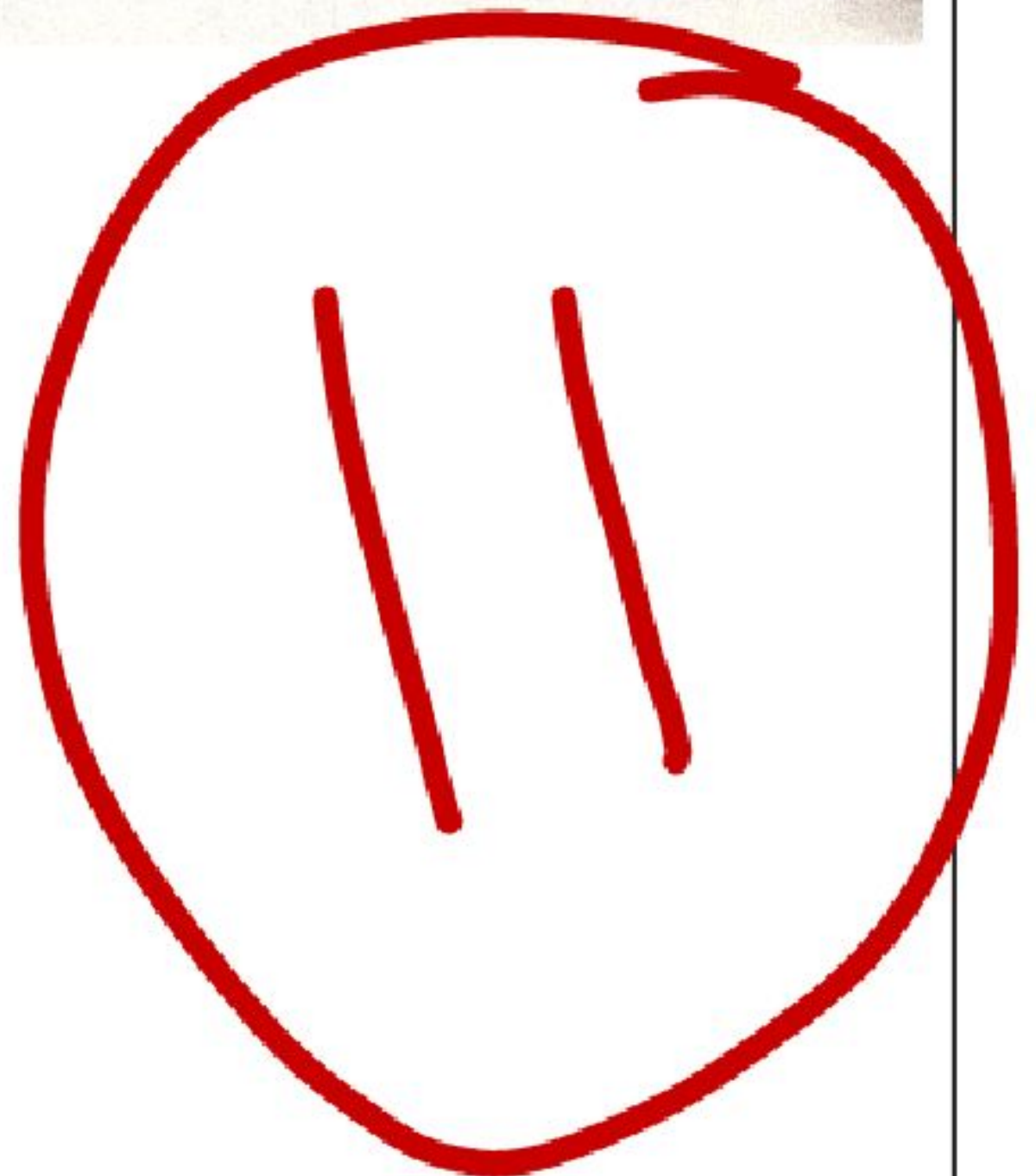
$$\therefore I_0^2 \times R_A = \frac{V_s^2}{R_v} \Rightarrow I_0^2 \times 0.1 = \frac{250^2}{6.5 \times 1000}$$

$$\boxed{I_0 = 9.81 \text{ Amps}}$$

Hence, $\% \text{ error}$ when pressure coil connected near load = 0.32%

$\% \text{ " " "$ current coil " " " = 0.48%

and current when both error are equal = 9.81 Amps



Q.1 (d) Simplify the Boolean expressions:

(i) $(X + Y)(X + \bar{Y})(\bar{X} + Z)$

(ii) $XYZ + X\bar{Y}Z + XY\bar{Z}$

[6 + 6 marks]

Ans. (d) (i) $(X + Y)(X + \bar{Y})(\bar{X} + Z) = (X \cdot X + X \cdot \bar{Y} + Y \cdot X + Y \cdot \bar{Y})(\bar{X} + Z)$
 $\therefore X \cdot X = X, Y \cdot \bar{Y} = 0$ So,
 $(X + X\bar{Y} + XY)(\bar{X} + Z) \Rightarrow (X(1 + \bar{Y}) + X\bar{Y})(\bar{X} + Z)$
 $\therefore \boxed{1 + \bar{Y} = 1}$ So, $(X + X\bar{Y})(\bar{X} + Z) \Rightarrow X(1 + \bar{Y})(\bar{X} + Z)$
 also $\boxed{1 + \bar{Y} = 1}$ So, $X(\bar{X} + Z) \Rightarrow X \cdot \bar{X} + XZ = XZ (\because X \cdot \bar{X} = 0)$

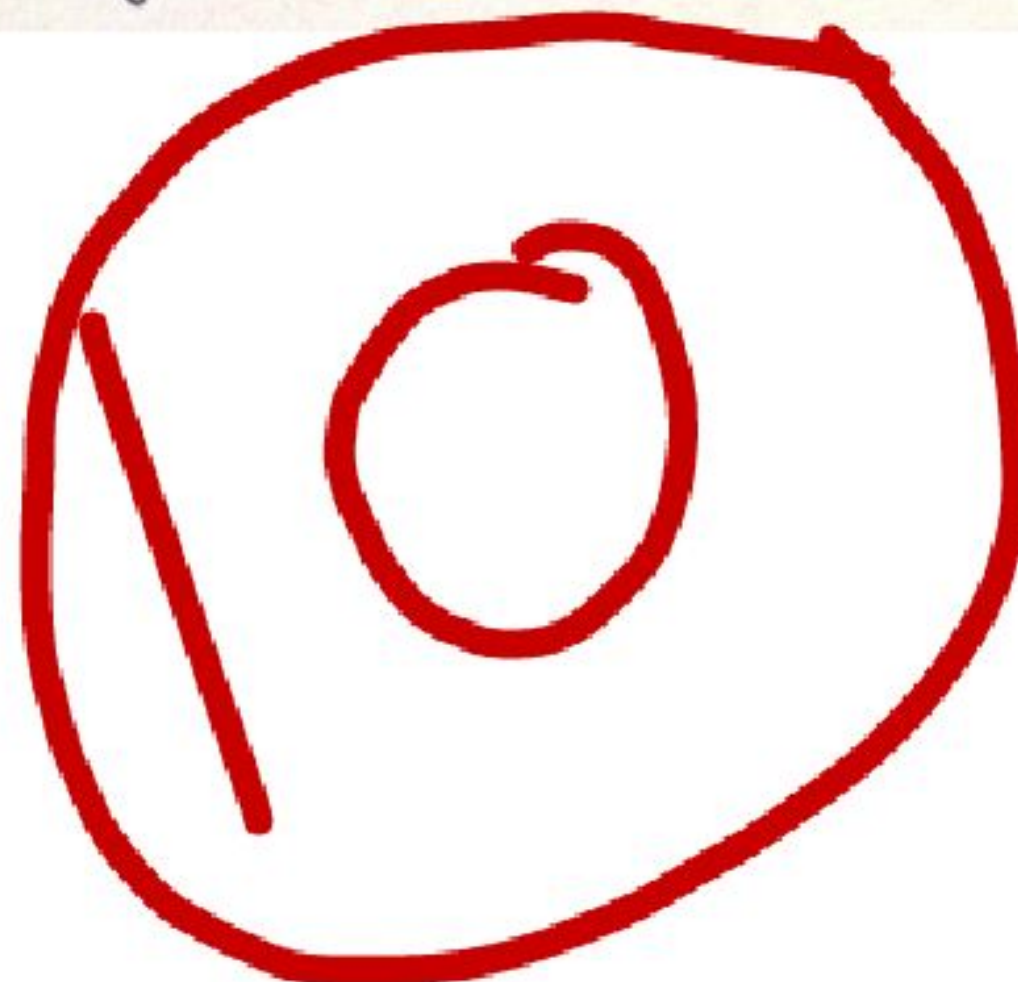
Hence after simplification result is XZ .

(ii) $XYZ + X\bar{Y}Z + XY\bar{Z} = XZ(Y + \bar{Y}) + XY\bar{Z} = XZ + XY\bar{Z}$

$\therefore Y + \bar{Y} = 1$ Now, $X[Z + Y\bar{Z}] = X[Z + Y]$

$\therefore \boxed{A + \bar{A}B = A + B}$, So, $XZ + XY\bar{Z} = X[Y + Z]$

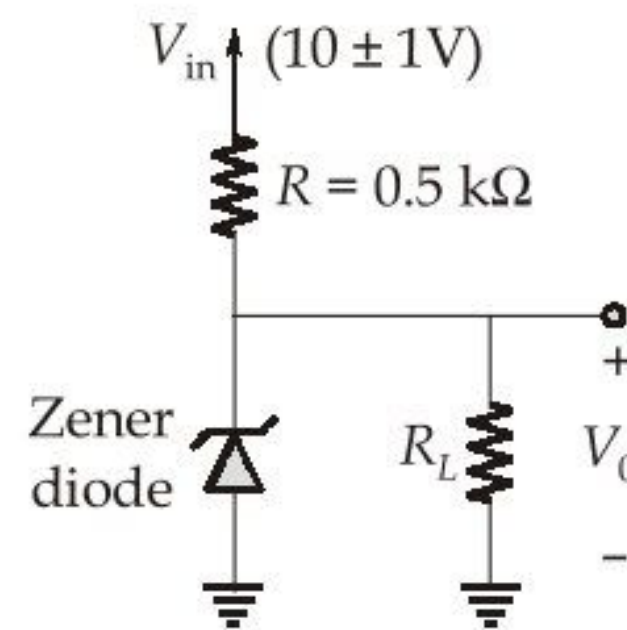
Hence after simplification, we get $X[Y + Z]$



Q.1 (e) The zener diode has specifications:

$$V_Z = 6.8 \text{ V at } I_Z = 5 \text{ mA}, r_Z = 20 \Omega \text{ and } I_{ZK} = 0.2 \text{ mA}$$

Find the line regulation resulting from $\pm 1 \text{ V}$ change in V_{in} . Also, find the load regulation for a load resistance R_L that draws 1 mA current.



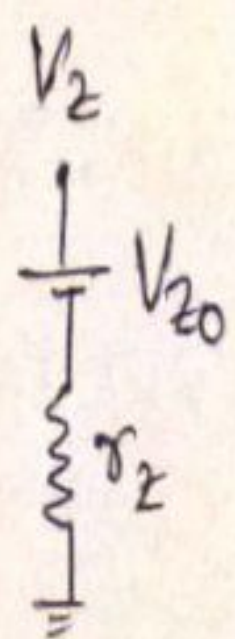
[12 marks]

Ans 1(e)

Given $V_Z = 6.8 \text{ V}$, $I_Z = 5 \text{ mA}$
 $r_Z = 20 \Omega$ and $I_{ZK} = 0.2 \text{ mA}$

$$\therefore \text{Line regulation} = \frac{\Delta V_o}{\Delta V_{in}} = \frac{r_Z}{R + r_Z} = \frac{20}{500 + 20} = 38.5 \text{ mV/V}$$

$$\text{So } \Delta V_o = \Delta V_{in} \times 38.5 \frac{\text{mV}}{\text{V}} = \pm 38.5 \frac{\text{mV}}{\text{V}}$$



for load take 1 mA current Zener current decrease
 by 1 mA

$$\text{Or } \Delta V_o = r_Z \Delta I_Z = 20 \times (-1) = -20 \text{ mV}$$

$$\text{So load regulation} = \frac{\Delta V_o}{I_{\text{Load}}} = \frac{-20 \text{ mV}}{1 \text{ mA}} = -20 \frac{\text{mV}}{\text{mA}}$$

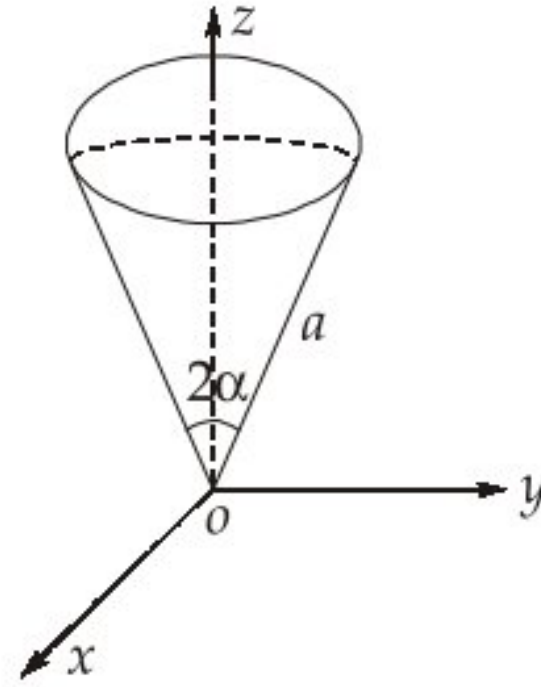
- Q.2 (a)
- (i) What is swamping resistor? Why is it used in ammeters?
 - (ii) A circuit consisting of a coil, a resistance and a variable capacitor connected in series is tuned to resonance using a Q -meter. If the frequency is 500 kHz, the resistance 0.5Ω the variable capacitor set to 350 pF and the Q -meter indicates 90, then find the effective inductance and resistance of the coil.
 - (iii) A 0-1 A moving-iron ammeter has an internal resistance of $50 \text{ m}\Omega$ and inductance of 0.1 mH. A shunt coil is connected to extend its range 0 - 10 A for all operating frequencies. Find the time constant and resistance of the shunt coil.

[5 + 5 + 10 marks]

- Q.2 (b)
- (i) Explain the two sources of magnetic moments for electrons.
 - (ii) Briefly describe the phenomenon of magnetic hysteresis and why it occurs for ferromagnetic and ferrimagnetic materials?
 - (iii) A ferromagnetic material has a remanence of 1.0 Tesla and a coercivity of 15000 A/m. Saturation is achieved at a magnetic field strength of 25000 A/m, at which the flux density is 1.25 Teslas. Sketch the hysteresis curve and from the plot, find the energy loss per cycle of the material.

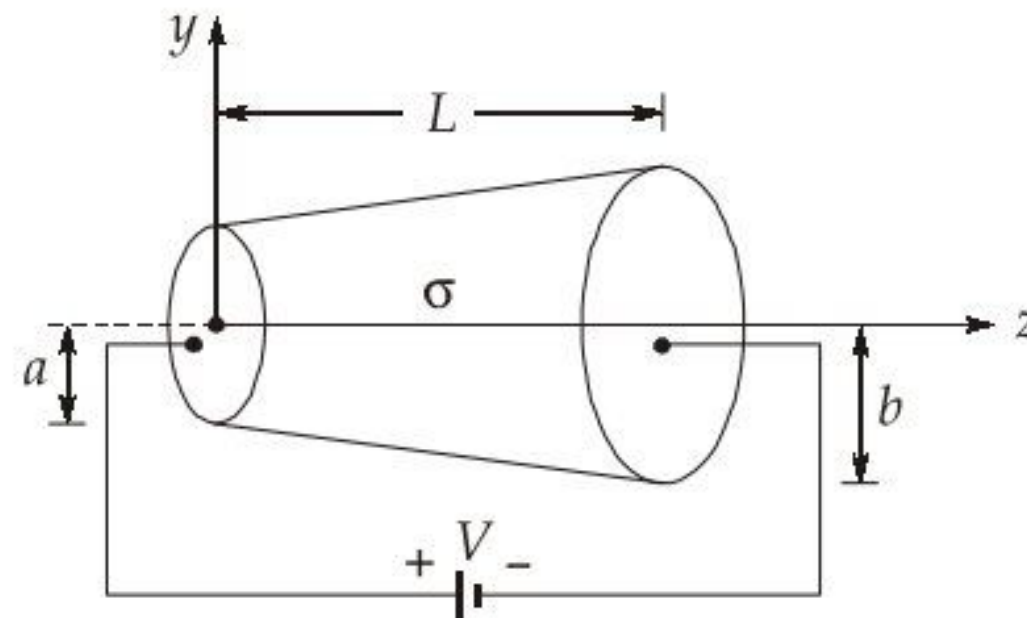
[6 + 6 + 8 marks]

- Q.2 (c) Consider a solid in the shape of a cone. It is bounded by a sphere of radius ' a ' centered at the origin. The cone has vertex at origin, vertex angle ' 2α ' and slant height ' a '
- (i) Find the mass and centre of mass (C.O.M.) of the solid. (Assume density = 1)
- (ii) Let U be the boundary of the conical lower surface, S the upper spherical cap of the solid. Use the divergence theorem to compute the upward flux of $\vec{F} = z\hat{K}$ through U and S . [For part (ii) take $a = \sqrt{2}$, vertex angle to be $\frac{\pi}{2}$].



[8 + 12 marks]

- Q.3 (a) (i) Point charges $Q_1 = 1 \text{ nC}$, $Q_2 = -2 \text{ nC}$, $Q_3 = 3 \text{ nC}$ and $Q_4 = -4 \text{ nC}$ are positioned one at a time and in that order at $(0, 0, 0)$, $(1, 0, 0)$, $(0, 0, -1)$ and $(0, 0, 1)$ respectively. Calculate the energy in the system after each charge is positioned.
- (ii) Current flows through a truncated circular cone of conductivity $\sigma = 10^{-4} \text{ (S/m)}$. Left radius of cone is $a = 1 \text{ cm}$, right radius of the cone is $b = 2 \text{ cm}$ and length of the cone is $L = 0.2 \text{ m}$. Assume that this resistance is fed by DC voltage as shown, what is the resistance of this cone structure?



[8 + 12 marks]

Ans 3 (a) given $Q_1 = 1 \text{ nC}$, $Q_2 = -2 \text{ nC}$, $Q_3 = 3 \text{ nC}$, $Q_4 = -4 \text{ nC}$
at point A, B, C, D respectively

Let initially system energy = 0 as no charge
is present initially.

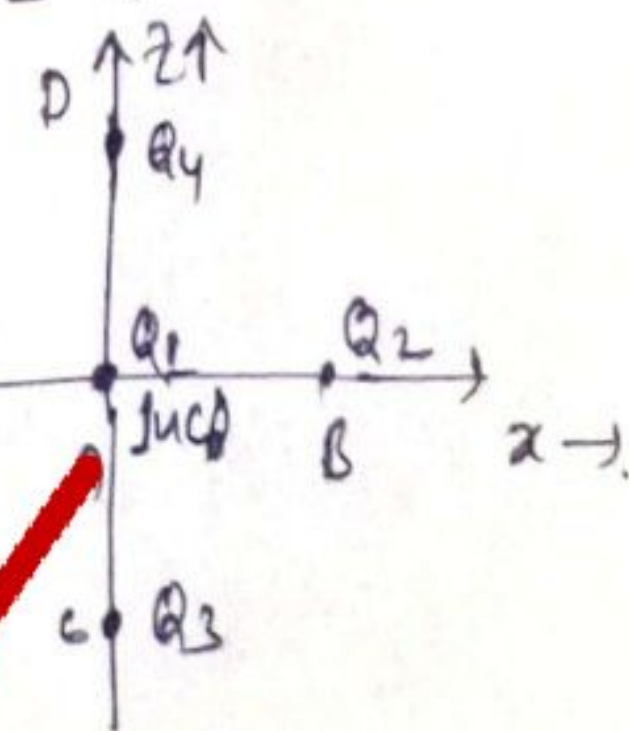
Let us bring charge Q_1 at point A.

Now also, system energy = 0.

Now bring charge Q_2 at point B

$$\text{System energy} = \frac{k Q_1 Q_2}{r_{12}}$$

$$\text{where } k = \frac{1}{4\pi\epsilon_0}$$



$$E = \frac{9 \times 10^9 \times 1 \times 10^{-9} \times (-2 \times 10^{-9})}{1} = -18 \text{ nJ}$$

Let us bring charge Q_3 at point C.

$$\text{Energy of system} = \frac{k Q_1 Q_2}{r_{Q_1 Q_2}} + \frac{k Q_1 Q_3}{r_{Q_1 Q_3}} + \frac{k Q_2 Q_3}{r_{Q_2 Q_3}}$$

$$= 9 \times 10^9 \left[\frac{(1 \times 10^{-9})(-2 \times 10^{-9})}{1} + \frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(1)} + \frac{(3 \times 10^{-9})(-2 \times 10^{-9})}{1} \right]$$

$$= -29.18 \text{ nJ}$$

Let us bring Q_4 at point D then Energy of system

$$E = \frac{k Q_1 Q_2}{r_{Q_1 Q_2}} + \frac{k Q_1 Q_3}{r_{Q_1 Q_3}} + \frac{k Q_1 Q_4}{r_{Q_1 Q_4}} + \frac{k Q_2 Q_3}{r_{Q_2 Q_3}} + \frac{k Q_2 Q_4}{r_{Q_2 Q_4}} + \frac{k Q_3 Q_4}{r_{Q_3 Q_4}}$$

$$E = 9 \times 10^9 \left[\frac{(1 \times 10^{-9})(-2 \times 10^{-9})}{1} + \frac{(1 \times 10^{-9})(3 \times 10^{-9})}{1} + \frac{(1 \times 10^{-9})(-4 \times 10^{-9})}{1} + \frac{(-2 \times 10^{-9})(3 \times 10^{-9})}{1} \right. \\ \left. + \frac{(-2 \times 10^{-9})(-4 \times 10^{-9})}{\sqrt{2}} + \frac{(3 \times 10^{-9})(-4 \times 10^{-9})}{(2)} \right]$$

$$\text{Energy of system} = -68.27 \text{ nJ}$$

(i) Given, $\sigma = 10^{-4} \text{ (S/m)}$, $a = 1 \text{ cm}$, $b = 2 \text{ cm}$, $L = 0.2 \text{ m}$

$$\therefore \vec{E} = \text{Electric field} = \frac{V}{L} a \hat{z}$$

$$\text{and } \vec{J} = \sigma \vec{E} = 10^{-4} \times \frac{V}{L} a \hat{z}$$

$$\therefore R = \oint \frac{1}{A} = \frac{1}{\sigma} \frac{1}{A}$$

$$\text{So, Resistance, } R = \int_0^L \frac{1}{\sigma} \frac{dz}{A}$$

$$\text{and Area } A = \pi x^2$$

$$R = \frac{1}{\sigma} \int_0^L \frac{dz}{\pi x^2} \quad \text{--- (2)}$$

By Symmetry $\frac{x}{p+z} = \frac{b}{p+L} = \frac{a}{p}$

$$\Rightarrow x = a + \frac{z}{L}(b-a) \quad \text{put this in \& \textcircled{1} we get}$$

$$R = \frac{1}{\sigma} \int_0^L \frac{dz}{\pi \left(a + \frac{z}{L}(b-a) \right)^2} = \frac{-1}{\pi \sigma} \left[\frac{1}{a + \frac{z}{L}(b-a)} \right] \Bigg|_0^L$$

$$R = \frac{1}{\pi \sigma} \cdot \left[\frac{1}{a} - \frac{1}{b} \right] \times \frac{L}{(b-a)} = \frac{1}{10^{-4} \pi} \left[\frac{1}{0.01} - \frac{1}{0.02} \right] \times \frac{0.02}{0.01}$$

$$\boxed{R = 3.183 \text{ M}\Omega}$$

Q.3 (b)

(i) Differentiate between

1. Circuit switching and packet switching.
2. IPv4 and IPv6.

(ii) What is the TCP/IP model? Explain the protocol layers for this model.

[10 + 10 marks]

Ans 3 (b) Circuit Switching. Packet Switching

- In circuit switching there are 3 Phases:→
 - 1) Connection Establishment.
 - 2) Data Transfer
 - 3) Connection Released
- In circuit switching each data unit know the entire path address which is provided by the source.
- In circuit switching, data is processed at source system only.
- Delay between data units in circuit switching is uniform.
- Resource reservation is feature of circuit switching because the path is fixed for data transmission.
- Circuit switching is more reliable
- Waste of resources are more in circuit switching.
- It is not a store forward technique.
- Transmission of data is done by the source.
- Congestion can occur during connection establishment time, there might be a case will requesting for channel the channel is already occupied.
- In packet switching directly data transfer takes place.
- In packet switching each data unit just know the final destination address intermediate path is decided by the routers.
- In packet switching data is processed at all intermediate node including source system.
- Delay between data units in packet switching is not uniform
- There is no resource reservation because bandwidth is shared among the users.
- Packet switching is less reliable
- Less wastage of resources as compared to circuit switching.
- It is a store and forward technique.
- Transmission of data is done not only by source but also by intermediate routers.
- Congestion can occur during data transfer phase, large number of packets come in no time.

(ii)

IPV 4

- IPV4 has 32 bit address length.
- It support manual and DHCP address configuration.
- End to end connection integrity is unachievable.
- It can generate 4.29×10^9 address space.
- Security feature is dependent on application.
- It has header of 20 - 60 bytes.

IPV 6

- It has 128 bit address length.
- It supports auto and renumbering address configuration.
- End to End connection integrity is achievable.
- It can generate quite large i.e. 3.4×10^{38} address space.
- Security feature is ~~is~~ built in IPV6.
- It has header of 40 bytes fixed.

Ans 3(b) (ii) TCP/IP stands for "Transmission Control Protocol/Internet protocol". It is a concise version of the OSI model.

- TCP/IP is more reliable.
- TCP/IP does not have very strict boundaries.
- TCP/IP follows horizontal approach.
- TCP/IP both session and presentation layer in the application layer itself.

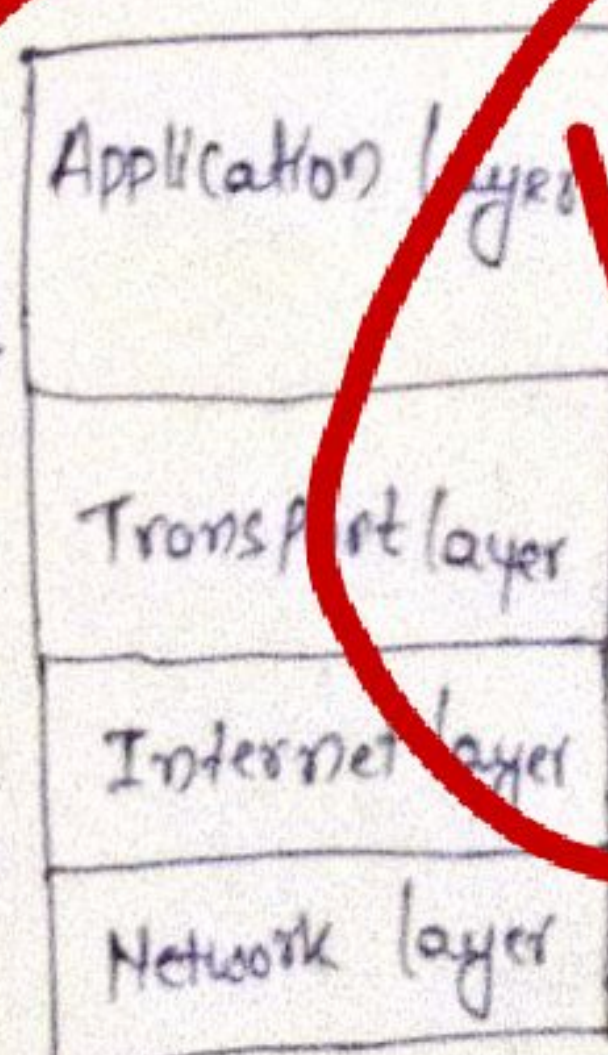
TCP/IP has 4 layers

- Network layer → Responsible for the transmission for the between two device on the same network

- Internet layer → • To move packets from source to destination.

- To provide internetworking

- Transport layer → • To provide reliable process to process message delivery and error delivery.



TCP/IP layer

- Application layer → • To allow access to network resources.

- Presentation and session layer of OSI is built in application layer of TCP/IP Model.

- Q.3 (c) (i) In a low voltage Schering bridge designed for the measurement of permittivity, the branch ab consists of two electrodes between which the specimen under test may be inserted; arm bc is a non-reactive resistor $R_3 = 5000 \Omega$ in parallel with a standard capacitor C_3 , arm cd is a non-reactive resistor $R_4 = 5000 \Omega$ in parallel with a standard capacitor C_4 , arm da is a standard air capacitor of capacitor C_2 . Without the specimen between electrodes, balance is obtained with $C_2 = 150 \text{ pF}$, $C_3 = C_4 = 120 \text{ pF}$. With the specimen inserter these values become $C_2 = 900 \text{ pF}$, $C_3 = 200 \text{ pF}$, $C_4 = 1000 \text{ pF}$. In each test $\omega = 5000 \text{ rad/s}$. Find the relative permittivity of the specimen.
- (ii) An analog voltmeter uses external multiplier settings with a multiplier setting of $20 \text{ k}\Omega$, it reads 440 V and with a multiplier setting of $80 \text{ k}\Omega$, it reads 352 V . Find the voltmeter reading for a multiplier setting of $40 \text{ k}\Omega$.

[12 + 8 marks]

Ans 3(c)

Case 1 Specimen not put between electrode

Given, $R_3 = 5000 \Omega$, $R_4 = 5000 \Omega$
 $C_2 = 150 \text{ pF}$, $C_3 = C_4 = 120 \text{ pF}$
 At balanced condition

$$C_1 = C_2 \frac{C_3}{C_4}$$

So $C_1 = 150 \times \frac{120}{120} = 150 \text{ pF}$

Case 2 When C_1 Specimen is put between electrode

$$C_1' = C_2 \frac{C_3}{C_4}$$

$$= 900 \times \frac{200}{1000} = 180 \text{ pF}$$

$\therefore C_1 \propto \epsilon_0$ & $C_1' \propto \epsilon_0 \epsilon_{r1}$ where ϵ_0 permittivity of free space & ϵ_{r1} : relative permittivity.

$$\text{So } \frac{C_1}{C_1'} = \frac{\epsilon_0}{\epsilon_0 \epsilon_{r1}}$$

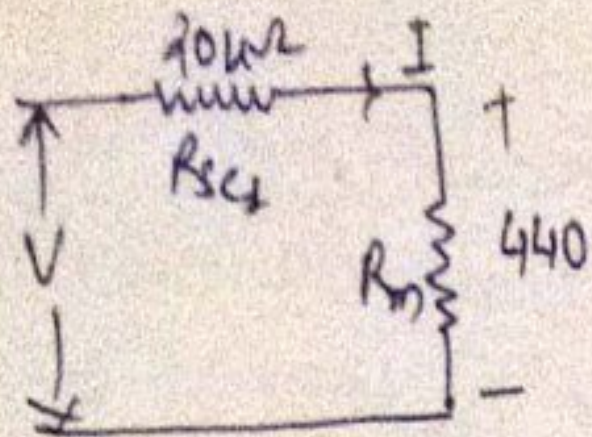
$$\Rightarrow \frac{150}{180} = \frac{1}{\epsilon_{r1}} \Rightarrow \epsilon_{r1} = \frac{180}{150} = 1.2$$

Hence Relative permittivity of specimen is 1.2

Ans (i) Given $R_{se1} = 20k\Omega$, $V_1 = 440V$

$R_{se2} = 80k\Omega$, $V_2 = 352V$

Case 1

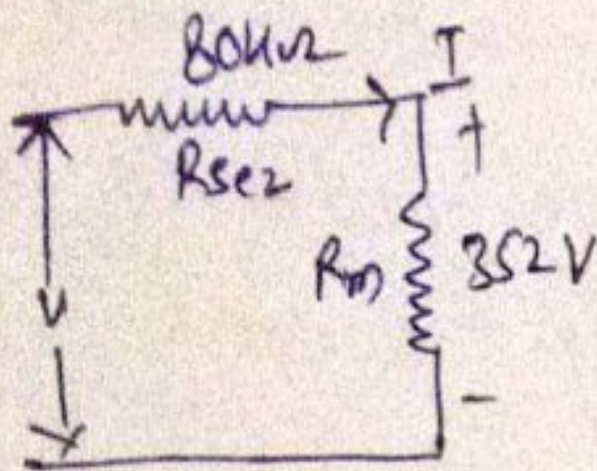


So, $I = \frac{440}{R_m}$ mA = Current through meter

$$\therefore \frac{440}{R_m} = \frac{V_m}{(20 + R_m)} \quad \text{--- (1)}$$

$$\Rightarrow 440(20 + R_m) = V(R_m)$$

Case 2



$$I = \frac{352}{R_m} = \frac{V_m}{(80 + R_m)} \quad \text{--- (2)}$$

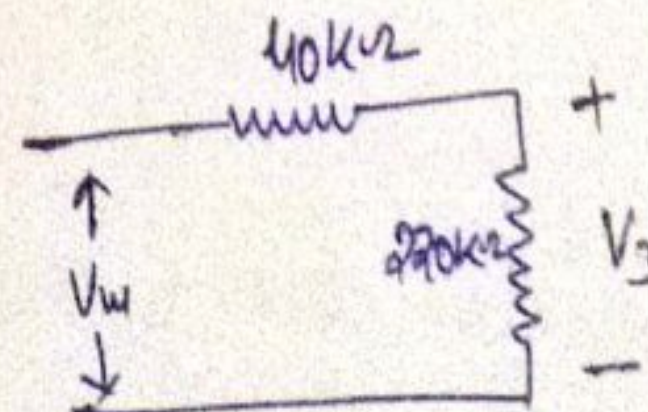
Divide Eq (1) by Eq (2) we get

$$\frac{440}{352} = \frac{(80 + R_m)}{(20 + R_m)} \Rightarrow \boxed{R_m = 220k\Omega}$$

put this value of R_m in Eq (1) $\Rightarrow \frac{440^2}{220} = \frac{V_m}{(20 + 220)}$

$$\Rightarrow \boxed{V_m = 480 \text{ Volts}}$$

So when $R_{se3} = 40k\Omega$



$$V_3 = V_m \times \frac{220}{220 + 40}$$

(By voltage division rule)

$$= 480 \times \frac{220}{260} = 406.154 \text{ Volts}$$

Hence Voltmeter reads 406.154 Volts.

Q.4 (a) Function $f(t)$ is given as,

$$f(t) = 1, \quad 0 \leq t < \frac{\pi}{2}$$

$$= -1, \quad \frac{\pi}{2} \leq t < \pi \quad \text{Also, } f(t) = f(t + \pi).$$

(i) Find the Fourier series of $f(t)$.

(ii) Find a solution to $\ddot{x}(t) + x(t) = f(t)$.

[20 marks]

Ans (a) Given (i) $f(t) = f(t + \pi)$ i.e. period of $f(t) = T = \pi$
 $\omega T = 2\pi \Rightarrow \omega = 2$ rad/sec

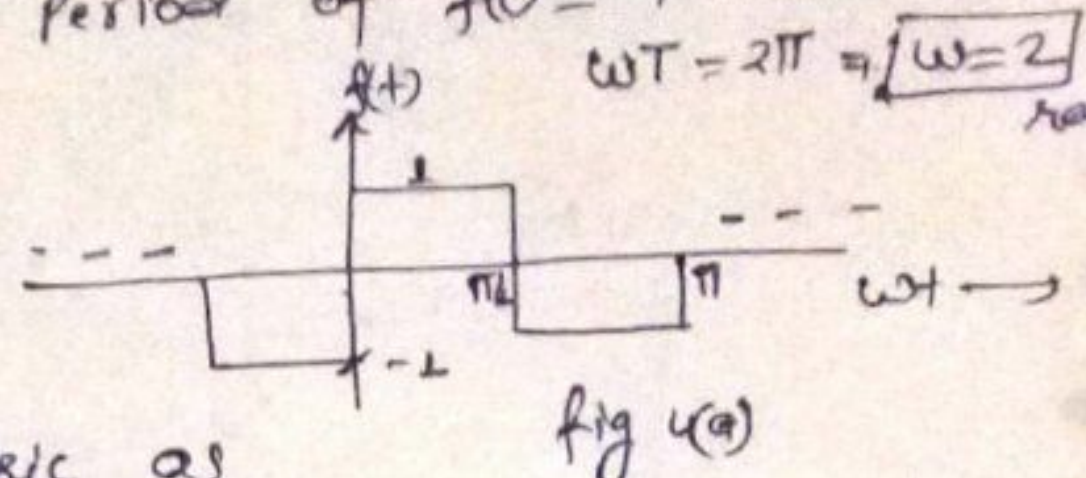
$$f(t) = \begin{cases} 1 & 0 \leq t < \pi/2 \\ -1 & \pi/2 \leq t < \pi \end{cases}$$


fig 4(a)

So, $\therefore f(t)$ is Half wave symmetric as per figure 4(a) & definition of $f(t)$.

So, $a_0 = 0$, $a_n = 0$, but $b_n \neq 0$

Therefore $f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t$

Now $b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t dt = \frac{2}{\pi} \int_0^{\pi} f(t) \sin n\omega t dt$

$b_n = \frac{2}{\pi} \times 2 \int_0^{\pi/2} 1 \sin n\omega t dt$ [$\because f(t)$ is H.W.S. area of both half equal]

$b_n = \frac{4}{\pi} \left(-\frac{\cos n\omega t}{n} \right) \Big|_0^{\pi/2} = \frac{4}{n\pi} [1 - \cos n\pi/2]$

$b_n = \frac{4}{n\pi}$, n is odd

$= 0$, n is even

[\because Even Harmonic doesn't exist when H.W.S. is there.]

So $f(t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} \sin(n\omega t) = \frac{4}{\pi} \sum_{n=1,3,5}^{\infty} \sin(n\omega t)$

$f(t) = \frac{4}{\pi} \sum_{n=1,3,5}^{\infty} \sin(2nt)$ $\because \omega = 2$ rad/sec

(ii) $\ddot{x}(t) + x(t) = f(t)$

Complementary solution :-

$$m^2 + 1 = 0$$

$$m^2 = -1 \Rightarrow m = \pm i$$

So Generalized solution of $x(t) = C_1 \cos t + C_2 \sin t$

Particular solution of $x(t)$:-

$$(D^2+1)x(t) = f(t) \Rightarrow x(t) = \frac{f(t)}{D^2+1}$$

$$x(t) = \frac{\frac{4}{n\pi} \sum_{n=1,3,5} \sin n(2t)}{(D^2+1)} \quad \text{let } n=1$$

$$\text{So } x(t) = \frac{\frac{4}{\pi} \sin 2t}{D^2+1} \Rightarrow \frac{\frac{4}{\pi} \sin 2t}{-4+1} \Rightarrow -\frac{4}{3\pi} \sin 2t$$

$$\text{Similarly } x(t) = \frac{\frac{4}{n\pi} \sum_{n=1,3,5} \sin(2n)t}{D^2+1} \Rightarrow \frac{\frac{4}{n\pi} \sum_{n=1,3,5} \sin(2n)t}{(-4n^2+1)}$$

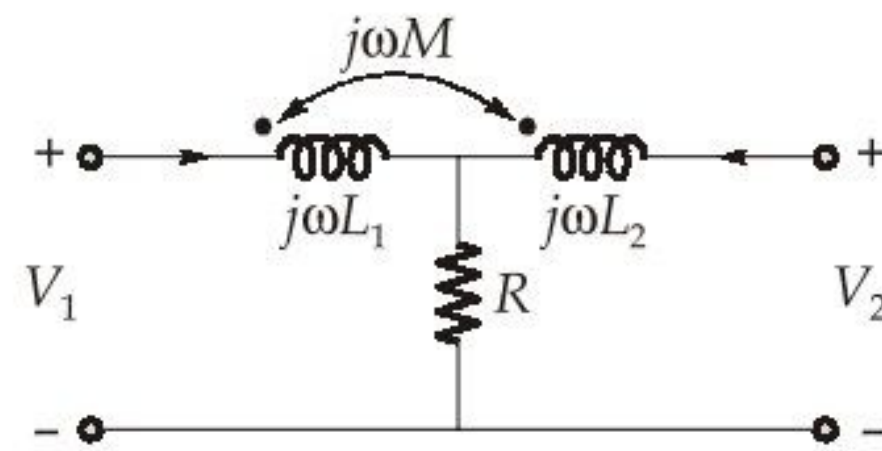
$$\text{So } x(t) = -\frac{4}{n\pi(4n^2-1)} \sum_{n=1,3,5,\dots}^{\infty} \sin(2nt)$$

Hence complete solution of $x(t) = C \cdot I + P \cdot I$.

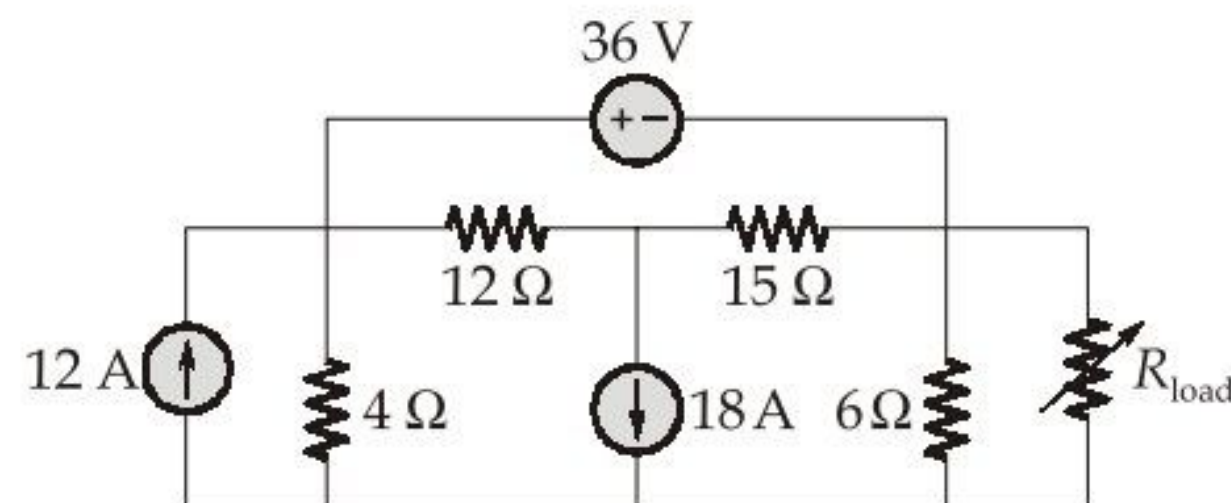
$$x(t) = C_1 \cos t + C_2 \sin t + \left(-\frac{4}{n\pi}\right) \times \frac{1}{(4n^2-1)} \sum_{n=1,3,5}^{\infty} \sin 2nt$$

2

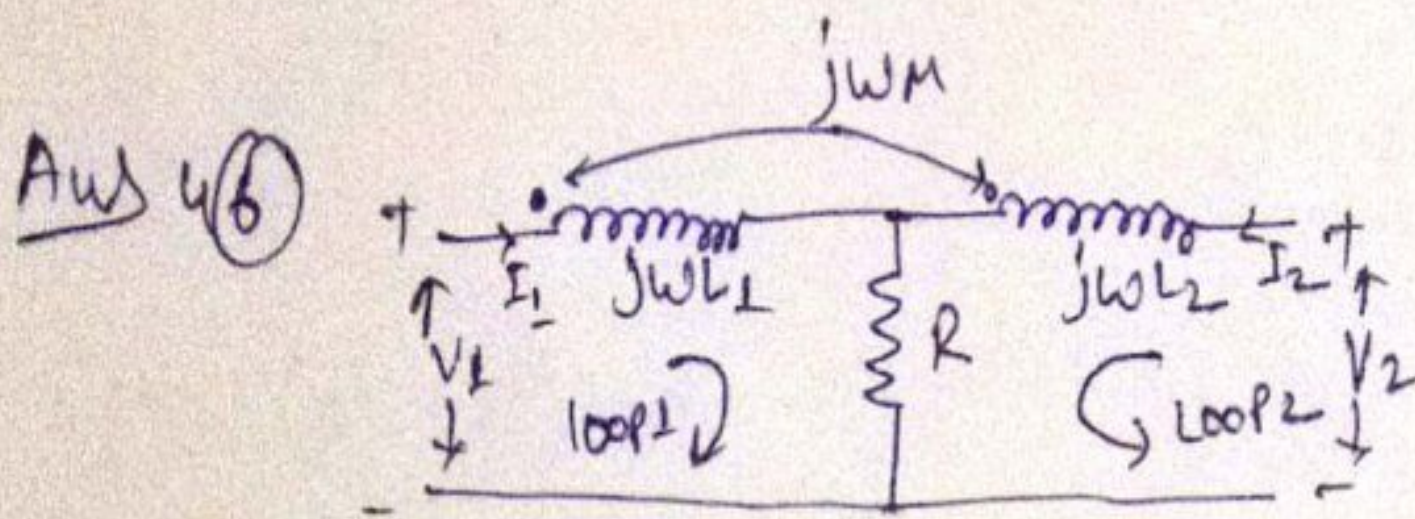
Q.4 (b) (i) Find the z-parameters for the 2-port network shown,



(ii) For the circuit shown, what should be the value of load resistor R_{Load} so that it absorbs maximum power and then, what would be the power absorbed by R_{Load} ?



[10 + 10 marks]



Applying KVL in loop 1 we get

$$V_1 = (R + j\omega L_1) I_1 + (R - j\omega M) I_2 \quad \text{--- (1)}$$

Similarly KVL in loop 2 we get

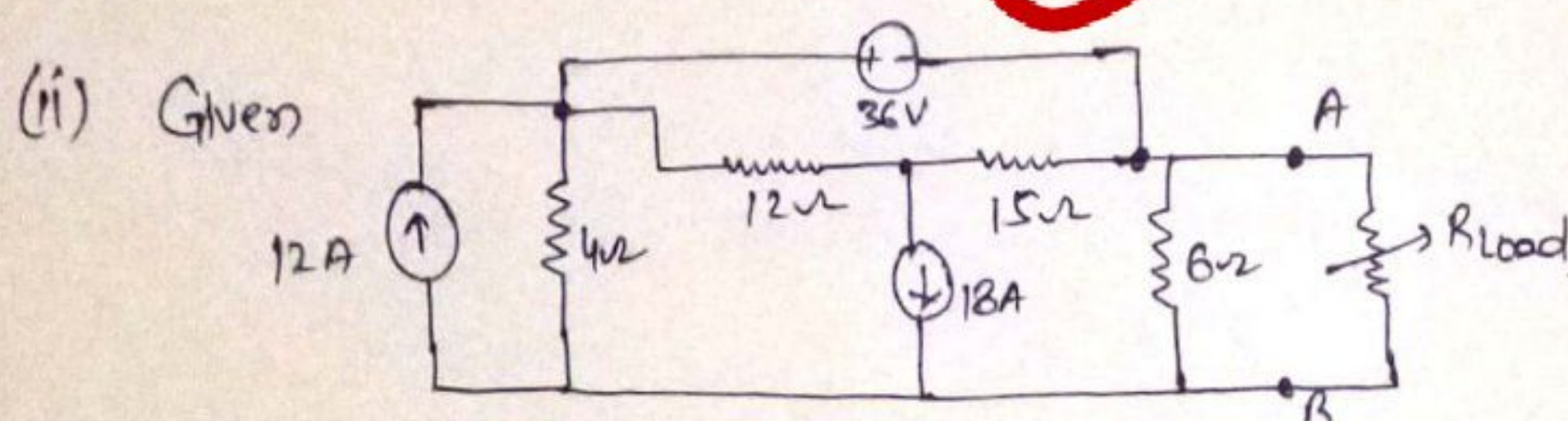
$$V_2 = (R - j\omega M) I_1 + (R + j\omega L_2) I_2 \quad \text{--- (2)}$$

On comparing Eq (1) & (2) with standard two port network

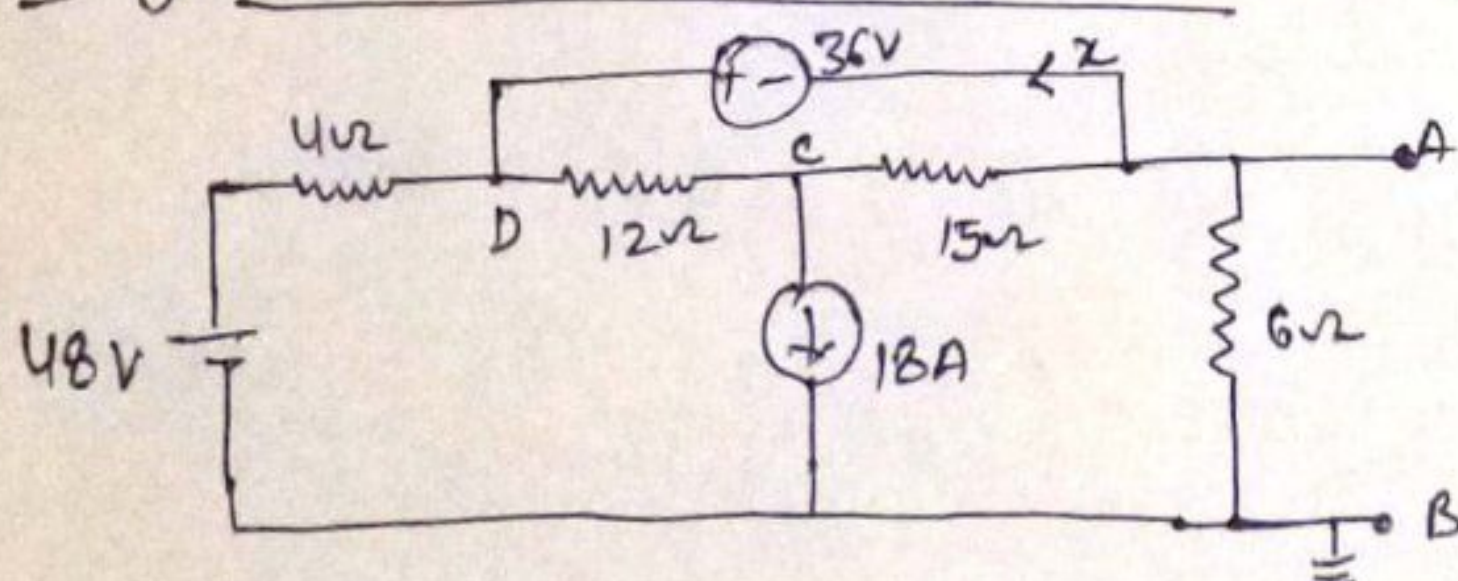
we get $z_{11} = (R + j\omega L_1) \Omega$, $z_{12} = (R - j\omega M) \Omega$

$z_{21} = (R - j\omega M) \Omega$, $z_{22} = (R + j\omega L_2) \Omega$

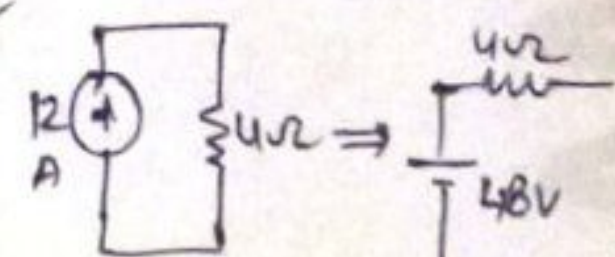
So $Z_{Matrix} = \begin{bmatrix} (R + j\omega L_1) & (R - j\omega M) \\ (R - j\omega M) & (R + j\omega L_2) \end{bmatrix}$



Using Thevenin's theorem to find open circuit voltage V_{Th}



Source transformation done



Applying KCL at Node C

$$18 + \frac{V_C - V_A}{15} + \frac{V_C - V_D}{12} = 0$$

$\therefore V_D = (36 + V_A) \text{ Volts}$

$$\text{So, } V_C = \frac{\frac{V_A}{15} + \frac{V_D}{12} - 18}{\left(\frac{1}{15} + \frac{1}{12}\right)} = \frac{V_A \left[\frac{1}{15} + \frac{1}{12}\right] + \frac{36}{12} - 18}{\left[\frac{1}{15} + \frac{1}{12}\right]}$$

$V_C = (V_A - 100) \text{ Volts} \quad \text{--- (1)}$

Applying KCL at Node A, we get

$$\frac{V_A - V_B}{6} + \frac{V_A - V_C}{15} + x = 0 \Rightarrow V_A \left[\frac{1}{6} + \frac{1}{15} \right] = \frac{V_C}{15} - x$$

$$\Rightarrow V_A - V_C = 100 \quad \text{from Eq (1)}$$

$$\text{So } \frac{V_A}{6} = -\frac{100}{15} - x \Rightarrow V_A = -(40 + 6x) \quad \text{--- (2)}$$

Applying KCL at Node D we get

$$\frac{V_D - 48}{4} + \frac{V_D - V_C}{12} = x \Rightarrow \frac{36 + V_A - 48}{4} + \frac{36 + V_A - V_C}{12} = x$$

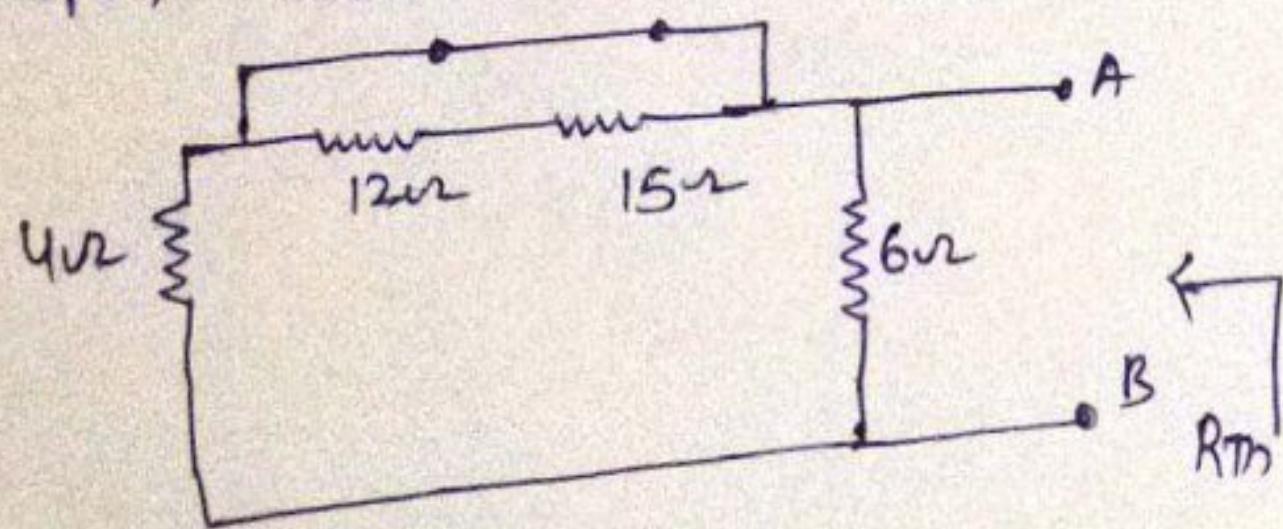
$$\Rightarrow V_A \left[\frac{1}{4} + \frac{1}{12} \right] = -\frac{25}{3} + x \Rightarrow V_A = -25 + 3x \quad \text{--- (3)}$$

From Eq (2) and (3) we get $V_A = -10 \text{ Volts}$

$$\text{So } V_{th} = V_{AB} = V_A - V_B = -10 \text{ Volts.}$$

For Thevenin's Resistance R_{th}

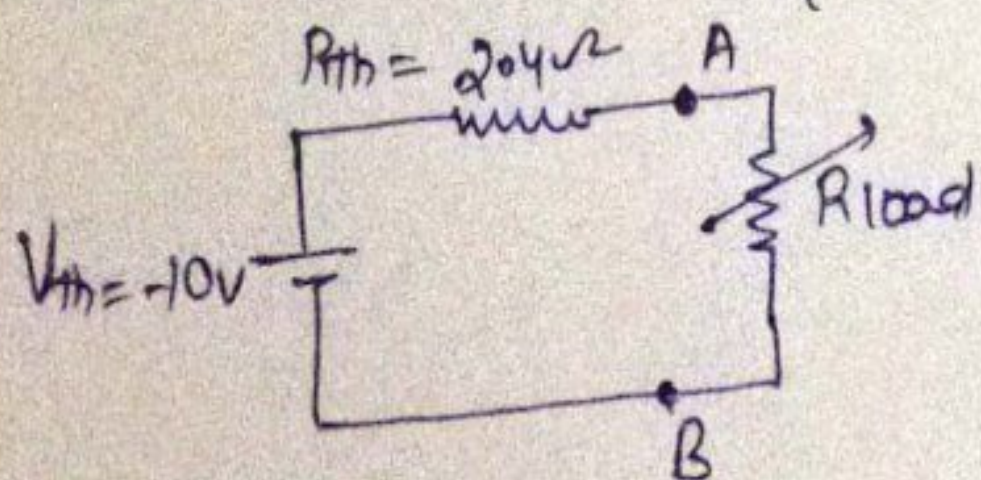
• open circuit current source, short circuit voltage source



$$\text{So } R_{th} = (6 \parallel 4) \Omega$$

$$R_{th} = \frac{6 \times 4}{6 + 4} = 2.4 \Omega$$

Hence Thevenin's Equivalent



do for Maximum Power transfer

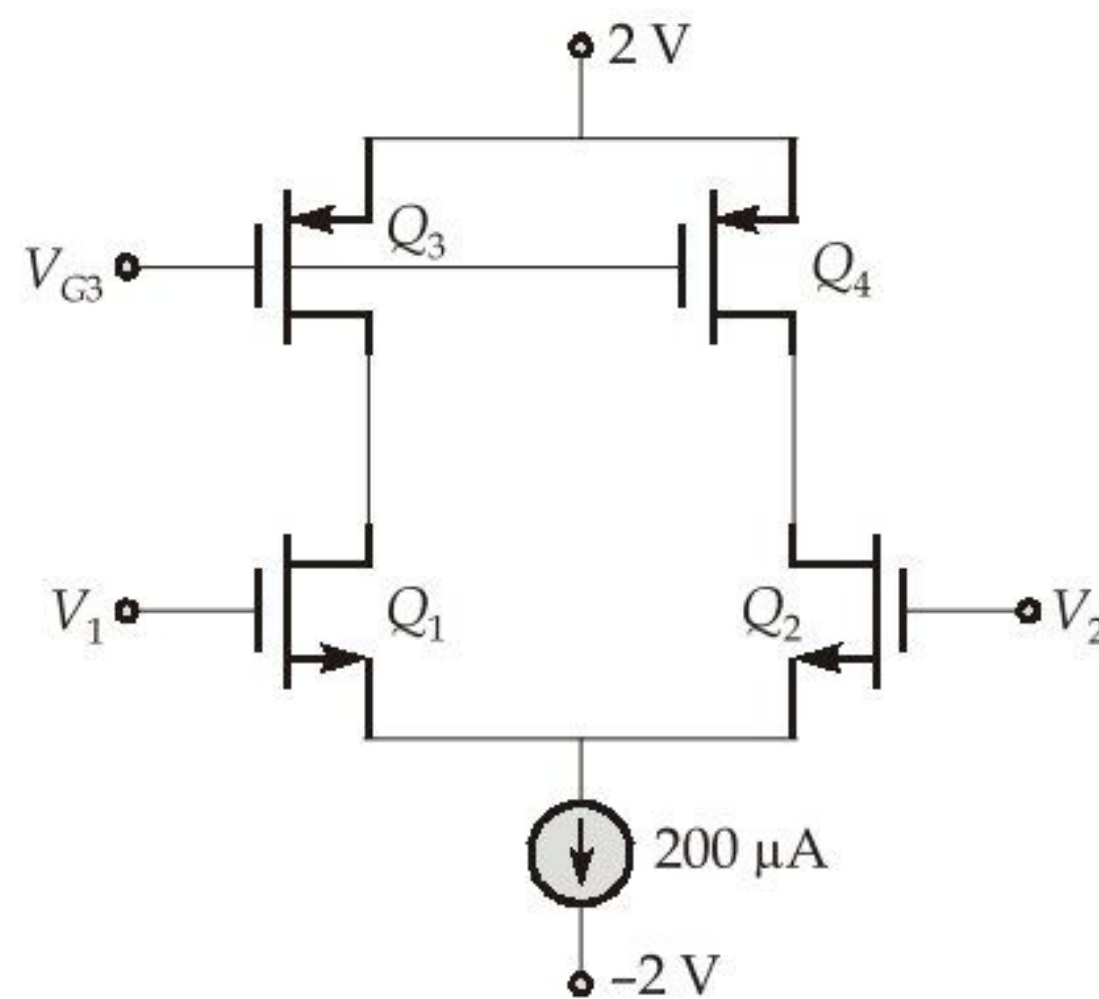
$$R_{load} = R_{th} = 2.4 \Omega$$

and maximum Power absorbed
by load

$$= \left(\frac{V_{th}}{2} \right)^2 \times \frac{1}{R_{load}}$$

$$= \left(\frac{-10}{2} \right)^2 \times \frac{1}{2.4} = 10.4167 \text{ W}$$

- Q.4 (c) (i) What are the common ways to improve the CMRR of an emitter (source coupled) differential amplifier?
- (ii) Find the differential gain and (W/L) of all transistors in the circuit below, Q_3 and Q_4 are matched, Q_1 and Q_2 are matched, all transistors have $V_{OV} = 0.2$ V, $\mu_n C_{ox} = 400 \mu\text{A}/\text{V}^2$, $\mu_p C_{ox} = 100 \mu\text{A}/\text{V}^2$ and $V_{An} = |V_{Ap}| = 3.6$ V (Ignore channel width modulation).



[5 + 15 marks]

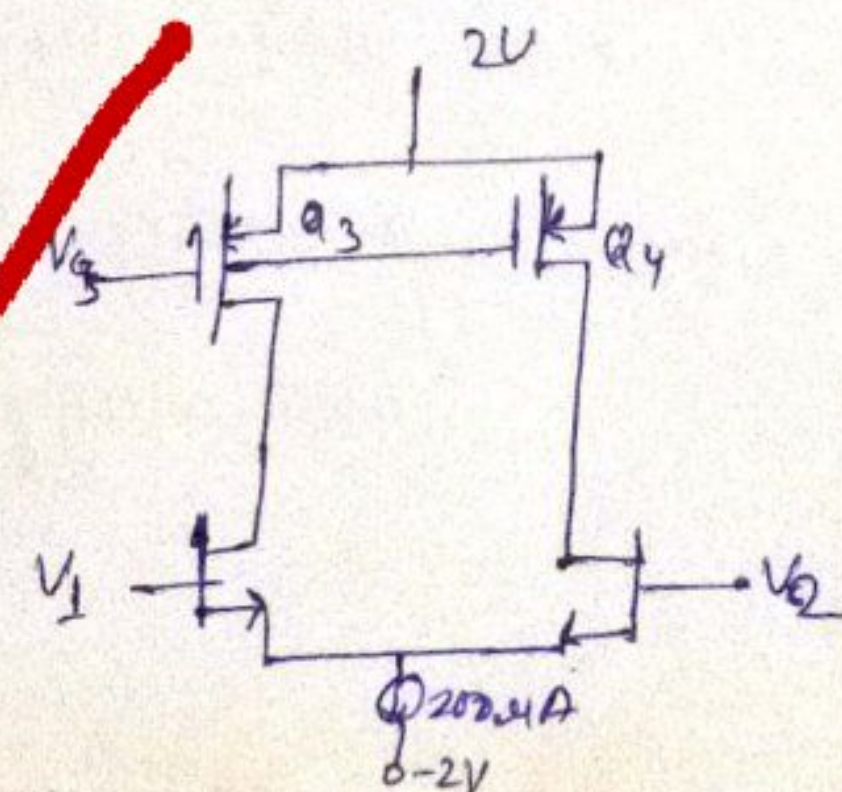
Ans 4(c) ways to improve CMRR:

- CMRR improves when emitter resistance R_E is higher. But voltage source is also loaded if R_E is higher. It is better to replace R_E with a current source.
- CMRR is also improved by adding a follower i.e. adding emitter followers which would reduce output impedance $R_C \approx R_C / \beta$.

(ii) Given, $V_{OV} = 0.2$ V, $\mu_n C_{ox} = 400 \mu\text{A}/\text{V}^2$, $\mu_p C_{ox} = 100 \mu\text{A}/\text{V}^2$ and $V_{An} = |V_{Ap}| = 3.6$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = 100 \mu\text{A}$$

$$\therefore I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right) V_{OV}^2$$



$$100 \times 10^{-6} = 0.5 \times 400 \times 10^{-6} \left(\frac{W}{L} \right)_1 0.2^2 \Rightarrow \left(\frac{W}{L} \right)_1 = 12.5$$

$$\text{also } \left(\frac{W}{L} \right)_2 = 12.5$$

$$\text{Now } I_{D3} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L} \right)_3 V_{ov}^2 \Rightarrow 100 \times 10^{-6} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L} \right)_3 0.2^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_3 = \left(\frac{W}{L} \right)_4 = 50$$

$$\text{So gain } A_d = \frac{V_{\text{output}}}{V_{\text{in}}} = g_{m1} (r_{o1} \parallel r_{o3})$$

$$\Rightarrow g_{m1} = \frac{I_{D1}}{0.5V} = 10^{-3} \text{ A/V}$$

$$\Rightarrow r_{o3} = \frac{V_A}{I_{D1}} = \frac{3.6}{100 \mu\text{A}} = 36 \text{ k}\Omega = r_{o1}$$

$$\text{So differential gain, } A_d = -g_{m1} (r_{o1} \parallel r_{o3})$$

$$A_d = -10^{-3} (36 \text{ k} \parallel 36 \text{ k}) = -18$$

$$\text{So gain } |A_d| = 18$$

Section-B

Q.5 (a)

A 4-way set-associated cache memory unit with a capacity of 16 kB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. Find the length (number of bits) of TAG field.

[12 marks]

Ans 5 (a) Given 4way set associated cache memory = 16kB

Block size = 8 words, each of size 4 Block.

word length = 32 bits and physical address = 4GB.

Number of blocks in cache memory = $N = \frac{\text{Size of Cache Memory}}{\text{Block size}}$

$$\text{So } N = \frac{16 \times 1024 \text{ B}}{8 \times 4 \text{ B}} = 512 \quad [\because 4 \text{ bytes} = 1 \text{ word}]$$

and Number of sets, $S = \frac{\text{Number of blocks in cache}}{\text{No. of block in a set}} = \frac{N}{K}$

$$S = \frac{512}{4} = 128$$

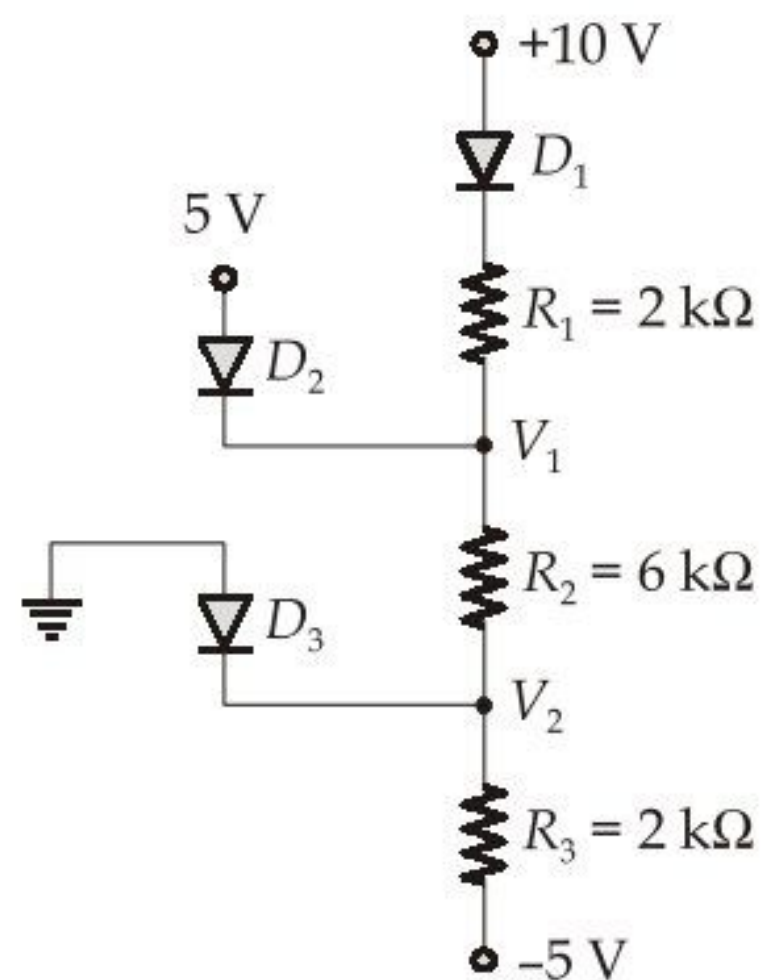
Now, physical address = 4 GB = 2^{32} B

$$\text{So size of set} = \frac{\text{physical address}}{\text{No. of set}} = \frac{2^{32} \text{ B}}{128} = 2^{25} \text{ B}$$

$$= 2^{23} \text{ words} = 2^{20} \text{ blocks}$$

So each set can access 2^{20} blocks. So tag bits needed to identify each block is 20 bits.

Q.5 (b) For the diode circuit shown, the potential barrier for diode is 0.7 V. Find V_1 and V_2 .



[12 marks]

Ans 5 (b) Given

Given diode barrier potential is 0.7V.
 \therefore Max. system voltage is +10V so Diode D_1 is ON.
Assuming D_1 ON and D_3 ON, D_2 OFF
 So, $V_2 = -0.7V$, Applying KVL from A to C
 $10 - 0.7 - 2 \times 10^3 \times I - 6 \times 10^3 \times I = V_2 = -0.7$
 So $I_{\text{current}} = \frac{10 - 0.7 - (-0.7)}{(2 + 6) \times 10^3}$

$$\therefore I = 1.25 \text{ mA. So, } V_1 = 10 - 0.7 - 2 \times 10^3 \times 1.25 \times 10^{-3}$$

$$V_1 = 6.8V \quad (\because V_+ = 5V, V_- = V_1 = 6.8V \text{ so diode OFF})$$

Hence our assumption was right therefore the

Potential $V_1 = 6.8 \text{ Volts}$

and $V_2 = -0.7 \text{ Volts}$

Q.5 (c) (i) Evaluate $\int_C \frac{|z|e^z}{z^2} dz$, where C is the circle with radius 2 and center at origin.

(ii) Evaluate $\int_C \frac{2z+1}{z^2+z} dz$ using Cauchy's integral formula, where C is $|z| = 0.5$.

[6 + 6 marks]

Ans 5c (i) $\int_C \frac{|z|e^z}{z^2} dz$, $C =$ circle of $r=2$ at centre at origin. $\Rightarrow |z|=2$

So $\int_C \frac{2e^z}{z^2} dz$ So $f(z) = \frac{2e^z}{z^2}$ has pole at $z=0$

By Taylor series expansion of $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$

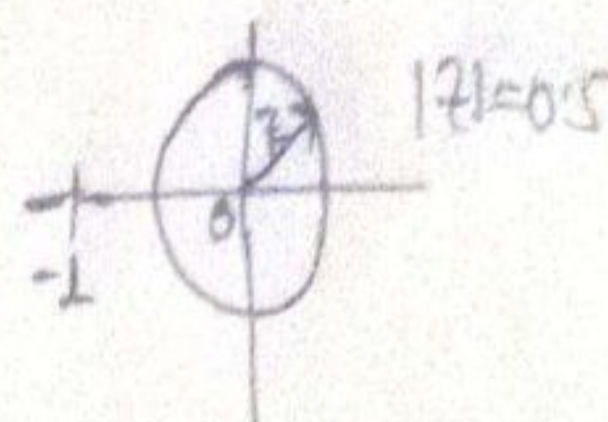
So $\frac{2e^z}{z^2} = \frac{2 \left[1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots \right]}{z^2} = 2 \left[\frac{1}{z^2} + \frac{1}{z} + \frac{1}{2!} + \frac{z}{3!} + \dots \right]$

$\lim_{z \rightarrow 0} \text{Res } f(z) \Rightarrow 2$

So By residue theorem

$$\int_C \frac{|z|e^z}{z^2} dz = 2 \times 2\pi i = 4\pi i$$

Ans (ii) $I = \int \frac{2z+1}{z^2+z} dz$, $|z| = 0.5$



$\Rightarrow \int \frac{2z+1}{z(z+1)} dz$ So pole at $z=0$, $z=-1$

$\therefore z=0$ lie inside circle & $z=-1$ lie outside circle

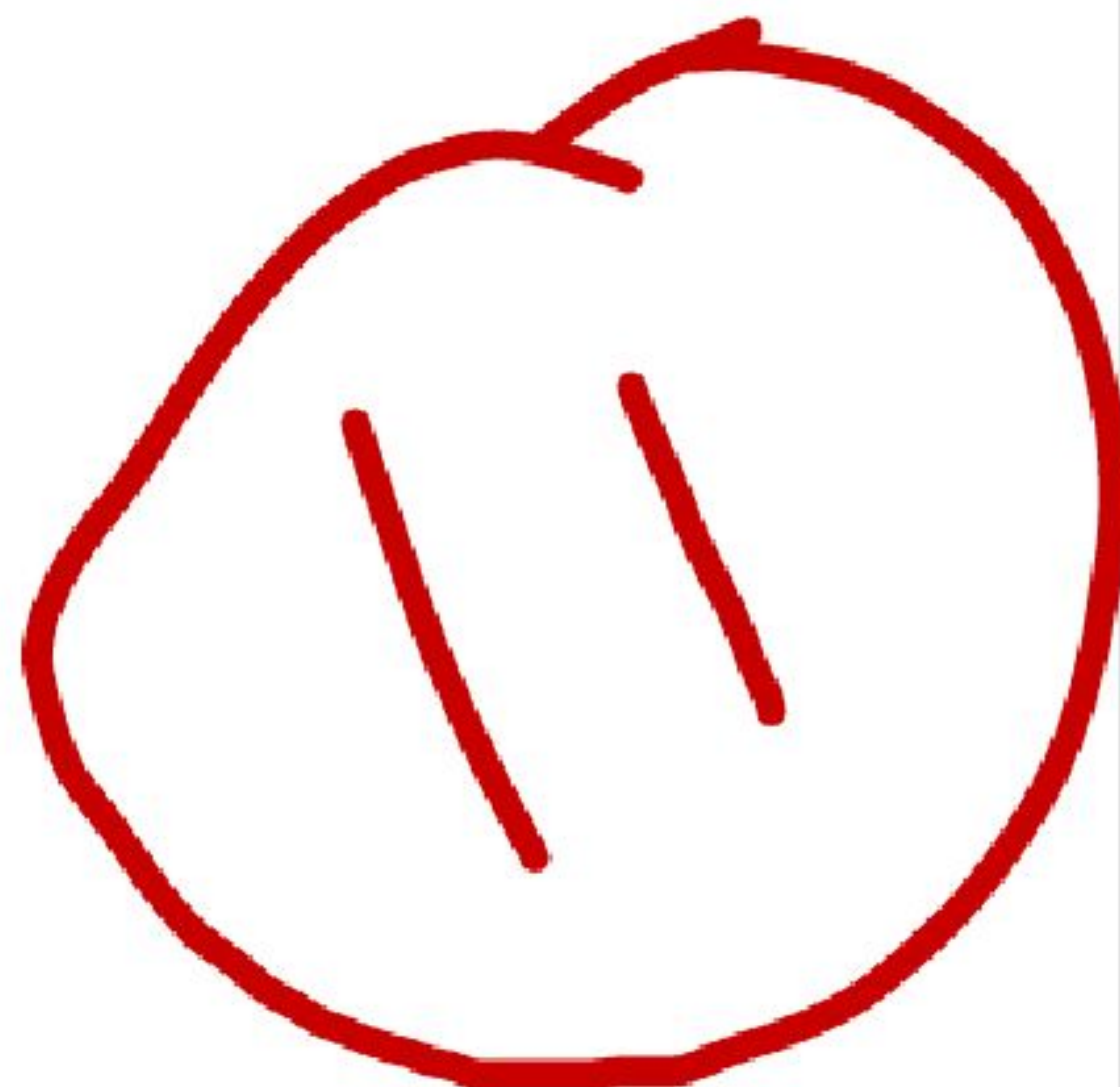
So $\text{Res } f(z) \Big|_{z=-1} = 0$ (\therefore)

Using Cauchy integral formula.

Now, $I = 2\pi i \text{Res}(f(z)) \Big|_{z=0} = 2\pi i \times \frac{2z+1}{(z+1)} \Big|_{z=0}$

$= 2\pi i \times \frac{(2/0)+1}{0+1} = 2\pi i$

Hence integral of I is $= 2\pi i$



- Q.5 (d) Two elements A and B form a compound AB which crystallizes in the Sodium Chloride structure. Assuming the atoms may be considered as hard spheres of radii r_a and r_b , show that atoms along a cube edge cannot touch each other as soon as the ratio of the radii is larger than 2.414.

[12 marks]

Ans (d). As NaCl has F.C.C. structure

Cation Na^+ lie inside the void created by Cl^- ions.

Let a be side length of lattice.

r_a = Radius of Na^+ and r_b = radius of Cl^-

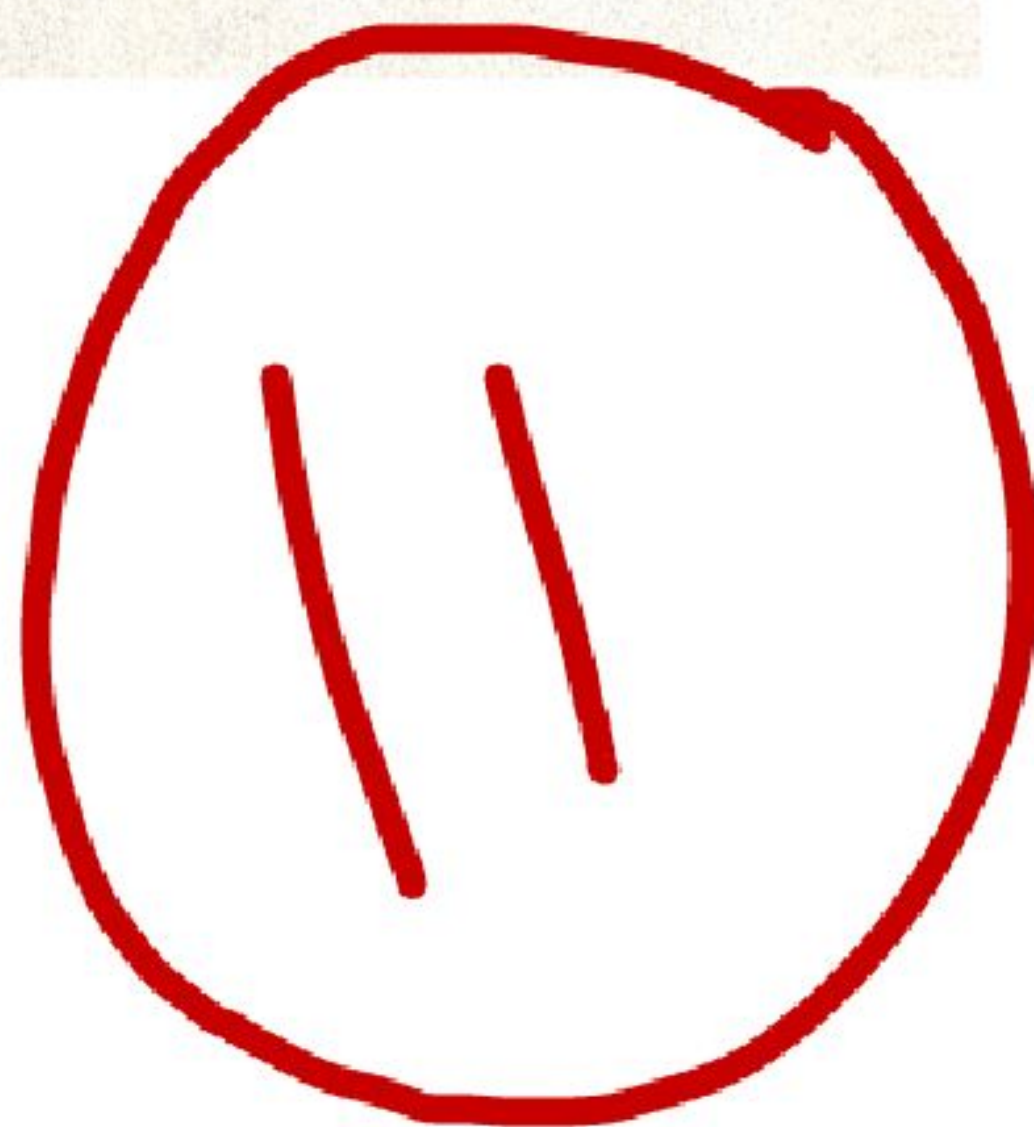
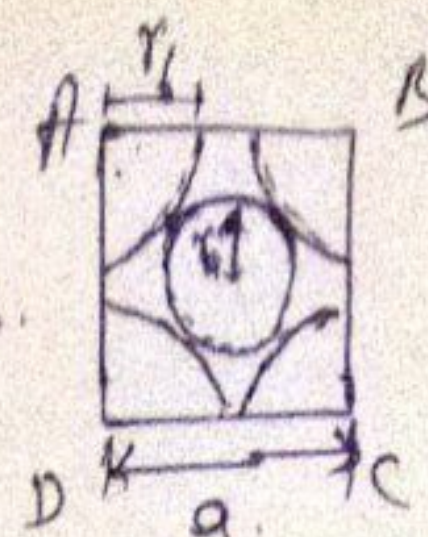
So, for A and B to touch each other, Diagonal length AC
 $\leq 2r_a + 2r_b$

$$\text{or } \sqrt{2}a \leq 2(r_a + r_b) \Rightarrow \sqrt{2} \times 2r_b \leq 2(r_a + r_b)$$

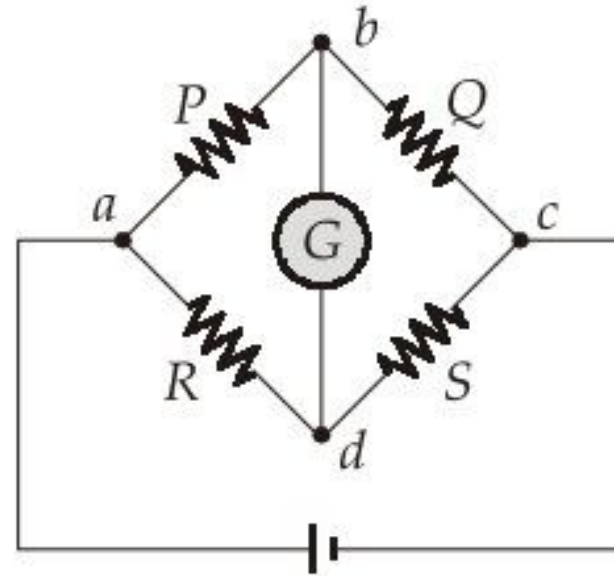
$$\Rightarrow \sqrt{2} \leq \left(\frac{r_a}{r_b} + 1\right) \Rightarrow \frac{r_a}{r_b} \geq \sqrt{2} - 1$$

$$\text{So, } \frac{r_b}{r_a} \leq \frac{1}{\sqrt{2} - 1} = 2.414$$

Hence ratio of radii is 2.414 for atoms to touch each other.



- Q.5 (e) In a Wheatstone bridge, the values of resistances of various arms are $P = 1000 \Omega$, $Q = 100 \Omega$, $R = 2005 \Omega$ and $S = 200 \Omega$. The battery has an emf of 5 V and negligible internal resistance. The galvanometer has a current sensitivity of $10 \text{ mm}/\mu\text{A}$ and an internal resistance of 100Ω . Calculate the deflection of galvanometer and the sensitivity of the bridge in terms of deflection per limit change in resistance.



[12 marks]

Ans Given

Given $P = 1000 \Omega$, $Q = 100 \Omega$
 $R = 2005 \Omega$, $S = 200 \Omega$

So, $V_b = 5 \times \frac{Q}{P+Q} = 5 \times \frac{100}{1000+100}$
 $= 0.4545 \text{ Volts}$

$V_d = 5 \times \frac{S}{R+S} = \frac{5 \times 200}{2005+200} = 0.4535 \text{ Volts}$

Hence $V_b - V_d = 1.0307 \times 10^{-3} \text{ Volts}$, Given Galvanometer, $R_g = 100 \Omega$

Internal resistance of bridge, $R' = \frac{RS}{R+S} + \frac{PQ}{P+Q}$

$= \frac{2005 \times 200}{2005+200} + \frac{1000 \times 100}{1000+100} = 272.77 \Omega$

So current through galvanometer, $I_D = \frac{V_{bd}}{R' + R_g}$

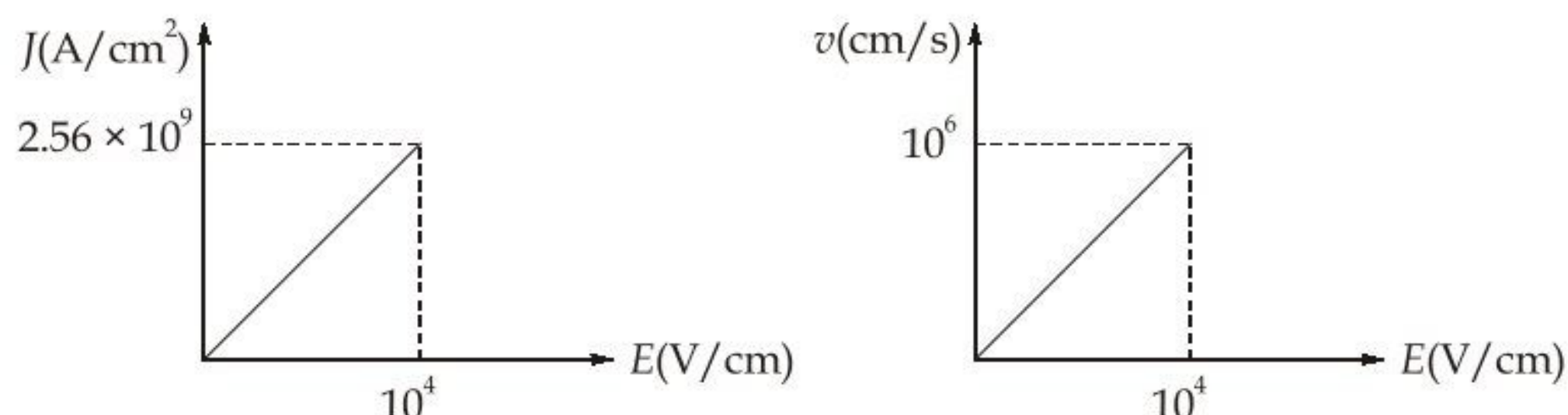
$\therefore I_D = (1.0307 \times 10^{-3}) / (272.77 + 100) = 2.77 \mu\text{A}$

Deflection of galvanometer $= S_I \times I_D = 10 \times 2.77 = 27.7 \text{ mm}$

Sensitivity of bridge, $S_B = \frac{\theta}{\Delta R} = \frac{27.7}{5} = 5.54 \text{ mm}/\Omega$

- Q.6 (a) (i) For an unknown material, which has a simple cubic lattice and lattice constant $a = 0.5 \text{ nm}$, the J vs E and v vs E plots are given. Using the classical Drude model, determine the valence of atoms in this material.

Also, redraw the plots J vs E and v vs E , if the valence is now 3 and structure is BCC.



- (ii) For a p - n germanium abrupt junction, let the conductivity of the bulk p -region be $10^4 \Omega^{-1} \text{ m}^{-1}$ and that of bulk n -region is $100 \Omega^{-1} \text{ m}^{-1}$. In thermal equilibrium, the voltage drop across the barrier layer is 0.5 V . Calculate the capacitance of the junction if its circular cross section has a diameter of 0.15 mm . Also, calculate the capacitance for a reverse voltage of 3 V . ($\mu_e = 0.36 \text{ m}^2/\text{V-s}$, $\mu_h = 0.17 \text{ m}^2/\text{V-s}$).

- (iii) Why are metals transparent to high-frequency X-ray and γ -ray radiations?

[8 + 8 + 4 marks]

Ans 6 (a) Given J vs E and v vs E plots.

From J vs E plot: $J = \sigma E$ where σ is conductivity.

From v vs E plot: $v = \mu E$ where μ is mobility.

Let n be the no. of valence electron then

$\sigma = \frac{n}{V_c} \times q \times \mu$ where $V_c = 0.5 \times 10^{-9} \text{ m}^3$

$2.56 \times 10^7 = \frac{n}{0.5 \times 10^{-9}} \times 1.6 \times 10^{-19} \times 10^{-2}$

$\Rightarrow n = 2$ Hence valency of atom in material is 2.

Given valency $n = 3$ and structure is BCC.

for BCC effective number of atoms in unit cell $A = 2$

So, $\sigma_{\text{new}} = \frac{3}{2} \sigma \times 2 = 3\sigma$

but V_D vs E remains same $\therefore \mu$ is not changing

$\sigma = \frac{N e^2 \tau}{m_e}$ and $N = \frac{A n}{a^3}$

So $\sigma \propto A n$

Redrawn plots for $n = 3$ and BCC structure:

(ii) Given, $\sigma_p = 10^4 (\Omega \cdot m)^{-1}$, $\sigma_n = 100 (\Omega \cdot m)^{-1}$, $V_D = 0.5V$

Diometer = $0.15mm$, $\mu_e = 0.36 \frac{m^2}{V \cdot s}$, $\mu_h = 0.17 \frac{m^2}{V \cdot s}$

$$\therefore \sigma_p = n_h e \mu_h \Rightarrow n_h = \frac{\sigma_p}{e \mu_h} = \frac{10^4}{1.6 \times 10^{-19} \times 0.17} = 3.677 \times 10^{23} m^{-3}$$

Similarly $\sigma_n = n_e e \mu_e \Rightarrow n_e = \frac{\sigma_n}{e \mu_e} = \frac{10^2}{1.6 \times 10^{-19} \times 0.36} = 1.736 \times 10^{21} m^{-3}$

Voltage drop, $V_D = 0.5V$

width of depletion region, $W = \sqrt{\frac{2\epsilon}{e} V_D \left[\frac{1}{n_e} + \frac{1}{n_h} \right]}$

$$\Rightarrow W = 0.179 \mu m$$

\therefore Capacitance, $C = \frac{\epsilon A}{W} = \frac{8.85 \times 10^{-12} \times \pi \left(\frac{0.15 \times 10^{-3}}{4} \right)^2}{179 \times 10^{-6}}$

$$C = 0.874 pF$$

for 3V reverse bias voltage depletion region width

$$W = \sqrt{\frac{2 \times \epsilon}{e} [V_D + |V_R|] \left[\frac{1}{n_e} + \frac{1}{n_h} \right]}$$

$$W = \sqrt{\frac{2 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19}} [0.5 + 3] \left[\frac{1}{3.677 \times 10^{23}} + \frac{1}{1.736 \times 10^{23}} \right]}$$

$$W_2 = 0.473 \mu m$$

So capacitance $C_2 = \frac{\epsilon A}{W_2} = \frac{8.85 \times 10^{-12} \times \pi \left(\frac{0.15 \times 10^{-3}}{4} \right)^2}{0.473 \times 10^{-6}}$

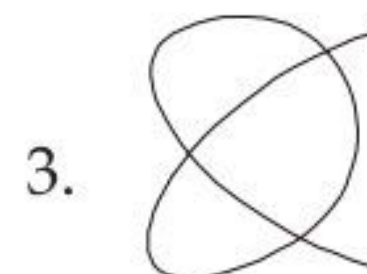
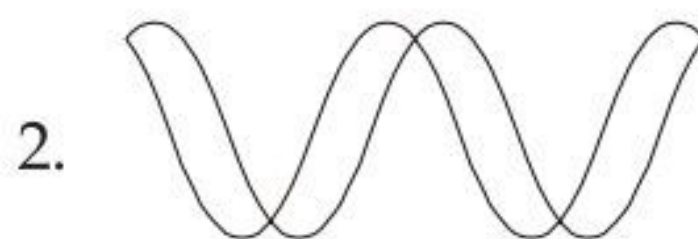
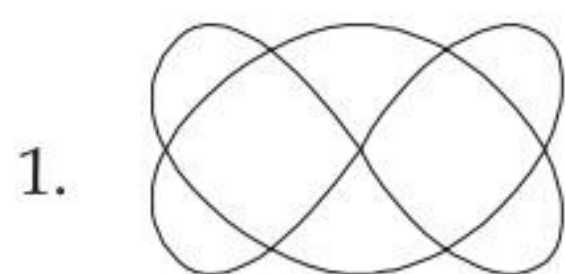
$$\Rightarrow C_2 = 0.331 pF$$

(iii) High energy E.M. radiation penetrate metal due to its high energy. It see metal as individual nucleus and electrons. Thus photon scattering is very low, also photon converts to electron and positron in electric field of atomic nucleus and can radiate photon.

Q.6 (b) (i) Explain the role of following parts in a Cathode-Ray Oscilloscope (CRO).

1. Time Base Generator
2. Blanking Circuit

(ii) In the screen-pattern oscillograms shown, a sine-wave signal of unknown frequency is connected to the vertical input terminals of the oscilloscope and a 60 Hz voltage is connected to the horizontal input. State the frequency of unknown signal in each case.

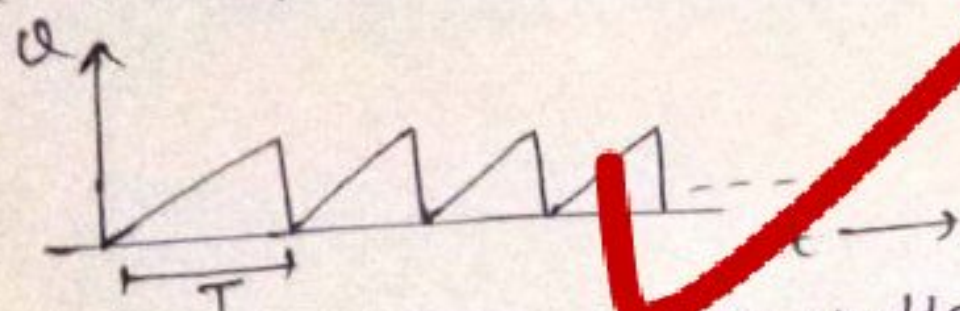


(iii) Explain the working principle of electromagnetic flow meters.

[8 + 6 + 6 marks]

Ans (i) (ii) (i) Time Base Generator → It is specifically designed electronic function generator to produce various time base signals by generating varying voltage or current.

- This generated linear time-varying voltages are used in cathode ray tube to deflect the electron beam in a horizontal direction.

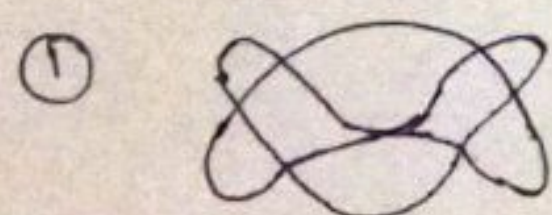


A Saw tooth voltage waveform.

(ii) Blanking Circuit → A sweep waveform delivered from a sawtooth waveform generator forms the trace across the CRT screen by dragging the electron beam across the screen face inside CRT.

- During sweep time, the spot is made visible during linear side of waveform, left to right.
- The retrace is quick but still visible the line from right to left. The line is unnecessary so this part of retrace is made invisible by applying a negative voltage on the control grid of CRT which effectively cut off the beam during the short retrace interval.

(ii) Given $f_x = 60 \text{ Hz}$

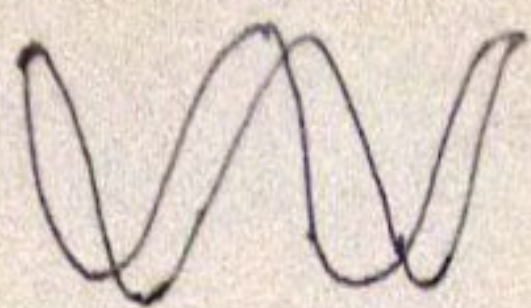


$$\therefore \frac{f_y}{f_x} = \frac{\text{Horizontal tangent cut at curve}}{\text{Vertical tangent cut at curve}}$$

$$\frac{f_y}{60} = \frac{6}{2}$$

$$\Rightarrow f_y = 180 \text{ Hz}$$

②.



$$\frac{f_y}{f_x} = \frac{\text{Horizontal tangent cut on curve}}{\text{Vertical tangent cut on curve}}$$

$$\Rightarrow \frac{f_y}{f_x} = \frac{8}{2} \Rightarrow f_y = 4 \times 60 = 240 \text{ Hz}$$

3.



$$\frac{f_y}{f_x} = \frac{\text{Horizontal tangent cut on the curve}}{\text{Vertical tangent cut on the curve}}$$

$$f_y = 60 \times \frac{3}{4} \Rightarrow 45 \text{ Hz}$$

(iii) Electromagnetic flow meter \Rightarrow Electromagnetic flow meters are based on 'Faraday Law induction'. These meters are called magflow or electromagnetic flow meters.

• A magnetic field is applied to the metering tube, which results in a potential difference proportional to flow velocity, perpendicular to the flux lines

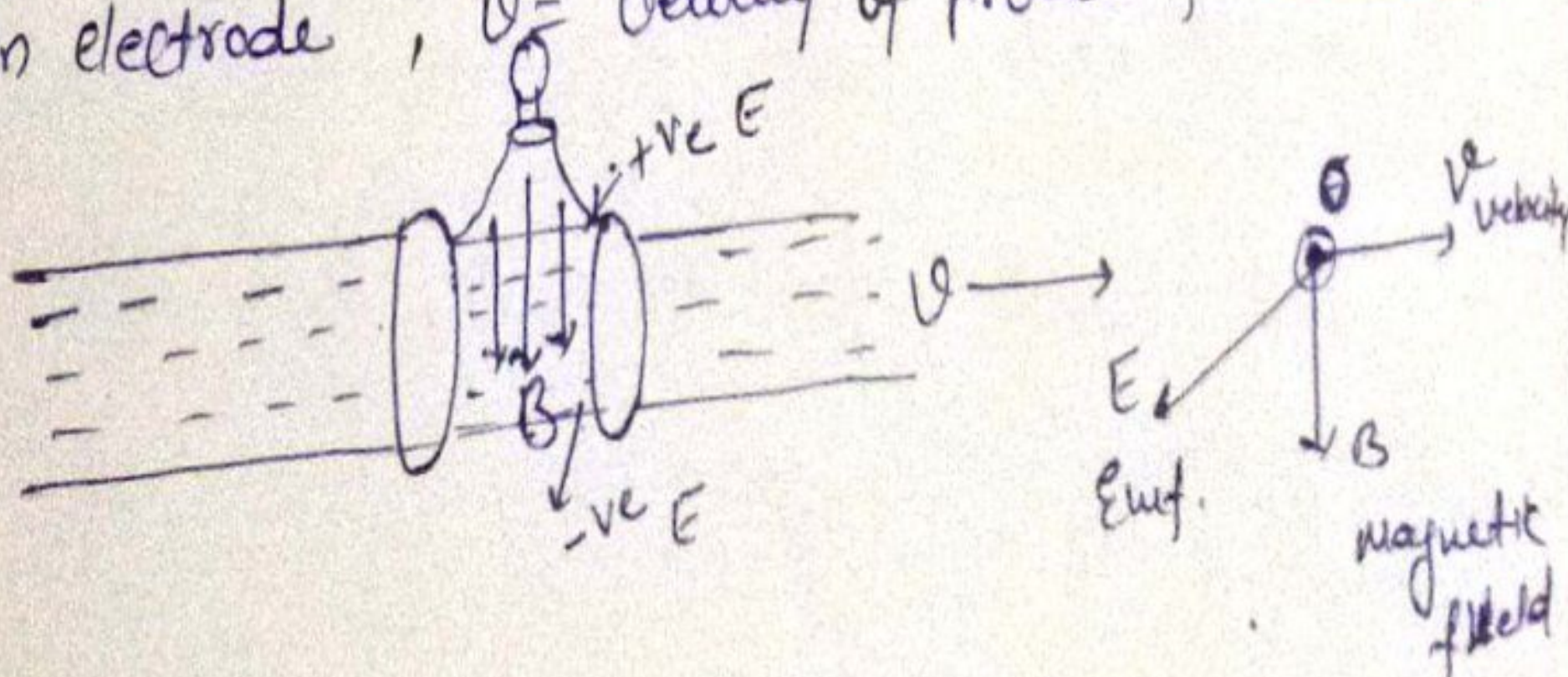
• The physical principle at work is electromagnetic induction.

$$E = k B D V$$

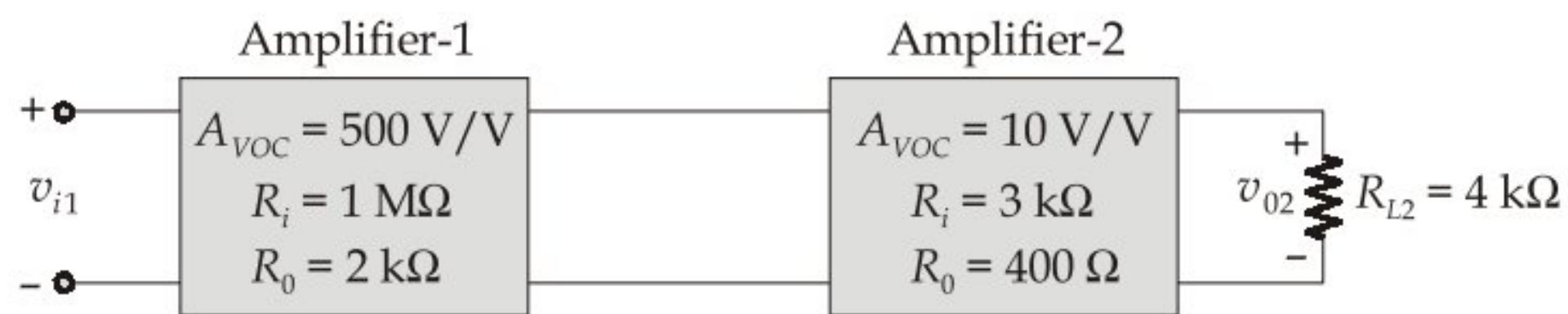
Where E = induced voltage

k = Proportionality constant, B = Magnetic field strength,

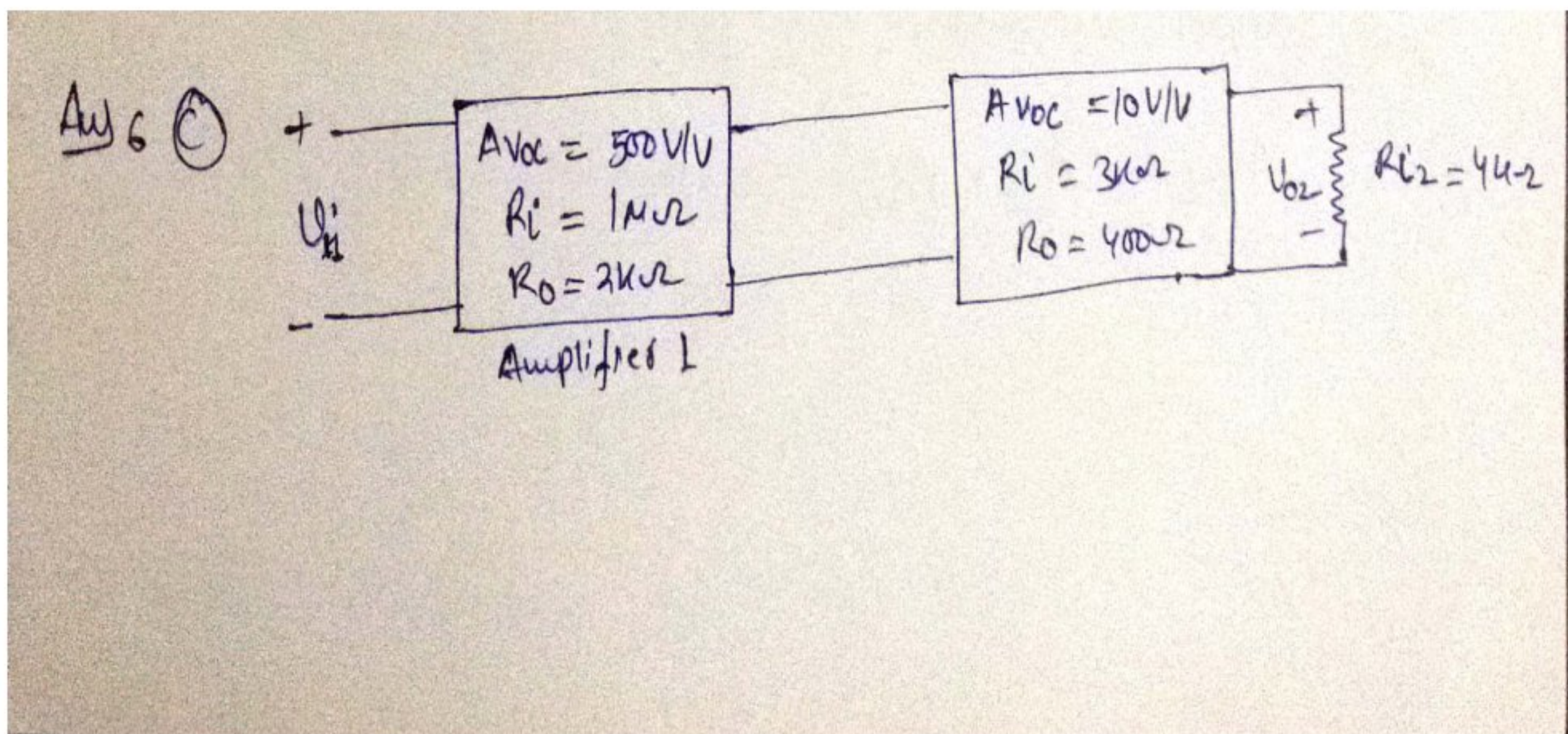
D = distance between electrode, V = Velocity of process fluid



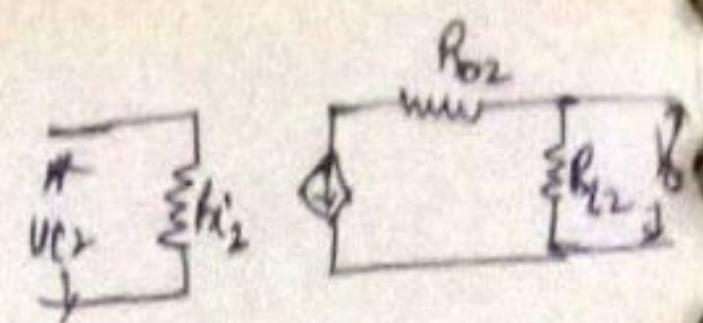
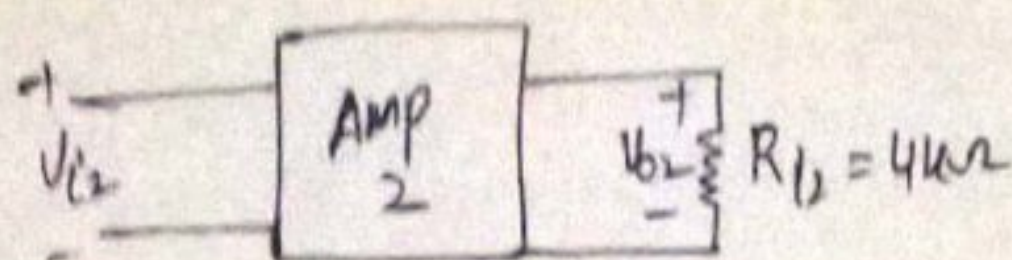
- Q.6 (c) Find the A_v , A_i , R_i , R_o and A_{VOC} of the equivalent amplifier by series cascading Amplifier-1 and Amplifier-2.



[20 marks]



Consider Amplifier 2

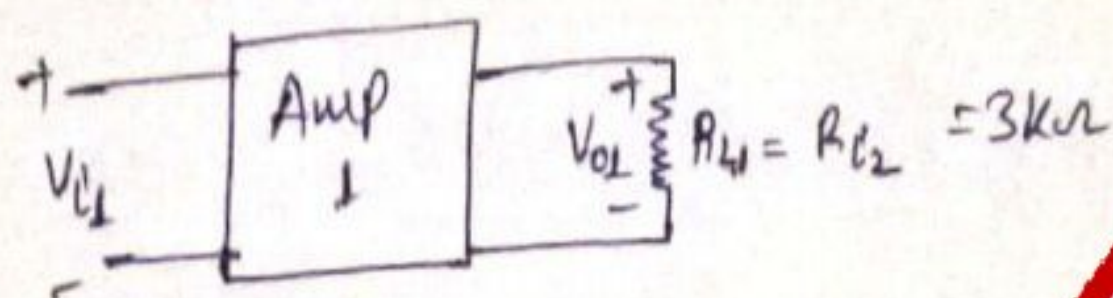


$$\Rightarrow A_{V2} = \frac{V_{o2}}{V_{i2}} = A_{VOC} \times \frac{R_{L2}}{R_{L2} + R_{o2}} = 10 \times \frac{4 \times 10^3}{4 \times 10^3 + 400}$$

$$\Rightarrow A_{V2} = 9.1 \frac{V}{V}$$

$$\text{Current gain } A_{I2} = \frac{I_{o2}}{I_{i2}} = A_{V2} \times \frac{R_{i2}}{R_{L2}} = 9.1 \times \frac{3 \times 10^3}{4 \times 10^3} = 6.82$$

Consider Amplifier 1



$$\text{So } A_{V1} = \frac{V_{o1}}{V_{i1}} = A_{VOC1} \times \frac{R_{L1}}{R_{L1} + R_{o1}}$$

$$\Rightarrow A_{V1} = 500 \times \frac{3 \times 10^3}{(3 \times 10^3 + 2 \times 10^3)} = 300$$

$$\text{and current gain } A_{I1} = \frac{I_{o1}}{I_{i1}} = A_{V1} \times \frac{R_{i1}}{R_{L1}} = 300 \times \frac{10^4}{3 \times 10^3} = 10^5$$

$$\text{So overall Voltage gain, } A_V = A_{V1} \cdot A_{V2} = 300 \times 9.1 = 2730$$

$$\text{and Current gain, } A_I = A_{I1} \cdot A_{I2} = 10^5 \times 6.82 = 6.82 \times 10^5$$

Input Resistance $R_i = R_{i1} = 1M\Omega$

and output resistance, $R_{o2} = 400\Omega$

$$A_{VOC} = A_{V1} \times A_{VOC2} = 300 \times 10 = 3000$$

Q.7 (a)

(i) Briefly explain the use of following network devices:

1. Repeater

2. Bridge

3. Router

(ii) What is a process? Explain the different process states with the help of a process life cycle.

[9 + 11 marks]

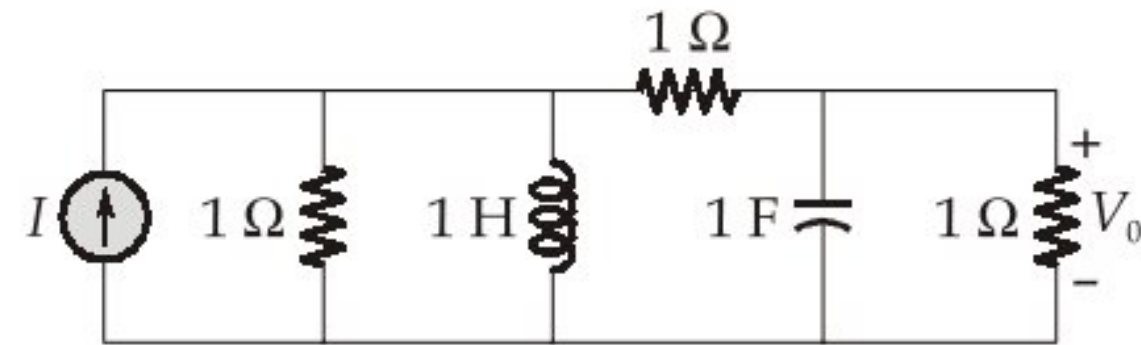
Q.7 (b)

- (i) Find the Taylor series of $\ln(1 + x)$ centered at $a = 0$.
- (ii) Determine the radius of convergence of this Taylor series.
- (iii) Use the first two non-zero terms of the power series you found in (i) to approximate $\ln \frac{3}{2}$.
- (iv) Give an upper bound on the error in your approximation in (iii) using Taylor's inequality.

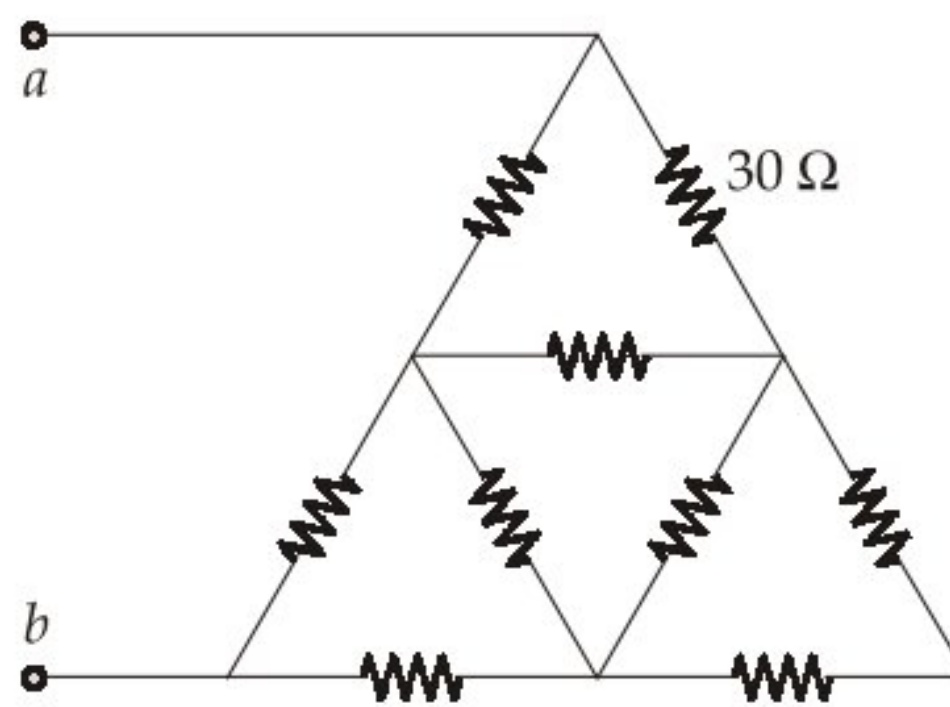
[8 + 4 + 2 + 6 marks]

Q.7 (c) (i) For the network shown, find

1. the transfer function $H(\omega) = \frac{V_0(\omega)}{I(\omega)}$.
2. the magnitude of H at $\omega_0 = 1 \text{ rad/s}$.



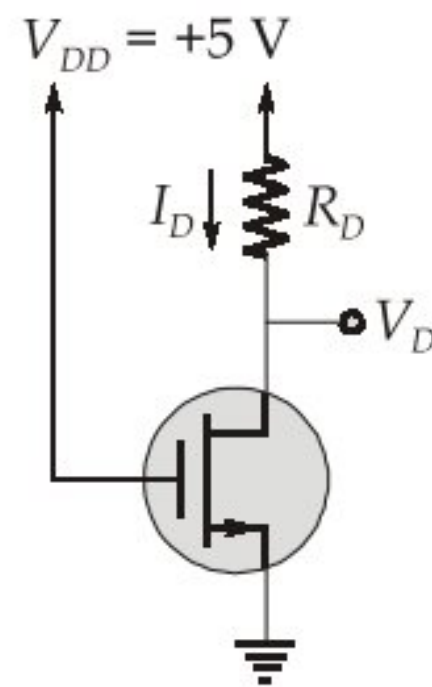
(ii) Find the equivalent resistance between terminals a and b (All resistances are equal).



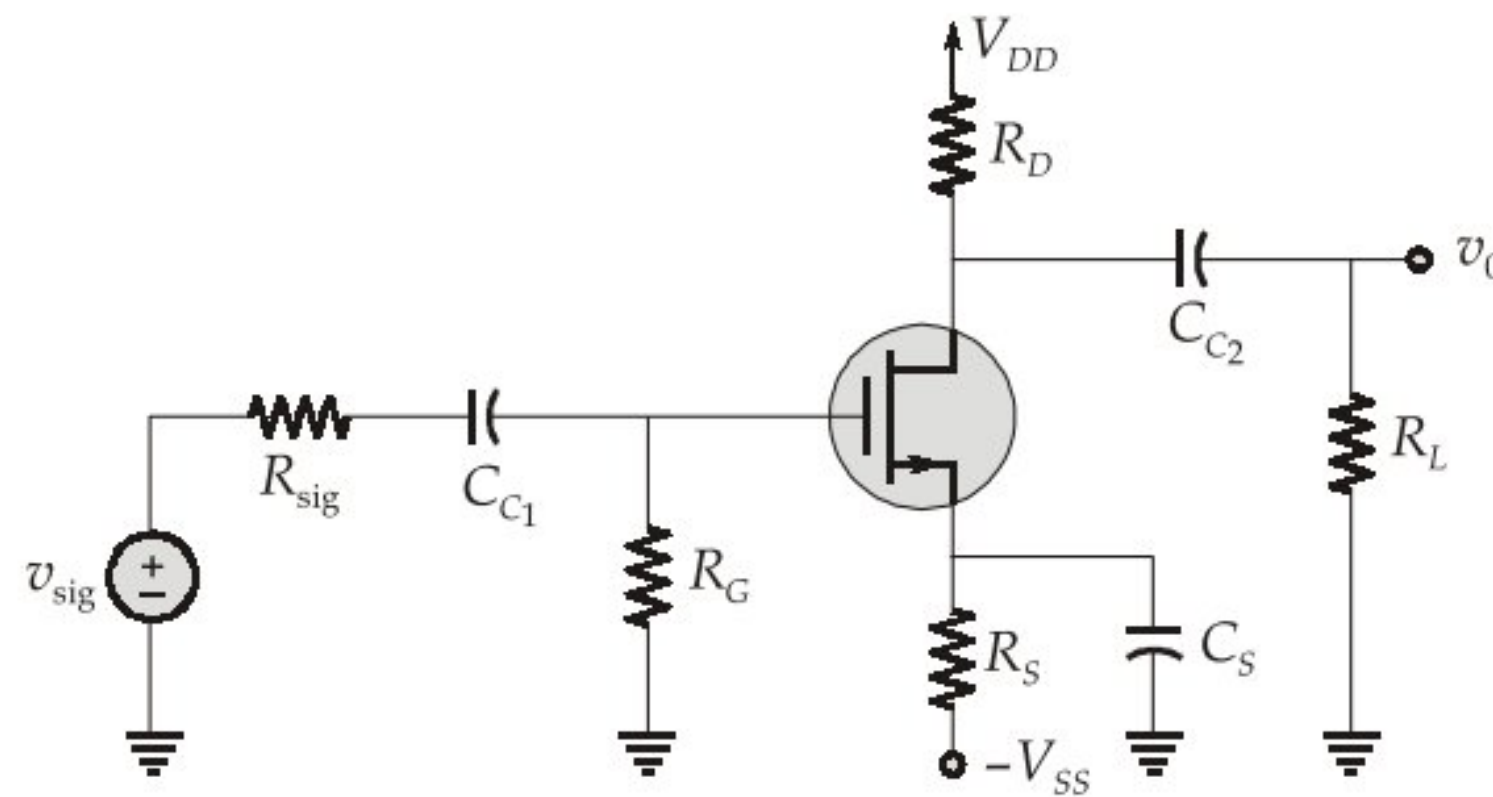
[12 + 8 marks]

- Q.8 (a) (i) Find R_D to establish a drain voltage of 0.1 V. What is the effective resistance between drain and source at this operating point?

$$V_t = 1 \text{ V and } k'_n \left(\frac{W}{L} \right) = 1 \text{ mA/V}^2.$$



- (ii) For $R_G = 4.7 \text{ M}\Omega$, $R_D = R_S = R_L = 15 \text{ k}\Omega$, $R_{\text{sig}} = 100 \text{ k}\Omega$ and $g_m = 1 \text{ mA/V}$. Find C_S , C_{C1} and C_{C2} for a lower cut off frequency $f_L = f_{P2}$ of 20 Hz and other pole frequencies, $f_{P1} = 1 \text{ Hz}$ and $f_{P3} = 4 \text{ Hz}$.



[8 + 12 marks]

Q.8 (b) (i) Given page reference string : 1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6. Compare the number of page faults for LRU and FIFO page replacement algorithms (4 frames page).

(ii) Run through the given code and state the output.

```
#include <stdio.h>
int foo (int* a, int* b)
{
    int sum = *a + *b;
    *b = *a;
    return *a = sum - *b;
}
int main ( )
{
    int i = 0, j = 1, k = 2, l;
    l = i++ || foo (&j, &k);
    printf ("%d %d %d %d", i, j, k, l),
    return 0;
}
```

[12 + 8 marks]

- Q.8 (c) Suppose a dielectric has a complex dielectric constant given by $\epsilon_r^* = \epsilon_{ei} + \epsilon_{ro}^*$ where ϵ_{ro}^* refers to the dipole orientations and ϵ_{ei} is a real quantity referring to the electronic and ionic polarizations. Assume that ϵ_{ro}^* is determined by a simple relaxation time τ . Consider the space between two parallel metal plates filled with this dielectric. If the distance between the plates is 1 m, show that effective admittance of the condenser per m^2 plate area is equal to

$$Y^* = j\omega\epsilon_0 \left[\epsilon_{ei} + 1 + \frac{\epsilon_{ro} - 1}{1 + j\omega\tau} \right]$$

[20 marks]

Space for Rough Work

Space for Rough Work

Space for Rough Work
