Name:-

Roll No:-

Test 11 :- Full Length Test
(Paper II)

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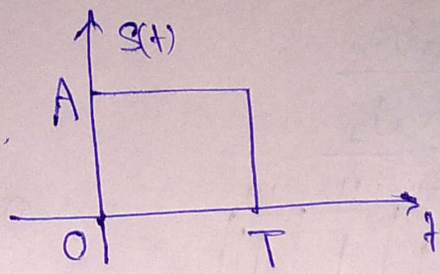
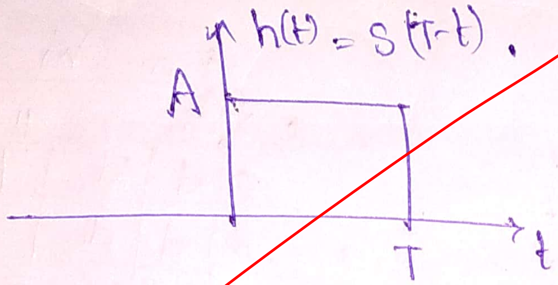
(1)(a) Given Signal $s(t)$,



Therefore, matched filter

$$h(t) = \cancel{s(t-T)}\ s^*(T-t) = s(T-t).$$

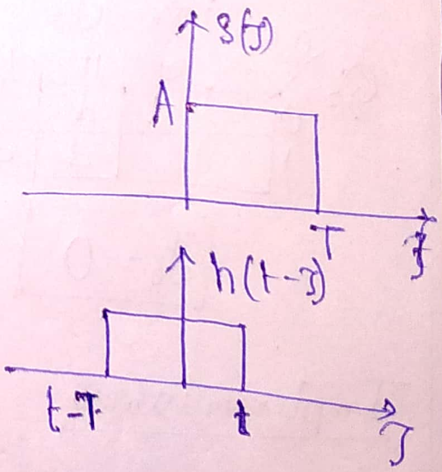
Plot of the matched filter, $h(t)$



Output response,

$$y(t) = s(t) * h(t).$$

$$\Rightarrow \quad y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau.$$



For $t < 0$,

$$y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = 0$$

For $0 \le t \le T$

$$y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = \int_{0}^{t} A \cdot A \cdot d\tau = A^2 t$$

For $T < t \leq 2T$

$$y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau.$$

$$= \int_{t-T}^{2T} A.A.\, d\tau.$$

$$= A^2 [\tau]_{t-T}^{2T}$$

$$= A^2 [2T-(t-T)]$$

$$= A^2 (2T-t).$$

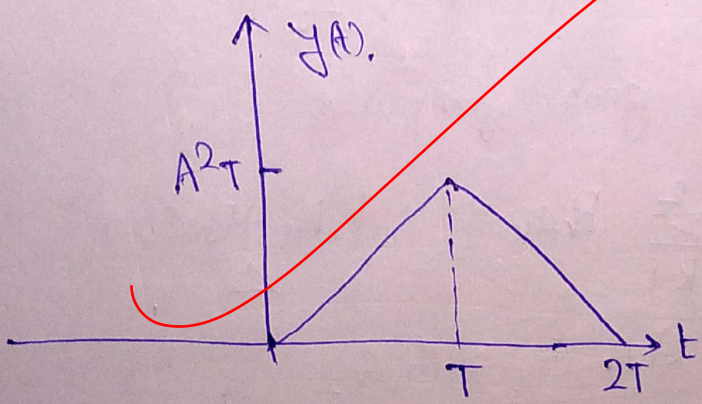
For $t > 2T$

$$y(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

$$= 0$$

$$\text{Output } y(t) = \begin{cases} A^2 t & \text{for } 0 \leq t \leq T \\ A^2(2T-t) & \text{for } T < t \leq 2T \\ 0 & \text{otherwise.} \end{cases}$$

Therefore sketch of output ~~pto~~ response $y(t)$



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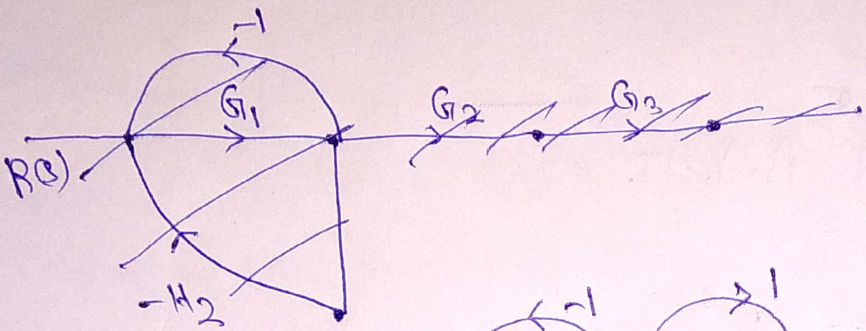
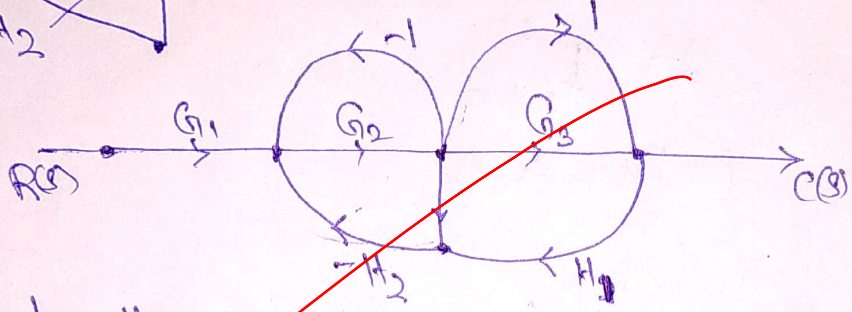
(1)(b) Signal flow graph of the given block diagram





Forward path

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_2$

Two non touching loop

Nil.

Individual loop

$L_1 = -G_2$

$L_2 = -G_2 H_2$

$L_3 = -H_1 H_2 G_2 G_3$

$L_4 = -H_1 H_2 G_2$

$\Delta = 1 - \left(\text{Sum of Individual loop gain}\right) + \left(\text{Sum of two non touching loop gain}\right) - - -$

$= 1 - (L_1 + L_2 + L_3 + L_4)$

$\Delta = 1 + G_2 + G_2 H_2 + H_1 H_2 G_2 + H_1 H_2 G_2 G_3$

$\Delta_1 = 1 \ , \ \Delta_2 = 1$

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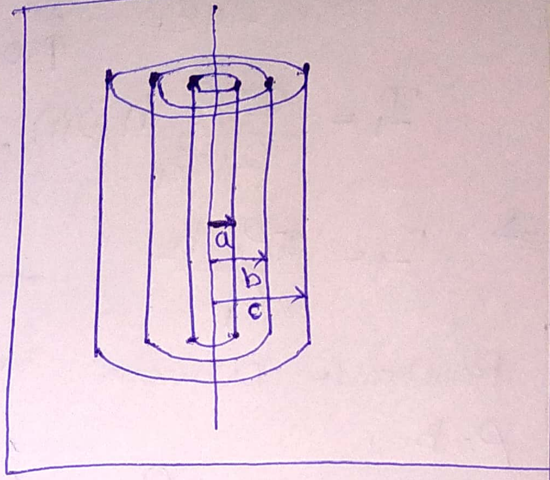
By using Mason's gain formula

$$\frac{C(S)}{R(S)} = T(S) = \frac{1}{\Delta} \sum_{K=1}^{2} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}.$$

$$\boxed{T(S) = \frac{G_1 G_2 G_3 + G_1 G_2}{1 + G_2 + G_2 H_2 + H_1 H_2 G_2 G_3 + H_1 H_2 G_2}}$$

(1)(c)

Let ρ_L (C/m) line charge density is present on the inner core of radius (a).



Therefore,

$\vec{E}$, in the medium between conductor $a<\rho<b$ by using Gauss law.

$$\oiint \vec{D}.\vec{ds} = Q_{enc}.$$

$$D_\rho(2\pi\rho h) = \rho_L h$$

$$\Rightarrow D_\rho = \frac{\rho_L}{2\pi\rho} \qquad \vec{D} = D_\rho \hat{a}_\rho = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

$$\vec{E}_1 = \frac{\vec{D}}{\epsilon_1} = \frac{\rho_L}{2\pi\rho\epsilon_1}\hat{a}_\rho$$

$$\Rightarrow \vec{E}_1 = \frac{\rho_L}{2\pi\epsilon_1\rho}\hat{a}_\rho \quad \text{———} (1)$$

Potential between surface ($\rho=a$ and $\rho=b$).

$$V_{ab} = -\int_b^a \vec{E}_1.\vec{dl} = -\int_a^b \frac{\rho_L}{2\pi\epsilon_1\rho}\,d\rho$$

$$V_{ab} = -\frac{\rho_L}{2\pi\epsilon_1}[\ln\rho]_b^a$$

$$V_{ab} = \frac{\rho_L}{2\pi\epsilon_1}\ln(b/a) \quad \text{———} (2)$$

Current crossing the conductor surface with conductivity σ_1

$$I_1 = \iint \vec{J}_1.\vec{ds} = \int_{\theta=0}^{2\pi}\int_{\phi=0}^{\,h} \sigma_1 E_1 .\,\rho\, d\theta\, dz$$

$$I_1 = \frac{\sigma_1 \rho_L}{2\pi\epsilon_1} \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{h} dz$$

$$I_1 = \frac{\sigma_1 \rho_L h (2\pi)}{2\pi \epsilon_1}.$$

$$\Rightarrow \quad I_1 = \frac{\sigma_1 \rho_L h}{\epsilon_1} \quad \text{———} (3)$$

Resistance between the conductor surface $\rho = a$ and $\rho = b$,

$$R_1 = \frac{V_1}{I_1} = \frac{\frac{\rho_L}{2\pi\epsilon_1} \ln(b/a)}{\frac{\sigma_1 \rho_L h}{\epsilon_1}}$$

$$\Rightarrow \quad R_1 = \frac{\ln(b/a)}{2\pi \sigma_1 h} \quad \text{———} (4)$$

~~Resistance~~

Similarly for surfaces $\rho = b$ and $\rho = c$ resistance.

$$R_2 = \frac{\ln(c/b)}{2\pi \sigma_2 h} \quad \text{———} (5).$$

~~Equiv.~~ Net resistance between surfaces $\rho = a$ and $\rho = c$,

$$R = R_1 + R_2 = \frac{\ln(b/a)}{2\pi\sigma_1 h} + \frac{\ln(c/b)}{2\pi\sigma_2 h}$$

Resistance per unit length ~~fo~~ i.e. $h = 1$

$$r/ = \cancel{\frac{\ln(b/a)}{2\pi\sigma_1}} \qquad \boxed{r = \frac{\ln(b/a)}{2\pi\sigma_1} + \frac{\ln(c/b)}{2\pi\sigma_2}}$$

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1(e)

Given that,

$T_{CPU} = 3$ sec.

$T_{WC} = 4.5$ sec.

$P_1 = 10 \times 10^6$ FLOP/sec

I/O operation time

$T_{I/O} = T_{WC} - T_{CPU} = 4.5 - 3$

$\Rightarrow T_{I/O} = 1.5$ sec

When processor of six time faster replaced

$T_{CPU} = \frac{3}{6} = 0.5$ sec.

$\Rightarrow \boxed{T_{CPU} = 0.5 \text{ sec}}$

$T_{WC} = T_{CPU} + T_{I/O} = 0.5 + 1.5$

$\boxed{T_{WC} = 2 \text{ sec}}$

Performance,

$\cancel{P_2 = 6P_1 = 6 \times 10 \times 10^6}$

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Previously, FLOP done in $T_{WC} = 4.5$ sec.

$= 10 \times 10^6 \times 4.5$ FLOPS

$= 45 \times 10^6$ FLOPS.

Now due to faster processor same nos of FLOPS done in new $T_{WC} = 2$ sec.

Therefore,

New $P = \frac{45 \times 10^6}{2}$ FLOP/sec

$\boxed{P = 22.5 \times 10^6 \text{ FLOP/sec}}$

(2) (a) Given, ~~s~~ $m'(t) = A \sin w_m(t)$.

Output of Square law device,

$$m(t) = [m'(t)]^2$$

$$m(t) = [A \sin w_m(t)]^2$$

$$m(t) = \frac{A^2}{2} - \frac{A^2}{2} \cos(2 w_m t) \quad \text{---------- (1)}$$

When signal m(t) ~~FM~~ frequency modulated a carrier with f_c. The FM signal.

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k \int m(t)\, dt\right) \quad \text{---------- (2)}$$

$$S_{FM}(t) = A_c \cos\left\{2\pi f_c t + k \int \left(\frac{A^2}{2} - \frac{A^2}{2} \cos 2 w_m t\right) dt\right\}$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k \frac{A^2}{2} t - \frac{2\pi k A^2}{2} \frac{\sin 2 W_m t}{2 W_m}\right]$$

$$S_{FM}(t) = A_c \operatorname{Re}\left[\exp\left\{j\left(2\pi f_c t + 2\pi k \frac{A^2 t}{2} - \frac{2\pi k A^2}{4 W_m} \sin 2 w_m t\right)\right\}\right]$$

$$S_{FM}(t) = A_c \operatorname{Re}\left[\exp\left\{j\left(2\pi f_c t + \beta t - \beta_1 \sin 2 w_m t\right)\right\}\right]$$

[Where,

$\beta = \pi A^2 k$

and $\beta_1 = \frac{\beta}{2 W_m}$]

$$S_{FM}(t) = A_c \operatorname{Re}[C(t)] \quad \text{---------- (1)}$$

$$C(t) = e^{j 2\pi f_c t} \cdot e^{j\beta t} \cdot e^{-j\beta_1 \sin 2 w_m t} \quad \text{---------- (2)}$$

$C'(t) = e^{-j\beta_1 \sin 2\omega_m t}$ is a periodic signal,

$$C'(t) = \sum_{n=-\infty}^{\infty} C_n' e^{j\frac{2\pi n t}{T_0}}$$

Where, $$C_n' = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{-j\beta_1 \sin 2\omega_m t} e^{-j\frac{2\pi n t}{T_0}}$$

$$\because \left[T_0 = \frac{2\pi}{2\omega_m} = \frac{\pi}{\omega_m}\right]$$

$$C_n' = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{-j(\beta_1 \sin 2\pi 2 f_m t - 2\pi n f_m t)} dt$$

$$= \frac{1}{2\pi}(-1)^n \int_{-\pi}^{\pi} e^{\{j(\beta_1 \sin\theta - n\theta)\}} d\theta.$$

$$C_n' = (-1)^n J_n(\beta_1)$$

Therefore,

$$C'(t) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta_1) e^{j\frac{2\pi n t}{T_0}}. \quad \text{——— (3)}$$

$$[T_0 = \pi/\omega_m]$$

By equation (1), (2) and (3) we get

$$C(t) = e^{j2\pi f_c t} \cdot e^{j\beta t} \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta_1) e^{jn2\omega_m t}.$$

$$C(t) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta_1) e^{j(2\pi f_c t + \beta t + 2n\omega_m t)} \quad \text{——— (4)}$$

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From, eqn (1) and (4) we get,

$$S_{FM}(t) = A_c \, Re[C(t)]$$

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta_1) \cos[2\pi f_c t + (2n\omega_m + \beta)t]$$

?

<proved>

(2)

(b) (i) Given,

$$H(z) = 3 + \frac{4z}{z-\frac{1}{2}} - \frac{2}{z-\frac{1}{4}}$$

$$H(z) = 3 + 4(1-\tfrac{1}{2}z^{-1})^{-1} - 2z^{-1}(1-\tfrac{1}{4}z^{-1})$$

$$\Rightarrow H(z) = 3 + 4\left[1+\tfrac{1}{2}z^{-1}+(\tfrac{1}{2}z^{-1})^2+(\tfrac{1}{2}z^{-1})^3+\dots\right] - 2z^{-1}\left[1+\tfrac{1}{4}z^{-1}+(\tfrac{1}{4}z^{-1})^2+\dots\right]$$

As the filter containing infinite number of impulses, it is a IIR filter.

(ii)

$$H(z) = 3 + \frac{4z}{(z-\frac{1}{2})} - \frac{2}{(z-\frac{1}{4})}.$$

$$\Rightarrow H(z) = \frac{3(z-\frac{1}{2})(z-\frac{1}{4}) + 4z(z-\frac{1}{4}) - 2(z-\frac{1}{2})}{(z-\frac{1}{2})(z-\frac{1}{4})}$$

$$\Rightarrow H(z) = \frac{3(z^2-\frac{3}{4}z+\frac{1}{8}) + (4z^2-z) - 2z+1}{z^2-\frac{3}{4}z+\frac{1}{8}}$$

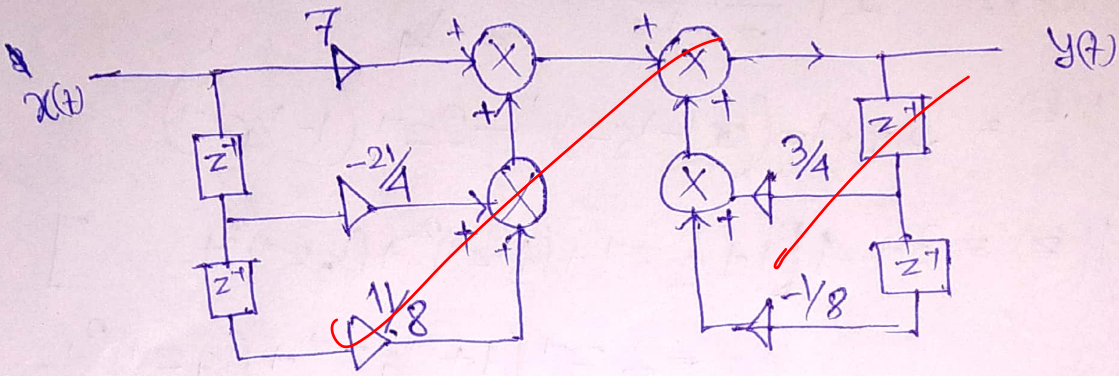
$$H(z) = \frac{7z^2-\frac{21}{4}z+\frac{11}{8}}{z^2-\frac{3}{4}z+\frac{1}{8}} = \frac{7-\frac{21}{4}z^{-1}+\frac{11}{8}z^{-2}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \quad \text{——(1)}$$

$$\cancel{H(z) = \frac{11z^{-2}-42z^{-1}+56}{z^{-2}-6z^{-1}+8}} \quad \text{——(1)}$$

$$\cancel{H(z) = (11z^{-2}-42z^{-1}+56) \times \frac{1}{z^{-2}-6z^{-1}+8}}$$

$$H(z) = \frac{\frac{11}{8}z^{-2}-\frac{21}{4}z^{-1}+7}{1-(\frac{3}{4}z^{-1}-\frac{1}{8}z^{-2})}$$

Filter $H(z)$ Can be realize in direct form I structure as

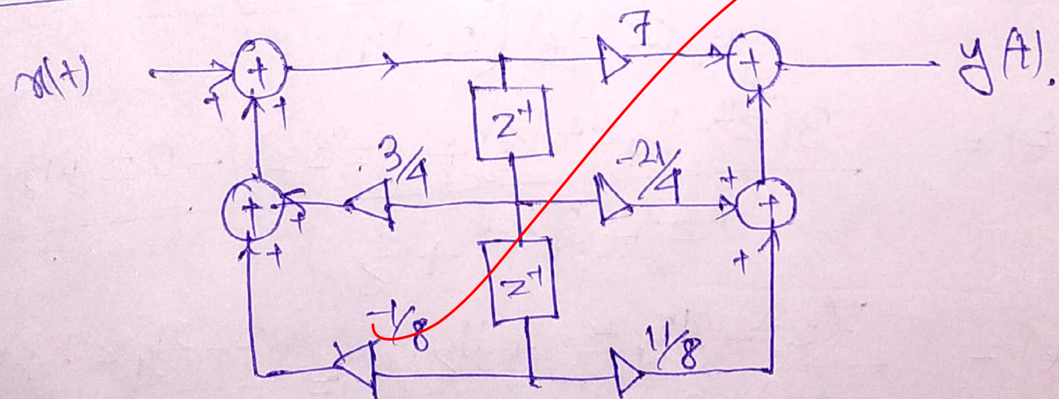


(iii)

From the eqn (1)

$$H(z) = \frac{7 - \frac{21}{4}z^{-1} + \frac{1}{8}z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Filter $H(z)$ Can be realize in direct form II Canonical form as



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(2)
(c) Given, $\dot{x}_1 = -\frac{3}{2}x_1 + \frac{1}{2}x_2 + U$

$\dot{x}_2 = -\frac{1}{2}x_1 - \frac{1}{2}x_2 + U.$

$y = x_1 + x_2$

Can be written into matrix form,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-3/2 & 1/2\\ -1/2 & -1/2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}U.$$

$$[y] = [1 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Hence,
$$A = \begin{bmatrix}-3/2 & 1/2\\ -1/2 & -1/2\end{bmatrix}, \quad B = \begin{bmatrix}1\\ 1\end{bmatrix}, \quad C = [1 \quad 1]$$

therefore,

$$[SI - A] = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}-3/2 & 1/2\\ -1/2 & -1/2\end{bmatrix}$$

$$\Rightarrow [SI-A]^{-1} = \begin{bmatrix}S+3/2 & S-1/2\\ S+1/2 & S+1/2\end{bmatrix}^{-1} = \frac{\begin{bmatrix}S+1/2 & -(S-1/2)\\ -(S+1/2) & S+3/2\end{bmatrix}}{(S+3/2)(S+1/2)-(S+1/2)(S-1/2)}$$

$$= \frac{1}{2S+1}\begin{bmatrix}S+1/2 & -(S-1/2)\\ -(S+1/2) & S+3/2\end{bmatrix}$$

$$y(s) = C[SI-A]B\ U(s)$$

$$\Rightarrow = [1 \quad 1]\frac{\begin{bmatrix}S+1/2 & -(S-1/2)\\ -(S+1/2) & S+3/2\end{bmatrix}}{(S^2+2S+3/4)-(S^2-1/4)}\begin{bmatrix}1\\ 1\end{bmatrix}U(s)$$

$$y(s) = \frac{[1 \quad 1]\begin{bmatrix}1\\ 1\end{bmatrix}}{(2S+1)}U(s) = \frac{2}{2S+1}U(s)$$

(i) Transfer function,

$$T(s) = \frac{Y(s)}{U(s)} = \frac{2}{(2s+1)} \quad \text{——— (1)}$$

(ii)

We know,

$$\dot{x} = Ax + B$$

$$\Rightarrow \quad sX(s) - X(0) - A\,X(s) = B$$

$$\Rightarrow \quad X(s) = [B + X(0)]\,[sI - A]^{-1}$$

Given $x_1(0) = 1$ and $x_2(0) = -1$

therefore, $[X(0)] = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

And,

$$Y(s) = C\,[sI - A^{-1}]\,B\,U(s) + C\,[sI - A^{-1}]\,[X(0)] \quad \text{——— (2)}$$

for step input $U(s) = 1/s$.

From eqn (1) and (2) we get,

$$Y(s) = \frac{2}{(2s+1)} \cdot \frac{1}{s} + [1 \quad 1] \frac{\begin{bmatrix} s+1/2 & -(s-1/2) \\ -(s+1/2) & (s+3/2) \end{bmatrix}}{2s+1} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{2}{s(2s+1)} + [1 \quad 1] \frac{\begin{bmatrix} 2s \\ -2s-2 \end{bmatrix}}{2s+1}$$

$-\frac{1}{2} - \frac{3}{2}$

$$= \frac{2}{s(2s+1)} + \frac{(-2)}{2s+1}$$

$$Y(s) = \frac{2}{s} - \frac{4}{(2s+1)} - \frac{2}{2s+1} = \frac{2}{s} - \frac{3}{s+1/2}$$

Taking inverse Laplace transform

$$\boxed{y(t) = (2 - 3e^{-1/2 t})\,u(t).}$$

(iii) $A = \begin{bmatrix} -3/2 & 1/2 \\ -1/2 & -1/2 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -3/2 & 1/2 \\ -1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\cancel{A} \cdot [B : AB] = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$|B : AB| = -1+1 = 0$$

As $|B : AB|$ is equal to zero, the given system is not controllable

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(3)
(a)

(i) Given System,

$$G(s) = \frac{K}{s^2(s+2)(s+5)}$$

Poles
$P = 0, 0, -2, -5.$

Number of asymptotes

$$P - Z = 4 - 0 = 4.$$

Centroid

$$\sigma_A = \frac{\sum \text{Real part of poles} - \sum \text{Real part of zero}}{P - Z}$$

$$= \frac{+0+0-2-5}{4-0} = -7/4 = -1.75.$$

Angle of asymptotes

$$= \frac{(2q+1)}{P-Z} 180^\circ \qquad q = 0, 1, 2, 3 \ldots$$

$$= \frac{(2q+1)\,180^\circ}{4}$$

$$= (2q+1)\,45^\circ$$

$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Break away point

$$1 + G(s) = 0.$$

$$\Rightarrow 1 + \frac{k}{s^2(s+2)(s+5)} = 0.$$

$$\Rightarrow K = -(s^4 + 7s^3 + 10s^2)$$

$$\Rightarrow \frac{dk}{ds} = -(4s^3 + 21s^2 + 20s) = 0$$

$$\Rightarrow s(4s^2 + 21s + 20) = 0.$$

$$s = 0, -1.25, -4$$

~~Here~~ $s = 0, -4$ are the valid point

Intersection on imaginary axis

$$1 + G(s) = 0,$$

$$\Rightarrow 1 + \frac{k}{s^2(s+2)(s+5)} = 0,$$

$$\Rightarrow s^4 + 7s^3 + 10s^2 + k = 0.$$

(ii) RH table

s^4	1	10	K
s^3	7	0	
s^2	10	K	
s^1	$-\frac{7K}{10}$		
s^0	K		

System is stable for, ~~$-7K$~~

(i) $-\frac{7k}{10} > 0$

$k < 0$

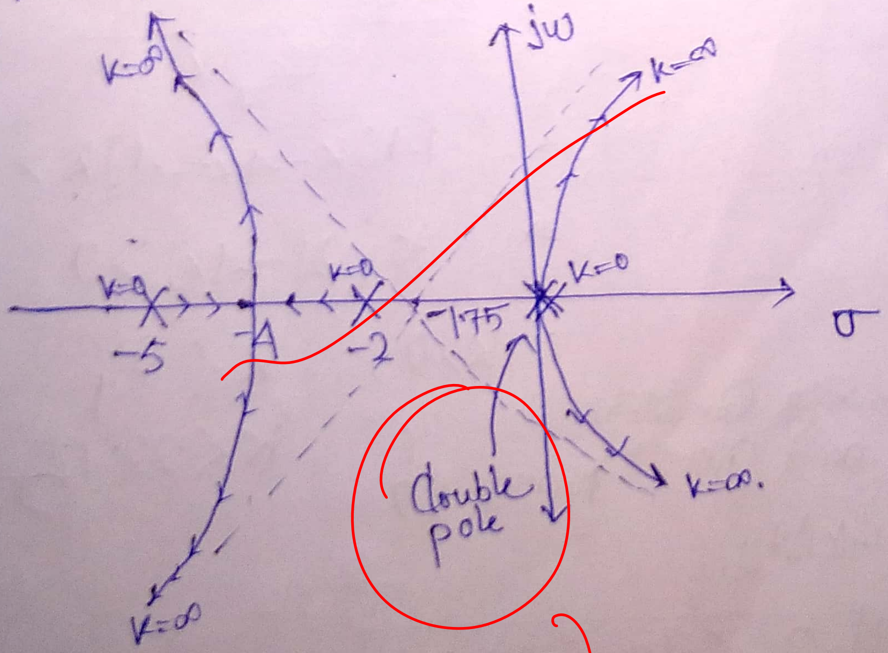
(ii) $k > 0$

Therefore the system is not stable for any value of k.

There is no intersection on imaginary axis.

(i) Root locus

~~RH~~ plot can be drawn as



(iii) When feedback changed to $H(s) = 1+2s$.

Characteristics equation,

$$1 + G(s)H(s) = 0.$$

$$\Rightarrow 1 + \frac{(1+2s)k}{s^2(s+2)(s+5)} = 0.$$

$$\Rightarrow s^4 + 7s^3 + 10s^2 + 2sk + k = 0.$$

Routh table

s^4	1	10	k
s^3	7	2k	
s^2	$\frac{70-2k}{7}$	K	
s^1	$\frac{\frac{70-2k}{7}\times 2k - 7k}{\frac{70-2k}{7}}$		
s^0	K		

(i) $\frac{70-2k}{7} > 0$

$k < 35$

(ii) $k > 0$

(iii) $\frac{\frac{70-2k}{7}2k - 7k}{\frac{70-2k}{7}} > 0$

$\Rightarrow 140k - 4k^2 - 49k > 0$

$\Rightarrow k(91-4k) > 0$

$\therefore k>0, \quad k < 91/4$

$k < 22.75$

By Combining Conditions (i), (ii) and (iii) range of k for stability.

$$\boxed{0 < k < 22.75}$$

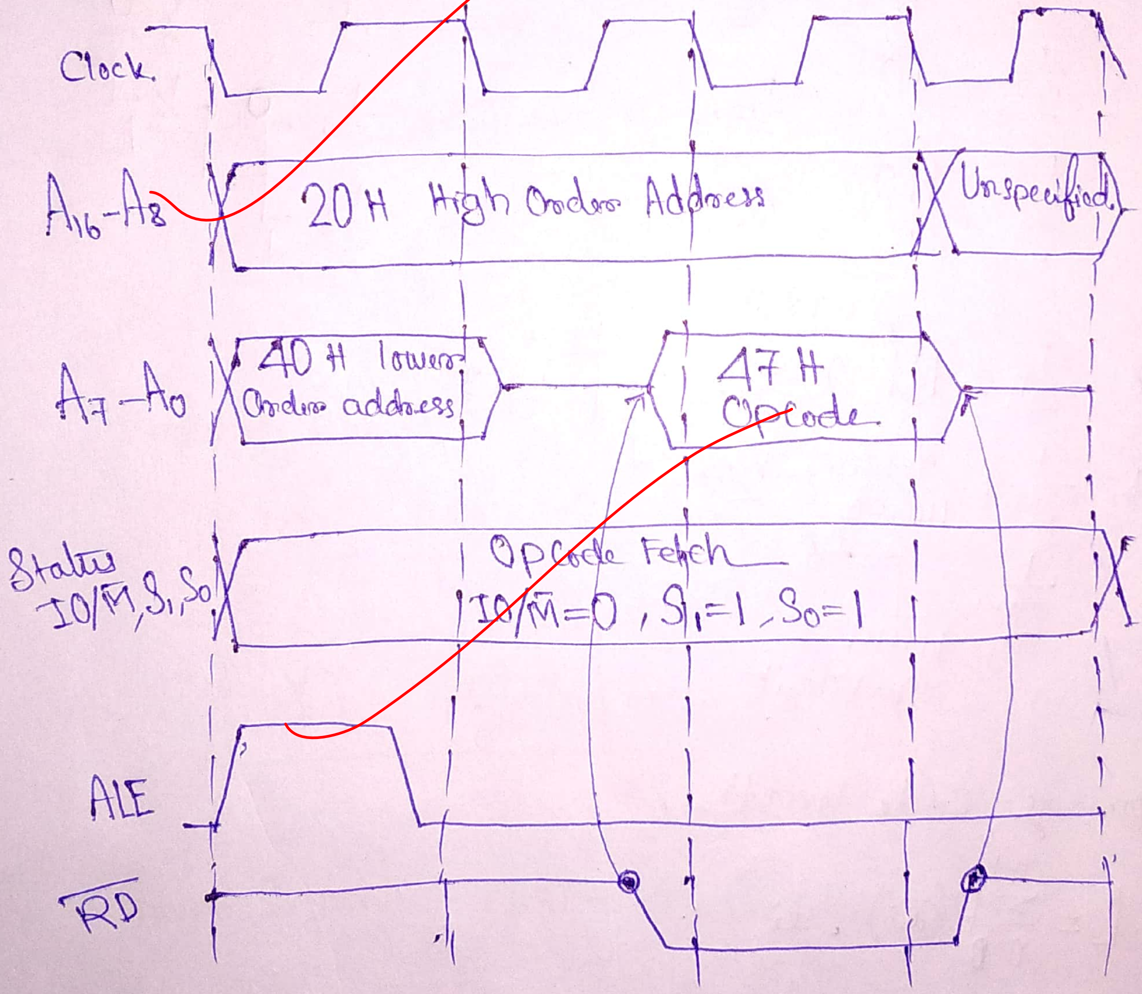
2

(3)
(b)

2040H $\boxed{47\ H}$ ← MOV B,A.

ACC $\boxed{D5\ H}$

Timing diagram of instruction MOV B,A.



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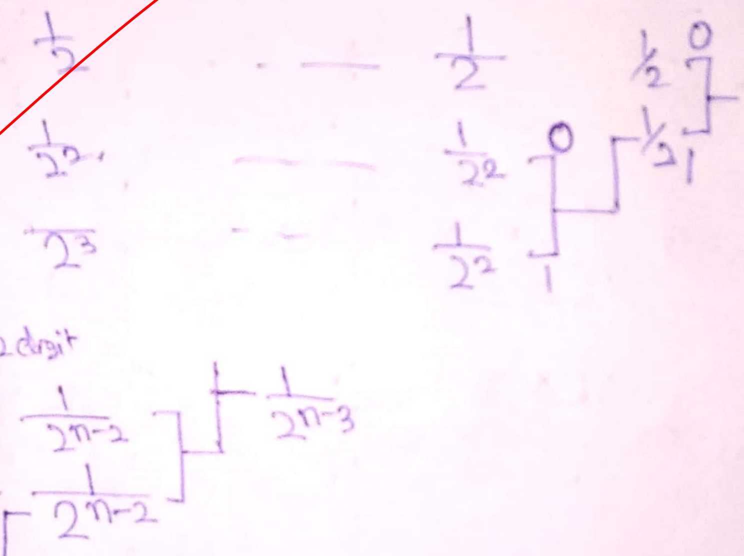
(3) Given that

(c) (i) Probabilities $= \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \cdots \frac{1}{2^{n-2}}, \frac{1}{2^{n-1}}, \frac{1}{2^{n-1}} \right\}$

$= \left\{ \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots, \frac{1}{2^{n-2}}, \frac{1}{2^{n-1}}, \frac{1}{2^{n-1}} \right\}$

Symbol	Probabilities	Code
S_0	$\frac{1}{2}$	0
S_1	$\frac{1}{2^2}$	10
S_2	$\frac{1}{2^3}$	110
⋮	⋮	⋮
S_{n-3}	$\frac{1}{2^{n-2}}$	111...0 (n-2 digit)
S_{n-2}	$\frac{1}{2^{n-1}}$	11111...0 (n-1 digit)
S_{n-1}	$\frac{1}{2^{n-1}}$	11111...1 (n-1 digit)



Average code length,

$$L = \sum_{n=0}^{n-1} P(x_i) \cdot l_i$$

$$= \frac{1}{2} \times 1 + \frac{1}{2^2} \times 2 + \frac{1}{2^3} \times 3 + \cdots + \frac{n-2}{2^{n-2}} + \frac{(n-1)}{2^{n-1}} + \frac{(n-1)}{2^{n-1}} \quad \text{(1)}$$

$$L = \frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2} \quad \text{[When } n \to \infty \text{]}$$

$\left[\because a + 2ax + 3ax^2 + 4ax^3 + \cdots = \frac{a}{(1-x)^2}\right]$ for $x<1$

$L = 2$ bits/symbol.

Entropy

$$H(x) = -\sum_{i=0}^{n-1} P_i \log_2 P_i$$

$$= -\left[\frac{1}{2}\log_2 \frac{1}{2} + \frac{1}{2^2}\log_2 \frac{1}{2^2} + \frac{1}{2^3}\log_2 \frac{1}{2^3} + \cdots + \frac{1}{2^{n-1}}\log_2 \frac{1}{2^{n-1}}\right]$$

$$H(x) = +\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \cdots + \frac{n-2}{2^{n-2}} + \frac{n-1}{2^{n-1}} + \frac{n-1}{2^{n-1}} \qquad \text{——— (2)}$$

From eqn(1) and (2) we get,

$$\frac{H(x)}{L} = 1$$

⇒ Coding efficiency,

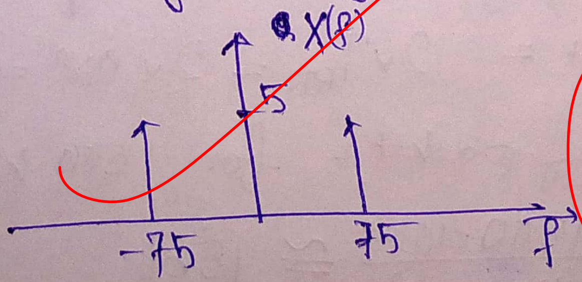
$$\eta = \frac{H(x)}{L} = 1 = 100\%.$$

$$\boxed{\eta = 100\%}$$

(ii) Given Signal $x(t) = 10 \cos 150\pi t$.

$$f = \frac{150\pi}{2\pi} = 75 \text{ Hz}.$$

Sketch of the given Signal



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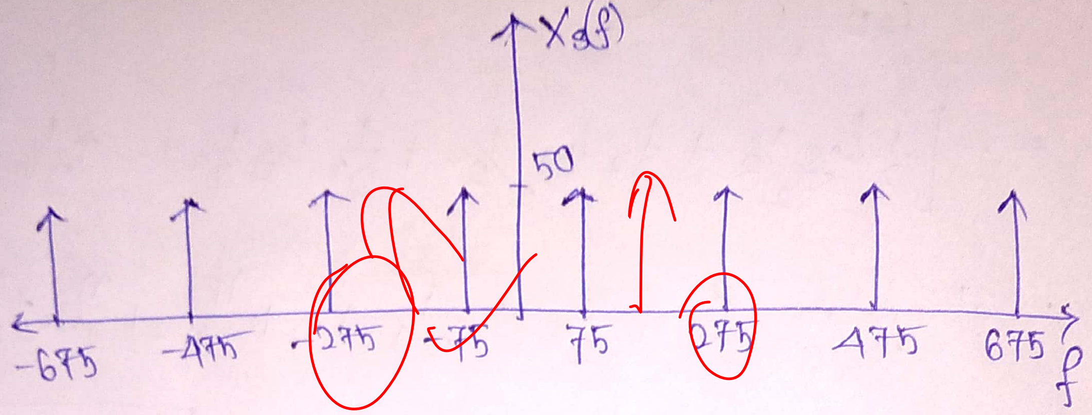
~~When the~~

Sketch of the sampled signal when it is sampled at 200 ~~Hz~~ sample /sec



Section - B

(5) (a)

One way delay $T_i = T_d = 0.4$ ms.

Time required to RTT

$$T_{RTT} = 2 \times T_d = 0.8 \text{ ms}.$$

As BW is 1Mbps, time required to send 1kB of packets data with header

$$T_t = \frac{1024 \times 8}{10^6} = 8.192 \text{ ms}$$

Wait time between two transmission of packet

$$T_{wait} = 2 \times T_{RTT} = 2 \times 0.8 = 1.6 \text{ msec}$$

Total number of packets for 1500 kB file

$$N_p = \frac{1500 \times 1024}{(1024-40)} \cong 1561$$

2

Hence total time required to transmit 1500 kB file

$$T_{total} = T_i + N_P T_t + (N_P - 1) T_W.$$

$$= 0.4 + 1561 \times 8.192 + (1561-1)\, 0.8 \quad mS.$$

$$\boxed{T_{total} = 14.036 \text{ Sec}}$$

(5) (b)

~~Page~~ Given that,

Mode TE_{10}

$a = 6$ cm.

$b = 4$ cm.

We know

~~$l_{max} = 8$~~

$l_{max} - l_{min} = \frac{\lambda}{4} = 8$ cm,

$\lambda = 32$ cm.

~~Cutoff wavelength of for~~

Cutoff frequency for TE_{10} mode,

$$f_c = \frac{u}{2a}$$

assume the wave guide is air filled, therefore,

$U = 3 \times 10^{10}$ cm/sec.

$$f_c = \frac{3 \times 10^{10}}{2 \times 6}$$

$$\Rightarrow f_c = 2.5 \text{ GHz}.$$

Cutoff wavelength is $\lambda_c = 2a = 2 \times 6$

$\lambda_c = 12$ cm

If the wave frequency is f

then,

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} = \frac{\lambda_c}{\sqrt{1-(f_c/f)^2}}$$

$$\Rightarrow \quad 32 = \frac{12}{\sqrt{1-\left(\frac{2.5}{f}\right)^2}}$$

$$\Rightarrow \quad \boxed{f = 2.697 \text{ GHz}}$$

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(5)(c) Given characteristics equation,

$$q(s) = s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0.$$

Routh-Hurwitz table

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	1	3		
s^2	6	16		
s^1	$\frac{1}{3}$			
s^0	16			

All the element of s^3 row becomes zero,

Aux eqn

$$A(s) = 2s^4 + 12s^2 + 16$$

$$A'(s) = 8s^3 + 24s + 0$$

$$= 8(s^3 + 3s + 0)$$

As the all terms of first colum is positive i.e. no sign change, ~~hence the system is stable~~ and s^3 row becomes zero (all terms), Hence the

System is marginally stable.

Given, characteristics equation

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0 \quad \text{--- (1)}$$

As the the s^3 row in the R-H becomes zero, therefore Auxilairy equation will be a factor of characteristics equation,. From eqn (1) we get,

$$s^2(s^4 + 6s^2 + 8) + 2s(s^4 + 6s^2 + 8) + 2(s^4 + 6s^2 + 8) = 0$$

$$\Rightarrow (s^4 + 6s^2 + 8)(s^2 + 2s + 2) = 0.$$

$$s = \pm j2, \pm j\sqrt{2}, -1+j, -1-j,$$

Therefore the roots are,

$$s = +j2, -j2, +j\sqrt{2}, -j\sqrt{2}, -1+j, -1, -j$$

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5 (d)

Given $d = 1000$ km.

$f = 2.65$ GHz

$R = 6378$ km,

$\mu = 3.986 \times 10^5 \ km^3/s^2$

Distance of the Satellite from the earth ~~orbit~~ center

$$r = d + R = (1000 + 6378) = 7378 \text{ km}.$$

Therefore velocity of the Satellite

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{7378}}$$

$$v = 7.35 \times 10^3 \ m/s.$$

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Therefore

Doppler shift in received signal

$$\Delta f' = \frac{f \times V}{c} = \frac{2.65\times10^{9} \times 7.35\times10^{3}}{3\times10^{8}}$$

$$\boxed{\Delta f = 64.927 \text{ kHz}}$$

5 (e)

Given,

$$F(z) = \frac{(z^3 - 3)}{z(z-0.25)(z-0.5)}$$

$$F(z) = \frac{A}{z} + \frac{B}{z-0.25} + \frac{C}{z-0.5}$$

By doing partial fractions,

$$A = z \cdot F(z)\Big|_{z=0} = \frac{(-3)}{(-0.25)(-0.5)} = -24$$

$$B = (z-0.25)\, F(z)\Big|_{z=0.25} = \frac{(0.25^3 - 3)}{0.25 \times (0.25-0.5)}$$

$$= 47.75$$

$$C = (z-0.5)\, F(z)\Big|_{z=0.5} = \frac{(0.5^3 - 3)}{(0.5)\,(0.5-0.25)}$$

$$= -23$$

Therefore,

$$F(z) = -\frac{24}{z} + \frac{47.75}{z-0.25} - \frac{23}{z-0.5}$$

$$F(z) = -24z^{-1} + \frac{47.75z^{-1}}{1-0.25z^{-1}} - \frac{23z^{-1}}{1-0.5z^{-1}} \qquad \text{——(1)}$$

ROC: $|z| \neq 0$ ROC: $|z| > |\frac{1}{4}|$ ROC: $|z| > \frac{1}{2}$

By taking inverse Z- ~~tem~~ transform of eqn

$$f(n) = -24\,\delta(n-1) + 47.75\left(\tfrac{1}{4}\right)^{n-1}u(n-1) - 23\left(\tfrac{1}{2}\right)^{n-1}u(n-1)$$

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$\left[\because \dfrac{1}{1-az^{-1}} \overset{IZT}{\rightleftharpoons} a^n u(n)\right.$

And, $\left. z^{-1}X(z) \overset{IZT}{\rightleftharpoons} x(n-1)\right]$

Therefore,

$$\boxed{f(n) = -24\,\delta(n-1) + 47.75\left(\tfrac{1}{4}\right)^{n-1}u(n-1) - 23\left(\tfrac{1}{2}\right)^{n-1}u(n-1)}$$

ROC: $|z| > \frac{1}{2}$

(5) (f) Characteristics of light detector

→ High sensitivity at the operating wavelength

→ High fidelity

→ Large electrical response to the received optical signal

→ Short response time to obtain a suitable bandwidth

→ A minimum or zero noise introduced by the detector

→ Stability of the performance characteristics

→ Small size

→ Low bias voltage.

→ High reliability

→ Low cost.

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(8) (a)

(i) Given $V = 4y(x^3 + 6x + 2)$

Electric field

$$\vec{E} = -\nabla . V$$

$$= -\left[\frac{\partial}{\partial x} V \hat{a_x} + \frac{\partial}{\partial y} V \hat{a_y} + \frac{\partial}{\partial z} V \hat{a_z}\right]$$

$$= -\left[\frac{\partial}{\partial x}\{4yx^3 + 24xy + 8y\}\hat{a_x} + \frac{\partial}{\partial y}\{4x^3y + 24xy + 8y\}\hat{a_y} + \frac{\partial}{\partial z}\{4x^3y + 24xy + 8y\}\hat{a_z}\right]$$

$$\boxed{\vec{E} = -(12x^2y + 24y)\hat{a_x} - (4x^3 + 24x + 8)\hat{a_y}}$$

(ii)

Flux density,

$$\vec{D} = \epsilon_r \epsilon_0 \vec{E}$$

[let assume the medium is vacuum i.e. $\epsilon_r = 1$]

$$\vec{D} = -\epsilon_0(12x^2y + 24y)\hat{a_x} - \epsilon_0(4x^3 + 24x + 8)\hat{a_y}$$

at surface $y=0$, is a conductor,

By using Gauss law.

$$\vec{D}|_{y=0} = -\epsilon_0(4x^3 + 24x + 8)\hat{a_y}$$

Total outwards flux through = Enclosed charge

$$\oint\!\!\oint \vec{D}\cdot\vec{ds} = Q_{enc}$$

$$\Rightarrow Q_{enc} = \iint \vec{D}\cdot\hat{a}_y \,.\, dx\,dz$$

$$= -\varepsilon_0 \int_{x=0}^{3} (4x^3+24x+8)\,dx \int_{z=0}^{2} dz$$

$$= -\varepsilon_0 \left[4\frac{x^4}{4} + \frac{24x^2}{2} + 8x\right]_0^3 * \left[z\right]_0^2$$

$$= -\varepsilon_0 \left[3^4 + 12\times3^2 + 8\times3\right]\times[2]$$

$$= -8.854\times10^{-12}\times 213\times 2$$

$$\boxed{Q = -3.771\ nC}$$

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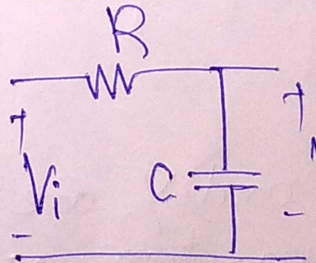
(8) (b)

Transfer function of the filter



$R = 1k\Omega$

$C = 1\mu F$

$RC = 10^3\times10^{-6} = 10^{-3}$ Sec

$f_c = \frac{1}{2\pi RC} = \frac{10^3}{2\pi}$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}}$$

$$H(s) = \frac{1}{RCs+1}$$

$$\Rightarrow H(\omega) = \frac{1}{j\omega RC+1}$$

$$|H(\omega)| = \frac{1}{\sqrt{\omega^2R^2C^2+1}}$$

$$|H(\omega)| = \frac{1}{\sqrt{\omega^2R^2C^2+1}}$$

$$|H(f)| = \frac{1}{\sqrt{(f/f_c)^2+1}}$$

$$\left[f_c = \frac{10^3}{2\pi}\ Hz\right]$$

(1)

Output PSD of noise,

$$(PSD)_o = |H(f)|^2 \cdot (PSD)_i$$

$$S_o(f) = \frac{1}{(f/f_c)^2+1} \cdot \frac{N_o}{2}$$

Given $\left[\frac{N_o}{2} = 10^{-6} \right]$

$$S_o(f) = \frac{10^{-6}}{(f/f_c)^2+1} = \frac{10^{-6} f_c^2}{f^2+f_c^2} = \frac{10^{-6} 4\pi^2 f_c}{(2\pi f)^2+(2\pi f_c)^2}$$

Auto correlation function of output

$$R_y(\tau) = F^{-1}[S_o(f)]$$

$$= F^{-1}\left[\frac{2(2\pi f_c)}{(2\pi f)^2+(2\pi f_c)^2} \cdot 10^{-6}\pi f_c\right]$$

$$\boxed{R_y(\tau) = 5\times10^{-4} e^{-10^3|\tau|}}$$

$\because f_c = \frac{10^3}{2\pi}$

$2\pi f_c = 10^3$

$\left\{ \because F^{-1}\left[\frac{2a}{(2\pi f)^2+a^2}\right] = e^{-a|\tau|} \right\}$ and $10^{-6}\pi f_c = 5\times10^{-4}$

X and Y are two random variable with separation 0.1

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8)
(c)

Criteria: Based on arrival time

Mode :- Pre emption.

Time Quantum :- 2 unit.

Ready Queue

P1	P2	P1	P3	P4	P2	P5	P1	P5

0 1 2 3 3 4 6 12

Process chart

P1	P2	P1	P3	P4	P2	P5	P1	P5

0 2 4 6 7 9 10 12 13 14.

Process	Completion Time (CT)	Turn around Time (TAT) (CT-AT)	Waiting Time (TAT-BT)
P_1	13	13-0=13	13-5=8
P_2	10	10-1=9	9-3=6.
P_3	7	7-2=5	5-1=4
P_4	9	9-3=6	6-2=4
P_5	14	14-4=10	10-3=7.

Average waiting time = $\frac{8+6+4+4+7}{5}$ = 5.8 unit

Average turn around time = $\frac{13+9+5+6+10}{5}$ = 8.6 unit

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8) (d)

Given that, $NA = 0.3$

$n_1 = 1.45$

$J_{mat} = 250 \text{ ps nm}^{-1}\text{ km}^{-1}$

$\Delta = \frac{(NA)^2}{2n_1^2} = \frac{0.3^2}{2\times1.45^2} = 0.0214$

2

(i) RMS Pulse broadening due to intramodal dispersion

$$\sigma_s = \frac{n_1 \Delta^2}{2\sqrt{3}\, C} = \frac{1.45\times0.0214^2}{2\sqrt{3}\times3\times10^8} \text{ S/m}$$

$$\sigma_s = 926.5 \text{ ps km}^{-1}$$

Total pulse broadening

$$\Delta T_{total} = \sqrt{\sigma_s^2 + (J_{mat}\times D)^2}$$

$$= \sqrt{(926.5)^2 + (250\times50)^2}$$

$$\boxed{\Delta T_{total} = 12.534 \text{ ns km}^{-1}}$$

[Given D = 50 nm]

(ii)

Bandwidth length product

$$BW\times L = \frac{1}{2\Delta t_{total}} = \frac{1}{2\times12.534\times10^{-9}}$$

$$\boxed{BW\times L = 39.89 \text{ MHz-Km}}$$