1) (a) Given: Input Voltage, $V_s = 20V$

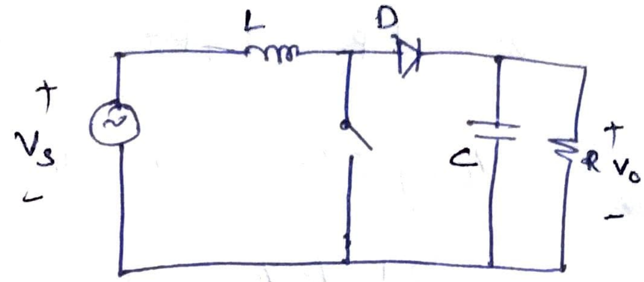
Duty ratio, $D = 0.6$

Inductance, $L = 100\mu H$

Resistance, $R = 50\Omega$

Capacitance, $C = 100\mu F$

Switching frequency, $f = 15KHz$



Boost converter

When SW is ON

$$V_s = L\frac{\Delta I}{\Delta T} = \frac{L\Delta I f}{D} \Rightarrow 20 = \frac{10^{-4} \times \Delta I \times 15\times10^3}{0.6}$$

$\Delta I = 8A.$

So, $\boxed{\frac{\Delta I}{2} = 4A.}$

Average inductor current, $i_{Lavg} = i_{in}$ (input current)

Considering continuous conduction –

Output Voltage, $\boxed{V_o = \frac{V_s}{1-D}} \Rightarrow \boxed{V_o = 50\ V}$

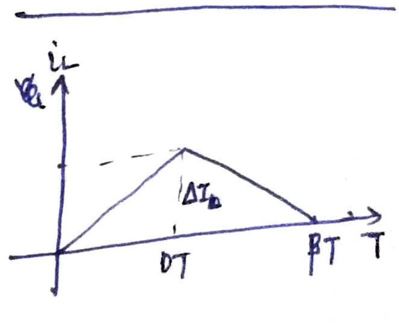
So, Output current, $\boxed{i_o = \frac{V_o}{R}} \Rightarrow \boxed{i_o = 1A.}$

So, $V_o i_o = V_s i_{in} \Rightarrow 50 = 20 \times i_{in}$

$\boxed{i_{in} = 2.5A}$ & $\boxed{i_{Lavg} = i_{in} = 2.5A}$

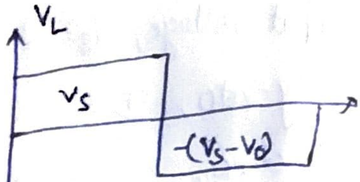
So, $i_{Lavg} < \frac{\Delta I}{2}$ Hence discontinuous conduction mode.

$$i_{Lavg} = \frac{1}{T} \times \left[\frac{1}{2} \times \Delta I_L \times \beta T\right]$$



$\boxed{i_{Lavg} = \frac{\Delta I_L \beta}{2}} \Rightarrow \boxed{i_{Lavg} = 4\beta}$

$2.5 = 4\beta \Rightarrow \boxed{\beta = 0.625}$



So, As, $V_{L\,avg} = 0$ ($V_{L_{avg}}$ is Inductor average Voltage)

$$\Rightarrow [V_s D + (V_s - V_0)(\beta - D)]\, T \times \frac{1}{T} = 0$$

$$\Rightarrow \cancel{V_s D} + V_s \beta - \cancel{V_s D} - V_0 \beta + V_0 D = 0$$

$$\Rightarrow \boxed{V_0 = V_s\left[\frac{\beta}{\beta - D}\right]}$$

$$\Rightarrow \boxed{V_0 = 500V}$$

Maximum inductor current, $\boxed{i_{L\max} = i_{L avg} + \frac{\Delta I_L}{2}}$

$i_{L max} = 2.5 + 4$

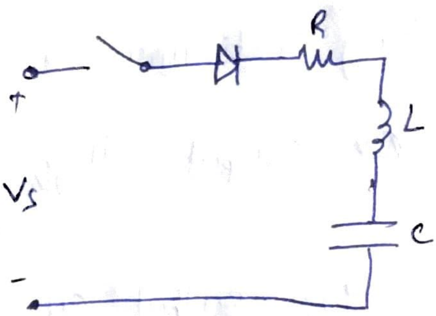
$\boxed{i_{L max} = 6.5 A}$

(b) Given: Resistance, $R = 10\,\Omega$

Inductance, $L = 1mH$

Capacitance, $C = 5\mu F$

Supply Voltage, $V_s = 230V$.



$\Rightarrow$ ~~$\frac{V_s}{S}$~~ By applying KVL & laplace transform in equation 1 we get -

$$\frac{V_s}{S} = LSI(S) + \frac{I(S)}{CS} + RI(S).$$

$$\Rightarrow \frac{V_s}{S} = I(S)\left[\frac{LCS^2 + RCS + 1}{CS}\right]$$

$$\Rightarrow I(s) = \frac{V_s \cancel{s}}{L\cancel{s}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} = \frac{V_s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$\Rightarrow I(s) = \frac{230}{10^{-3}\left[s^2 + 10^4 s + 2\times10^8\right]}$$

$$\Rightarrow I(s) = \frac{230\times10^3}{(s+5\times10^3)^2 + ((1.75)^{1/2}\times10^4)^2}$$

$$\Rightarrow i(t) = \frac{23\times10^4}{1.75^{1/2}\times10^4} e^{-5\times10^3 t} \cancel{\sin(5\times10^3 t)} \sin(1.323\times10^4 t)$$

$$\Rightarrow \boxed{i(t) = 17.386366\, e^{-5\times10^3 t} \sin(1.323\times10^4 t).}$$

Conduction of time of diode be t_c, then,

$$1.323\times10^4 \times t_c = \pi \Rightarrow \boxed{t_c = 0.2375 \text{ ms}}$$

$$\frac{di(t)}{dt} = 17.386366\times(-5\times10^3)\, e^{-5\times10^3 t} \sin(1.323\times10^4 t) + 17.386366\, e^{-5\times10^3 t} \cos(1.323\times10^4 t) \times 1.323\times10^4$$

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$$\left.\frac{di(t)}{dt}\right|_{t=0} = 17.386316 \times 1.323\times10^4.$$

$$\boxed{\left.\frac{di(t)}{dt}\right|_{t=0} = 230021.6222 \text{ A/s}}$$

1 (c) Given: $5x+3y+7z=4$

$3x+26y+2z=9$

$7x+2y+10z=5$

So,

$$[A]=\begin{bmatrix}5 & 3 & 7\\ 3 & 26 & 2\\ 7 & 2 & 10\end{bmatrix} \quad \& \quad B=\begin{bmatrix}4\\ 9\\ 5\end{bmatrix}$$

$$[A:B]=\left[\begin{array}{ccc|c}5 & 3 & 7 & 4\\ 3 & 26 & 2 & 9\\ 7 & 2 & 10 & 5\end{array}\right]$$

$$=\left[\begin{array}{ccc|c}5 & 3 & 7 & 4\\ 0 & 24.2 & -2.2 & 6.6\\ 0 & -2.2 & 0.2 & -0.6\end{array}\right]$$

$$=\left[\begin{array}{ccc|c}5 & 3 & 7 & 4\\ 0 & 24.2 & -2.2 & 6.6\\ 0 & 0 & 0 & 0\end{array}\right]$$

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Hence, Rank $[A]$ = Rank of $[A:B] \neq$ Number of Variables

~~But~~ Hence, infinite solution.

1 (d) Given: $f(z)=\dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ C: $|z|=2.5$

As, $z=3$ lies outside the contour C, so,

$$\oint f(z)\,dz = 2\pi i\left[\operatorname{Res}(f(z))\Big|_{z=1}+\operatorname{Res}(f(z))\Big|_{z=2}\right]$$

$$\operatorname{Res}(f(z))\Big|_{z=1} = \frac{1}{3!} \lim_{z\to 1} \frac{d^3}{dz^3} (z-1)^4 \frac{z^3}{(z-2)(z-3)(z-1)^4}$$

$$\operatorname{Res}(f(z))\Big|_{z=1} = \frac{1}{6} \lim_{z\to 1} \frac{d^3}{dz^3} \left[\frac{z^3}{z^2-5z+6}\right]$$

$$= \frac{1}{6} \lim_{z\to 1} \frac{d^3}{dz^3} \left[\frac{z^3}{z^2-5z+6}\right]$$

$$\frac{d}{dz}\left[\frac{z^3}{z^2-5z+6}\right] = \frac{3z^2(z^2-5z+6) - (2z-5)\times z^3}{(z^2-5z+6)^2}$$

$$= \frac{3z^4 - 15z^3 + 18z^2 - 2z^4 + 5z^3}{(z^2-5z+6)^2}$$

$$= \frac{z^4 - 10z^3 + 18z^2}{(z^2-5z+6)^2}$$

$$\frac{d^2}{dz^2}\left[\frac{z^3}{z^2-5z+6}\right] = \frac{(4z^3-30z^2+36z)(z^2-5z+6)^2 - 2(z^2-5z+6)(2z-5)(z^4-10z^3+18z^2)}{(z^2-5z+6)^4}$$

$$= X$$

$$= \frac{(4z^3-30z^2+36z)(z^2-5z+6) - 2(2z-5)(z^4-10z^3+18z^2)}{(z^2-5z+6)^3}$$

$$= \frac{4z^5 - 50z^4 + 24z^3 + 150z^3 - 180z^2 + 36z^3 - 180z^2 + 216z - (4z-10)(z^4-10z^3+18z^2)}{z^2-5z+6}$$

$$= 4z^5 - 50z^4 + 210z^3 - 360z^2 + 216z - 4z^5 + 40z^4 - 72z^3 - 10z^4$$

$$\frac{d^3}{dz^3}[X] = 37.875$$

$$\text{Res } f(z)\Big|_{z=2} = \lim_{z\to 2}(z-2)\frac{z^3}{(z-1)^4(z-2)(z-3)}$$

$$= \frac{8}{-1} = -8$$

5

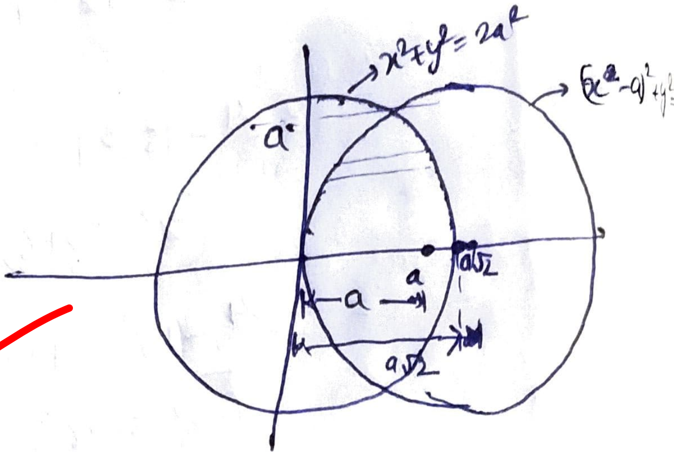
$$\boxed{\text{Res } f(z)\big|_{z=2} = -8}$$

So, $\oint f(z)\,dz = 2\pi j\,[37.8.75\ \phi -8]$

$$\boxed{\oint f(z)\,dz = 187.71 j}$$

① ⓔ $r = a\sqrt{2}$

$r = 2a\cos\theta$



$x^2+y^2 = 2a^2$

$(x-a)^2+y^2 = a^2.$

$x^2+y^2 = 2a^2$

$x^2+a^2-2ax+y^2 = a^2$

$\Rightarrow 2a^2 = 2ax$

$\Rightarrow \boxed{x=a}$

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Area, $A = 2\times\int_0^a \int_{\sqrt{2a^2-y^2}}^{\sqrt{a^2-y^2}+a} dx\,dy.$

$A = 2\times\int_0^a \left(\sqrt{a^2-y^2} + a - \sqrt{2a^2-y^2}\right) dy.$

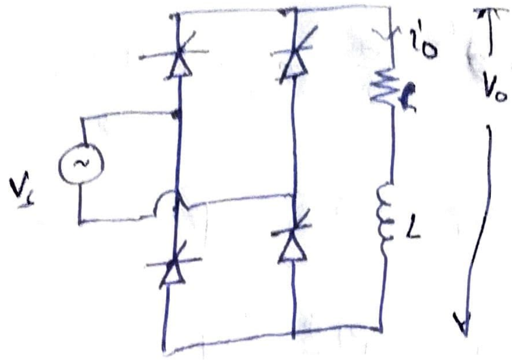
$A =$

① Given: $V_s = 120\sqrt{2}\ \sin(120\pi t)$

Resistance, $R = 10\,\Omega$

Inductance, $L = 20\,mH$

firing angle, $\alpha = 60°$



$$i_o = \frac{V_s}{|Z|}\sin(120\pi t - \phi) + A e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{20\times10^{-3}}{10} = 2\times10^{-3} = \frac{1}{500}$$

$$|Z| = \left[R^2 + (120\pi \times L)^2\right]^{1/2} \Rightarrow \boxed{|Z| = 12.524\,\Omega}$$

$$\phi = \tan^{-1}\left(\frac{120\pi \times L}{R}\right) = 37.0156°.$$

$$\Rightarrow i_o = 13.3504 \sin(120\pi t - 37.0156) + A e^{-t/\frac{1}{500}}$$

$$\Rightarrow \boxed{i_o = 13.5504 \sin(120\pi t - 37.0156) + A e^{-500t}\ A}$$

at, $120\pi t = \pi/3$, $i_o = 0\,A.$

$$t = \frac{1}{120\times3} = \frac{1}{360}$$

$$-5.291167 = A e^{-500\times\frac{1}{360}} \Rightarrow \boxed{A = 21.21965}$$

$$\Rightarrow \boxed{i_o = 13.5504 \sin(120\pi t - 37.0156) + 21.21965\, e^{-500t}\ A.}$$

Considering, i_o at $200°$ as initial condition, $120\pi t = 200 \times \frac{\pi}{180}$

$t = \frac{200}{120\pi} \times \frac{\pi}{180}$

$\boxed{i_o|_{200°} = 4.1723\,A}$ $\Rightarrow i_o = 13.5504 \sin(\beta - 37.0156) + 21.21965\, e^{-\frac{500}{120\pi}\beta}$

$\beta' = \frac{1}{9}\beta_0 + \frac{4.1723}{29.20841}$

$\beta_1 = 3.6335 \text{ rad.}$

$\beta_2 = \beta_1 \pm \frac{f(\beta_1)}{f'(\beta_1)} \Rightarrow \beta_2 = 3.6335 + \frac{(-7.284367)}{11.2173}$

$\beta_2 = 2.984113 \text{ rad}$

$\beta_3 = 2.984113 + \frac{(-7.17806)}{\quad}$

$$i_0 = 13.5504 \sin(\beta - 0.646) + 21.21965 \times e^{-\frac{\beta \times 500}{120\pi}}$$

$$\boxed{i_0|_\pi = 8.486767 \text{ A.}}$$

$$i_0' = 13.5504 \cos(\beta - 0.646) - \frac{500 \times 21.21965}{120\pi} e^{-\frac{\beta \times 500}{120\pi}}$$

$$i_0'|_\pi = 11.255946 \text{ A.}$$

$$\beta_1 = \beta_0 - \frac{i_0|_\pi}{i_0'|_\pi} \Rightarrow \beta_1 = \pi + 0.753981$$

$$\Rightarrow \boxed{\beta_1 = 3.895574 \text{ rad}}$$

$$\beta_2 = \beta_1 - \frac{i_0|_{\beta_1}}{i_0'|_{\beta_1}} = 3.895574 + \frac{-1.33872}{13.63206}$$

$$\boxed{\beta_2 = 3.79787 \text{ rad}}$$

$$\beta_3 = 3.79737 + \frac{5.98 \times 10^{-3}}{13.4326657} \Rightarrow \boxed{\beta_3 = 3.7978 \text{ rad}}$$

$$\boxed{\beta_3 = 214.5982^\circ}$$

where β_3 is extinction angle

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Average load current, $i_o = \frac{1}{\pi}\int_{\alpha} i_o \, d(\omega t)$

$$\boxed{i_o = 9.52054 \text{ A}}$$

(b) Single phase full bridge inverter.

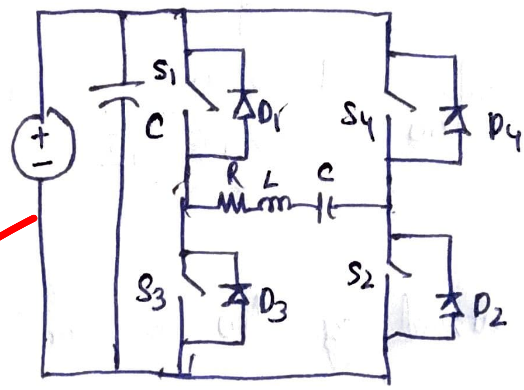


fig 1-Φ full bridge inverter

Given: Supply voltage, $V_s = 230$ V

Time, $T = 1$ ms

$\Rightarrow$ frequency, $f = 1$ KHz.

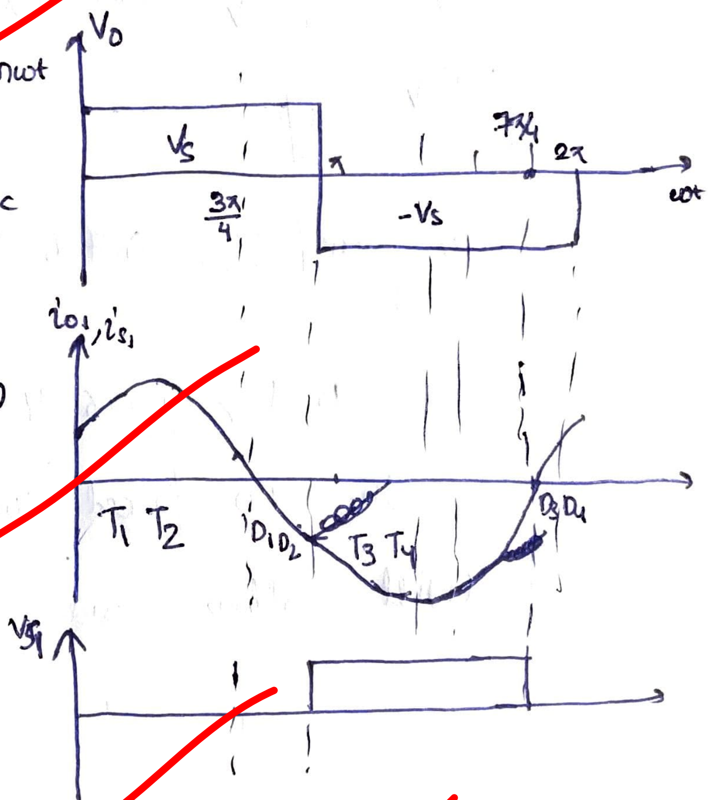
Resistance, $R = 1$

Inductive reactance, $X_L = \omega L = 6\,\Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = 7\,\Omega$.

$$V_o = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$a_0 = a_n = 0$ (as odd symmetric signal)



$$b_n = \frac{2}{\pi}\int_0^{\pi} V_s \sin n\omega t \, d(\omega t)$$

$$b_n = \frac{2V_s}{n\pi}\left[-\cos n\omega t\right]_0^{\pi}$$

$$\boxed{b_n = \frac{4V_s}{n\pi}}$$

$$\Rightarrow \boxed{V_o = \sum_{n=1}^{\infty} \frac{4V_s}{n\pi} \sin(n\omega t)}$$

Impedance (fundamental), $|Z_{01}| = \sqrt{1^2 + (6-7)^2} = \sqrt{2}$

$$\Rightarrow \boxed{|Z_{01}| = \sqrt{2}\,\Omega}$$

$\phi = \tan^{-1}\left(\frac{7-6}{1}\right) = 45^\circ$

$$\boxed{i_{o_1} = \frac{4V_s}{\pi|Z_{o,1}|}\sin(\omega t + 45^\circ)}$$

$$\Rightarrow \boxed{i_{o_1}\big|_{rms} = \frac{4\times 230}{\pi\times\sqrt{2}\times\sqrt{2}} = 146.4225\,A.}$$

(ii) Power delivered ~~by~~ due to fundamental component,

$$P_q = i_{o_1|rms}^2 \times R.$$

$$\boxed{P_{o_1} = 21439.57246\,W}$$

(iii) $\omega t_q = 2\pi f t_q = 2\times\pi\times 10^3\times 10^{-4}$

$$\boxed{\omega t_q = \frac{\pi}{5}}$$

$$\Rightarrow \boxed{\tan(\pi/5) = 0.72654}$$

as $1 > 0.72654$

So, no forced commutation, the circuit will follow load commutation.

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(2)(c) Given: $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$\cancel{\Rightarrow [\lambda I + A] = \begin{bmatrix} \lambda-3 & -1 & -4 \\ & -2 & \end{bmatrix}}$

$$[A-\lambda I] = \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix}$$

$$= (3-\lambda)\,[(2-\lambda)(5-\lambda)] - 1\,[0-0] + 4\,[0-0]$$

$$\Rightarrow (\lambda-3)(\lambda-2)(\lambda-5) = 0$$

Hence, Eigen Values, $\boxed{\lambda = 2, 3, 5}$

for $\lambda = 2$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow \quad 3x_1 + x_2 + 4x_3 = 2x_1 \Rightarrow x_1 + x_2 + 4x_3 = 0$ —(1)

$2x_2 + 6x_3 = 2x_2 \Rightarrow \boxed{x_3 = 0}$

$5x_3 = 2x_3 \Rightarrow x_3 = 0$

& $\boxed{x_1 = -x_2}$ from (1)

So, Eigen vector, $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

for $\lambda = 3$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$3x_1 + 2x_2 + 4x_3 = 3x_1$

$\Rightarrow \boxed{x_2 + 4x_3 = 0}$

$2x_2 + 6x_3 = 3x_2 \Rightarrow \boxed{x_2 = 6x_3}$

$\boxed{5x_3 = 3x_3}$ So, $x_3 = 0$

$\Rightarrow x_2 = 0$

$\Rightarrow x_1 = K$ (any arbitrary constant)

So, eigen vector, $v_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

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for $\underline{\lambda = 5}$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 5 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow 3x_1 + x_2 + 4x_3 = 5x_1$

$\Rightarrow \boxed{2x_1 = x_2 + 4x_3}$

$2x_2 + 6x_3 = 5x_2 \Rightarrow 3x_2 = 6x_3 \Rightarrow \boxed{x_2 = 2x_3}$

$\boxed{5x_3 = 5x_3}$ $\Rightarrow 2x_1 = 6x_3 \Rightarrow \boxed{x_1 = 3x_3}$

Let $x_1 = 1$

So, eigen vector, $v_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

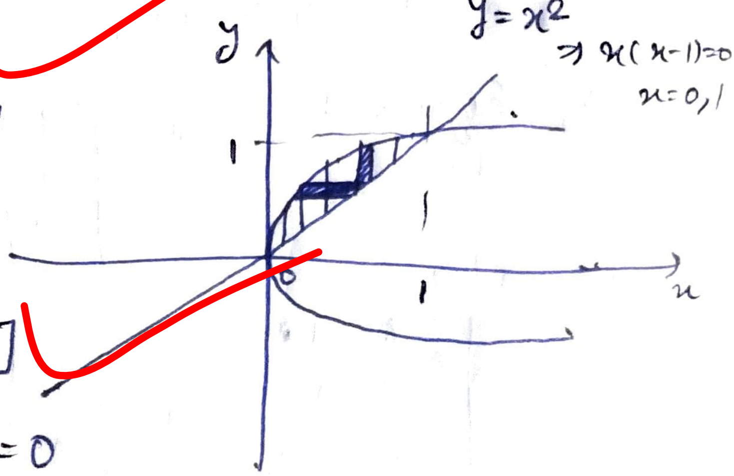
(ii) Given: $I = \int_C [(xy + y^2)\,dx + x^2\,dy]$ C: $y = x$

$y = x^2$

$\Rightarrow x(x-1) = 0$

$x = 0, 1$

$I = \int (xy + y^2)\,dx + \int x^2\,dy$



$I = \int_C [(xy + y^2)\,\hat{a}_x + x^2\,\hat{a}_y]$

$[dx\,\hat{a}_x + dy\,\hat{a}_y]$

$\Rightarrow x(x-1) = 0$

$\vec{f}(x,y) = (xy + y^2)\hat{a}_x + x^2\hat{a}_y$ [By using Stoke's theorem]

$$\vec{\nabla}\times\vec{f} = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy+y^2 & x^2 & 0 \end{bmatrix} = \hat{a}_z[2x - x - 2y]$$

$$= (x-2y)\hat{a}_z$$

$I_1 = \iint_S (\vec{\nabla}\times\vec{f})(dx\,dy\,\hat{a}_z)$ (By using stoke's theorem)

$$\Rightarrow I_1 = \int_0^1\int_x^{x^2} (x-2y)\,dy\,dx = \int_0^1\int_x^{x^2}(x-2y)\,dy\,dx$$

$$\Rightarrow I = \int_0^1 \left[(x^2 - 2xy)\right]_{\sqrt{y}}^{y} dy = \int_0^1 \left((y^2-2y^2) - \left[y - 2y^{3/2}\right]\right)dy$$

$$\Rightarrow I = \int_0^1 \left[-y^2 - y + 2y^{3/2}\right] dy$$

$$\Rightarrow I_1 = \int_0^1 \left[xy - y^2\right]_x^{x^2} dx = \int_0^1 x^3 - x^4 - \left[x^2 - x^2\right] dx$$

$$\Rightarrow I_1 = \frac{1}{20}$$

Green's theorem

Let $M = xy + y^2$ & $N = x^2$.

$$I_2 = \iint\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\,dx$$

$$I_2 = \iint (2x - x - 2y)\,dy\,dx$$

$$I_2 = \int_0^1\int_x^{x^2}(x-2y)\,dy\,dx$$

$$I_2 = \frac{1}{20}$$

As, $I_1 = I_2$, So, Green theorem is verified.

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(3) (a)(i) Given: $[D^2-4D+4]y = 8x^2 e^{2x}\sin 2x$

for complementary solution,

$$[D^2-4D+4]\, y_{CM} = 0$$

$$\Rightarrow D^2-4D+4=0 \Rightarrow (D-2)^2 = 0 \Rightarrow D = 2,2$$

$$\Rightarrow \boxed{y_{CM} = (A+Bx)e^{2x}}$$

for particular solution:

$$[D^2-4D+4]\, y_{PI} = 8x^2 e^{2x}\sin 2x$$

$$y_{PI} = \frac{8x^2 e^{2x}\sin 2x}{(D^2-4D+4)}$$

$$\Rightarrow y_{PI} = e^{2x}\frac{8x^2 \sin 2x}{(D+2)^2-4(D+2)+4} = e^{2x}\frac{8x^2\sin 2x}{D^2+4+4D-4D-8+4}$$

$$\Rightarrow y_{PI} = 8e^{2x}\frac{x^2 \sin 2x}{D^2}$$

$$\Rightarrow y_{PI} = 8e^{2x}\frac{1}{D}\int x^2 \sin 2x$$

$$I = \int x^2 \sin 2x$$

$$I = \frac{-x^2\cos 2x}{2} + \frac{2}{2}\int x\cos 2x\, dx$$

$$I = \frac{-x^2\cos 2x}{2} + \int x\cos 2x\, dx = \frac{-x^2}{2}\cos 2x + \frac{x\sin 2x}{2} - \frac{1}{2}\int \sin 2x\, dx$$

$$\boxed{I = \frac{-x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x.}$$

$$y_{PI} = 8e^{2x}\left[\int\left(\frac{-x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x\right)dx\right]$$

$$\Rightarrow y_{PI} = 8e^{2x}\left[-\frac{1}{2}\int x^2\cos 2x\,dx + \frac{1}{2}\int x\sin 2x\,dx + \frac{1}{8}\sin 2x\right]$$

$$I_1 = \int x^2\cos 2x = \frac{x^2\sin 2x}{2} - \int x\sin(2x)\,dx$$

$$\Rightarrow y_{PI} = 8e^{2x}\left[\frac{-x^2\sin 2x}{4} + \int x\sin 2x\,dx + \frac{1}{8}\sin 2x\right]$$

$$\Rightarrow y_{PI} = 8e^{2x}\left[\frac{-x^2\sin 2x}{4} - \frac{x\cos 2x}{2} + \frac{1}{2}\int\cos 2x\,dx + \frac{1}{8}\sin 2x\right]$$

$$\Rightarrow y_{PI} = 8e^{2x}\left[\frac{-x^2\sin 2x}{4} - \frac{x\cos 2x}{2} + \frac{3}{8}\sin 2x\right]$$

$$\Rightarrow \boxed{y_{PI} = \left[3\sin 2x - 4x\cos 2x - 2x^2\sin 2x\right]e^{2x}}$$

$$y = y_{CM} + y_{PI}$$

$$\boxed{y = (A+Bx)e^{2x} + e^{2x}(3\sin 2x - 4x\cos 2x - 2x^2\sin 2x)}$$

(iii)

Given: ~~for~~ $3x = \cos x + 1$

$$\Rightarrow \boxed{\cos x + 1 - 3x = f(x)}$$

$$\Rightarrow \boxed{f'(x) = -\sin x - 3}$$

Let, initial assumption, $x_0 = 0$

$$x_1 = 0 - \frac{f(0)}{f'(0)} = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow x_1 = 0 + \frac{2}{3} \Rightarrow \boxed{x_1 = \frac{2}{3}}$$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.607493}$$

$$\boxed{x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.607101}$$

$$\boxed{x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.607101}$$

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Hence, the root of ~~the~~ the given equation is $\boxed{x = 0.607101}$

(b) (i) Given: $f(x) = \begin{cases} Kx ; & 0 \le x \le 2 \\ 2K ; & 2 \le x < 4 \\ -Kx + 6K ; & 4 \le x < 6 \end{cases}$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_0^2 Kx\,dx + \int_2^4 2K\,dx + \int_4^6 (-Kx + 6K)\,dx = 1$$

$$\Rightarrow 2K + 2K[2] + K + 2K = 1.$$

$$\Rightarrow 8K = 1 \Rightarrow \boxed{K = \frac{1}{8}}$$

Mean of $f(x)$, $f(\bar{x}) = \int_0^6 x f(x)\,dx$

$$f(\bar{x}) = \int_0^2 \frac{x^2}{8}\,dx + \frac{2}{8}\int_2^4 x\,dx + \frac{1}{8}\int_4^6 (6-x)x\,dx$$

$$\Rightarrow f(\bar{x}) = \frac{1}{8}\left[\int_0^2 x^2\,dx + 2\int_2^4 x\,dx + \int_4^6 (6x - x^2)\,dx\right]$$

$$f(x) = \frac{1}{8}\left[\frac{8}{3} + 12 + \frac{28}{3}\right] \Rightarrow \boxed{f(x) = 3}$$

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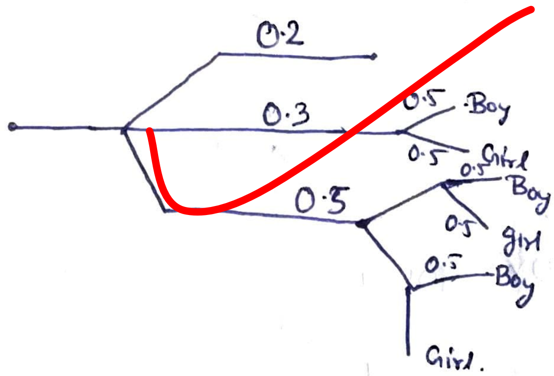
(ii) probablity ~~too~~ of having 0 child In a family, $p_0 = 0.2$

probablity of having 1 child in a family, $p_1 = 0.3$

probablity of having 2 childeren in a family, $p_2 = 0.5$.

probablity of being boy or girl, $p = 0.5$



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probablity of having no boy, $p_{nb} = p_0 + p_1 \times 0.5 + p_2 \times {}^2C_2\left(\frac{1}{2}\right)^2$

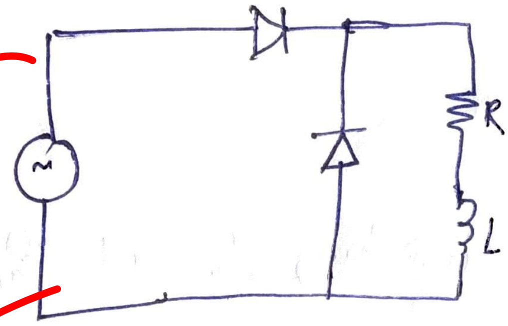
$$p_{nb} = 0.2 + 0.5 \times 0.3 + 0.5 \times \frac{1}{4}$$

$$\boxed{p_{nb} = 0.475}$$

③ © Given: $V_s = 240\sqrt{2}\, \sin(120\pi t)$

Resistance, $R = 8\,\Omega$

Inductance, $L \rightarrow \infty$



Average Output Voltage,

$$\boxed{V_0 = \frac{1}{2\pi}\int_0^{\pi} V_s \; d(\omega t)}$$

$$V_o = \frac{240\sqrt{2}}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$V_o = \frac{240\sqrt{2}}{2\pi} \times 2 \Rightarrow \boxed{V_o = 108.038\,V}$$

$$i_o = \frac{V_o}{R} \Rightarrow \boxed{i_o = 13.504745\,A}$$

Power absorbed by source,

$$P_o = V_o I_o$$

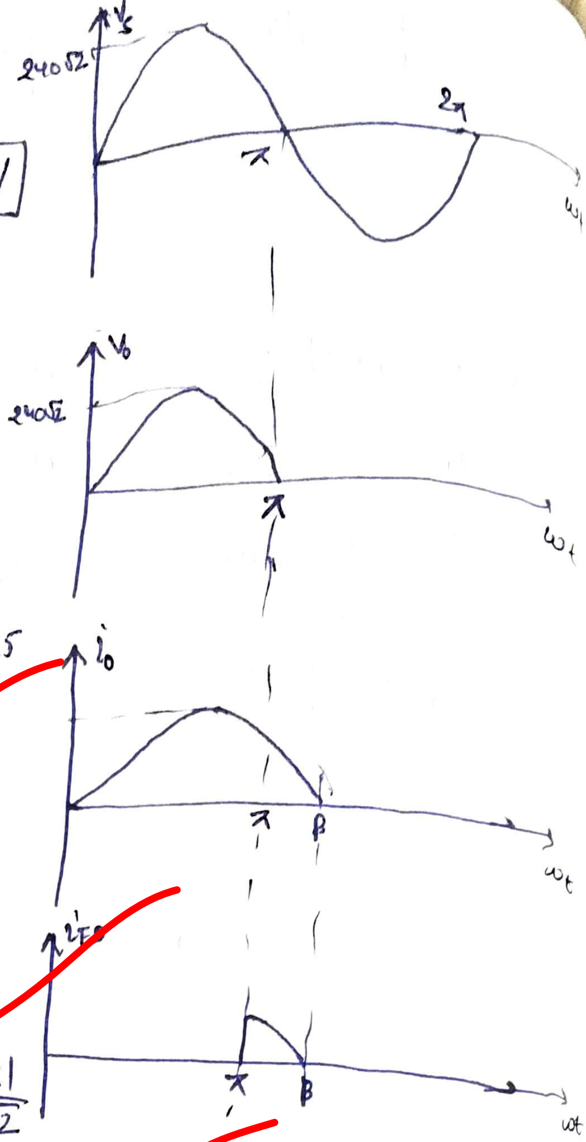
$$P_o = 108.038 \times 13.504745$$

$$\Rightarrow \boxed{P_o = 1459.02564\,W}$$

power factor,

$$\cos\phi = \frac{P_o}{240 \times 13.504745 \times \frac{1}{\sqrt{2}}}$$

$$\boxed{\cos\phi = 0.63662 \text{ (lagging)}}$$



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(iii) Considering for 2nd harmonic component only,

$$V_o = V_{o} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_2 = \frac{2}{\pi}\int_0^{\pi} 240\sqrt{2}\sin\omega t \cos 2\omega t \; d(\omega t)$$

$a_2 = -144.0506$ (Considering only for 2nd harmonic content)

$b_2 = \frac{2}{\pi}\int_0^{\pi} 240\sqrt{2}\ \sin\omega t\ \sin(2\omega t)\ d(\omega t)$

$b_2 = 0.$

$|I_2| = \left| \frac{-144.0506}{[8^2 + (2\omega L)^2]^{1/2}} \right|$

$|I_2| = \frac{144.0506}{\sqrt{64 + (240\pi \times L)^2}} \leq 0.1 \times 13.504745.$

$106.666^2 \leq 64 + (240\pi \times L)^2$

$\boxed{L \geq 0.1410726\ H.}$

5 (a) for region 1

Electric field, $\vec{E}_1 = 5V/m$

polarization, $\vec{P}_1 = 20pc/m^2$

polarization of region 2, $\vec{P}_2 = 50pc/m^2$

thickness, $t = 4$ cm

$\cancel{C_1 = \frac{\vec{P}_1}{\vec{E}_1 t} \Rightarrow \frac{\varepsilon_0 \varepsilon_r}{t} = \frac{20\times10^{-12}}{5\times t}}$

$\cancel{\Rightarrow 8.854\times10^{-12}\,\varepsilon_r = 4\times10^{-12}}$

Polarization, $\vec{P}_1 = \varepsilon_0(\varepsilon_r - 1)\vec{E}_1 \Rightarrow 20\times10^{-12} = 8.854(\varepsilon_{r_1}-1)\times10^{-12}\times5$

$\boxed{\varepsilon_{r_1} = 1.451773}$

ε_{r_1} is relative permeability of region 1.

$$\boxed{\vec{P}_2 = \varepsilon_0 (\varepsilon_{r_2} - 1) \vec{E}_1}$$ where ε_{r_2} is relative permeability of region2

$\Rightarrow 50 \times 10^{-12} = 8.854 \times 10^{-12} \times 5 \times (\varepsilon_{r_2} - 1)$

$\Rightarrow$ $\boxed{\varepsilon_{r_2} = 2.1294}$

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(B) Registers → These are the memory elements present inside the CPU for its internal operation and for arithmetic and logic operation

- They have the least latency time as compared to all other memories (main memory, secondary memory etc)
- Some of the registers present are general purpose register, accumulator, memory buffer register etc.

function of register

(1) Accumulator is mainly used in arithmetic and logic expression it behaves as primary data and also after evaluation the output is stored at the accumulator. example → ADD C

SUB B

etc.

(2) Memory register → These register directly point to the memory location address or it stores the memory address. Ex → CMP M

CM means data at memory

(3) General purpose register → These are generally used to hold secondary data and can be used as counters

Purpose of Memory buffer register

① There are two buffer register, MAR (Memory Address Register) MBR (memory buffer register)

② MAR is a 16 bit register used to hold the ~~data~~ memory address before it is placed on address lines.

③ MBR is a 8 bit register used to hold the data before ~~when~~ it is placed on the data lines or before it is retrived by the internal ~~instruction~~ register of CPU.

(c) Given → Double order pole at $s = -a$

Zero at, $s = -b$

$b - a = B$

$h(0^+) = 2.$

$$H(s) = \frac{K(s+a)}{(s+b)^2} \Rightarrow H(s) = K\left[\frac{A}{(s+b)} + \frac{C}{(s+b)^2}\right]$$

$$\frac{K(s+a)}{(s+b)^2} = K\left[\frac{A(s+b) + C}{(s+b)^2}\right]$$

$$As + bA + C = s + a$$

$$\Rightarrow \quad As = s \Rightarrow \boxed{A = 1}$$

$$bA + C = a \Rightarrow C = a - b \Rightarrow C = -B.$$

$$H(s) = K\left[\frac{1}{s+b} - \frac{B}{(s+b)^2}\right]$$

$$\boxed{h(t) = K\left[e^{-bt} - Bte^{-bt}\right]u(t)}$$

$h(0^+) = K \Rightarrow \boxed{K=2}$

$$\boxed{h(t) = 2\left[e^{-bt} - Bt\, e^{-bt}\right] u(t)}$$

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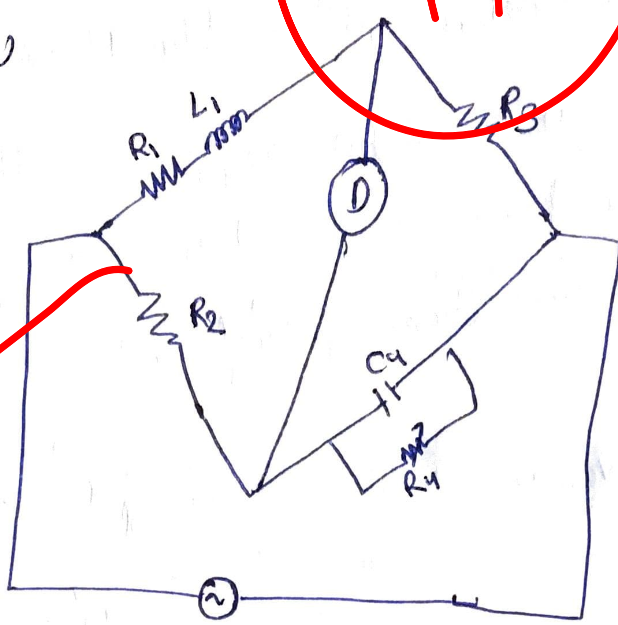
5 (d) $R_2 = 400\Omega$ (Given)

$R_3 = 600\Omega$

$R_4 = 1000\Omega$

$C_4 = 0.5\mu F$

frequency, $f = 1000Hz$



$Z_4 = R_4 \| \left(\frac{1}{j\omega C_4}\right)$

$$= \frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}$$

$$\boxed{Z_4 = \frac{R_4}{j\omega R_4 C_4 + 1}}$$

At bridge balance condition: $Z_1 Z_4 = Z_2 Z_3$

$$\Rightarrow \left[R_1 + j\omega L_1\right] \frac{R_4}{j\omega R_4 C_4 + 1} = R_2 R_3$$

$$\Rightarrow R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + R_2 R_3 j\omega R_4 C_4$$

By comparing real and imaginary parts

$$R_1 R_4 = R_2 R_3 \Rightarrow \boxed{R_1 = \frac{R_2 R_3}{R_4}}$$

$$\Rightarrow R_1 = \frac{400 \times 600}{1000} \Rightarrow \boxed{R_1 = 240\Omega}$$

And, $R_2 R_3 j\omega R_4 C_4 = j\omega L_1 R_4$

$$\boxed{L_1 = R_2 R_3 C_4}$$

$\Rightarrow \boxed{L_1 = 0.12H}$

Quality factor of coil, $\boxed{Q = \frac{\omega_0 L}{R}}$ $\Rightarrow$ & $\omega_0 = 2\pi f$

$Q = \pi \Rightarrow \boxed{Q = 3.1416}$

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Advantages

① The value of L_1 and R_1 in the expression are frequency independent.

② It can be used to find inductance of coils at wide range of audio frequency.

Disadvantages

① The Variable capacitor increases the cost of the bridge

② It can be used for coils having Quality factor,

$$\boxed{1 \leq Q \leq 10}$$

⑤ ⓔ Given: $x_c(t) = 10 \cos[2\pi \times 10^6 t + 0.1 \sin(10^3)\pi t]$

$K_p = 10$ rad/v

$K_f = 10\pi$ rad/s-volt

$\theta_i = 2\pi \times 10^6 t + 0.1 \sin(10^3 \pi t)$ = Instantaneous angle

$\frac{d\theta_i}{dt} = \Delta\omega_i = 2\pi \times 10^6 + 0.1 \times 10^3 \pi \cos(10^3 \pi t)$

ω_i is Instantaneous frequency deviation

Maximum frequency deviation, $\Delta\omega_i|_{max}$

$\Delta f_i|_{max} = ~~(10^6 + 50)~~$ Hz

for FM signal

$x_c(t) = 10\cos[2\pi 10^6 t + k_f \int m(t)\,dt]$

$m(t)$ is message signal

$k_f \int m(t)\,dt = 0.1 \sin(10^3 \pi t)$

$\Rightarrow \int m(t)\,dt = \frac{0.1}{10\pi} \sin 10^3 \pi t$

$\Rightarrow m(t) = \frac{0.1}{10\pi} \frac{d \sin(10^3\pi t)}{dt} = \frac{0.1}{10\pi} \times 10^3 \pi \cos 10^3 \pi$

$\Rightarrow \boxed{m(t) = 10\cos(10^3 \pi t) V}$

frequency deviation, $\Delta f = \frac{k_f A_m}{2\pi} \Rightarrow \boxed{\Delta f = 50 Hz.}$

for PM signal

, $x_c(t) = 10\cos[2\pi \times 10^6 t + k_p m(t)]$

$k_p m(t) = 0.1 \sin(10^3 \pi t)$

$\boxed{m(t) = 10^{-2} \sin(10^3 \pi t)}$

So, frequency deviation $\boxed{\Delta\omega = k_p A_m \omega_m}$

$\Rightarrow \boxed{\Delta f = k_p A_m f_m}$

$\Rightarrow \Delta f = 10 \times 10^{-2} \times \frac{10^3 \pi}{2\pi} \Rightarrow$

$\boxed{\Delta f = 50 Hz}$

7) (a) Program development cycle → It is used for developing a program for real life problem or application. It consist of various steps which result in the development of program.

It is a set standard of how to proceed for developing a program for given real life problem.

Steps involved in developing a program

(1) ~~...~~ Retriving input data → The data that has to be taken by the user as input should be retrived or gathered from the user in a proper format

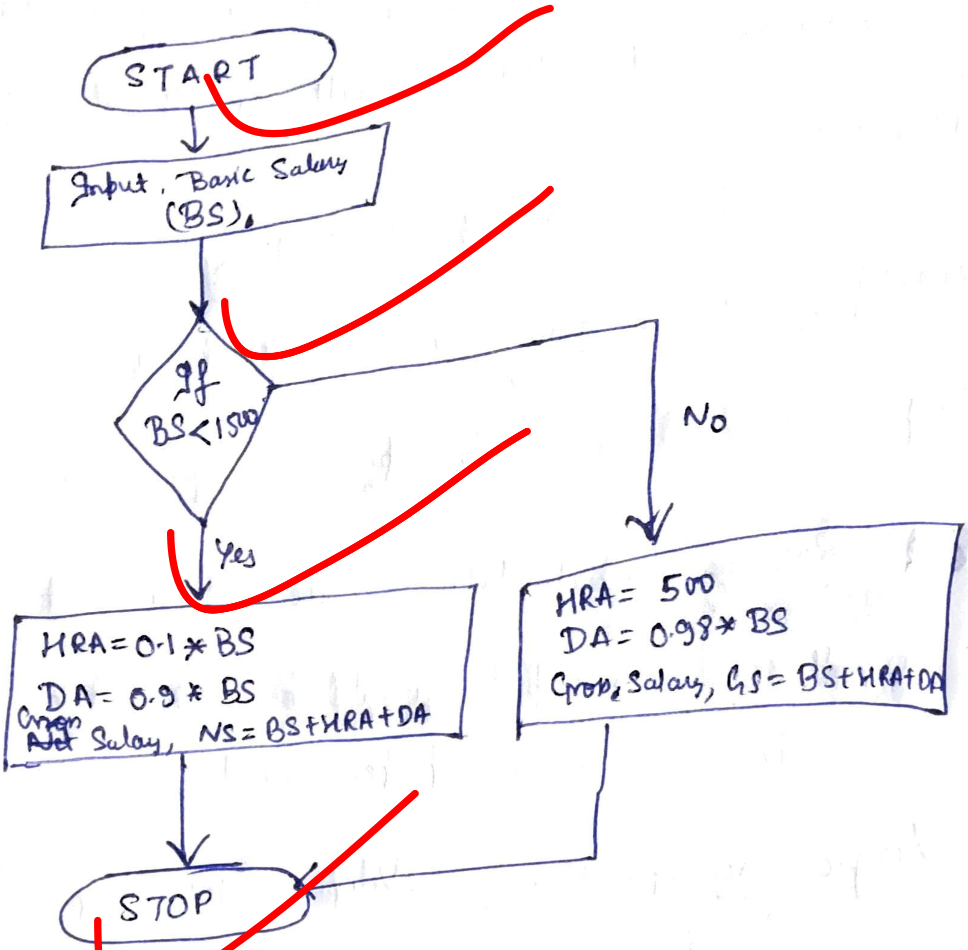
(2) Logic development → Developing the proper logic for the ~~execution of~~ execution of the program for its proper functioning.

(3) Initialization ~~...~~ → ~~It is used for a~~ The variables used in the program should be initialized with their type and value (if applicable).

(4) Execution → The program should be written in proper format so, that program runs as per the logic formed.

(5) Output generation and display → Output should be displayed in a manner, so, that it is easily understandable by the user.

(ii) flow chart ~~....~~



C-Program

```
# include <stdio.h>
# include <conio.h>
 Void main ( )
{
float HRA, DA, NS;
      int BS;
 printf ("\n Enter the basic Salary");
 Scanf ("%d", & BS);
  if ( BS < 1500)
  {
        HRA = 0.1 * BS ;
        DA = 0.9 * BS ;
        NS = BS + HRA + DA ;
  }
  else { HRA = 500
        DA = 0.98 * BS;
```

NS = BS + HRA + DA

}

printf ("Gross salary %f", & NS)

}

7 b $$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}nk}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(K)\, e^{+j\frac{2\pi}{N}kn}$$

$$\Rightarrow x_1(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(K)\, e^{+j\frac{2\pi}{N}kn}$$

$$x_2(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_2(K)\, e^{j\frac{2\pi}{N}kn}$$

$$\cancel{\Rightarrow x_1(n)\cdot x_2(n) = \frac{1}{N^2}\left(\sum_{k=0}^{N-1} X_1(K)\, e^{j\frac{2\pi}{N}kn}\right)\left(\sum_{k=0}^{N-1} X_2(K)\, e^{j\frac{2\pi}{N}kn}\right)}$$

we know that -

$$x_1(n) * x_2(n) = X_1(K)\, X_2(K)$$

$\Rightarrow$ By using duality property

$$\frac{1}{N}\left[X_1(-K) * X_2(-K)\right] = x_1(K)\, x_2(K)$$

As, it is a circular convolution, $X_1(-K) = X_1(K)$
$X_2(-K) = X_2(K)$

$$\Rightarrow \boxed{x_1(K)\, x_2(K) = \frac{1}{N}\left[X_1(K) * X_2(K)\right]}$$ (By interchanging variables)

$$\Rightarrow \boxed{x(n) = \frac{1}{N}\left[X_1(K) * X_2(K)\right]}$$

7 (c) To prove:

$$\sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)\, X_2^*(k)$$

$x(n) =$

$$x_1(n)\, x_2(n) = \frac{1}{N}\left[X_1(k) * X_2(k)\right]$$

$x_2^*(n) \xrightarrow{DFT} X_2^*(-k+iN)$ i is any arbitrary integer

$$x_1(n)\, x_2^*(n) = \frac{1}{N}\left[X_1(k) * X_2^*(-k+iN)\right]$$

$$= \frac{1}{N} X_1(k) * X_2^*(-k+iN)$$

$$\Rightarrow \sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) * X_2^*(-k+iN)$$

$$\Rightarrow \sum_{n=0}^{N-1} x_1(n)\, x_2^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)\, X_2^*(k)$$

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7) (a)(i) ~~Resis~~ Resistance, $R = \frac{\rho L}{A}$

Area, $A = \frac{\pi D^2}{4}$

$$\frac{\partial A}{\partial D} = \frac{2\pi D}{4} = \frac{\pi D}{2}$$

$$\ln R = \ln\left(\frac{\rho L}{A}\right)$$

$$\Rightarrow \ln R = \ln(\rho) + \ln(L) - \ln(A)$$

$$\Rightarrow \frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{1}{L}\partial L - \frac{1}{A}\frac{\partial A}{\partial D}\partial D$$

$$\Rightarrow \frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{1}{L}\partial L - \frac{4}{\pi D^2} \times \frac{\pi D}{2}\partial D$$

$$\Rightarrow \frac{\partial R}{R} = \frac{\partial \rho}{\rho} + \frac{1}{L}\partial L - \frac{2}{D}\partial D$$

$$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - \frac{2\Delta D}{D}$$

$$\Rightarrow \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = 1 + \frac{\Delta \rho}{\frac{\Delta L}{L}(\rho)} - 2\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}$$

Let $\boxed{\frac{\Delta L}{L} = \varepsilon \text{ (Strain)}}$

$$\Rightarrow \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = 1 + \frac{\Delta \rho}{\varepsilon \rho} + 2\frac{\left(-\frac{\Delta D}{D}\right)}{\frac{\Delta L}{L}}$$

$$\boxed{\nu = -\frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}} = \text{Poisson's ratio}}$$

$$\frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \text{Gauge factor } (G.f)$$

$$\Rightarrow \boxed{G.f = 1 + 2\nu + \frac{\Delta \rho}{\varepsilon \rho}}$$

(ii) Given: Nominal resistance, $R_1 = 500\,\Omega$

Gauge factor, $G_f = 4$

Other resistances, $R_2, R_3, R_4 = 500\,\Omega$

Strain, $\varepsilon = 100 \times 10^{-6}$ m/m $= 10^{-4}$ m/m

$$\boxed{G_f = \frac{\frac{\Delta R_1}{R_1}}{\varepsilon}}$$

ΔR_1 is change in resistance.

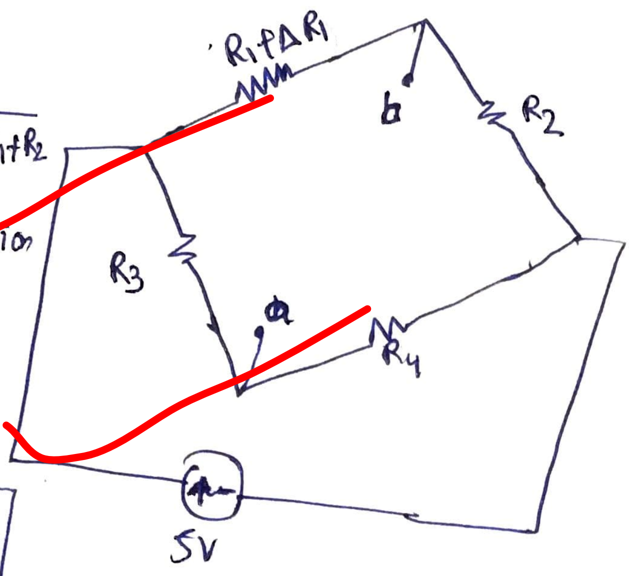
$$4 = \frac{\frac{\Delta R_1}{500}}{10^{-4}} \Rightarrow 4 \times 10^{-4} \times 500 = \Delta R_1$$

$$\boxed{\Delta R_1 = 0.2\,\Omega}$$

Supply voltage, $V = 5$ V (Given)

$$V_b = V \times \frac{R_2}{R_1 + \Delta R_1 + R_2}$$

(By voltage division rule)



$$V_b = 5 \times \frac{500}{1000.2}$$

$$\boxed{V_b = 2.4995001\text{ V}}$$

$V_a = V \times \frac{R_4}{R_3 + R_4}$ (By using voltage division rule)

$\Rightarrow V_a = 5 \times \frac{500}{1000}$ $\cancel{\boxed{V_a = 2.3V}}$ $\Rightarrow \boxed{V_a = 2.5V}$

So, $V_{ab} = V_a - V_b$

$\boxed{V_{ab} = 0.4999 \text{ mV}}$

If the voltmeter is connected across ab, we get,

Output voltage, $\boxed{V_{ab} = V_0 = 0.4999 mV}$

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