Name →

Roll NO →

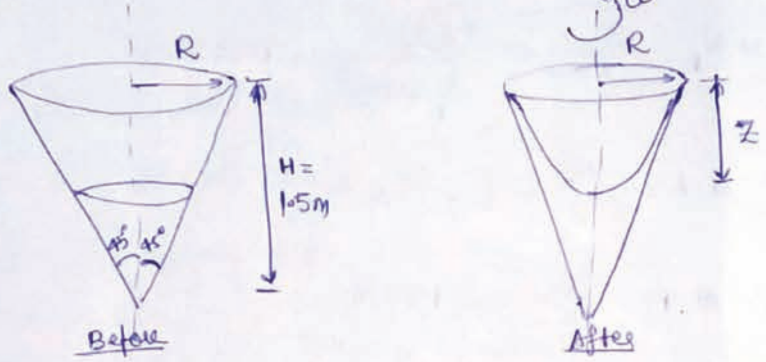
TEST No → 05

SUBJECT NAME → FLUID MECHANICS AND MACHINERY

BRANCH → CIVIL ENGG.

270/300

1 (a)



Volume of water = $\frac{1}{2} \times$ Volume of Cone

$\Rightarrow$ Volume of water = $\frac{1}{2}\left\{\frac{1}{3} \times \pi R^2 H\right\}$

Now

Volume of paraboloid = $\frac{1}{2} \times$ Volume of cone

$$\Rightarrow \frac{1}{2}\left\{\pi R^2 Z\right\} = \frac{1}{2}\left\{\frac{1}{3}\pi R^2 H\right\}$$

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$$\Rightarrow Z = \frac{H}{3} = \frac{1.5}{3} = 0.5\,m$$

For 90° cone, $R = H = 1.5\,m$

$$\therefore \frac{\omega^2 R^2}{2g} = (Z - Z_0)$$

$$\Rightarrow \frac{\omega^2 \times (1.5)^2}{2 \times 9.81} = 0.5$$

$\Rightarrow$ ω = Angular velocity to not spill out water = 2.088 radian/s

$\therefore$ Rotation per minutes = $\frac{60\omega}{2\pi} = 19.9 \approx 20$ rpm

1 (b)

Pressure head at $A = \frac{P_A}{\rho g} = 30m$

Pressure head at $B = \frac{P_B}{\rho g} = -4m$

Datum head of $A = Z_A = 2m$

Datum head of $B = Z_B = 0m$

Velocity at $A = V_A = \frac{Q}{A} = \frac{0.5}{\frac{\pi}{4} \times (0.4)^2} = 3.98 \, m/s$

Velocity at $B = V_B = \frac{Q}{Area} = \frac{0.5}{\frac{\pi}{4} \times (0.6)^2} = 1.768 \, m/s$

$\therefore$ Velocity head at $A = \frac{V_A^2}{2g} = 0.807 \, m$

Velocity head at $B = \frac{V_B^2}{2g} = 0.159 \, m.$

Applying bernaulli equation between A and B

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + Z_B + H_{turbine}$$

$$\Rightarrow 30 + 0.807 + 2 = -4 + 0.159 + 0 + H_{turbine}$$

$$\Rightarrow H_{turbine} = 36.648 \, m.$$

$\therefore$ Head used by turbine $= 36.648 \, m.$

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$\therefore$ Power available at turbine $= \gamma_w Q H = 9.81 \times 0.5 \times 36.648$

$= 179.76 \, kW.$

$\therefore$ Power Output $= 88\%$ of $179.76 \, kW = 158.19 \, kW$

$$\boxed{\text{Power Output} = 158.19 \, kW}$$

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1 (c) (i) Assumption in derivative of hydraulic jump.

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(i) Discharge is constant

(ii) Specific force remain constant.

(iii) frictionless channel

(iv) slope of channel bed is almost horizontal.

(v) All the energy get dissipated in only hydraulic jump.

1 (c) (ii)

Rectangular channel:

Prejump velocity $V_1 = 10 \, m/s$

Prejump depth $y_1 = 1m$

$\therefore F_1 =$ Prejump Froude no. $= \frac{V_1}{\sqrt{g y_1}} = \frac{10}{\sqrt{9.81 \times 1}} = 3.193$

Supercritical

Formula used :-

$$\therefore \quad \frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8F_1^2}\right]$$

$$\Rightarrow \quad \frac{y_2}{1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8(3.193)^2}\right]$$

$$\Rightarrow \quad y_2 = 4.043 \, m$$

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$\therefore$ $\boxed{\text{post jump depth of flow} = y_2 = 4.043m}$

Energy loss formula for rectangular channel :-

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2} = \frac{(4.043 - 1)^3}{4 \times 4.043 \times 1} = 1.742m$$

$\therefore$ $\boxed{\text{Energy loss} = \Delta E = 1.742m}$

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1(d)

Radius of pipe = R_0.

friction factor = $f = 0.02$

(i) $U^* = V\sqrt{\frac{f}{8}} = V\sqrt{\frac{0.02}{8}} = 0.05\,V$ $\boxed{U^* = 0.05V}$

And

$$\frac{u-V}{U^*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$$

$$\Rightarrow \frac{u-V}{0.05V} = 5.75 \log_{10}\left(\frac{y}{r_0}\right) + 3.75$$

At $y = r_0$ $\quad u = U_{max}$.

$$\therefore \frac{U_{max} - V}{0.05V} = 5.75 \log_{10}\left(\frac{r_0}{r_0}\right) + 3.75$$

$$\Rightarrow U_{max} = 3.75 \times 0.05V + V$$

$$\boxed{U_{max} = 1.1875V}$$

(ii)

For radial distance $= 0.3r_0$ $\quad y = 0.7r_0$

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$$\therefore \frac{u-V}{0.05V} = 5.75 \log_{10}\left(\frac{0.7r_0}{r_0}\right) + 3.75$$

$$\Rightarrow \frac{u-V}{0.05V} = 2.8593.$$

$$\Rightarrow u = 1.143\,V$$

$$\therefore \boxed{\frac{u}{V} = 1.143}$$

1 (e)

Specific Energy $E = y + \frac{V^2}{2g}$.

$$\Rightarrow E = y + \frac{Q^2}{2g A^2}$$

$$\Rightarrow E = y + \frac{Q^2}{2g [K_1 y^a]^2}$$

$$\Rightarrow E = y + \frac{Q^2}{2g K_1^2 y^{2a}}$$

At critical flow, specific energy become minimum for constant discharge.

$$\therefore \frac{dE}{dy} = 0.$$

$$\Rightarrow \frac{d}{dy}\left[y + \frac{Q^2}{2g K_1^2 y_c^{2a}} \right]_{y=y_c} = 0$$

$$\Rightarrow 1 + \frac{Q^2}{2g K_1^2} \cdot (-2a)\, y_c^{-2a-1} = 0$$

$$\Rightarrow 1 - \frac{Q^2}{2g K_1^2} \frac{2a}{y_c^{2a+1}} = 0$$

$$\Rightarrow y_c^{2a+1} = \frac{Q^2}{g} \times \frac{a}{K_1^2}$$

$$\Rightarrow \boxed{y_c = \left[\frac{Q^2}{g} * \frac{a}{K_1^2} \right]^{\frac{1}{2a+1}}}$$

Hence Proved

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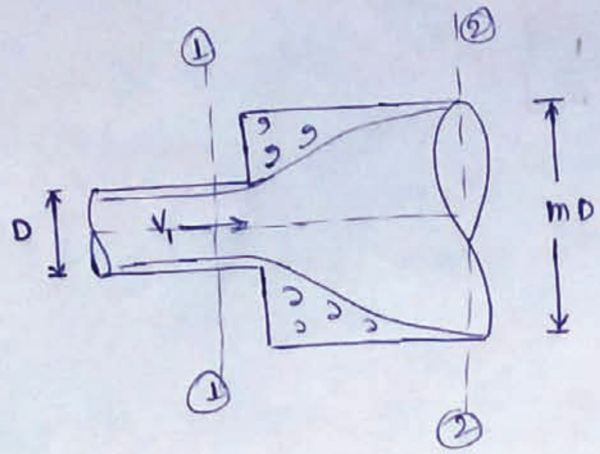
2(a)



Assume the pipe is horizontal.

D = Diameter of smaller dia pipe

V_1 = Velocity at section ①–①

V_2 = Velocity at section ②–②

Let discharge = Q.

Applying Energy equation between section ①-① and ②-②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1}^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{Z_2}^{0} + \text{Head loss due to expansion}$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2)^2}{2g}$$

$$\Rightarrow \frac{P_2 - P_1}{\rho \cancel{g}} = \frac{V_1^2 - V_2^2 - (V_1 - V_2)^2}{2\cancel{g}}$$

$$\Rightarrow \frac{P_2 - P_1}{\rho} = \frac{V_1^2 - V_2^2 - (V_1^2 + V_2^2 - 2V_1V_2)}{2}$$

$$\Rightarrow \frac{\Delta P}{\rho} = \frac{-2V_2^2 + 2V_1V_2}{2} \quad \text{—— ①}$$

From continuity : $Q = A_1 V_1 = A_2 V_2$

$$\therefore Q = \frac{\pi}{4} D^2 V_1 = \frac{\pi}{4} (mD)^2 V_2$$

$$\Rightarrow Q = cV_1 = m^2 c V_2 \qquad \text{where } c = \frac{\pi}{4} D^2$$

$$\therefore V_1 = \frac{Q}{c} \qquad V_2 = \frac{Q}{m^2 c}$$

Putting these values in ① we get

$$\Rightarrow \frac{\Delta P}{\rho} = -\left(\frac{Q}{m^2 c}\right)^2 + \left(\frac{Q}{c}\right)\left(\frac{Q}{m^2 c}\right)$$

$$\Rightarrow \frac{\Delta P}{\rho} = \frac{Q^2}{c^2}\left[-\frac{1}{m^4} + \frac{1}{m^2}\right]$$

$$\Rightarrow \Delta P = \frac{\rho Q^2}{c^2}\left[-\frac{1}{m^4} + \frac{1}{m^2}\right]$$

For maximum rise in pressure →

$$\frac{d(\Delta P)}{dm} = 0$$

$$\Rightarrow \frac{d}{dm}\left[-\frac{1}{m^4} + \frac{1}{m^2}\right] = 0$$

$$\Rightarrow \left[\frac{4}{m^5} - \frac{2}{m^3}\right] = 0$$

$$\Rightarrow m^2 = 2$$

$$\Rightarrow m = \sqrt{2}$$

$\therefore$ Ratio of diameter for maximum pressure rise $= \frac{\sqrt{2}}{1}$

Now

Rise in pressure $= \Delta P_{max} = \frac{\rho Q^2}{c^2}\left[-\frac{1}{(\sqrt{2})^4} + \frac{1}{(\sqrt{2})^2}\right]$

$$= \frac{\rho Q^2}{c^2}\left[-\frac{1}{4} + \frac{1}{2}\right]$$

$$= \frac{\rho Q^2}{4c^2}$$

$$= \frac{\rho Q^2}{4 \times \left(\frac{\pi}{4} D^2\right)^2} = \frac{\rho}{4} V_1^2$$

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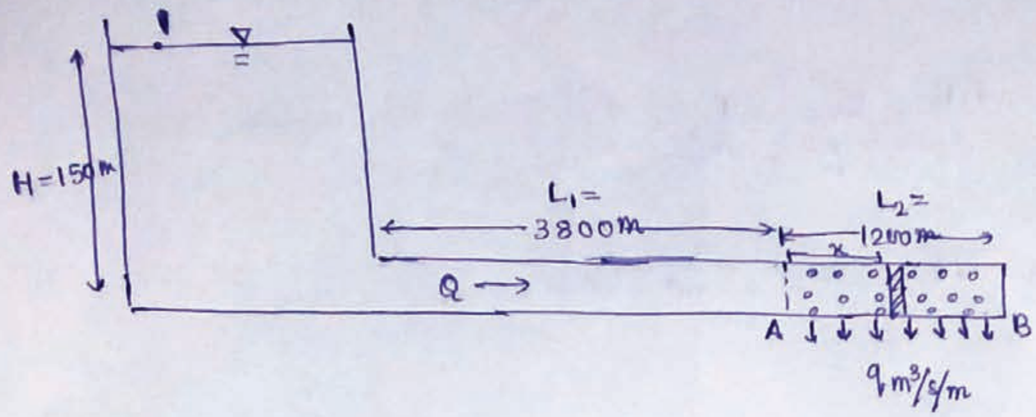
$\therefore$ $\boxed{\text{Maximum Rise in pressure} = \frac{\rho V_1^2}{4}}$

2(b)



Let the emission rate at last 1200 m is $q\ m^3/s$ per metre length.

Taking a small element dx at a distance x from (A).

$\therefore$ discharge from element $= q \cdot dx\ \ m^3/s$.

$\therefore$ head loss in element $= dh_{f_2} = \dfrac{8 f (dx) (Q - qx)^2}{\pi^2 g\ \ D}$

$$\Rightarrow dh_{f_2} = \frac{f (Q-qx)^2 dx}{12\cdot 1\ D^5}$$

$$\Rightarrow \int_0^{h_{f_2}} dh_{f_2} = \int_0^{L_2} \frac{f (Q-qx)^2}{12\cdot 1\ D^5} dx$$

$$\Rightarrow h_{f_2} = \frac{f}{12\cdot 1\ D^5} \left[\frac{(Q-qx)^3}{3(-q)} \right]_0^{L_2}$$

$$\Rightarrow h_{f_2} = \frac{f}{12\cdot 1\ D^5} \times \left[\cancel{(Q - qL_2)^3}^{\,0} - Q^3 \right] \frac{1}{3(-q)}$$

$$\Rightarrow h_{f_2} = \frac{f\ Q^3}{12\cdot 1\ D^5\ q} \cdot \frac{1}{3}$$

$$\Rightarrow h_{f_2} = \frac{f\ Q^2}{12\cdot 1\ D^5} \cdot \frac{qL_2}{q} \cdot \frac{1}{3}$$

$$\Rightarrow \boxed{h_{f_2} = \frac{f L_2 Q^2}{12\cdot 1\ D^5} \cdot \frac{1}{3}}$$

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Now Given: $q = \frac{0.088}{300}\ m^3/s/m$

$$\therefore Q = qL_2 = \frac{0.088}{300} \times 1200\ m^3/s = 0.352\ m^3/s$$

$$\therefore h_{f_2} = \frac{1}{3} \times \frac{f L_2 Q^2}{12.1 D^5} = \frac{1}{3} \times \frac{0.02 \times 1200 \times (0.352)^2}{12.1 \times (0.6)^5}$$

$$= 1.053\ m$$

$$h_{f_1} = \frac{f L_1 Q^2}{12.1 D^5} = \frac{0.02 \times 3800 \times (0.352)^2}{12.1 \times (0.6)^5} = 10.008\ m.$$

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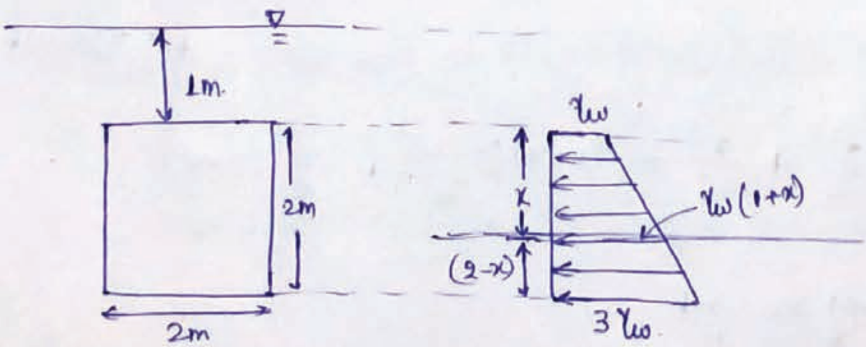
Applying energy equation between (1) and (B)

$$\cancel{\frac{P_1}{\rho g}}^{\,atm} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 = \frac{P_B}{\rho g} + \cancel{\frac{V_B^2}{2g}}^{\,0} + \cancel{z_2}^{\,0} + h_{f_1} + h_{f_2}$$

$$\Rightarrow 150 = \frac{P_B}{\rho g} + 10.008 + 1.053\ m.$$

$$\therefore \frac{P_B}{\rho g} = \text{Pressure head at dead end} = \boxed{138.939m}$$

2 (c)



(1) Let at depth x from top of plate where total pressure on top portion is equal to total pressure on bottom portion.

$$\therefore \frac{1}{2} \times (\gamma_w + \gamma_w(1+x)) \cdot x \cdot 2 = \frac{1}{2}(\gamma_w(1+x) + 3\gamma_w) \cdot (2-x) \cdot 2$$

$$\Rightarrow (1+1+x)x = (1+x+3)(2-x).$$

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$$\Rightarrow (2+x)x = (4+x)(2-x)$$

$$\Rightarrow \cancel{2x} + x^2 = 8 - 4x + \cancel{2x} - x^2$$

$$\Rightarrow 2x^2 + 4x - 8 = 0$$

$$\Rightarrow x^2 + 2x - 4 = 0$$

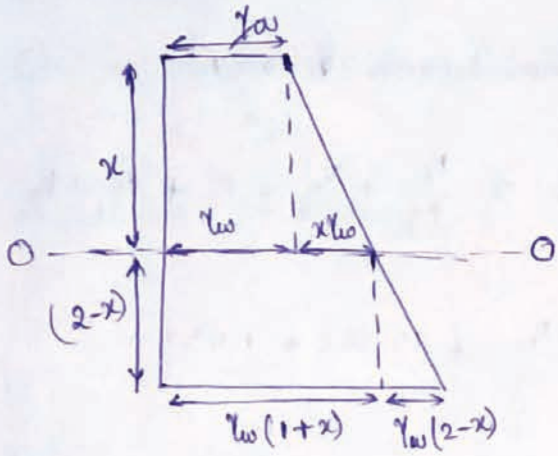
Solving we get

$$x = 1.236 \;,\; -3.236 \text{ (Not possible)}$$

$\therefore \boxed{x = 1.236}$ below top of plate

$\therefore$ depth below water surface $= 1 + 1.236 = \boxed{2.236\text{ m}}$

(ii)



Taking moment about 0 – 0.

$$\cancel{\gamma_w} \times x \times \cancel{2} \times \frac{x}{2} + \frac{1}{2} \times x\cancel{\gamma_w} \times x \times \cancel{2} \times \frac{1}{3}x = \cancel{\gamma_w}(1+x) \times (2-x) \times \cancel{2} \times \frac{(2-x)}{2}$$

$$+ \frac{1}{2}\cancel{\gamma_w}(2-x) \times (2-x) \times (2-x) \times \frac{2}{3} \times \cancel{2}$$

$$\Rightarrow \frac{x^2}{2} + \frac{x^3}{6} = (1+x)\frac{(2-x)^2}{2} + \frac{2}{6}(2-x)^3$$

Solving we get

$$x = 1.167\text{ m}$$

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$\therefore$ Depth below water surface for equal moment $= 1 + 1.167$

$= \boxed{2.167\text{ m}}$

4(a)

Net head available. $H = 300\ m$

Speed $N = 550$ rpm.

speed ratio $\phi = \frac{u}{\sqrt{2gH}} = 0.46$.

$C_v = 0.98$

$\frac{\text{jet dia}}{\text{wheel dia}} = \frac{1}{10}$

$\therefore\ u = 0.46\sqrt{2gH} = 0.46\sqrt{2\times 9.81\times 300} = 35.29\ m/s$

$\therefore\ \frac{\pi D N}{60} = 35.29$

$\Rightarrow\ \frac{\pi \times D \times 550}{60} = 35.29$

$\Rightarrow\ D = 1.225m$

$\therefore$ Wheel diameter $= 1.225m$

$\therefore$ jet diameter $= \frac{1.225}{10} = 0.1225m$.

Efficiency of nozzle $= \eta_{nozzle} = C_v^2 = (0.98)^2 = 0.96$.

Velocity at the exit of nozzle $V = C_v\sqrt{2gH} = 0.98\times\sqrt{2\times 9.81\times 300}$
$= 75.186\ m/s$.

$\therefore$ discharge through one jet $= \frac{\pi}{4}\times(0.1225)^2\times 75.186 = 0.886\ m^3/s$.

Now Power available $= \gamma_w Q H$

$\Rightarrow\ \frac{6000\times 10^3}{0.85} = 9.81\times 10^3\times Q\times 300$

$\Rightarrow\ Q = 2.3985\ m^3/s$

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(i) No. of jets required = $\dfrac{\text{Total discharge required}}{\text{discharge from one jet}}$

$$= \frac{2.3985}{0.886}$$

$$= 2.707 \simeq 3$$

Provide 3 no. of jet

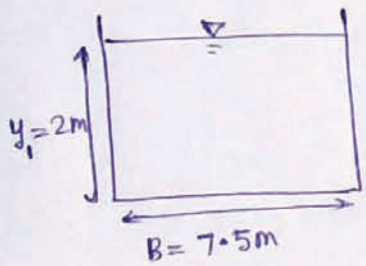
(ii) Diameter of jet = 0.1225 m (Calculated above)

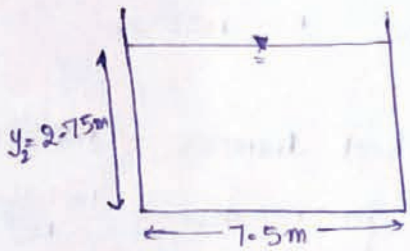
(iii) Diameter of wheel = 1.225 m (Calculated above)

(iv) Quantity of water required = 2.3985 m^3/s (calculated above)

4(b)

Rectangular channel:





Area $A_1 = B \times y_1 = 2 \times 7.5 = 15 m^2$

Perimeter $P_1 = 7.5 + 2 \times 2 = 11.5 m$

$\therefore$ hydraulic Radius $R_1 = \dfrac{A_1}{P_1}$

$$= \frac{15}{11.5} = 1.304 m$$

$$\therefore Q = \frac{1}{\eta} A_1 R_1^{2/3} S_{f_1}^{1/2}$$

$$= \frac{1}{0.02} \times 15 \times (1.304)^{2/3} \left(\frac{1}{3000}\right)^{1/2}$$

$$= 16.344 \, m^3/s$$

Now

$Q = 16.344 m^3/s$

$A_2 = 7.5 \times 2.75 = 20.625 m^2$

$P_2 = 7.5 + 2 \times 2.75 = 13 m$

$R_2 = \dfrac{A_2}{P_2} = \dfrac{20.625}{13} = 1.587 m.$

$$\therefore Q = \frac{1}{\eta} A_2 R_2^{2/3} S_{f_2}^{1/2}$$

$$\Rightarrow 16.344 = \frac{1}{0.02} \times 20.625 \times (1.587)^{2/3} \sqrt{S_{f_2}}$$

$$\Rightarrow S_{f_2} = \frac{1}{7369.6}$$

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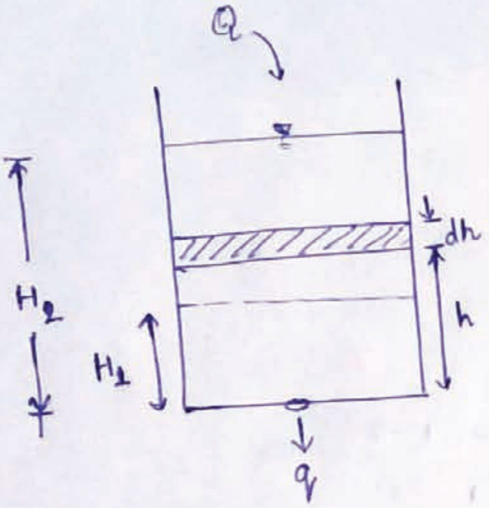
Now $$F_{r_2} = \frac{V_2}{\sqrt{g y_2}} = \frac{Q}{A_2\sqrt{g y_2}} = \frac{16.344}{20.625 \times \sqrt{9.81 \times 2.75}} = 0.1526$$

$$\therefore \frac{dy}{dx} = \frac{S_{f_1} - S_{f_2}}{1 - F_{r_2}^2} = \frac{\frac{1}{3000} - \frac{1}{7369.6}}{1 - (0.1526)^2} = 2.024 \times 10^{-4}$$

$\therefore$ Water surface slope = 1 in 4941.86

4(c)



Let an element of dh depth at a distance of h from bottom of tank

A = Area of tank (cross section)

a = crosssectional area of orifice.

Inlet − Outlet Volume = Rise in water level

$$\Rightarrow Q\,dt - q\,dt = A\,dh$$

$$\Rightarrow (Q - q)\,dt = A\,dh.$$

$$\Rightarrow \left(Q - C_d a\sqrt{2gh}\right) dt = A\,dh.$$

$$\Rightarrow dt = \frac{A\,dh}{\left(Q - C_d a\sqrt{2g}\sqrt{h}\right)}$$

$$\Rightarrow dt = \frac{A}{Q}\frac{dh}{\left(1 - \frac{C_d a\sqrt{2g}}{Q}\sqrt{h}\right)}$$

$\cancel{\text{Let}}$ $\cancel{\frac{C_d a\sqrt{2g}}{Q}} \cancel{= x}$

$$dt = \frac{\frac{A}{C_d a\sqrt{2g}} \, dh}{\left(\frac{Q}{C_d a\sqrt{2g}} - \sqrt{h}\right)}$$

Put $x^2 = h \Rightarrow 2x\,dx = dh$

$\therefore x = \sqrt{h}$

Limits : at $h = H_1$ $x = \sqrt{H_1}$

at $h = H_2$ $x = \sqrt{H_2}$

$$\therefore \quad dt = \frac{\frac{2A}{C_d a\sqrt{2g}} \, x\,dx}{\left(\frac{Q}{C_d a\sqrt{2g}} - x\right)}$$

Let $\frac{2A}{C_d a\sqrt{2g}} = \gamma$ and $\frac{Q}{C_d a\sqrt{2g}} = S$.

$$\therefore \quad dt = \frac{\gamma x\,dx}{(S-x)}$$

$$\int_{t_1}^{t_2} dt = \int_{\sqrt{H_1}}^{\sqrt{H_2}} \frac{\gamma x\,dx}{(S-x)}$$

$$\boxed{5}$$

$$\Rightarrow t_2 - t_1 = \gamma \int_{\sqrt{H_1}}^{\sqrt{H_2}} \frac{-(S-x) + S}{(S-x)} \, dx$$

$$\Rightarrow \Delta t = \gamma \int_{\sqrt{H_1}}^{\sqrt{H_2}} \left(\frac{S}{(S-x)} - 1\right) dx$$

$$\Rightarrow \Delta t = \gamma \left[S \frac{\ln(S-x)}{(-1)} - x \right]_{\sqrt{H_1}}^{\sqrt{H_2}}$$

$$\Rightarrow \Delta t = \gamma \left[S \ln\left(\frac{S-\sqrt{H_1}}{S-\sqrt{H_2}}\right) - \left(\sqrt{H_2} - \sqrt{H_1}\right) \right]$$

Now $\quad r = \dfrac{2A}{C_d a\sqrt{2g}} = \dfrac{A}{579.53}$

$$S = \frac{Q}{C_d a\sqrt{2g}} = \frac{Q}{289.77}$$

Putting we get

$$\Delta t = \frac{A}{579.53}\left[\frac{Q}{289.77}\ln\left(\frac{Q-289.77\sqrt{H_1}}{Q-289.77\sqrt{H_2}}\right)-\left(\sqrt{H_2}-\sqrt{H_1}\right)\right]$$

At $\Delta t = 107$ sec $\qquad H_2 = 0.76 \qquad H_1 = 0.6$.

$$\therefore\ 107 = \frac{A}{579.53}\left[\frac{Q}{289.77}\ln\left(\frac{Q-289.77\sqrt{0.6}}{Q-289.77\sqrt{0.76}}\right)-\left(\sqrt{0.76}-\sqrt{0.6}\right)\right] \quad \text{—(i)}$$

At $\Delta t = 120$ sec $\qquad H_2 = 1.28$ m $\qquad H_1 = 1.2$ m

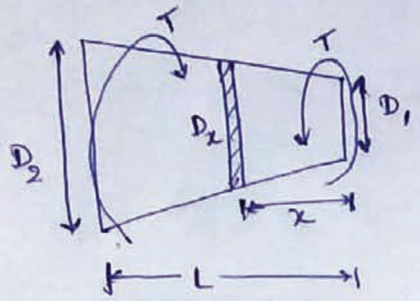
$$\therefore\ 120 = \frac{A}{579.53}\left[\frac{Q}{289.77}\ln\left(\frac{Q-289.77\sqrt{1.20}}{Q-289.77\sqrt{1.28}}\right)-\left(\sqrt{1.28}-\sqrt{1.2}\right)\right] \quad \text{—(ii)}$$

Solving (i) and (ii) we get

$$Q = \cancel{1.346\ m^3/s}\ \ 390\ m^3/s$$

$$A = 403352.88\ m^2$$

5(a)



Assuming an element of diameter D_x at a distance of x from smaller dia end..

Let $d\theta$ = rotation in element due to Torque T.

$$\therefore \quad d\theta = \frac{T\,dx}{G J_x}$$

$$\int_0^\theta d\theta = \frac{T}{G}\int_0^L \frac{dx}{\frac{\pi}{32} D_x^4}$$

Now
$$D_x = D_1 + \frac{D_2 - D_1}{L}x$$

$$\Rightarrow D_x = D_1 + Kx \qquad \text{where } k = \frac{D_2 - D_1}{L}$$

$$\therefore \quad \theta = \frac{T}{G} \times \frac{32}{\pi}\int_0^L \frac{dx}{(D_1 + kx)^4}$$

$$= \frac{32T}{\pi G}\left[-\frac{1}{(D_1+kx)^3 \times 3k}\right]_0^L$$

$$= \frac{32T}{\pi G}\left[\frac{1}{D_1^3} - \frac{1}{(D_1+kL)^3}\right] \times \frac{1}{3k}$$

$$= \frac{32T}{\pi G}\left[\frac{1}{D_1^3} - \frac{1}{D_2^3}\right] \times \frac{L}{3(D_2 - D_1)}$$

$$= \frac{32T}{\pi G}\left[\frac{D_2^3 - D_1^3}{D_1^3 D_2^3}\right] \times \frac{L}{3(D_2 - D_1)}$$

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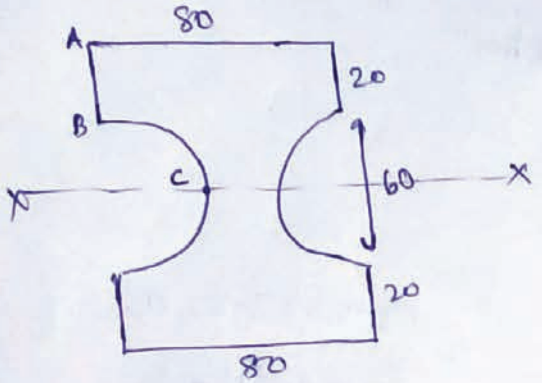
$$= \frac{32T}{\pi G}\left[\frac{D_2^2+D_1^2+D_1D_2}{D_1^3 D_2^3}\right] \times \frac{L}{3}$$

$$\therefore \text{ Angle of rotation} = \frac{32}{3\pi}\frac{TL}{G}\left[\frac{D_2^2+D_1^2+D_1D_2}{D_1^3 D_2^3}\right]$$

5(b)



$$I_{xx} = \frac{80 \times 100^3}{12} - \frac{\pi}{64} \times (60)^4$$

$$= 6.03 \times 10^6 \ mm^4$$

Shear force $S = 20 kN$

Shear stress at B :- (just above)

$$\tau = \frac{SA\bar{y}}{Ib} = \frac{20 \times 10^3 \times (80 \times 20) \times (30+10)}{6.03 \times 10^6 \times 80} = 2.653 \ N/mm^2$$

Shear stress at B (just below).

$$\tau = 2.653 \times \frac{80}{80} = 2.653 \ N/mm^2$$

Shear stress at C :

$$\tau = \frac{20 \times 10^3 \times (80 \times 50 - 0.5\pi (30)^2) \times \bar{y}}{6.03 \times 10^6 \times 20}$$

$$= 0.4289 \ \bar{y}$$

for $\bar{y}$

$$\bar{y} = \frac{80 \times 50 \times 25 - 0.5\pi \times 30^2 \times \frac{4 \times 30}{3\pi}}{(80 \times 50 - 0.5\pi \ 30^2)}$$

$$= 31.71 \ mm.$$

$$\therefore \tau = 0.4289 \times 31.71 = 13.6 \ N/mm^2$$

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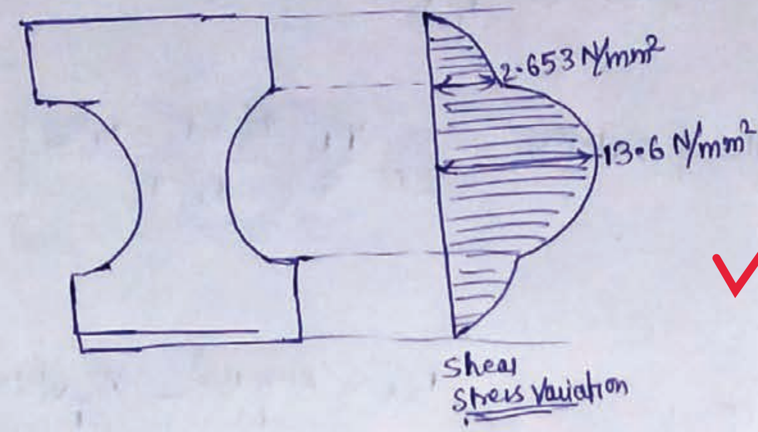
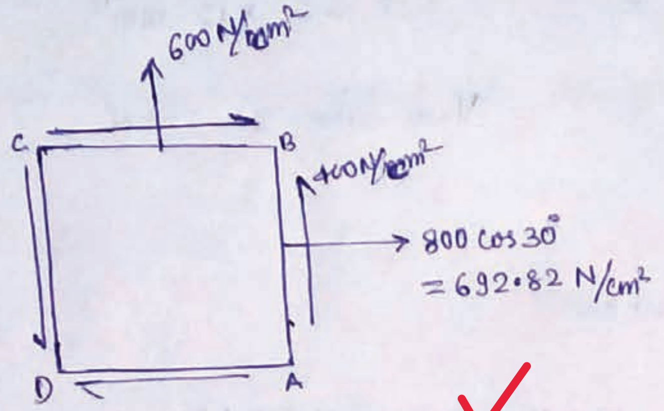


5(c)



$\sigma_x = 692.82 \text{ N/cm}^2$

$\sigma_y = 600 \text{ N/cm}^2$

$\tau_{xy} = 400 \text{ N/cm}^2$

Although tensor can not be resolved into horizontal and vertical component But since it is a 2D problem. so, we can resolve it into components.

(i)

$$\text{Resultant stress on plane BC} = \sqrt{\sigma_y^2 + \tau_{xy}^2}$$

$$= \sqrt{(600)^2 + (400)^2}$$

$$= 721.11 \text{ N/cm}^2$$

(ii)

$$\sigma_{1/2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1/2} = \left(\frac{692.82 + 600}{2}\right) \pm \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$\sigma_I = 1049.09 \text{ N/cm}^2 \quad \sigma_2 = 243.73 \text{ N/cm}^2$$

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$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 400}{692.82 - 600} = 8.62$$

$$\theta_p = 41.69^\circ \qquad 131.69^\circ$$

Check for orientation:-

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta_p + \tau_{xy} \sin 2\theta_p$$

$$= \frac{692.82 + 600}{2} + \frac{692.82 - 600}{2} \cos(2 \times 41.69) + 400 \sin(2 \times 41.69)$$

$$= 1049.09 \text{ N/mm}^2 = \sigma_{max}$$

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So, at $\theta = 41.69^\circ$ $\quad \sigma_{max} = 1049.09 \text{ N/mm}^2$

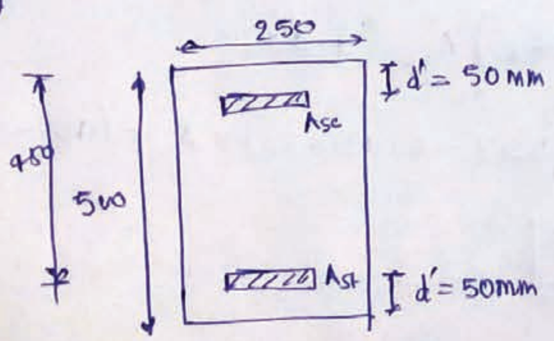
at $\theta = 131.69^\circ$ $\quad \sigma_{min} = 243.73 \text{ N/mm}^2$

(iii) $$\tau_{max} = \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2} = 402.68 \text{ N/mm}^2$$

Orientation $= 41.69^\circ + 45^\circ$ and $131.69 + 45^\circ$

$= 86.69^\circ$ and 176.69°

5(d)



$d = 450$ mm

$b = 250$ mm

$d' = 50$ mm.

$f_{ck} = 20$ MPa.

$f_y = 415$ MPa.

$f_{sc} = 353$ MPa

$$M_{u,lim} = 0.138 f_{ck} b d^2 = 0.138 \times 20 \times 250 \times 450^2$$

$$= 139.725 \text{ kN-m}$$

$\because$ $M_u = 225 \text{ kN-m} > M_{u,lim}$ $\Rightarrow$ Doubly reinforced beam required.

$\therefore$ For A_{st_1}:

$$M_{u,lim} = 0.87 f_y A_{st_1} \times [d - 0.42 x_{u,lim}]$$

$$\Rightarrow 139.725 \times 10^6 = 0.87 \times 415 \times A_{st_1} \times [450 - 0.42 \times 0.48 \times 450]$$

$$\Rightarrow A_{st_1} = 1077.14 \text{ mm}^2$$

For A_{st_2}:

$$M_u - M_{u,lim} = 0.87 f_y A_{st_2} (d - d')$$

$$\Rightarrow (225 - 139.725) 10^6 = 0.87 \times 415 \times A_{st_2} \times (450 - 50)$$

$$\Rightarrow A_{st_2} = 590.47 \text{ mm}^2$$

$$\therefore A_{st} = A_{st_1} + A_{st_2} = 1077.14 + 590.47 = \boxed{1667.61 \text{ mm}^2}$$

For A_{sc}:

$$M_u - M_{u,lim} = (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$$

$$\Rightarrow (225 - 139.725) \times 10^6 = (353 - 0.45 \times 20) \times A_{sc} \times (450 - 50)$$

$$\Rightarrow A_{sc} = \boxed{619.73 \text{ mm}^2}$$

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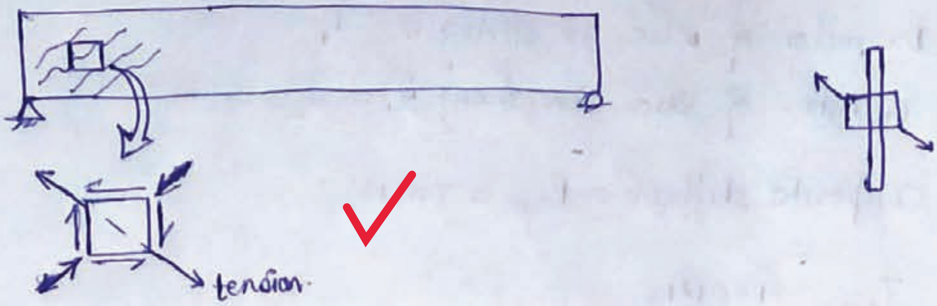
5(e) (i) When the shear force is in excessive amount, the diagonal tension get developed in concrete. Concrete being weak in excess tension get failed ~~despite~~ without the shear reinforcement in beam.



As shown in the figure, an element at neutral axis is in pure shear (at support). It causes tension in diagonal direction at an angle of 45°. This tension tries to impart crack in beam at 45° angle. To avoid such crack due to diagonal tension, shear reinforcement is provided vertically so that it can take the diagonal tension an protect the concrete from cracking. The spacing near support or heavy loading (concentrated) is less.

(ii)

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Limit state Method	Working stress Method
(i) less cross section area of beam.	(i) Large cross section area of beam.
(ii) Use of ultimate strength analysis. steel is allowed to stress upto ultimate.	(ii) Use of elasticity analysis method. steel can not be stress beyond elastic strength.
(iii) economical	(iii) uneconomical.
(iv) More ductile design	(iv) Less ductile design.
(v) strain diagram is mainly important to find stresses in different fibres	(v) Stress diagram is used to find stresses in different fibres
(vi) Mostly used now a days and effective in earthquake design.	(vi) Mainly used in water tank where high Factor of safety is required.

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7 (a)

No. of Coils $n = 12$

Mean diameter = 25 mm

Mean Radius $R = \frac{25}{2} = 12.5$ mm.

Diameter of wire in spring ① = d_1

Diameter of wire in spring ② = $d_2 = 2.5$ mm.

Composite stiffness = $k_{eq} = 700$ N/m $= 0.7$ N/mm

$\tau_{max} = 180$ MPa.

$G = 80$ MPa

Stiffness in spring ① $= k_1 = \frac{G d_1^4}{64 R^3 n} = \frac{80 \times 10^3 \times d_1^4}{64 \times (12.5)^3 \times 12} = \frac{4}{75} d_1^4$ N/mm

Stiffness in spring ② $= k_2 = \frac{G d_2^4}{64 R^3 n} = \frac{80 \times 10^3 \times (2.5)^4}{64 \times (12.5)^3 \times 12} = \frac{25}{12}$ N/mm

$\therefore \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$

$\Rightarrow \frac{1}{0.7} = \frac{75}{4 d_1^4} + \frac{12}{25}$

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$\Rightarrow \boxed{d_1 = 2.109 \text{ mm.}}$

For spring ①

$\tau_{max} = \frac{16 P R}{\pi d_1^3} \Rightarrow 180 = \frac{16 \times P \times 12.5}{\pi \times (2.109)^3} \Rightarrow P = 26.52$ N

For spring ②

$\tau_{max} = \frac{16 P R}{\pi d_2^3} \Rightarrow 180 = \frac{16 \times P \times 12.5}{\pi \times (2.5)^3} \Rightarrow P = 44.18$ N.

$\therefore$ Safe load = min^m {26.52 N, 44.18 N} = $\boxed{26.52 \text{ N}}$

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7 (b)

(I) Data:- Factored load $P_u = 1500$ kN

Clear Cover = 40mm.

$f_{ck} = 25$ MPa.

$f_y = 415$ MPa.

(II) Dimensions —

Assuming 2% longitudinal reinforcement

$A_c = 0.98\ A_g$.

$A_{sc} = 0.02\ A_g$

$A_g \rightarrow$ gross area

$\therefore\ P_u = 1.05\left[0.4 f_{ck} A_c + 0.67 f_y A_{sc}\right]$

$\Rightarrow 1500 \times 10^3 = 1.05\left[0.4 \times 25 \times 0.98\ A_g + 0.67 \times 415 \times 0.02\ A_g\right]$

$\Rightarrow\ A_g = 92999.8\ mm^2$

Provide 350 mm dia ~~helical~~ size of column.

$\therefore\ A_{sc} = 2\%$ of $\frac{\pi}{4} \times 350^2 = 1924.23\ mm^2$

Provide 6 – 22 mm dia longitudinal bar

(III) Helical reinforcement:

· Diameter of Core = D_c = $350 - 2 \times 40 = 270$ mm

Area of Core = $\frac{\pi}{4} \times D_c^2 = \frac{\pi}{4} \times 270^2 = 57255.5\ mm^2$

Volume of Core per metre length = $V_c = 57255.5 \times 10^3\ mm^3$

Assuming 8mm dia spiral.

Volume of spiral per metre length $= V_{hs} = \pi(350 - 2\times40 - 8)\times\frac{1000}{p}\times\frac{\pi}{4}\times8^2$

$$= \frac{4137 3381.65}{p}$$

Now

$$\frac{V_{hs}}{V_c} = 0.36\frac{f_{ck}}{f_y}\left[\frac{A_g}{A_c} - 1\right]$$

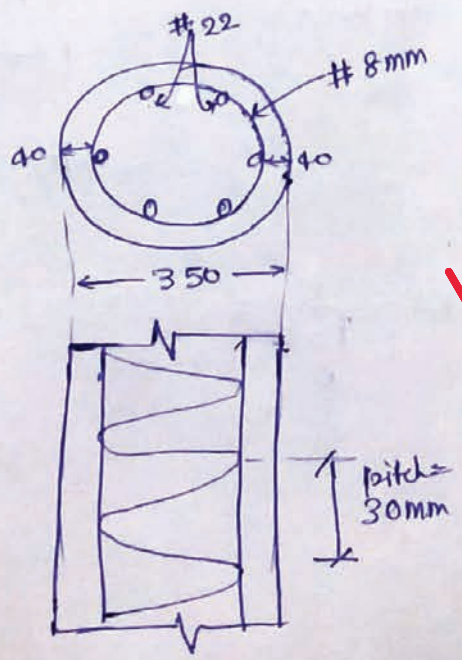
$$\Rightarrow \frac{41373381.65}{p\times57255.5\times10^3} = 0.36\times\frac{25}{415}\left[\frac{\frac{\pi}{4}\times350^2}{57255.5} - 1\right]$$

$$\Rightarrow p = 49 \text{ mm}$$

$$\text{Maximum pitch} = \min\begin{cases} 75\text{mm} \\ \frac{d_c}{6} = \frac{270}{6} = 45\text{ mm} \end{cases}$$

$$\text{Minimum pitch} = \max\begin{cases} 25\text{mm} \\ 3\phi_h = 3\times8 = 24\text{mm} \end{cases}$$

∴ pitch ⇒ Provide 8 mm ϕ spiral tie bar at the pitch of 30 mm



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7(c)

$$I_{xx} = 8.391 \times 10^6 + 2\left[\frac{120 \times 12^3}{12} + 120 \times 12 \times 81^2\right] = 27.32 \times 10^6 \text{ mm}^4$$

$$I_{yy} = 0.948 \times 10^6 + 2\left[\frac{\cancel{12} \times 120^3}{\cancel{12}}\right] = 4.404 \times 10^6 \text{ mm}^2$$

$\therefore I_{min} = 4.404 \times 10^6 \text{ mm}^2$

Area $A = 2167 + 2[120 \times 12] = 5047 \text{ mm}^2$

$$\therefore r_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{4.404 \times 10^6}{5047}} = 29.54 \text{ mm}$$

$$\text{Length effective} = L_e = \frac{L}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828 \text{ m}$$

$$\therefore \text{slenderness ratio} = \lambda = \frac{L_e}{r_{min}} = \frac{2828}{29.54} = 95.75$$

Now

Rankine Constant $= \alpha = \frac{1}{7500}$

Crushing stress $= f_c = 315$ MPa.

Using Rankines' formula: $P = \frac{f_c A}{1 + \alpha \lambda^2} = \frac{315 \times 5047}{1 + \frac{1}{7500} \times (95.75)^2}$

$= 715.35$ KN

$$\therefore \text{Safe Load} = \frac{P}{FOS} = \frac{715.35}{3.5} = 204.386 \text{ KN}$$

$\boxed{20}$