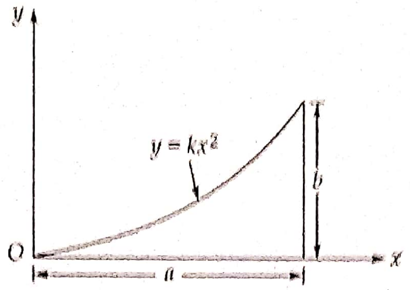
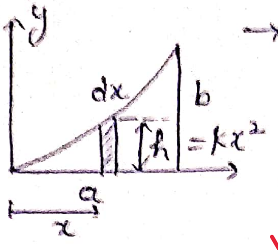
total marks= 238

Section A : Strength of Materials and Mechanics

Q.1 (a) For the given figure, find the ratio of a to b so that centroids, $\bar{x} = \bar{y}$?



[12 marks]



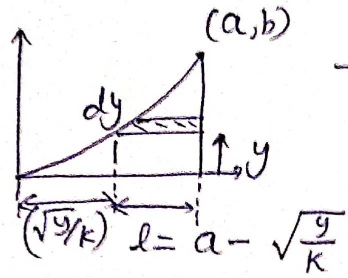
→ Let us consider a strip of dx thickness at a distance x from origin

→ centroid of the small strip is at a distance of x from origin.

→ Height of small area $= y = kx^2$, $dA = (kx^2)\,dx$

→ Let $\bar{x}$ and $\bar{y}$ be the centroid of whole area –

then $\bar{x} = \dfrac{\int_0^a x\,dA}{\int_0^a dA} = \dfrac{\int_0^a x\,kx^2\,dx}{\int_0^a kx^2\,dx} = \dfrac{k\,x^4/4}{k\,x^3/3} = \dfrac{3a}{4}$



→ Now taking a small area of dy thickness, at a distance y.

$$dA = l\,dy = \left(a - \sqrt{\tfrac{y}{K}}\right)dy$$

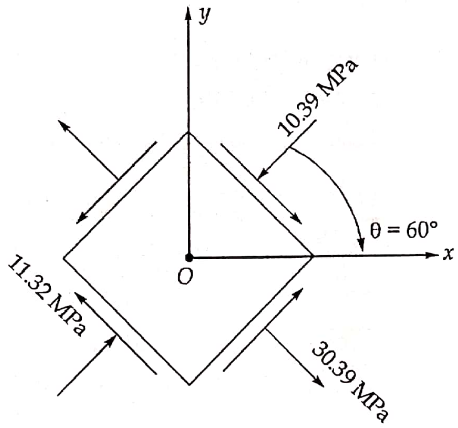
12

$$\bar{y} = \frac{\int_0^b y\,dA}{\int_0^b dA} = \frac{\int_0^b y\left(a - \sqrt{\frac{y}{K}}\right)dy}{\int_0^b \left(a - \sqrt{\frac{y}{K}}\right)dy} = \frac{\left[\frac{a y^2}{2} - \frac{1}{\sqrt{K}}\frac{y^{2.5}}{2.5}\right]_0^b}{\left[ay - \frac{1}{\sqrt{K}}\frac{y^{1.5}}{1.5}\right]_0^b}$$

$\bar{y} = \dfrac{\frac{ab^2}{2} - \frac{b^{2.5}}{2.5\sqrt{K}}}{ab - \frac{b^{1.5}}{1.5\sqrt{K}}}$ and $b = Ka^2 \Rightarrow \bar{y} = \dfrac{\frac{ab^2}{2} - \frac{b^{2.5}}{2.5(b^{0.5}/a)}}{ab - \frac{b^{1.5}}{1.5(b^{0.5}/a)}}$

solving we get $\bar{y} = \dfrac{3b}{10}$, $\bar{x} = \bar{y} \Rightarrow \boxed{\dfrac{a}{b} = 0.4}$ * Ans.

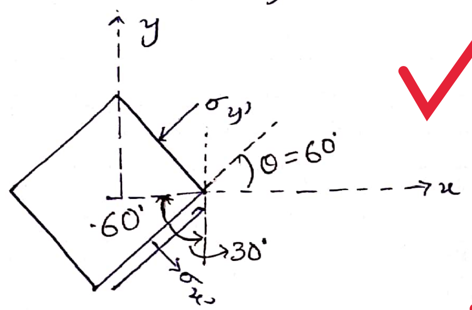
Q.1 (b) An element in plane stress is rotated through a known angle θ. On the rotated element, the normal and shear stresses have the magnitudes and directions as shown in the figure. Determine the normal and shear stresses on the element whose sides are parallel to *x*, *y*-axis. Also find the principal stresses.



Given —

[12 marks]

$\sigma_{x'} = 30.39\,MPa$, $\sigma_{y'} = -10.39\,MPa$, $\tau_{x'y'} = -11.32\,MPa$ (C.C.W)



Assuming x' and y' as original refrence plane axes —

$$\sigma_n = \frac{\sigma_{x'}+\sigma_{y'}}{2} \cancel{\cos 2\theta} + \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$\tau_s = -\left(\frac{\sigma_{x'}-\sigma_{y'}}{2}\right)\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$\theta = -30°$ (ccw) for plane $\parallel^{l}$ to x and y axes -

$$\sigma_x = 10 + 20.39\cos(-60°) - 11.32\sin(-60°) = \boxed{30\,MPa = \sigma_x}$$

$$\tau_s = -(20.39)\sin(-60°) - 11.32\cos(-60°) = \boxed{12\,MPa = \tau_s}$$

6

for σ_y $\theta = +60°$ (CW).

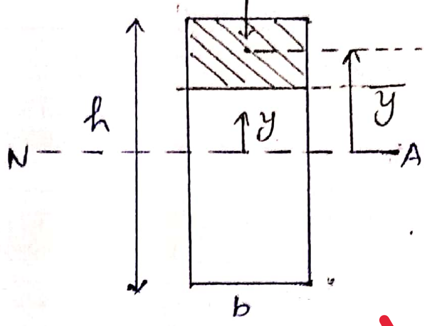
$$\boxed{\sigma_y = 10\,MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\frac{(\sigma_x-\sigma_y)^2}{4} + \tau_{xy}^2} = \frac{30+10}{2} \pm \sqrt{\frac{(30-10)^2}{2^2} + 12^2}$$

$$\sigma_{1,2} = 20 \pm 15.62 \quad \boxed{\sigma_1 = 35.62\,MPa} \quad \boxed{\sigma_2 = 4.38\,MPa}$$

Q.1 (c) A beam of rectangular cross-section of width b mm and depth h mm. This cross-section carries a transverse load of P Newton. Show that the shear stress variation along the depth of beam is parabolic and its maximum value is at the center with magnitude 1.5 times of average shear stress in the cross-section.

[12 marks]



Given -

Transvers load = P

average shear stress $= \frac{P}{A}$

$\tau_{avg} = \frac{P}{b \times h}$

We know that shear stress at a location y from NA is given by $\tau = \frac{P}{I}\frac{A\bar{y}}{b}$

where I = area Moment of Inertia about NA $= \frac{bh^3}{12}$

A = area above $y = b \times \left(\frac{h}{2} - y\right)$ [Shaded area)

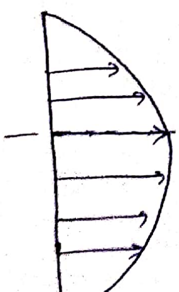
$\bar{y}$ = centroid of shaded area $= \frac{y + \left(\frac{h}{2} - y\right)}{2} = \frac{y + \frac{h}{2}}{2}$

b = width of area (A)

Parabolic eq^n

$$\tau = \frac{P}{\frac{bh^3}{\cancel{12}\,6}} \frac{\cancel{b}\left(\frac{h}{2} - y\right)}{\cancel{b}} \frac{\left(\frac{h}{2} + y\right)}{\cancel{2}} = \boxed{\frac{6P}{bh^3}\left(\frac{h^2}{4} - y^2\right) = \tau}$$

at centre $\tau_{NA} = \frac{6P}{bh^3}\left(\frac{h^2}{4} - 0\right) = \frac{3P}{2 \times bh} = \boxed{1.5\,\tau_{avg.} = \tau_{NA}}$



at top $y = \frac{h}{2}$, $\rightarrow \tau = 0$

$\tau_{max} = \tau_{NA} = 1.5\,\tau_{avg.}$

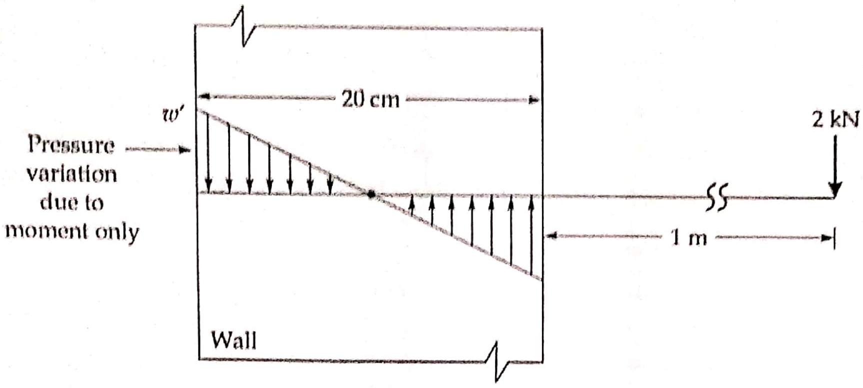
12

at Bottom $y = -\frac{h}{2}$, $\rightarrow \tau = 0$

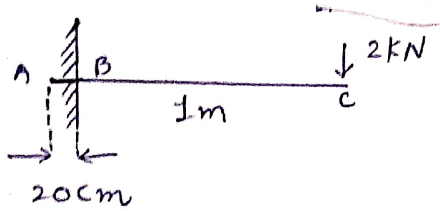
→ from the above equations it is clear that shear stress distribution is parabolic.

Do not write in this margin

Q.1 (d) A cantilever beam of span 1 m is built into a wall of 20 cm thickness. The beam carries a concentrated load of 2 kN at the free end. Assume that the pressure exerted by the wall on the beam due to moment is linearly varying along the 20 cm thickness of the wall into which the beam is embedded as shown in figure. Find the pressure intensity in kN/m by the wall on the beam due to moment and reaction.

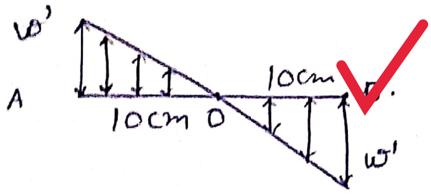


[12 marks]

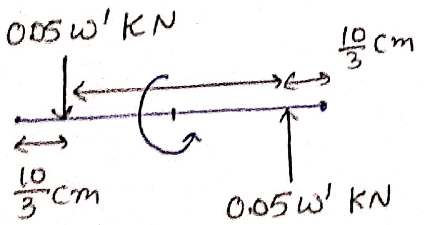


→ AB is embeded into wall.

→ let Pressure intensity be w' $\frac{KN}{m}$ and it is acting linearly with Max^m Magnitude of w' at A and B.



→ It can be considered on centroid of triangles and converting it into forces of Magnitude = area.



area $= \frac{1}{2} \times w' \times \frac{10}{100} = 0.05\, w'$ KN

5 Moment created $= M = 0.05 w' \times \frac{40}{3 \times 100}$

$M = \frac{20}{3} w'$ N-m

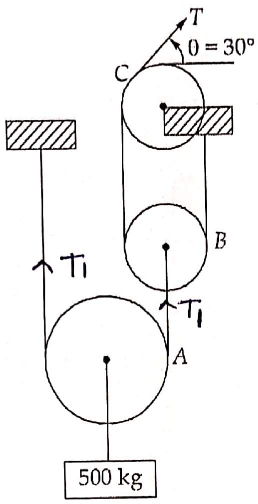
Due to concentrated load of 2 KN Moment created

$= 2 \times 1000 \times 1 = 2000$ N-m

this Moment is resisted by wall only $= 2000 = \frac{20}{3} w'$

$\boxed{w' = 300\ kN/m}$ $\boxed{M_{reaction} = 2000\ N\text{-}m}$ (CW) $\boxed{R = 2\ KN \uparrow}$

Q.1 (e) Calculate the tension T in the cable which supports the 500 kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing and the weights of all parts are small compared with the load. Find the magnitude of total force on the bearing of pulley C.



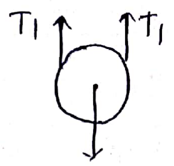
[12 marks]

assumptions-

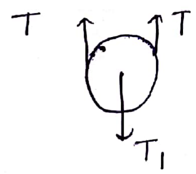
– pulleyes are frictionless and Massless

→ asssuming tension in the first rope be T_1

then $2T_1 = 500g \Rightarrow T_1 = 2452.5$ N



pulley A

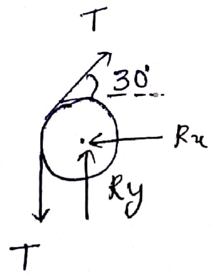


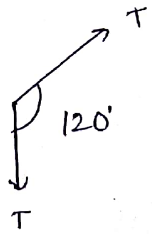
$\Rightarrow 2T = T_1$

$\boxed{T = 1226.25\ N}$

Pulley B

Pulley C →





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total Reaction R is given by = $R = \sqrt{T_1^2 + T_2^2 + 2T_1T_2\cos\theta}$

$T_1 = T_2 = T, = 1226.25N, \quad \theta = 120°$

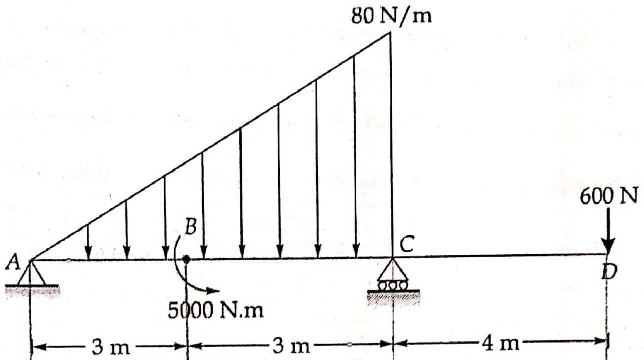
$R = \sqrt{T^2 + T^2 + 2T^2\cos(120°)} = T\sqrt{2(1+\cos 120°)} = T$

$\boxed{R = 1226.25N}$

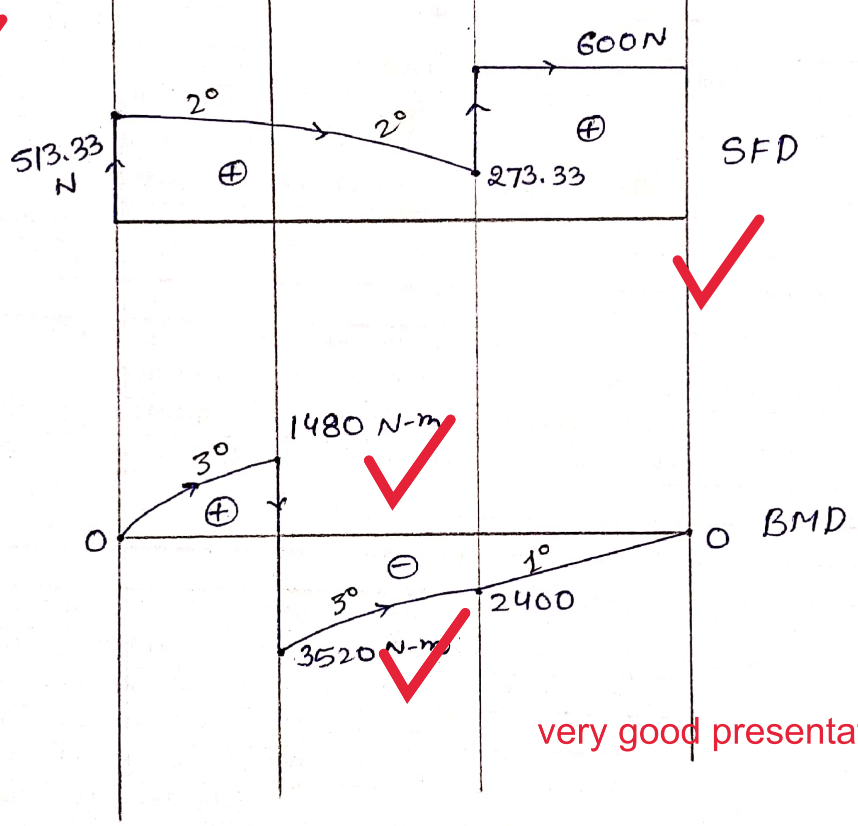
Q.2 (a) A beam ABCD is loaded as shown in figure given below. The beam is of square cross section 100 mm × 100 mm.

(i) Sketch the S.F. and B.M. diagram for the beam.

(ii) Determine the maximum bending stress in the beam.



[15+5 marks]



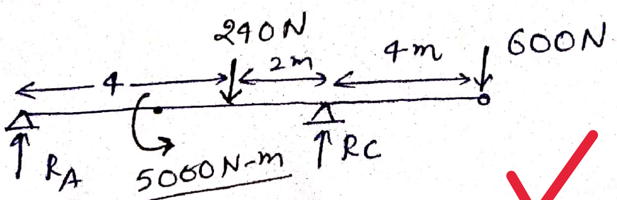
very good presentation

Let Reactions at A and C be R_A and R_C –

Taking Moments about point A → $M_A = 0$

Uniformly varying load can be considered on centroid of triangle whose magnitude = area of Δ.

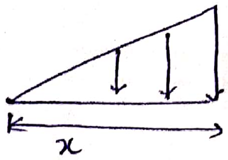
$= \frac{1}{2} \times 6 \times 80 = 240N$



$R_C \times 6 + 5000 - 240\times 4 - 600 \times 10 = 0$

$R_C = 326.67 N$

$R_A + R_C = 600 + 240 = 840 N$, $R_A = 513.33 N$



— Shear force at a distance x from A is given By.

(AC)

$(SF)_x = R_A - \frac{1}{2}\times x \times \frac{80}{6}x = 513.33 - 6.67x^2$ $x < 6m$

for CD portion → $(SF)_x = 513.33 - 240 + 326.67$
$= 600 N$

for $SF_x = 0$ in AC portion → $x = \sqrt{\frac{513.33}{6.67}} = \underline{8.7727m > 6}$

Hence $SF_x \neq 0$ in AC portion

$(SF)_C = 273.33 N$

BM in portion → AB ⇒ $M_x = R_A \times x - \frac{1}{2}\times x \times \frac{80x}{6} \times \frac{x}{3}$

$M_x = 513.33x - 2.22x^3$

$M_B \Rightarrow (x = 3m) = 1480.05 N\text{-}m$

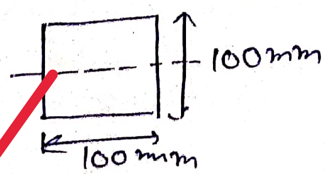
$M_x \Rightarrow (BC) = 513.33x - 2.22x^3 - 5000$

$M_C = (x = 6m) \Rightarrow -2400 N\text{-}m$

→ $M_{max} = -3520 N\text{-}m$ (at B).

$\sigma_{b,max} = \frac{M_{max}}{Z} = \frac{3520 \times 10^3 N\text{-}mm}{100 \times \frac{100^2}{6}} = 21.12 MPa$

$Z = \frac{bd^2}{6}$



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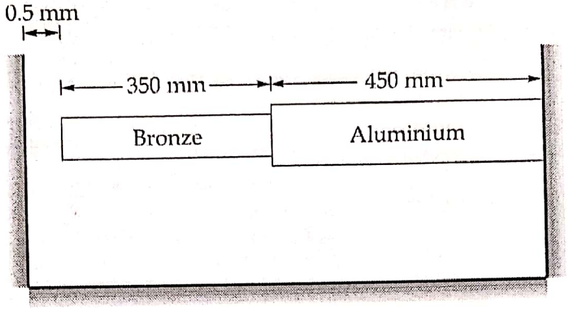
Max^m Bending stress = $\boxed{21.12 MPa}$

Q.2 (b) Knowing that a 0.5 mm gap exists when the temperature is 75°F. Determine the forces in the bars shown after a temperature rise of 180°F and the corresponding change in length of the bronze bar.



Bronze :	Aluminium :
$A = 1500\ mm^2$	$A = 1800\ mm^2$
$E = 103.42$ GPa	$E = 73.08$ GPa
$\alpha = 12 \times 10^{-6}/°F$	$\alpha = 12.9 \times 10^{-6}/°F$

[20 marks]

Given →

$\Delta T_{rise} = 180° F$

Increase in length due to temperature rise ⇒

Bronze Bar → $\Delta l = l \alpha \Delta T$

$= 350 \times 12 \times 10^{-6} \times 180 = 0.756\ mm$

Aluminium Bar $\Delta l = 450 \times 12.9 \times 10^{-6} \times 180 = 1.0449\ mm$

Since change in length is greater than gap 0.5mm

Both Bars will be compressed due reaction forces.

Let P be the reaction force - P (Newtons)

then Bronzle Bare $\Delta l = \frac{-Pl}{AE} = \frac{-P \times (350)}{1500 \times (103.42) \times 10^3}$

$\Delta l\ (aluminium) = \frac{-P \times 450}{1800 \times 73.08 \times 10^3} = -3.421 \times 10^{-3} \times P\ mm$

20

$(\Delta l)_{Bronze} = -2.256 \times 10^{-3} \times P\ mm$

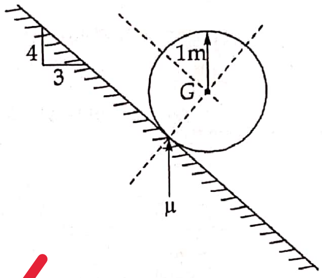
$(\Delta l)_{temp} - (\Delta l)_{force} = (\Delta l)_{gap}$

$0.756 + 1.0449 - (2.256 + 3.421) \times 10^{-3} P = 0.5$

$\boxed{P = 229.15 N}$ $\quad \Delta l_{bronze} = 0.756 - 2.256 \times 0.22915 =$

$\boxed{\Delta l_b = 0.239\ mm}$

Q.2 (c) The given sphere of radius 1 m and weight 160 N rests on an inclined plane. If the friction between the plane and sphere is (μ = 0.5). Find the magnitude and directions of the angular acceleration of sphere, the acceleration of its mass center and the friction force between the sphere and the plane. If the friction between the plane and sphere changes to (μ = 0.25). What will be the linear acceleration and angular acceleration of sphere? (Assume $g = 10\ m/s^2$)



[20 marks]

Given–

case-(i) $\mu = 0.5$

$R = 1m$

$wt = 160N$

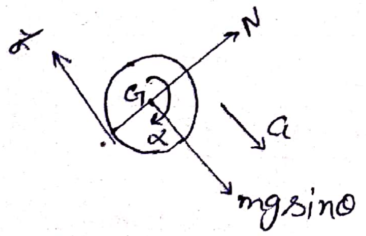
$\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

$m = \frac{wt}{g} = 16\ kg$, Moment of Inertia about $CG = \frac{2}{5}mR^2$

$I_{sphere} = \frac{2}{5} \times 16 \times 1^2 = 6.4\ kg\text{-}m^2$

Normal Reaction $= N = mg\cos\theta = 160 \times \frac{3}{5} = 96N$

Max^m value of friction force $= f_{max} = \mu N = 48N$



FBD.

→ Let angular acceleration of sphere about its centre be α and Linear acceleration $= a$.

→ let friction force be $= f$

→ Writing Force balance eq^n and Moments about G.

$mg\sin\theta - f = ma$

$160 \times \frac{4}{5} - f = 16 \times a$ — (i)

$f \times R = I\alpha \Rightarrow f \times 1 = 6.4 \times \alpha$ — (ii)

→ Assuming sphere is rolling without slipping on inclined plane –

$a = \frac{\alpha}{R} = \frac{\alpha}{1} = \alpha$

and solving eq^n – (i) and (ii) with $\alpha = a$ Gives–

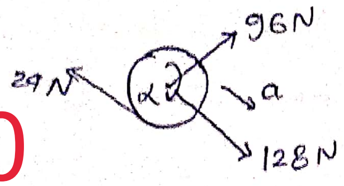
$\boxed{f = 36.57\ N}$ and $\boxed{a = \frac{40}{7}\ m/sec^2}$, $\boxed{\alpha = \frac{40}{7}\ rad/sec^2}$

f is less than f_{max} (48 N), Means our assumption was correct and sphere rolls without slipping.

→ sphere will slide only if friction force is insufficient to prevent sliding, But in this case friction is sufficiently large to prevent sliding.

Case – (ii) $\mu = 0.25$ $f_{max} = \mu N = 24\ N$

In this case, since friction required for sliding is 36.57 N and f_{max} value is 24 N, it will slide and friction force acting will be 24 N.



writing force and Moment eq^{n}s –

$128 - 24 = 16 \times a$

$\boxed{a = 6.5\ m/sec^2}$

$T = I\alpha$ Gives → $24 \times 1 = 6.4 \times \alpha$

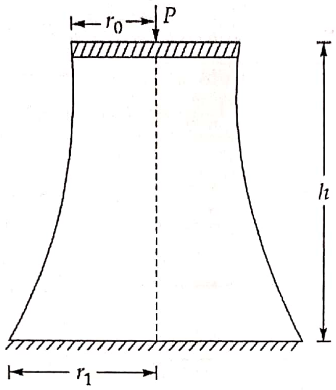
$\boxed{\alpha = 3.75\ rad/sec^2}$

20

very good

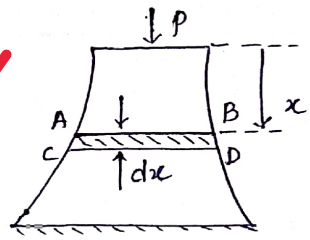
Q.3 (a) A pillar of varying circular cross-section carry a load of P Newton and having uniform stress of σ throughout the pillar. γ is the specific weight of the material and h is the height of the pillar. Prove that volume of the pillar of uniform allowable stress is

$$V = \frac{P}{\gamma}\left[\exp\left(\frac{\gamma h}{\sigma}\right) - 1\right]$$



[20 marks]

Consider an element of small thickness dx at a distance x from top.



→ Let A_x be the area of c/s at distance x and A_{x+dx} be the area of c/s at distance $(x+dx)$ from top.

→ Volume of small area ~~and~~ thickness dx = $A_x\,dx$

→ weight of this element = $\gamma A_x\,dx$ (ABCD)

→ Let V_x be the volume above this element (AB) and V_{x+dx} be the volume above element whose height is $(x+dx)$ from top. (above CD)

→ Load acting on this element = $P + V_x \cdot \gamma$ (AB)

→ Load acting a on lower element $(x+dx)$ element = $P + V_{x+dx}\,\gamma$ (CD)

→ Let σ_x be the stress induced in element AB and σ_{x+dx} be the stress induced in element CD.

$$\sigma_x = \frac{P + \gamma V_x}{A_x} \quad , \quad \sigma_{x+dx} = \frac{P + \gamma V_{x+dx}}{A_{x+dx}}$$

But σ is uniform throughout the pillar $\rightarrow \sigma_x = \sigma_{x+dx} = \sigma$

$$A_x = \frac{P + \gamma V_x}{\sigma} \quad , \quad A_{x+dx} = \frac{P + \gamma V_{x+dx}}{\sigma}$$

But $A_{x+dx} = A_x + dA_x$ and $V_{x+dx} = V_x + A_x dx$

$$A_x + dA_x = \frac{P + \gamma V_x + \gamma A_x dx}{\sigma} = \frac{P + \gamma V_x}{\sigma} + \frac{\gamma A_x dx}{\sigma}$$

$$dA_x = \frac{\gamma A_x dx}{\sigma}$$

$$\int_{A_0}^{A_x} \frac{dA_x}{A_x} = \int_0^x \frac{\gamma}{\sigma} dx$$

$$(\ln A_x)_{A_0}^{A_x} = \frac{\gamma}{\sigma}(x-0)$$

$$\ln \frac{A_x}{A_0} = \frac{\gamma}{\sigma} x \Rightarrow A_x = e^{\frac{\gamma}{\sigma} x} \cdot A_0$$

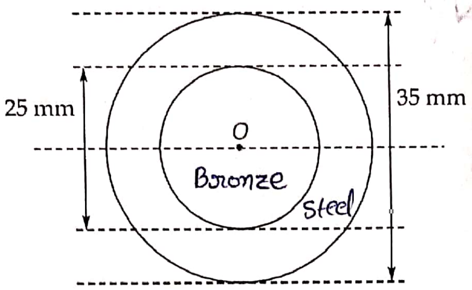
20 Volume of the pillar $= \int_0^h A_x dx = \int_0^h A_0 e^{\frac{\gamma}{\sigma} x} dx$

$$V = \frac{A_0}{\left(\frac{\gamma}{\sigma}\right)} \left[e^{\frac{\gamma}{\sigma} x}\right]_0^h = \frac{A_0 \sigma}{\gamma} \left[e^{\frac{\gamma h}{\sigma}} - e^0\right]$$

stress at top fiber $= \sigma = \frac{P}{A_0} \Rightarrow P = A_0 \sigma$

Volume $= \boxed{V = \frac{P}{\gamma} \left[e^{\left(\frac{\gamma h}{\sigma}\right)} - 1\right]}$ * Hence Proved

Q.3 (b) A solid rod of bronze 25 mm in diameter is surrounded by a fitting steel cylinder of external diameter 35 mm. If the permissible bending stresses in bronze and steel are 90 N/mm^2 and 120 N/mm^2 respectively, find the moment of resistance of composite section. Young's modulus for steel may be taken as 1.75 times that of bronze.



[20 marks]

Assuming that composite section will act like a single c/s in Bending →

$$I_{Bronze} = \frac{\pi d_i^4}{64} = \frac{\pi (25)^4}{64} = 19174.8 \text{ mm}^4$$

$$I_{steel} = \frac{\pi (d_o^4 - d_i^4)}{64} = \frac{\pi (35^4 - 25^4)}{64} = 54487 \text{ mm}^4$$

$y_{max,B} = \frac{25}{2} = 12.5 mm$, $y_{max,steel} = \frac{35}{2} = 17.5 mm$

$E_{steel} = 1.75\, E_{Bronze}$, $\sigma_{b,max} = 90$ MPa, $\sigma_{s,max} = 120$ MPa

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

R = Radius of Curvature

→ R will be same for both Bronze and steel, since section is behaving as composit section.

case-(1) → Assuming Bronze will Reach upto Maximum bending stress limit. $\sigma_{b,max} = 90$ MPa

$$\frac{M_B}{19174.8} = \frac{90}{12.5} \Rightarrow M_B = 138.06 \text{ N-m}$$

$$\frac{1}{R} = \frac{M_B}{I_B E_B} = \frac{M_s}{I_s E_s}$$

$$\frac{138.06}{19174.8 \times \cancel{1.75} E_s/1.75} = \frac{M_s}{54487 \times E_s} \Rightarrow M_s = \cancel{224.18} \; 686.55 \text{ N-m}$$

$$\frac{\sigma_{b,s,max}}{y_{max}} = \frac{M_s}{I_s} = \frac{\cancel{224.18}\; 686.55 \times 10^3 \text{ N-mm}}{54487 \text{ mm}^4} \Rightarrow \sigma_{b,max,s} = \cancel{72.0 \text{ MPa}} \; 220.5 \text{ MPa}$$

Max^m stress induced in steel section $= \cancel{70}\ 220$ MPa $\not< 120$ MPa.
It is a unsafe condition. Hence it is Not possible.
and total Moment of Resistance $= M_B + M_S = \boxed{\cancel{362.24 N\text{-}m}}$

case-(ii) $\quad \sigma_{max,s} = 120$ MPa

$$\frac{M_S}{54487} = \frac{120}{17.5} \rightarrow M_S = 373.62\ N\text{-}m$$

$$\frac{M_S}{I_S E_S} = \frac{M_B}{I_B E_B} \longrightarrow M_B = \frac{373.62 \times 19174.8}{54487} \times \frac{E_B}{1.75 E_B} = 75.13\ N\text{-}m$$

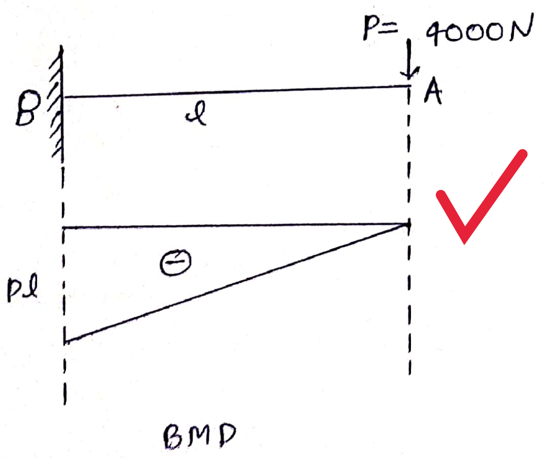
20

$$\frac{M_B}{I_B} = \frac{\sigma_{max}}{y_{max}} \Rightarrow \sigma_{max} = \frac{75.13 \times 10^3\ N\text{-}mm}{19174.8} \times 12.5 = 49\ MPa$$

$\sigma_{b,max,\ Bronze} = 49 MPa < 90 MPa \rightarrow$ Hence it is safe condn.

total Resistance Moment $= M_B + M_S = \boxed{448.75\ N\text{-}m}$ **

Q.3 (c) A cantilever of uniform strength is to be turned from a mild steel bar of 50 mm diameter. A load of 4000 N is to be supported from the free end, and the maximum bending stress is limited to 70 N/mm². Determine the maximum length of the cantilever and its end deflection if E = 205,000 N/mm². **[20 marks]**



Given–

$\sigma_{b,max} = 70$ MPa

$E_{ms} = 205$ GPa

$d = 50$ mm

Let length of Beam $= l$ m.

$P = 4000$ N

$$I_{NA} = \frac{\pi d^4}{64} = \frac{\pi (50)^4}{64} = 306796.15\ mm^4$$

Maximum Bending Moment will be at free end-

$M_{max} = P \times l$

Do not write in this margin

$$\frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \sigma_{b,max} = \frac{M_{max} \cdot y_{max}}{I_{NA}}$$

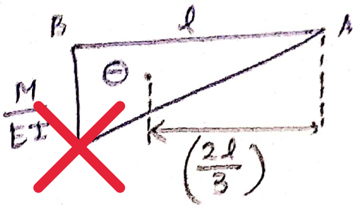
$$70 = \frac{(4000 \times l) \times 1000 \text{ N-mm} \times 25 \text{ mm}}{306796.15}$$

$\boxed{l = 0.21475 \text{ m}}$ Max^m length of the Beam

Deflection at freeend (A) can be found by eq^n-

$$\Delta_A = \cancel{\Delta_B}^{0} + \cancel{\theta_B}^{0} \cdot AB + \Delta_{A/B}$$

$\Delta_{A/B}$ = first Moment of area of $\frac{M}{EI}$ diagram between A and B, assuming A as origin. $= A\bar{x}$



$$\Delta_{A/B} = \frac{1}{2} \cdot \frac{M}{EI} \cdot l \times \frac{2l}{3} \quad (\text{mm})$$

$$= \frac{1}{2} \times \frac{4000 \times l^3 \times 2/3}{205 \times 10^3 \times 306796.15}, \quad l = 214.75 \text{ mm}$$

$\Delta_{A/B} = 0.21 \text{ mm}$

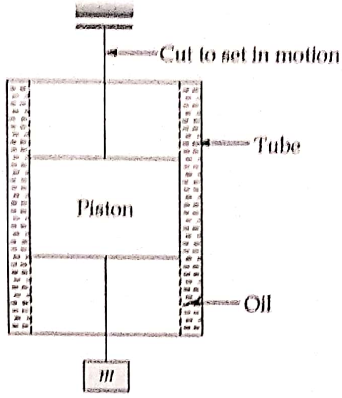
Deflection at free end $= \boxed{\delta_A = 0.21 \text{ mm}}$ **

3

moment of inertia will vary

Section B : F.M. and Turbo Machinery-1; HMT-2 + Theory of Machines-2

Q.5 (a) (i) A 73 mm diameter aluminium (SG = 2.64) piston of 100 mm length resides in a stationary 75 mm inner diameter steel tube lined with oil at 25°C of viscosity 0.13 Pa-s. A mass $m = 2$ kg is suspended from the end of the piston. The piston is set into motion by cutting a support cord. What is the terminal velocity of mass m? Assume a linear velocity profile within the oil.



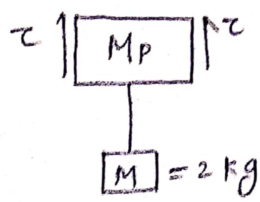
(ii) Assume you are performing an experiment and you intend to gently place several steel needles on the free surface of the water in a large tank. The needles come in two lengths. Some are 5 cm long and some are 10 cm long. Needles of each length are available with diameter of 1 mm, 2.5 mm and 5 mm. Make a prediction as to which needles, if any will float. Given specific gravity of steel is 7.83 and surface tension of water is 0.0728 N/m.

[6 + 6 = 12 marks]

(i) Mass of the piston $= m_p = \frac{\pi d_i^2}{4} \times l \times \rho$

$$= \pi \frac{(0.073)^2}{4} \times (0.1) \times 2.64 \times 1000 = 1.105 \text{ kg.}$$

→ considering piston and mass (m) to be a system moving with terminal velocity v, and at terminal velocity acceleration of system = 0.



thickness of film = $t = \frac{D_o - D_i}{2}$

Given → $\mu = 0.13$ Pa-sec.

and assuming linear velocity profile within oil.

$$\tau_w = \mu \frac{du}{dy} = 0.13 \times \frac{(v - 0)}{\left(\frac{75-73}{2 \times 1000}\right)} = 130 v \ \frac{N}{m^2}$$

Balancing Force ⇒ $F_{viscous} = F_{gravity}$

$$130 v \times \pi d_i l = (m + m_p) g$$

$$v = \frac{(2 + 1.105) \times 9.81}{\pi (0.073) \times (0.1) \times 130} = \boxed{10.21 \text{ m/sec.}}$$ *

(ii) Given –

Needles length = 5 cm, 10 cm

diameter = 1 mm, 2.5 mm, 5 mm

S.G. steel = 7.83

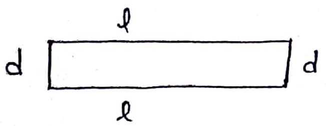
$\sigma = 0.0728\ N/m$

weight of each Needle $= \frac{\pi d^2}{4} \times l \times 7.83 \rho_w \times g$

Max^m Bouyouncy Force $F_B = \frac{\pi d^2}{4} \times l \times \rho_w \times g$

Max^m force due to surface tension $= (\sigma L \cos\theta)_{max}$

$= \sigma$ (Perimeter length)$_{max}$



$l_{max} = 2(l+d)$

For Needles to float $F_B + F_{ST} \geq F_{wt}$

$$\frac{\pi d^2}{4} l \rho_w g + \sigma \times 2(l+d) \geq \frac{\pi d^2}{4} \times l \times 7.83 \rho_w g$$

$$2\sigma(l+d) \geq \frac{\pi d^2}{4} \times l \times 6.83 \rho_w g$$

$l >> d \rightarrow$

$$2 \times 0.0728\ \cancel{l} \geq \frac{\pi}{4}(d^2) \times \cancel{l} \times 6.83 \times 1000 \times 9.81$$

12 $$d^2 \leq 2.766 \times 10^{-6}$$

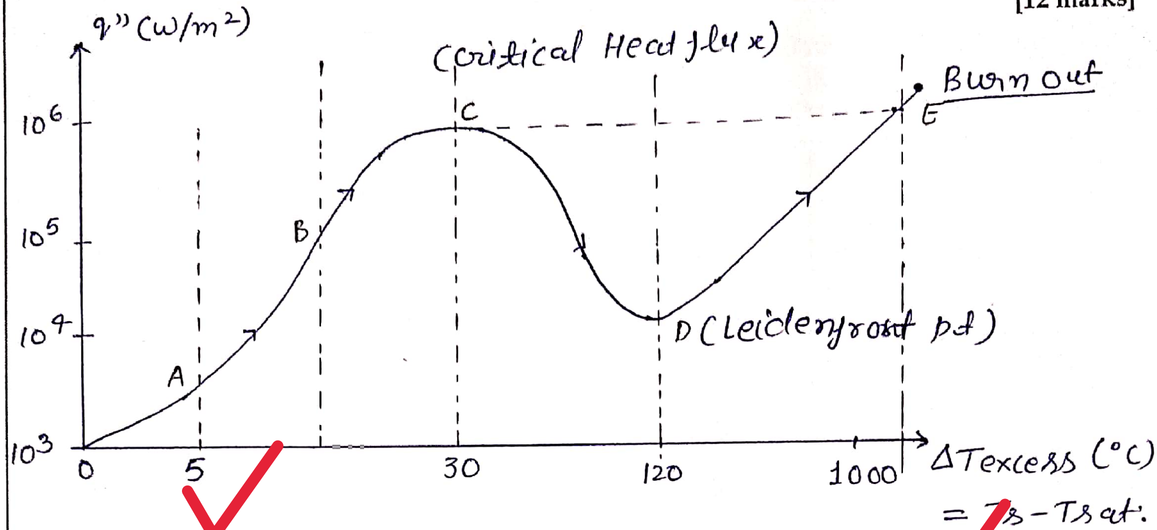
$$\boxed{d \leq 1.6633\ mm}$$

Hence Needles of 1 mm diameters will float in optimum conditions, of any length such that $l >> d$.

good approach

Q.5 (b) Draw the boiling curve and identify the different boiling regimes for water. Also, explain the characteristics of each regime.

[12 marks]



→ above figure shows pool Boiling or Natural Boiling of stationary fluids such as water.

→ x-axis represents temp. difference between surface and saturation temp at a particular pressure. and y-axis represents heat flux through the surface to stationary fluid.

OAB Region → (OA) It is Natural convection Boiling region. (AB) In this region bubbles forming due to vaporization of water quickly collapse and do not rise upto free surface of water.

AB Region → In this region Nucleate Boiling takes place and point A is called ONB (onset of nucleate Boiling). and q'' is very small

BC Region – In this region bubbles formed rise upto free surface. Het transfer rate increases upto satisfactory level. This is Most desirable region for boiling. As Bubbles travel upto, it increases convective heat transfer rate. Maximum heat transfer occurs at pt C.

Region CD → It is called transistion boiling. In this region both film boiling and nucleate boiling occurs partially. Heat transfer rate decreases because excess formation of liquid to vapour bubbles, surface is covered with more bubbles, which decreases q'' due to very less convection/conduction.

12

Region (D-E) – In region DE film Boiling takes place. In this region Radiation heat flux dominates due to very high surface temp. and heat flux increases. But it is unstable region and Not-desirable and Burn-out can take place in this region due to very high temp. of surface.

Q.5 (c) A cam of circular arc type is to operate a flat-faced follower of a four-stroke engine. The exhaust valve opens 50° before top dead centre and closes 15° after bottom dead centre. The valve lift is 10 mm, base circle radius of cam is 20 mm and nose radius is 3 mm. Calculate the maximum velocity, acceleration and retardation, if cam rotates at 1800 rpm.
Write down the parameters which defines the size of cam.

[10+2 = 12 marks]

For circular arc type Cam –

displacement is given by –

when follower is on flank

$x = (R - r_1)(1 - \cos\theta)$

where R = radius of circular flank

r_1 = Base circle radius r_2 = Nose Radius

2

when follower is on Nose –

$x = (r_2 - r_1) + L\cos(\alpha - \theta)$

→ the size of the cam is specified by its Base circle radius, which is minimum radius of circle which touches the cam profile drawn from centre of Rotation.

Q.5 (d) A centrifugal pump having three stages in parallel delivers 360 m^3 of water per hour, against a head of 16 m when running at a speed of 1500 rpm. Diameter of its impeller being 150 mm.
A multistage pump, geometrically similar to the one given above, but having stages in series is to be designed to run at 1200 rpm and to deliver 450 m^3/hr of water against a head of 140 m. Find the impeller diameter and number of stages required.

[12 marks]

Given → Assuming all pumps are identical-

case-(i) Pumps are in parallel → $Q_{total} = n\,Q_{1,pump}$.

$n = 3$ stages , $H_m = H_1 = H_2 = \cdots H_n$

$Q = 360\,m^3/hour$, $Q/pump = \frac{360}{3} = 120\,m^3/hour$

$H_m = 16\,m$

$N = 1500\,rpm$, $D_{impeller} = 150\,mm$

case-(ii) Pumps are in series $H_{total} = n \times H_m$

$Q = Q_1 = Q_2 = \cdots Q_n$

H_m (Head generated by 1 pump) $= \frac{140}{n}\,m$

$Q = 450\,m^3/hr$

$N = 1200\,rpm$

For Geometrically similar pumps -

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \Rightarrow \frac{120}{1500 \times 150^3} = \frac{450}{1200 \times D_2^3}$$

$\boxed{D_2 = 251\,mm}$ *

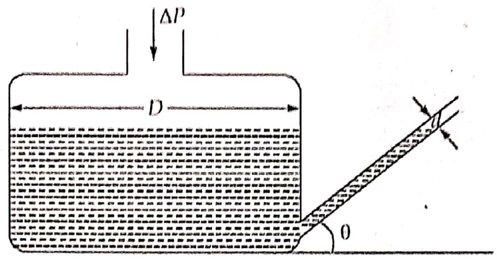
and $\frac{H_{m1}}{N_1^2 D_1^2} = \frac{H_{m2}}{N_2^2 D_2^2} \Rightarrow$

12

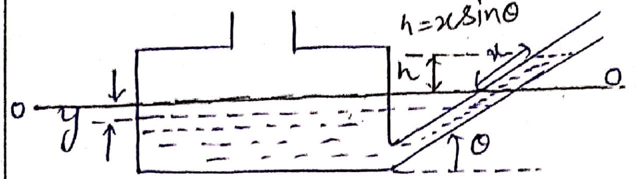
$$\frac{16}{1500^2 \times 150^2} = \frac{(140/n)}{1200^2 \times (251)^2} \Rightarrow n = 4.88$$

$\boxed{n = 5}$ = 5 stages are required in series flow pump.
*

Q.5 (e) The inclined tube manometer shown has $D = 76$ mm and $d = 8$ mm, and is filled with Meriam red oil having specific gravity 0.827. What is the angle θ, that will give a 15 cm oil deflection along the inclined tube for an applied pressure of 25 mm of water (gauge)? Also determine the sensitivity of this manometer.



[12 marks]



Initial level of oil

Let initial level of oil in the tank and manometer tube is denoted by o-o line, at this time there is no pressure difference so oil in both will be at equal level (o-o line).

→ after applying ΔP, Pressure of 25 mm of water let level in tank goes down by y mm then level in manometer goes up by x mm.

$$\frac{\pi D^2}{4} y = \frac{\pi d^2}{4} x \Rightarrow x = 90.25\, y \text{ mm}$$

Taking all pressure as gauge pressures-

$$\rho_{oil}\, g (x \sin\theta + y) = \rho_w \times g \times 25$$

8

given oil deflection $= x = 150$ mm, $y = 1.66$ mm

$$150 \sin\theta + 1.66 = \frac{25}{0.827}$$

$$\sin\theta = 0.190$$

$$\boxed{\theta = 10.98^\circ}$$

check solution

sensitivity of Manometer is given by change in x per unit pressure rise $\Delta x \propto \frac{\Delta P}{\sin\theta}$

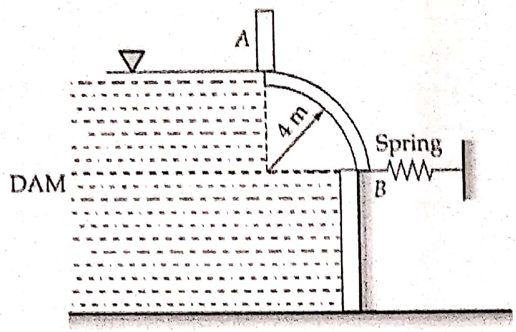
$$\boxed{\text{sensitivity} = \frac{1}{\sin\theta}} = \boxed{5.25}$$

Q.6 (a) A 4 m long quarter-circular gate of radius 4 m and of negligible weight is hinged about its upper edge *A* as shown in figure. The gate controls the flow of water over the edge at *B*, where the gate is pressed by a spring. Determine the minimum spring force required to keep the gate closed when the water level rises to *A* at the upper edge of the gate. If the spring constant is 25 MN/m, what is the leakage rate (in m^3/s) from the dam assuming rectangular cross-section of area of leakage?

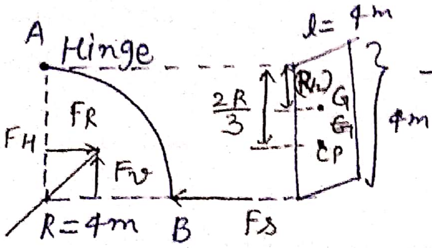
Given –

length = 4 m

$K = 25 \frac{MN}{m}$



[20 marks]



→ Horizontal Force is given by Projecting the curved submerged surface on a vertical plane and finding force on this Rectangular plane.

$F_H = \rho_w g\, H_{centroid}\ \text{Area}$, $H_G = \frac{2R}{32}$

$= 1000 \times 9.81 \times \frac{\cancel{2}}{\cancel{3}} \times \frac{4}{2} \times 4 \times 4 = \cancel{418.56\ kN} = 313.92\ kN$

this horizontal force (F_H) will pass through centre of pressure (CP) of Rectangular section and height of cp is given by –

$$\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

where $\bar{x} = \frac{R}{2} = 2m$

θ = angle with vertical = $90°$

A = area $= 4 \times 4 = 16 m^2$

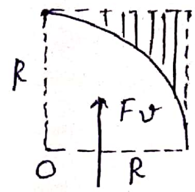
I_G = area moment of Inertia about centroid $= \frac{bd^3}{12}$

$I_G = 4 \times \frac{4^3}{12} = \frac{64}{3} m^4$

$$\bar{h} = 2 + \frac{64 \times 1}{3 \times 16 \times 2} = 2 + \frac{2}{3} = \frac{8}{3}\,m$$

For vertical force $-F_v$

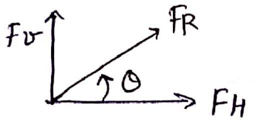
Magnitude of vertical force is given wt of shaded area.



$$F_v = \rho_w \times g \times \left(R^2 - \frac{\pi R^2}{4}\right) \cdot l = 1000 \times 9.81 \times 4^2 \left(1 - \frac{\pi}{4}\right) \times 4 = 134.735\ kN$$

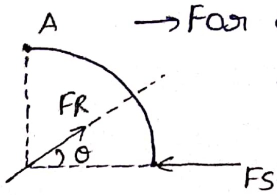
$$F_R = \sqrt{F_H^2 + F_v^2} = \sqrt{313.92^2 + 134.735^2} = 341.613\ kN$$

→ Resultant force will always through centre of arc (O), as pressure force always acts normal to surface and each normal to the arc passes through centre O.



$$\tan\theta = \frac{F_v}{F_H} = \frac{134.735}{313.92}$$

$$\theta = 23.23^\circ$$



→ For door to remain closed, $M_A = 0$.

→ Taking Moments about hinge A.

$$F_R \cdot \cancel{R}/\cos\theta = F_s \times \cancel{R}$$

$$F_s = \text{spring force} = \frac{341.613 \times \cos 23.23^\circ}{1} = \boxed{313.92\ kN} = F_s$$

$$\text{Deflection of spring} = \Delta x = \frac{F_s}{k} = \frac{313.92}{25 \times 10^3} = 12.56\ mm$$

→ Now water will leak through gap created by spring at point B.

very good

→ area of Gap $= \Delta x \times l = 0.05022\ m^2$

20

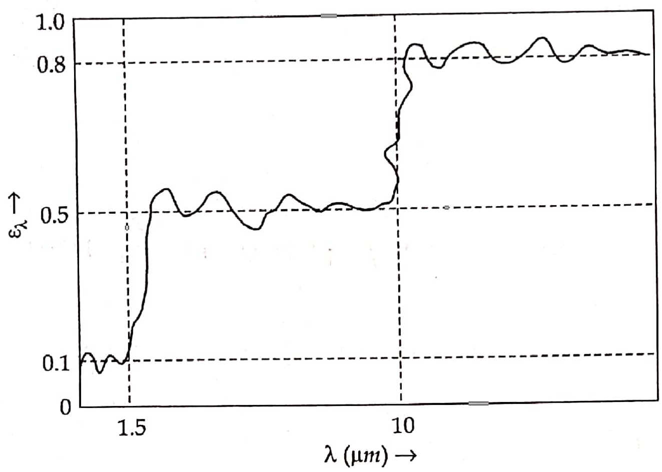
→ applying Bernouli eqn b/w pt A and B −

$$\frac{V_A^2}{2g} + Z_A + \frac{P_A}{\rho g} = \frac{V_B^2}{2g} + Z_B + \frac{P_B}{\rho g}, \quad V_A \simeq 0, \quad P_A = P_B, \quad Z_A - Z_B = R$$

$$V_B = \sqrt{2gR} = \sqrt{2 \times 9.81 \times 4} = 8.86\ m/sec.$$

Volume flow rate $Q = Av = \boxed{0.444\ m^3/sec. = Q_{leakage}}$

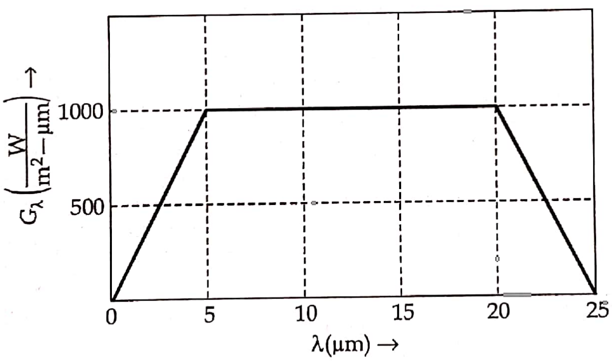
Q.6 (b) (i) A diffuse, fire brick wall of temperature, T_s = 500 K has the spectral emissivity shown and is exposed to a bed of coal at 2000 K.



λT = 600 μmK	$F_{0-\lambda}$ = 0.000
λT = 750 μmK	$F_{0-\lambda}$ = 0.000
λT = 3000 μmK	$F_{0-\lambda}$ = 0.273
λT = 5000 μmK	$F_{0-\lambda}$ = 0.634
λT = 5800 μmK	$F_{0-\lambda}$ = 0.720
λT = 20000 μmK	$F_{0-\lambda}$ = 0.986

Determine the total hemispherical emissivity and emissive power of the fire brick wall. What is the total absorptivity of the wall to irradiation resulting from emission by the coal?

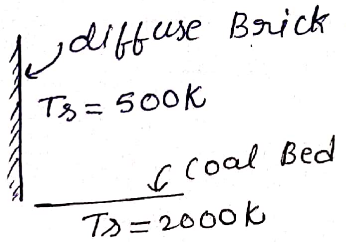
(ii) The spectral distribution of surface irradiation is as follows :



What is the total irradiation?

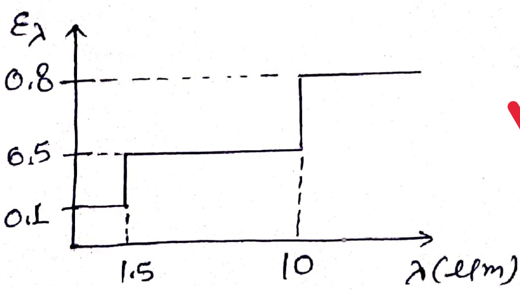
[15+5 = 20 marks]

(i)



→ assuming average spectral emissivity (ε_λ) and Drawing it,

→ Below graph is obtained for average ε_λ.



→ let total hemispherical emissibity be ε –

then $\varepsilon = \dfrac{E}{E_b}$

$$\varepsilon = \frac{\int_0^{\infty} E_\lambda d\lambda}{E_b} = \frac{\int_0^{\infty} \varepsilon_\lambda E_{b\lambda} d\lambda}{E_b}$$

$$= \frac{\int_0^{1.5} 0.1\, E_{b\lambda} d\lambda + \int_{1.5}^{10} 0.5\, E_{b\lambda} d\lambda + \int_{10}^{\infty} 0.8\, E_{b\lambda} d\lambda}{E_b}$$

$$= 0.1\, F_{0-1.5} + 0.5\, F_{1.5-10} + 0.8\, F_{10-\infty}$$

$F_{0-\lambda}$ = fraction of Energy lying b/w 0 to λ w.r.t Black Body.

$$F_{0-\lambda} = \frac{\int_0^{\lambda} \varepsilon_\lambda E_{b\lambda} d\lambda}{\int_0^{\infty} E_{b\lambda} d\lambda}, \quad T_s = 500\,K$$

for $\lambda = 1.5\,\mu m$, $\lambda T = 750.0$

$F_{0-1.5} = 0$

and $\lambda = 10\,\mu m$, $\lambda T = 5000$

$F_{0-10} = 0.634$

$$\varepsilon = 0.1 \times 0 + 0.5(0.634 - 0) + 0.8(1 - 0.634)$$

$\boxed{\varepsilon = 0.6098}$, emissive power $= \varepsilon \sigma T_s^4 = 0.6098 \times 5.67 \times 5^4$

$$\boxed{E = 2160.98\ W/m^2}$$

at steady state
Heat absorbed = Heat emitted

$$\alpha \times \sigma T^4 = 2160.98$$

$$\alpha = \frac{2160.98}{5.67 \times (20)^4} = \boxed{2.38 \times 10^{-3} = \text{total absorptivity}}$$

(ii) Total irradiation on surface $(G) = \int_0^{\infty} G_\lambda d\lambda$

14 = area of G_λ and λ diagram $= \frac{1}{2} \times (25 + 15) \times 1000 \ \frac{W}{m^2}$

$$\boxed{G = \frac{20\,kW}{m^2}}$$

Q.6 (c) A team of students while designing various parameters for formula student race car, came across the following parameters of a car.

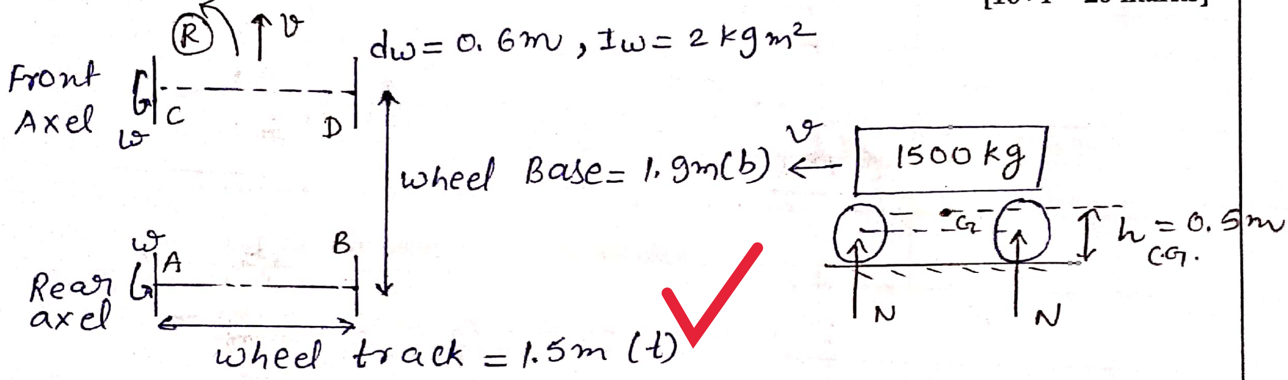
- Moment of inertia of each wheel (I_w) = 2 kg.m^2
- Effective diameter of wheel (d_w) = 0.6 m
- M.O.I. of rotating part of engine (I_e) = 1.25 kg.m^2
- Gear ratio of back axel (G) = 3 : 1
- Mass of automobile (M) = 1500 kg
- C.G. above the road level ($h_{C.G.}$) = 0.5 m
- Wheel track and wheel base (t, b) = 1.5 m and 1.9 m (respectively)

If engine axis is parallel to rear axle and the crank shaft rotation is in the same direction as the wheels, then determine limiting speed (in km/h) of the vehicle around a curve of 100 m mean radius, for all four wheels to maintain contact with the road surface.

Also, comment how variation of above parameter will increase the cornering speed.

[16+4 = 20 marks]

$d_w = 0.6\,m$, $I_w = 2\,kg\,m^2$



Let car takes a left turn with velocity v and turning Radius R ⇒ $\omega_p = \frac{V}{R} = \frac{v}{100\,m}$

Engine speed = 3 x speed of wheels in same dirn

$= 3\omega$

Now taking all the forces on car and wheels.

centrifugal force acting outwards (→) = Fc

$$F_c = \frac{MV^2}{R} = \frac{1500 \times V^2}{100} = 15V^2\ N,$$

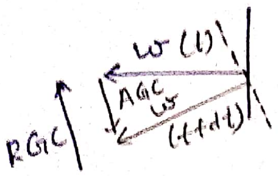
gyroscopic couple on wheels and engine = $C = I\omega\omega_p$

$$C = 4I_w\,\omega\,\omega_p + I_e \times 3\omega \times \omega_p$$

$$= 4 \times 2 \times \omega \times \frac{V}{100} + 1.25 \times 3 \times \omega \times \frac{V}{100} = \frac{47 v\omega}{400}\ N\text{-}m$$

assuming No slipping → $V = \omega R_{wheels}$ ⇒

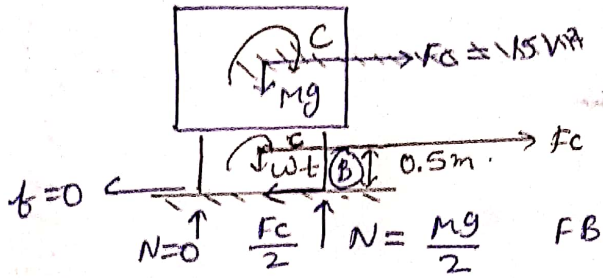
$V = \omega \times 0.3 = 0.3\omega$ m/sec.



— Due to left turn Reactive gyroscopic couple will try to lift wheels A and C (inner wheels).

→ centrifugal force will be balanced by friction force.

→ For limiting case Normal force on inner wheels = 0



FBD of 1 wheel (B).

8 Balancing Moments –

$$C + F_c \times h = N \times \frac{t}{2}$$ check solution

$$\frac{47}{100} \times v \times \frac{v}{0.3} + 15v^2 \times 0.5 = \frac{(1500) \times 9.81 \times 1.5}{2}$$

solving $\frac{136 v^2}{15} = 11036.25 \longrightarrow v = 34.88$ m/sec

$v = 34.88 \times \frac{18}{5} = \boxed{125.6 \text{ km/hr}}$ ** limiting speed

→ for large turning radius (R), ω_p becomes very less, thus gyroscopic couple produced is small and cornering speed will be high.

→ for sharp turns (small turning Radius), ω_p increases, which increase value of RGC, thus automobile starts to topple about outer wheels at high speeds, thus cornering speed limit decreases for small R.