NAME :

BATCH : MAINS A

ROLL :

total marks
265

Q1). a). Let Link AR rotate about a fixed point A.
P is the point on a slider on the link.

At any given instant,

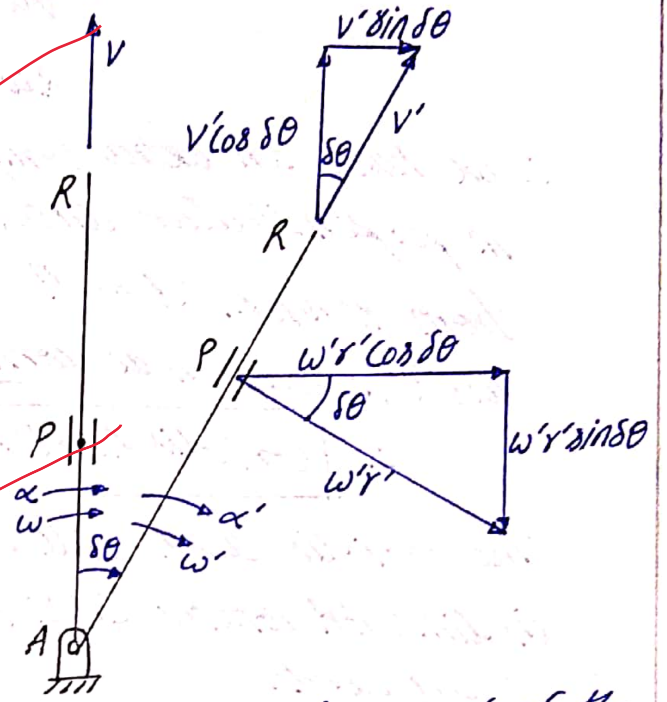
ω = angular velocity of the link.

α = angular acceleration of the link.

v = linear velocity of the slider

a = linear acc^n of the slider.

r = radial distance of point P.



In short time δt, let $\delta\theta$ be angular displacement of the link and δr, the radial displacement of the slider in the ~~downward direction~~ outward direction.

$\omega' = \omega + \alpha\delta t$ $\quad v' = v + a\delta t$ $\quad r' = r + \delta r$

① Acceleration parallel to AR

Initial velocity of P along AR = v

Final velocity of P along AR = $v'\cos\delta\theta - \omega' r'\sin\delta\theta$

$\therefore$ Acc^n of P along AR $= \dfrac{(v+a\delta t)\cos\delta\theta - (\omega+\alpha\delta t)(r+\delta r)\sin\delta\theta - v}{\delta t}$

as $\delta t \to 0 \Rightarrow \cos\delta\theta \to 1$ and $\sin\delta\theta \approx \delta\theta$

$\Rightarrow$ Acc^n along AR $= a - \omega r\dfrac{d\theta}{dt} = a - \omega^2 r$

This is the centripetal acc^n of slider.

② Acceleration of P perpendicular to AR

Initial velocity of P $\perp$ to AR = ωr

Final velocity of P $\perp$ to AR = $v'\sin\delta\theta + \omega' r'\cos\delta\theta$

$\therefore$ Acc^n of P $\perp$ to AR $= \dfrac{(v+a\delta t)\sin\delta\theta - (\omega+\alpha\delta t)(r+\delta r)\cos\delta\theta - \omega r}{\delta t}$

as $\delta t \to 0$

$\Rightarrow \cos\delta\theta \to 1$ & $\sin\delta\theta \to \delta\theta$

⇒ Acc^n of $P \perp$ to $AR = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\alpha$

$= v\omega + \omega v + r\alpha$

$= 2\omega v$ + tangential acc^n

∴ we have an extra component of acc^n of P in perpendicular direction to link AR apart from tangential acc^n. This component of acc^n is known as <u>coriolis acc^n</u>.

∴ $\boxed{\text{Coriolis Acceleration} = 2\omega v}$

The coriolis acc^n will be +ve if:-

(i) link AR rotates C.W. and slider moves radially outwards.

(ii) link AR rotates C.C.W. and slider moves radially inwards.

otherwise, the coriolis component is (-ve).

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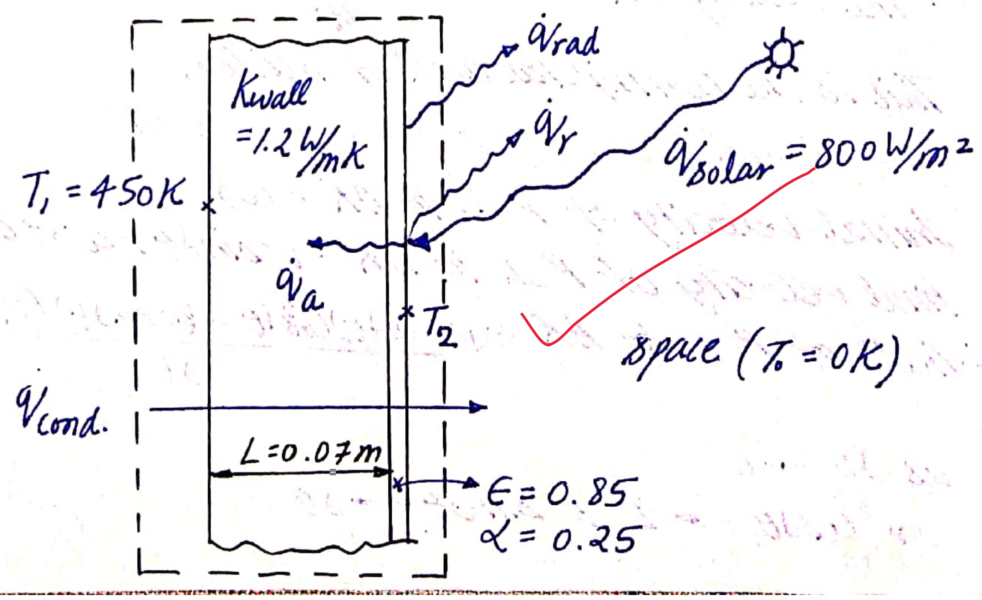
Thus, these observations can be summarised by following rule:-

The direction of the coriolis acc^n component is obtained by rotating the radial velocity vector 'v' through 90° in the direction of rotation of the link.

Q1)(b) <u>Assumption</u>:- There is no heat generation in the wall.



Assuming the thickness of porcelain ~~tile~~ tiles to be negligible $\therefore$ thermal resistance of porcelain tiles is negligible.

Assuming $\tau = 0 \Rightarrow \alpha + \rho = 1$

Absorbed solar radiation, $\dot{q}_a = \alpha \dot{q}_{solar}$

Reflected solar radiation, $\dot{q}_r = \rho \dot{q}_{solar}$

$$\dot{q}_{solar} = \dot{q}_a + \dot{q}_r$$

$$\cancel{\dot{q}_{solar}} \Rightarrow \dot{q}_a = \dot{q}_{solar} - \dot{q}_r \quad \text{-(I)}$$

At steady state, for the boundary shown in fig.

$$\dot{q}_{in} = \dot{q}_{out}$$

$$\Rightarrow \dot{q}_{solar} = \dot{q}_{rad} + \dot{q}_r + \dot{q}_{cond}$$

$$\Rightarrow \dot{q}_{cond} = \dot{q}_{solar} - \dot{q}_{rad} - \dot{q}_r$$

$$\Rightarrow \dot{q}_{cond} = \dot{q}_a - \dot{q}_{rad} \quad (\text{from eq (I)})$$

$$\Rightarrow \frac{K_w (T_2 - T_1)}{L} = \alpha \dot{q}_{solar} - \epsilon \sigma T_2^4 \quad \text{(II)}$$

$$\Rightarrow \frac{1.2}{0.07}(T_2 - 450) = 0.25 \times 800 - (4.8195 \times 10^{-8} \times T_2^4)$$

$$\Rightarrow 4.8195 \times 10^{-8} T_2^4 + 17.1428 T_2 = 7914.2857$$

solving the eq. we get $\boxed{T_2 = 393.95K}$

$$\therefore \boxed{\dot{q}_{cond.} = \frac{K}{L}(T_2 - T_1) = 960.85 \ W/m^2}$$

* If there is no incident light :- $\dot{q}_{solar} = 0$

by eq. (II) $\frac{K(T_2 - T_1)}{L} = -\epsilon \sigma T_2^4$

$$4.8195 \times 10^{-8} T_2^4 + 17.428 T_2 = 7714.28$$

$$\Rightarrow \boxed{T_2 = 383.08 \ K}$$

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comments on results

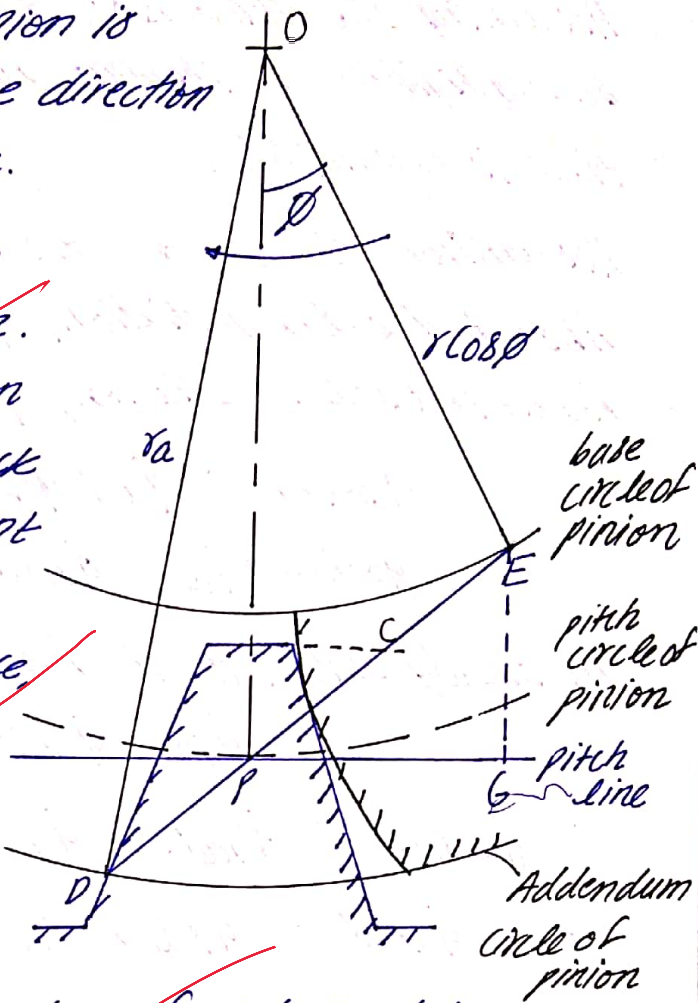
Q1. c. (i) In the figure the pinion is rotating in clockwise direction and driving the rack.

P → pitch point

PE → line of action.

Engagement of pinion tooth with tip of rack tooth occurs at point C on line of action.

To avoid interference, the maximum addendum of rack can be increased in such a way that point C coincides with E. Thus, addendum of rack must be less than GE.



Let a_r = fractional addendum for the rack.

$$GE = PE \sin\phi = (r\sin\phi)\sin\phi = r\sin^2\phi = \frac{mt}{2}\sin\phi$$

where t = no. of teeth on pinion.

To avoid interference, $GE \geq a_r m$

$$\Rightarrow \frac{mt}{2}\sin^2\phi \geq a_r m \quad \text{or} \quad t \geq \frac{2a_r}{\sin^2\phi}$$

$$\therefore \boxed{t_{min} = \frac{2a_r}{\sin^2\phi}}$$

(ii) The following four method can be used to avoid interference :-

(1) Undercutting of gears

(2) Increasing pressure angle (ϕ)

(3) Stubbing the teeth

(4) Increase the no. of teeth (No limitations)

explain at least all methods

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Q1.(d) given,

diameter of hole = 400mm = d

thickness of ~~hole = 15~~ plate, $t = 15mm$

No. of holes/min = 30

$\therefore$ time taken for one hole

= cycle time = $t_c = \frac{60}{30}$ = 2seconds

Energy/mm^2 = 0.6 Kgfm/mm^2 = 5.886 Nm/mm^2

Time taken for punching operation = t_a = 0.1 second

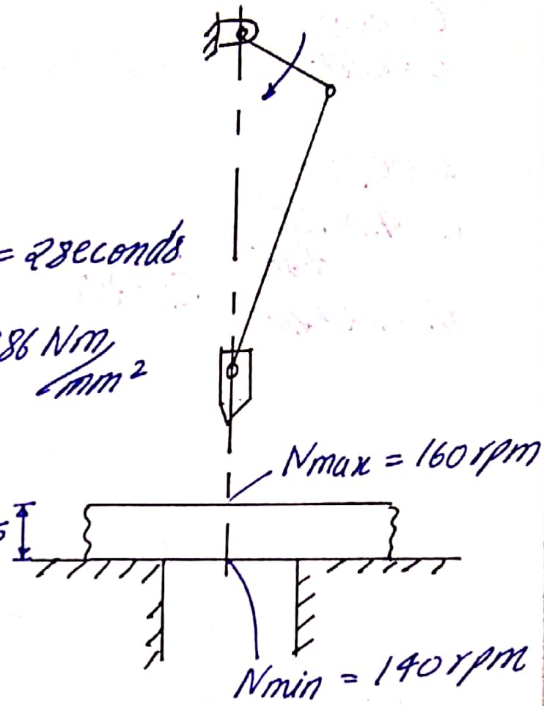

fig. schematic diagram for the given punching machine

~~Energy~~

Energy/hole = Energy/mm^2 $\times \pi d t$

$\Rightarrow$ Energy/hole = 110948.48 Nm

Power of the motor, $P = \frac{\text{Energy/hole}}{t_c} = \underline{\underline{55474\ W}}$

Maximum fluctuation of Energy

$$\Delta E = P(t_c - t_a) = 105401.06\ Nm$$

Also $\Delta E = \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$

where I = moment of Inertia

$\omega = \frac{2\pi N}{60}$ = angular velocity

$$\Rightarrow \Delta E = \frac{1}{2} I \left(\frac{2\pi}{60}\right)^2 (N_{max}^2 - N_{min}^2)$$

$$\Rightarrow \Delta E = \frac{1}{2} I \left(\frac{2\pi}{60}\right)^2 (160^2 - 140^2)$$

$$\Rightarrow I = 3203.8\ Kg\text{-}m^2$$

$\therefore$ The flywheel required for the given operation must have a moment of Inertia :-

$$\boxed{I = 3203.8\ kg\text{-}m^2}$$

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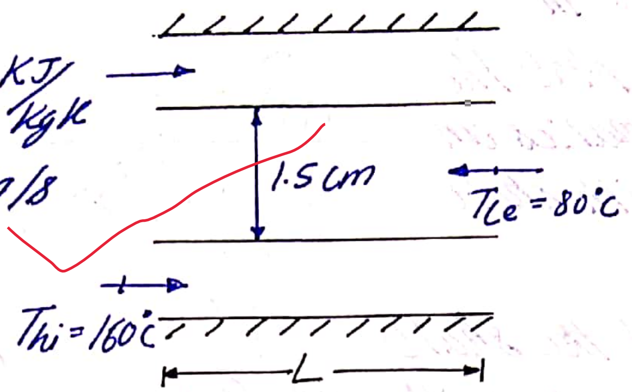
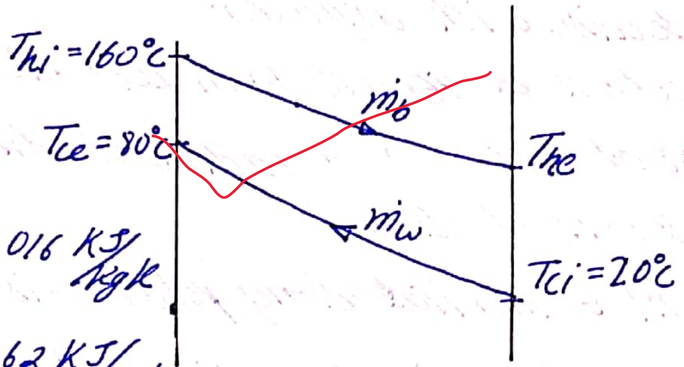
Q.V.e) Assumption:- (i) There is no heat flow to surroundings.
(ii) steady flow

brine:
$C = 4.31\ KJ/KgK$
$\dot{m}_b = 2\ kg/s$

water:
$C = 4.18\ KJ/KgK$
$\dot{m}_w = 1.2\ Kg/s$



$i \rightarrow$ inlet
$e \rightarrow$ exit
$w \rightarrow$ water
$b \rightarrow$ brine



$(\dot{m}C)_w = 5.016\ KJ/KgK$

$(\dot{m}C)_b = 8.62\ KJ/KgK$

$\therefore\ C = \dfrac{(\dot{m}C)_w}{(\dot{m}C)_b} = 0.5819$

$\because (\dot{m}C)_w < (\dot{m}C)_b \Rightarrow \epsilon = \dfrac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} = \underline{0.4285}$

It is given that, $NTU = \dfrac{1}{C-1} \ln\left(\dfrac{\epsilon - 1}{\epsilon C - 1}\right)$

putting value of ϵ & C in above eq^n:-

$NTU = \underline{0.6523}$

Also $NTU = \dfrac{UA}{(\dot{m}C)_{min}} = \dfrac{UA}{(\dot{m}C)_w}$ $\quad U = 640\ W/m^2K = 0.64\ KW/m^2K$ (given)

$\Rightarrow A = \underline{5.112\ m^2}$

$\because$ heat transfer takes place along the inner tube surface. $\therefore A = \pi d L$ $\quad d = 0.015\ m$ (given)

$\Rightarrow$ Length of the given heat exchanger

$\boxed{L = 108.5\ m}$

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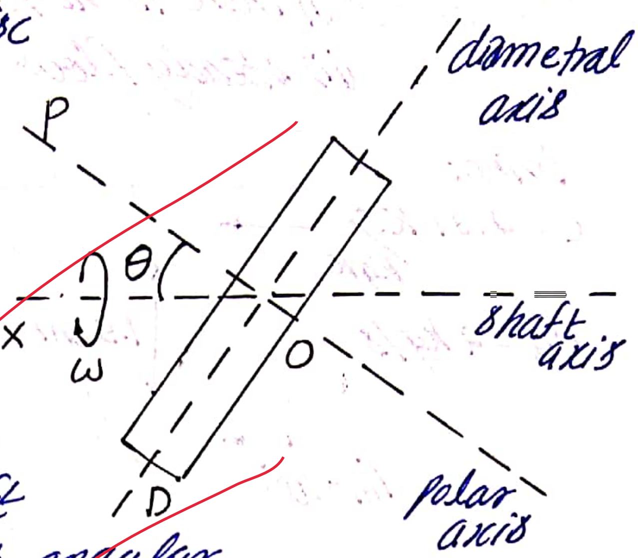
Q.2 (a) Consider a circular disc fixed rigidly to a rotating shaft in such a way that the polar axis makes an angle θ with shaft axis.



Assume that the shaft rotates clockwise with angular velocity ω when viewed from left side.

$m \rightarrow$ mass of disc $\quad r \rightarrow$ radius of disc

$t \rightarrow$ thickness of disc.

I_p = moment of inertia along polar axis $= \frac{mr^2}{2}$

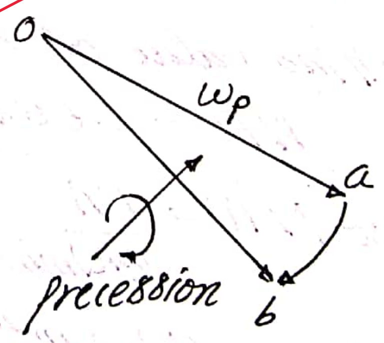
I_d = moment of inertia along dimetral axis

$$I_d = m\left(\frac{r^2}{4} + \frac{t^2}{12}\right)$$

$$I_d = \frac{mr^2}{4} \quad \text{(neglecting thickness 't')}$$

(i) <u>spinning about polar axis:-</u>

Angular vel. of spin = Angular velocity of disc about OP $= \omega\cos\theta$



Angular velocity of precession = Angular velocity about OD $= \omega\sin\theta$

$\therefore$ gyroscopic couple $= I_p \times \omega\cos\theta \times \omega\sin\theta$

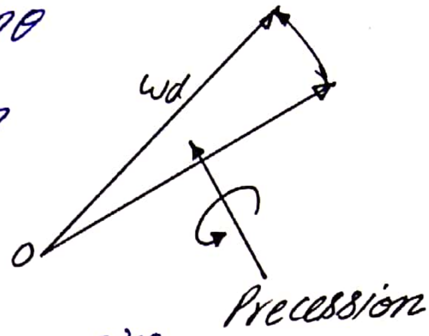
$\Rightarrow$ gyroscopic couple $= \frac{1}{2} I_p \sin 2\theta$

Its effect is to rotate the disc counter clockwise when viewed from top.

(ii) Spinning about diametral axis

Angular velocity of spin
= Angular vel. about OD = $\omega \sin\theta$

Angular velocity of precession
= Angular velocity of disc
about OP = $\omega \cos\theta$



$\therefore$ gyroscopic couple = $I_d \omega \cos\theta \, \omega \sin\theta$

$\Rightarrow$ gyroscopic couple = $\frac{1}{2} I_d \omega^2 \sin 2\theta$

Its effect is to rotate the disc clockwise when viewed from the top.

$\therefore$ Resultant gyroscopic couple on disc

$$C = \frac{1}{2}(I_p - I_d)\omega^2 \sin 2\theta$$

$$C = \frac{1}{2}\left(\frac{mr^2}{2} - \frac{mr^2}{4}\right)\omega^2 \sin 2\theta$$

$$\boxed{C = \frac{mr^2}{8}\omega^2 \sin 2\theta}$$ hence proved.

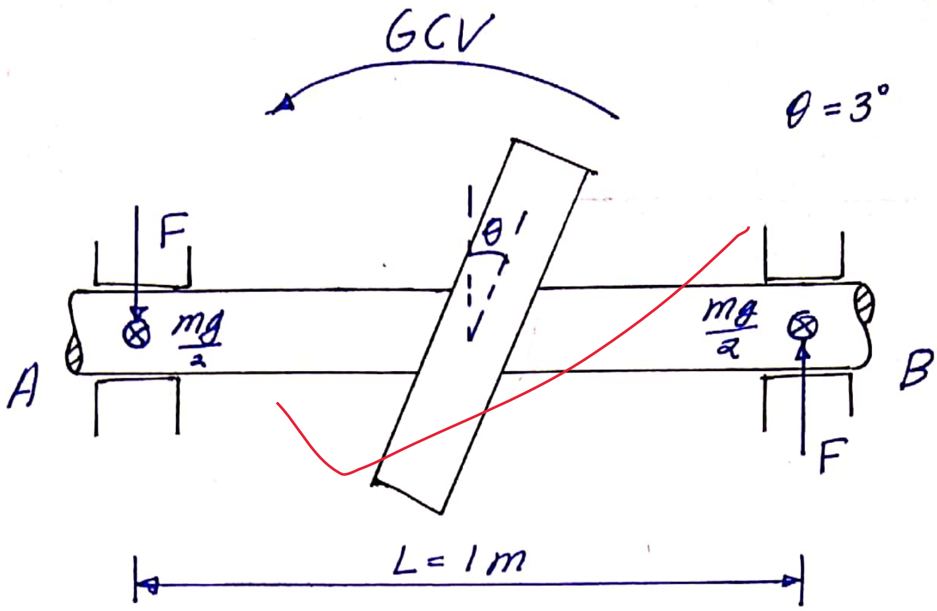
$\because I_p > I_d \;\therefore$ net effect will be counter clockwise rotation of the disc when viewed from top.

It is given that, $m = 40\,kg$ $\quad r = 0.25\,m$
$\theta = 3^\circ \quad N = 1000\,rpm$

$$\omega = \frac{2\pi N}{60} = 104.719\ rad/s$$

$$\Rightarrow C = \frac{40(0.25)^2}{8}(104.719)^2 \sin 6^\circ$$

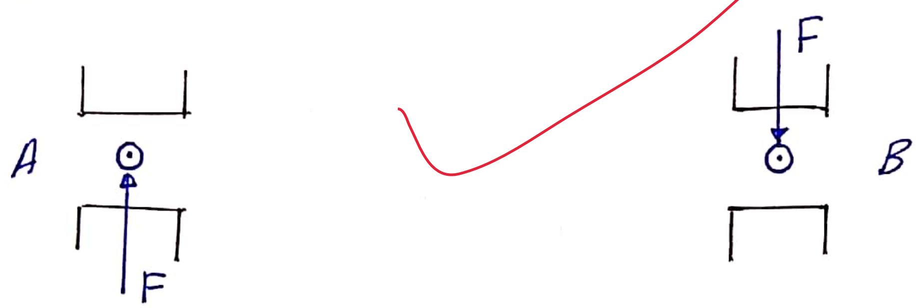
$$\Rightarrow \boxed{C = 358.213\ N\text{-}m}$$ counter clockwise when viewed from top.



$F \times L = C \Rightarrow F = 358.213\,N$

$\frac{mg}{2} = 196.2\,N$

The reaction on bearing A & B is due to Reactive GCV:-



LHS view:-

$R_A = \sqrt{F^2 + \left(\frac{mg}{2}\right)^2}$ $\qquad$ $R_B = \sqrt{F^2 + \left(\frac{mg}{2}\right)^2}$

$\Rightarrow \boxed{R_A = R_B = 408.42\,N}$

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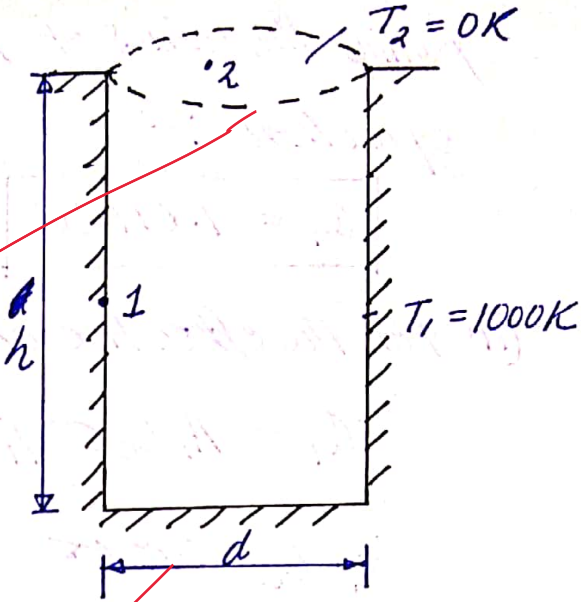
Q2(b) (i) $h = 24\,mm$

$d = 6\,mm$

$\epsilon_1 = 0.8$

$\epsilon_2 = 1$ (blackbody)



$F_{12} + F_{11} = 1$

$\Rightarrow F_{11} = 1 - F_{12}$

Also, $F_{2-1} + F_{2-2} = 1$

$F_{2-2} = 0$ (flat surface)

$\Rightarrow F_{2-1} = 1$

$A_1 F_{1-2} = A_2 F_{2-1}$

$\Rightarrow F_{1-2} = \frac{A_2}{A_1} F_{2-1} = \frac{A_2}{A_1}$

$\Rightarrow F_{1-1} = 1 - \frac{A_2}{A_1} = 1 - \frac{\frac{\pi}{4} d^2}{\frac{\pi}{4} d^2 + \pi d h}$

$\Rightarrow \boxed{F_{1-1} = \frac{4h}{4h+d}}$ —(1)

$\therefore$ for the given cylindrical cavity

$$F_{1-1} = \frac{4(0.024)}{4(0.024) + 0.006} = 0.94$$

$$A_1 = \frac{\pi}{4} d^2 + \pi d h = 4.8066 \times 10^{-4}\, m^2$$

The net radiative heat transfer from a cavity can be calculated using the following formula:-

$$Q_1 = A_1 \epsilon_1 \sigma T_1^4 \left(\frac{1 - F_{1-1}}{1 - (1-\epsilon_1) F_{1-1}} \right)$$

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putting the values of A_1, ϵ_1, F_{1-1}, T_1 in above equation :-

$$\boxed{Q_1 = 1.611\ W}$$

(ii) For the effective emissivity (ϵ_e) of the cavity

$$Q_1 = A_1 \epsilon_e \sigma T_1^4$$

$$\Rightarrow \cancel{\epsilon_e = 0.05}$$

$$\Rightarrow 1.611 = 4.8066 \times 10^{-4} (\epsilon_e) 5.67 \times 10^{-8} (1000)^4$$

$$\Rightarrow \boxed{\epsilon_e = 0.059}$$

(iii) we know that

$$Q_1 = A_1 \epsilon_e \sigma T_1^4 = A_1 \epsilon_1 \sigma T_1^4 \left(\frac{1 - F_{1-1}}{1 - (1-\epsilon_1) F_{1-1}} \right)$$

$$\Rightarrow \epsilon_e = \epsilon_1 \left(\frac{1 - F_{1-1}}{1 - (1-\epsilon_1) F_{1-1}} \right)$$

~~ϵ_1~~ from eq. (1) $F_{1-1} = \frac{4h}{4h+d}$

putting. F_{1-1} in above eq. :-

$$\Rightarrow \epsilon_e = \epsilon_1 \left(\frac{1 - \left(\frac{4h}{4h+d}\right)}{1 - (1-\epsilon_1)\left(\frac{4h}{4h+d}\right)} \right)$$

$$\Rightarrow \boxed{\epsilon_e = \frac{\epsilon_1 d}{d + 4h\epsilon_1}}$$

from this eq.^{n} we can say that :-

(i) if $h = 0 \Rightarrow \epsilon_e = \epsilon_1$

(ii) if $h \to \infty \Rightarrow \epsilon_e \to 0$

$$\Rightarrow \boxed{0 < \epsilon_e < \epsilon_1}$$

A cavity of infinite depth will note radiate any heat.

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refer solution

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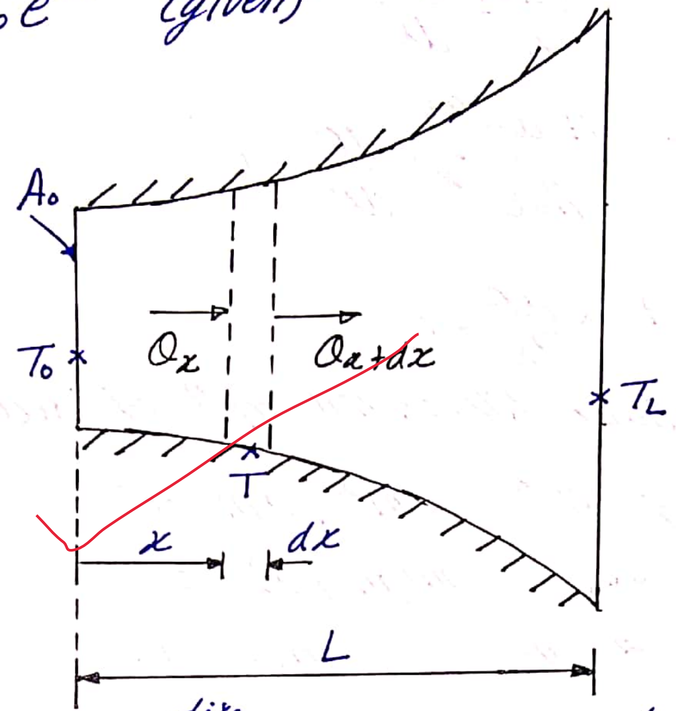
Q.2(c) Assumption:- heat conduction is in x direction only.

$A = A_0 e^{ax}$ (given)



$T_0 > T_L$ (given)

$\theta_0 = 0$

$\theta_L = T_0 - T_L$

(i) Assuming a ~~strip~~ disc of thickness dx at distance x ~~dx~~ from left side of rod.

Energy balance for the ~~strip~~ disc :-

$$Q_x = Q_{x+dx} \quad (\because \text{lateral surface is insulated})$$

$$\Rightarrow -KA\frac{dT}{dx} = -KA\frac{dT}{dx} + \frac{d}{dx}\left(-KA\frac{dT}{dx}\right)$$

$$\Rightarrow KA\frac{d^2T}{dx^2} + \left(\frac{dA}{dx}\right)K\frac{dT}{dx} = 0 \quad (\because A = f(x))$$

Let $\theta = T_0 - T$

$$\Rightarrow \frac{d\theta}{dx} = \frac{-dT}{dx}$$

$$\Rightarrow \frac{d^2\theta}{dx^2} = \frac{-d^2T}{dx^2}$$

$$\Rightarrow KA\frac{d^2\theta}{dx^2} + \left(\frac{dA}{dx}\right)K\frac{d\theta}{dx} = 0$$

dividing the equation by KA :-

$$\Rightarrow \frac{d^2\theta}{dx^2} + \frac{(dA/dx)}{A}\frac{d\theta}{dx} = 0$$

$$\Rightarrow \frac{d^2\theta}{dx^2} + \frac{A_0 a e^{ax}}{A_0 e^{ax}} \cdot \frac{d\theta}{dx} = 0$$

$$\Rightarrow \frac{d^2\theta}{dx^2} + a\frac{d\theta}{dx} = 0$$

It is a linear diffrential eq^n of second order.

Auxillary eq^n $\Rightarrow$ $m^2 + am = 0$

$$m(m+a) = 0$$

$$\Rightarrow m_1 = 0, \quad m_2 = -a$$

$\therefore$ the solution of given eq^n is:-

$$\theta = C_1 e^0 + C_2 e^{-ax}$$

$$\Rightarrow \theta = C_1 + C_2 e^{-ax} \quad \text{(i)}$$

Boundary condition:-

at $x=0$ $\theta=0$ at $x=L$ $\theta=\theta_L = T_0 - T_L$

$C_1 + C_2 = 0$ (ii) $\theta_L = C_1 + C_2 e^{-aL}$ (iii)

subtracting eq (ii) from eq. (iii)

we get, $C_2 = \dfrac{\theta_L}{e^{-aL}-1}$

from eq. (ii) $C_1 = -C_2 = \dfrac{-\theta_L}{e^{-aL}-1}$

putting values of C_1 & C_2 in eq. (i)

$$\theta = \frac{-\theta_L}{(e^{-aL}-1)} + \frac{\theta_L e^{-ax}}{(e^{-aL}-1)}$$

$$\Rightarrow \theta = \frac{\theta_L(e^{-ax}-1)}{(e^{-aL}-1)}$$

$$\Rightarrow T_0 - T = \frac{(T_0-T_L)(e^{-ax}-1)}{(e^{-aL}-1)}$$

$$\Rightarrow \boxed{T = T_0 - \frac{(T_0-T_L)(e^{-ax}-1)}{(e^{-aL}-1)}}$$

hence proved.

(ii) Assumption:- steady state

Now heat generation is taking place in the rod as per the following given eq^n:-

$$\dot{q} = \dot{q}_0 e^{-ax}$$

from energy balance of the disc :-

$$\dot{q}_{cond.} + \dot{q}_{gen} = \dot{q}_{storage}$$

$$\dot{q}_{storage} = 0 \quad (\text{steady state})$$

$$\Rightarrow \dot{q}_{cond.} + \dot{q}_{gen} = 0$$

$$\Rightarrow Q_x - Q_{x+dx} + \dot{q}_{gen} = 0$$

$$\Rightarrow KA\frac{d^2T}{dx^2} + \left(\frac{dA}{dx}\right)K\frac{dT}{dx} + \dot{q}_0 e^{-ax} A\,dx = 0$$

dividing by KA :-

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{(dA/dx)}{A}\frac{dT}{dx} + \frac{\dot{q}_0}{k}e^{-ax}dx = 0$$

$$\Rightarrow \frac{d^2T}{dx^2} + a\frac{dT}{dx} + \frac{\dot{q}_0}{k}e^{-ax}dx = 0$$

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Integrating the eq.^{n} we get :-

$$\frac{dT}{dx} = -aT + \frac{q_0}{aK} e^{-ax} + C \quad -(1) \quad (C = \text{const.})$$

It is given that at $x=0$ the rod is insulated

$$\therefore (q_x)_{x=0} = 0 \Rightarrow 0 = -KA\left(\frac{dT}{dx}\right)_{x=0}$$

$$\Rightarrow \left(\frac{dT}{dx}\right)_{x=0} = 0 \quad \& \ (T)_{x=0} = T_0$$

using this boundary cond^n in eq. (1) :-

$$0 = -aT_0 + \frac{q_0}{aK} + C$$

$$\Rightarrow C = aT_0 - \frac{q_0}{aK}$$

using this in eq. (1) :-

$$\frac{dT}{dx} = -aT + \frac{q_0}{aK}(e^{-ax}) + aT_0 - \frac{q_0}{aK}$$

$$\Rightarrow \frac{dT}{dx} = a(\theta) + \frac{q_0}{aK}(e^{-ax} - 1) \quad -(11)$$

from fourier law for conduction:-

$$q_x = -KA\frac{dT}{dx}$$

from eq. (11)

$$q_x = -KA_0 e^{ax}\left(a\theta + \frac{q_0}{aK}(e^{-ax} - 1)\right)$$

$$\Rightarrow \boxed{q_x = -aKA_0 e^{ax}\theta + \frac{q_0 A_0}{a}(e^{ax} - 1)}$$

The above eq^n is the desired eq^n for q_x

refer solution

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Q.5(a) Let a reversible heat Engine work between two reservoirs. The heat engine absorbs Q_H heat from the ~~le~~ reservoir at temp. T_H. It produces work W and rejects Q_L heat to low temp. reservoir at T_L temp.

$$W = Q_H - Q_L \quad (I)$$

efficiency $\eta = 1 - \frac{Q_L}{Q_H}$

$\because$ the engine is reversible $\therefore \frac{Q_L}{Q_H} = \frac{T_L}{T_H}$.

(by thermodynamic temperature scale)

$\Rightarrow \eta = 1 - \frac{T_L}{T_H}$

Now, if temp. of low temp. reservoir, i.e, $T_L = 0K$

$\Rightarrow \eta = 1 \quad \Rightarrow \quad W = Q_H \quad (II)$

from eq. (I) & (II) $Q_L = 0$.

$\therefore$ If $T_L = 0K$ then heat rejected to the low temp. reservoir also becomes zero and the ~~given~~ assumed heat engine will interact with single thermal reservoir while producing work. Thus, the engine will violate the kelvin Planck statement of second law.

Such a heat engine is impossible as it will convert the unavailable energy in Q_H into available energy completely, which will lead to reduction in the entropy of the universe.

Thus it is impossible to achieve absolute zero temperature.

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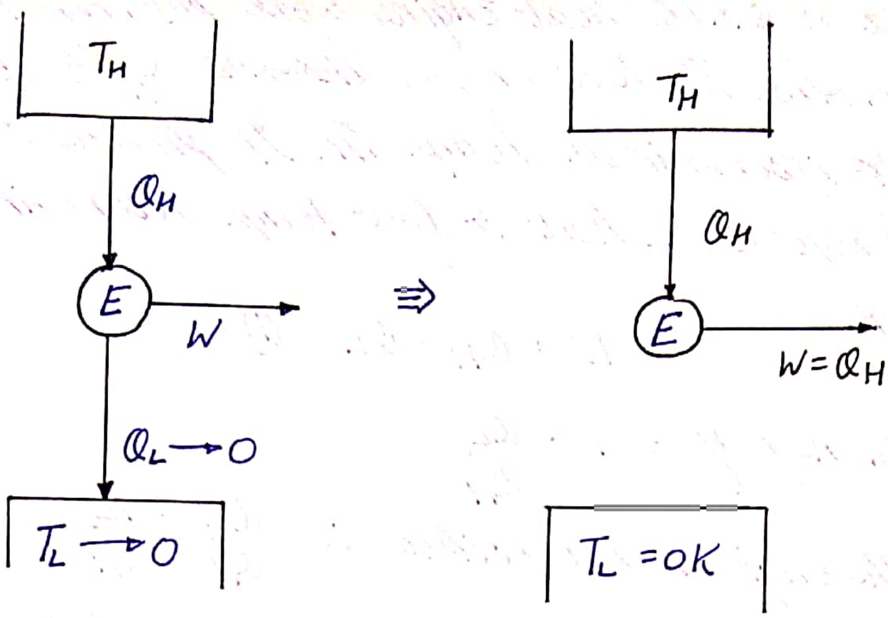
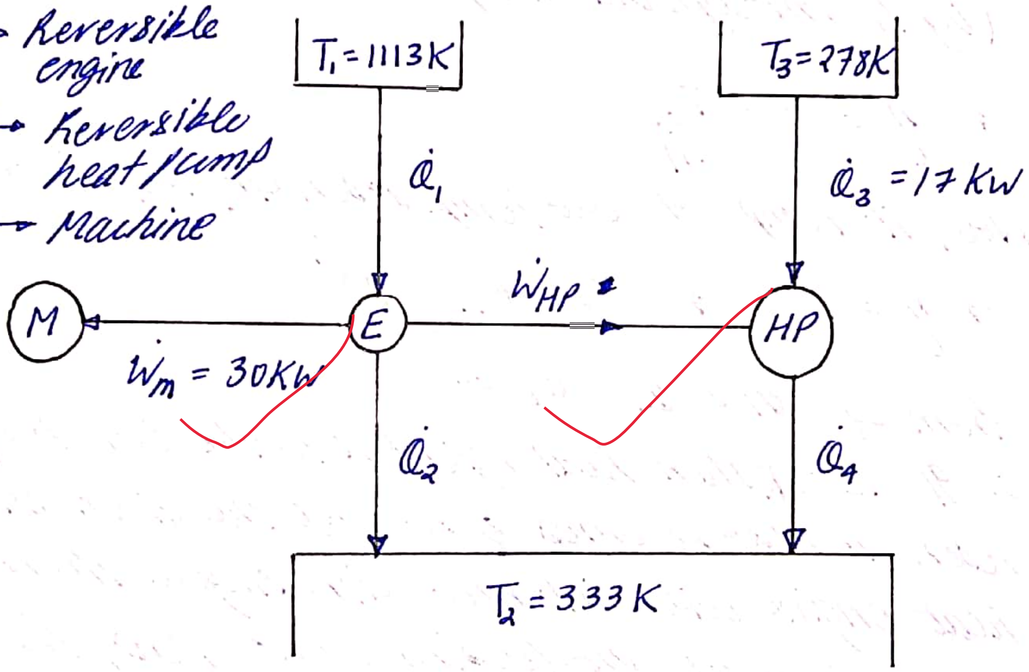
for more understanding refer solution



Q.5(b)

E → Reversible engine
HP → Reversible heat pump
M → Machine



$$\dot{W}_E = \dot{W}_m + \dot{W}_{HP} = 30 + \dot{W}_{HP} \quad \text{(I)}$$

$$\eta_E = 1 - \frac{T_2}{T_1} = 0.7 = \frac{\dot{W}_E}{\dot{Q}_1} \quad \text{(II)}$$

$$(COP)_{HP} = \frac{T_2}{(T_2 - T_3)} = 6.054$$

$$\text{also } (COP)_{HP} = \frac{\dot{Q}_4}{\dot{W}_{HP}} = \frac{\dot{Q}_3 + \dot{W}_{HP}}{\dot{W}_{HP}} = \frac{\dot{Q}_3}{\dot{W}_{HP}} + 1$$

$$\Rightarrow 6.054 = \frac{\dot{Q}_3}{\dot{W}_{HP}} + 1$$

$$\Rightarrow \dot{W}_{HP} = \underline{3.36 \; KW}$$

$\therefore$ from eq. (i)

$$\dot{W}_E = 33.36\ KW$$

using value of $\dot{W}_E$ in eq (ii)

$$\dot{Q}_1 = 47.66\ KW$$

$$\dot{Q}_2 = \dot{Q}_1 - \dot{W}_E = 14.297\ KW$$

$$\dot{Q}_4 = \dot{Q}_3 + \dot{W}_{HP} = 20.363\ KW$$

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(i) Rate of heat supply from 840°C source $= \dot{Q}_1$

$$\boxed{\dot{Q}_1 = 47.66\ KW}$$

(ii) Rate of heat rejection to sink at 60°C $= \dot{Q}_2 + \dot{Q}_4$

$$\boxed{\dot{Q}_2 + \dot{Q}_4 = 34.66\ KW}$$

Q.5(c) (i)

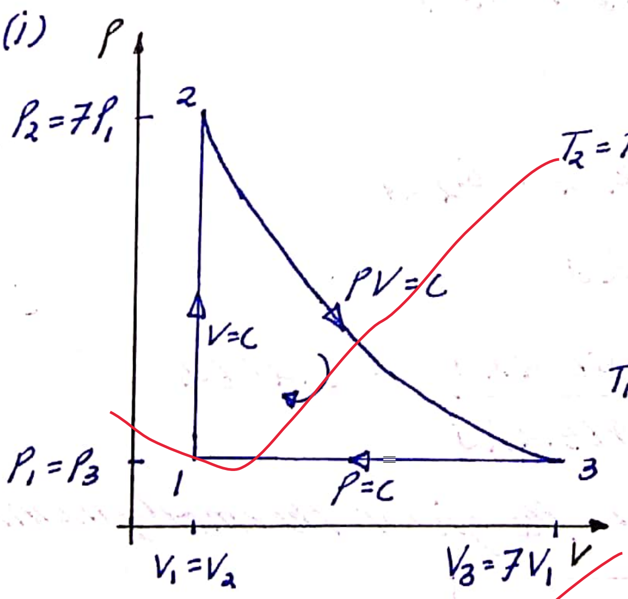

fig. P-V diagram

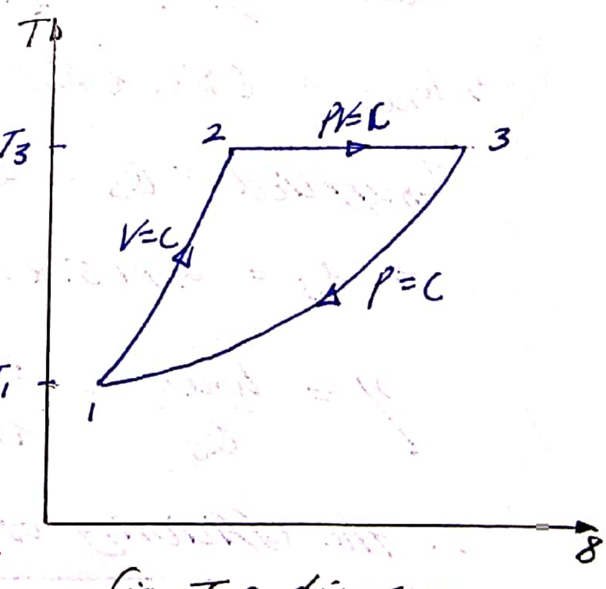

fig. T-s diagram

(ii) Assumption:- Air is an ideal gas

Air
$m = 1\ Kg$
$P_1 = 100\ KPa$
$T_1 = 300\ K$

$$V_1 = \frac{mRT_1}{P_1} = 0.861\ m^3 = V_2$$

$$V_3 = 7V_1 = 6.027\ m^3$$

for process $1 \to 2$ $V=C$ $\therefore \frac{P_2}{P_1} = \frac{T_2}{T_1}$

$\Rightarrow T_2 = 2100\,K$

<u>Process $1 \to 2$</u> $W_{12} = 0$ $(\because V=C)$

$Q_{12} = mC_V(T_2 - T_1) = +1292.4\,KJ$

<u>Process $2 \to 3$</u> $\cancel{Q_{12}}$ $Q_{23} = (\Delta U)_{23} + W_{23}$

$(\Delta U)_{2-3} = 0$ $(\because T = C)$

$\Rightarrow Q_{2-3} = W_{2-3} = mP_2V_2 \ln\left(\frac{V_3}{V_2}\right)$

$\Rightarrow Q_{2-3} = W_{2-3} = +1172.80\,KJ$

<u>Process $3 \to 1$</u> ~~$Q_{3-1} = 0$ (∵~~

$W_{3-1} = P_1(V_1 - V_3) = -516.6\,KJ$

$W_{net} = \cancel{W_{1-2}}^{\,0} + W_{2-3} + W_{3-1}$

$\Rightarrow W_{net} = 656.2\,KJ$

$Q_{supplied} = Q_S = Q_{1-2} + Q_{2-3}$

$\Rightarrow Q_S = 2465.2\,KJ$

$\eta = \frac{W_{net}}{Q_S} = \frac{656.2\,KJ}{2465.2\,KJ} = 0.2662$

$\therefore$ the efficiency of the given thermodynamic cycle is $\boxed{\eta = 26.62\%}$

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Q.5).(d) specified capacity = 160 TR

given,

Cooling water flow rate = 20 litre/s

$\therefore \dot{m}_{water} = 20\ kg/s$

$(T_{in})_w = 25°C \qquad (T_{out})_w = 35°C$

$P_{motor} = 300\ KW \quad (\eta_{mech} = 92\%)$

Assumption:- There is no heat loss to the surrounding in the condenser.

$\therefore$ heat lost by working fluid = heat gained by water

$$\dot{Q}_c = \dot{m}_w C_w (T_{out} - T_{in}) \qquad C_w = 4.18\ KJ/kgK$$

$$\Rightarrow \dot{Q}_c = 836\ KW$$

$$\eta_{mech} = \frac{\text{Work of compressor } (\dot{W}_c)}{\text{Power of motor } (P)}$$

$$\Rightarrow \dot{W}_c = \eta_{mech} P_{motor} = 276\ KW$$

$$\dot{Q}_c = \dot{W}_c + \dot{Q}_E$$

where $\dot{Q}_E$ = heat absorbed in ~~ev~~ evapourator or refrigeration capacity.

$$\dot{Q}_E = \dot{Q}_c - \dot{W}_c = 560\ KW = 160\ TR$$

$\therefore$ The actual refrigerating capacity of the given air conditioning system is as per specifications.

$$\boxed{RC = 160\ TR}$$

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Q.5(c) In the condenser of a refrigeration system, heat of the refrigerent is rejected. The refrigerent vapour at discharge of compressor is superheated. Desuperheating of the vapour takes place in first few coils of condenser. Then the saturated vapour is converted to saturated liquid by rejecting latent heat of condensation. In some condensers subcooling may also take place. The liquid refrigerant thus obtained can be used in evapourator to absorb heat.

* The type of condensers based on cooling medium:-

(i) Air cooled condenser
(ii) Water cooled condenser
(iii) Evapourative condenser.

Evapourative condenser

The refrigerant first rejects its heat to water and then water rejects its heat to air, mainly in the form of evapourated water.
Air leaves with high humidity as in a cooling tower.

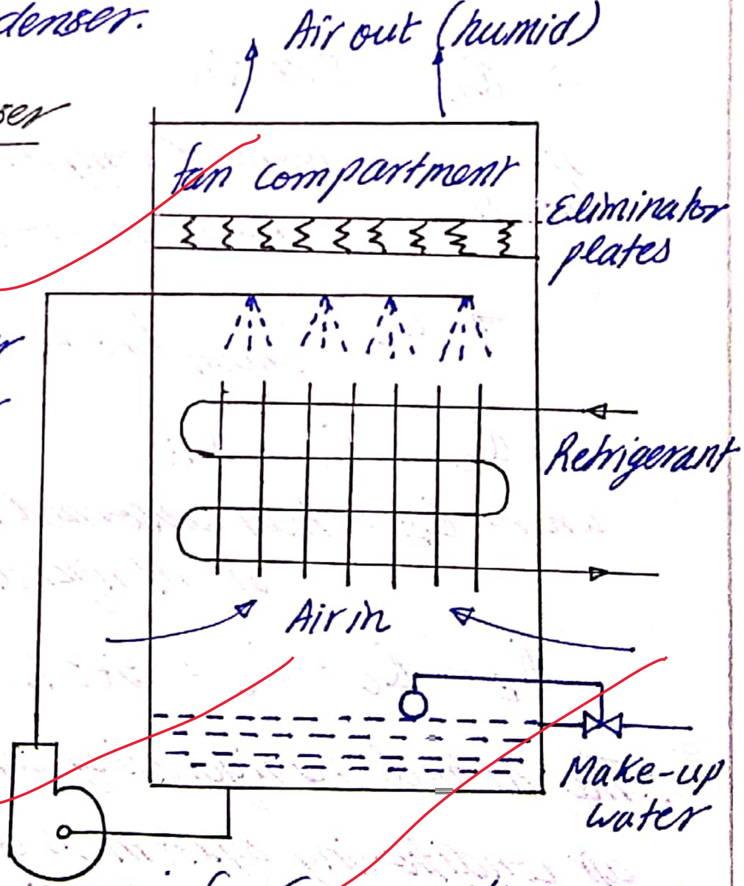

fig. Evapourative Condenser

Thus an evaporative condenser combines the functions of condenser and cooling tower.
Evaporative condensers are commonly used in large ammonia plants.

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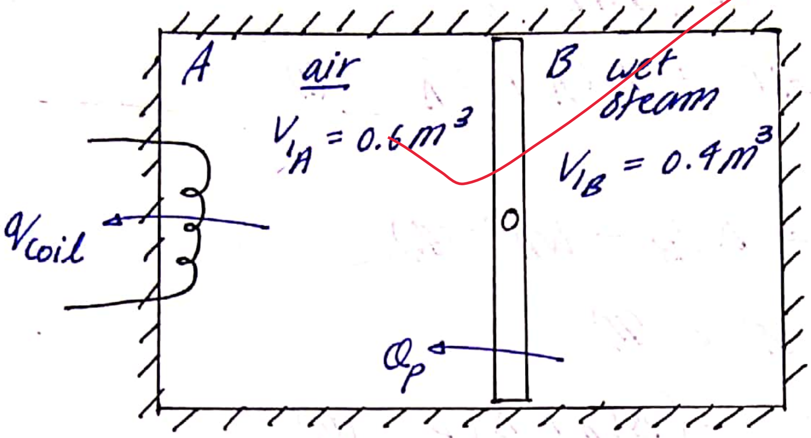
Q7(a) Assumption:- (i) Air is an Ideal gas.
(ii) Coefficient of expansion of piston material is negligible. (piston dimensions remain constant w.r.t. temp.)



$V_T = V_{1A} + V_{1B}$
$V_T = 1 m^3$

$\because$ piston will be in rest only if pressure on both sides is equal $\therefore$ $P_{1A} = P_{1B} = P_1 = 800 KPa$
$P_{2A} = P_{2B} = P_2 = 600 KPa$

$\because$ piston is diabatic and conducts perfectly
$\therefore$ temperature on both sides is equal at equilibrium state

$T_{1A} = T_{1B} = (T_{sat})_{800 KPa} = T_1 = 170.41°C$

$T_{2A} = T_{2B} = (T_{sat})_{600 KPa} = T_2 = 158.83°C$

for part A, $m_a = \frac{P_1 V_{1A}}{R T_1} = 3.7718 Kg$

$V_{2A} = \frac{m_a R T_2}{P_2} = 0.779 m^3$

$V_T = V_{2A} + V_{2B}$

$\Rightarrow V_{2B} = V_T - V_{2A} = 0.221 m^3$

Now, $v_{2B} = v_{f_{2B}} = 0.001101 m^3/kg$ (from table)

$\Rightarrow \boxed{m_B = \frac{V_{2B}}{v_{2B}} = 200.7266 Kg}$

work done by air on piston = work done by piston on water

$\Rightarrow \quad W_B = -W_A$

for part A $\quad \Delta KE = 0, \Delta PE = 0$

$$Q_p - Q_{coil} = \Delta U_A + W_A \quad \text{— (I)}$$

for part B

$$-Q_p = \Delta U_B - W_A \quad \text{— (II)}$$

from eq. (I) & (II)

$$Q_{coil} = -(\Delta U_A + \Delta U_B) \quad \text{— (III)}$$

Now, $v_{1B} = \frac{V_{1B}}{m_B} = \frac{0.4}{200.73} = 0.0019928 \ m^3/kg$

$$v_{1B} = v_{f1B} + x(v_{g1B} - v_{f1B})$$

$\Rightarrow x = 0.003669$ (using values from table)

$\therefore \quad u_{1B} = u_{f1B} + x(u_{fg_{1B}})$

$\Rightarrow \quad u_{1B} = 726.78 \ KJ/kg$ (using values from table)

$$u_{2B} = u_{f2B} = 669.72 \ KJ/kg$$

$$\Delta U_B = m_B(u_{2B} - u_{1B}) = -11453.466 \ KJ$$

$$\Delta U_A = m_A C_v (T_2 - T_1) = -31.36 \ KJ$$

$\Rightarrow Q_{coil} = -(-11453.466 - 31.36)$

$\Rightarrow \boxed{Q_{coil} = 11.48 \ MJ}$

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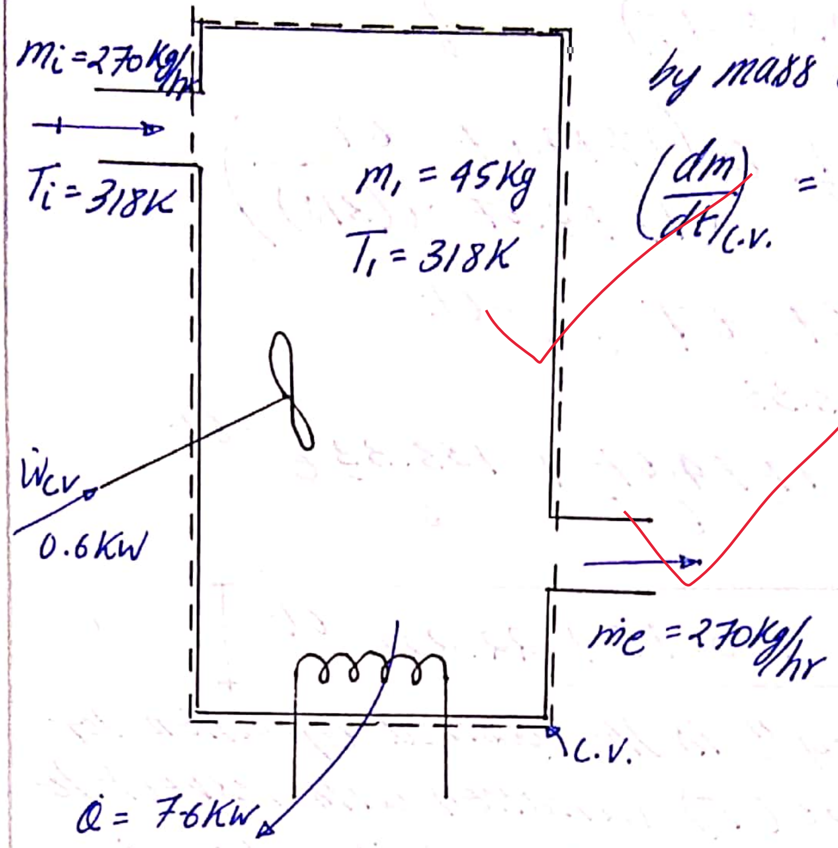
Q.7(b) Assumption:- specific heat of water, $C_w = 4.2\ KJ/kgK$



by mass conservation:-

$$\left(\frac{dm}{dt}\right)_{c.v.} = \dot{m}_i - \dot{m}_e = 0$$

by energy conservation:- ($\Delta KE = 0$, $\Delta PE = 0$)

$$\left(\frac{dU}{dt}\right)_{c.v.} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{c.v.}$$

$$\Rightarrow \left(\frac{dU}{dt}\right)_{c.v.} = 270\, C_w (318) + (-7.6 \times 3600\ KJ/hr)$$

$$- 270 C_w T - (-0.6 \times 3600\ KJ/hr)$$

$$\Rightarrow \left(\frac{dU}{dt}\right)_{c.v.} = 335412 - 1134T$$

$$U = mcT$$

$$\frac{dU}{dt} = mc\frac{dT}{dt} \quad \because \text{mass is constant} \left(\frac{dm}{dt} = 0\right)$$

$$\Rightarrow 45 \times 4.2 \frac{dT}{dt} = 335412 - 1134T$$

$$\Rightarrow \frac{dT}{dt} = 1774.67 - 6T$$

$$\Rightarrow \int_{318}^{T} \frac{1}{(1774.67 - 6T)}\, dT = \int_{0}^{t} dt$$

$$\Rightarrow t = -\frac{1}{6} \ln\left(\frac{1774.67 - 6T}{1774.67 - 6(318)}\right)$$

$$\Rightarrow \frac{1774.67 - 6T}{-133.33} = e^{-6t}$$

$$\Rightarrow T = \frac{1774.67}{6} + \frac{133.33}{6} e^{-6t}$$

$$\Rightarrow \boxed{T = 295.77 + 22.22 e^{-6t}}$$

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The above eqⁿ is the desired equation for temperature (T) variation with respect to time (t), where time (t) is in hours.

Q.7 (c). factors affecting performance of VCRS cycle :-

(i) <u>Decrease in evaporator pressure</u>

↑ → increases

↓ → decreases.

<u>Effects</u> :-

(1) Refrigeration effect ↓

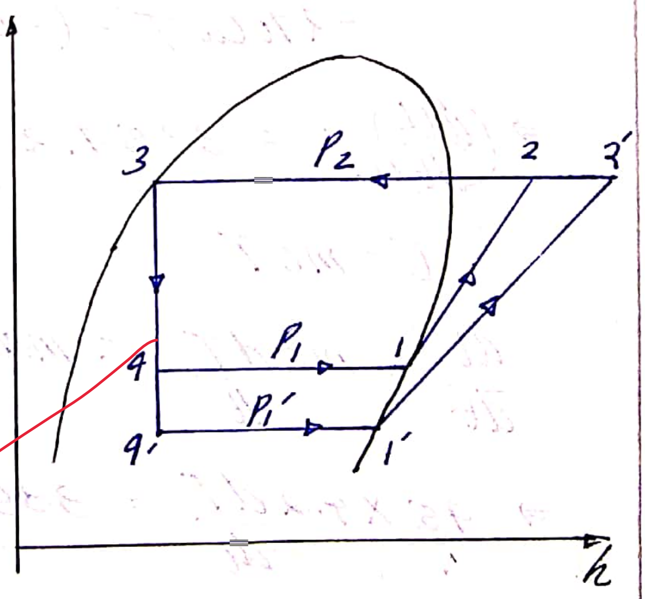
(2) Compressor work ↑

(3) specific volume ↑

∴ compressor size ↑

(4) volumetric efficiency of compressor ↓

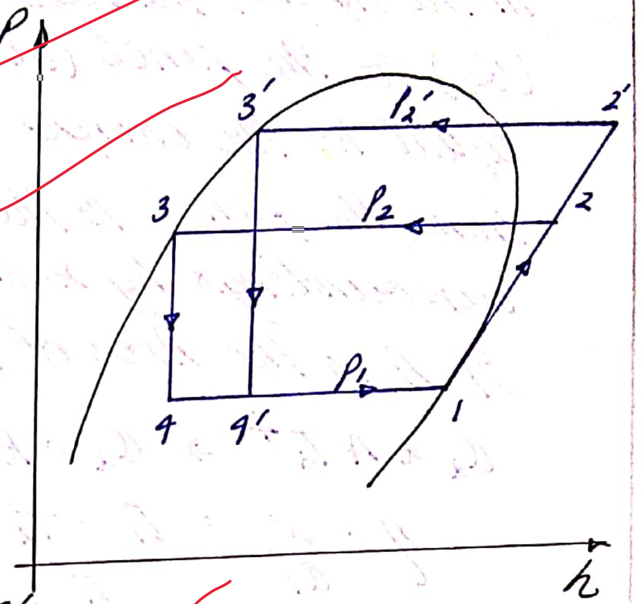
(5) COP ↓



(ii) Increase in condenser pressure

Effects:-

(1) Refrigeration effect ↓
(2) Compressor work ↑
(3) volumetric efficiency of compressor ↓
(4) COP ↓

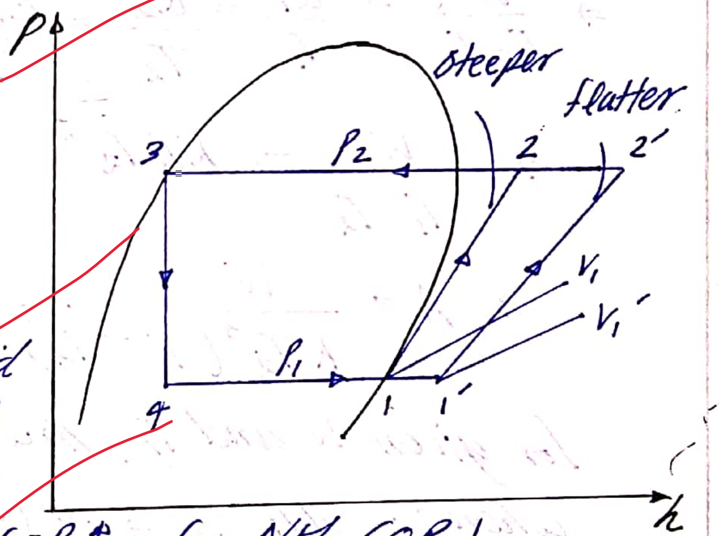


(iii) Suction vapour superheat

(a) on evaporator side:-

effect:-

(1) ref. effect ↑
(2) specific volume ↑
∴ compresser size ↑
(3) Ensures complete vapourisation of liquid before entering comp.
(4) COP may increase or decrease. for R-134, COP ↑ : for NH_3, COP ↓



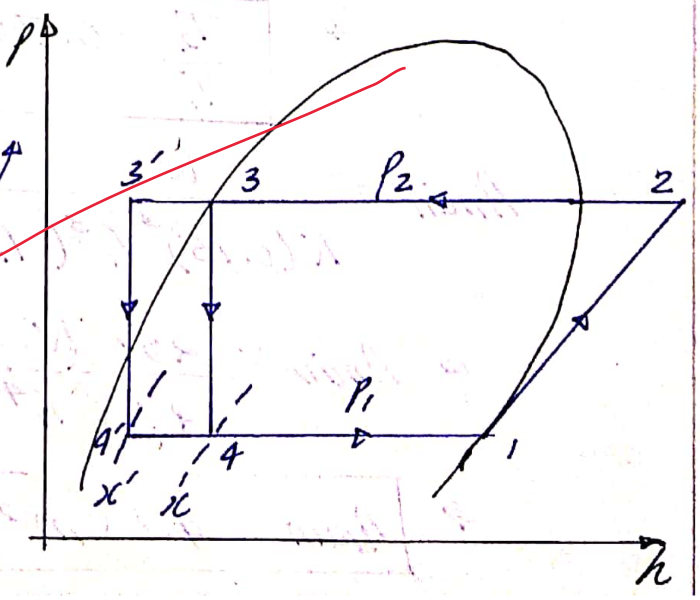
(b) ~~on condenser~~ out side evaporator:-

effects:- (i) Refrigeration effect remains same
(ii) Work required by compressor ↑
(iii) COP ↓

(iv) liquid subcooling

effects:-

(i) Refrigeration effect ↑
(ii) Compressor work remains same
(iii) COP ↑



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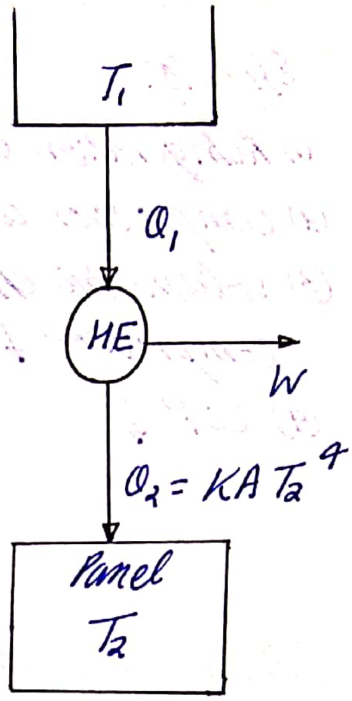
Q.8(a) for the heat engine as shown in the figure, the heat rejected (Q_2) to the panel (at T_2) is equal to the energy emitted from the panel to the surroundings by radiation. If A is the Area of panel



$$Q_2 \propto A T_2^4 \Rightarrow Q_2 = KAT_2^4$$

$$\eta = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$\Rightarrow \frac{W}{T_1 - T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{KAT_2^4}{T_2}$$

$$\Rightarrow \frac{W}{T_1 - T_2} = KAT_2^3$$

$$\Rightarrow A = \frac{W}{KT_2^3(T_1 - T_2)} = \frac{W}{K(T_1T_2^3 - T_2^4)}$$

for given W and T_1, Area will be minimum when :-

$$\frac{dA}{dT_2} = 0 \Rightarrow \frac{-W}{K}(3T_1T_2^2 - 4T_2^3)(T_1T_2^3 - T_2^4)^{-2} = 0$$

$$\therefore (T_1T_2^3 - T_2^4)^{-2} \neq 0$$

$$\Rightarrow 3T_1T_2^2 = 4T_2^3$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = 0.75}$$ hence proved.

$$A_{min} = \frac{W}{K(0.75)^3T_1^3(T_1 - 0.75T_1)}$$

$$\Rightarrow A_{min} = \frac{256\,W}{27KT_1^4}$$

$\left\{ \begin{array}{l} W = 10\,KW;\ T_1 = 800K \\ K = 5.67 \times 10^{-8}\ W/m^2K \end{array} \right\}$ given

$$\Rightarrow \boxed{A_{min} = 4.08\,m^2}$$

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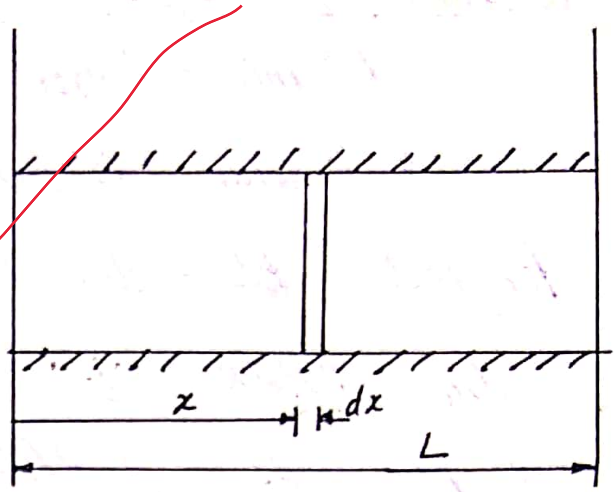
Q8(b)

final temp. is T_3

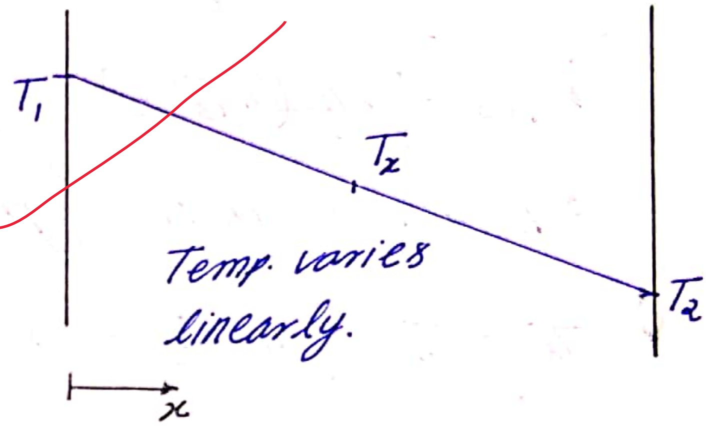
$$T_x = T_1 - \left(\frac{T_1 - T_2}{L}\right)x$$

considering a disc of thickness dx at x distance from left side.



$dm = \rho A dx$

$dU = dm\,C(T_3 - T_x)$



$$\Rightarrow dU = (\rho A dx)\,C\left[T_3 - T_1 + \left(\frac{T_1 - T_2}{L}\right)x\right]$$

by integration,

$$\Delta U = \int dU = \int_0^L \rho A C\left[T_3 - T_1 + \left(\frac{T_1 - T_2}{L}\right)x\right]dx$$

$$\Rightarrow \Delta U = \rho A C\left[(T_3 - T_1)x + \left(\frac{T_1 - T_2}{L}\right)\frac{x^2}{2}\right]_0^L$$

$$\Rightarrow \Delta U = (\rho A C L)\left[(T_3 - T_1) + \left(\frac{T_1 - T_2}{2}\right)\right]$$

Applying first law of thermodynamics on the rod :- $Q = \Delta U + W$

$Q = 0$ (insulated) $W = 0$

$\Rightarrow \Delta U = 0$

$$\Rightarrow (\rho A C L)\left[(T_3 - T_1) + \left(\frac{T_1 - T_2}{2}\right)\right] = 0$$

$$\Rightarrow \boxed{T_3 = \frac{T_1 + T_2}{2}}$$

Entropy ~~ang~~ analysis:-

$$\Delta S_{univ} = \Delta S_{rod} + \cancel{\Delta S_{surr.}}^{\,0} \text{ (insulated)}$$

$$\Rightarrow \Delta S_{univ} = \Delta S_{rod}.$$

for rod:- $ds = dmc \ln\left(\frac{T_3}{T_x}\right)$

$$dm = \rho A dx$$

$$\Rightarrow \Delta S = \int ds = \int_0^L \rho A C \ln\left(\frac{T_3}{T_x}\right) dx$$

$$\Rightarrow \Delta S = \left[\rho AC(\ln T_3) x\right]_0^L - \int_0^L \rho AC(\ln T_x) dx$$

$$\Rightarrow \Delta S = \rho ACL \ln T_3 - \int_0^L \rho AC(\ln T_3) dx$$

$$T_x = T_1 - \left(\frac{T_1 - T_2}{L}\right)x \Rightarrow dT_x = -\left(\frac{T_1 - T_2}{L}\right)dx$$

$$\Rightarrow dx = \frac{-L}{(T_1 - T_2)} dT_x$$

$$\Rightarrow \Delta S = \rho ACL \ln T_3 - \int_0^L \rho AC(\ln T_x)\left(\frac{-L}{T_1 - T_2}\right) dT_x$$

$$\Rightarrow \Delta S = \rho AC \ln T_3 - \left(\frac{\rho ACL}{T_1 - T_2}\right) \int_{T_2}^{T_1} (\ln T_x) dT_x$$

$$\Rightarrow \Delta S = \rho ACL \ln T_3 - \left(\frac{\rho ACL}{T_1 - T_2}\right)\left[T_x \ln T_x - T_x\right]_{T_2}^{T_1}$$

$$\Rightarrow \Delta S = mc \ln T_3 - \left(\frac{mc}{T_1 - T_2}\right)\left[T_1 \ln T_1 - T_2 \ln T_2 - (T_1 - T_2)\right]$$

$$(\because \rho AL = m)$$

$$\Rightarrow \Delta S = mc \ln T_3 - mc\left[\left(\frac{T_1 \ln T_1}{T_1 - T_2}\right) - \left(\frac{T_2 \ln T_2}{T_1 - T_2}\right) - 1\right]$$

$$\Rightarrow \boxed{\Delta S = \Delta S_{univ} = mc\left[1 + \ln T_3 + \frac{T_2 \ln T_2}{T_1 - T_2} - \frac{T_1 \ln T_1}{T_1 - T_2}\right]}$$

Hence Proved.

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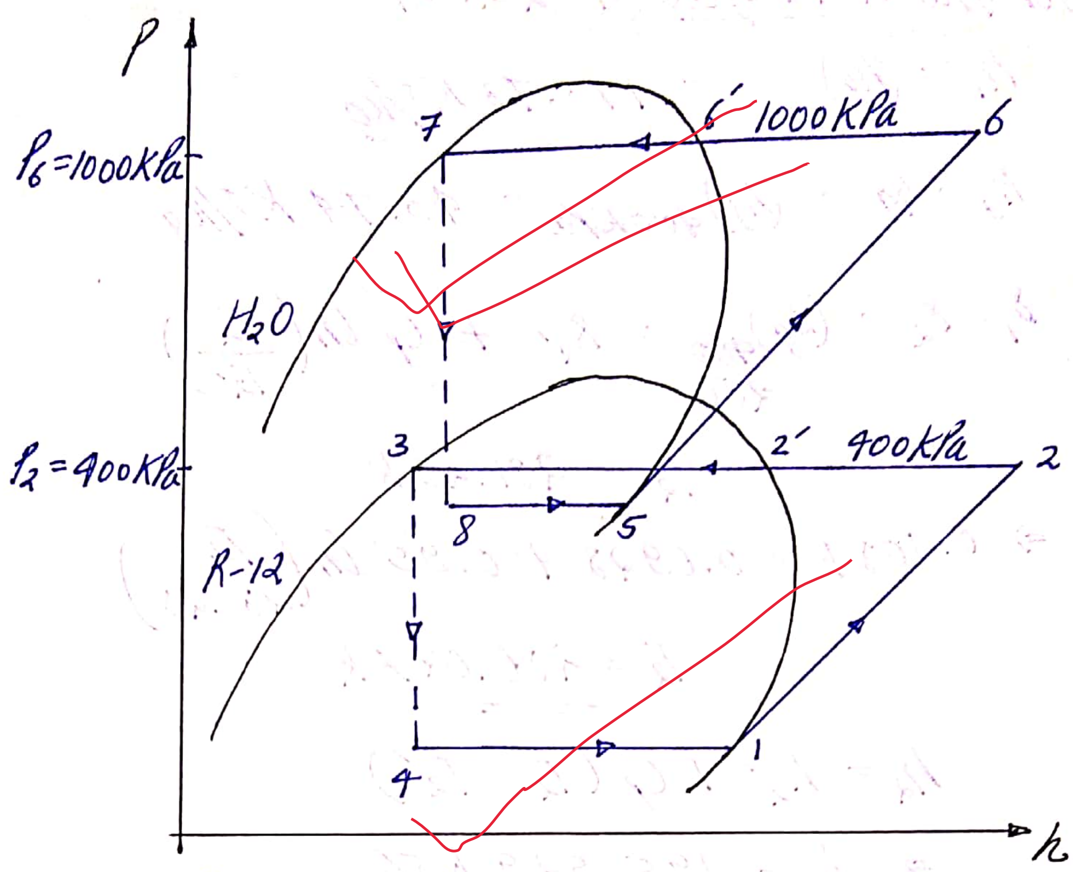
Q.8 (c)



given $(C_p)_{vapour}$ for $H_2O = 2.1\ KJ/kgK$

$(C_p)_{vapour}$ for R-12 $= 0.95\ KJ/kgK$

for H_2O cycle 5-6-7-8

$h_5 = (h_g)_{5°C} = 2510.1\ KJ/kg$

$h_8 = h_7 = (h_f)_{1000KPa} = 762.51\ KJ/kg$

$s_6 = s_5 = (s_g)_{5°C} = (s_g)_{1000KPa} + C_p \ln\left(\frac{T_6}{T_6'}\right)$

$T_6' = 452.88\,K$ (from table)

$\Rightarrow 9.0249 = 6.5850 + 2.1 \ln\left(\frac{T_6}{452.88}\right)$

$\Rightarrow T_6 = 1447.34\,K$

$\Rightarrow h_6 = h_{6'} + 2.1\,(T_6 - T_6')$

$\Rightarrow h_6 = \underline{4865.466}\ KJ/kg.$

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for the R-12 cycle 1-2-3-4

$h_1 = (h_g)_{-20°C} = 178.73 \text{ KJ/kg}$

$h_3 = h_4 = (h_f)_{400kPa} = 43.74 \text{ KJ/kg}$

$s_1 = s_2 = (s_g)_{-20°C} = s_{2'} + C_p \ln\left(\frac{T_2}{T_{2'}}\right)$

$T_{2'} = 281.28 \text{ K}$

$\Rightarrow 0.7087 = 0.6928 + 0.95 \ln\left(\frac{T_2}{281.28}\right)$

$T_2 = \underline{286.027} \text{ K}$

$h_2 = h_{2'} + C_p(T_2 - T_{2'})$

$\Rightarrow h_2 = 195.529 \text{ KJ/kg}$

∴ mass flow rates:-

mass flow rate of R-12 $= \dot{m}_R = \frac{RC}{(h_1 - h_4)}$

$RC = 10 \text{ TR} = 350 \text{ KW}$

$\Rightarrow \boxed{\dot{m}_R = \underline{2.59 \text{ Kg/s}} = 155.4 \text{ Kg/min}}$

mass flow rate of water $= \dot{m}_w = \frac{\dot{m}_R (h_2 - h_4)}{(h_5 - h_8)}$

$\Rightarrow \boxed{\dot{m}_w = 0.225 \text{ Kg/s} = 13.51 \text{ Kg/min}}$

$COP = \frac{RC}{\dot{W}_1 + \dot{W}_2}$

$\dot{W}_1 = \dot{m}_R (h_2 - h_1) = 43.51 \text{ KW}$

$\dot{W}_2 = \dot{m}_w (h_6 - h_5) = 530.43 \text{ KW}$

$\Rightarrow \boxed{COP = \frac{350}{573.94} = \underline{0.61}}$

∴ The COP of cascade system is $\underline{0.61}$

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