NAME —

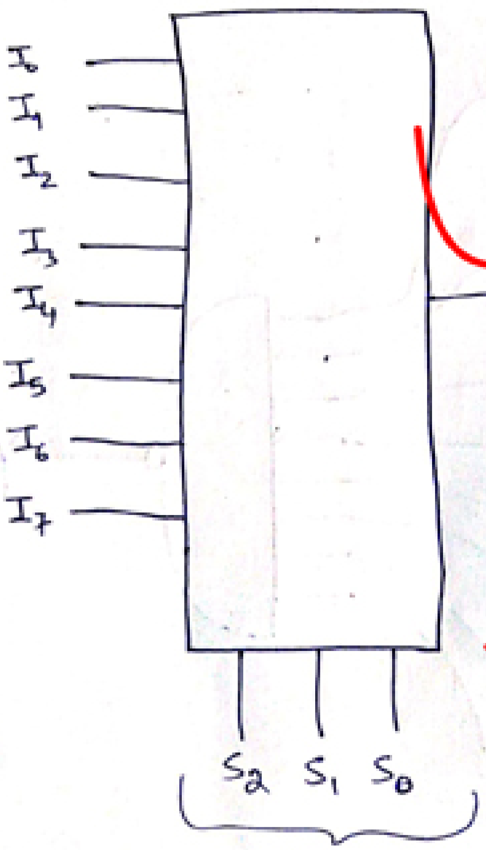
ROLL NO —

TEST NO — 1

SUBJECT NAME — Digital Electronics + Control Systems

Ans 1 (a) Design the 8x1 multiplexer using NAND gates only.

→



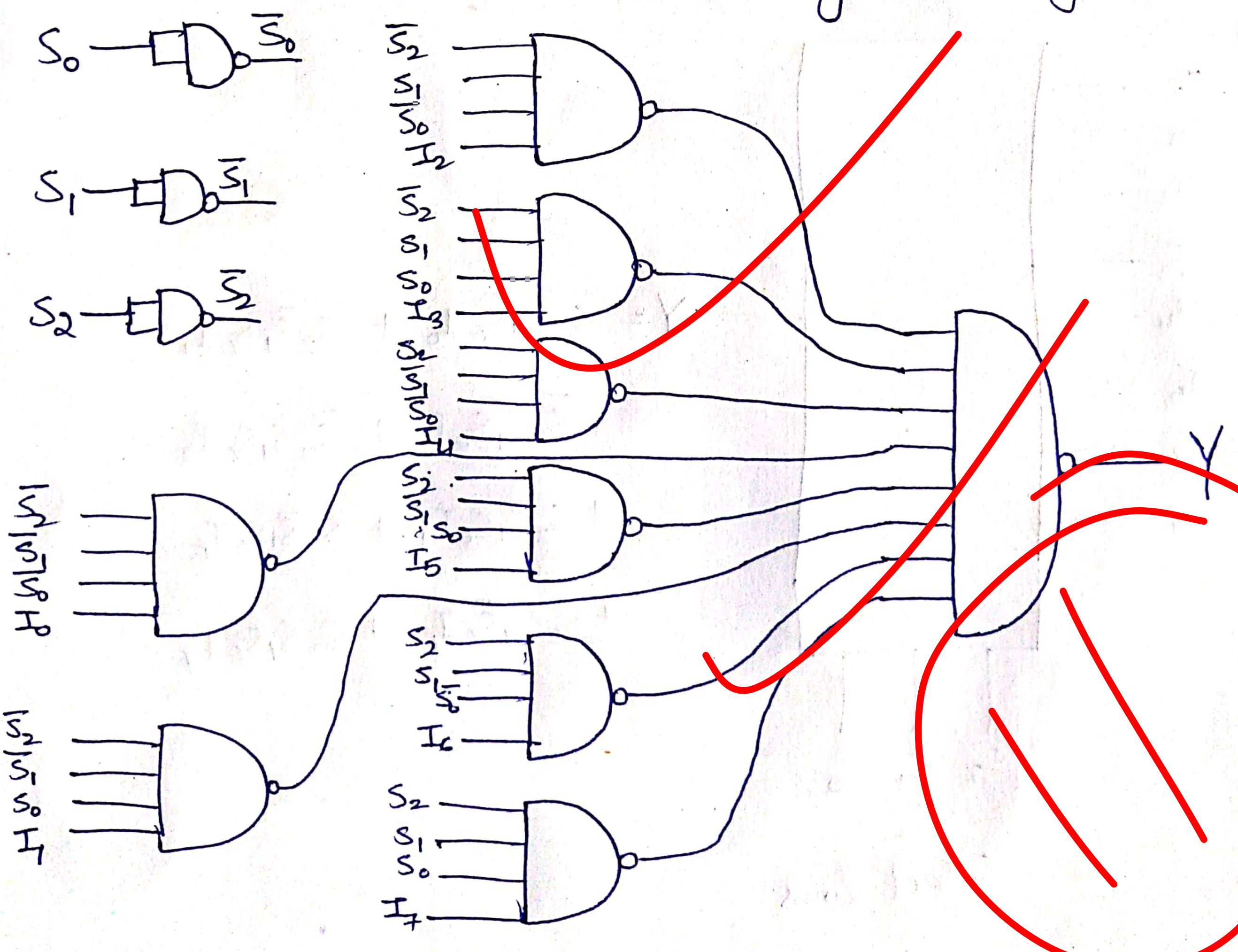
$$Y = \bar{S}_2\bar{S}_1\bar{S}_0 I_0 + \bar{S}_2\bar{S}_1 S_0 I_1 + \bar{S}_2 S_1 \bar{S}_0 I_2 + \bar{S}_2 S_1 S_0 I_3 + S_2\bar{S}_1\bar{S}_0 I_4 + S_2\bar{S}_1 S_0 I_5 + S_2 S_1 \bar{S}_0 I_6 + S_2 S_1 S_0 I_7$$

$$\overline{\overline{Y}} = \overline{\overline{\bar{S}_2\bar{S}_1\bar{S}_0 I_0 + \bar{S}_2\bar{S}_1 S_0 I_1 + \bar{S}_2 S_1\bar{S}_0 I_2 + \bar{S}_2 S_1 S_0 I_3 + S_2\bar{S}_1\bar{S}_0 I_4 + S_2\bar{S}_1 S_0 I_5 + S_2 S_1\bar{S}_0 I_6 + S_2 S_1 S_0 I_7}}$$

Applying De-morgan's law

$$Y = \left(\overline{(\bar{S}_2\bar{S}_1\bar{S}_0 I_0)} \cdot \overline{(\bar{S}_2\bar{S}_1 S_0 I_1)} \cdot \overline{(\bar{S}_2 S_1\bar{S}_0 I_2)} \cdot \overline{(\bar{S}_2 S_1 S_0 I_3)} \cdot \overline{(S_2\bar{S}_1\bar{S}_0 I_4)} \cdot \overline{(S_2\bar{S}_1 S_0 I_5)} \cdot \overline{(S_2 S_1\bar{S}_0 I_6)} \cdot \overline{(S_2 S_1 S_0 I_7)}\right)'$$

Implimenting the Output Y using NAND gates.



Ans 1 b) For a 8-bit counter type analog to digital converter using a 2MHz clock.

Determine:

i> Maximum conversion rate

As frequency given $= 2\times10^6$ Hz

Time period $T = \frac{1}{2\times10^6} = 0.5\,\mu sec.$

max^m conversion rate $= \frac{1}{T} = \frac{1}{0.5} = \boxed{2\times10^6 \text{ per sec}}$

ii> Maximum conversion time $= (2^n-1)T$

$= (2^8-1)\,0.5\,\mu sec$

$\boxed{= 127.5\,\mu sec}$

iii> Average conversion time = ~~Minimum~~

Avg. conversion time $= \frac{\text{Min}^m \text{ con. time} + \text{Max}^m \text{ con. time}}{2}$

$= \frac{T + (2^n-1)T}{2}$

$= \frac{(0.5+127.5)\,\mu sec}{2}$

$\boxed{= 64\,\mu sec}$

Ans 1 c) Convert a D-flip flop to function as a SR flip-flop. Draw circuit.

→ Characteristic table of SR

S	R	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	X

Excitation table of D flip flop

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

D = S + RQ

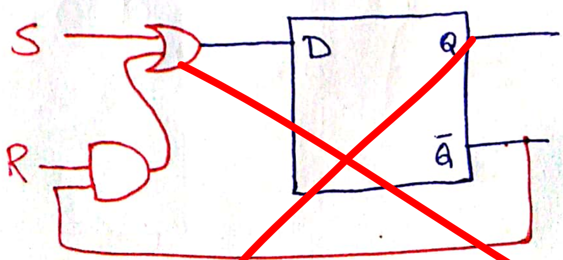
Now to convert D flip flop to SR flip flop —

S	R	Q_n	Q_{n+1}	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	X	X
1	1	1	X	X

9

S \ RQ_n	00	01	11	10
$\bar{S}$		1		
S	1	1	X	X

$\boxed{D = S + \bar{R}Q_n}$



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Ans 1 d) Reduce the expression $f(A,B,C,D) = \Pi M(2,8,9,10,11,12,14)$ and implement the same using NOR gates.

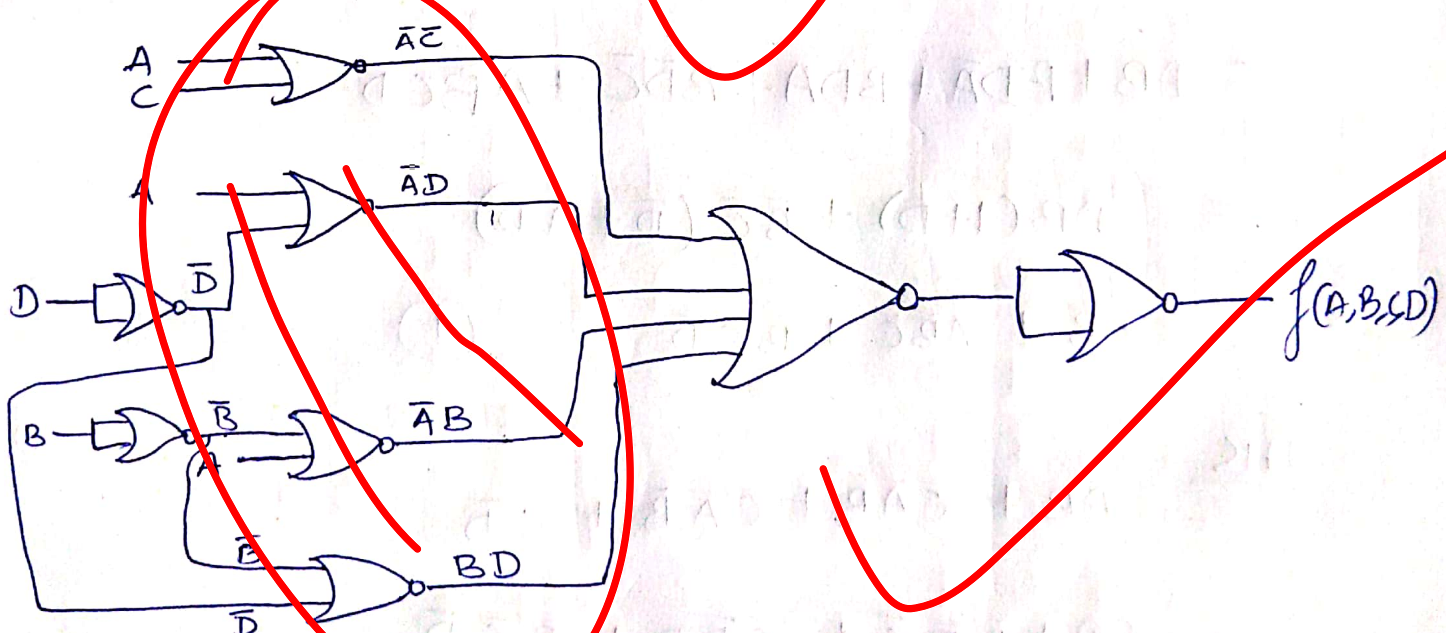
A→

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	1 (3)	(2)
$\bar{A}B$	1 (4)	1 (5)	1 (7)	1 (6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	(8)	(9)	(11)	(10)

$\Pi(M)(2,8,9,10,11,12,14)$

$= \Sigma m(0,1,3,4,5,6,7,13,15)$

$f(A,B,C,D) = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + BD = \bar{A}(B+\bar{C}+D) + BD$



Ans 1 e) If $\bar{A}B + C\bar{D} = 0$, then prove that

$$AB + \bar{C}(\bar{A}+\bar{D}) = AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$$

$\bar{A}B + C\bar{D} = 0$ means $\bar{A}B = 0$

$C\bar{D} = 0$

taking LHS

$$AB + \bar{C}\bar{A} + \bar{C}\bar{D}$$

RHS

$$AB + BD + \bar{B}\bar{D} + \bar{A}\bar{C}D$$

$$= AB + BD + \cancel{\bar{B}\bar{D}C}^{\,0} + \bar{B}\bar{D}\bar{C} + \cancel{\bar{A}\bar{C}DB}^{\,0} + \bar{A}\bar{C}D\bar{B}$$

$$= AB + \cancel{BD\bar{A}}^{\,0} + BDA + \bar{B}\bar{D}\bar{C} + \bar{A}\bar{B}\bar{C}D$$

$$= AB(1+D) + \bar{B}\bar{C}(\bar{D} + \bar{A}D)$$

$$= AB + \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C}\bar{D} \quad \text{—— (1)}$$

LHS

$$= AB + \cancel{\bar{C}\bar{A}B}^{\,0} + \bar{C}\bar{A}\bar{B} + \bar{C}\bar{D}$$

$$= AB + \bar{A}\bar{B}\bar{C} + \bar{C}\bar{D}B + \bar{B}\bar{C}\bar{D}$$

$$= AB + \bar{A}\bar{B}\bar{C} + \bar{B}\bar{C}\bar{D} \quad \text{—— (2)}$$

eqn (1) = (2) Hence proved.

Ans 4 a) What is Gray code? Give applications of Gray code?

→ It is a binary code. It is a way of generating binary numbers such that every step has a change of single bit only. i.e Hamming distance of this code is 1.

Applications

- Karnaugh Map
- Rotary encoders

(As there is only one digit change from one step to next step. Its easy to find the next location).

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Ans 4 b) Obtain minimal SOP expression for outputs of combinational circuit that produce the 2's complement of a 4 bit binary number ?

Let the 4 bit binary nos are ABCD.

and the corresponding 2's complement = PQRS

So

A	B	C	D	P	Q	R	S
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

Now we find the expression for P, Q, R, S using K-map.

(For P)

AB \ CD	00	01	11	10
00		1	1	1
01	1	1	1	1
11				
10	1			

(For Q)

AB \ CD	00	01	11	10
00		1	1	1
01	1			
11	1			
10		1	1	1

$P = \bar{A}B + \bar{A}D + \bar{A}C + A\bar{B}\bar{C}\bar{D}$

$Q = \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$

$$\boxed{P = \bar{A}(B+C+D) + A\bar{B}\bar{C}\bar{D}}$$

$$\boxed{Q = \bar{B}(C+D) + B(\bar{C}\bar{D})}$$

(For R)

AB \ CD	00	01	11	10
00		1		1
01		1		1
11		1		1
10		1		1

(For S)

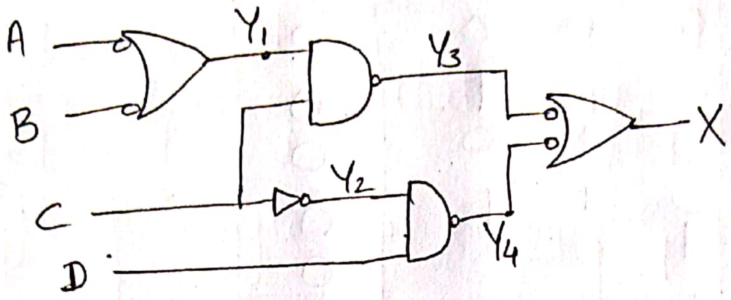
AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10		1	1	

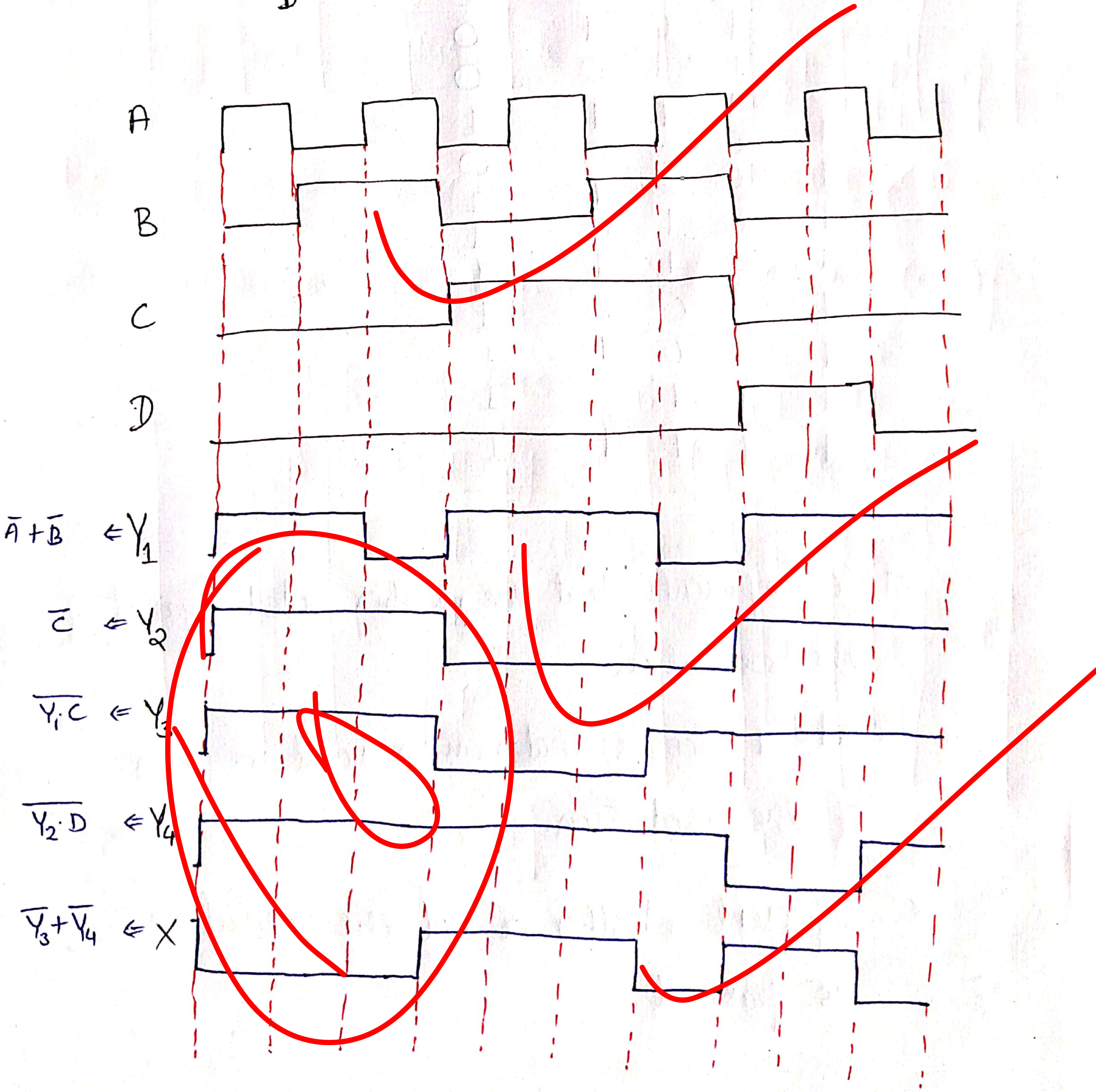
$$\boxed{R = \bar{C}D + C\bar{D} = C \oplus D}$$

$$\boxed{S = D}$$

Ans 4 c)

Consider the logic circuit shown below. Waveforms of A, B, C, D are shown below.





Ans 4 d) The possible no. of cases are given in Truth table form.

P (30)	Q (25)	R (23)	S (22)	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Let '0' represents that the member voted against the resolution.

and '1' denotes that member voted in favor of the resolution.

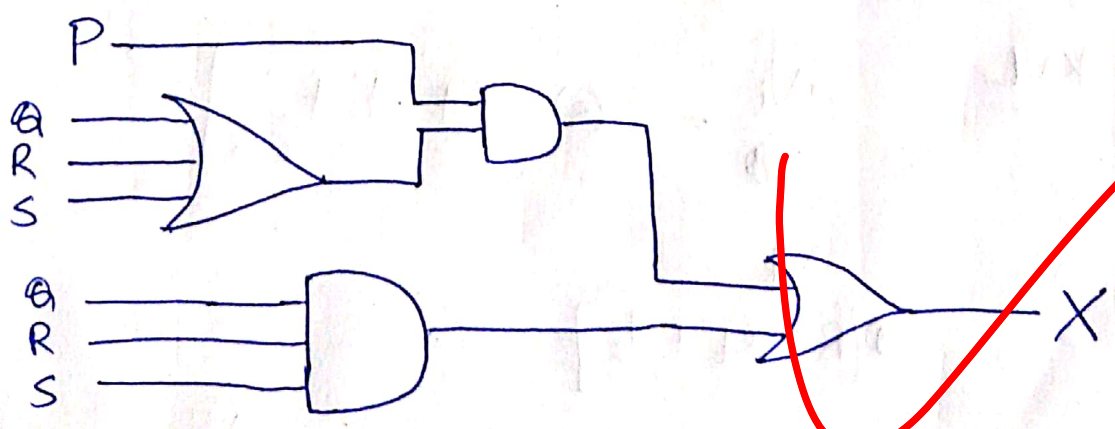
X represent wheather or not the resolution is passed.

when sum of the votes are greater than or equal to 50% in favor of resolution then we take $X = 1$.

PQ \ RS	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$	(0)	(1)	(3)	(2)
$\bar{P}Q$	(4)	(5)	1 (7)	(6)
PQ	1 (12)	1 (13)	1 (15)	1 (14)
$P\bar{Q}$	(8)	1 (9)	1 (11)	1 (10)

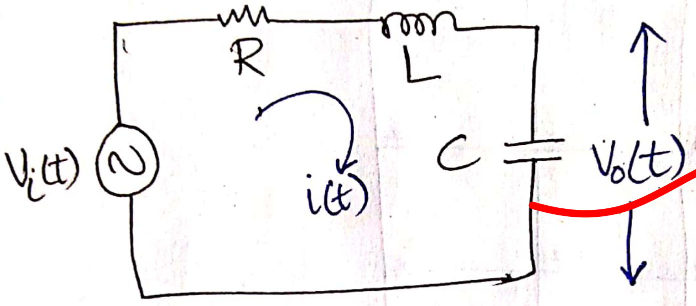
$$X = PQ + PS + PR + QRS$$

$$\boxed{X = P(Q+R+S) + QRS}$$

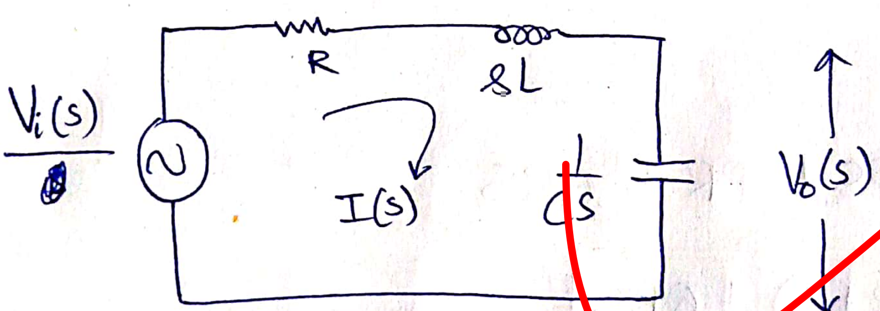


Ans 5 a)

If $V_i(t)$ is a step voltage in the network shown in figure, find the value of resistor such that a 20% overshoot in voltage will be seen across the capacitor if $C = 10^{-6}$ F and $L = 0.5H$.



Applying Laplace to ckt and redrawing –



By KVL

$$\frac{V_i}{s} = I(s)\left[R + sL + \frac{1}{Cs}\right]$$

$$I(s) = \frac{V_i}{s\left[R + sL + \frac{1}{Cs}\right]}$$

Also, $V_o(s) = I(s) \cdot \frac{1}{Cs} = \frac{1}{sC}\left[\frac{V_i}{sR + s^2L + 1/C}\right]$

$$V_o(s) = \frac{V_i(s)}{s^2LC + sRC + 1}$$

$$\text{Transfer function} = \frac{V_o}{V_i} = \frac{1/LC}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Compairing with $s^2 + 2\xi\omega_n s + \omega_n^2$

$$\frac{R}{L} = 2\xi\omega_n \qquad \omega_n^2 = \frac{1}{LC} = \frac{1}{0.5\times10^{-6}}$$

$$\omega_n = \sqrt{2\times10^6} = 1.414 \text{ Krad/s}$$

given: $M_p = 20\%$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$0.2 = e^{-\pi/\tan\theta}$$

$$\ln(0.2) = -\pi/\tan\theta$$

$$\tan\theta = 1.952$$

$$\theta = 62.874^\circ$$

$$\cos\theta = \xi = 0.456$$

$$2\xi\omega_n = \frac{R}{L}$$

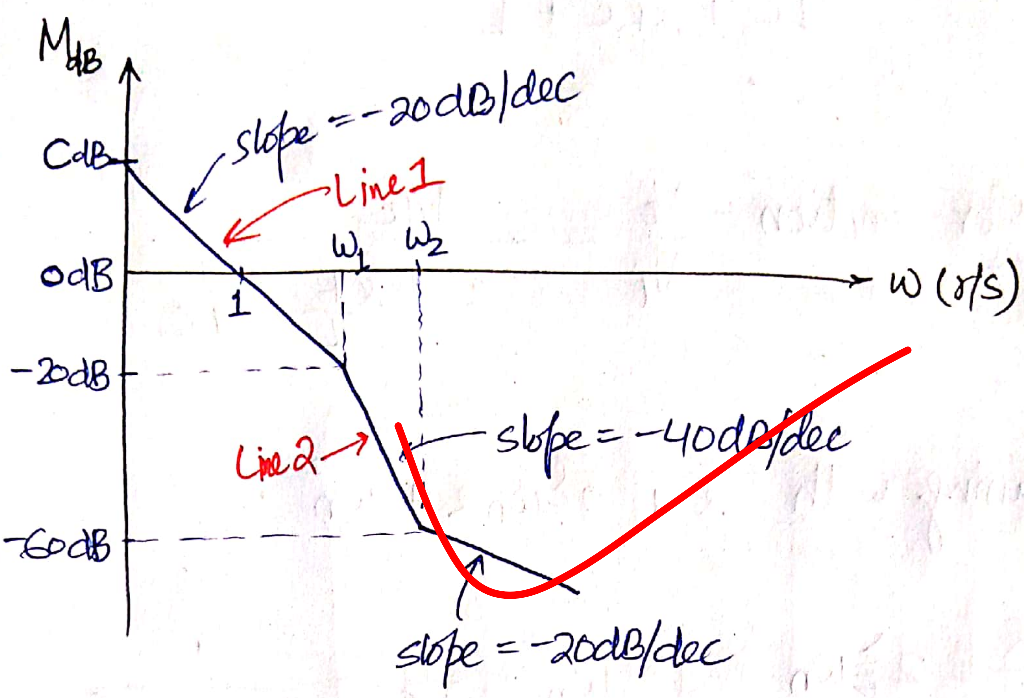
$$R = L \cdot 2\xi\omega_n$$

$$R = 0.5\times2\times0.456\times1.414\times10^3$$

$$\boxed{R = 644.81\ \Omega}$$

2.

Ans 5. b) Determine the Transfer Function.



From the Bode plot it is clear that we have a pole at origin, then another pole at $\omega=\omega_1$ and a zero at $\omega=\omega_2$.

So the transfer function would be -

$$T(s) = \frac{K(1+s/\omega_2)}{s(1+s/\omega_1)}$$

$$T(j\omega) = \frac{K(1+j\omega/\omega_2)}{j\omega(1+j\omega/\omega_1)}$$

$$\text{Slope of line 2} = \frac{-20-(-60)}{\log\omega_1 - \log\omega_2} = -40 \text{ dB/dec}$$

$$\frac{40}{\log\omega_2 - \log\omega_1} = 40$$

$$\log\omega_2 = \log\omega_1 + 1$$

$$\log\omega_2 = \log\omega_1 + \log 10$$

$$\boxed{\omega_2 = 10\omega_1} \quad \text{——— (1)}$$

<u>Slope of Line 1</u> $= \dfrac{-20}{\log\omega_1 - \log 1} = -20$

$$\log\omega_1 = \log 1 + 1 \qquad \boxed{\omega_1 = 10 \text{ rad/s}}$$

from eqⁿ (1) $\boxed{\omega_2 = 100 \text{ rad/s}}$

<u>Eqⁿ of Line 1 can be written as</u>

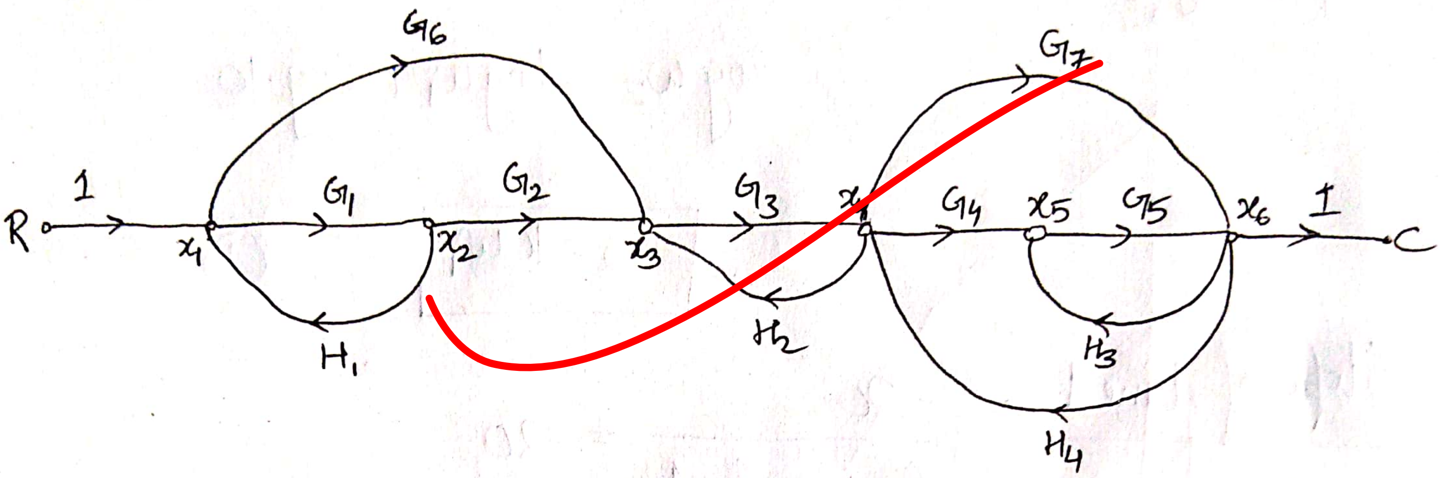
$$-20 = -20\log\omega_1 + C$$

$$C = 0$$

$$C = 20\log K \qquad \boxed{K = 1}$$

$$\boxed{\text{Transfer Function} \Rightarrow T(s) = \frac{\left(1 + \frac{s}{100}\right)}{s\left(1 + s/10\right)}}$$

Ans 5 c) By Mason's gain formula, Determine $\frac{C}{R}$



Mason's Gain Formula : $\frac{C}{R} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$

P_i = Forward path

Δ_i = Δ corresponding to P_i

$\Delta = 1 - \left(\begin{matrix}\text{Sum of}\\ \text{total loops}\end{matrix}\right) + \left(\begin{matrix}\text{Sum of}\\ \text{Two non-touching loops}\end{matrix}\right) - \left(\begin{matrix}\text{Sum of}\\ \text{Three non-touching loops}\end{matrix}\right) + \ldots$

Total loops :

$L_1 = G_1 H_1$

$L_2 = G_3 H_2$

$L_3 = G_5 H_3$

$L_4 = G_4 G_5 H_4$

$L_5 = G_7 H_4$

Forward paths: $P_1 = G_1G_2G_3G_4G_5$ $\quad \Delta_1 = 1 -$ loops not touching P_1

$\Delta_1 = 1$

$P_2 = G_6G_3G_4G_5 \quad \Delta_2 = 1$

$P_3 = G_1G_2G_3G_7 \quad \Delta_3 = 1$

$P_4 = G_6G_3G_7 \quad \Delta_4 = 1$

Two Non-touching loops: $L_1L_3 = G_1H_1G_5H_3$

$L_1L_2 = G_1H_1G_3H_2$

$L_1L_4 = G_1H_1G_4G_5H_4$ $\quad L_1L_5 = G_1H_1G_7H_4$

$L_2L_3 = G_3H_2G_5H_3$

Three non-touching loops: $L_1L_2L_3 = G_1H_1G_3H_2G_5H_3$

Now the transfer function $\frac{C}{R}$ using Mason's Gain formula is —

$$\frac{C}{R} = \frac{P_1 + P_2 + P_3 + P_4}{1 - L_1 - L_2 - L_3 - L_4 - L_5 + L_1L_2 + L_2L_3 + L_1L_4 + L_1L_5 - L_1L_2L_3}$$

$$\boxed{\frac{C}{R} = \frac{G_1G_2G_3G_4G_5 + G_3G_4G_5G_6 + G_1G_2G_3G_7 + G_3G_6G_7}{1 - G_1H_1 - G_3H_2 - G_5H_3 - G_4G_5H_4 - G_7H_4 + G_1H_1\left[G_3H_2 + G_5H_3 + G_4G_5H_4 + G_7H_4\right] - G_1H_1G_3H_2G_5H_3}}$$

10

Ans 5 d). Given: 2% tolerance

$$R(s) = \frac{1}{s} \longrightarrow \boxed{\frac{15}{s+15}} \longrightarrow C(s).$$

$$C(s) = \frac{15}{s^2+15s} = \frac{A}{s} + \frac{B}{s+15}$$

Using partial fraction $A = 1$, $B = -1$

$$C(s) = \frac{1}{s} - \frac{1}{s+15}$$

Taking Laplace inverse

$$\boxed{c(t) = u(t)\left[1 - e^{-15t}\right]}$$

So, $\boxed{\text{Time Constant} = \frac{1}{15} \text{ sec}}$

Rise time $\Rightarrow$ $1 - e^{-15t} = 0.9$

$e^{-15t} = 0.1$ $\quad -15t = \ln(0.1)$

Rise time = $\boxed{t_s = 0.1535 \text{ sec}}$

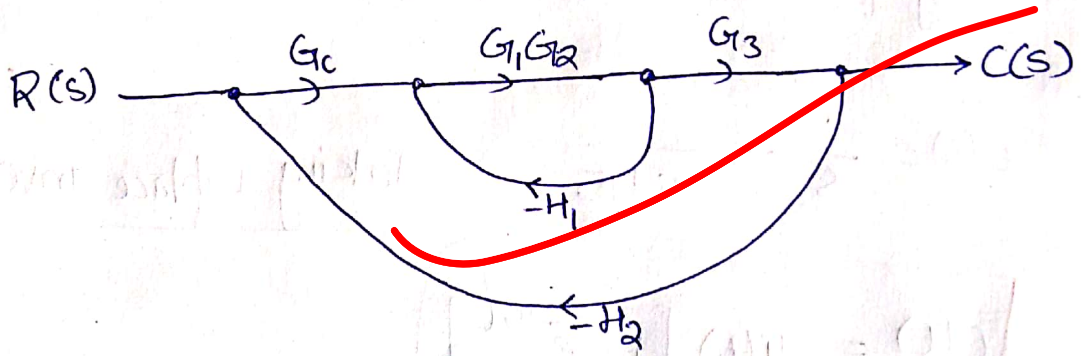
Settling time $\Rightarrow$ $1 - e^{-15t} = 0.98$

$-15t = \ln(0.02)$ $\quad \boxed{t_s = 0.26 \text{ sec}}$

Ans 5 e) $\left.\dfrac{C(s)}{R(s)}\right|_{D(s)=0}$ and $\left.\dfrac{C(s)}{D(s)}\right|_{R(s)=0}$

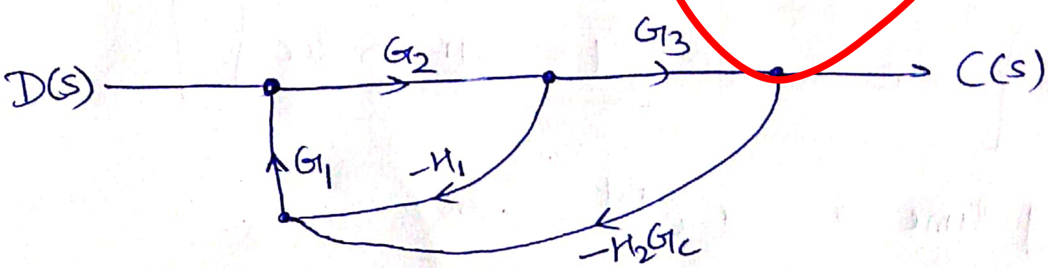
Considering $D(s)=0$

Signal flow graph



$$\boxed{\left.\frac{C(s)}{R(s)}\right|_{D(s)=0} = \frac{G_c G_1 G_2 G_3}{1 + G_1 G_2 H_1 + G_c G_1 G_2 G_3 H_2}}$$

Now taking $R(s)=0$.



$$\left.\frac{C(s)}{D(s)}\right|_{R(s)=0} = \frac{G_2 G_3}{1+G_1G_2H_1+G_1G_2G_3G_cH_2}$$

Here Transfer functions are calculated by using Mason's Gain formula.

$$\frac{C}{R} = \frac{\sum_{K=1}^{n} P_K \Delta_K}{\Delta}$$

P_K = Forward path Gain

Δ_K = Δ corresponding to P_K

$\Delta = 1 - \left(\text{Sum of total loops}\right) + \left(\text{Sum of two non-touching loops}\right) - \left(\text{Sum of three non-touching loops}\right) + \cdots$

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Ans 6 a) $$T(s) = \frac{73.626}{(s+3)(s^2+4s+24.542)}$$

Unit step response $C(s)$.

$$C(s) = \frac{73.626}{s(s+3)(s^2+4s+24.542)}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{Cs+D}{(s+2)^2+20.542}$$

Using partial fraction, & solving

$A=1$ $B=-1.14$ $C=0.14$ $D=-2.856$

$$C(s) = \frac{1}{s} - \frac{1.14}{s+3} + \frac{0.14s-2.856}{(s+2)^2+20.542}$$

$$= \frac{1}{s} - \frac{1.14}{s+3} + 0.14\left[\frac{s-20.4}{(s+2)^2+20.542}\right]$$

$$C(s) = \frac{1}{s} - \frac{1.14}{s+3} + 0.14\left[\frac{s+2}{(s+2)^2+20.542} - (4.94)\frac{4.532}{(s+2)^2+20.542}\right]$$

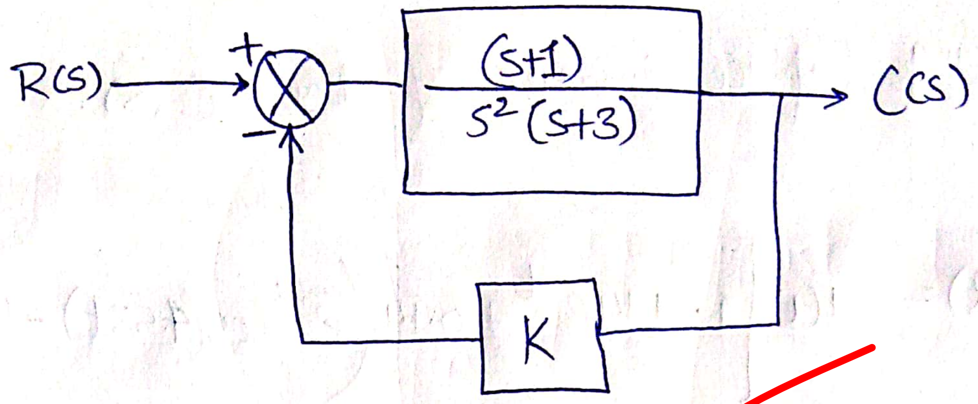
Taking Laplace inverse of the $C(s)$.

$$c(t) = u(t)\left[1 - 1.14e^{-3t} + 0.14\left(e^{-2t}\cos(4.53t) - 4.94e^{-2t}\sin(4.53t)\right)\right]$$

$$\boxed{c(t) = \left[1 - 1.14e^{-3t} + e^{-2t}\left(0.14\cos(4.53t) - 0.692\sin(4.53t)\right)\right]}$$

12

Ans 6 b)



To find the type of the system, we need to convert it into a unity feedback system.

$$G(s) = \frac{S+1}{s^2(S+3)} \qquad H(S) = K$$

$$\text{Open Loop TF} = \frac{G(S)}{1+GH-G(S)}$$

$$= \frac{S+1/s^2(S+3)}{1+\frac{K(S+1)}{s^2(S+3)} - \frac{S+1}{s^2(S+3)}}$$

$$= \frac{S+1}{s^2(S+3)+K(S+1)-(S+1)}$$

$$\boxed{G'(S) = \frac{S+1}{s^2(S+3)+(S+1)(K-1)}}$$

i) $\boxed{\text{From } G'(s) \text{ Type of System } = 0}$

ii) For $e_{ss} = 0.001$ when $R(s) = \frac{1}{s}$ $K = ?$

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G'(s) H(s)} \qquad [H(s) = 1]$$

$$= \lim_{s \to 0} \frac{s \cdot 1/s}{1 + \frac{s+1}{s^2(s+3) + (s+1)(K-1)}}$$

$$= \lim_{s \to 0} \frac{1}{1 + \frac{1}{K-1}}$$

$$e_{ss} = \frac{K-1}{K} = 0.001$$

$$0.001K = K - 1$$

$$0.999K = 1$$

$$\boxed{K = 1.001} = \frac{1000}{999}$$

Ans6 c)

A/c to Nyquist criteria, $N = P - Z$

N = no. of encirclements by Nyquist plot of ~~origin~~ (-1,0) point.

P = RHS poles of open loop Transfer funᶜ.

Z = RHS zeros of characteristic eqⁿ

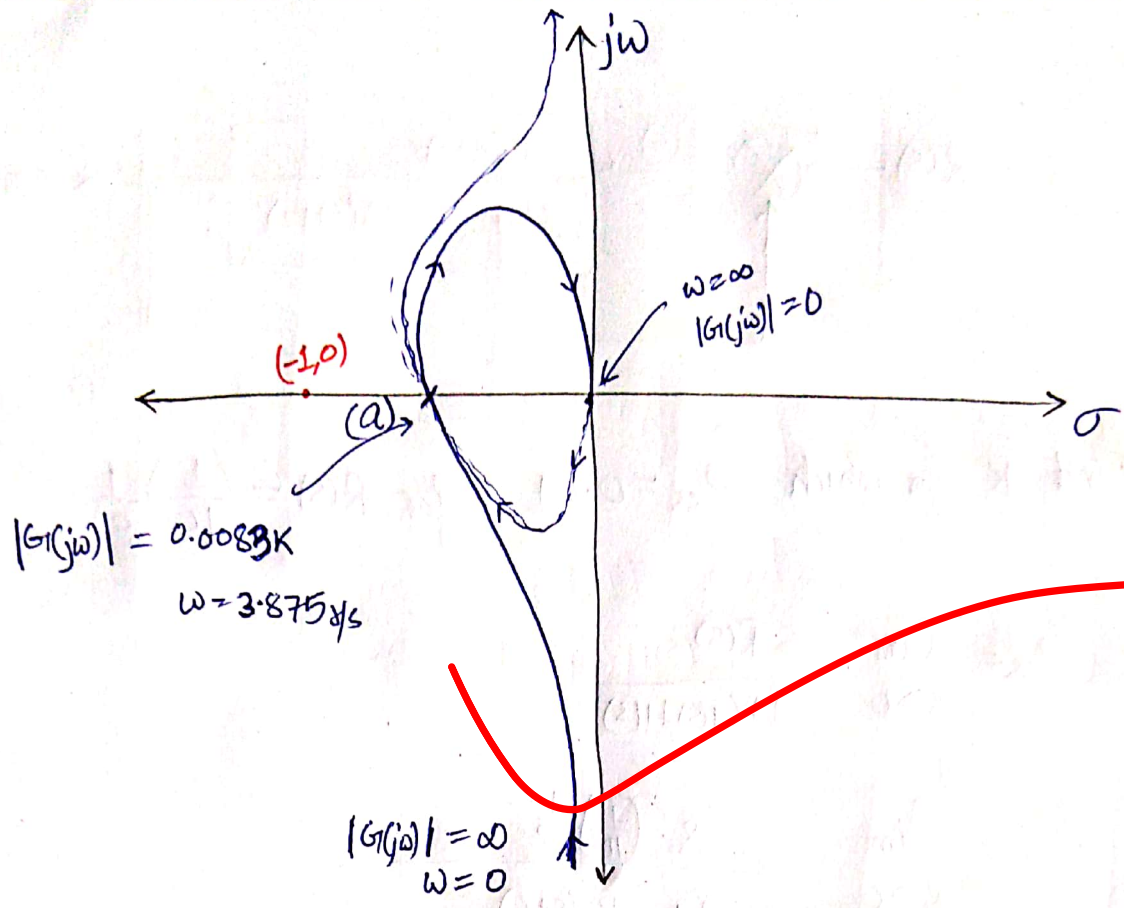
$$|G(j\omega)| = \frac{K}{\omega\sqrt{\omega^2+9}\cdot\sqrt{\omega^2+25}}$$

$$\angle G(j\omega) = -90 - \tan^{-1}(\omega/3) - \tan^{-1}(\omega/5)$$

ω	0	1	3.875	10	∞
$\lvert G(j\omega)\rvert$	∞	0.062	0.0083K	0.00085K	0
$\angle G(j\omega)$	-90°	-120°	-180°	-226.7°	$+90^\circ$

P = 0 For stability Z = 0

means $\boxed{N = 0}$ there should be no encirclements of (-1, 0) pt.



For stability pt ⓐ should be on ~~left~~ Right side of point (-1,0) on real axis

means $0.0083K < 1$

$$K < \frac{1}{0.0083} = 120.482$$

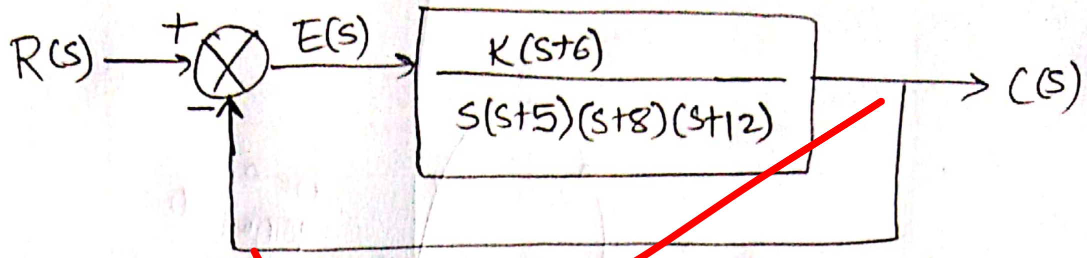
Also $K > 0$

$$= \boxed{\text{Range of K} \quad 0 < K < 120.482}$$

$$\boxed{\begin{array}{l} K_{marginal} = 120.482 \\ W_{oscillation} = 3.875 \text{ rad/sec} \end{array}}$$

[from table]

Ans 7 a)



i> Find K for which $e_{ss} = 0.01$ for $R(s) = \left(\frac{1}{10}\right)\frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} \frac{s\,R(s)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{s \cdot \left(\frac{1}{10}\right)\frac{1}{s^2}}{1 + \frac{K(s+6)}{s(s+5)(s+8)(s+12)}}$$

$$= \lim_{s \to 0} \frac{(1/10)}{s + \frac{K(s+6)}{(s+5)(s+8)(s+12)}}$$

$$e_{ss} = \frac{(1/10)}{0 + \frac{K \cdot \cancel{6}}{5 \times 8 \times \cancel{12}\,2}} = \frac{8}{K} = 0.01$$

$$\boxed{K = \frac{8}{0.01} = 800}$$

Ans 7 a) ii>

$e_{ss} = \frac{8}{K}$

e_{ss} will be minimum for maximum value of K.

To find maximum possible value of K we use Routh Hurwitz criteria.

Characteristic eq^n = $s(s+5)(s+8)(s+12) + K(s+6)$

$= s^4 + 25s^3 + 196s^2 + (480+K)s + 6K$

$$
\begin{array}{c|ccc}
s^4 & 1 & 196 & 6K \\
s^3 & 25 & (480+K) & 0 \\
s^2 & \dfrac{196\times25-(480+K)}{25} & 6K & \\
s^1 & \dfrac{\left(\dfrac{4420-K}{25}\right)\times(480+K) - 150K}{\left(\dfrac{4420-K}{25}\right)} & 0 & \\
s^0 & 6K & &
\end{array}
$$

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$6K > 0 \Rightarrow \boxed{K > 0}$ — condition ①

$4420 - K > 0$ $\boxed{K < 4420}$ — condition ②

$$\left(\frac{4420-K}{25}\right)\cdot(480+K) > 150K$$

$$2121600 + 4420K - 480K - K^2 > 3750K$$

$$-K^2 + 190K + 212160 > 0$$

$$-375.3 < K < 565.3$$

$\boxed{K < 565.3}$ — condition ③

from combining condition ①, ② and ③

$$\boxed{0 < K < 565.3}$$

$\underline{K_{max} = 565.3}$

So,

$$\boxed{e_{ss\,(minimum)} = \frac{8}{565.3} = 0.01415}$$

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Ans 7 b)

Bandwidth is that frequency at which Magnitude becomes $\frac{1}{\sqrt{2}}$ of its ~~peak~~ DC value.

Let $\frac{C(s)}{R(s)} = T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$|T(s)| = \frac{\omega_n^2}{-\omega^2 + j2\xi\omega_n \cdot \omega + \omega_n^2}$$

$$|T(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\xi\omega_n\omega)^2}} = \frac{1}{\sqrt{2}}\left(\text{DC value of } T(s)\right)$$

$$\frac{1}{\sqrt{2}} = \frac{\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\xi\omega_n\omega)^2}} \quad \left[\because \text{DC value of } |T(j\omega)| = 1\right]$$

$$(\omega_n^2-\omega^2)^2 + 4\xi^2\omega_n^2\omega^2 = 2\omega_n^4$$

$$\omega_n^4 + \omega^4 - 2\omega_n^2\omega^2 + 4\xi^2\omega_n^2\omega^2 = 2\omega_n^4$$

$$\omega^4 - 2\omega^2\omega_n^2(1-2\xi^2) - \omega_n^4 = 0$$

Let $\omega^4 = x^2$, $\omega^2 = x$ | $x^2 - x\left(2\omega_n^2(1-2\xi^2)\right) - \omega_n^4 = 0$

Sol^n of $x = \dfrac{+2\omega_n^2(1-2\xi^2) \pm \sqrt{4\omega_n^4(1+4\xi^4-4\xi^2)+4\omega_n^4}}{2}$

$$= \omega_n^2(1-2\xi^2) \pm \sqrt{\omega_n^4(4\xi^4-4\xi^2+1)+\omega_n^4}$$

$$= \omega_n^2(1-2\xi^2) \pm \sqrt{\omega_n^4(4\xi^4-4\xi^2+2)}$$

$$x = \omega_n^2\left[1-2\xi^2 \pm \sqrt{4\xi^4-4\xi^2+2}\right]$$

$$\omega_b = x^{1/2} = \omega_n\left(1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}\right)^{1/2} \quad \text{—— (1)}$$

As $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$$\boxed{\omega_b = \frac{\pi}{t_p\sqrt{1-\xi^2}}\left(1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}\right)^{1/2}}$$

Hence proved.

From eq^n (1)

$$\underbrace{\frac{\omega_b}{\omega_n}}_{y} = \left[1-2\xi^2+\sqrt{4\xi^4-4\xi^2+2}\right]^{1/2}$$

taking $\xi = x$

Now plotting x vs y graph

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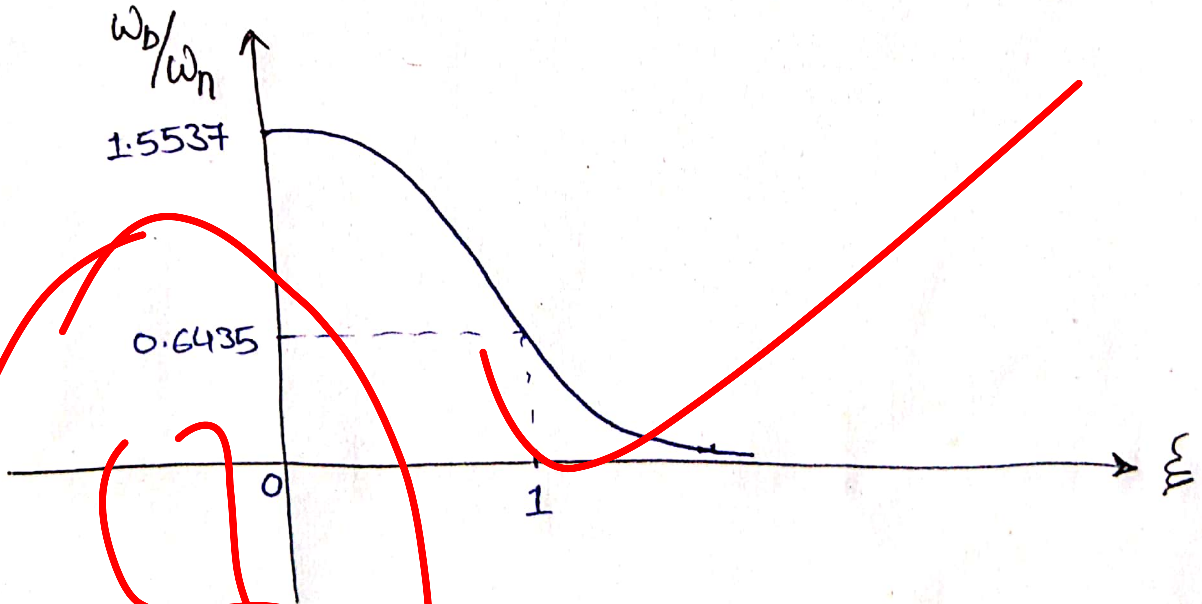
$\frac{\omega_b}{\omega_n}$	1.5537	1.272	1.01	0.6435	0.2665	0.05
ξ	0	0.5	0.7	1	2	10



Ans 7 c)

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

For Controllability, we find $|Q_c|$.

$$Q_c = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -5 \end{bmatrix}$$

$$|Q_c| = 2(-5) + 4(0) + 0 = -10 \neq 0$$

$$\boxed{\text{As } |Q_c| \neq 0 \text{ System is Controllable}}$$

Now for Observability we find $|Q_o|$.

$$Q_o = \begin{bmatrix} C^T & A^TC^T & (A^T)^2C^T \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -3 & 5 \\ 0 & -1 & 0 \end{bmatrix}$$

$$|Q_o| = 1(0+5)+1(0)+0 = 5 \neq 0$$

$\boxed{\text{As } |Q_o| \neq 0 \text{ System is Observable}}$

Hence,

<u>System is both Controllable and Observable</u>