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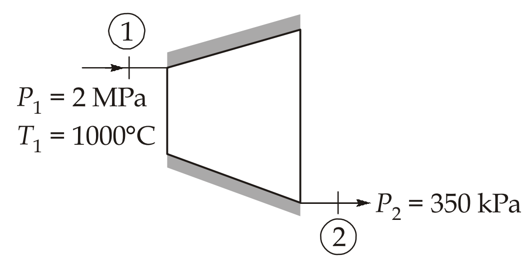
Detailed Solutions

SSC-JE 2018 Mains Test Series (PAPER-II)

Mechanical Engineering Test No : 7

Q.1 (a) Solution:



Given: Inlet temperature, $T_1 = 1273$ K

Mass flow rate, $\dot{m} = 0.5$ kg/s

Power developed, $\dot{W} = 120$ kW

$\gamma = 1.67$

$c_p = 0.52$ kJ/kgK

Using SFEE,

$$\dot{m}\{h_1 + (KE)_1 + (PE)_1\} + \dot{Q} = \dot{m}\{h_2 + (KE)_2 + (PE)_2\} + \dot{W} \quad ...(1)$$

Neglecting change in KE and PE.

$$\dot{Q} = 0 \quad \{\because \text{ insulated turbine}\}$$

From eq. (1)

$$h_1 = h_2 + \frac{\dot{W}}{\dot{m}}$$

$$c_p T_1 = c_p T_2 + \frac{\dot{W}}{\dot{m}}$$

$$T_2 = 1273 - \frac{120}{0.5 \times 0.52}$$

$$T_2 = 811.46\,\text{K}$$

Q.1 (b) Solution:

At inlet condition: $p_1 = 500$ kPa

$T_1 = 520°\text{C} = (520 + 273)\,\text{K} = 793$ K

At exit condition: $p_2 = 100$ kPa

$T_2 = 300°\text{C} = (300 + 273)\,\text{K} = 573$ K

Heat lost: $q = 10$ kJ/kg

Surroundings condition: $p_0 = 98$ kPa

$T_0 = 20°\text{C} = (20 + 273)\,\text{K} = 293$ K

$c_p = 1.005$ kJ/kgK

$R = 0.287$ kJ/kgK

The actual work is calculated by application of steady flow energy equation.

According to steady flow energy equation per unit mass

$$h_1 + \frac{V_1^2}{2} + gz_1 + q = h_2 + \frac{V_2^2}{2} + gz_2 + w$$

Neglecting the change in kinetic and potential energies.

$$h_1 + q = h_2 + w$$

or

$$w = (h_1 - h_2) + q$$

$$= c_p(T_1 - T_2) + q$$

$$= 1.005(793 - 573) - 10 = 211.1 \text{ kJ/kg}$$

Maximum work output per unit mass,

$$w_{max} = (h_1 - h_2) + T_0\,\Delta s_{sys}$$

$$= c_p(T_1 - T_2) - T_0\left(c_p \log_e \frac{T_1}{T_2} - R\log_e \frac{p_1}{p_2}\right)$$

$$= 1.005(793 - 573) - 293\left(1.005 \log_e \frac{793}{573} - 0.287\log_e \frac{500}{100}\right)$$

$$= 221.1 - 293(0.3265 - 0.4619)$$

$$= 221.1 - 293(-0.1354) = 221.1 + 39.67$$

$$= 260.77 \text{ kJ/kg}$$

(i) Irreversibility: $I = w_{max} - w = 260.77 - 211.1 =$ **49.67 kJ/kg**

OR

$$I = T_0\,\Delta s_{uni} = T_0[\Delta s_{sys} + \Delta s_{surr}]$$

$$= T_0\left[c_p \log_e \frac{T_2}{T_1} - R\log_e \frac{p_2}{p_1} + \frac{q}{T_0}\right]$$

$$= 293\left[1.005 \log_e \frac{573}{793} - 0.287 \log_e \frac{100}{500} + \frac{10}{293}\right]$$

$$= 293[-0.3265 + 0.4619 + 0.034] = \mathbf{49.66\ kJ/kg}$$

(ii) Decrease in availability or change in availability

$$\psi_1 - \psi_2 = w_{max} = \mathbf{260.77\ kJ/kg}$$

(iii) Maximum work : $w_{max} = \mathbf{260.77\ kJ/kg}$

Q.1 (c) Solution:

Let T_f = final temperature of mixture

or $3 \times c_p(T_f - 40) = 7 \times c_p \times (100 - T_f)$

or $3T_f - 120 = 700 - 7T_f$

or $10T_f = 820$

or $T_f = 82°C = 82 + 273 = \mathbf{355\ K}$ **Ans. (i)**

The change in entropy: $\Delta S = m_1 c_p \ln\left(\frac{T_f}{T_1}\right) + m_2 c_p \ln\left(\frac{T_f}{T_2}\right)$

$$\Delta S = 3 \times 4.18 \ln\left(\frac{355}{273+40}\right) + 7 \times 4.18 \times \ln\left(\frac{355}{273+100}\right)$$

$$\Delta S = 3 \times 4.18 \times \ln\left(\frac{355}{313}\right) + 7 \times 4.18 \times \ln\left(\frac{355}{373}\right)$$

$$\Delta S = 1.57897 - 1.44722$$

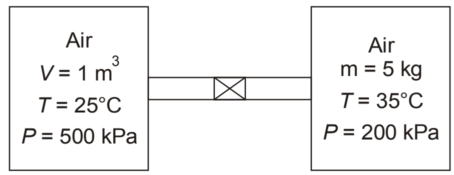
$= \mathbf{0.13175\ kJ/kgK}$ **Ans. (ii)**

Unavailable energy $= T_0\,(\Delta S)_{40}$

$$= 298 \times 3 \times 4.18 \times \ln\left(\frac{355}{313}\right)$$

$= \mathbf{470.53\ kJ}$ **Ans. (iii)**

Q.1 (d) Solution:



$$v_B = \left(\frac{m_1RT_1}{P_1}\right)_B = \frac{5\times 0.287\times 308}{200} = 2.21\ \text{m}^3$$

$$m_A = \left(\frac{P_1V}{RT_1}\right)_A = \frac{500\times 1}{0.287\times 298} = 5.846\ \text{kg}$$

$$V = V_A + V_B = 1 + 2.21 = 3.21\ \text{m}^3$$

$$m = m_A + m_B = 5.846 + 5 = 10.846\ \text{kg}$$

Final equilibrium pressure becomes

$$P_2 = \frac{mRT_2}{V} = \frac{10.846\times 0.287\times 293}{3.21}$$

$$= 284.1\ \text{kPa}$$

Q.2 (a) Solution:

$$\eta_{bth} = \frac{\text{B.P.}}{m_f\times \text{C.V.}} = \frac{16.65\times 1000\times 3600}{11400\times 7.5\times 717\times 4.184}$$

$$= 0.2336 = 23.36\,\%$$

$$\text{B.P.} = \frac{2\pi NT}{60}$$

$$T = (52\times 9.8)\times\left(\frac{75.5}{2}+\frac{2.5}{2}\right)$$

$$= 19874.4\ \text{N.cm} = 198.744\ \text{Nm}$$

$$\text{B.P.} = \frac{2\times\pi\times 2400\times 198.744}{60\times 3}$$

$$= 16.65\ \text{kW}$$

$$\eta_{mech} = \frac{\text{B.P.}}{I.\text{P.}}$$

$$0.80 = \frac{16.65}{I.P}$$

or $$IP = 20.81242 \text{ kW} = 20812.42 \text{ W}$$

$$IP = \frac{P_m LANK}{60 \times 2} = \frac{P_m \times 0.09 \times \pi \times 0.064^2}{4 \times 60 \times 2}$$

$$20812.42 = \frac{P_m \times 0.09 \times \pi \times 0.064^2 \times 2400 \times 4}{4 \times 60 \times 2}$$

or $$P_m = \mathbf{8.99 \text{ bar}}$$

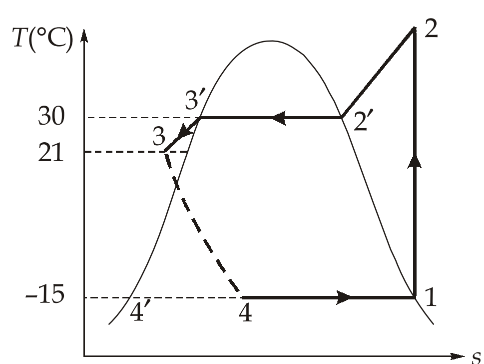
Q.2 (b) Solution:

Compression process 1 - 2 is isentropic

So, $$s_1 = s_2$$

$$5.8285 = 5.2623 + 3.252 \ln\left(\frac{T_2}{T_2'}\right)$$

$$\ln\left(\frac{T_2}{303}\right) = 0.1740$$



$$T_2 = 360.587 \text{ K}$$

Now, $$h_2 = h_2' + c_{P_v}(T_2 - T_2')$$

$$= 1485.93 + 3.252(360.587 - 303)$$

$$h_2 = 1673.20 \text{ kJ/kg}$$

Again, $$h_3 = h_3' - c_{P_l}(T_3' - T_3)$$

$$= 342.08 - 4.843(303 - 294)$$

$$h_3 = 298.493 \text{ kJ/kg}$$

And expansion process 3 - 4 is isenthalpic so,

$$h_3 = h_4 = 298.493 \text{ kJ/kg}$$

(a)Displacement volume/min, $V_D = \frac{\pi}{4} \times D^2 \times L \times N \times \eta_{vol} \times \text{Number of cylinder}$

$$= \frac{\pi \times 15 \times 15 \times 14 \times 250 \times 0.79 \times 2}{4 \times 100 \times 100 \times 100} = 0.977 \text{ m}^3/\text{min}$$

Therefore, mass of refrigerant flowing per min,

$$\dot{m} = \frac{V_D}{v_1} = \frac{0.977}{0.50905} = 1.9193 \text{ kg/min}$$

(b) The tons of Refrigeration,

$$\text{RE} = \frac{\dot{m}(h_1 - h_4)}{210} = \frac{1.9193 \times (1444.19 - 298.493)}{210}$$

$$= 10.47 \text{ tons of refrigeration}$$

(c) Coefficient of performance, $\text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1444.19 - 298.493}{1673.20 - 1444.19} = \frac{1145.697}{229.01} = 5.00$

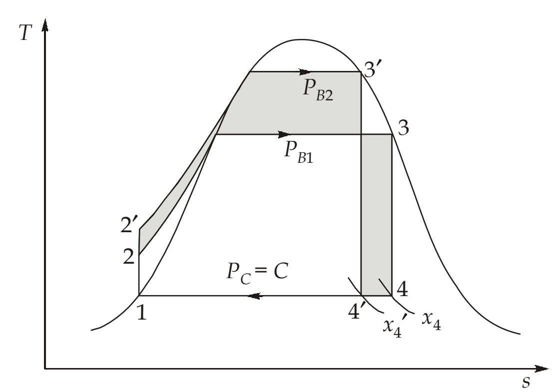
Q.2 (c) Solution:

Where: W_P – Pump work, W_T – Turbine work, $W_{net} = (W_T - W_P)$

Q_S – Heat supplied, Q_R – Heat rejected, T_{ma} – Mean temperature at heat addition,

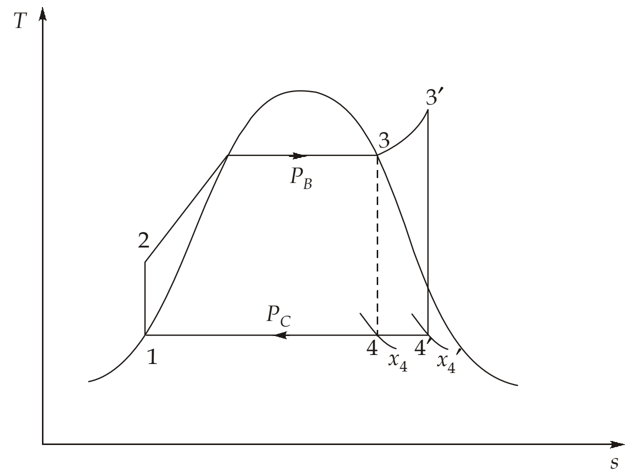
T_{mr} – Mean temperature at heat rejection, η – Efficiency of cycle, x – dryness fraction.

By increasing boiler pressure ($P_B\uparrow$):



- $W_P - \uparrow$
- $W_T - \uparrow\uparrow$
- $W_{net} - \uparrow$
- Q_S – cannot say ($\downarrow$ normally)
- $Q_R - \downarrow$
- $T_{ma} - \uparrow$
- T_{mr} – Same
- $\eta - \uparrow$
- $x - \downarrow$
- Moisture content will increase.

By superheating :

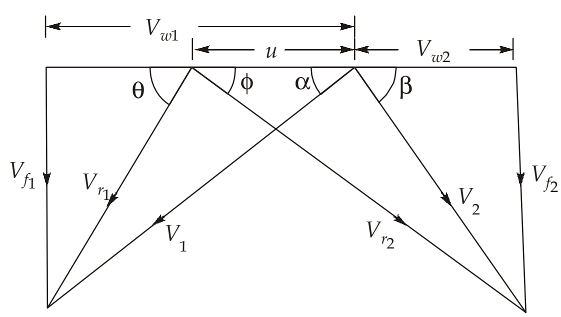


- W_P – Same
- W_T – ↑
- W_{net} – ↑
- Q_S – ↑
- Q_R – ↑
- T_{ma} – ↑
- T_{mr} – Same
- η – ↑
- x – ↑
- Moisture content will decrease

Q.2 (d) Solution:

It is the ratio of work done by the blade to the actual energy at the inlet of blade.

$$\eta_b = \frac{\dot{m}(V_{w1}+V_{w2})\times u}{\frac{1}{2}\dot{m}V_1^2} = \frac{2(V_{w1}+V_{w2})\times u}{V_1^2}$$



From velocity diagram,

$$V_{w1} + V_{w2} = V_{r1}\cos\theta + V_{r2}\cos\phi$$

$$= V_{r1}\cos\theta\left[1+\left(\frac{V_{r2}}{V_{r1}}\right)\times\left(\frac{\cos\phi}{\cos\theta}\right)\right]$$

Let, $\quad \dfrac{V_{r2}}{V_{r1}} = k$ and $\dfrac{\cos\phi}{\cos\theta} = x$

$$V_{w1} + V_{w2} = V_{r1}\cos\theta(1 + kx) \quad ...(2)$$

Now, $\quad V_{r1}\cos\theta = V_{w1} - u$

$$V_{r1}\cos\theta = (V_1\cos\alpha - u) \quad ...(3)$$

Now, from eq. (2) and (3)

$$V_{w1} + V_{w2} = (V_1\cos\alpha - u)(1 + kx)$$

$$\eta_b = \frac{2(V_1\cos\alpha - u)(1+kx)\times u}{V_1^2}$$

$$\eta_b = \frac{2\left(V_1 u\cos\alpha - u^2\right)(1+kx)}{V_1^2} \qquad \left(\text{Let, } \rho = \frac{u}{V_1}\right)$$

$$\eta_b = \frac{2V_1^2\left(\frac{u}{V_1}\cos\alpha - \left(\frac{u}{V_1}\right)^2\right)(1+kx)}{V_1^2}$$

$$\eta_b = \frac{2\left(\rho\cos\alpha - \rho^2\right)(1+kx)}{1}$$

Now for maximum blade efficiency,

$$\frac{d\eta_b}{d\rho} = 0$$

$$2(1 + kx)(\cos\alpha - 2\rho) = 0$$

$$\rho = \frac{\cos\alpha}{2}$$

$$\frac{d^2\eta_b}{d\rho^2} = -4(1 + kx)$$

$$\left(\frac{d^2\eta_b}{d\rho^2} < 0, \text{so it is condition for maximization}\right)$$

Condition for maximum blade efficiency, $\rho = \frac{\cos\alpha}{2} = \frac{u}{V_1}$

Now,

$$(\eta_b)_{\max} = 2\left[\frac{\cos\alpha}{2}\times\cos\alpha - \frac{\cos^2\alpha}{4}\right](1+kx)$$

$$= 2\left[\frac{\cos^2\alpha}{4}\right](1+kx)$$

$$(\eta_b)_{\max} = \frac{\cos^2\alpha}{2}(1+kx)$$

If blades are frictionless, $V_{r1} = V_{r2}$

$$k = 1$$

If blades are symmetrical, $\theta = \phi$

$$\Rightarrow \quad x = 1$$

$$(\eta_b)_{max} = \frac{\cos^2\alpha}{2}(1+1) = \cos^2\alpha$$

Maximum blade efficiency of impulse turbine, $(\eta_b)_{max} = \cos^2\alpha$

Q.3 (a) Solution:

Let the equation of velocity profile

$$u = Ay^2 + By + C$$

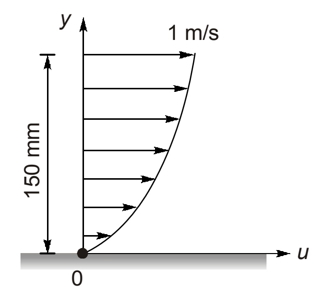
Now apply boundary condition

(i) $u = 0$ at $y = 0 \Rightarrow c = 0$

(ii) $u = 1$ m/s at $y = 0.15$ m

$$1 = 0.15^2 \times A + 0.15\,B \quad \text{...(ii)}$$

(iii) at $y = 0.15$ m at $\frac{du}{dy} = 0$



$$\frac{du}{dy} = 2Ay + B$$

$$2A \times 0.15 + B = 0 \quad \text{...(iii)}$$

From Eq. (ii) and (iii), we get,

$$A = -44.4; \quad B = 13.33$$

So velocity profile will be given as

$$u = -44.4\,y^2 + 13.33\,y$$

(a) at $y = 0$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0 + 13.33 = 13.33 \text{ sec}^{-1}$

$$\text{Shear stress, } \tau = \mu\frac{du}{dy} = 0.804 \times 13.33 = 10.8 \text{ N/m}^2$$

(b) at $y = 100$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0.1 + 13.33 = 4.45 \text{ sec}^{-1}$

$$\tau = \mu\frac{du}{dy} = 0.804 \times 4.45 = 3.575 \text{ N/m}^2$$

(c) at $y = 150$ mm $\quad \frac{du}{dy} = -2 \times 44.4 \times 0.15 + 13.33 = 0$

$$\tau = 0$$

Q.3 (b) Solution:

A draft tube is a pipe or passage of gradually increasing cross-sectional area which connects the runner exit to the tail race.

The draft tube has two purposes as follows:

(a) It permits a negative or suction head to be established at the runner exit, thus making it possible to install the turbine above the tail level without loss of head.

(b) It converts a large proportion of kinetic energy rejected from the runner into useful pressure energy i.e., it acts as a recuperator of pressure energy.

Q.3 (c) Solution:

Diameter of impeller at outlet, D_2 = 300 mm = 0.3 m

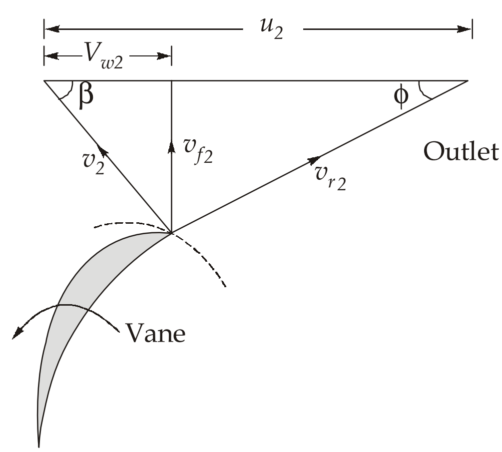
Width of impeller at outlet, B_2 = 60 mm = 0.06 m

Discharge through the pump, Q = 164 litres/s = 0.164 m^3/s

Manometric efficiency, η_m = 85% = 0.85

Effective vane outlet angle, ϕ = 30°

Speed of rotation, N = 900 rpm



Peripheral or tangential velocity at outlet of impeller,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 900}{60}$$

$$u_2 = 14.137 \text{ m/s}$$

Velocity of flow at outlet, $$v_{f2} = \frac{Q}{\pi D_2 B_2}$$

$$v_{f2} = \frac{0.164}{\pi \times 0.3 \times 0.06}$$

$$v_{f2} = 2.9 \text{ m/s}$$

from velocity triangle at outlet as shown above,

$$V_{w_2} = u_2 - \frac{V_{f_2}}{\tan\phi} = 14.137 - \frac{2.9}{\tan 30^\circ} = 9.114 \text{ m/s}$$

$$\text{Manometric efficiency, } \eta_m = \frac{gH_m}{V_{w2}u_2}$$

$$H_m = \frac{V_{w2} \times u_2 \times \eta_m}{g} = \frac{9.114 \times 14.137 \times 0.85}{9.81} = 11.164 \text{ m}$$

$$\text{Specific speed, } N_S = \frac{N\sqrt{Q}}{(H_m)^{3/4}}$$

$$= \frac{900\sqrt{0.164}}{(11.164)^{0.75}} = 59.676$$

Q.3 (d) Solution:

Given data:

Generator output power: $P_G = 40 \text{ MW} = 40 \times 10^6 \text{ W}$

Effective head: $H = 400 \text{ m}$

Generator efficiency: $\eta_G = 95\,\% = 0.95$

Hydraulic efficiency: $\eta_H = 88\,\% = 0.88$

$C_v = 0.97$

$K_u = 0.46$

$\frac{D}{d} = 12$

A double overhung Pelton wheel has two runner keyed on the two ends of the shaft and the generator lies between them. Thus, the generator is driven by two Pelton turbines.

$$\text{Generator efficiency: } \eta_G = \frac{\text{Generator output power}}{\text{Turbines output power}}$$

$$\eta_G = \frac{P_G}{P_{\text{Net}}}$$

$$0.95 = \frac{40 \times 10^6}{P_{\text{Net}}}$$

or $$P_{\text{Net}} = \frac{40 \times 10^6}{0.95} = 42.10 \times 10^6 \text{ W}$$

Power output of each turbine: $$P = \frac{P_{\text{Net}}}{2} = \frac{42.10 \times 10^6}{2}$$

$$= 21.05 \times 10^6 \text{ W}$$

Now, the peripheral speed: $$u = K_u \sqrt{2gH}$$

$$= 0.46 \times \sqrt{2 \times 9.81 \times 400} = 40.75 \text{ m/s}$$

The absolute velocity at inlet: $$V_i = C_v \sqrt{2gH}$$

$$= 0.97 \times \sqrt{2 \times 9.81 \times 400} = 85.93 \text{ m/s}$$

We know that hydraulic efficiency: $$\eta_H = \frac{\rho Q[V_{wi} \pm V_{wo}]}{\frac{1}{2}\rho Q V_i^2}$$

Let mechanical efficiency: $$\eta_m = 100\,\% = 1$$

$\therefore$ $$\eta_H = \frac{\text{Power output of each turbine: } P}{\frac{1}{2}\rho Q V_i^2}$$

$$0.88 = \frac{21.05 \times 10^6}{\frac{1}{2} \times 1000 \times Q \times (85.93)^2}$$

or $$Q = 6.47 \text{ m}^3/\text{s}$$

also $$Q = \frac{\pi}{4} d^2 \times V_i$$

$\therefore$ $$6.47 = \frac{3.14}{4} \times d^2 \times 85.93$$

or $$d^2 = 0.09591$$

or $$d = \mathbf{0.30969 \text{ m}}$$

$$\frac{D}{d} = 12$$

or $D = 12 \times d$

$= 12 \times 0.30969 = \mathbf{3.71\ m}$

also $$u = \frac{\pi DN}{60}$$

$$40.75 = \frac{3.14 \times 3.71 \times N}{60}$$

or $N = 209.88$ rpm

Since the electricity is produced at constant frequency, $f = 50$ Hz, the synchronous speed N is determined by

$$N = \frac{120f}{p}$$

where p = Number of poles

or $$p = \frac{120f}{N} = \frac{120 \times 50}{209.88} = 28.58 \simeq 28$$

Therefore, Synchronous speed: $N = \dfrac{120 \times 50}{28} = 214.128$ rpm

Specific speed: $N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$ (SIunits)

where $N = 214.28$ rpm

$P = 20.05 \times 10^3$ kW

$H = 400$ m

$\therefore$ $$N_s = \frac{214.28\sqrt{20.05 \times 10^3}}{(400)^{5/4}}$$

$= \mathbf{16.96}$ **(SI units)**

Q.4 (a) Solution:

(i) Normalizing is final heat treatment process which is given to a product which are subjected to relatively high stresses. This process consists of heating steel to a temperature 40 - 50°C above the line where austenile is stable. It is done for refining grain structure and for improving the mechanical properties. Normalizing done by air quenching produces microstructures consisting of ferrite and pearlite for hypo-eutectic steel and pearlite and cementite for hypoeutectoid steels.

(ii) 1. Normalizing requires a heating range which is about 40°C above that of annealing.

2. Mechanical properties of steels are better than those produced by annealing.

3. Heat transfer process is of short duration due to increased rate of cooling of the metal in air.

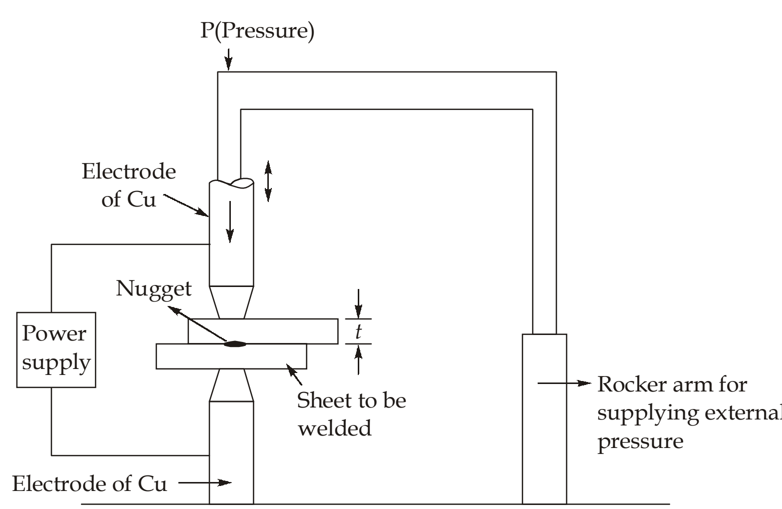
4. If mechanical properties is not the main aim of the heat treatment, better machinability and removal of internal stresses is possible in annealing than obtained by normalizing.

(iii) **Induction hardening:** An alternating current of high frequency pass through an induction coil enclosing the steel part to be heat treated. The induced emf heats the steel. The depth up to which the heat penetrates and raises the temperature above A_{C_3} is inversely proportional to the square root of the ac frequency. Corresponding, the hardened depth decreases with increasing frequency. The heating time is usually a few seconds. Immediately after heating, water jets are activated to quench the surface. Martensite is produced at the surface, making it hard and wear resistant. The microstructure of the core remains unaltered.

Flame hardening: For large work pieces, such as mill rolls, large gears and complicated cross-sections, induction heating is not easy to apply. In such cases, flame hardening is done by means of an oxyacetylene torch. Heating should be done rapidly by the torch and the surface quenched, before appreciable heat transfer to the core occurs.

Q.4 (b) Solution:

Setup for spot welding:



Spot welding: For joining of sheet metals to produce a lap joint this welding technique is used. In this method two sheets will be held between two copper electrodes. A very high rate of current is supplied through the electrodes for a small fraction of time. At the contact of two workpieces (sheets) due to more contact resistance heat will be generated. After getting sufficient amount of heat by switching off the power supply external pressure can be applied to produce the joint between two workpieces below the electrodes known as 'Nugget'. There is no overlapping of nugget in this welding. It is a type of resistance welding. There is no gas defect in the weld.

Drawbacks: (1) Leakproof joint is not possible.

(2) There is a possibility of indentation between electrode and workpiece.

Application: It is used for lap joining of metal sheets in automobile bodies and refrigerator bodies.

Pressure V/s time graph:

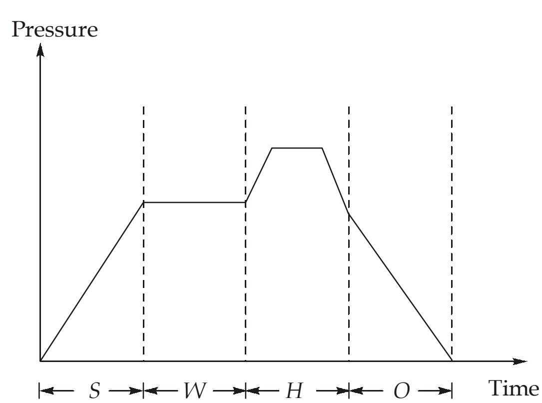
S $\rightarrow$ Squeeze time (pressure applied by electrode).

W $\rightarrow$ Weld time (current supply and liquid metal production).

H $\rightarrow$ Hold time (initially pressure increases by rocker arm then maintained constant for joint formation and after that rocker arm pressure relieved).

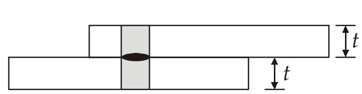
O $\rightarrow$ Off time (Electrode pressure relieved).

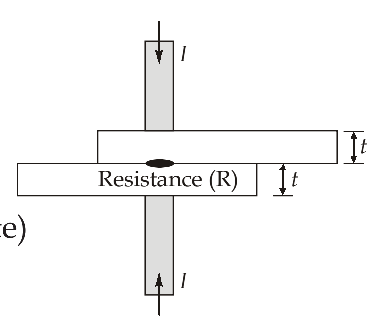
Cycle time = Squeeze time + Weld time + Hold time + Off time



Melting efficiency:

Diameter of nugget, $d_n = 6\sqrt{t}$





Height of nugget, $h_n = 2(t - \text{Indentation per plate})$

$h_n = 2(t - \text{Indentation})$

$$\text{Volume of nugget, } V = \frac{\pi}{4} d_n^2 h_n$$

$$\text{Mass of nugget, } m = V \times \rho = \left(\frac{\pi}{4} d_n^2 h_n\right)\rho$$

$$\text{Heat required to melt, } H_m = mc\Delta T + mL = m(c\Delta T + L)$$

$$\text{Heat supplied, } H_s = I^2 Rt$$

$$\text{Melting efficiency, } \eta_m = \frac{H_m}{H_s}$$

(Where, I = Current supplied; R = Resistance; t = time of current supply)

Process Parameters:

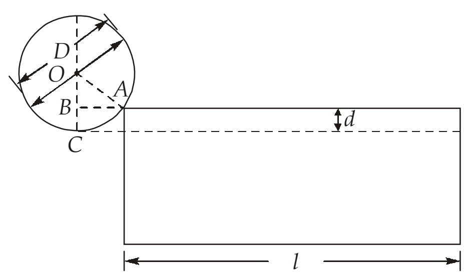
Thickness of sheet = 1 to 2 mm

Current, (I) = 10000 to 50000 Ampere

Time of current supply, t = 0.01 to 0.5 seconds

Q.4 (c) Solution:

Given, D = 120 mm, l = 150 mm, d = 20 mm, z = 10 teeth, f = 0.20 mm per teeth, V = 40 m/min



$$\text{Compulsory approach, } AB = \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{D}{2} - d\right)^2} = \sqrt{60^2 - (60-20)^2} = 44.72 \text{ mm}$$

$$\text{Cutting velocity, } V = \frac{\pi DN}{1000}$$

$$40 = \frac{\pi \times 120 \times N}{1000}$$

$$N = 106.10 \text{ rpm}$$

Table feed = $f \times z \times N$ = 0.20 × 10 × 106.10 = 212.2 mm/min

Total cutter travel = Length of workpiece + Compulsory approach

= 150 + 44.72 = 194.72 mm

$$\text{Slot cutting time, } T = \frac{\text{Total cutter travel}}{\text{Table feed}} = \frac{194.72}{212.2}$$

$$= 0.9176 \text{ minutes} \simeq 55.05 \text{ s}$$

Q.4 (d) Solution:

We have to determine V/A ratio for the casting plate.

$$\text{Volume, } V = 8 \times 13 \times 2.5 = 260 \text{ cm}^3$$

$$\text{Surface area, } A = 2(8 \times 13 + 8 \times 2.5 + 13 \times 2.5)$$

$$= 2(104 + 20 + 32.5) = 313 \text{ cm}^2$$

$$\text{Solidification time of casting, } t_s = 1.6 \text{ min}$$

$$\text{From Chvorinovs's rule, } t_s = k\left(\frac{V}{A}\right)^2$$

$$k = \frac{t_s}{\left(\frac{V}{A}\right)^2} = \frac{1.6}{\left(\frac{260}{313}\right)^2} = 2.3188 \text{ min/cm}^2$$

$$\text{for Riser, Height (H)} = \text{Diameter (D)}$$

$$\text{Volume of riser, } V_R = \frac{\pi}{4}D^2H = \frac{\pi}{4}D^3$$

$$\text{Area of riser, } A_R = \pi DH + 2\times\frac{\pi D^2}{4} = 1.5\pi D^2$$

$$\frac{V_R}{A_R} = \frac{D}{6}$$

$$\text{for riser, } t_s = 2.0 \text{ min}$$

$$t_s = k\left(\frac{V}{A}\right)_R^2$$

$$= \left(\frac{D}{6}\right)^2 \times k = 2.3188 \times \left(\frac{D}{6}\right)^2$$

$$D = \sqrt{31.0506} = 5.5723 \text{ cm}$$

Riser dimensions, $D = 5.57$ cm Answer

$H = 5.57$ cm Answer

Check: Volume of riser = $\frac{\pi}{4}D^3$ = 135.89 cm^3 which is just 47% of the volume of cast plate, though its solidification time is 25% longer.

Q.5 (a) Solution:

Mobility or Degree of freedom: The minimum number of independent variables required to define the position or motion of the system is known as mobility (or) degree of freedom of the system.

- Suppose, a planar mechanism having (l) links is to be designed. Before any connections are made. The system of l links will have a total ($3l$) degree of freedom
- As every mechanism will always be considered to be fixed to the ground. This leaves the system with total number of degree of freedom, $F = 3(l - 1)$
- Each one degree of freedom joint removes two degree of freedom from the system.
- Similarly, each two degree of freedom joints removes one degree of freedom from the system.
- Then the total mobility of the system is given by Kutzback equation.

$$F = 3(l - 1) - 2J\text{-h} \quad \text{... (i)}$$

F - The mobility or number of degree of freedom.

l - The total number of links including the ground.

J - The number of one degree of freedom joints.

h - The number of two degree of freedom joints or higher pair.

In those mechanism in which degree of freedom is one and number of higher pair is zero then it leads to Gruebler's criteria.

$$\begin{aligned} F &= 3(l - 1) - 2\text{J-h} \\ 1 &= 3(l - 1) - 2J - 0 \\ 3l - 3 - 2J - 1 &= 0 \\ 3l - 2J - 4 &= 0 \\ 3l &= 2(J + 2) \quad \text{... (ii)} \end{aligned}$$

This equation is known by Gruebler's equation which is derived from Kutzback equation (i). It is clear that, number of links should be in even number to satisfy the above equation (ii). Now, a closed chain can not be formed from two links therefore, the minimum four number of links are required in order to obtain a mechanism.

Gruebler's equation will only be satisfied with planar mechanism in which number of higher pair is zero and the degree of freedom is one. But equation (i) which is Kutzback equation can apply to any planar mechanism and there is no restriction about number of higher and lower pair.

Q.5 (b) Solution:

$$\text{Power} = 8 \text{ kW}$$

Therefore $$[T_1 - T_2]v = 8000 \quad \text{...(i)}$$

$$[T_1 - T_2]\frac{\pi \times D \times N}{60} = 8000$$

$$[T_1 - T_2] \times \frac{\pi \times 1.1 \times 200}{60} = 8000$$

$$T_1 - T_2 = 694.49 \text{ N}$$

Also $$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_1}{T_2} = e^{0.25 \times \frac{\pi}{180} \times 160}$$

$$\frac{T_1}{T_2} = 2$$

$$T_1 = 2T_2$$

$$2T_2 - T_2 = 694.49$$

$$T_2 = 694.49 \text{ N}$$

$$T_1 = 2 \times 694.49$$

$$= 1388.98 \text{ N}$$

Let the length of the belt be "l" m and width be b "m".

$$m = \rho \times b \times t$$

$$m = 0.001 \times 10^6 \times b \times 8 \times 10^{-3}$$

$$m = 8b \text{ kg}$$

$$\text{Mass per unit length of belt} = 8b \text{ kg}$$

$$\text{Centrifugal tension} = mv^2$$

$$= 8b \times \left(\frac{\pi DN}{60}\right)^2$$

$$= 8 \times b \times \left(\frac{\pi \times 1.1 \times 200}{60}\right)^2 = 1061.53 \text{ b N}$$

$$\text{Maximum tension} = T_1 + T_c$$
$$= (1388.98 + 1061.53\text{ b})\text{N}$$

$$\text{Maximum force from limiting condition} = \sigma_{safe} \times \text{area}$$
$$= 2.2 \times (b \times 1000) \times 8 = 17600\text{ b N}$$
$$1388.98 + 1061.53\text{ b} = 17600\,b$$
$$1388.98 = 16538.47\,b$$
$$b = 83.98\text{ mm}$$

Q.5 (c) Solution:

There are two basic modes of gear tooth failure

- breakage of the tooth due to static and dynamic loads
- Surface destruction.

The complete breakage of the tooth can be avoided by adjusting the parameter in gear design, such as the module and the face width, so that the beam strength of the gear tooth is more than the sum of static and dynamic loads.

The surface destruction or tooth wear is classified according to the basis of their primary causes. The principal types of gear tooth wear are as follows:

1. **Abrasive wear:** Foreign particles in the lubricant, such as dirt, rust, weld, spatter, or metallic debris can scratch or Brinell the tooth surface. Remedies against this type of wear are provision of oil filters, increasing surface hardness, and use of high viscosity.
2. **Corrosive wear:** The corrosion of the tooth surface is caused by corrosive elements, such as extreme pressure additives present in lubricating oils and foreign materials due to external contamination. Remedies against this type of wear are providing complete enclosure for the gears free from external contamination, selection of proper additives.
3. **Initial pitting.** The initial or correction pitting is a localized phenomenon, characterized by small pits at high spots. The remedies against initial pitting are precise machining of gears, adjusting the correct alignment of gears so that the load is uniformly distributed.
4. **Scoring:** Excessive surface pressure, high surface speed and inadequate supply of lubricant result in the breakdown of the oil film. This results in excessive frictional heat and overheating of the meshing teeth. Scoring can be avoided by selecting the parameters, such as surface speed, surface pressure and the flow of lubricant in such a way that the resulting temperature at the contacting surfaces is within permissible limits.

Q.5 (d) Solution:

Both gears are made of some material hence pinion is weaker

(i) beam strength $= \sigma_b \times Y \times b \times m$

$$= \frac{S_{ut}}{3} \times 0.337 \times 30 \times 3 = \frac{600}{3} \times 0.337 \times 30 \times 3 = 6066 \text{ N}$$

(ii) Velocity factor $= C_v$

$$\text{Velocity of pinion} = \frac{\pi D_P \times N_P}{60} = \frac{\pi \times m \times T_P \times N_P}{60}$$

$$= \frac{\pi \times 3 \times 24 \times 1200}{60000} = 4.524 \text{ m/s}$$

$$C_v = \frac{3}{3+v}$$

if v < 5 m/s

$$= \frac{3}{3+4.524} = 0.3987$$

(iii) Rated power

$$P_{\text{eff}} = \frac{c_s}{c_v} \times P_t$$

$$\frac{\text{Beam strength}}{\text{factor of safety}} = \frac{c_s}{c_v} \times P_t$$

$$\frac{6066}{1.5} = \frac{1.5}{0.3987} \times P_t$$

$$P_t = 1074.98 \text{ N}$$

$$\text{Power} = P_t \times \text{velocity}$$

$$= 1074.98 \times 4.524$$

$$= 4863.189 \text{ W}$$

$$= 4.86 \text{ kW}$$

Q.6 (a) Solution:

Laws of Coulomb Friction: The friction, that exists between two surfaces which are not lubricated, is known as solid friction. The two surfaces may be at rest or one of the surface is moving and other surface is at rest. The fallowing are the laws of Coulomb's friction :

1. The force of friction acts in the opposite direction in which the surface is having a tendency to move.
2. The force of friction is equal to the force applied to the surface, as long as the surface is at rest.

3. When the surface is at the verge of motion, the force of friction is maximum and this maximum frictional force is called the limiting frictional force.
4. The limiting frictional force bears a constant ratio to the normal reaction between two surfaces.
5. The limiting frictional force does not depend upon the shape and areas of the surfaces in contact.
6. The ratio between limiting friction and normal reaction is slightly less when the two surfaces are in motion.
7. The force of friction is independent of the velocity of sliding.

The above Laws of solid friction are also called laws of static and dynamic friction.

Q.6 (b) Solution:

$$V = 72 \text{ km/hr} = 20 \text{ m/s}$$
$$d = 100 \text{ cm}$$
$$\text{Radius, } r = \frac{d}{2} = 50 \text{ cm} = 0.5 \text{ m}$$
$$S = 30 \text{ m}$$
$$\text{Angular velocity, w} = \frac{V}{r} = \frac{20}{0.5} = 40 \text{ rad/ s}$$

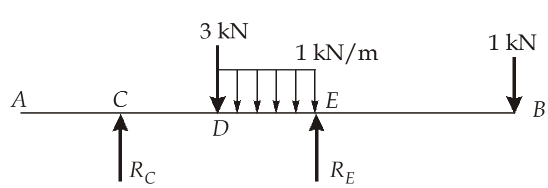
Also,
$$V^2 = u^2 + 2as$$
$$0 = (20)^2 + (2a \times 30)$$

$$\Rightarrow \quad a = -\frac{20}{3} \text{ m/ s}^2$$

$$\text{Angular acceleration, a} = \frac{a}{r} = -\frac{20}{3 \times 0.5} = -\frac{40}{3} \text{ rad/ s}^2 \quad (\text{– indicates retardation})$$

Q.6 (c) Solution:

FBD of AB



Taking moment about *C*,

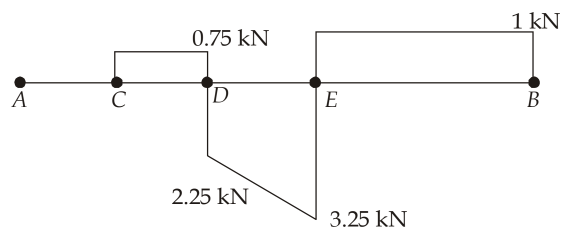
$$-3\times1-1\times1\times\frac{3}{2}+R_E+2-1\times4 = 0$$

$$R_E = \frac{8.5}{2} = 4.25 \text{ kN}$$

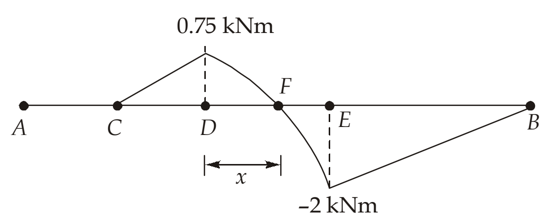
Equating vertical forces,

$$R_C + R_E = 3 + 1 \times 1 + 1$$
$$R_C = 0.75 \text{ kN}$$

***SFD* of *AB*,**



***BMD* of *AB*,**



Point of contraflexure is in between D and E. Let's take F to be at x distance from D then.

$$R_c \times (1 + x) = 3 \times x + 1 \times x \times \frac{x}{2}$$

$$0.75 + 0.75x = 3x + \frac{x^2}{2}$$

$$\Rightarrow \qquad x^2 + 6x - 1.5x - 1.5 = 0$$

$$x = 0.311$$

So from point A, point of contraflexure is at $(2 + x) = 2.311$ m distance. Maximum moment is 2 kNm and will be acting at E.

Q.6 (d) Solution:

Given: $\sigma_x = -60$ MPa, $\sigma_y = -80$ MPa, $\tau_{xy} = 50$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60-80}{2} \pm \sqrt{\left(\frac{-60+80}{2}\right)^2 + (50)^2}$$

$$= -70 \pm \sqrt{(10)^2 + (50)^2} = -70 \pm \sqrt{2600}$$

$$\sigma_2 = -19 \text{ MPa and } \sigma_1 = -121 \text{ MPa} \qquad \text{...(i)}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{2 \times 50}{(-60+80)} = 5$$

$$\theta_p = 39.34° \text{ and } 129.34°$$

Maximum In-plane shear stress is given by

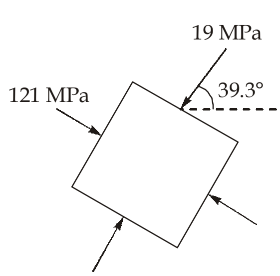
$$\tau_{max} = \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$= \left[\left(\frac{-60+80}{2}\right)^2 + 50^2\right]^{1/2} = 51 \text{ MPa} \qquad \text{...(ii)}$$

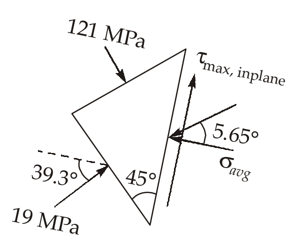
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} = \frac{-20}{50} \times \frac{1}{2} = -0.2$$

$$\theta_s = -5.65° \text{ and } 84.3°$$

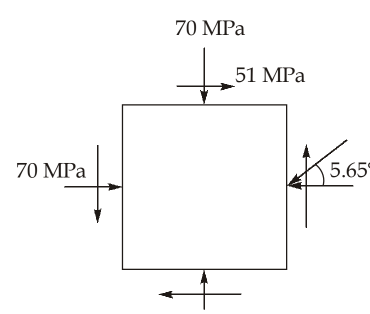
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = -70 \text{ MPa}$$



Orientation for $\sigma_{1,2}$



Position of $\tau_{max,\ implane}$



Avg. stress

❍❍❍❍