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Detailed Solutions

SSC-JE 2018
Mains Test Series
(PAPER-II)

Mechanical Engineering
Test No : 6

Q.1 (a) Solution:

Given data:

60% of volume of the spherical float is immersed in water

Diameter of valve: $d = 15 \text{ mm} = 0.015 \text{ m}$

$\therefore$ Area of the valve: $a = \frac{\pi}{4}d^2 = \frac{3.14}{4} \times (0.015)^2 = 1.766 \times 10^{-4} \text{ m}^2$

Pressure required to closed the valve,

$$p = 147 \text{ kN/m}^2 = 147 \times 10^3 \text{ N/m}^2$$

$\therefore$ Force exerted by fluid on the valve when closed,

$$F = p \times a = 147 \times 10^3 \times 1.766 \times 10^{-4} \text{ N} = 25.96 \text{ N}$$

Let V = volume of float

Volume of float immersed in water:

$$\forall = 60\% \text{ of volume of float} = 0.6V$$

$\therefore$ Buoyant force: F_B = weight of liquid displaced by the body

$$= \rho g \forall = 1000 \times 9.8 \times 0.6V = 5886V \text{ N}$$

Now taking moment about hinge point is zero

$$F_B \times 0.5 - F \times 0.150 = 0$$

$$5886 \times V \times 0.5 - 25.96 \times 0.150 = 0$$

$$5886 \times V \times 0.5 = 4.04976$$

$$V = 1.37606 \times 10^{-3} \text{ m}^3$$

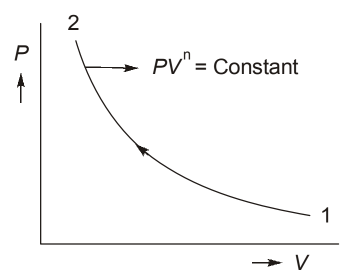
Volume of float: $$V = \frac{\pi}{6}D^3$$

$\therefore$ $$1.37606 \times 10^{-3} = \frac{3.14}{6} \times D^3$$

$$D^3 = 2.6294 \times 10^{-3} \text{ m}^3$$

or Diameter of the float: $D = 0.13802$ m **= 138.02 mm**

Q.1 (b) Solution:



$$V_1 = 1 \text{ m}^3$$
$$P_1 = 1.5 \text{ bar}$$
$$T_1 = 20°\text{C} = 273 + 20 = 293 \text{ k}$$
$$P_2 = 6 \text{ bar}$$
$$T_2 = 120°\text{C} = 273 + 120 = 399 \text{ k}$$

(i) $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

or $$\frac{n-1}{n}\ln\left(\frac{P_2}{P_1}\right) = \ln\left(\frac{T_2}{T_1}\right)$$

or $$\frac{n-1}{n} = \frac{\ln(T_2/T_1)}{\ln(P_2/P_1)} = \frac{\ln(393/293)}{\ln(6/1.5)} = 0.2118$$

or $$1 - \frac{1}{n} = 0.2118$$

or $$\frac{1}{n} = 0.7882$$

or $$n = 1.2687$$

(ii) $$m = \frac{P_1V_1}{RT_1} = \frac{1.5\times10^5\times1}{287\times293} = 1.784 \text{ kg}$$

$$\text{Work done (W)} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mR(T_1-T_2)}{n-1}$$

$$W = \frac{1.784\times0.287(293-393)}{(1.2687-1)} = -190.54 \text{ kJ}$$

$$Q = \Delta U + W = mc_v(T_2 - T_1) + W$$

$$Q = 1.784 \times 0.718(393 - 293) - 190.54 = -62.45 \text{ kJ}$$

(iii) $$S_2 - S_1 = \frac{(n-\gamma)}{(\gamma-1)(n-1)} mR \ln\frac{T_2}{T_1}$$

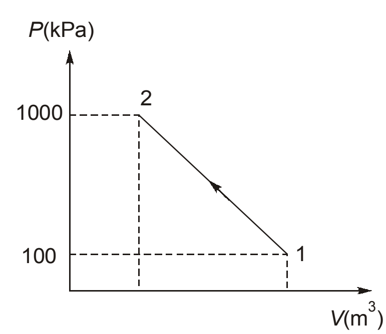
$$= \frac{(1.2687-1.4)}{0.4\times0.2687}\times1.784\times0.287\ \ln\frac{393}{293}$$

$$S_2 - S_1 = -0.18366 \text{ kJ/K}$$

Q.1 (c) Solution:

(i) $$R = \frac{R_0}{M_{co_2}} = \frac{8.314}{44} = 0.1889 \text{ kJ/kgK}$$

Taking CO_2 as the system, we can assume it as a closed system since no mass crosses the bounders of the system.



$$V_1 = \frac{mRT_1}{P_1} = \frac{0.1889\times298}{100} = 0.5629 \text{ m}^3$$

$$V_2 = \frac{mRT_2}{P_2} = \frac{0.1889\times573}{1000} = 0.1082 \text{ m}^3$$

From the first law :

$$Q - W = \Delta U = mc_v(T_2 - T_1)$$

Due to spring, pressure changes linearly with volume and the work done is equal to the area under line 1 → 2 :

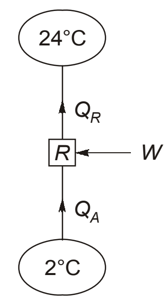
$$W = \text{Area} = \frac{P_1 + P_2}{2}(V_2 - V_1)$$

$$= \left(\frac{100 + 1000}{2}\right)(0.1082 - 0.5629) = -250.1 \text{ kJ}$$

$$Q = W + mc_v(T_2 - T_1)$$

$$= -250.1 + 0.657(300 - 25) = -69.4 \text{ kJ}$$

(ii) COP of refrigerator as per inventor's claim = 13.5



$$\text{COP}_{max} = \text{COP}_{reversible} = \frac{1}{\frac{T_H}{T_L} - 1}$$

$$\text{COP}_R = \frac{1}{\frac{(24 + 273)}{(2 + 273)} - 1}$$

$$\text{COP}_R = \frac{1}{\frac{297}{275} - 1} = 12.5$$

This is the highest COP a refrigerator can have when absorbing heat from a cool medium at 2°C and rejecting it to a warmer medium at 24°C. Since the COP claimed by the inventor is above this maximum value, the claim is FALSE.

Q.1 (d) Solution:

The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_2}{T_1}$$

Let T_2 be decreased by ΔT with T_1 remaining the same,

$$\eta_1 = 1 - \frac{T_2 - \Delta T}{T_1}$$

If T_1 is increased by the same ΔT, T_2 remaining the same.

$$\eta_2 = 1 - \frac{T_2}{T_1 + \Delta T}$$

Then,
$$\eta_1 - \eta_2 = \frac{T_2}{(T_1 + \Delta T)} - \frac{T_2 - \Delta T}{T_1} = \frac{(T_1 - T_2)\Delta T + \Delta T^2}{T_1(T_1 + \Delta T)}$$

Since, $T_1 > T_2, (\eta_1 - \eta_2) > 0$

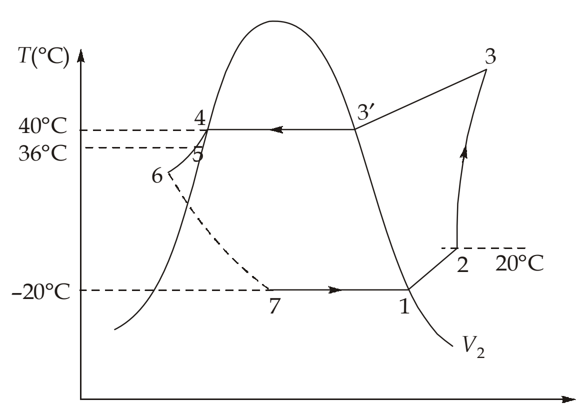
So, the more effective way to increase the cycle efficiency is to decrease T_2.

Q.2 (a) Solution:

1. **Basic cycle:** S.I. engine works on otto cycle whereas C.I. engine works on diesel cycle.
2. **Fuel:** Fuel used in S.I. engine should be of high self ignition temperature and high octane number (petrol). In C.I. engine the fuel should have low self ignition temperature and high cetane number.
3. **Introduction of fuel in cylinder:** In S.I. engine a mixture of air and fuel mixed through carburettor is introduced in the cylinder. In C.I. engine the fuel is injected directly into the combustion chamber of the end of compression stroke.
4. **Ignition:** In S.I. engine there is spark ignition, whereas in C.I. engine, ignition is through compression.
5. **Compression ratio:** In S.I. engine usually the compression ratio is 6-11 which is restricted by detonation. In C.I. engine compression ratio is around 13-22. Higher compression ratio reduces knocking but is restricted by mechanical and thermal stresses.
6. **Speed:** S.I. engine operates at a higher speed (2000 - 6000 rpm) as compared to C.I. engine (400 – 1200) rpm.
7. **Efficiency:** Higher efficiency of S.I. engine for same compression ratio and heat input than C.I. engine.
8. **Weight:** Weight per unit power in S.I. is lower as compared to C.I. engine.

Q.2 (b) Solution:

1.



Enthalpy change of vapour refrigerant in heat exchanger = Enthalpy change of liquid refrigerant in heat exchanger

$$\dot{m}(h_2 - h_1) = \dot{m}c_{pl}(T_5 - T_6)$$

$$423.9 - 397.5 = 1.37 \times (36 - T_6)$$

$$T_6 = 16.73°\text{C}$$

2. Now, Refrigerant capacity = $\dot{m}$ × Refrigeration effect

$$7.5 \times 3.5(\text{kW}) = \dot{m} \times (h_1 - h_7)$$

$$7.5 \times 3.5 = \dot{m} \times (397.5 - h_7) \quad \text{...(i)}$$

For enthalpy at point 7,

$$h_7 = h_f - c_{pl}(T_4 - T_6)$$

$$= 249.08 - 1.37(40 - 16.73)$$

$$= 217.2001 \text{ kJ/kg}$$

Put value of h_7 in equation (i),

$$7.5 \times 3.5 = \dot{m}(397.5 - 217.2001)$$

$$\dot{m} = 0.1456 \text{ kg/s}$$

$$\dot{m} = 8.735 \text{ kg/min}$$

Q.2 (c) Solution:

Based on the method of exposing the water surface to the atmospheric air and air circulation inside the cooling tower, they are classified as:

1. Natural Draught type and
2. Mechanical Draught type.

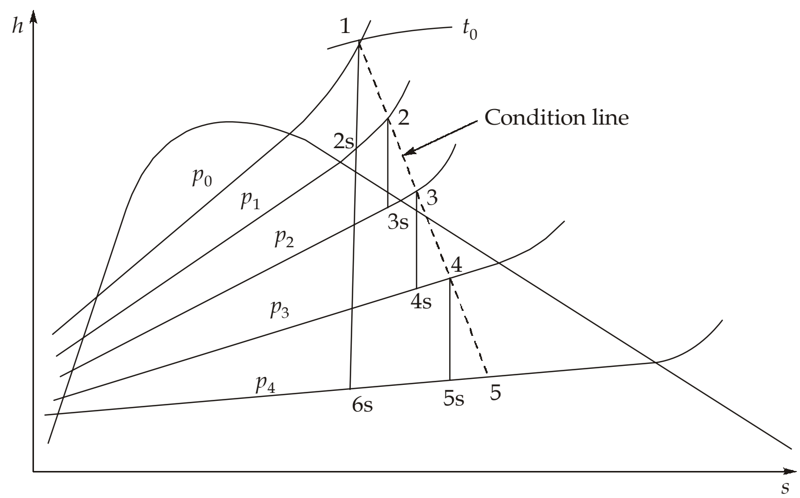
In Natural Draught Type, air flow is not created artificially/mechanically. Water is sprayed up and the atmospheric air is allowed to remove the heat from the water naturally. Efficiency of this type of tower is mainly dependent on wind conditions. It cannot be effectively used for higher capacities beyond say 200 TR, temperature drop of 5 to 6°C is only possible and it requires 2 to 3 times more space than the Mechanical Draught Type tower. However, its advantage is it is inexpensive.

In Mechanical Draught type tower, fans/mechanical equipment is used to create air flow artificially. Air draught is either induced or forced and air flow is either counter flow or cross flow.

In Induced Drought design, a fan is mounted on the top of the tower to generate draught. It requires balancing, periodical maintenance, generate continuous humming noise during operation and some vibration too.

In Forced Air Draught type tower, fan is mounted at the bottom and the air draught is forced in to the bottom of the tower and discharged out from the top. This tower's efficiency is slightly better the induced draught type and can be installed indoor. However, it requires more power to force the air from the bottom of the tower up(due to extra resistance against the gravity) and it takes more time to recirculate the water. Fan size will have to be limited though and it generates continuous humming noise and some vibration during the operation.

Q.2 (d) Solution:



Condition line for a four stage turbine

Consider a 4-stage turbine,

i. The actual and isentropic expansion is shown in the Mollier diagram.

Reheat factor (RF) is defined as the ratio of cumulative heat drop, Δh_c to the direct isentropic or Rankine heat drop, Δh_{isen},

$$RF = \frac{\Delta h_c}{\Delta h_{isen}}$$

$$RF = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{h_1 - h_{6s}} \quad \text{...(1)}$$

It is given that stage efficiency is same for all stages,

$$\eta_{\text{st}} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{h_4 - h_5}{h_4 - h_{5s}}$$

Substituting, $$(h_1 - h_{2s}) = \frac{(h_1 - h_2)}{\eta_{st}}, (h_2 - h_{3s}) = \frac{(h_2 - h_3)}{\eta_{st}}$$

$$(h_3 - h_{4s}) = \frac{(h_3 - h_4)}{\eta_{st}} \text{ and } (h_4 - h_{5s}) = \frac{h_4 - h_5}{\eta_{st}}$$

in eq. (1) we get

$$RF = \frac{\frac{h_1 - h_2}{\eta_{st}} + \frac{h_2 - h_3}{\eta_{st}} + \frac{h_3 - h_4}{\eta_{st}} + \frac{h_4 - h_5}{\eta_{st}}}{h_1 - h_{6s}}$$

or,

$$RF = \left(\frac{h_1 - h_5}{h_1 - h_{6s}}\right) \times \frac{1}{\eta_{st}} = \frac{\eta_{\text{internal}}}{\eta_{st}}$$

where,

$$\eta_{\text{internal}} = \left(\frac{h_1 - h_5}{h_1 - h_{6s}}\right)$$

$$\therefore \quad \eta_{\text{internal}} = \text{R.F.} \times \eta_{\text{st}}$$

Q.3 (a) Solution:

Given:

$$\text{Scale ratio, } L_r = \frac{D_m}{D_p} = \frac{1}{10}$$

Prototype	Model
$P_p = 7355\,\text{kW}$	$H_m = 6\,\text{m}$
$H_p = 10$ m	$\eta_{\text{om}} = 88\%$
$N_p = 100\,\text{rpm}$	
$\eta_{op} = 92\%$	

Effective heads available for the model and prototype may be obtained as:

$$(H_p)_{\text{eff}} = \eta_{\text{op}} \times H_p = 0.92 \times 10 = 9.2 \text{ m}$$

$$(H_m)_{\text{eff}} = \eta_{\text{om}} \times H_m = 0.88 \times 6 = 5.28 \text{ m}$$

Running speed of the model:

$$\frac{(H_m)_{\text{eff}}}{N_m^2 D_m^2} = \frac{(H_p)_{\text{eff}}}{N_p^2 D_p^2}$$

$$N_m = N_p \left(\frac{D_p}{D_m}\right) \times \left[\frac{(H_m)_{\text{eff}}}{(H_p)_{\text{eff}}}\right]^{1/2}$$

$$N_m = 100 \times 10 \times \left(\frac{5.28}{9.2}\right)^{1/2} = 757.6 \text{ rpm}$$

Q.3 (b) Solution:

Given:

Dia. of prototype, D_P = 1.5 m

Viscosity of fluid, μ_P = 3×10^{-2} poise

Q for prototype, Q_P = 3000 lit/s 3.0 m^3/s

Sp. gr. of oil, S_P = 0.9

∴ Density of oil, $\rho_P = S_P \times 1000 = 0.9 \times 1000 = 900\ kg/m^3$

Dia. of the model, D_m = 15 cm = 0.15 m

Viscosity of water at 20°C = 0.01 poise = 1×10^{-2} poise or $\mu_m = 1 \times 10^{-2}$ poise

Density of water or ρ_m = 1000 kg/m^3

For pipe flow, the dynamic similarity will be obtained if the Reynold's number in the model and prototype are equal

Hence using equation, $\dfrac{\rho_m V_m D_m}{\mu_m} = \dfrac{\rho_P V_P D_P}{\mu_P}$ {For pipe, linear dimension in D}

$$\frac{V_m}{V_P} = \frac{\rho_P}{\rho_m} \times \frac{D_P}{D_m} \times \frac{\mu_m}{\mu_P}$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

But $$V_P = \frac{\text{Rate of flow in prototype}}{\text{Area of prototype}}$$

$$= \frac{3.0}{\frac{\pi}{4}(D_P)^2} = \frac{3.0}{\frac{\pi}{4}(1.5)^2}$$

$$= \frac{3.0 \times 4}{\pi \times 2.25} = 1.697\ m/s$$

∴ $V_m = 3.0 \times V_P = 3.0 \times 1.697$ **= 5.091 m/s**

Rate of flow through model, $Q_m = A_m \times V_m = \dfrac{\pi}{4}(D_m)^2 \times V_m = \dfrac{\pi}{4}(0.15)^2 \times 5.091\ m^3/s$

$= 0.0899\ m^3/s = 0.0899 \times 1000$ lit/s **= 89.9 lit/s**

Q.3 (c) Solution:

Speed, N_1 = 180 rpm

Power, P_1 = 5150.25 kW

Head, H_1 = 220 m

Overall efficiency, η_0 = 80% = 0.8

speed ratio, ϕ = 0.47

New head of water, H_2 = 140 m

$$\eta_{\text{overall}} = \frac{P_1}{gQ_1 H_1} = \frac{5150.25}{9.81\times Q_1\times 220}$$

or $$Q_1 = \frac{5150.25}{220\times 9.81\times 0.8} = 2.983\ \text{m}^3/\text{s}$$

Unit speed, $$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{180}{\sqrt{220}} = \mathbf{12.1356\ rpm}$$ **Ans.**

Unit discharge, $$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{2.983}{\sqrt{220}} = \mathbf{0.201\ m^3/s}$$ **Ans.**

Unit power, $$P_u = \frac{P_1}{H_1^{3/2}} = \frac{5150.25}{(220)^{3/2}} = \mathbf{1.578\ kW}$$ **Ans.**

For speed, $$\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

or $$N_2 = N_1\sqrt{\frac{H_2}{H_1}} = 180\sqrt{\frac{140}{220}} = \mathbf{143.6\ rpm}$$ **Ans.**

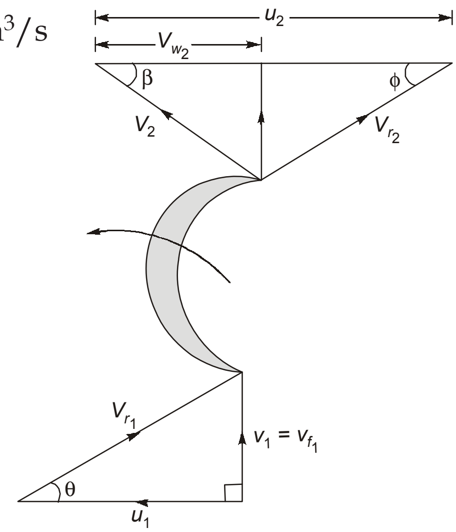
For discharge, $$\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

or $$Q_2 = Q_1\sqrt{\frac{H_2}{H_1}} = 2.983\sqrt{\frac{140}{220}} = \mathbf{2.379\ m^3/s}$$ **Ans.**

For power, $$\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$$

or $$P_2 = P_1\left(\frac{H_2}{H_1}\right)^{3/2} = 5150.25\left(\frac{140}{220}\right)^{1.5} = \mathbf{2614.48\ kW}$$

Q.3 (d) Solution:



$$\text{Diameter at outlet, } D_2 = 1.2 \text{ m}$$

$$\text{Speed, } N = 200 \text{ rpm}$$

$$\text{Discharge, } Q = 1880 \text{ L/S or } 1.88 \text{ m}^3/\text{s}$$

$$\text{Manometric head, } H_m = 6 \text{ m}$$

$$\text{Vane outlet angle, } \phi = 26°$$

$$\text{Velocity of flow at outlet, } V_{f2} = 2.5 \text{ m/s}$$

$$\text{Diameter at inlet, } D_1 = 0.6 \text{ m}$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$= \frac{3.14 \times 1.2 \times 200}{60}$$

$$= 12.56 \text{ m/s}$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

or
$$u_2 - V_{w2} = \frac{V_{f2}}{\tan 26°}$$

$$= \frac{2.5}{\tan 26°} = 5.12$$

∴
$$V_{w2} = u_2 - 5.13 = 12.56 - 5.13 = 7.43 \text{ m/s}$$

$$\eta_{\text{manometric}} = \frac{gH_m}{V_{w2}u_2} = \frac{9.81 \times 6}{7.43 \times 12.56} = 0.63 \text{ or } 63\%$$

Least speed to start the pump:

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

or
$$\frac{\omega^2}{2g}(r_2^2 - r_1^2) = 6$$

or
$$\frac{\omega^2}{2 \times 9.81}(0.6^2 - 0.3^2) = 6$$

or
$$\omega^2 = \frac{6 \times 2 \times 9.81}{0.36 - 0.09} = 436$$

$$\omega = 20.88 \text{ rad/s} = \frac{2\pi N}{60}$$

$$N = \frac{60 \times 20.88}{2 \times 3.14} = \mathbf{200 \text{ rpm}}$$

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Q.4 (a) Solution:

(i) Desirable properties are :

- Hardness
- Hot hardness / red hardness
- Wear resistance
- Toughness
- Low friction
- Better thermal characteristics

High speed steel (HSS) :

- It has high cutting speed capability
- It has high hardness, hot hardness, good wear resistance, high toughness and reasonable cost.
- Toughness of HSS is highest among all the cutting tool materials.
- The hardness of HSS falls rapidly beyond 650°C.
- Cutting speed is limited to 0.5 to 0.75 m/s.

Ceramic tools :

- Ceramics are essentially alumina (Al_2O_3) based high refractory materials introduced specifically for high speed machining of difficult to machine materials and cast iron.
- it can withstand very high temperatures , are chemically more stable and have higher wear resistance than the other cutting tool materials.
- It has excellent fracture toughness
- Ceramic tools are not suitable for low cutting speeds or for intermittent cutting.
- Negative rake angles is used so that less force is applied directly to the ceramic tip.

(ii) In pearlite there is eulectoid composition, so α and Fe_3C phases will be present. As per lever rule, the fulcrum of the line eulectoid point.

$$m_\alpha = \frac{6.67 - 0.8}{6.67 - 0.02} = \mathbf{0.883 \text{ or } 88.3\%}$$

$$m_{Fe_3C} = 1 - 0.883 = \mathbf{0.117 \text{ or } 11.7\%}$$

Q.4 (b) Solution:

Non destructive inspection techniques are necessary for creating a confidence when using a cast product. Some techniques used for testing the various kinds of defects are listed below.

1. **Visual inspection**: Common defects such as rough surfaces (fused sand), obvious shifts, omission of cores, and surface cracks can be detected by a visual inspection of casting. Cracks may also be detected by hitting the casting with a mallet and listening to the quality of the tone.
2. **Pressure test**: The pressure test is conducted on a casting to be used as a pressure vessel. In this, first all the flanges and ports are blocked. Then, the casting is filled with water, oil or compressed air. Thereafter, the casting is submerged in a soap solution when any leak will be evident by the bubbles that come out.
3. **Magnetic particle inspection**: The magnetic particle test is conducted to check for very small voids and cracks at or just below the surface of a casting of a ferromagnetic material. The test involves inducing a magnetic field through the section under inspection. The powdered ferromagnetic material is spread out onto the surface. The presence of voids or cracks in the section results in a change in the permeability of the surface; this, in turn, cause a leakage in the magnetic field. The powdered particles offer a low resistance path to the leakage. Thus, the particles accumulate on the disrupted magnetic field, outlining the boundary of discontinuity.
4. **Dye penetrant inspection**: The dye-penetrant method is used to detect invisible surface defects in a nonmagnetic casting. The casting is brushed with, sprayed with, or dipped into a dye containing a fluorescent material. The surface to be inspected is then wiped, dried and viewed in darkness. The discontinuities in the surface will then be readily visible.
5. **Radiographic examination**: The radiographic method is expensive and is used only for subsurface exploration. In this, both X- and γ-ray are used. With γ-rays, more than one film can be exposed simultaneously; however, X-ray pictures are more distinct. Various defects, e.g., voids, nonmetallic inclusions, porosity, cracks and tears can be detected by this method. On the exposed film, the defects, being less dense, appear darker in contrast to the surrounding.
6. **Ultrasonic inspection**: In the ultrasonic method, an oscillator is used to send an ultrasonic signal through the casting. Such a signal is readily transmitted through a homogeneous medium. However, on encountering a discontinuity, the signal is reflected back. This reflected signal is then detected by an ultrasonic detector. The time interval between sending the signal and receiving its reflection determines the location of the discontinuity. The method is not very suitable for a material with a high damping capacity (e.g., cast iron) because in such a case the signal gets considerably weakened over some distance.

Q.4 (c) Solution:

Detrimental effects of the high cutting temperature on the

Machined product:

- Welding of chips to the work piece.
- Dimensional errors due to stresses generated
- Low surface finish

On the cutting tool:

- Welding of chips to tool
- Excessive tool were and its affects on the life of a tool.
- Overheated at isolated points and localized phase transformation can occur.
- Softening of the surface of the tool and frequently very small cracks will be formed, that results in surface transformation.

(ii) Such cutting temperature can be reduced by using cutting fluids without sacrificing productivity.

Reasons:

- Cutting fluids reduce the temperature either by dissipating generated heat through cooling or by reducing frictional heat generation through lubricant action.
- Results in low power consumption, increased tool life, better surface finish and dimensional accuracy, possibility of using higher cutting speeds and feeds.
- It facilitates removal of chips.

Q.4 (d) Solution:

Given: Current, $I = 12000\,\text{A}$

Resistance, $R = 180\ \mu\Omega = 180 \times 10^{-6}\ \Omega$

Time, $T = 0.1$ s

Thickness of sheet, $t = 1.5$ mm

Indentation = 10% of thickness = 0.1 × 1.5 = 0.15 mm

Heat required for melting = 1680 J/gm

Melting efficiency, $\eta = 64\%$

Now, heat supplied, $H_s = I^2RT = (12000)^2 \times 180 \times 10^{-6} \times 0.1 = 2592$ J

$$\text{We know that, } \eta = \frac{\text{Heat required for melting}}{\text{Heat supplied}}$$

$$H_m = (H_s) \times \eta$$

$$H_m = 2592 \times 0.64$$

$$H_m = 1658.88\ \text{J}$$

$$\text{Mass of nugget} = \frac{H_m}{\text{Heat required for melting one gram of steel}}$$

$$= \frac{1658.88}{1680} = 0.98743 \text{ gm}$$

$$\text{Height of nugget, } h_n = 2 \text{ (Thickness – Indentation)}$$

$$= 2(1.5 - 0.15) = 2.7 \text{ mm}$$

$$\text{Diameter of nugget, } d_n = 6\sqrt{t} = 6\times\sqrt{1.5} = 7.348 \text{ mm}$$

$$\text{Volume of nugget, } V_n = \frac{\pi}{4}d_n^2 \times h_n$$

$$= \frac{\pi}{4}\times(7.348)^2\times(2.7)$$

$$V_n = 114.496 \text{ mm}^3 = 114.496 \times 10^{-3} \text{ cm}^3$$

$$\text{Mass of nugget} = \text{Density} \times \text{Volume of nugget}$$

$$0.98743 = \rho \times (114.496 \times 10^{-3})$$

$$\rho = 8.6244 \text{ g/cm}^3 = 8624.1 \text{ kg/m}^3$$

Q.5 (a) Solution:

From the turning moment equation, $T_{mean} = 10000$ N.m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 300}{60} = 31.4 \text{ rad/s}$$

$$\text{Power of engine, } P = T\omega$$

$$= 10000 \times 31.4 = \mathbf{314 \text{ kW}}$$ **Ans. (i)**

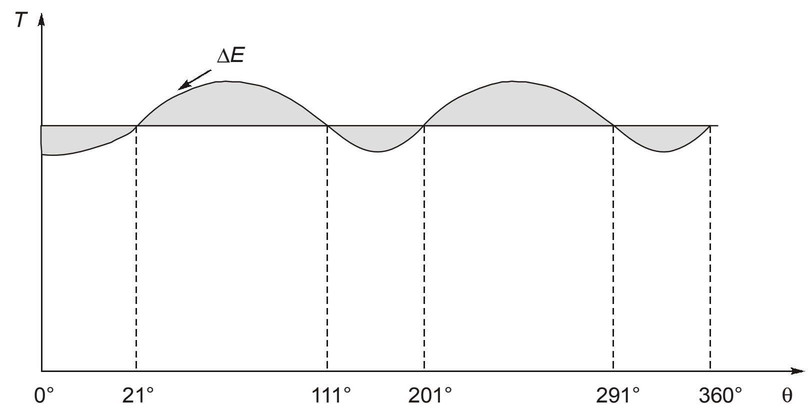
$$T = 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta$$

$$T - T_m = \Delta T = 2000 \sin 2\theta - 1800 \cos 2\theta$$

$$\Delta T = 0$$

$$\tan 2\theta = \frac{1800}{2000} = 0.9$$

$$\theta = \frac{n\pi}{2} + 21°, n = 0, 1, 2$$



Maximum fluctuation of energy, ΔE

$$= \int_{21^\circ}^{111^\circ} \Delta T d\theta = \int_{21^\circ}^{111^\circ} (2000 \sin 2\theta - 1800 \cos 2\theta) \, d\theta$$

$$= 1000 |-\cos 2\theta|_{21^\circ}^{111^\circ} - 900 |\sin 2\theta|_{21^\circ}^{111^\circ}$$

$$= 1486.3 + 900 \times 1.3383 = 2690.735 \text{ N.m}$$

$$\Delta E = I\omega^2 C_s$$

$$C_s = 0.5, I = ?$$

$$2690.735 = I \times (31.4)^2 \times \frac{0.5}{100}$$

$$I = \mathbf{545.81 \text{ kg.m}^2}$$ **Ans.(ii)**

$$\text{At } \theta = 45°$$

$$\Delta T = 2000 \sin 90° - 1800 \cos 90°$$

$$\Delta T = 2000 \text{ N.m}$$

$$\Delta T = I\alpha$$

$$\alpha = \frac{\Delta T}{I} = \frac{2000}{545.81} = \mathbf{3.6643 \text{ rad/s}^2}$$ **Ans.(iii)**

Q.5 (b) Solution:

$$`T_A = 80 \; T_B = 200$$

Now, $$T_B = 2\left[\frac{T_A}{2} + T_C\right]$$

or $$200 = 2\left[\frac{80}{2} + T_C\right] \text{ or } T_C = 60$$

Action	a	A	C/D	B
a fixed, A + 1 rev.	0	1	$-\frac{80}{60}$	$-\frac{80}{60}\times\frac{60}{200}$
a fixed, A + x rev.	0	x	$-\frac{4x}{3}$	$-\frac{2x}{5}$
Add y	y	$y+x$	$y-\frac{4x}{3}$	$y-\frac{2x}{5}$

(i) From the given conditions,

$$y + x = 100 \quad \text{or} \quad y = 100 - x$$

and
$$y - \frac{2x}{5} = -50$$

or
$$100 - x - \frac{2x}{5} = -50 \quad \text{or} \quad x = 107.1 \text{ and } y = -7.1$$

Thus, Speed of arm a = 7.1 rpm counterclockwise

(ii)
$$y + x = 100 \quad \text{or} \quad y = 100 - x$$

and
$$y - \frac{2x}{5} = 0$$

or
$$100 - x - \frac{2x}{5} = 0$$

or
$$x = 71.4$$

and
$$y = 28.6$$

Thus, Speed of arm 'y' = 28.6 rpm clockwise

Q.5 (c) Solution:

$$\text{Power, } P = 8 \text{ kW},$$
$$\mu = 0.3,$$
$$N = 1000 \text{ rpm}$$
$$R_m = \frac{R_0 + R_i}{2} = 4.5(R_0 - R_i)$$
$$R_0 + R_i = 9\,R_0 - 9\,R_i$$

or
$$8R_0 = 10\,R_i$$

or
$$R_0 = 1.25\,R_i$$

In case of power transmission through a clutch, it is safer to use the expressions obtained by uniform wear theory.

$$P = T\omega$$
$$8000 = T \times \frac{2\pi \times 1000}{60}$$

or $$T = 76.39 \text{ N.m}$$

$$\text{Torque, } T = \frac{\mu F}{2}(R_0 + R_i) \times n \qquad (n = \text{number of surfaces})$$

$$T = \frac{\mu}{2}[2\pi p_i R_i (R_0 + R_i)](R_0 + R_i)n$$

$$T = \frac{0.3}{2} \times 2[2 \times 3.14 \times 70 \times 10^3 \times R_i \times 0.25 R_i](2.25 R_i)$$

$$76.39 = 74220.126\, R_i^3$$

$$R_i^3 = 1.0292 \times 10^{-2}$$

$$R_i = \mathbf{0.1009 \text{ m or } 101 \text{ mm}}$$

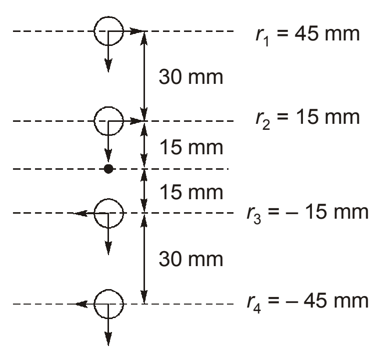
$$R_0 = 1.25\, R_i = \mathbf{126.25 \text{ mm}}$$

$$\text{mean radius, } R_m = \frac{R_0 + R_i}{2} = \frac{126.25 + 1.1}{2} = 113.625 \text{ mm}$$

$$\text{width} = \frac{R_m}{4.5} = \frac{113.625}{4.5} = \mathbf{25.25 \text{ mm}}$$

Q.5 (d) Solution:

$$\text{Primary force on each rivet} = \frac{20}{4}\text{kN} = 5 \text{ kN}$$



$$F \times e = \frac{F_{s1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right]$$

$$20 \times \frac{80}{1000} = \frac{F_{s1}}{0.045} \times \left[(0.045)^2 + (0.015)^2 + (0.015)^2 + (0.045)^2\right]$$

$$F_{s1} = \frac{20 \times 80 \times 0.045}{1000 \times 4.5 \times 10^{-3}} = 16 \text{ kN}$$

$$F = \sqrt{F_p^2 + F_{J_1}^2 + 2F_P \times Fs_1 \times \cos 90} = \sqrt{\left(\frac{20}{4}\right)^2 + 16^2}$$

$$= 16.76 \text{ kN} = 16.8 \text{ kN}$$

Q.6 (a) Solution:

As the given cone is symmetrical about xy-plane, z coordinate of centroid is zero. Now consider an half disk element at distance x of radius $r = \frac{ax}{h}$ whose centroid lies at $\frac{4r}{3\pi}$.

$$\bar{y} = \frac{\int y\,dV}{V} \text{ where } V \text{ is volume}$$

$$= \frac{\int \frac{4r}{3\pi} \times \frac{\pi r^2}{2} dx}{\frac{1}{3}\frac{\pi a^2 h}{2}} = \frac{\frac{2}{3}}{\frac{\pi a^2 h}{6}} \int \left(\frac{ax}{h}\right)^3 dx$$

$$= \frac{4}{\pi a^2 h} \times \frac{a^3}{h^3} \times \frac{h^4}{4} = \frac{a}{\pi}$$

$$\bar{x} = \frac{\int x\,dV}{\frac{\pi a^2 h}{6}} = \frac{6}{\pi a^2 h} \int \frac{x \pi r^2}{2} dx = \frac{3}{a^2 h} \int x \times \left(\frac{ax}{h}\right)^2 dx$$

$$= \frac{3}{h^3} \times \frac{h^4}{4} = \frac{3h}{4}$$

Q.6 (b) Solution:

Since all three forces are in equilibrium

then $\quad \vec{F_1} + \vec{F_2} + \vec{F_3} = 0 \quad$ ....(i)

where $\quad |\vec{F_1}| = 7 \text{ N}$

$$|\vec{F_2}| = 5 \text{ N}$$

$$|\vec{F_3}| = 3 \text{ N}$$

From equation (i), $\quad \vec{F_1} = -(\vec{F_2} + \vec{F_3})$

or $\quad (\vec{F_2} + \vec{F_3}) = -\vec{F_1}$

Squaring both sides, we get, $(\vec{F_2} + \vec{F_3})^2 = (-\vec{F_1})^2$

Also, $\quad (\vec{A})^2 = |\vec{A}|^2 = A^2$

$\Rightarrow \quad F_2^2 + F_3^2 + 2F_2F_3\cos\theta = F_1^2$

where θ is angle between F_2 and F_3

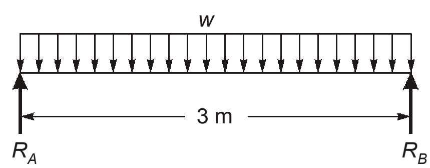
$$5^2 + 3^2 + 2 \times 5 \times 3 \times \cos\theta = 7^2$$

$$\cos\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 60°$$

Hence angle between 5 N and 3 N forces is 60°.

Q.6 (c) Solution:

Let the diameter of rod be "D" m.



$$\text{Weight of rod} = \rho \times \text{volume} \times g$$

$$= 7860 \times \frac{\pi}{4} \times D^2 \times 3 \times 9.81$$

$$= 181,678.15\ D^2 N$$

$$\text{Weight per unit length of rod} = \frac{181,678.15 D^2}{3} N/m$$

$$= 60,559.38\ D^2 N/m$$

Maximum bending moment for simply supported and uniformly loaded beam

$$= \frac{wl^2}{8} = \frac{60559.38 D^2 \times 9}{8}$$

$$= 68129.303\ D^2 (N.m.)$$

$$\sigma_{max} = \frac{M_{max}}{Z}$$

$$28 \times 10^6 = \frac{68129.303 D^2}{Z}$$

$$Z = \frac{I}{y} = \frac{\pi}{64} \frac{D^4}{\frac{D}{2}} = \frac{\pi}{32} D^3$$

$$28 \times 10^6 = \frac{68129.303 D^2}{\frac{\pi}{32} \times D^3}$$

$$D = 0.0248 \text{ m} = 24.78 \text{ mm}$$

Q.6 (d) Solution:

$$\text{Diameter of rod, } d = 3 \text{ cm} = 30 \text{ mm}$$

$$\text{Area of the rod, } A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times 900 = 225\pi \text{ mm}^2$$

$$\text{Initial temperature, } T_1 = 90°\text{C}$$

$$\text{Final temperature, } T_2 = 25°\text{C}$$

$$\text{Fall in temperature, } \Delta T = T_1 - T_2 = 90 - 25 = 65°\text{C}$$

$$\text{Modulus of elasticity, } E = 2 \times 10^5 \text{ MPa}$$

$$\text{Coefficient of linear expansion, } \alpha = 12 \times 10^{-6} \text{ /°C}$$

(i) When the ends do not yield

$$\begin{aligned} \text{Stress} &= \alpha \Delta T E \\ &= 12 \times 10^{-6} \times 65 \times 2 \times 10^5 \text{ MPa} \\ &= 156 \text{ MPa (Tensile)} \end{aligned}$$

$$\begin{aligned} \text{Pull in the rod} &= \text{stress} \times \text{area} \\ &= 156 \times 225\pi = 110269.9 \text{ N or } 110.27 \text{ kN} \end{aligned}$$

(ii) When the ends yield by 0.15 cm

$$\delta = 0.15 \text{ cm} = 1.5 \text{ mm}$$

The stress when the ends yield is given by equation

$$\begin{aligned} \text{Stress} &= \left(\frac{(\alpha TL - \delta)}{L}\right)\times E = \frac{12\times 10^{-6}\times 65\times 5000 - 1.5}{5000}\times 2\times 10^5 \\ &= \left(\frac{3.9-1.5}{5000}\right)\times 2\times 10^5 = 96 \text{ MPa} \quad \text{(Tensile)} \end{aligned}$$

$$\begin{aligned} \text{Pull in the rod} &= \text{stress} \times \text{area} \\ &= 96 \times 225\pi = 67858.4 \text{ N} = 67.86 \text{ kN} \end{aligned}$$

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