Detailed Solutions

SSC-JE 2018 Mains Test Series (PAPER-II)

Mechanical Engineering Test No : 1

Q.1 (a) Solution:

For polytropic process,

$$P_1V_1^n = P_2V_2^n = P_3V_3^n$$

$$n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{100}{95}\right)}{\ln\left(\frac{0.1043671}{0.1}\right)} = 1.2$$

$$\text{Work done} = \frac{P_1V_1 - P_3V_3}{n-1} = 0.90835 \text{ kJ}$$

Now, Work done = (Mean effective pressure) × (Swept volume)

$$0.90835 = 80\times\frac{\pi}{4}\times d^2\times L$$

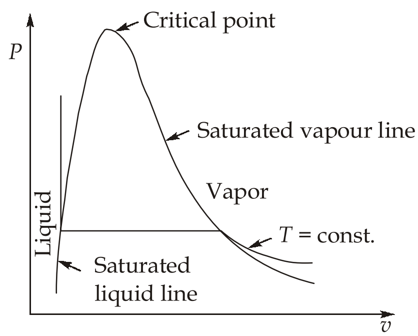
$$0.90835 = 80\times\frac{\pi}{4}\times 0.2^2\times L$$

$$L = 0.361 \text{ m}$$

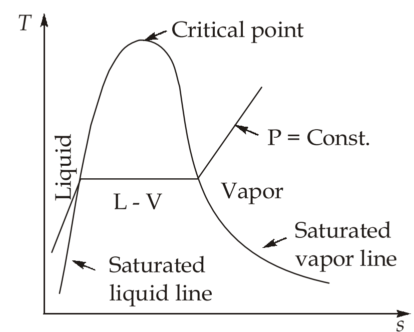
$$L = 361 \text{ mm}$$

Q.1 (b) Solution:

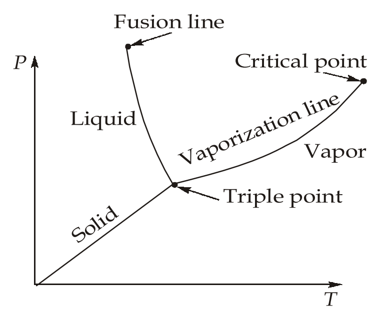
Water is a substance which expands upon freeezing.



Phase equilibrium diagram for water on P-v plane



Phase equilibrium diagram for water on T-s plane



Phase equilibrium diagram for water on P-T plane

Q.1 (c) Solution:

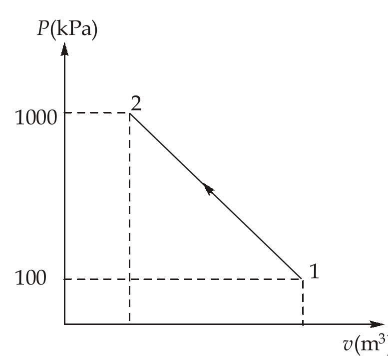
Assumptions are:

(i) Here, CO_2 gas is enclosed in a closed system as no mass crosses the boundary of the system.

(ii) CO_2 is an ideal gas since it is at a high temperature relative to its critical temperature of 304.2 K.

(iii) *KE* and *PE* changes are negligible.

Pressure changes linearly with volume and the work done is equal to the area under the process line 1-2:



As per 1st law of thermodynamics,

$$Q = W + \Delta U$$

$$Q = W + mc_v(T_2 - T_1) \quad \text{...(i)}$$

$$\text{Initial volume, } V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.1889 \times 298}{100} = 0.5629 \text{ m}^3$$

$$\text{Final volume, } V_2 = \frac{mRT_2}{P_2} = \frac{1 \times 0.1889 \times 573}{1000} = 0.1082 \text{ m}^3$$

$$W = \text{Area} = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$

$$= \left(\frac{100 + 1000}{2}\right) \times (0.1082 - 0.5629)$$

$$W = -250.1 \text{ kJ}$$

$$\text{Work input} = 250.1 \text{ kJ}$$ Answer

$$\text{From equation (i): } Q = -250.1 + 0.657(300 - 25) = -69.4 \text{ kJ}$$

$$Q_{out} = 69.4 \text{ kJ}$$ Answer

Q.1 (d) Solution:

Given data:

$$p_1 = 5 \text{ bar}$$

$$T_1 = 500 \text{ K}$$

$$V_1 = 50 \text{ m/s}$$

$$c_p = 1.005 \text{ kJ/kgK}$$

$$c_v = 0.718 \text{ kJ/kgK}$$

$$\therefore \quad R = c_p - c_v = 1.005 - 0.718 = 0.287 \text{ kJ/kgK}$$

$$\text{Surrounding condition: } p_0 = 1 \text{ bar}$$

$$T_0 = 300 \text{ K}$$

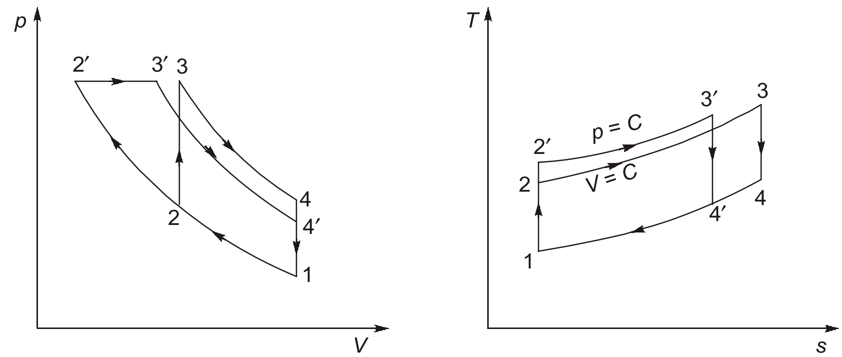
$$\text{Available energy} = (h_1 - h_0) + \frac{V_1^2 - V_0^2}{2000} - T_0\left[c_p \log_e \frac{T_1}{T_0} - R \log_e \frac{p_1}{p_0}\right]$$

$$= c_p(T_1 - T_0) + \frac{V_1^2 - 0}{2000} - 300\left[1.005 \log_e \frac{500}{300} - 0.287 \log_e \frac{5}{1}\right]$$

$$= 1.005(500 - 300) + \frac{(50)^2}{2000} - 300[0.5133 - 0.4619]$$

$$= 201 + 1.25 - 15.42 = \mathbf{186.83 \text{ kJ/kg}}$$

Q.2 (a) Solution:



The Otto and Diesel cycles are shown in p-V and T-s diagram of above figure, according to given condition of the same maximum cycle pressure and heat input. These cycles are shown in figure. as

Otto cycle: 1-2-3-4-1

Diesel cycle: 1-2′-3′-4′–1

$$\text{The thermal efficiency} = 1 - \frac{\text{heat rejection}}{\text{heat input}}$$

As the heat input is constant. The thermal efficiency of the cycle is dependent only one the heat rejection, lower the heat rejection, higher the thermal efficiency. It is cleared from T-s diagram, heat rejection is higher in the Otto cycle and lower in the Diesel cycle.

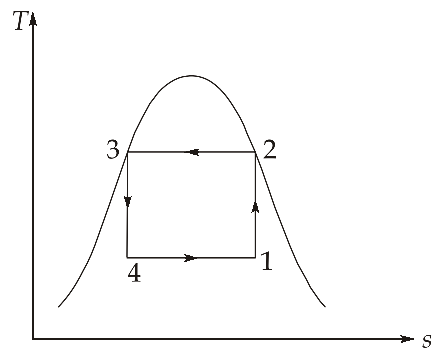
i.e., $$(Q_{4\text{-}1})_{Otto} > (Q_{4'\text{-}1})_{Diesel}$$

Thus, the thermal efficiency of the Diesel cycle is more than Otto cycle at given condition of the same maximum cycle pressure and heat input

i.e., $$\eta_{Diesel} > \eta_{Otto}$$

Q.2 (b) Solution:

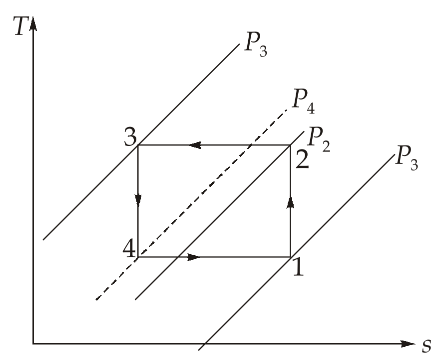
In the reversed Carnot cycle with vapour as refrigerant, the isothermal processes of condensation and evaporation are internally reversible process, and they are easily achievable in practice although there may be some problem in having only partial evaporation. However, isentropic compression and expansion processes have some limitations. In brief, it is difficult to design an expander to handle a mixture of largely liquid and partly vapour for the process 3-4. Also, because of the internal irreversibilities in the compressor and the expander, the actual COP of the Carnot cycle is very low, though the ideal cycle COP is the maximum. A cycle which is closest to the reversed Carnot vapour cycle is the vapour compression cycle.



There are two drawbacks of reversed Carnot cycle with gas as a refrigerant.

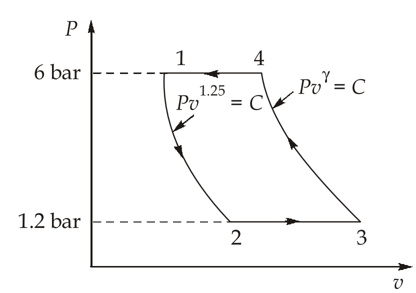
(i) Firstly, it is not possible to devise, in practice, isothermal processes of heat absorption and rejection, 4-1 and 2-3 in figure with gas as the working substance. These are impractical as those will be infinitely slow.

(ii) Secondly, the cycle on P-V diagram is very narrow since the volume is changing both during the reversible isothermal and reversible adiabatic process. Drawn correctly to scale, the Carnot P-V diagram is much thin. As a result, the stroke volume of the cylinder is very large. The cycle, therefore, suffers from poor actual COP as a result of irreversibilities of the compressor and expander.



Q.2 (c) Solution:

$$T_3 = 15 + 273 = 288\text{K}$$
$$T_1 = 25 + 273 = 298\text{K}$$
$$P_1 = P_4 = 6 \text{ bar}$$
$$P_2 = P_3 = 1.2 \text{ bar}$$



For isentropic compression process 3 → 4 :

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6}{1.2}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.2857} = 1.5838$$

$$T_4 = T_3 \times 1.5838$$

$$T_4 = 288 \times 1.5838 = 456.1344 \text{ K}$$

For isentropic expansion (1 → 2) process:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{1.25-1}{1.25}} = \left(\frac{1.2}{6}\right)^{0.2} = 0.72478$$

$$T_2 = T_1 \times 0.72478$$

$$T_2 = 298 \times 0.72478$$

$$T_2 = 215.984 \text{ K}$$

(i) Work done per kg of air flow:

$$W_{net} = W_{comp.} - W_{exp.}$$

$$= \frac{\gamma}{\gamma-1}R(T_4 - T_3) - \frac{n}{n-1}R(T_1 - T_2)$$

$$= \frac{1.4}{0.4}\times 0.287(456.1344 - 288) - \frac{1.25}{0.25}\times 0.287(298 - 215.984)$$

$$= 168.891 - 117.693 = 51.198 \text{ kJ/kg}$$ Answer (i)

(ii) Refrigerating effect $= c_p(T_3 - T_2)$

$$= 1.003(288 - 215.984) = 72.232 \text{ kJ/kg}$$ Answer (ii)

(iii) $$\text{COP} = \frac{\text{Refrigerating effect}}{\text{Work done}}$$

$$= \frac{72.232}{51.198} = 1.41$$ Answer (iii)

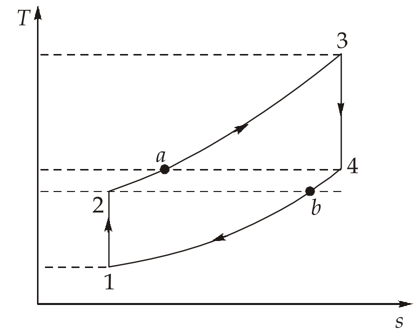
(iv) Refrigerating capacity $= \dfrac{\dot{m}\times\text{Refrigerating effect}}{3.5}$ tonnes

$$= \frac{72.232\times 90}{3600\times 3.5} = 0.515 \text{ tonnes}$$ Answer (iv)

Q.2 (d) Solution:

Regeneration: Regeneration is a process in which high temperature exhaust gases coming out from the turbine is utilised for heating cold air coming out from the compressor and before entering the combustion chamber. This preheating of cold air decreases the fuel requirement thus increasing the efficiency.

Effects of regeneration:



1. Compressor work → Remains same
2. Turbine work → Remains same
3. Net work output → Remains same
4. Heat supplied → Decreases
5. Heat rejected → Decreases
6. Mean temperature of heat addition → Increases
7. Mean temperature of heat rejection → Decreases
8. Specific fuel consumption → Decreases
9. Efficiency → Increases.

Ideal regenerative cycle: In an ideal regenerative cycle cold air is heated upto turbine exit temperature in a regenerator. It is possible for an infinitely large heat exchanger and at that point its effectiveness will be 100% and T_a will become equal to T_4 and T_b will become equal to T_2.

$$\eta = 1-\frac{Q_R}{Q_S}$$

$$\eta = 1-\frac{(T_b-T_1)}{(T_3-T_a)}$$

For ideal regenerative cycle, $T_a = T_4$, $T_b = T_2$

$$\eta = 1-\frac{(T_2-T_1)}{(T_3-T_4)}$$

$$= 1-\frac{T_1\left[(T_2/T_1)-1\right]}{T_3\left[(1-T_4/T_3)\right]}$$

$$= 1-\left(\frac{T_1}{T_3}\times\frac{T_2}{T_1}\right)\frac{[1-T_1/T_2]}{[1-T_4/T_3]}$$

In Brayton cycle, $$\frac{T_3}{T_4} = (r_p)^{\gamma-1/\gamma}$$

$$\frac{T_2}{T_1} = (r_p)^{\gamma-1/\gamma}$$

Now, $$\frac{T_3}{T_4} = \frac{T_2}{T_1} \text{ or } \frac{T_4}{T_3} = \frac{T_1}{T_2} \quad \text{also} \quad 1-\frac{T_4}{T_3} = 1-\frac{T_1}{T_2}$$

$$\eta = 1 - \frac{T_1}{T_3}(r_p)^{\gamma-1/\gamma} \times 1$$

$$\eta_{\text{Ideal regen.}} = 1 - \frac{T_1}{T_3}(r_p)^{\gamma-1/\gamma}$$

Q.3 (a) Solution:

As per given information,

$$p = c\rho^k$$

$$dp = c(k\rho^{k-1}\,d\rho)$$

differential form of Bernoulli's equation

$$\int \frac{dp}{\rho} + \int v\,dv + \int g\,dz = \text{constant}$$

$$\int \frac{c\left(k\rho^{k-1}\cdot\,d\rho\right)}{\rho} + \int v\,dv + \int g\,dz = \text{constant}$$

$$ck\int \rho^{k-2}\,d\rho + \frac{v^2}{2} + gz = \text{constant}$$

$$c.k\frac{\rho^{k-2+1}}{(k-2+1)} + \frac{v^2}{2} + gz = \text{constant}$$

$$\frac{kc\rho^k}{\rho(k-1)} + \frac{v^2}{2} + gz = \text{constant} \qquad (p = c\rho^k)$$

$$\frac{kp}{\rho(k-1)} + \frac{v^2}{2} + gz = \text{constant.}$$

Q.3 (b) Solution:

$$\rho = 900 \text{ kg/m}^3;$$

$$\mu = 10 \text{ poise} = 1 \text{ N s/m}^2;$$

$$D = 110 \text{ mm} = 0.11 \text{ m}$$

$$U_{max} = 2 \text{ m/s};$$

$$U_{max} = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)R^2$$

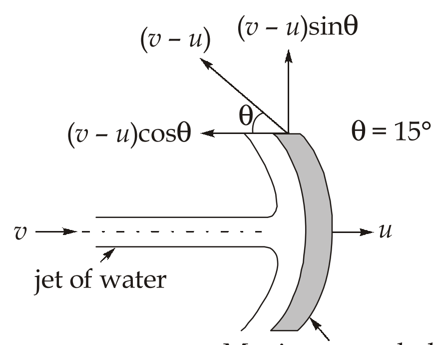
(i) $$\left(\frac{-dp}{dx}\right) = \frac{4\mu \times U_{max}}{R^2} = \frac{4 \times 1 \times 2}{0.055^2} = 2644.6 \text{ N/m}^3;$$

(ii) $$\tau_0 = \left(\frac{-dp}{dx}\right) \times \frac{R}{2} = 2644.6 \times \frac{0.055}{2} = 72.72 \text{ N/m}^2;$$

(iii) $$Re = \frac{\rho \times \bar{\mu} \times D}{\mu} = \frac{900 \times 1 \times 0.11}{1} = 99; \text{ and}$$

(iv) $$u = \frac{1}{4\mu}\left(\frac{-dp}{dx}\right)(R^2 - r^2)$$
$$= (2644.6)(0.055^2 - 0.025^2) = 1.586 \text{ m/s}$$

Q.3 (c) Solution:



F_x = Force exerted in x-direction

[Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$F_x = \rho a[(v - u)[(v - u) - (-(v - u)\cos\theta)]$$

$$F_x = \rho a(v - u)^2 [1 + \cos\theta]$$

Diameter of the jet, d = 10 cm = 0.1 m

Area of jet, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(0.1)^2 = 7.854 \times 10^{-3}$ m²

Velocity of the jet, v = 20 m/s

Velocity of the plate, u = 8 m/s

Angle of deflection of the jet = 165°

Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°$$

$$F_x = 1000 \times 7.854 \times 10^{-3} \times (20 - 8)^2[1 + \cos 15°]$$

$$F_x = 7.854 \times 144 \times 1.966$$

$$F_x = 2223.5 \text{ N} \qquad \text{Answer (1)}$$

Work done by the jet on the plate per second = $F_x \times 4$

$$W = 2223.5 \times 8 = 17788 \text{ N m/s}$$

Work done by the jet on the plate per second = $\dfrac{17788}{1000} = 17.8 \text{ kW}$ Answer (2)

$$\text{Efficiency of the jet} = \frac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$$

$$\eta = \frac{17788}{\frac{1}{2}(\rho a v)v^2} = \frac{2\times 17788}{1000\times 7.854\times 10^{-3}\times 20^3}$$

$$\eta = 0.5662 \text{ or } 56.62\% \qquad \text{Answer (3)}$$

Q.3 (d) Solution:

Given data:

60 % of volume of the spherical float is immersed in water

Diameter of valve: $d = 15 \text{ mm} = 0.015 \text{ m}$

∴ Area of the valve: $a = \dfrac{\pi}{4}d^2 = \dfrac{3.14}{4}\times(0.015)^2 = 1.766 \times 10^{-4} \text{ m}^2$

Pressure required to closed the valve,

$$p = 147 \text{ kN/m}^2 = 147 \times 10^3 \text{ N/m}^2$$

∴ Force exerted by fluid on the valve when closed,

$$F = p \times a = 147 \times 10^3 \times 1.766 \times 10^{-4} \text{ N} = 25.96 \text{ N}$$

Let V = volume of float

Volume of float immersed in water: $\forall$ = 60 % of volume of float = $0.6V$

∴ Buoyant force: F_B = weight of liquid displaced by the body

$$= \rho g \forall = 1000 \times 9.8 \times 0.6V = 5886V \text{ N}$$

Now taking moment about hinge point is zero

$$F_B \times 0.5 - F \times 0.150 = 0$$

$$5886 \times V \times 0.5 - 25.96 \times 0.150 = 0$$

$$5886 \times V \times 0.5 = 4.04976$$

$$V = 1.37606 \times 10^{-3} \text{ m}^3$$

Volume of float: $V = \dfrac{\pi}{6}D^3$

$$\therefore \quad 1.37606 \times 10^{-3} = \frac{3.14}{6} \times D^3$$

$$D^3 = 2.6294 \times 10^{-3} \text{ m}^3$$

or Diameter of the float: $D = 0.13802 \text{ m}$ **= 138.02 mm**

Q.4 (a) Solution:

(i) Important properties of stainless steels are:

- Good corrosion and oxidation resistance
- Good creep strength
- Corrosion resistance can further be enhanced by nickel and molybdenum
- It possesses magnetic properties
- Can frequently be used to elevated temperatures
- Good abrasion resistance

(ii)

(1) Ferritic stainless steel:

AlSl 409 : 0.08% C, 9.0 % Cr, 1 % Mn, 0.5 % Ni, 0.75 % Ti

AlSl 446 : 0.20% C, 25% Cr, 1.5% Mn

Applications : Valve bodies, combustion chambers, automotive exhaust components etc.

(2) Martensitic stainless steel :

AlSl 410 : 0.15 % C, 12.5% Cr, 1 % Mn

AlSl 440 B : 0.75 % C, 17 % Cr

Applications : Rifle barrels, cutlery, jet engine parts etc.

(3) Austenitic stainless steels:

AlSl 304 : 0.08% C, 19 % Cr, 2 % Ni, 2 % Mn

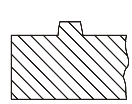
AlSl 316 L : 0.03 % C, 17 % Cr, 12 % Ni, 2.5 % Mo, 2 % Mn

Applications : Cryogenic vessels, welding construction etc.

Q.4 (b) Solution:

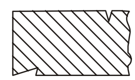
Types of Castings Defects:

(i) Drop: Irregular projections on the top of casting caused by dropping of sand from cope.



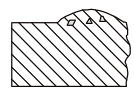
Drop

(ii) Buckle: V-shaped depressions occurring on flat castings due to expansions of sand at the mould face before liquid metal solidifies.



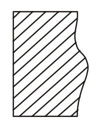
Buckle

(iii) Scab: Protruding surface of casting at roof.



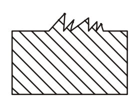
Scab

(iv) Swell: Liquid metal displaces the sand at the wall regions due to hydrostatic pressure.



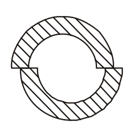
Swell

(v) Penetration: Due to improper ramming of sand, liquid metal penetrates into the sand.



Penetration

(vi) Mould Shift: Due to misalignment between the two halves.



Mould shift

Q.4 (c) Solution:

$$\text{The rpm of the grinding wheel} = \frac{1000 \times 20 \times 60}{\pi \times 250} = 1528 \text{ rpm}$$

$$\text{Total approach distance} = 250 \text{ mm}$$

$$\text{Time for one pass} = \frac{250 + 250}{10 \times 1000} = 0.05 \text{ minutes}$$

For transverse-feed rate of 5 mm/pass

$$\text{Number of passes required} = \frac{100}{5} = 20$$

$$\text{Total grinding} = 20 \times 0.05 = 1 \text{ minute.}$$

(i) Given, $W = 100$ mm and $D = 200$ mm

Total approach distance, $A = 200$ mm

$$\text{Total machining time} = \frac{250+200}{10\times 2000} = 0.045 \text{ minutes}$$

Q.4 (d) Solution:

Grinding wheels wear by three different mechanisms as described below:

1. **Attritious wear**: The cutting edges of a sharp grain becomes dull by attrition (known as attritious wear), developing a wear flat that is similar to flank wear in cutting tools. Wear is caused by the interaction of the grain with the workpiece material, resulting in complex physical and chemical reactions.

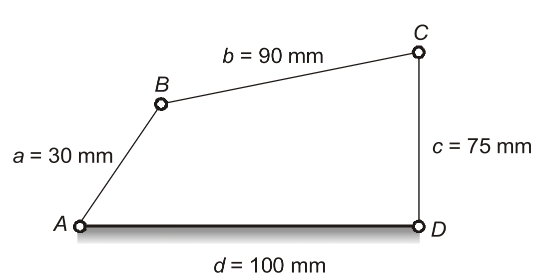
   These reactions involve diffusion, chemical degradation or decomposition of the grain, fracture at a microscopic scale, plastic deformation, and melting. Attritious wear as low when the two materials are chemically inert with respect to each other.

2. **Grain fracture**: Abrasive grains are brittle so during grinding their fracture may occur, their fracture characteristics in grinding are important. If the wear flat caused by attritious wear is excessive, the grain becomes dull, and the grinding operation becomes inefficient and produce high temperature.

3. **Bond fracture**: The strength of the bond (grade) is a significant parameter in grinding.

   If the bond is too strong, dull grains can't be dislodged so that new grains come above and start cutting. On the other hand, if the bond is too weak the grains are easily dislodged and the wear rate of wheel increases.

Q.5 (a) Solution:



$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

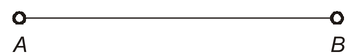
At $\theta = 180°$, Transmission angle is maximum

$$a^2 + d^2 + 2ad = b^2 + c^2 - 2bc\cos\mu$$

$$(a+d)^2 = b^2 + c^2 - 2bc\cos\mu$$

$$\cos\mu = \frac{b^2 + c^2 - (a+d)^2}{2bc}$$

$$\mu = \cos^{-1}\left[\frac{\{b^2 + c^2 - (a+d)^2\}}{2bc}\right]$$



$$\mu_{max} = \cos^{-1}\left[\frac{90^2 + 75^2 - 130^2}{2 \times 90 \times 75}\right]$$

$$\mu_{max} = 103.6$$

At $\theta = 0°$ transmission angle is

$$a^2 + d^2 - 2ad\cos\theta = b^2 + c^2 - 2bc\cos\mu$$

$$a^2 + d^2 - 2ad = b^2 + c^2 - 2ad\cos\mu$$

$$(a-d)^2 = b^2 + c^2 - 2bc\cos\mu$$

$$\cos\mu = \frac{b^2 + c^2 - (a-d)^2}{2bc} = \frac{90^2 + 75^2 - (30-100)^2}{2 \times 90 \times 75}$$

$$\mu = \cos[0.6537]$$

$$\mu_{min} = 49.18°$$

Q.5 (b) Solution:

(i)

As pressure angle is the angle made by the direction of motion of follower and the normal to the pitch curve at any point (or in other words the measure of steepness of the cam profile), its significance can be stated as:

- Higher value of pressure angle increases the horizontal force acting on the follower (as higher is the side thrust). This could jam the translating follower in its guide ways.
- Lower value of pressure angle increases the weight of cam and it becomes bulky, thus increasing the inertial force.

Following are the consideration of pressure angle from performance point of view:

(i) Usually the pressure angle should be as small as possible within the limits of design and

(ii) The pressure angle can be reduced by increasing the cam size or by adjusting the offset.

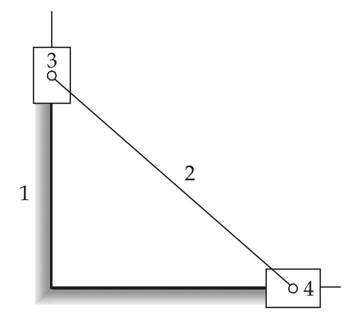
(ii)

1. **Trace point**: It is a point on the follower, and it is used to generate the pitch curve. Its motion describing the movement of the follower. For a knife-edge follower, the trace point is at knife-edge. For a roller follower the trace point is at the roller center, and for a flat-face follower, it is at the point of contact between the follower and the cam surface when the contact is along

the base circle of the cam. It should be not that the trace point is not necessarily the point of contact for all other positions of the cam.

2. **Pitch point**: The point corresponds to the point of maximum pressure angle is called pitch point, and a circle drawn with its centre at the cam centre, to pass through the pitch point, is known as the pitch circle.
3. **Prime circle**: The prime circle is the smallest circle that can be drawn so as to be tangential to the pitch curve, with its centre at the cam centre. For a roller follower, the radius of the prime circle will be equal to the radius of the base circle plus that of the roller where as for knife-edge follower the prime circle will coincides with the base circle.

Q.5 (c) Solution:

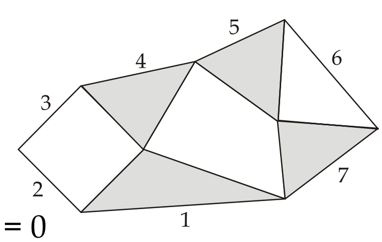


Number of links = 4

Number of joints = 4

Mobility, M = 3(*n* – 1) – 2j

= 3(4 – 1) – 2 × 4 = 9 – 8 = 1 Answer

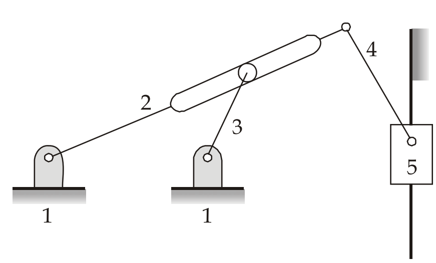


Number of links, n = 7

Number of joints, j = 9

$M = 3(n - 1) - 2j$

$= 3(7 - 1) - 2 \times 9 = 18 - 18 = 0$



Number of links, n = 5

Number of lower pairs, j = 5

Number of pairs with 2 DOF = 1 (higher pair)

$$M = 3(n-1) - 2j - h$$
$$= 3(5-1) - 2 \times 5 - 1$$
$$= 3 \times 4 - 10 - 1 = 1$$

Answer

Q.5 (d) Solution:

Diameter of pulley, D = 0.9 m, N = 200 rpm,

Power, P = 7.5 kW, T = 145 N in 10 mm width.

$$T_1 = 2T_2$$

Velocity of the pulley or belt,

$$V = \frac{\pi DN}{60} = \frac{3.14 \times 0.9 \times 200}{60} = 9.426 \text{ m/s}$$

Tight side tension = T_1

Slack side tension = T_2

Power transmitted, $P = (T_1 - T_2)v$

$$T_1 - T_2 = \frac{7.5 \times 10^3}{9.426} = 796 \text{ N}$$

$\therefore$ $T_1 = 2T_2$

$$2T_2 - T_2 = 796 \text{ N} \Rightarrow T_2 = 796 \text{ N}$$
$$T_1 = 2T_2 = 2 \times 796 = 1592 \text{ N}$$

Since velocity of belt < 10 m/s, centrifugal tension need not be considered.

Let, b = width of belt, $T_1 = \frac{145}{10} = 14.5 \text{N/mm width}$

$$b = \frac{T_1}{14.5} = \frac{1592}{14.5} = 109.8 \text{ mm}$$

The standard width of the belt, b = **112 mm** **Ans.**

Q.6 (a) Solution:

As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.

(i) Rectangular portion

$$a_1 = 100 \times 50 = 5000 \text{ mm}^2$$

$$x_1 = 25 + \frac{100}{2} = 75 \text{ mm}$$

and $$y_1 = \frac{50}{2} = 25 \text{ mm}$$

(ii) Semicircular portion

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982 \text{ mm}^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4 \text{ mm}$$

and $$y_2 = \frac{50}{2} = 25 \text{ mm}$$

(iii) Triangular portion

$$a_3 = \frac{50 \times 50}{2} = 1250 \text{ mm}^2$$

$$x_3 = 25 + 50 + 25 = 100 \text{ mm}$$

and $$y_3 = 50 + \frac{50}{3} = 66.7 \text{ mm}$$

We know that distance between centre of gravity of the section and left edge of the circular portion,

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250}$$

$$= 71.1 \text{ mm}$$

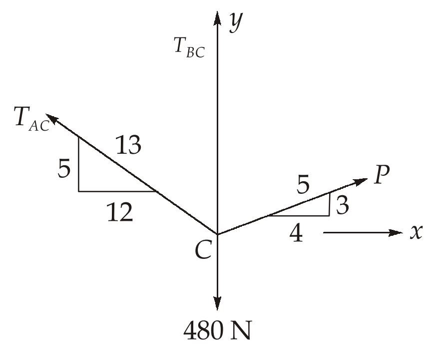
Similarly, distance between centre of gravity of the section and bottom face of the rectangular portion,

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} \text{ mm}$$

$$= 32.2 \text{ mm}$$

Q.6 (b) Solution:



FBD of point C

For horizontal force equilibrium, $\Sigma F_x = 0$

$$-\frac{12}{13}T_{AC} + \frac{4}{5}P = 0$$

$$T_{AC} = \frac{13}{15}P \quad \text{...(i)}$$

For vertical force equilibrium, $\Sigma F_y = 0$

$$\frac{5}{13}T_{AC} + T_{BC} + \frac{3}{5}P - 480 = 0$$

$$\left(\frac{5}{13}\right)\left(\frac{13}{15}\right)P + T_{BC} + \frac{3}{5}P - 480 = 0 \quad \text{[from equation (i)]}$$

$$T_{BC} + \frac{P}{3} + \frac{3P}{5} = 480$$

$$T_{BC} = 480 - \frac{14}{15}P \quad \text{....(ii)}$$

From equation (i), $T_{AC} > 0$ which requires $P > 0$

From equation (ii), $T_{BC} > 0$, requires $\frac{14P}{15} < 480$, $P < 514.29$ N

Hence allowable range for force : $0 < P < 514$ N Ans.

Q.6 (c) Solution:

$$\text{Tensile stress in rod 1} = \frac{P}{\left(\frac{\pi(0.3)^2}{4}\right)} \text{ kN/m}^2$$

$$\text{Compressive stress in rod 2} = \frac{Q-P}{\left(\frac{\pi(0.8)^2}{4}\right)} \text{ kN/m}^2$$

If both are equal then

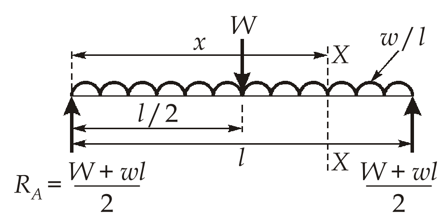
$$\frac{P}{\left(\frac{\pi(0.3)^2}{4}\right)} = \frac{5-P}{\left(\frac{\pi(0.8)^2}{4}\right)}$$

$$\Rightarrow \quad \frac{64P}{9} = 5 - P$$

$$\Rightarrow \quad P = \frac{5\times 9}{73} = 0.6164 \text{ kN}$$

Q.6 (d) Solution:

Let the fictitious load "W" act at $x = l/2$



$$M = \frac{W+wl}{2}\cdot x - \frac{wx^2}{2} \qquad \forall x \in (l/2, l)$$

$$M = \frac{Wx}{2} + \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial W} = \frac{x}{2} + 0 - 0$$

$$\frac{\partial U}{\partial W} = 2\times\int_0^{l/2}\frac{M}{EI}\cdot\frac{\partial M}{\partial W}\cdot dx = \frac{2}{EI}\int_0^{l/2}\left[\left(\frac{W+wl}{2}\right)x - \frac{wx^2}{2}\right]\cdot\frac{x}{2}dx$$

Putting $W = 0$

$$\frac{\partial U}{\partial W} = \frac{2}{EI}\cdot\int_0^{l/2}\left\{\frac{wlx}{2} - \frac{wx^2}{2}\right\}\frac{x^2}{2}dx$$

$$\frac{\partial U}{\partial W} = \frac{2}{EI}\cdot\left[\int_0^{l/2}\left\{\frac{wlx^2}{4} - \frac{wx^3}{4}\right\}dx\right]$$

$$= \frac{2}{EI} \cdot \left[\frac{wl}{4} \cdot \left(\frac{l}{2}\right)^3 \cdot \frac{1}{3} - \frac{w}{4} \cdot \left(\frac{l}{2}\right)^4 \cdot \frac{1}{4} \right]$$

$$= \frac{2}{EI}\left[\frac{w \cdot l^4}{96} - \frac{wl^4}{256}\right] = \frac{2 \times wl^4}{EI}\left[\frac{256 - 96}{256 \times 96}\right] = \frac{5wl^4}{384EI}$$

Deflection at the centre of the simply supported beam carrying a uniformly distributed load of intensity w over the full span is $\frac{5wl^4}{384EI}$.

❍❍❍❍