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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-14: Full Syllabus Test

Paper-I

Name : Kartikeya Singh

Roll No : E E 1 9 M B D L A 7 3 2

Test Centres					Student's Signature
Delhi ☐	Bhopal ☐	Noida ☐	Jaipur ☐	Indore ☐	
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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	58
Q.2	48
Q.3	
Q.4	45
Section-B	
Q.5	55
Q.6	
Q.7	42
Q.8	
Total Marks Obtained	248

Signature of Evaluator: Gupta

Cross Checked by: Saurabh Kumar

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Section-A

(a) A voltage of $V = 2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t$ is applied to a series circuit having $R = 10\ \Omega$, $C = 30\ \mu F$ and a variable inductance.

(i) Find the value of inductance so as to give resonance at 3rd harmonic frequency.

(ii) What are the rms values of voltage and current with this inductance in the circuit? (Take $\omega = 300$ rad/sec)

[12 marks]

at third harmonic frequency, resonance occurs

so

$$\omega_r = 3 \times 300 = 900 \text{ rad/sec}$$

So at resonance $\omega_r L = \frac{1}{\omega_r C}$

$$L = \frac{1}{\omega_r^2 C} = \frac{1}{(900)^2 \times 30 \times 10^{-6}}$$

$$L = 0.041152 \text{ H}$$

→ RMS input voltage

$$= \sqrt{\left(\frac{2000}{\sqrt{2}}\right)^2 + \left(\frac{400}{\sqrt{2}}\right)^2 + \left(\frac{100}{\sqrt{2}}\right)^2}$$

$$= 1443.95 \text{ Volts}$$

→ Current at fundmental frequeny

$$Z_1 = 10 + j\left(300 \times 0.041152 - \frac{1}{300 \times 30 \times 10^{-6}}\right)$$

$$Z_1 = 99.270 \angle 84.21\ \Omega$$

$$I_{1peak} = \frac{V_1}{Z_1} = \frac{2000}{99.270 \angle -84.21} = 20.14 \angle 84.21 \text{ Amp}$$

→ current at 3rd harmonic frequeny

$$Z_3 = 10 + j\left(900 \times 0.041152 - \frac{1}{900 \times 30 \times 10^{-6}}\right) = 10\Omega$$

$$I_{3(P)} = \frac{400}{10} = 40 \angle 0° \text{ Amp}$$

→ Current at 5th harmonic frequeny

$$Z_5 = 10 + j\left(1500 \times 0.041152 - \frac{1}{1500 \times 30 \times 10^{-6}}\right)$$

$Z_3 = 40.7517 \angle 75.79 \ \Omega$

$I_{3\,(peak)} = \frac{V_3}{Z_3} = \frac{100}{40.7517\angle 75.79} = 2.4538 \angle -75.79 A$

hence
RMS current

$I_{rms} = \sqrt{\frac{(20.14)^2}{2} + \frac{(40)^2}{2} + \frac{(2.4538)^2}{2}}$

$I_{rms} = \underline{31.714 \text{ amp}}$

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Q.1 (b) **(i)** If an NaC*l* crystal is subjected to an electric field of 1000 V/m and the resulting polarization is 4.3×10^{-8} C/m^2. Calculate the relative permittivity of NaC*l*.

[4 marks]

Electric field $E = 1000$ V/m

Polarization $P = 4.3 \times 10^{-8}$ C/m^2

$P = \varepsilon_0(\varepsilon_r - 1)E$

$4.3 \times 10^{-8} = 8.85 \times 10^{-12}(\varepsilon_r - 1) \times 1000$

$\varepsilon_r - 1 = 4.8587$

$\boxed{\varepsilon_r = 5.8587}$

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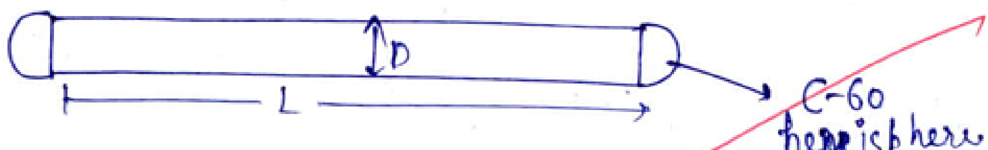
(b) **(ii)** What are carbon nanotubes? How they are different from fullerenes? Mention properties of carbon nanotubes and some of the possible areas related to its application?

[8 marks]

→ Carbon nanotubes are basically long rods of small diameter made up of carbon and both ends are capped with hemispheres of C-60 fulerene.



→ Carbon nanotubes have high length to diameter ratio that is $L >> D$.

→ Carbon nanotube is a single molecule made up of several atoms (million of atoms)

→ Carbon nanotubes have very high mechanical strength.

→ Carbon nanotube can be or can not be electrical conductors.

→ Fullerene (C-60) is a molecule of spherical shape ~~whe~~ means it is a 3D type of ~~nanomaterial~~ structure. where as nanotubes are 1-D type of structure.

→ Carbon nanotubes have various type of applications.

→ they can be used to make high strength and ~~sus~~ high load carrying ropes.

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Q.1 (c) Express the matrix, $A = \begin{bmatrix} 1+i & 2 & 5-.5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ as the sum of Hermitian matrix and Skew-Hermitian matrix.

[12 marks]

$A^\theta =$ Conjugate of $A = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix}$

$(A^\theta)^T = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$

Let $A = A_{\text{skew Hermi}} + A_{\text{Hermi}}$

$A_{\text{skew hermitian}} = \dfrac{A-(A^\theta)^T}{2} = \begin{bmatrix} i & 1+i & 3-2i \\ -1+i & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$

minus

$A_{\text{Hermitian}} = \dfrac{A+(A^\theta)^T}{2} = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 7 \end{bmatrix}$

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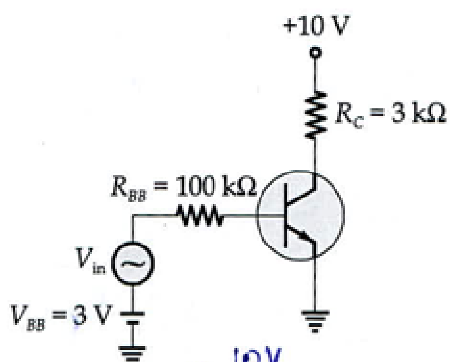
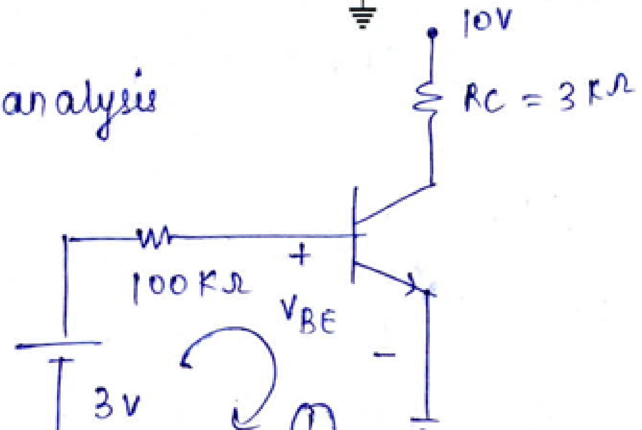
d) In the circuit shown below, determine the voltage gain of the transistor amplifier. (Take $\beta = 100$, $V_T = 25$ mV and draw the ac equivalent circuit).



[12 marks]

DC analysis

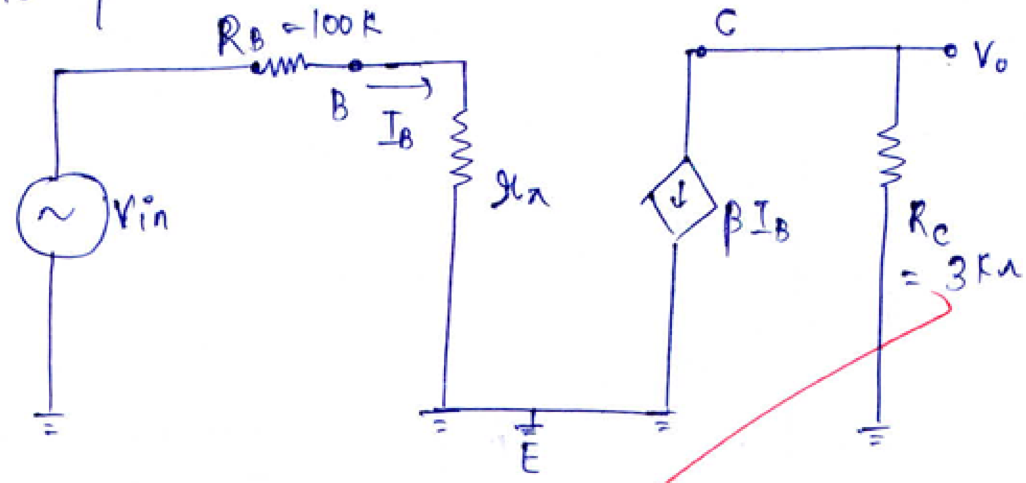


KVL in loop (1)

$$I_B = \frac{3 - 0.7}{100K} = 0.023 \text{ mA}$$

and $$r_\pi = \frac{V_T}{I_B} = \frac{25 \text{ mV}}{0.023 \text{ mA}} = 1086.95\,\Omega$$

AC equivalent



Now on output side

$$V_o = -\beta I_B \times R_C \quad \text{——} (I)$$

On input side

$$V_{in} = I_B \left[R_B + r_\pi \right] \quad \text{——} (II)$$

hence voltage gain

$$A_V = \frac{V_o}{V_{in}} = \frac{-\beta I_B R_C}{I_B (R_B + r_\pi)}$$

$$A_V = \frac{-\beta R_C}{R_B + r_\pi}$$

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$$A_V = -\frac{100 \times 3K}{100K + 1.086K}$$

$$A_V = -2.9677$$

1 (e) Explain how thermistor can be used for temperature measurement. The resistance at temperature, T Kelvin is given by

$$R_T = R_0 e^{\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)}$$

Where, $R_0 = 1050\ \Omega$ at 27° C

the corresponding, $\beta = 3140$

What is the temperature when thermistor resistance is 2330 Ω?

[12 marks]

→ Thermistor is a type of transducer whose resistance changes with temperature. Most of thermistors have -ve temperature coefficient of resistance. (although positive temperature coefficient thermistors are also there).

→ This property of change in resistance with temperature is used in thermistors to measure the temperature.

→ for a thermistor $R_T = R_0\, \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)$

if resistance at T_0 and at T can be measured precisely, then temperature T can be found.

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<u>Numerical</u>

$$R_T = R_0\, e^{\beta(1/T - 1/T_0)}$$

$\beta = 3140$

$R_0 = 1050\,\Omega$ at $T_0 = 273 + 27 = 300K.$

$R_T = 2330\,\Omega$ hence

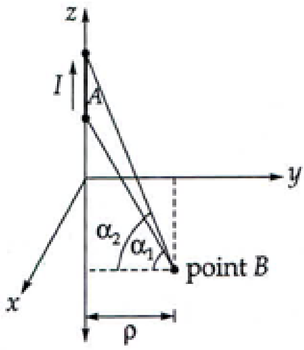
$$2330 = 1050\, e^{3140\left(\frac{1}{T} - \frac{1}{300}\right)}$$

$$3140\left(1/T - 1/300\right) = \ln\left(\frac{2330}{1050}\right)$$

$$1/T - 1/300 = \frac{0.797}{3140}$$

$$T = \underline{278.77\ K}$$

Q.2 (a) For the finite-length current element on the z-axis, as shown in below figure, use the Biot-savart law to derive $H_\phi = \frac{1}{4\pi\rho}\{\sin\alpha_2 - \sin\alpha_1\}\hat{a}_\phi$.

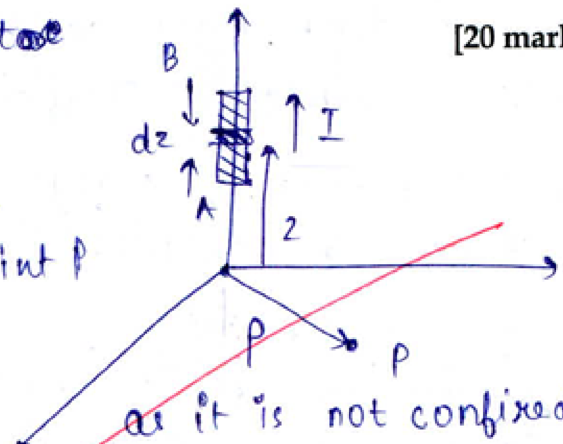


[20 marks]

biot savart law

$$\vec{dH} = \frac{I\vec{dl} \times \vec{a_R}}{4\pi R^2}$$

consider a small differential current element as $I\,\vec{dz}\,\vec{a_z}$ now -



distance unit vector $\vec{a_R}$ from current element to field point P is given as

$$\vec{a_R} = \frac{(z'-z)\vec{a_z} + \rho\vec{a_\rho}}{\sqrt{\rho^2 + (z'-z)^2}}$$

as it is not confirmed that point is in xy plane or not so I.m considering point P as (ρ, ϕ, z')

hence field intensity at P due to current element

$$\vec{dH} = \frac{I\,dz\,\vec{a_z} \times ((z'-z)\vec{a_z} + \rho\vec{a_\rho})}{4\pi[\rho^2 + (z'-z)^2]^{3/2}}$$

$$\vec{dH} = \frac{I\,\vec{dz} \times \rho\,\vec{a_z} \times \vec{a_\rho}}{4\pi[\rho^2 + (z'-z)^2]^{3/2}}$$

$$\vec{dH} = \frac{I\rho\,dz\,\vec{a_\phi}}{4\pi[\rho^2 + (z'-z)^2]^{3/2}}$$

$z_A \longrightarrow z_A$
$\alpha \longrightarrow \alpha_1$

say $z' - z = \rho\tan\alpha$

$z_B \longrightarrow z_B$

$-dz = \rho\sec^2\alpha\,d\alpha$ $\quad \alpha \longrightarrow \alpha_2$

$$\vec{H} = -\int_{\alpha \text{ at } A}^{\alpha \text{ at } B} \frac{I \cancel{\rho} \times \cancel{\rho} \sec^2\alpha \, d\alpha}{\rho \cancel{\rho^3} \sec^3\alpha \times 4\pi} \vec{a_\phi}$$

$$\vec{H} = +\frac{I}{4\pi\rho} \int_{\alpha_2}^{\alpha_1} \cos\alpha \, d\alpha . \, \vec{a_\phi}$$

$$\vec{H} = +\frac{I}{4\pi\rho} \left[\sin\alpha\right]_{\alpha_2}^{\alpha_1} \vec{a_\phi}$$

$$\boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_1 - \sin\alpha_2\right] \vec{a_\phi}}$$

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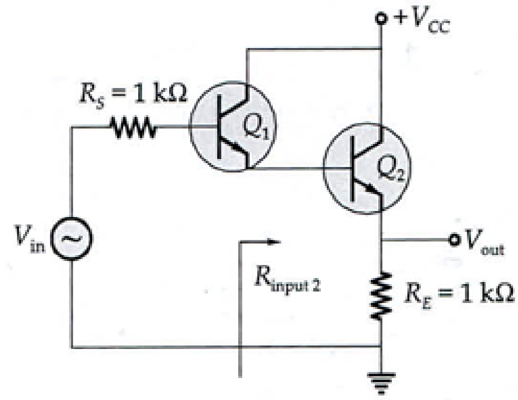
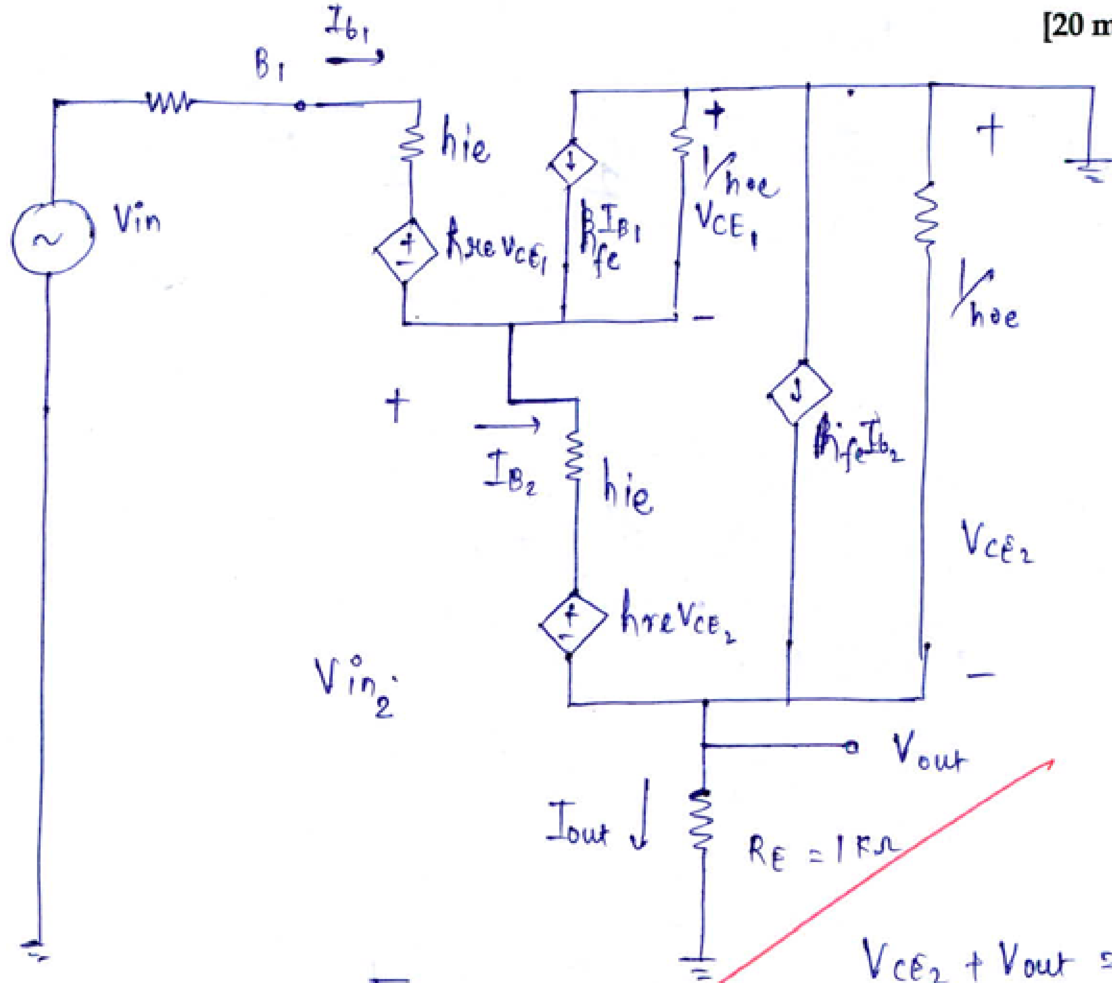
Q.2 (b) For the circuit shown below, calculate the values by deriving the expression of A_{I2}, $R_{\text{input 2}}$ and A_{V2} (where I and V stands for current and voltage respectively). Assume the following h-parameters for both the transistors:

$h_{ie} = 1.1\ \text{k}\Omega$, $h_{fe} = 50$, $h_{re} = 2.5 \times 10^{-4}$ and $h_{0e} = 25\ \mu\text{A/V}$



[20 marks]



$V_{CE_2} + V_{out} = 0$

$V_{out} = -V_{CE_2}$

Now ~~V_{in_2}~~ = ~~V_{out}~~

$$I_{out} = I_{B_2} + h_{fe} I_{b_2} + V_{CE_2} h_{oe}$$

$$I_{out} = I_{B_2}(1 + h_{fe}) + (-V_{out}) h_{oe}$$

$$I_{out} = I_{B_2}(1 + h_{fe}) + (-I_{out} R_E) \times h_{oe}$$

$$I_{out}(1 + h_{oe} R_E) = I_{B_2}(1 + h_{fe})$$

$$\frac{I_{out}}{I_{B_2}} = A_{I_2} = \frac{1 + h_{fe}}{1 + h_{oe} R_E} = \frac{1 + 50}{1 + 25 \times 10^{-6} \times 1 \times 10^{3}}$$

$$A_{I_2} = \underline{49.75}$$

→ Now $R_{in_2} = \frac{V_{in_2}}{I_{B_2}} =$

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$$V_{in_2} = I_{B_2} h_{ie} + h_{re} V_{CE_2} + V_{out}$$

$$V_{in} = I_{B_2} h_{ie} - h_{re} V_{out} + V_{out}.$$

$$V_{in} = I_{B_2} h_{ie} + (1 - h_{re}) \times I_{out} R_E$$

$$V_{in} = I_{B_2} h_{ie} + (1 - h_{re}) \times A_{I_2} I_{B_2} R_E$$

$$R_{in_2} = \frac{V_{in}}{I_{B_2}} = h_{ie} + (1 - h_{re}) A_{I_2} R_E$$

$$R_{in_2} = 1.1K + (1 - 2.5 \times 10^{-4}) \times 49.75 \times 1K$$

$$R_{in_2} = \underline{50.8375K}$$

→ and $A_{v_2} = \frac{V_{out}}{V_{in_2}}$

$$A_{v_2} = \frac{I_{out} R_E}{R_{in_2} I_{B_2}} = \frac{A_{I_2} \times R_E}{R_{in_2}}$$

$$A_{v_2} = \frac{49.75 \times 1K}{50.8375K}$$

$$= \underline{0.97860}$$

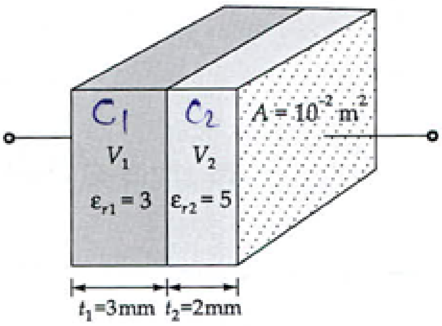
Q.2 (c) (i) A parallel plate capacitor has two dielectrics with relative permittivity ϵ_{r1} and ϵ_{r2} placed in between plates as shown in figure. The specification of capacitor are given below:

Plate surface area = $100\ \text{cm}^2$

Dielectric 1: $\varepsilon_{r1} = 3$; $t_1 = 3$ mm

Dielectric 2: $\varepsilon_{r2} = 5$; $t_2 = 2$ mm

If a potential difference of 100 V is applied across the plates, find the total energy stored in capacitor, capacitance of both dielectrics and also the potential gradient in each dielectric.



[10 marks]

① Capacitance $C_1 = \dfrac{\varepsilon_0 \varepsilon_{r1} A}{d}$

$$C_1 = \frac{8.85\times10^{-12} \times 10^{-2} \times 3}{3\times10^{-3}} = 88.5\ \text{PF}$$

Capacitance $C_2 = \dfrac{\varepsilon_0 \varepsilon_{r2} A}{d}$

$$C_2 = \frac{8.85\times10^{-12} \times 5\times10^{-2}}{2\times10^{-3}} = 221.25\ \text{pF}$$

Now voltage across C_1

$$V_1 = \frac{V \times C_2}{C_1 + C_2} = \frac{100 \times 221.25}{221.25 + 88.5} = 71.428\ \text{Volt}$$

$$V_2 = 100 - 71.428 = 28.5714\ \text{Volts}$$

potential gradient in Dielectric (1).

$$E_1 = \frac{V_1}{d_1} = \frac{71.428}{3 \times 10^{-3}} = 23.8 \text{ KV/m}$$

potential gradient in dielectric (2).

$$E_2 = \frac{V_2}{d_2} = \frac{28.5714}{2 \times 10^{-3}} = 14.2855 \text{ KV/m}$$

$\longrightarrow$ Energy stored

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$$\Rightarrow \frac{1}{2} C V^2 = \frac{1}{2} C_{eq} \times V^2$$

$V = 100 \text{ V}$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{88.5 \times 221.25}{88.5 + 221.25}$$

$$= 63.257 \text{ pF}$$

$$\text{energy stored} = \frac{1}{2}\, 63.257 \times 10^{-12} \times (100)^2$$

$$= 316285.71 \times 10^{-12}$$

$$= 316.285 \text{ nJ}$$

Q.2 (c) **(ii)** A specimen of alloy steel is used as lamination which are 0.4 mm thick and carry maximum flux density of 1 Wb/m^2 at a frequency of 60 Hz. If the resistivity of alloy steel is 16 $\mu\ \Omega$-cm with density of 7.8 gm/cm^3 and hysteresis loss occurring in every cycle is 240 W/m^3. Calculate eddy current loss per m^3, hysteresis loss in watt/kg and core loss per kg occurring in the specimen.

[10 marks]

$\pi \; B^2 f^2 t^2$

Hysteresis losses

$P_H = 240 \; W/m^3$

and density

$\rho = 7.8 \; gm/cm^3$

$\rho = \dfrac{7.8 \times 10^{-3} \; Kg}{10^{-6} \; m^3}$

$\rho = 7.8 \times 10^3 \; Kg/m^3$

hence hysteresis losses per Kg

$P_{H/Kg} = \dfrac{240}{7.8 \times 10^3} \; \dfrac{W/m^3}{Kg/m^3}$

$= 0.030769 \; W/Kg$

eddy current losses

$P_e = \dfrac{\pi^2 t^2 f^2 B^2}{\rho} \; W/m^3$

$P_e = \dfrac{\pi^2 \times (0.4 \times 10^{-3})^2 \times 60^2 \times 1^2}{16 \times 10^{-8}}$

$P_e = 35530.6 W/m^3$

eddy current loss per Kg

$P_{e/Kg} = \dfrac{35530.6}{7.8 \times 10^3}$

$P_{e/Kg} = 4.555 \; Watt/Kg$

hence core loss per Kg

$P_{c/Kg} = P_{H/Kg} + P_{e/Kg} = 4.5859 \; Watt/Kg$

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3 (a) A computer system has a level-1 instruction cache (I-cache), a level-1 data cache (D-cache) and a level-2 cache (L2-cache) with the following specifications:

	Capacity	Mapping method	Block size
I - cache	4 K words	Direct mapping	4 words
D - cache	4 K words	2 way set-associative mapping	4 words
L2 - cache	64 K words	4 way set associative mapping	16 words

The length of the physical address of a word in the main memory is 30-bits. Calculate the capacity of the tag memory in the I-cache, D-cache and L2 cache.

[20 marks]

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3 (b) An electromagnetic flowmeter is used to measure the average flow rate of an effluent in a pipe of 50 mm diameter. The velocity profile is symmetrical and can be assumed uniform. The flux density in the liquid has a peak value of 0.1 Wb/m^2, the output from the flow meter electrodes is taken to an amplifier of gain 1000 and impedance between the electrodes is 250 kΩ. The input impedance of the meter is 2.5 MΩ.

(i) Determine the effluent average velocity when the peak to peak voltage at the amplifier output is 0.2 V.

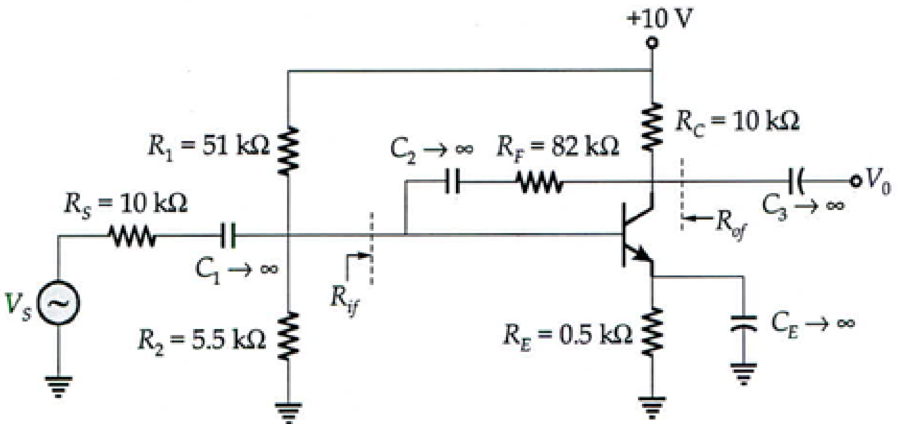
(ii) If the effluent conductivity decreases by 20 percent with the same flow rate, calculate the percentage change in reading of the amplifier output.

[20 marks]

Q.3 (c) By changing order of integration find $I = \int_0^1 \int_{y^2}^{2-y} (x+y)\,dx\,dy$.

[20 marks]

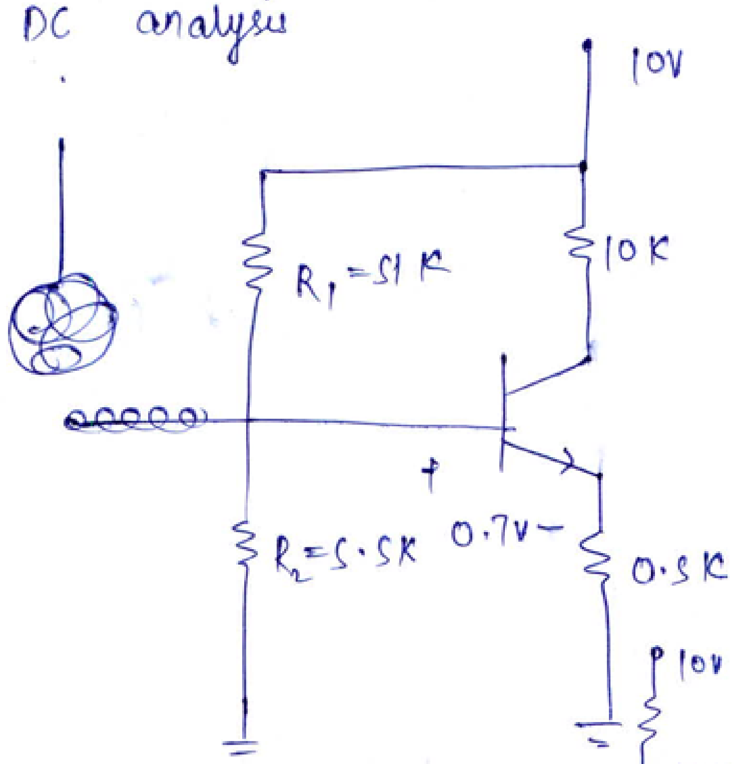
Q.4 (a) Consider the circuit shown in the figure below:



If $\beta = 100$, $V_{BE(on)} = 0.7$ V and $V_A = \infty$, then calculate the small signal input resistance R_{if} and small signal output resistance R_{of}. (Assume $V_T = 26$ mV)

[20 marks]

DC analysis

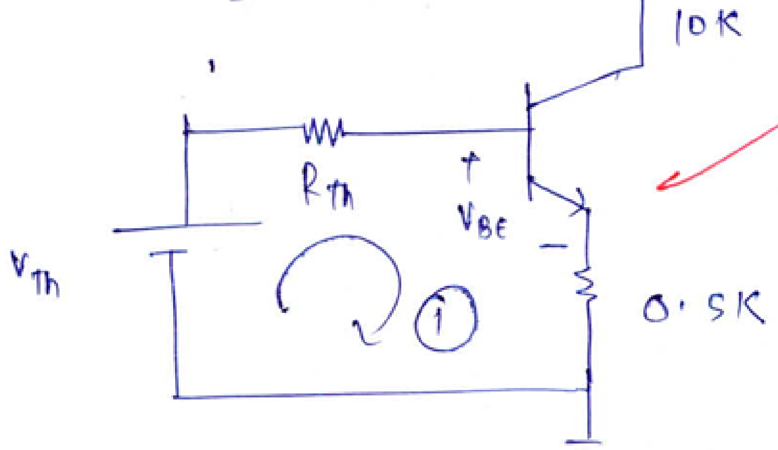


$$V_{Th} = \frac{10 \times 5.5}{5.5 + 51}$$

$V_{Th} = 0.97341$ Volts

$$R_{Th} = \frac{51 \times 5.5}{51 + 5.5}$$

$R_{Th} = 4.9646$ KΩ



applying KVL to loop ①

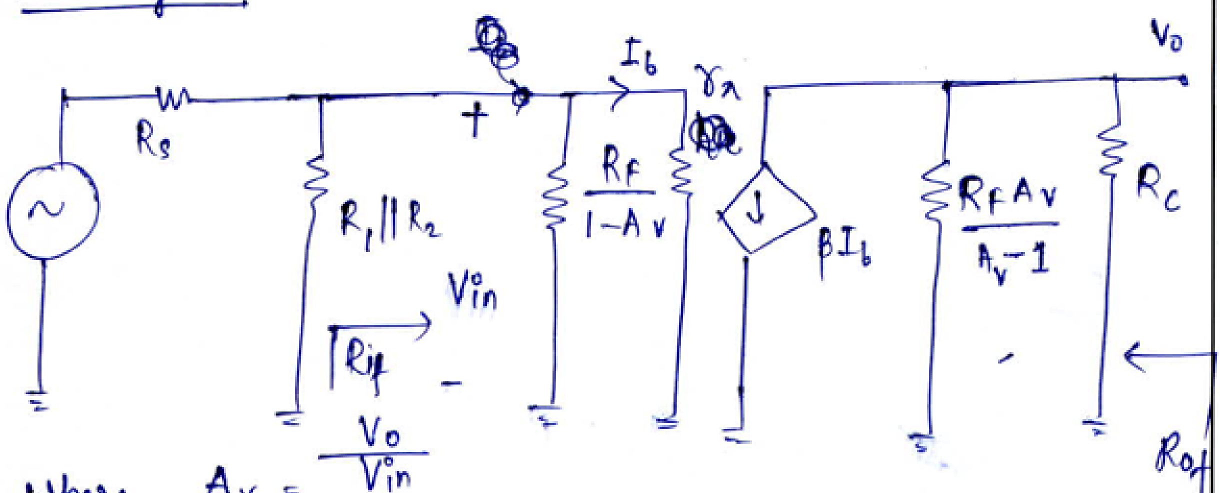
$V_{Th} = R_{Th} i_b + V_{BE} + R_E I_E$

$$i_b = \frac{V_{Th} - V_{BE}}{R_{Th} + (1+\beta) R_E} = \frac{0.97341 - 0.7}{4.9646K + 101 \times 0.5K}$$

$I_b = 4.9294 \times 10^{-3}$ mA

hence $r_\pi = \frac{V_T}{I_b} = \frac{26 \times m V}{4.929 \times 10^{-3} mA} = 5274.9\ \Omega$
$= 5.274\ k\Omega$

AC Analysis



where $A_V = \frac{V_o}{V_{in}}$

Hence considering A_V to be very high $(A_V >> 1)$

$\frac{R_F A_V}{A_V - 1} \approx R_F$

hence $V_o = -\beta I_b \times (R_F \| R_C)$

and $V_{in} = I_b r_\pi$

hence $A_V = \frac{V_o}{V_{in}} = \frac{-\beta (R_F \| R_C)}{r_\pi}$

$A_V = -\frac{100 \times [82 \| 10]}{5.274} = -168.999$

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so Input resistance

$R_{if} = r_\pi \| \frac{R_F}{1 - A_V} = 5.274 \| \frac{82}{1 + 168.999}$

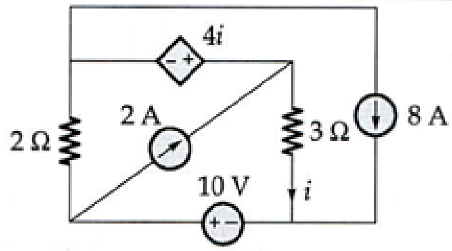
$R_{if} = 5.274 \| .4823$

$= 0.4419\ k\Omega = 441.9\ \Omega$

$R_{of} = R_C \| \frac{R_F A_V}{A_V - 1} \approx R_C \| R_F \quad (A_V >> 1)$

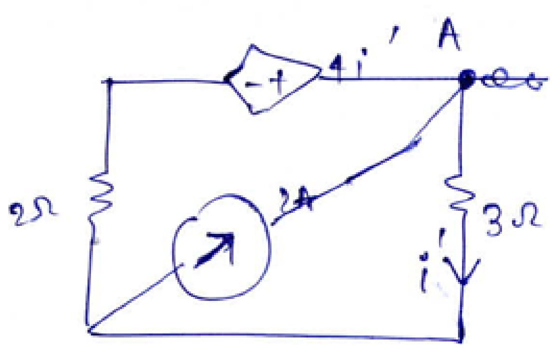
$R_{of} = 10K \| 82K \approx 8.913\ k\Omega$

4 (b) (i) Find the current i in the circuit shown in figure using the superposition theorem.



[10 marks]

Consider 2A source



apply Nodal analysis at A

$$2 = \frac{V_A - 4i'}{2} + \frac{V_A}{3}$$

$$2 = \frac{V_A}{2} + \frac{V_A}{3} - 2i'$$

and $i' = V_A/3$

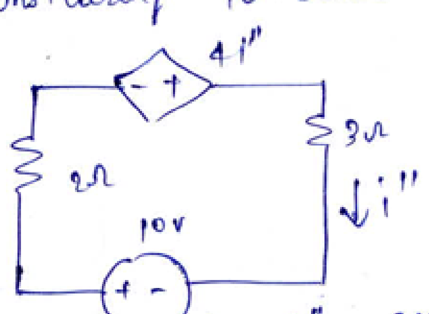
$$2 = \frac{V_A}{2} + \frac{V_A}{3} - \frac{2V_A}{3}$$

$2 = V_A/6 \Rightarrow V_A = 6$

and $i' = V_A/3 = 2A$

4

Considering 10V source

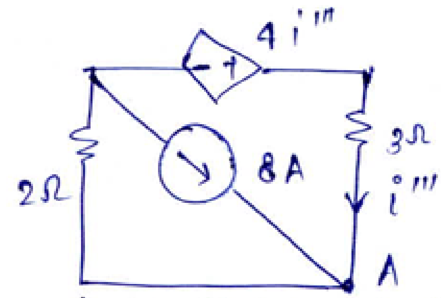


$10 = 2i'' - 4i'' + 3i''$

$10 = i''$

$i'' = 10A$

Considering 8 A source,



applying Nodal at A

$$8 = \frac{V_A}{2} + \frac{V_A - 4i'''}{2}$$

and $i''' = -\frac{(V_A - 4i''')}{3}$

$-3i''' = V_A - 4i'''$

$i''' = V_A$.

hence $i''' = V_A = -16A$.

hence total current

$i = i' + i'' + i'''$

$= 6 + 10 - 16$

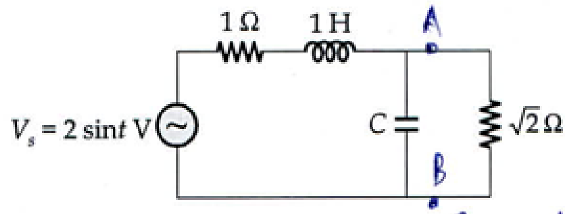
$= 0A$ //

Q.4 (b) **(ii)** Find the value of C that will cause maximum power in the $\sqrt{2}\,\Omega$ load.

$X_L = 1j$

$\omega = 1$,

$X_C = \frac{-j}{C}$



[10 marks]

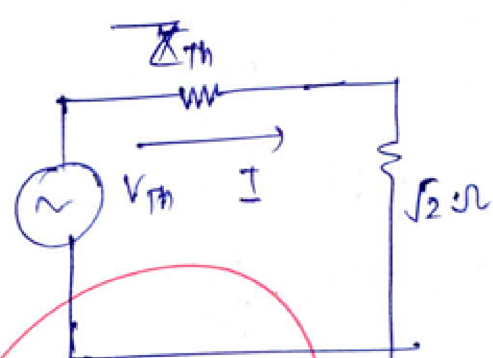
Load is fixed and source impedance is variable. ~~Now X_{Th} across √2Ω should be minimum~~ so across AB finding V_{Th} & R_{Th}.

$$V_{Th} = \frac{2X_C}{1 + 1j + X_C} = \frac{-2j/C}{1 + 1j - j/C} = \frac{-2j}{C + Cj - j}$$

$$V_{Th} = \frac{2}{1 - C + Cj}$$

and $$Z_{Th} = \frac{-j/C \times (1 + 1j)}{1 + 1j - j/C} = \frac{-j(1 + 1j)}{C + Cj - j}$$

$$Z_{Th} = \frac{(1 + 1j)}{jC - C + 1}$$



so for max power transfer to √2Ω, I must be maximum

$$I = \frac{V_{Th}}{Z_{Th} + R_L} = \frac{\dfrac{+2}{-C + Cj + 1}}{\dfrac{1 + 1j}{1 - C + jC} + \sqrt{2}}$$

$$I = \frac{+2}{1 + 1j + \sqrt{2}(1 - C + jC)}$$

$$\Rightarrow |I|^2 = \frac{4}{[1 + \sqrt{2}(1 - C)]^2 + [(1 + \sqrt{2}$$

10

I is max when denominator is min

$$\frac{d}{dC}\left[\left((1 + \sqrt{2}) - \sqrt{2}C\right)^2 + (1 + \sqrt{2}C)^2\right] = 0$$

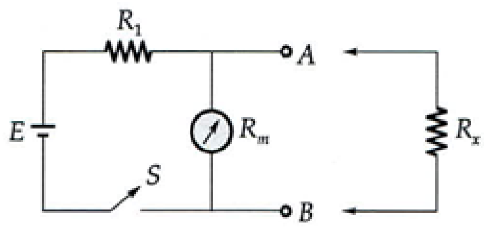
$$2(1 + \sqrt{2} - \sqrt{2}C) \times (-\sqrt{2}) + 2(1 + \sqrt{2}C)\sqrt{2} = 0$$

$$\cancel{1} + \sqrt{2}C = \cancel{1} + \sqrt{2} - \sqrt{2}C$$

$$C = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}\,F$$

$$\boxed{C = \frac{1}{2}F}$$

.4 (c) The shunt type ohmmeter is shown below. Determine the resistance R_x at which the meter shows half scale deflection.



[20 marks]

→ this is shunt type meter, gives full scale deflection when $R_x = \infty$

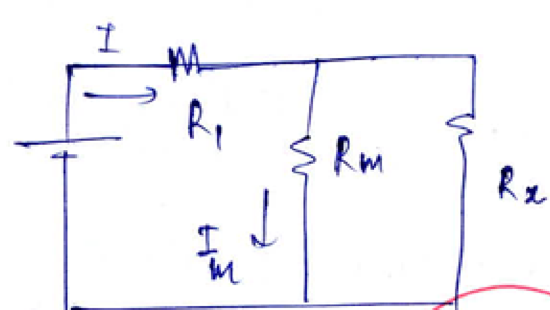
$$I_{fs} = \frac{E}{(R_1 + R_m)}$$

→ at half scale ~~at~~ current half of I_{fs}

$$I_m = \frac{I_{fs}}{2} = \frac{E}{(R_1 + R_m) \times 2}$$

→ So meter current is $I_m = \frac{E}{2(R_1 + R_m)}$ at half scale deflection

from the circuit



$$I = \frac{E}{R_1 + \frac{R_m R_x}{R_m + R_x}}$$

$$I_m = \frac{R_x}{\cancel{R_m + R_x}} \times \frac{E\cancel{(R_m + R_x)}}{R_m(R_1 + R_x) + R_1 R_x}$$

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$$\frac{\cancel{E}}{(R_1 + R_m)2} = \frac{\cancel{E} R_x}{(R_m + R_1) R_x + R_1 R_m}$$

$$(R_m + R_1) R_x + R_1 R_m = 2 R_x (R_1 + R_m)$$

$$\boxed{R_x = \frac{R_1 R_m}{R_1 + R_m}}$$ for half scale deflection.

Section-B

Q.5 (a) An air filled parallel plate capacitor is made of circular discs of area 2 m^2. The spacing between the discs is 0.1 m. If a voltage 20 cos $10^3 t$ volts is applied across the capacitor plates, find the displacement current density and the magnetic field between the capacitor plate. Also, show that the current flowing between the capacitor terminal is equal to the displacement current?

[12 marks]

Displacement current density

$$\vec{J} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\vec{J} = \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{V}{d}\right)$$

$$J = \frac{\varepsilon_0}{d}\frac{\partial V}{\partial t} = \frac{\varepsilon_0}{d} \times \frac{\partial}{\partial t}(20 \cos 10^3 t)$$

$$J = \frac{8.85 \times 10^{-12}}{0.1} \times 20 \times 10^3 (-\sin 10^3 t)$$

10

$$J = -1.776 \times 10^{-6} \sin 10^3 t$$

$$J = -1.776 \sin 10^3 t \ \mu A/m^2$$

$$I_D = J \times \text{Area} = -1.776 \times 2 \sin 10^3 t \ \mu A$$

$$= -3.54 \sin 10^3 t \ \mu A$$

Now capacitor current

$$I_C = C\frac{dV}{dt} \quad \text{where} \quad C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2}{0.1}$$

$$C = 1.77 \times 10^{-10}\ F$$

$$I_C = 1.77 \times 10^{-10} \times -20 \times 10^3 \sin 10^3 t$$

$$I_C = -3.54 \sin 10^3 t \ \mu A$$

hence for capacitor

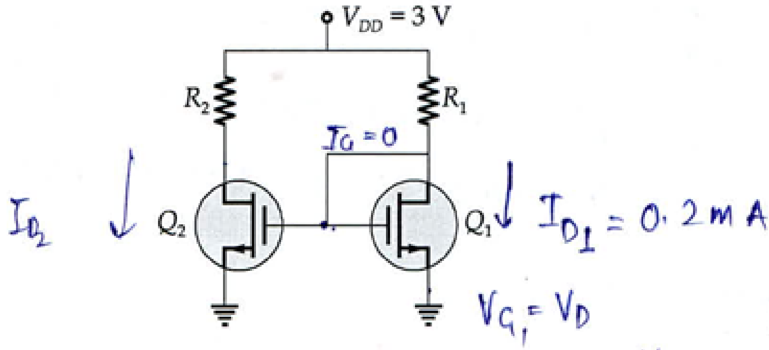
$$I_C = I_D = -3.54 \sin 10^3 t \ \mu A$$

Do not write in this margin

5 (b) Consider the circuit shown in figure. Let Q_1 and Q_2 have $V_t = 0.6$ V, $\mu_n C_{OX} = 200\ \mu A/V^2$, $L_1 = L_2 = 0.8\ \mu m$, $W_1 = 8\ \mu m$ and $\lambda = 0$.

(i) Find the value of R_1, required to establish a current of 0.2 mA in Q_1.

(ii) Find W_2 and R_2 so that Q_2 operates in the saturation region with a current of 0.5 mA and a drain voltage of 1 V.



$V_{G_1} = V_D$

$V_{S_1} = 0 \quad V_{GS_1} = V_D$

[12 marks]

(i) $I_{D_1} = 0.2 \times 10^{-3}$ A

$$I_D = \frac{\mu_n C_{ox} W_1}{2 L_1}\left[V_{GS} - V_T\right]^2$$

$$0.2 \times 10^{-3} = \frac{200 \times 10^{-6} \times 8 \times 10^{-6}}{2 \times 0.8 \times 10^{-6}}\left[V_D - 0.6\right]^2$$

$$0.2 = (V_D - 0.6)^2$$

$V_D = 1.04721$ volts

hence $R_1 = \frac{V_{DD} - V_D}{I_{D_1}}$

$$R_1 = \frac{3 - 1.04721}{0.2 \times 10^{-3}}$$

$R_1 = 9.763\ k\Omega$

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(ii) for Q_2 $V_D = 1V$

for $R_2 = \frac{V_{DD} - V_D}{I_{D_2}} = \frac{3 - 1}{0.5 \times 10^{-3}} = 4\ k\Omega$

and $I_{D_2} = \frac{200 \times 10^{-6} \times W_2}{2 \times 0.8 \times 10^{-6}}\left[V_{GS_2} - V_t\right]^2$

$$\frac{0.5 \times 10^{-3} \times 2 \times 0.8 \times 10^{-6}}{200 \times 10^{-6}} = W_2\left[V_{D_1} - 0 - V_t\right]^2$$

$$W_2 = \frac{4 \times 10^{-6}}{(1.04721 - 0.6)^2} = 20.019\ \mu m$$

Do not write in this ma

Q.5 (c) What is paging in operating system? What is the difference between simple paging and virtual memory paging?

[12 marks]

→ Paging :- Paging is the concept of bringing data in pages from secondary storage to main memory on demand by CPU while executing a program. Paging concept allows the CPU to execute of program which can't be accomodated in the main memory.

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→ In simple paging concept, Data is brought to memory in the page form but CPU does not generates any logical address. it searches main memory and if is found that data is not ~~mm~~ in main memory, CPU traps OS and OS searches secondary memory and brings data from ~~ma~~ there into main memory.

→ In virtual memory paging, the program length is much more than the size of main memory hence it can't be accomodated in the main memory so it is stored in the secondary memory and it is brought to main memory in the form of pages when required. here CPU generates a virtual address which is mapped into the ~~logi~~ phisical address with the help of the page table maintained by OS.

(d) Using Cauchy integral formula, evaluate $\int_c \frac{dz}{(z+1)^2(z-2)}$ where C the circle $|z| = \frac{3}{2}$.

[12 marks]

Poles of the function.

$z = -1$ and $z = 2$

here $z = -1$ is inside the contour so

Residue at $z = -1$ (double order)

$$= \lim_{z \to -1} \frac{1}{(2-1)!} \frac{d}{dz}\left[(z+1)^2 \times \frac{1}{(z+1)^2 (z-2)}\right]$$

$$= \lim_{z \to -1} \frac{1}{1!} \frac{d}{dz}\left[\frac{1}{(z-2)}\right]$$

$$= \lim_{z \to -1} -\frac{1}{(z-2)^2} = -\frac{1}{9}$$

hence $\int_c \frac{dz}{(z+1)^2 (z-2)} = 2\pi i \times \sum$ Residue of poles inside the contour

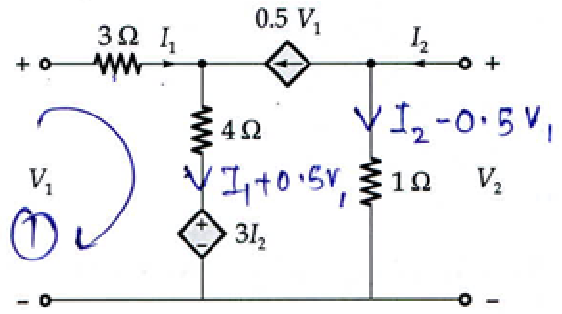
$$= 2\pi i \times \left(-\frac{1}{9}\right)$$

$$= \frac{-2\pi i}{9}$$

12

Q.5 (e) Find the h-parameters for the two-port network shown in figure.

V_1, I_2



[12 marks]

KVL in loop 1

$$V_1 = 3I_1 + 4(I_1 + 0.5V_1) + 3I_2$$

$$V_1 = 7I_1 + 2V_1 + 3I_2$$

$$-V_1 = 7I_1 + 3I_2 \quad \text{—— (1)}$$

KVL in loop 2

$$V_2 = 1 \times (I_2 - 0.5V_1) = I_2 - 0.5V_1$$

$$V_2 = I_2 - 0.5[-7I_1 - 3I_2] \quad \text{(from 1)}$$

$$V_2 = I_2 + 3.5I_1 + 1.5I_2$$

$$I_2 = \frac{1}{2.5}(V_2 - 3.5I_1)$$

$$I_2 = \frac{V_2}{2.5} - \frac{3.5}{2.5}I_1$$

$$I_2 = -1.4I_1 + 0.4V_2 \quad \text{—— (ii)}$$

Now subsituting eq (ii) in eq (1)

$$-V_1 = 7I_1 + 3(-1.4I_1 + 0.4V_2)$$

$$-V_1 = 2.8I_1 + 1.2V_2$$

$$V_1 = -2.8 I_1 - 1.2 V_2 \quad \text{——— (iii)}$$

Comparing with

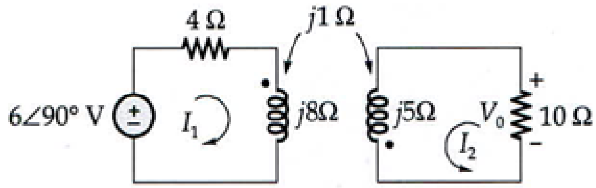
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$[h] = \begin{bmatrix} -2.8 & -1.2 \\ -1.4 & 0.4 \end{bmatrix}$$

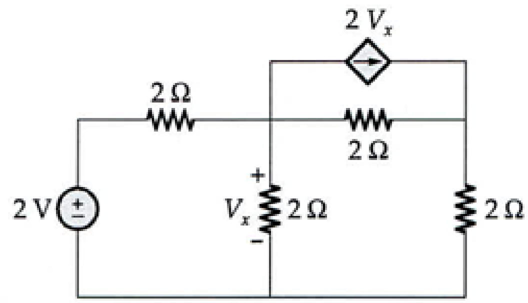
12

Q.6 (a) (i) Solve for I_1, I_2 and V_0 in figure using the T-equivalent circuit for the linear transformer.



[10 marks]

(a) (ii) For the network shown in figure, draw the oriented graph, select a suitable tree and obtain the fundamental cutset matrix. Determine the node equations and find voltage V_x.



[10 marks]

Q.6 (b) Why application of external electric field has different effect on polarization in case of solids, liquids and gases?

A homogeneous sphere of radius R is kept in vacuum with no applied field. If P is the polarization of sphere prove that field inside the sphere is given by $E_s = P/3\epsilon_0$ where ϵ_0 is permittivity in vacuum. Also derive expression for total internal field in a cubic system.

[20 marks]

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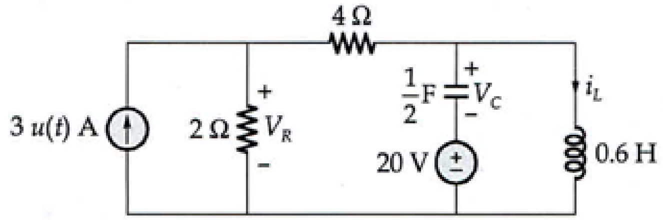
(c) Solve $(D^2 + 2D - 1)y = (x + e^x)^2 + \cos 2x \cosh x$.

[20 marks]

(a) In the circuit of figure,



Calculate:

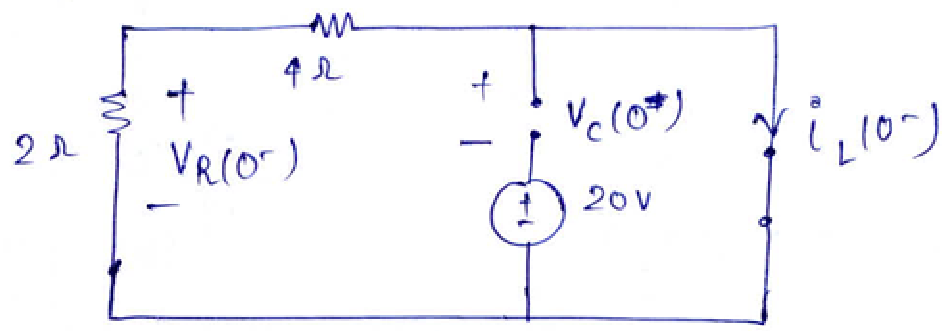
(i) $i_L(0^+), V_C(0^+), V_R(0^+)$

(ii) $\frac{di_L(0^+)}{dt}, \frac{dV_C(0^+)}{dt}, \frac{dV_R(0^+)}{dt}$

(iii) $i_L(\infty)$, $V_C(\infty)$, $V_R(\infty)$

[20 marks]

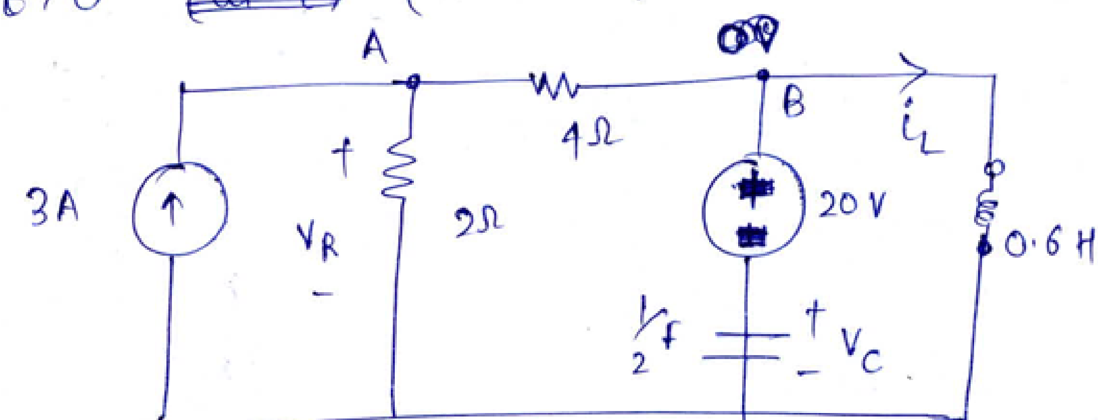
for $t < 0$ (at $t = 0^-$)



here $V_C(0^-) = -20$ V

$i_L(0^-) = 0$ Amp.

at $t > 0$ ~~at t>~~ (at $t = 0^+$)



$$\boxed{\begin{aligned} V_C(0^-) &= V_C(0^+) = -20\text{ V} \\ i_L(0^-) &= i_L(0^+) = 0\text{ A} \end{aligned}}$$

and, $V_B(0^+) = 20 + V_C(0^+) = 0$ V

hence applying nodal at A

$$3 = \frac{V_R}{2} + \frac{V_R - V_B}{4} = \frac{V_R}{2} + \frac{V_R}{4}$$

$$\boxed{V_R(0^+) = 4\ \text{Volts.}}$$

→ Now at $t = 0^+$

$$V_B = 0\,V = \frac{L\,di_L(0^+)}{dt}$$

$$\boxed{\frac{di_L}{dt}(0^+) = 0\ A/sec}$$

→ at $t = 0^+$

$$\frac{V_R}{4} = i_L + i_C \quad \text{—— (I)}$$

$$\frac{V_R}{4} = 0 + i_C$$

$$i_C = \frac{4}{4} = 1\ Amp.$$

$$\frac{C\,dV_C(0^+)}{dt} = 1\ Amp$$

$$\boxed{\frac{dV_C}{dt}(0^+) = \frac{1}{C} = 2\ V/sec}$$

~~Now from (I)~~

$$\cancel{\frac{dV_R}{dt} = 4\left(\frac{di_L}{dt} + \frac{di_C}{dt}\right)}$$

$$\cancel{\frac{dV_R}{dt} = 4\left(0 + \frac{di_C}{dt}\right)}$$

→ $V_R = 2 \times (3 - i_C)$

$$\frac{dV_R}{dt} = -2\frac{di_C}{dt} \quad \text{—— (II)}$$

$$\cancel{= -2 \times C\frac{d^2V_C}{dt^2}}$$

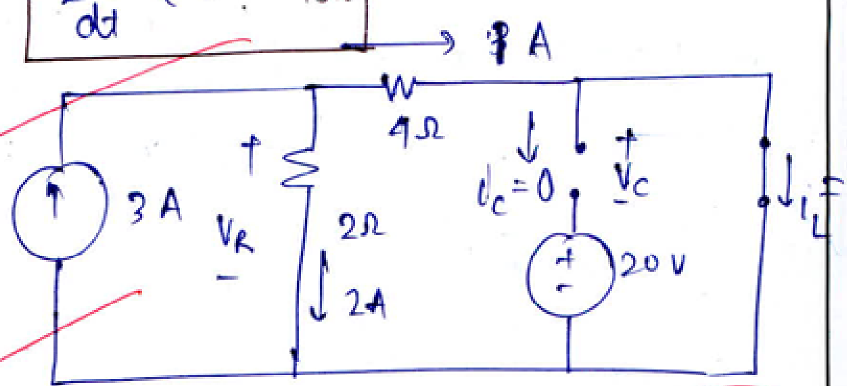
and also $\frac{dV_R}{dt} = 4\left(\frac{di}{dt} + \frac{di_C}{dt}\right)$

$$\frac{dV_R}{dt} = 4\frac{di_C}{dt} \quad \text{—— (III)}$$

from eq (II) and (III) $\boxed{\frac{dV_R}{dt}(0^+) = 0\ V/sec}$

→ at $t = \infty$

$V_C(\infty) = -20\,V$



$$i_L(\infty) = \frac{3 \times 2}{4 + 2}$$

$i_L(\infty) = 1\,A$

and $V_R(\infty) = 2 \times (3 - 1)$

$$= 2 \times 2$$

$V_R(\infty) = 4\,V$

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b) **(i)** In Newton Raphson's method if E_n is error in solution in n^{th} iteration i.e. $x_n = \alpha + E_n$ then show that,

$$E_{n+1} = \frac{E_n^2}{2} \cdot \frac{f''(\alpha)}{f'(\alpha)}$$

Where α is root of $f(x) = 0$.

(ii) Using Netwon Raphson's method, find real root of $3x = \cos x + 1$. Answer upto four decimal place. Given $0 \le x \le 1$.

[12 + 8 marks]

$\cancel{x_n = x_{n-1}} - $

$E_n = x_n - \alpha$

$E_{n+1} = x_{n+1} - \alpha$

$$x(n+1) = x(n) - \frac{f(x_n)}{f'(x_n)}$$

$\cancel{x_{n+1} - \alpha = x(n) - \alpha} - f$

$E_{n+1} = x_{n+1} - \alpha$

$$E_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \alpha$$

$$E_{n+1} = E_n - \frac{f(x_n)}{f'(x_n)}$$

(ii) $f(x) = 3x - \cos x - 1$

$f'(x) = 3 + \sin x$

Let initial guess is $x_0 = 1$

Ist iteration $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = 1 - \dfrac{f(1)}{f'(1)} = .0.62$

second iteration

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x)} = 0.62 - \dfrac{f(.62)}{f'(.62)}$

$x_2 = 0.607120$

third iteration

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 0.60712 - \dfrac{f(.60712)}{f'(.60712)}$

$x_3 = 0.60710$

hence answer upto four decimal place is

$x = \underline{0.6071}$

9

c) A resistor has a nominal value of $10\Omega \pm 1\%$. A voltage is applied across the resistor and the power consumed in the resistor is calculated in two ways:

(i) from $P = \frac{E^2}{R}$ and **(ii)** from $P = EI$

Calculate the uncertainty in the power determination in each case when the measured values of E and I are:

$$E = 100\text{ V} \pm 1\% \text{ and } I = 10\text{ A} \pm 1\%$$

[20 marks]

Now $R = 10\Omega \pm \frac{1\times 10}{100}$ $\quad E = 100 \pm \frac{1\times 100}{100}$

$R = 10 \pm 0.1\,\Omega$ $\quad E = 100 \pm 1\,V$

$I = 10 + \frac{1\times 10}{100} = 10 \pm 0.1$ Amp

(i) $P = \frac{E^2}{R}$ $\quad U_p = \sqrt{\left(\frac{\partial P}{\partial E}\right)^2 U_E^2 + \left(\frac{\partial P}{\partial R}\right)^2 U_R^2}$

$\frac{\partial P}{\partial E} = \frac{2E}{R} = \frac{2\times 100}{10} = 20$

$\frac{\partial P}{\partial R} = -\frac{E^2}{R^2} = \frac{-100\times 100}{10\times 10} = -100$

$U_p = \sqrt{(20)^2 \times 1^2 + (-100)^2 \times 0.1^2}$

$U_p = 22.36$ $\underline{\text{Watt}}$

(ii) $P = EI$

$\frac{\partial P}{\partial E} = I = 10$ $\quad \frac{\partial P}{\partial I} = E = 100$

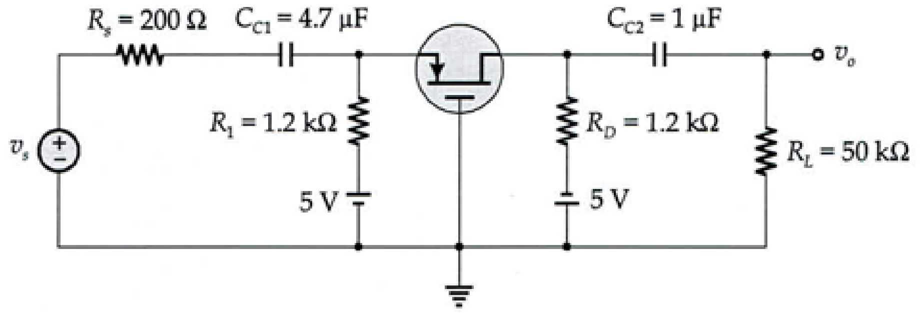
$U_p = \sqrt{\left(\frac{\partial P}{\partial E}\right)^2 \cdot U_E^2 + \left(\frac{\partial P}{\partial I}\right)^2 U_I^2}$

$$U_P = \sqrt{(10)^2 \times 1^2 + (100)^2 \times (0.1)^2}$$

$$U_P = 14.142 \text{ Watt}$$

14

a) The PMOS transistor of the circuit shown in the figure below has $V_{tp} = -1.5$ V, $K_p = \frac{\mu_p C_{ox}}{2}\left(\frac{W}{L}\right) = 1\,\text{mA/V}^2$ and $\lambda = 0$.



(i) Determine the quiescent and small-signal parameters of the transistor.

(ii) Find the time constants associated with C_{C1} and C_{C2}.

(iii) Is there a dominant pole frequency? Estimate the lower cut-off frequency of the amplifier.

[20 marks]

b) An analytic function $f(z)$ is defined as $f(z) = u + iv$.

Given that, $u + v = \dfrac{(\sin x + \cos y)\cdot(\sin 2x + \cos 2y)}{\sin x + \cos 2y}$. Find $f'(0)$.

[20 marks]

:) Write a 'C' program to arrange the given numbers in ascending order using Merge sorting.

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

$f(x_n$

$\frac{f''(\alpha)}{2}$

$E_{n+1} =$

$f(\alpha) = 0$