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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-12: Full Syllabus Test

Paper-I

Name : Kartikeya Singh

Roll No : E E 1 9 M B D L A 7 3 2

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Student's Signature

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	35
Q.2	44
Q.3	54
Q.4	
Section-B	
Q.5	54
Q.6	
Q.7	
Q.8	56
Total Marks Obtained	243

Signature of Evaluator: Sourabh kumar

Cross Checked by: Vipin

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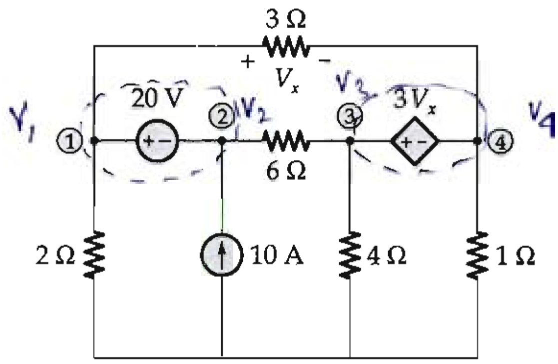
Section-A

a) Write an algorithm of bubble sort technique to arrange the number in ascending order.

[12 marks]

Q.1 (b) Find the node voltages in the circuit shown below:



[12 marks]

Let Node voltages are V_1, V_2, V_3, V_4

applying KCL at supernode (1, 2)

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} - 10 + \frac{V_2 - V_3}{6} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{3}\right) + \frac{V_2}{6} - \frac{V_3}{6} - \frac{V_4}{3} = 10 \quad \text{—— (i)}$$

and $V_1 - V_2 = 20.$ —— (ii)

applying KCL at supernode (3, 4)

$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0$$

$$-\frac{V_1}{3} - \frac{V_2}{6} + V_3\left(\frac{1}{6} + \frac{1}{4}\right) + V_4\left(1 + \frac{1}{3}\right) = 0 \quad \text{—— (iii)}$$

and $V_3 - V_4 = 3V_x$ where $V_x = V_1 - V_4$

$$V_3 - V_4 = 3(V_1 - V_4)$$

$$3V_1 - V_3 - 2V_4 = 0 \quad \text{—— (iv)}$$

from equation (i) and (ii) substituting V_2 value

$$V_1\left(\frac{1}{2} + \frac{1}{3}\right) + \frac{V_1 - 20}{6} - \frac{V_3}{6} - \frac{V_4}{3} = 10$$

$$V_1\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{6}\right) - \frac{V_3}{6} - \frac{V_4}{3} = 10 + \frac{20}{6}$$

$$V_1\left(\frac{6}{6}\right) - \frac{V_3}{6} - \frac{V_4}{3} = \frac{40}{3} \quad \text{—— (v)}$$

from equation (ii) and (iii)

$$-\frac{V_1}{3} - \frac{(V_1-20)}{6} + V_3\left(\frac{1}{6}+\frac{1}{4}\right) + V_4\left(1+\frac{1}{3}\right) = 0$$

$$-V_1/2 + V_3(5/12) + V_4(4/3) = -20/6 \quad \text{—— (vi)}$$

Solving (iv), (v) and (vi)

$V_1 = 80/3$ Volts $= 26.67$ Volts and $V_2 = V_1 - 20 = 20/3$ Volts $= 6.67$ Volts

$V_3 = \frac{520}{3}$ Volts $= 173.3$ Volts

$V_4 = -140/3$ Volts $= -46.67$ Volts

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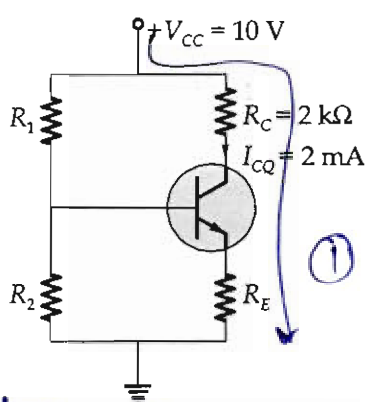
Good

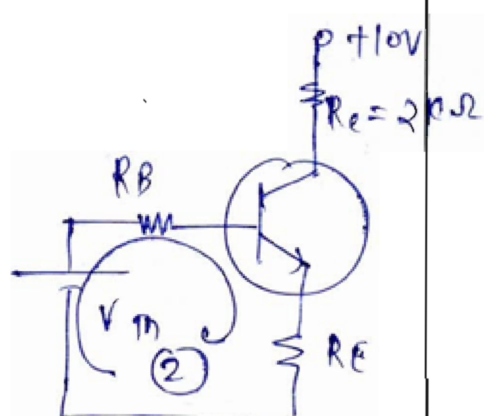
(c) A Silicon npn transistor operated with self bias gives $V_{CEQ} = 5$ V, $I_{CQ} = 2$ mA for $V_{CC} = 10$ V and $R_C = 2$ kΩ. If β for the transistor is 50, $V_{BE} = 0.7$ V and stability factor S is 5, then determine the values of biasing resistors R_1, R_2 and R_E.

$$V_{Th} = \frac{10 \times R_2}{R_1 + R_2}$$

and

$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$





[12 marks]

applying KVL in loop (1)

$$10 = I_C R_C + V_{CEQ} + R_E I_E$$

$$10 - 5 = 2 \times 2 + \frac{2 \times 51 \times R_E}{50}$$

$R_E = 0.490$ kΩ or 490Ω

Now stability factor $S = 5$

$$\frac{1+\beta}{1+\beta\frac{R_E}{R_E + R_B}} = 5$$

$$\frac{51}{1+\frac{50 R_E}{R_E+R_B}} = 5$$

$$\frac{R_E}{R_E+R_B} = 0.184$$

$$R_B \times 0.184 = R_E(1-.184)$$

$$R_B = \frac{R_E \times 0.816}{.184}$$

$$R_B = \underline{2.173\ K\Omega}$$

Now applying KVL in loop ②

$$V_{th} = I_B R_B + V_{BE} + I_E R_E$$

$$= \frac{2 \times 2.173}{50} + 0.7 + \frac{2\times51}{50} \times .490$$

$$= 1.78652 \text{ Volts}$$

Now $\frac{V_{CC} \times R_2}{R_1+R_2} = V_{th}$

$$\frac{V_{CC} \times R_B}{R_1} = V_{th}$$

$$R_1 = \frac{V_{CC} R_B}{V_{th}} = \frac{10 \times 2.173}{1.78652}$$

$$\boxed{R_1 = 12.1633\ K\Omega}$$

Now

$$R_B = \frac{R_1 R_2}{R_1+R_2}$$

$$2.173 \times (R_1+R_2) = R_1 R_2$$

$$\frac{2.173 \times R_1}{R_1 - 2.173} = R_2$$

$$R_2 = \frac{2.173 \times 12.1633}{12.1633 - 2.173}$$

$$\boxed{R_2 = 2.6456\ K\Omega}$$

Good

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d) State the significance of correlation coefficient and hence find correlation coefficient 'r' for following data:

Observation	1	2	3	4	5	6	7	8	9	10
x	12	13	10	9	20	7	4	22	15	23
y	50	54	48	47	70	20	15	40	35	37

[12 marks]

Correlation coefficient is

$$r = \frac{Covar(xy)}{\sqrt{Var x \; Var y}}.$$

x	y	xy	$(x-\bar{x})$ $d(x)$	$(y-\bar{y})$ $d(y)$	$(xy-\overline{xy})$ $d(xy)$
12	50	600	−1.5	8.4	−6.1
13	54	702	−0.5	12.4	95.9
10	48	480	−3.5	6.4	−126.1
9	47	423	−4.5	5.4	−183.1
20	70	1400	+6.5	28.4	793.9
7	20	140	−6.5	−21.6	−466.1
4	15	60	−9.5	−26.6	−546.1
22	40	880	8.5	−1.6	273.9
15	35	525	1.5	−6.6	−81.1
23	37	851	9.5	−4.6	244.9
$\Sigma x = 135$	$\Sigma y = 416$	$\Sigma xy = 6061$			

$\bar{x} = \frac{135}{10} = 13.5$

$\bar{y} = \frac{416}{10} = 41.6$

$\overline{xy} = \frac{6061}{10} = 606.1$

Variance $x = \frac{\Sigma [d(x)]^2}{n}$

$= 37.45$

Variance $y = \frac{\Sigma d[y]^2}{n}$

$= 234.24$

Covariance of $xy = \frac{\sum d(xy)^2}{n}$

$= ~~13~~ \quad 134598.69$

hence $\quad r = \dfrac{134598.69}{\sqrt{234.24 \times 37.45}}$

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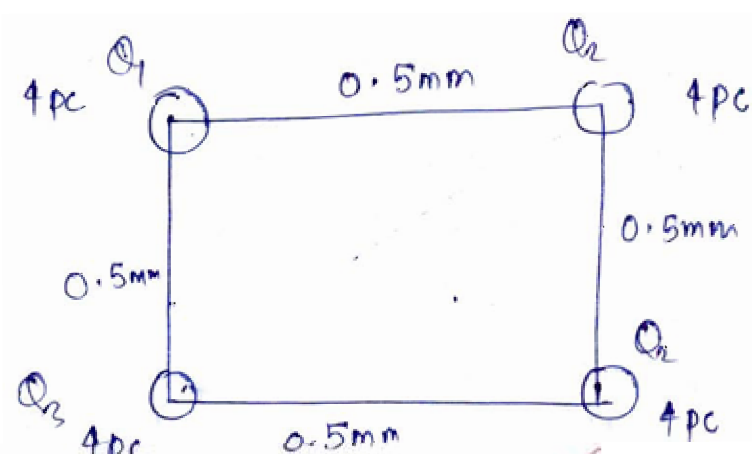
$r = \underline{1437.09}$

Q.1 (e) Four identical point charges of 4 pC each are located at the corners of a square, 0.5 mm on a side, in free space. How much work must be done to move one charge to a point equidistant from the other three and in the same plane?

[12 marks]



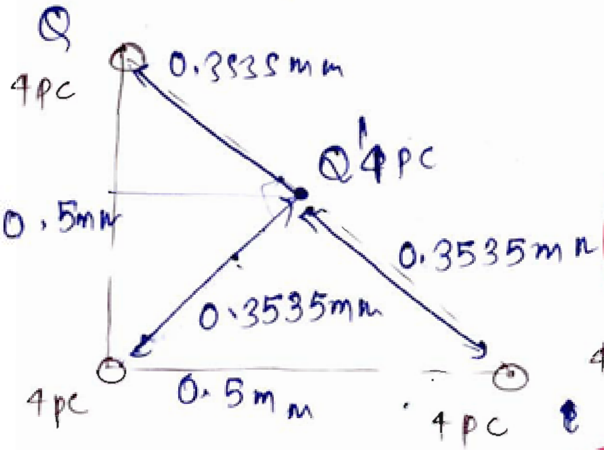
energy of system

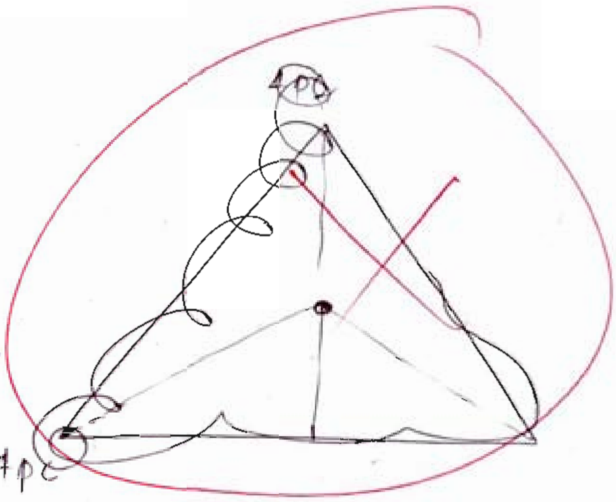
$$\Rightarrow 4 \times \frac{1}{2} Q_E \left[\frac{Q}{4\pi\varepsilon_0 \times 0.5\times10^{-3}} + \frac{Q}{4\pi\varepsilon_0 \times 0.5\times10^{-3}} + \frac{Q}{4\pi\varepsilon_0 \frac{0.5}{\sqrt{2}}\times10^{-3}} \right]$$

$$\Rightarrow 4 \times \frac{1}{2} \frac{Q^2}{4\pi\varepsilon_0} \left[\frac{1000}{0.5} + \frac{1000}{0.5} + \frac{1000}{0.5\sqrt{2}} \right]$$

$$= 2 \times 16\times10^{-24} \times 9\times10^{9}\left[4000 + \frac{4000}{\sqrt{2}}\right]$$

$$= 1.966 \text{ nJ}$$





Energy of system

$$= \frac{1}{2} Q\left[\frac{Q \times 3}{4\pi\varepsilon_0 \times .3535\times10^{-3}}\right]$$

$$+ \frac{1}{2} Q\left[\frac{Q}{4\pi\varepsilon_0 \times .3535\times10^{-3}} + \frac{Q\times10^{3}}{4\pi\varepsilon_0 \times .5\sqrt{2}} + \frac{Q}{4\pi\varepsilon_0 \times .5\times10^{-3}}\right]\times 2$$

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$$+ \frac{1}{2} Q\left[\frac{Q\times10^{3}}{4\pi\varepsilon_0 \times .5}\times 2 + \frac{Q\times10^{3}}{4\pi\varepsilon_0 \times .3535}\right]$$

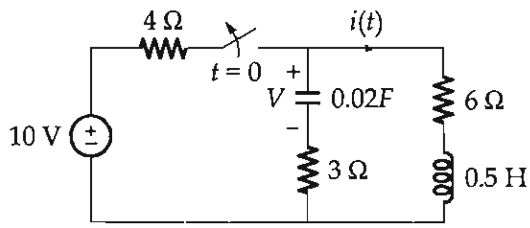
$$= \frac{1}{2} Q^{2}\times\frac{1}{4\pi\varepsilon_0}\left[\frac{3000}{.3535} + \frac{2000}{.3535} + \frac{2000}{.5\sqrt{2}} + \frac{2000}{.5} + \frac{2000}{.5} + \frac{1000}{.3535}\right]$$

$$= \frac{1}{2}\times16\times9\times10^{-24}\times10^{9}\left[27801.55\right]$$

$$= 2.0017\times10^{-9} = 2.0017 \text{ nJ}$$

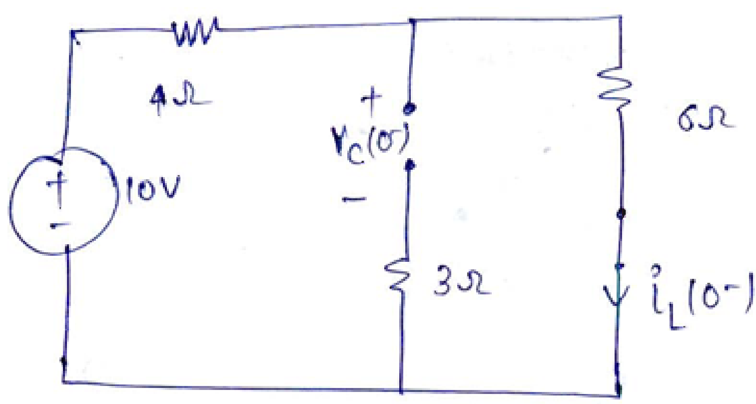
hence Work done $= 2.0017 - 1.966 = 0.0357 \text{ nJ}$

Q.2 (a) Find $i(t)$ in the circuit of figure. Assume that the circuit has reached steady state at $t = 0^-$.



[20 marks]

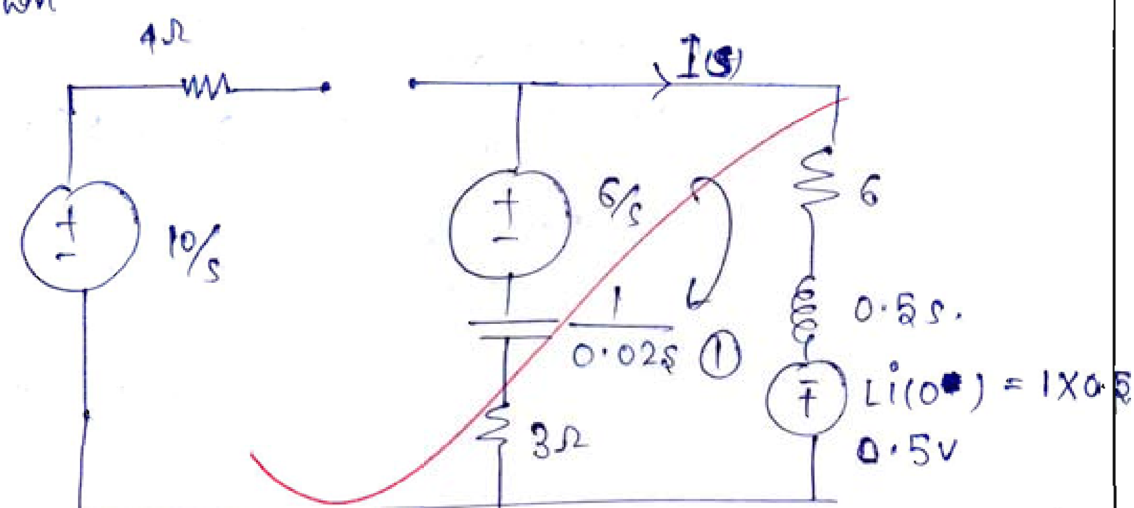
before $t = 0$ (at $t = 0^-$)



$$i_L(0^-) = \frac{10}{4+6} = 1\,A$$

and $V_c(0^-) = 1 \times 6 = 6$ Volts.

for $t > 0$, transformed circuit is as shown



applying KVL in loop ①

$$-\frac{6}{s} + 6I(s) + 0{\cdot}5s\,I(s) - 0{\cdot}5 + 3I(s) + \frac{1}{0{\cdot}02s} I(s) = 0$$

$$\frac{6}{s} + 0.5 = \left(6 + \frac{s}{2} + 3 + \frac{50}{s}\right) I(s)$$

$$\frac{6 + 0.5s}{\cancel{s}} = \left(\frac{18s + s^2 + 100}{2\cancel{s}}\right) I(s)$$

$$I(s) = \frac{12 + s}{s^2 + 18s + 100}$$

$$I(s) = \frac{s + 12}{(s+9)^2 + (\sqrt{19})^2}$$

$$I(s) = \frac{s+9}{(s+9)^2 + (4.358)^2} + \frac{3}{(s+9)^2 + (4.358)^2}$$

$$I(s) = \frac{s+9}{(s+9)^2 + (4.358)^2} + \frac{3 \times}{4.358} \frac{4.358}{(s+9)^2 + (4.358)^2}$$

applying ILT

$$i(t) = e^{-9t}\left(\cos 4.358t + 0.68838 \sin 4.358t\right)$$

ampere for $t > 0$

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Q.2 (b) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix},$$

Hence compute A^{-1}.

[20 marks]

Caley hamilton theorem states that every square matrix satisfies it characteristic equation.

Characteristic equation

$\Rightarrow |(\lambda I - A)| = 0$

$$\left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda-2 & 1 & -1 \\ 1 & \lambda-2 & 1 \\ -1 & 1 & \lambda-2 \end{vmatrix} = 0$$

$(\lambda-2)[(\lambda-2)^2 - 1] - 1[\lambda - 2 + 1] - 1[1 + \lambda - 2] = 0$

$(\lambda-2)[(\lambda-2)^2 - 1] - 2[\lambda - 1] = 0$

$(\lambda-2)[\lambda^2 + 3 - 4\lambda] - 2(\lambda-1) = 0$

$\lambda^3 + 3\lambda - 4\lambda^2 - 2\lambda^2 - 6 + 8\lambda - 2\lambda + 2 = 0$

$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

Now matrix A satisfies this eq^n then

$$A^3 - 6A^2 + 9A - 4I = 0 \quad \text{——} (1)$$

$$A^3 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

Now from eq (1)

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Now from equation (1) multiplying with A^{-1}

$$0 = A^2 - 6A + 9I - 4A^{-1}$$

$$A^{-1} = \frac{1}{4}(A^2 - 6A + 9I) = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

Q.2 (c) Name the law which states that in case of metals, the ratio of heat conductivity 'k' to electrical conductivity 'σ' at a constant temperature is same. A copper disk with diameter of 3 cm and thickness of 20 mm has a resistivity of 70 n-Ωm. If the disk is used as heat sink for electronic device at rate of 12 W then calculate the drop in temperature across the disk neglecting heat loss from surface.
(Take $L = 2.23 \times 10^{-8}\ W\text{-}\Omega/K^2$)

[20 marks]

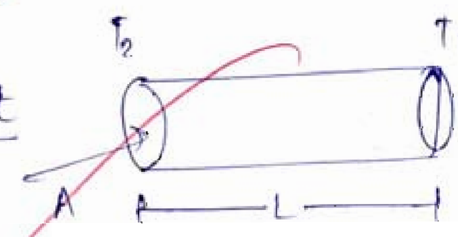
i) Wiedmann Franz law states that ratio of heat conductivity K to electrical conductivity σ at a constant temperature is same and is given by

$$\boxed{\frac{K}{\sigma} = LT} \quad \text{—— (i)}$$

where L is Lorentz number and T = absolute temperature.

ii) Heat transfer is given as

$$Q = \frac{KA(T_2 - T_1) \times t}{L}$$



Rate of heat transfer

$$\frac{Q}{t} = \frac{KA(T_2 - T_1)}{L} \quad \text{—— (ii)}$$

Now using equation (i)

$$\frac{K}{\sigma} = LT$$

$$K = \sigma LT = \frac{1}{70 \times 10^{-9}} \times 2.23 \times 10^{-8} \times 273$$

$$K = 86.97 \ \frac{W}{Km}$$

Now

from eq (ii)

$$\frac{Q}{t} = \frac{KA(T_2 - T_1)}{t \text{ (thickness)}}$$

$$12 = \frac{86.97 \times \pi \times \frac{9 \times 10^{-4}}{4} (\Delta T)}{20 \times 10^{-3}}$$

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$$\Delta T = \frac{12 \times 20 \times 10^{-3}}{86.97 \times \pi \times \frac{9 \times 10^{-4}}{4}}$$

$$\Delta T = \underline{\underline{3.9039}} \ K$$

Q.3 (a) A moving coil instrument whose resistance is 25 Ω gives a full scale deflection with a current of 1 mA. This instrument is to be used with a manganin shunt to extend its range to 100 mA. Calculate the error caused by a 10° C rise in temperature when:

(i) Copper moving coil is connected directly across the manganin shunt.

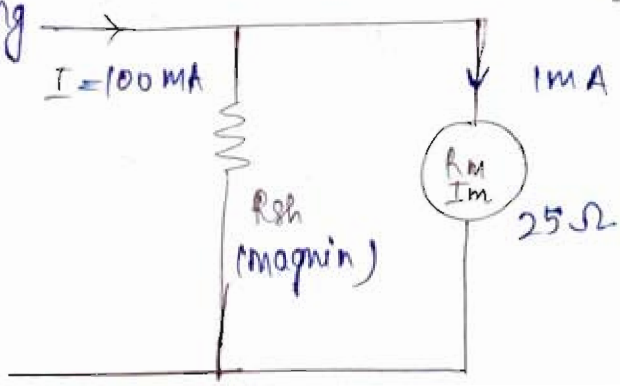
(ii) A 75-ohm manganin resistance is used in series with the instrument moving coil.

The temperature coefficient of copper is 0.004/° C and that of manganin is 0.00015/°C.

[20 marks]

① Without swamping

Now value of shunt resistance



$$R_{sh} = \frac{R_m}{m-1}$$

$$= \frac{25}{\left(\frac{100}{1} - 1\right)} = \frac{25}{99} = 0.2525\ \Omega$$

after 10°c rise in temperature

$$R_{sh}' = 0.2525(1 + 0.00015 \times 10)$$

$$= 0.2529\ \Omega$$

$$R_m' = 25(1 + 0.004 \times 10) = 26\ \Omega$$

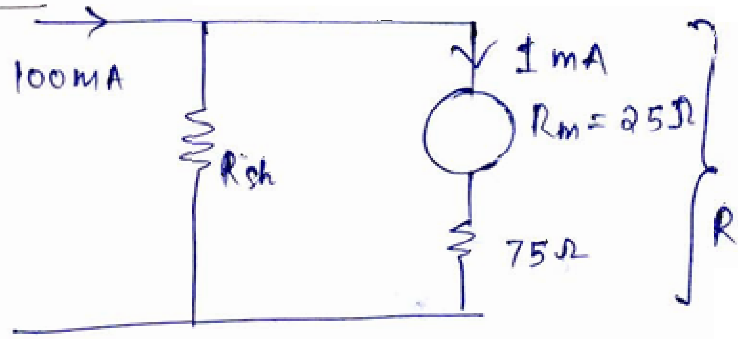
Now current in the meter –

$$I_m' = \frac{100 \times 0.2529}{0.2529 + 26} = 0.9633\ \text{mA}$$

$$\%\ \text{error} = \frac{0.9633 - 1}{1} \times 100 = -3.667\%$$

With swamping resistor -

Value of shunt required



$$R_{sh} = \frac{R_1}{m-1}$$

$$R_{sh} = \frac{25+75}{\frac{100}{1}-1} = \frac{100}{99} = 1.010\,\Omega$$

Now after 10°C rise in temperature

$$R_{sh}' = 1.01\left[1 + 0.00015 \times 10\right] = 1.0116\,\Omega$$

$$R_m' = 25(1 + 0.004 \times 10) = 26\,\Omega$$

$$R_{swamp}' = 75(1 + 0.00015 \times 10) = 75.1125\,\Omega$$

$$R_1' = R_m' + R_{swamp}' = 101.1125\,\Omega$$

Good

Now current in the meter -

$$I_m' = \frac{100 \times 1.0116}{1.0116 + 101.1125} = 0.990559$$

$$\% \text{ error} = \frac{0.990559 - 1}{1} \times 100$$

$$= \underline{-0.944\%}$$

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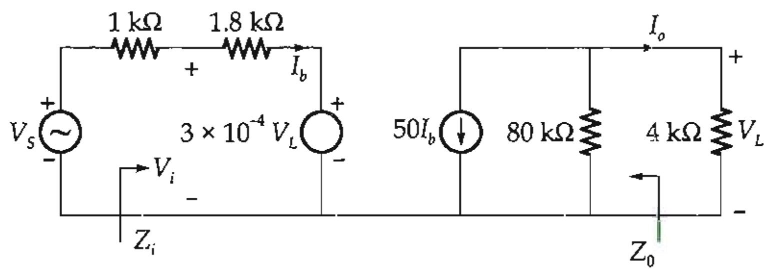
Q.3 (b) The small signal h-parameter ac equivalent circuit of a certain transistor connected in CE configuration is shown below:



Determine:

(i) Current gain A_i **(ii)** Voltage gain $\frac{V_L}{V_S}$

(iii) Input impedance Z_{in} **(iv)** Output impedance Z_{out}

[20 marks]

(i) Current gain $A_i = \frac{I_0}{I_b}$

Now $I_0 = -\frac{50 I_b \times 80}{80+4} = -47.619 I_b$

hence $A_i = \frac{I_0}{I_b} = -47.619$

(ii) Voltage gain

at O/p side

$V_L = -50 I_b \times \frac{80 \times 4\,K}{84}$ —— ①

at Input side

$V_S = (1+1.8)K I_b + 3 \times 10^{-4} V_L$

$V_S = 2.8K I_b + \frac{3 \times 10^{-4} \times (-50) \times 80 \times 4\,K}{84} I_b$

$V_s = 2.7428\,K\,I_b$ —— (ii)

hence voltage gain —

$$A_V = \frac{V_L}{V_S} = \frac{\cancel{2.7428\times 84}}{\cancel{50\times 80\times 4}} = \frac{-50\times 80\times 4K\times I_b}{84\times 2.7428\,K\,I_b}$$

$$A_V = \underline{-69.44}$$

(iii) Input resistance

$$Z_{in} = \frac{V_{in}}{I_b}$$

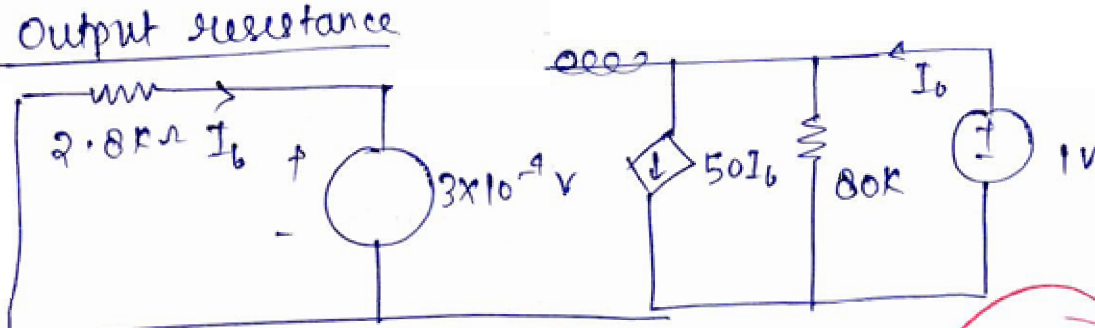
Now $V_{in} = 1.8 I_b + 3\times10^{-4} V_L$

$$V_{in} = 1.8 I_b + 3\times10^{-4}\times\frac{-50 I_b\times 80\times 4}{84}$$

$$\frac{V_{in}}{I_b} = 1.8 - \frac{3\times10^{-4}\times 50\times 80\times 4}{84}$$

$$\frac{V_{in}}{I_b} = Z_{in} = 1.7428\,K\Omega$$

(iv) Output resistance



Input side $I_b = \dfrac{-3\times10^{-4}}{2.8K}$

Output side $I_0 = \dfrac{1}{80K} + 50 I_b$

$$I_0 = \frac{1}{80K} + \frac{50(-3\times10^{-4})}{2.8K} = \frac{1}{140K}$$

hence $R_0 = \dfrac{1}{I_0} = \underline{140K\Omega}$

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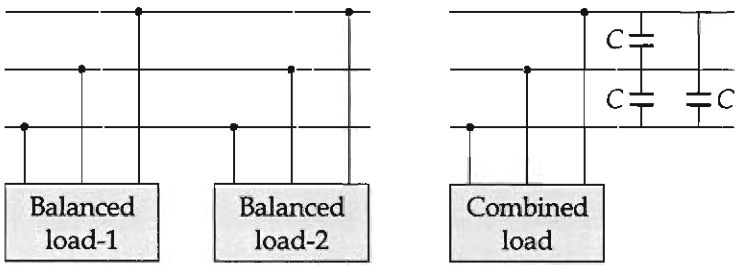
Q.3 (c) Two balanced loads are connected to a 240 kV rms, 60 Hz line as shown in figure. Load-1 draws 30 kW at a power factor of 0.6 lagging. While load 2 draws 45 kVAR at a power factor of 0.8 lagging. Assuming the *abc* sequence,

Determine :

(i) The complex, real and reactive power absorbed by the combined load,

(ii) The line currents and

(iii) The kVAR rating of the three capacitors Δ-connected in parallel with the load that will raise the power factor to 0.9 lagging and the capacitance of each capacitor.



[20 marks]

(1) for load (1)

Complex power

$$S_1 = \frac{P_1}{pf_1} \angle \cos^{-1} pf$$

$$S_1 = \frac{30}{0.6} \angle \cos^{-1} 0.6 = 50 \angle 53.13^\circ \text{ kVA}$$

$$S_1 = 30 + j40 \text{ kVA}$$

for load (2)

complex power $S_2 = \cancel{\frac{P_2}{pf_2} \angle \cos^{-1} pf}$.

$$S_2 = P_2 + jQ_2$$

$$= Q_2/\tan\phi + jQ_2$$

$$S_2 = 45/\tan\cos^{-1} 0.8 + j45$$

$$S_2 = 60.00 + j45 \text{ kVA}$$

$$= 75 \angle 36.86 \text{ kVA}$$

for combined load

$S = S_1 + S_2$

$= 30+j40 + 60+j45$

$S = 90 + j85 = 123.79 \angle 43.36$ KVA

and $P = 90$ KW

$Q = 85$ KVAR (lagging)

(ii) line currents

$S = \sqrt{3} V_L I_L$

hence for load (1)

$S_1 = \sqrt{3} V_L I_{L1}$

$I_{L1} = \frac{S_1}{\sqrt{3} V_L} \angle -\cos^{-1} pf$

$I_{L1} = \frac{50}{\sqrt{3} \times 240} \angle -53.13$

$I_{L1} = 0.12028 \angle -53.13$ A

for Load 2

$S_2 = \sqrt{3} V_L I_{L2}$

$I_{L2} = \frac{S_2}{\sqrt{3} V_L} \angle -\cos^{-1} pf$

$I_{L2} = \frac{75}{\sqrt{3} \times 240} \angle -\cos^{-1} 0.8$

$I_{L2} = 0.18042 \angle -36.86$ A

Now combined / Total line Current $= I_{L1} + I_{L2}$

$= 0.2977 \angle -43.35$ A

(iii) KVAR rating of capacitors

New pf = 0.9

KVAR rating $Q = P_1 [\tan\phi_1 - \tan\phi_2]$

$Q = 90 [\tan 43.36 - \tan \cos^{-1} 0.9]$

$Q = 41.40$ KVAR

hence capacitor bank must supply 41.4 KVAR

Now capacitors are connected in Δ

$$\frac{41.4}{3} = \frac{V_L^2}{X_C}$$

$$\frac{41.4}{3} = \omega C V_L^2$$

$$C = \frac{(41.4/3) \times 10^3}{(240 \times 10^3)^2 \times 120\pi}$$

$$C = 6.355 \times 10^{-10} \text{ F}$$

$$C = 635.5 \text{ PF}$$

18

(a) What is Hall effect? How it can be used to determine the carrier concentration in semi-conductors, explain with help of derivation?

[20 marks]

(b) Explain the working of Anderson's bridge. Derive its balance equation and give its application.

[20 marks]

(c) Evaluate $\int_S \vec{F} \cdot N ds$ where, $\vec{F} = 2x^2y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ and S is closed surface of region in the first octant bounded by cylinder $y^2 + z^2 = 9$ and planes $x = 0$, $x = 2$, $y = 0$ and $z = 0$.

20 marks]

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Section-B

Q.5 (a) Find the value of $\int_{0.2}^{1.4}\left(\sin x - \log_e x + e^x\right)dx$

(i) Trapezoidal method

(ii) Simpson's $\frac{1}{3}$rd rule

(iii) Simpson's $\frac{3}{8}$th rule

Taking step size $h = 0.2$

Also calculate the integral using integral calculus and comment on accuracy of trapezoidal, simpons plane.

[12 marks]

actual value

$= (\cos x)_{1.4}^{0.2} - (x\ln x - x)_{0.2}^{1.4} + (e^x)_{0.2}^{1.4}$

$= \underline{4.05094}$

(i) Trapezoidal Rule.

$$I = \frac{h}{2}\left[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

~~Bcz~~ given $h = 0.2$

and

y_0	y_1	y_2	y_3	y_4	y_5	y_6
$(x=0.2)$	$(x=0.4)$	$(x=0.6)$	$(x=0.8)$	$(x=1.0)$	$(x=1.2)$	$(x=1.4)$
3.0295	2.79753	2.89758	3.1660	3.5597	4.0698	4.704

hence

$$I = \frac{0.2}{2}\left[3.0295 + 4.7041 + 2(2.79753 + 2.89758 + 3.166 + 3.5597 + 4.0698)\right]$$

$$I = \underline{4.071482}$$

$\text{Error} = 4.071482 - 4.05094 = \underline{0.012082}$

Simponson rule $(1/3^{rd})$

$$I = \frac{h}{3}\left[y_0 + y_6 + 3(y_2 + y_4) + 2(y_1 + y_3 + y_5)\right]$$

$$I = \frac{0.2}{3}\left[3.0295 + 4.7041 + 3(2.89758 + 3.5597) + 2(2.79753 + 3.166 + 4.0698)\right]$$

$$I = \underline{3.1448}$$

error $= 3.1448 - 4.05094$
$= -0.90614$

Simpson $3/8$ th rule

$$I = \frac{3h}{8}\left[y_0 + y_6 + 4(y_2 + y_4 + y_5) \; 2(y_3)\right]$$

$$I = \frac{3 \times 0.2}{8}\left[3.0295 + 4.7041 + 4(2.89758 + 3.5597 + 4.0698) + 2 \times 3.166\right]$$

$$I = \underline{4.213044}$$

error $= 4.05094 - 4.213044$
$= \underline{-0.1621}$

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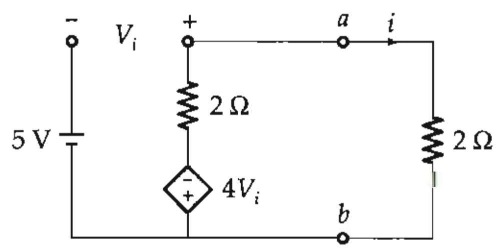
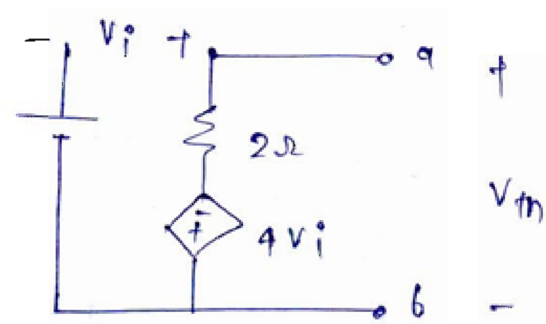
hence accuracy of trapezoidal method is higher than simposon here.

Q.5 (b) For the circuit shown in figure, find the current i in the 2 Ω resistor by using Thevenin's theorem.

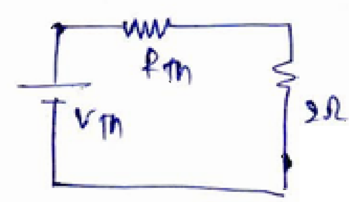


[12 marks]

V_{Th}



Now Thevenin equivalent



Now $V_{Th} = -4V_i$

and $V_i = V_{Th} - 5$

so $V_{Th} = -4(V_{Th} - 5)$

$V_{th} \times 5 = 20$

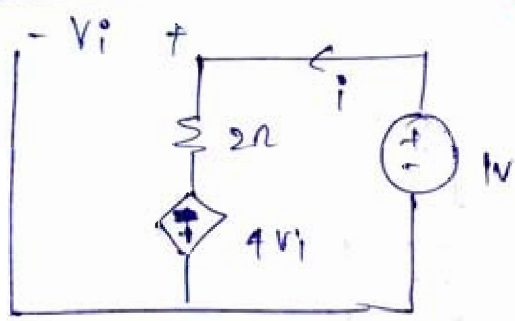
$V_{Th} = 4$ V

$i = \dfrac{V_{Th}}{R_{th} + 2}$

$i = \dfrac{4}{0.4 + 2}$

$\boxed{i = 1.67\text{ A}}$

and $R_{Th} =$



$i = \dfrac{1 + 4V_i}{2}$ and $V_i = 1 - 0 = 1$V

$i = \dfrac{1 + 4 \times 1}{2} = \dfrac{5}{2} = 2.5$ A

hence $R = \dfrac{1}{2.5} = 0.4\ \Omega$

11

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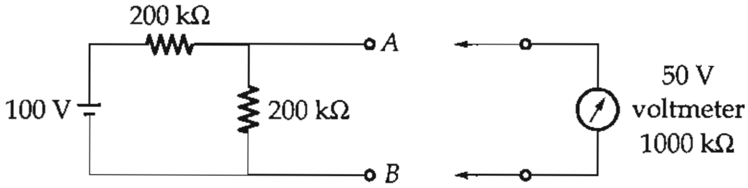
c) A 50 V range voltmeter is connected across the terminals A and B of the circuit shown in figure below. Find the reading of the voltmeter under open circuit and loaded conditions. Find the accuracy and the loading error. The voltmeter has a resistance of 1000 kΩ.



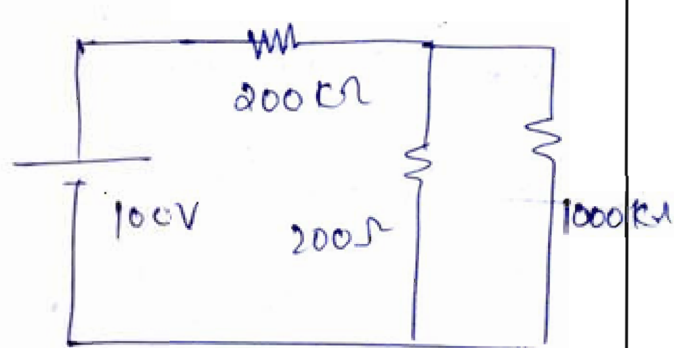
[12 marks]

True voltage across AB –

$$= \frac{100 \times 200}{200 + 200} = 50\text{V}$$ (under open circuit condition)

Voltmeter reads

Under loaded condition



Voltmeter reads.

$$= \frac{100 \times (1000 \| 200)}{1000\|200 + 200}$$

$$= \frac{100 \times 500/3}{500/3 + 200}$$

$$= 45.45 \text{ Volts}$$

hence error due to loading $E_r = \frac{V_m - V_T}{V_T} \times 100$

$$= \frac{45.45 - 50}{50} \times 100$$

$$= -9.09\%$$

hence accuracy $= 100 - 9.09$

$$= 90.90\%$$

11

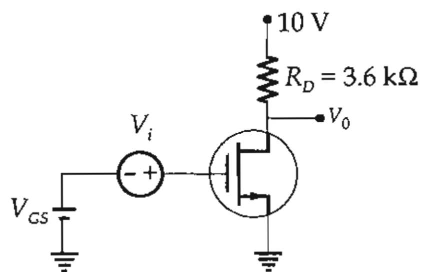
Q.5 (d) Consider the FET amplifier in figure with $V_t = 2$ V, $K_n'\left(\frac{W}{L}\right) = 1$ mA/V², $V_{gs} = 4$ V, $V_{DD} = 10$ V and $R_D = 3.6$ kΩ.

(i) Find the DC quantities I_D and V_D.

(ii) Calculate the value of g_m at the bias point.

(iii) Calculate the value of the voltage gain.

(iv) If the MOSFET has $\lambda = 0.01$ V⁻¹ find r_0 at the bias point and calculate the voltage gain.



[12 marks]

(i) DC Analysis

~~Eq~~ Drain current equation

$$I_D = \left(K'\frac{W}{L}\right) \times \frac{1}{2}(V_{GS} - V_T)^2$$

$$I_D = \frac{1}{2}(4-2)^2 = \frac{1}{2} \times 4 = 2\text{ mA}$$

Now $V_D = 10 - I_D R_D$

$$V_D = 10 - 3.6 \times 2$$

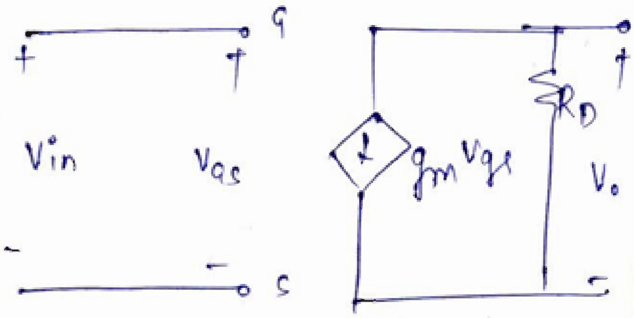
$$V_D = 2.8 \text{ Volts}$$

(ii) $$g_m = \frac{\partial I_D}{\partial V_{GS}} = \left(K'\frac{W}{L}\right)\cancel{\frac{1}{2}} \times \cancel{2}(V_{GS} - V_T)$$

$$= 1 \times (4-2)$$

$$g_m = 2 \text{ mA / volts}$$

Voltage gain (without r_o)



$$\frac{V_o}{V_{in}} = -g_m R_D$$

$$= -2\times10^{-3} \times 3.6\times10^{3}$$

$$= -7.2$$

Voltage gain with r_o

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.01\times2\times10^{-3}}$$

$$r_o = 50\,k\Omega$$

hence

$$\text{gain } \frac{V_o}{V_{in}} = -g_m(R_D \| r_o)$$

$$= -2\times10^{-3}\,[3.6 \| 50]\times10^{3}$$

$$= -6.7164$$

Good

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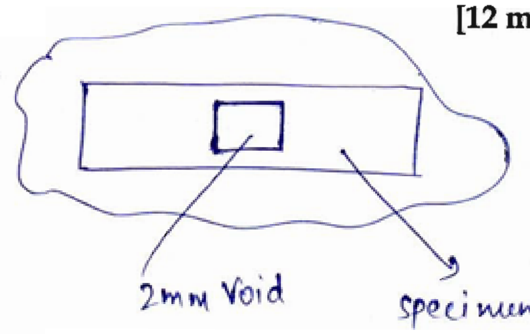
(e) A solid dielectric specimen of dielectric constant of 3.6 has a void of 2 mm thickness. If the specimen is 2 cm thick and subjected to voltage of 200 kV (rms). The air trapped in void has dielectric strength of 30 kV (peak)/cm. Calculate the voltage at which internal discharges will occur.

[12 marks]

$$C_{specimen} = \frac{\varepsilon_0 \varepsilon_r A}{d_1}$$



$$C_{void} = \frac{\varepsilon_0 A}{d_2}$$

Now voltage across void.

$$V_2 = \frac{V_{supply} \times C_{specimen}}{C_{specimen} + C_{void}}$$

$$V_2 = \frac{V_s \cancel{200\sqrt{2}} \times \frac{\varepsilon_0 \varepsilon_r A}{d_1}}{\frac{\varepsilon_0 \varepsilon_r A}{d_1} + \frac{\varepsilon_0 A}{d_2}}$$

$$V_2 = \frac{V_s \cancel{200\sqrt{2}}\ \varepsilon_r/d_1}{\varepsilon_r/d_1 + \frac{1}{d_2}}$$

$$V_2' = \frac{V_s \cancel{200\sqrt{2}}\ \varepsilon_r}{\varepsilon_r + d_1/d_2}$$

$$V_2 = \frac{V_s \cancel{200\sqrt{2}}\ \varepsilon_r}{\varepsilon_r + d_1/d_2} \quad \text{KV (peak)}$$

Now electric field in the void :

$$E_2 = \frac{V_2}{d_2} = \frac{\cancel{200\sqrt{2}}\ \varepsilon_r V_s}{d_2(\varepsilon_r + d_1/d_2)}$$

$$E_2 = \frac{\cancel{200\sqrt{2}}\ \varepsilon_r V_s}{\varepsilon_r d_2 + d_1} \quad \cancel{(\text{KV peak})/m} \quad (\text{peak value}).$$

$$\frac{30}{10^{-2}} = \frac{\cancel{200\sqrt{2}} \times 3.6\ V_s}{3.6 \times 2 \times 10^{-3} + (2\times10^{-2} - 2\times10^{-3})}$$

$$\frac{30}{10^{-2}} = \frac{\cancel{200\sqrt{2}} \times 3.6\ V_s}{3.6 \times 2\times10^{-3} + 0.018}$$

$$V_s = \frac{30 \times 0.0252 \times 100}{3.6}$$

$$V_s = 21 \text{ KV (peak)}$$

hence $V_s = 14.84$ KV (rms)

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(a) Prove that,

$$\int_a^x \int_a^x \ldots \int_a^x f(x)dx^n = \frac{1}{(n-1)!}\int_a^x (x-t)^{n-1} f(t)dt$$

Where, $n \in I^+$ and a is constant.

[20 marks]

b) **(i)** Consider an IEEE single precision floating point number as shown below:

0 1 0 0 0 0 1 1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

What is the octal equivalent of the given number?

(ii) Explain various techniques used for passing the parameters to a function.

[20 marks]

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c) A test voltage is applied for several minutes between the conductor of a 400 meter length of cable and earth. The galvanometer connected in series reads 250 divisions, the value of universal shunt being 2.5 with a standard resistance of 1 MΩ in circuit, the scale reading is 350, the value of shunt multiplier being 1000. Calculate the insulation resistance of the cable. What would be the insulation resistance of same cable of length 1000 meter?

[20 marks]

a) Show that the direction of angular momentum and the dipole moment are always opposite to each other leading to precise motion of electron around nucleus with help of mathematical derivation in connection to Bohr magneton. Also define Larmor's frequency and how it affects the angular frequency of electron around nucleus and its dipole moment.

[20 marks]

b) The input waveform shown in figure (a) is applied to the circuit shown in figure (b).

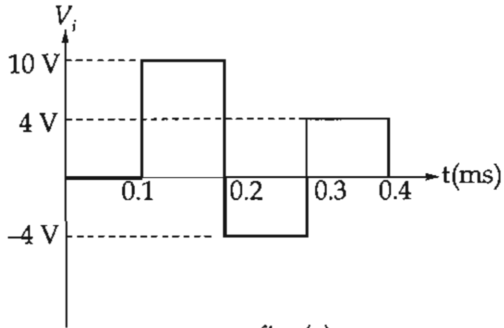


fig. (a)

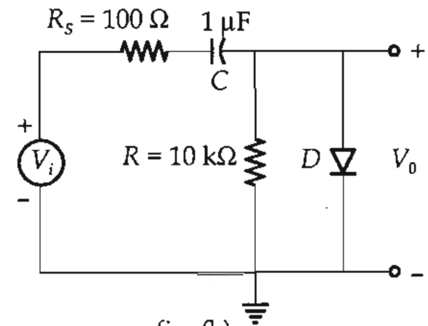


fig. (b)

If forward resistance of diode is $R_f = 100\ \Omega$, reverse resistance of diode, $R_r = \infty$ and cutin voltage $V_\gamma = 0$ then draw the output voltage waveform and lable all voltages.

[20 marks]

:) A cubical region of space is defined by the surfaces $x = 1.8$, $y = 1.8$, $z = 1.8$, $x = 2$; $y = 2$ and $z = 2$. If $D = 3y^2\hat{a}_x + 3x^2y\,\hat{a}_y\ \mathrm{C/m^2}$.

Find:

(i) The exact value of the total charge enclosed within the cube by surface integration.

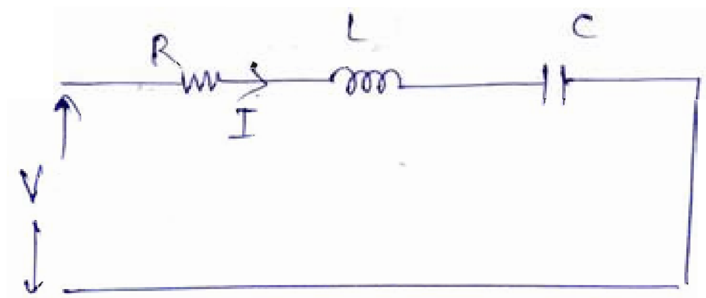
(ii) An approximate value for the enclosed charge by evaluating $\nabla \cdot D$ at the center of the cube.

[14 + 6 marks]

1) **(i)** Show that the sum of energy stored by the inductor and the capacitor connected in series at resonance is constant and is given by LI_{rms}^2.

(ii) Show that the sum of energy stored by the inductor and the capacitor in a parallel RLC circuit at resonance is constant and is given by CV_{rms}^2.

[20 marks]



$V = V_m \sin\omega t \qquad I = \frac{V_m}{R}\sin\omega t$ at resonance

Energy stored in $L = \frac{1}{2} L I^2 = \frac{1}{2} L \frac{V_m^2}{R^2}\sin^2\omega t$

~~$E = \frac{1}{2} L I$~~

Energy stored in capacitor $= \frac{1}{2} C V_C^2$

$V_C = -jX_C \bar{I}$

$= \frac{V_m}{RWC}\sin(\omega t - 90^\circ)$

$= -\frac{V_m}{RWC}\cos\omega t$

$= \cancel{\frac{1}{2} \times C \times (IX_C)^2}$

$= \cancel{\frac{1}{2} C \times \frac{I^2}{\omega^2 C^2}} = \frac{1}{2}\frac{1}{\omega^2 C}\frac{V_m^2}{R}\sin^2\omega t$

Sum of energy

$\cancel{= \left(\frac{1}{2}\frac{L}{R} + \frac{1}{2\omega^2 CR}\right)\frac{V_m^2 \sin^2\omega t}{}}$

$\cancel{= \left(\frac{V_m}{R}\right)^2 \times \frac{1}{2R}\left[L + \frac{1}{\omega^2 C}\right]}\sin^2\omega t$

$= \frac{V_m^2}{} \times$

$\cancel{V_C = \frac{1}{C}\int_0^t i\,dt = \frac{1}{C}\int_0^t \frac{V_m}{R}\sin\omega t\,dt}$

$\cancel{= \frac{V_m}{RC\omega}(\cos\omega t)_t^0 = \frac{V_m}{RC\omega}(1-\cos\omega t)}$

Energy in Capacitor

$= \frac{1}{2} C V_c^2$

$= \frac{1}{2} \times C \times \frac{V_m^2}{R^2C^2\omega^2} (\cos\omega t)^2$

$= \frac{1}{2R^2C\omega^2} V_m^2 (\cos^2\omega t)$

sum of energy

$= \frac{1}{2} V_m^2 \left[\frac{\cos^2\omega t}{R^2C\omega^2} + \frac{\sin^2\omega t\, L}{R^2}\right]$

$= \frac{1}{2} \frac{V_m^2 L}{R^2} \left[\frac{1}{\omega^2CL}\cos^2\omega t + \sin^2\omega t\right]$

at resonance

$\omega^2LC = 1 \quad (\omega L - \frac{1}{\omega C})$

$= \frac{1}{2} \frac{V_m^2}{R^2} L \left[\cos^2\omega t + \sin^2\omega t\right]$

$= \frac{1}{2}\left(\frac{V_m}{R}\right)^2 \times L$

$= L \times \left(\frac{V_m}{R\sqrt{2}}\right)^2$

$= I_{rms}^2 L$

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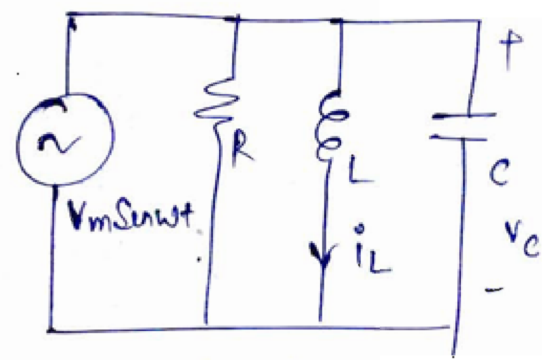
Parallel circuit



$V_c = V_m \sin\omega t$

and $i_L = \frac{V_m}{j\omega L}$

$i_L = \frac{V_m}{\omega L} \angle -90°$

$i_L = \frac{V_m}{\omega L} \sin(\omega t - 90)$

$i_L = -\frac{V_m}{\omega L} \cos\omega t$

hence

Energy in inductor $= \frac{1}{2} L i_L^2$

$= \frac{1}{2} \frac{V_m^2}{\omega^2 \cancel{L^2}} \cos^2\omega t \times \cancel{L}$

$E_L = \frac{1}{2} V_m C^2 \cos^2\omega t$

(because $\frac{1}{\omega C} = \omega L$) $\Rightarrow \omega$

Energy in capacitor $= \frac{1}{2} C V_c^2$

$E_C = \frac{1}{2} C \times V_m^2 \sin^2\omega t$

hence total energy $= E_L + E_C$

$= \frac{1}{2} C V_m^2 \sin^2\omega t + \frac{1}{2} C V_m^2 \cos^2\omega t$

$= \frac{1}{2} C V_m^2$

$= C V_{rms}^2$

(i) Consider the process arrival time and burst time chart given below:

Process	1	2	3	4	5
Arrival Time	0	0	6	7	8
Burst Time	16	10	4	6	10

Show how the above processes will be scheduled according to round robin algorithm. Also find waiting time of each process. Time quantum = 5 units.

(ii) What is paging and segmentation in operating system? Illustrate how virtual address is mapped to physical address using paging and segmentation with the help of a diagram?

[20 marks]

Ready queue

P_2 P_1 P_2 P_3 P_4 P_5 P_1 P_4 P_5 P_1 P_1

P_2	P_1	P_2	P_3	P_4	P_5	P_1	P_4	P_5	P_1	P_1
5	10	15	19	24	29	34	35	40	45	46

	CT	TAT CT − AT	WT TAT − BT
P_1	46	46	30
P_2	15	15	5
P_3	19	13	9
P_4	35	28	22
P_5	40	32	22

Avg WT time

$$= \frac{30+5+9+22+22}{5}$$

$= 17.6$ units

→ ~~B~~ Paging and segmetation is a technique of virtual memory management where data is placed in the secondary memory and with the help of paging and segmentation it is brought into main memory.

Suppose there is 4 blocks of 2KB in main memory and there are 16 blocks of 2KB in virtual / secondary memory. Now

8 KB of main memory requires address field as follows

block field	Block ~~field~~ offset
2 bits	11 bits

← 13 bits →

Physical address.

CPU generates ~~V~~irtual address which is as follows according to our example →

Page field	block offset.
4 bits	11 bits

← 15 bit →

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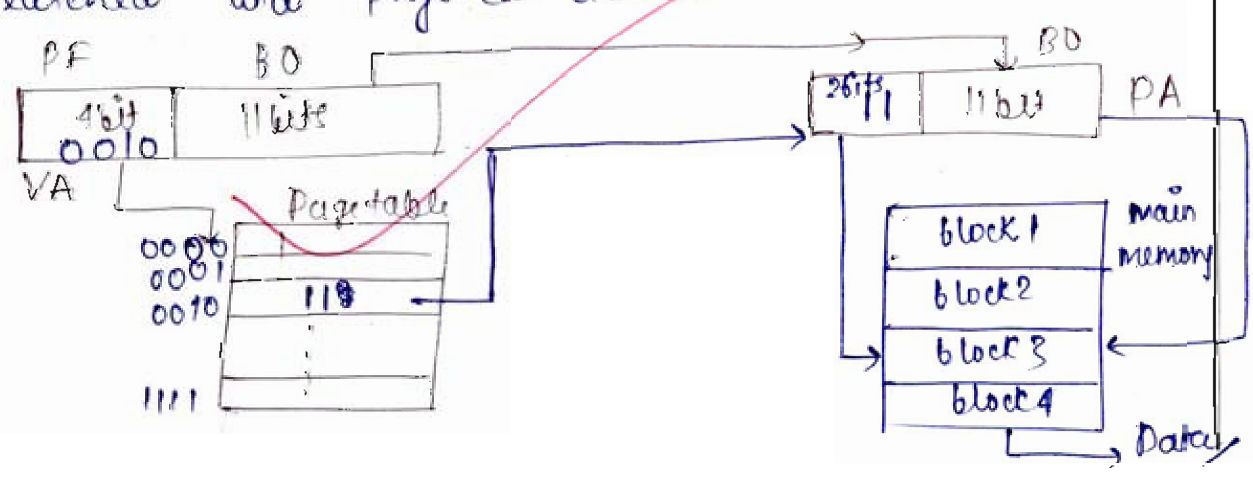
Now Page field of virtual address is connected to page table where block No. of main memory is searched and physical address is assembled



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A full-wave rectifier is fed from a transformer having a center-tapped secondary winding. The rms voltage from either end of secondary to center tap is 30 V. If the diode forward resistance is 2 Ω and that of the half secondary is 8 Ω, then for a load of 1 kΩ.

Calculate:

(i) Power delivered to load

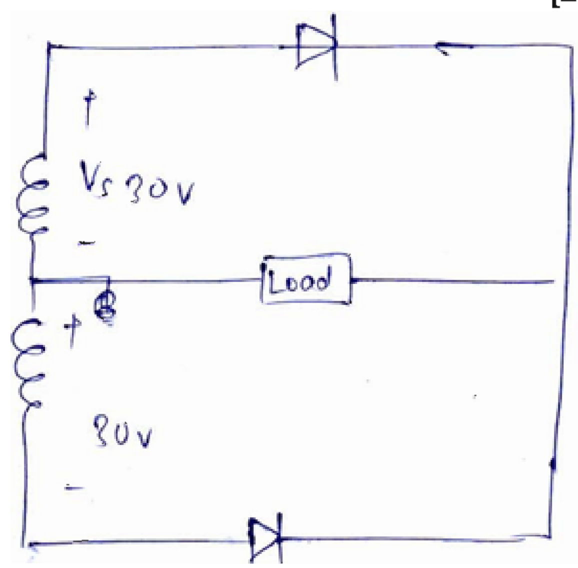
(ii) % Regulation at full load

(iii) Efficiency of rectification

(iv) Transformer utilization factor of secondary

[20 marks]

Power delivered to load



$= I_{rms}^2 R_L$

$= \left(\dfrac{V_{rms}/\sqrt{2}}{R_f + R_{s/2} + R_L}\right)^2 R_L$

$= \left(\dfrac{30}{2+8+1000}\right)^2 \times 1000$

$= 0.88226$ Watt

(ii) % Regulation $= \dfrac{V_{NL} - V_{FL}}{V_{FL}} \times 100$

$$= \frac{\dfrac{2V_m}{\pi} - \dfrac{2V_m R_L}{\pi(R_f + R_{s/2} + R_L)}}{\dfrac{2V_m R_L}{(R_f + R_{s/2} + R_L)\pi}} \times 100$$

$$= \frac{\dfrac{R_s}{2} + R_f}{R_L} \times 100$$

% Regulation $= \dfrac{R_s/2 + R_f}{R_L} \times 100$

$= \dfrac{8 + 2}{1000} \times 100$

$= \dfrac{10}{1000} \times 100$

$= 1\%$

Efficiency of rectification $= \dfrac{I_{DC}^2 R_L}{I_{rms}^2 (R_L + R_s/2 + R_f)}$

$= \dfrac{\left[\dfrac{2V_m}{\pi(R_L + R_s/2 + R_f)}\right]^2 \times R_L}{\left(\dfrac{V_m}{\sqrt{2}(R_L + R_s/2 + R_f)}\right)^2 \times (R_L + R_s/2 + R_f)} \times 100$

$= \dfrac{4 \times 2 R_L}{\pi^2 (R_L + R_s/2 + R_f)} \times 100$

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$= \dfrac{8}{\pi^2\left[1 + \dfrac{R_s/2 + R_f)}{R_L}\right]} \times 100$

$= \dfrac{8}{\pi^2\left[1 + \dfrac{10}{1000}\right]} \times 100 = \underline{80.254\%}$

TUF of secondary $\Rightarrow \dfrac{P_{dc}}{P_{s_1} + P_{s_2}}$

$\Rightarrow \dfrac{I_{dc}^2 R_L}{\dfrac{V_m}{\sqrt{2}} \times \dfrac{I_m}{2} + \dfrac{V_m}{\sqrt{2}} \times \dfrac{I_m}{2}}$

$\Rightarrow \dfrac{4 V_m^2 R_L / \pi^2 (R_L + R_s/2 + R_f)^2}{V_m \times \dfrac{V_m}{R_L + R_s/2 + R_f} \times 1/\sqrt{2}} = \dfrac{4\sqrt{2}}{\pi^2} \times \dfrac{R_L}{(R_L + R_s/2 + R_f)} = \dfrac{0.573}{1 + \dfrac{10}{1000}} = 0.$

Space for Rough Work

$$2i + 4v_i + 2i = 0$$

$$4i + 4[2i - 5] = 0$$

$$4i + 8i = 20$$

$$i = \frac{20}{12}$$

$$= 10/$$