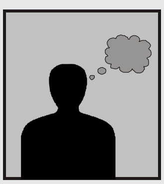
2020

RANK Improvement WORKBOOK



Answer key and Hint of Objective & Conventional *Questions*

Mechanical Engineering

Engineering Mechanics

MADE EASY Publications

1 System of forces, Centoriod, MOI

LEVEL 1 Objective Questions

1.	(c)
2.	(b)
3.	(a)
4.	(c)
5.	(b)
6.	(c)
7.	(b)
8.	(b)
9.	(34)
10.	(10.48)
11.	(3.28)
12.	(b)
13.	(6)
14.	(72.38)
15.	(4.26)
16.	(–9.5)
17.	(5)

LEVEL 2 Objective Questions

18.	(a)
19.	(c)
20.	(b)
21.	(b)
22.	(a)
23.	(b)
24.	(a)
25.	(b)
26.	(b)
27.	(c)
28.	(b)
29.	(c)
30.	(b)
31.	(c)
32.	(b)
33.	(c)
34.	(a)
35.	(c)
36.	(c)
37.	(d)
38.	(b)

■■■■

LEVEL 3 Conventional Questions

Solution : 39

(a) Points $P(6, -4)$ m and $Q(-3, 6)$ m have position vectors as follows

$$R_1 = 6i - 4j \text{ m}, \quad R_2 = -3i + 6j \text{ m}$$

Displacement vector, $S_{PQ} = R_2 - R_1 = (-3i + 6j) - (6i - 4j) \text{ m} = -9i + 10j \text{ m}$

$$|S_{PQ}| = \sqrt{(-9)^2 + (10)^2} = 13.453 \text{ m}$$

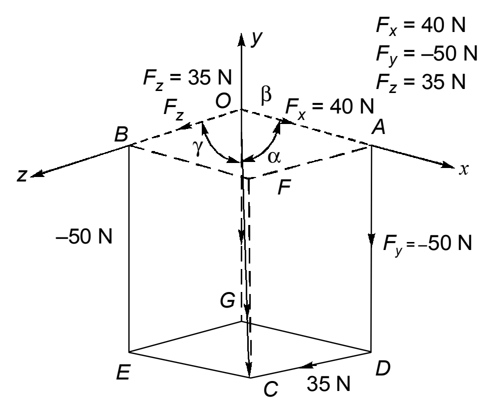
Direction cosines, $l = \cos\alpha = \dfrac{-9}{13.453} = -0.668$, $\quad m = \cos\beta = \dfrac{+10}{13.453} = +0.743$

Unit vector along S_{PQ}, $\bar{r} = -0.668i + 0.743j$

Solution : 40

Shown force vector, $F = 40i - 50j + 35k$ (to some suitable scale)

$$F = \sqrt{40^2 + (-50)^2 + 35^2}$$
$$= \sqrt{1600 + 2500 + 1225}$$
$$= 72.97 \text{ N}$$



Direction cosines

$$l = \cos\alpha = \frac{40}{72.97} = +0.548$$

$$m = \cos\beta = -\frac{50}{72.97} = -0.685$$

$$n = \cos\gamma = \frac{35}{72.97} = +0.48$$

Solution : 41

Force, $P = 30$ N

Moment arm of couple $= 2$ m (BC or FE)

Moment of the couple, $C = 30 \times 2 = +60$ Nm (ccw)

Couple vector

Force along FB, $P = \left(\dfrac{2i - 2j}{\sqrt{4+4}}\right) \times 30$

$$= (21.21i - 21.21j) \text{ N}$$

Position vector CB, $r = +2k$ m

Couple vector, $C = r \times P$

$$= (2k) \times (21.21i - 21.21j)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 2 \\ 21.21 & -21.21 & 0 \end{vmatrix} = 42.42j + 42.42i \text{ Nm}$$

Solution : 42

$$F_1 = 400 \text{ N acts downwards at } A\,(2, 0, 3) \text{ m}$$
$$F_2 = 150 \text{ N acts upwards at } B\,(4, 0, 0)$$
$$F_3 = 200 \text{ N acts upwards at } C\,(4, 0, 4) \text{ m}$$

Replacing each force by an equal force at origin and a couple.

Resultant force,

$$F_R = -F_1 + F_2 + F_3$$
$$= -400 + 150 + 200 = -50 \text{ N}$$
$$= -50j \text{ N (at the origin)}$$

Position vectors, $r_1 = 2i + 3k$

$$r_2 = 4i, \quad r_3 = 4i + 4k$$

Say the simplest resultant passes through coordinates $\bar{x}, \bar{z}$, then taking moments about z-axis

$$-400 \times 2 + 150 \times 4 + 200 \times 4 = -50\bar{x} = F_R.\bar{x}$$
$$-800 + 600 + 800 = -50\bar{x}$$
$$600 = -50\bar{x}$$
$$\bar{x} = -12 \text{ m} \qquad ...(1)$$

Taking moments about x-axis

$$+3 \times 400 + 150 \times 0 - 200 \times 4 = +50\bar{z}, \text{ taking ccw moments as positive}$$
$$400 = +50\bar{z}$$
$$\bar{z} = +8 \text{ m}$$

But the simplest resultant of the force system is 50 N ↓ acting at point (–12, 0, 8) m.

If this resultant is shifted to origin then

$$F_R = -50j \text{ N} + \text{couple } C_R = +400i + 600k \text{ Nm}, \; |C_R| = 721 \text{ Nm.}$$

Solution : 43

Intensity of loading, $w = ax^2$

$$w = 0 \text{ at } x = 0$$
$$w = 4 \text{ kN/m at } x = 4 \text{ m}$$

Therefore, $4 = a \times 4^2$

$$a = 0.25$$

or $w = 0.25x^2$ kN/m

$$\text{Total distributed load} = \int_0^4 w\,dx = \int_0^4 0.25x^2\,dx = \left|\frac{x^3}{12}\right|_0^4 = \frac{64}{12} = 5.33 \text{ kN}$$

Moment of distributed load, about end A

$$M_w = \int_0^4 xw\,dx = \int_0^4 0.25x^3\,dx = \left|\frac{x^4}{16}\right|_0^4 = 16 \text{ kNm}$$

F_R, simplest resultant of forces on beam,

$$F_R = 5.33 + 20 = 25.33 \text{ kN}$$

Taking moments about end A,

$$M_A = M_w + 20 \times 6 = 16 + 120 = 136 \text{ kNm } (cw)$$

$$\bar{x} F_R = 136$$

$$\bar{x} = \frac{136}{25.33} = 5.37 \text{ m from end } A$$

Simplest resultant is 25.33 kN acting at a distance of 5.37 m from A.

Solution : 44

Let us take origin at A. All the forces can be shifted to the origin, (equivalent to a force and a couple at origin). Resultant force,

$$F_R = 80i + 100j - 60j \text{ N} = 80i + 40j \text{ N}$$

$$|F_R| = \sqrt{80^2 + 40^2} = 89.4 \text{ N}$$

Moments about origin,

$$M_A = 100 \text{ N} \times 3 \text{ m} \,(ccw) - 150 \text{ Nm} \,(cw)$$

or $\qquad C_R = 150 \text{ Nm} \,(ccw)$ resultant couple

Note that forces 60 N vertical and 80 N horizontal are passing through A, these forces will not produce any moment.

$$C_R = 150 \text{ Nm} \,(ccw) \text{ in plane } xy$$

(Direction of couple vector along z-axis)

Say the resultant F_R passes through $\bar{x}$ and $\bar{y}$, then

$$F_{Rx} = 80 \text{ Nm}$$

$$F_{Ry} = 40 \text{ Nm}$$

$$F_{Rx} \times \bar{y} \,(ccw) - F_{Ry} . \bar{x} \,(ccw) = -150 \text{ Nm} \,(cw) \text{ for balancing}$$

$$80\bar{y} - 40\bar{x} = -150 \text{ Nm}$$

If we take $\qquad \bar{x} = 0, \; \bar{y} = -1.875 \text{ m}$

$$\bar{y} = 0, \; \bar{x} = +3.75 \text{ m}$$

Solution : 45

Let us locate the centroid of the section. The section is symmetrical about yy-axis which passes through the centroid of the section. Consider a reference axis $x'x'$ at the lower edge of the web. The section can be divided into two areas

$$A_1 = b \times d; \; A_2 = B \times t \text{ and total area } A = A_1 + A_2$$

Then distance of G from lower edge $(x'x')$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}, \text{ where } y_1 = \frac{d}{2} \text{ and } y_2 = d + \frac{t}{2}$$

$$= \frac{bd\left(\frac{d}{2}\right) + B.t\left(d + \frac{t}{2}\right)}{bd + Bt}$$

Moment of inertia of the flange about xx-axis

$$= \frac{Bt^3}{12} + Bt \times h_2^2 = \frac{Bt^3}{12} + Bt\left(d + \frac{t}{2} - \bar{y}\right)^2$$

Using parallel axes theorem.

Moment of inertia of the web about xx-axis

$$= \frac{bd^3}{12} + bd \times h_1^2 = \frac{bd^3}{12} + bd\left(\bar{y} - \frac{d}{2}\right)^2$$

using parallel axis theorem because

$$h_1 = \bar{y} - \frac{d}{2} \text{ and } h_2 = d + \frac{t}{2} - \bar{y}$$

Therefore moment of inertia of T-section about centroidal axis xx

$$I_{xx} = \frac{Bt^3}{12} + Bt\left(d + \frac{t}{2} - \bar{y}\right)^2 + \frac{bd^3}{12} + bd\left(\bar{y} - \frac{d}{2}\right)^2$$

Let us take some numerical values

$$B = 6 \text{ cm}, \quad t = 1 \text{ cm}; \quad d = 6 \text{ cm and } b = 1 \text{ cm}$$

Then $A_1 = A_2 = 6 \text{ cm}^2$

$$y_1 = 3 \text{ cm}; \quad y_2 = 6 + \frac{1}{2} = 6.5 \text{ cm}$$

$$\bar{y} = \frac{6 \times 3 + 6 \times 6.5}{6 + 6} = \frac{18 + 39}{12} = 4.75 \text{ cm}$$

Moment of inertia, $I_{xx} = \frac{6 \times 1^3}{12} + 6 \times 1(6.5 - 4.75)^2 + \frac{1 \times 6^3}{12} + 6\,(4.75 - 3)^2$

$$= 0.5 + 18.375 + 18 + 18.375 = 55.25 \text{ cm}^4.$$

Solution : 46

Let us break up the section into 3 rectangular strips, I, II and III as shown and write the co-ordinates of their centroids with respect to the given set of x and y axis.

Strip	Area	$\bar{x}$	$\bar{y}$	$A\bar{x}\bar{y}$
I.	20 cm^2	5 cm	1 cm	100 cm^4
II.	8 cm^2	0.5 cm	6 cm	24 cm^4
III.	8 cm^2	2 cm	11 cm	176 cm^4

Remember that the product of inertia of these rectangular strips about their principal axes passing through the respective centroids is zero, because rectangular strips have two axes of symmetry.

$$(I_{xy})_{\text{I}} = 0 + 100 \text{ cm}^4 \quad \text{(Using the parallel axis theorem for product of inertia)}$$

$$(I_{xy})_{\text{II}} = 0 + 24 \text{ cm}^4$$

$$(I_{xy})_{\text{III}} = 0 + 176 \text{ cm}^4$$

$$I_{xy} = 300 \text{ cm}^4$$

To determine, $I_{\bar{x}\bar{y}}$, let us first determine the position of the centroid of the section

$$\bar{x} = \frac{20\times5+8\times0.5+8\times2}{20+8+8} = 3.333 \text{ cm}$$

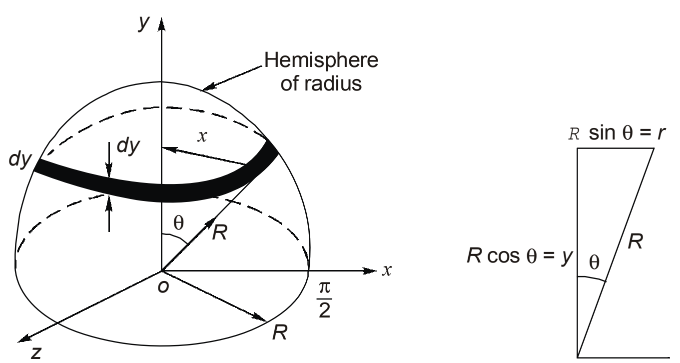
$$\bar{y} = \frac{20\times1+8\times6+8\times11}{20+8+8} = 4.333 \text{ cm}$$

Area of the cross-section,

$$A = 20+8+8 = 36 \text{ cm}^2$$

$$I_{\bar{x}\bar{y}} = I_{xy} - A\bar{x}\bar{y} = 300-36\times3.333\times4.333 = -220 \text{ cm}^4$$

Solution : 47



(a) (b)

Figure (a) shows a hemisphere of radius R, its base is in the x-z plane and hemisphere is symmetrical about oy-axis; therefore its centre of gravity will lie on the axis oy.

Consider an element of thickness dy at a height y from the zx plane.

Radius of small element, $r = R\sin\theta$

Height, $y = R\cos\theta$

as shown in figure (b)

Volume of the small element,

$$dV = \pi(R\sin\theta)^2\, dy$$

$$= \pi R^2 \sin^2\theta\, dy \qquad \ldots(1)$$

Now, $y = R\cos\theta$

So, $dy = -R\sin\theta . d\theta \qquad \ldots(2)$

Putting the value in Equation (1)

$$dV = \pi R^2 \sin^2\theta\,(-R\sin\theta)\,d\theta = -\pi R^3 \sin^3\theta\, d\theta$$

First moment of the volume about xz plane

$$M_{xz} = \bar{y}V = \int y\,dV = \int_{\pi/2}^{0} (-\pi R^3 \sin^3\theta)(R\cos\theta)\,d\theta$$

$$= \int_{\pi/2}^{0} -\pi R^4 \sin^3\theta \cos\theta \, d\theta = -\pi R^4 \left| \frac{\sin^4\theta}{4} \right|_{\pi/2}^{0} = +\frac{\pi R^4}{4}$$

Volume of the hemisphere, $V = \frac{2}{3}\pi R^3$

Distance of CG from xz plane,

$$\bar{y} = \frac{\pi R^4}{4} \times \frac{3}{2\pi R^3} = \frac{3}{8}R$$

Other distances $\bar{x} = \bar{z} = 0$

Solution : 48

Total area, $A = A_1 - A_2 - A_3$

A_1 = rectangle $140 \times 200 = 28000\ \text{mm}^2$

A_2 = circle, radius 50 mm = $7854\ \text{mm}^2$

A_3 = rectangle 40×60 mm = $2400\ \text{mm}^2$

$A = 28000 - 7854 - 2400 = 17746\ \text{mm}^2$

$$\bar{x} = \frac{28000 \times 100 - 7854 \times 100 - 2400 \times 180}{17746}$$

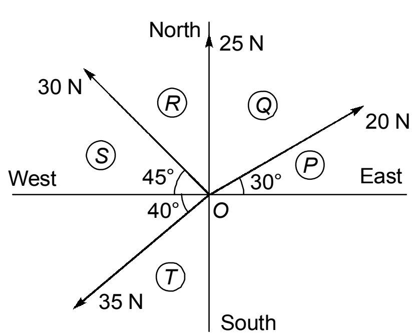
$$= \frac{2800000 - 785400 - 432000}{17746} = \frac{1582600}{17746} = 89.18 \text{ mm from } A$$

$$\bar{y} = \frac{28000 \times 70 - 7854 \times 70 - 2400 \times 110}{17746}$$

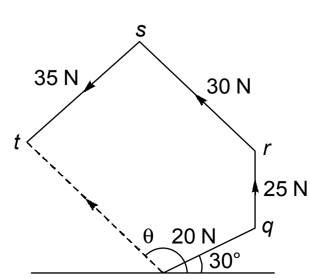
$$= \frac{1960000 - 549780 - 264000}{17746}$$

$$= \frac{1146220}{17746} = 64.59 \text{ mm from } A$$

Solution : 49



(a) Space diagram



(b) Vector diagram

First of all, draw the space diagram for the given system of forces (acting at point O) and name the forces according to Bow's notations as shown in figure (a). The 20 N force is named as pq, the 25 N force as qr, 30 N force as rs and 35 N force as st.

Now draw the vector diagram for the given system of forces as shown in figure (b) and as discussed below:

1. Select some suitable point p and draw pq equal to 20 N to some suitable scale and parallel to the force PQ.
2. Through q, draw qr equal to 25 N to the scale and parallel to the force QR of the space diagram.
3. Now through r, draw rs equal to 30 N to the scale and parallel to the force RS of the space diagram.
4. similarly, through s, draw st equal to 35 N to the scale and parallel to the force ST of the space diagram.
5. Join pt, which gives the magnitude as well as direction of the resultant force.
6. By measurement, we find that the magnitude of the resultant force is equal to 45.6 N and acting at an angle of 132 with the horizontal i.e. East-West line.

Solution : 50

Magnitude of the resultant of force

Resolving forces horizontally,

$$\Sigma H = 25 - 20 = 5 \text{ kgf}$$

and now resolving the force vertically

$$\Sigma V = (-50) + (-35) = -85 \text{ kgf}$$

∴ Magnitude of the resultant force

$$P = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(5)^2 + (-85)^2} \text{ kgf}$$

$$= 85.15 \text{ kgf}$$

Direction of the resultant force

Let θ = Angle which are resultant force makes with the horizontal

We know that, $$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-85}{5} = -17$$

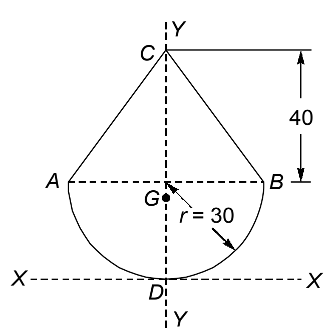
or $$\theta = -86° 38'$$

Since ΣH is positive and ΣV is negative, therefore lies between 270° and 360°. Therefore actual angle of the resultant force

$$= 360° - 86°38' = 273° 22'$$

Solution : 51

As the body is symmetrical about Y-Y axis, therefore its C.G. will lies on this axis as shown in figure. Let bottom of the hemisphere (D) be the point of reference.



(i) Hemisphere

$$v_1 = \frac{2\pi}{3} \times r^3 = \frac{2\pi}{3}(30)^3 \text{ mm}^3$$
$$= 18000\,\pi \text{ mm}^3$$
$$y_1 = \frac{5r}{8} = \frac{5 \times 30}{8} = 18.75 \text{ mm}$$

(ii) Right circular cone

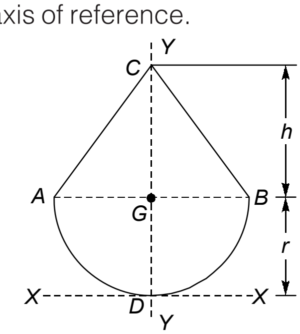
$$v_2 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3}(30)^2 \times 50 \text{ mm}^3$$
$$= 15000\,\pi \text{ mm}^3$$
$$y_2 = 30 + \frac{40}{4} = 40 \text{ mm}$$

We know that distance between C.G. of the body and bottom of the hemisphere D,

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$
$$= \frac{(18000\pi \times 18.75) + (15000\pi \times 40)}{18000\pi + 15000\pi} = 28.4 \text{ mm}$$

Solution : 52

First of all, let us find the C.G. of the body. As the body is symmetrical about Y-Y axis, therefore its C.G will lie on this axis as shown in figure. Now consider two parts of the body i.e. hemisphere and cone. Let bottom of the hemisphere (D) be the axis of reference.



(i) Hemisphere
$$v_1 = \frac{2\pi}{3} \times r^3$$
$$y_1 = \frac{5r}{8}$$

(ii) Cone
$$v_2 = \frac{\pi}{3} \times r^2 \times h$$
$$y_2 = r + \frac{h}{4}$$

We know that the distance between C.G. of the body and bottom of the hemisphere D,

$$\bar{y} = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

$$= \frac{\left(\frac{2\pi}{3}\times r^3\times\frac{5r}{8}\right)+\left(\frac{\pi}{3}\times r^2\times h\right)\left(r+\frac{h}{4}\right)}{\left(\frac{2\pi}{3}\times r^3\right)+\left(\frac{\pi}{3}\times r^2\times h\right)}$$

Now for stable equilibrium, we know that the C.G of the body should preferably be below the common face *AB*, or maximum it may coincide with it. Therefore substituting $\bar{y}$ equal to r in the above equation,

$$r = \frac{\left(\frac{2\pi}{3}\times r^3\times\frac{5r}{8}\right)+\left(\frac{\pi}{3}\times r^2\times h\right)\left(r+\frac{h}{4}\right)}{\left(\frac{2\pi}{3}\times r^3\right)+\left(\frac{\pi}{3}\times r^2\times h\right)}$$

$$\frac{2\pi}{3}\times r^4+\frac{\pi}{3}\times r^3\times h = \frac{5\pi}{12}\times r^4+\frac{\pi}{3}\times r^3\times h+\frac{\pi}{12}\times r^2\times h^2$$

Dividing both sides by πr^2,

$$\frac{2r^2}{3}+\frac{rh}{3} = \frac{5r^2}{12}+\frac{rh}{3}+\frac{h^2}{12}$$

$$\frac{3r^2}{12} = \frac{h^2}{12}$$

$$3r^2 = h^2$$

or $$h = 1.732\,r$$

Solution : 53

First of all, let us find out C.G. of the section. As the section is symmetrical about *Y-Y* axis, therefore its C.G. will lie on the axis. Let BC be the axis of reference.

(i) Triangular section

$$a_1 = \frac{100\times 90}{2} = 4500 \text{ mm}^2$$

$$y_1 = \frac{90}{3} = 30 \text{ mm}$$

(ii) Rectangular hole

$$a_2 = 30\times 20 = 600 \text{ mm}^2$$

$$y_2 = 30+\frac{30}{2} = 45 \text{ mm}$$

We know that the distance between the C.G. of the section and the base *BC*,

$$\bar{y} = \frac{a_1y_1-a_2y_2}{a_1-a_2} = \frac{(4500\times 30)-(600\times 45)}{4500-600}\text{mm} = 27.7 \text{ mm}$$

⇒ Moment of inertia about centre of gravity

We know that the moment of inertia of a triangular section about an axis through its C.G. or parallel to *X-X* axis,

$$I_{G1} = \frac{bh^3}{36} = \frac{100(90)^3}{36} = 2.025\times 10^6 \text{ mm}^4$$

and distance between C.G. of the section and *X-X* axis

$$h_1 = 30 - 27.7 = 2.3 \text{ mm}$$

∴ Moment of intertia of the triangular section about X-X axis

$$= I_{G1} + a \cdot h_1^2$$

$$= 2.025 \times 10^6 + [4500 \times (2.3)^2] \text{ mm}^4$$

$$= 2.049 \times 10^6 \text{ mm}^4 \quad \text{...(i)}$$

Now moment of inertia of the rectangular section about an axis passing through its C.G. and parallel to X-X axis,

$$I_{G2} = \frac{bd^3}{12} = \frac{20(30)^3}{12} = 45000 \text{ mm}^4$$

and distance between C.G. of the section and X-X axis,

$$h_2 = 45 - 27.7 = 17.3 \text{ mm}$$

∴ moment of inertia of the rectangular section about X-X axis,

$$= I_{G2} + a \cdot h_2^2$$

$$= 45000 + [600 \times (17.3)^2] \text{ mm}^4$$

$$= 0.225 \times 10^6 \text{ mm}^4 \quad \text{...(ii)}$$

Now moment of inertia of the whole section about X-X axis,

$$I_{XX} = 2.049 \times 10^6 - 0.225 \times 10^6 \text{ mm}^4$$

$$= 1.824 \times 10^6 \text{ mm}^4$$

⇒ Moment of inertia about the base AB

We know that M.I. of triangular section ABC about its base BC,

$$I = \frac{bh^3}{12} = \frac{100(90)^3}{12} = 6.075 \times 10^6 \text{ mm}^4$$

and M.I. of the rectangular section about an axis passing through its C.G. and parallel to X-X axis,

$$I_{G3} = \frac{bd^3}{12} = \frac{20(30)^3}{12} = 45000 \text{ mm}^4$$

and distance between the C.G. of the section and base BC,

$$h_3 = 30 + \frac{30}{2} = 45 \text{ mm}$$

∴ M.I. of the rectangular section about base BC

$$= I_{G3} + a \cdot h_3^2$$

$$= 45000 + [600 \times (45)^2] \text{ mm}^4$$

$$= 1.26 \times 10^6 \text{ mm}^4$$

Now moment of inertia of whole section about its base BC,

$$= 6.075 \times 10^6 - 1.26 \times 10^6 = 4.815 \times 10^6 \text{ mm}^4$$

Solution : 54

As the section is symmetrical about Y-Y axis, therefore C.G. of the section will lie on this axis. Let bottom of the section be the axis of reference.

(i) Semicircle,
$$a_1 = \frac{\pi r^2}{2} = \frac{\pi(6)^2}{2} = 56.55 \text{ cm}^2$$
$$y_1 = \frac{4r}{3\pi} = \frac{4\times 6}{3\pi} = 2.55 \text{ cm}$$

(ii) Rectangular hole,
$$a_2 = 4\times 2 = 8 \text{ cm}^2$$
$$y_2 = \frac{2}{2} = 1 \text{ cm}$$

We know that distance between C.G of the section and its bottom,
$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2} = \frac{(56.55\times 2.55)-(8\times 1)}{56.55-8} = 2.8 \text{ cm}$$

Moment of inertia about horizontal centroidal axis

We know that M.I. of a semicircular section about an axis passing through its C.G. and parallel to horizontal axis,
$$I_{G1} = \frac{\pi}{8}r^4 - \frac{\pi}{2}\times r^2\times\left(\frac{4r}{3\pi}\right)^2 = 0.11\, r^4 = 0.11(6)^4 = 142.6 \text{ cm}^4$$

and distance between the C.G. of the semicircular section and horizontal centroidal axis,
$$h_1 = 2.8 - 2.55 = 0.25 \text{ cm}$$

∴ M.I. of the semicircular section about horizontal centroidal axis
$$= I_{G1} + a_1\cdot h_1^2$$
$$= 142.6 + [56.55\,(0.25)^2] = 146.1 \text{ cm}^4$$

Similarly, M.I. of the rectangular section about an axis passing through its C.G and parallel to horizontal axis,
$$I_{G2} = \frac{bd^3}{12} = \frac{4(2)^3}{12} = 2.67 \text{ cm}^4$$

and distance between C.G. of the rectangular section and horizontal axis,
$$h_2 = 2.8 - 1 = 1.8 \text{ cm}$$

∴ M.I. of the rectangular section about horizontal centroidal axis,
$$= I_{G2} + a_2\cdot h_2^2$$
$$= 2.67 + [8(1.8)^2] = 28.59 \text{ cm}^4$$

Now moment of inertia of the whole section about the horizontal centroidal axis,
$$I_{XX} = 146.1 - 28.59 = 117.51 \text{ cm}^4$$

Moment of inertia about vertical centroidal axis

We know that moment of inertia of the beam section about vertical centroidal axis,
$$I_{YY} = \frac{\pi}{8}(r)^4 - \frac{db^3}{12} = \frac{\pi}{8}(6)^4 - \frac{2(4)^3}{12}\text{cm}^4$$
$$= 498.3 \text{ cm}^4$$

■■■■

2 Equilibrium of Rigid Bodies

LEVEL 1 Objective Questions

1.	(a)
2.	(a)
3.	(b)
4.	(c)
5.	(b)
6.	(d)
7.	(a)
8.	(c)
9.	(340)
10.	(a)
11.	(500)
12.	(87.5)
13.	(a)

LEVEL 2 Objective Questions

14.	(c)
15.	(481)
16.	(70.71)
17.	(755.51)
18.	(1911)
19.	(c)
20.	(c)
21.	(c)
22.	(b)
23.	(b)

■■■■

LEVEL 3 Conventional Questions

Solution : 24

Consider plane *CBDA*,

$$CB = CA = \text{length of guy wires.}$$

CD is altitude of triangle *CBDA*, where

$$CD = \sqrt{CO^2 + OD^2} = \sqrt{20^2 + 2^2}$$

$$\sqrt{404}\ \text{m} = 20.1\ \text{m}$$

Figure (a) shows plane *CBDA*, in which

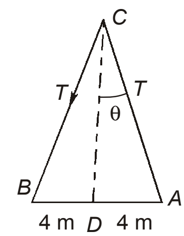
$$BD = DA = 4\ \text{m}$$

$$CD = 20.1\ \text{m}$$

$$\tan\theta = \frac{4}{20.1} = 0.199$$

$$\theta = 11.255^\circ$$

$$\cos\theta = 0.980$$



Plane *CBDA*

Fig. (a)

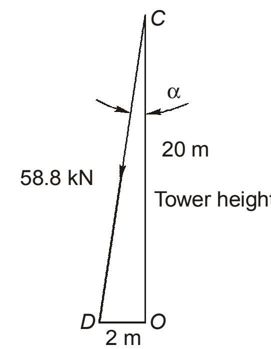


Fig. (b)

Component of tensions along *CD* is added,

$$T_{CD} = 2 \times T \times \cos\theta = 2 \times 30 \times 0.980 = 58.8\ \text{kN}$$

Now let us consider triangle *CDO*, in which

$$\tan\alpha = \frac{2}{20} = 0.1$$

$$\alpha = 5.71^\circ$$

$$\sin\alpha = 0.0995$$

Component of 58.8 kN along horizontal direction

$$= 58.8 \times \sin\alpha = 5.85\ \text{kN}$$

A horizontal force of 5.85 kN can be applied at point *C* of tower.

Solution : 25

Same cable *BD* passes over pulley and a load 500 N is applied at end of cable.

Tension, $T_3 = 500\ \text{N} = $ Tension in part *DE* of same cable.

Equilibrium at point B

$$T_3 \sin 30^\circ = T_2 \sin 45^\circ$$

$$500 \times 0.5 = T_2 \times 0.707$$

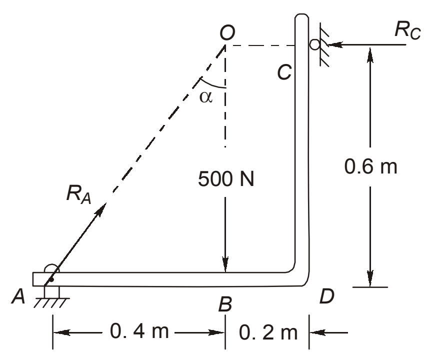
$$\text{Tension, } T_2 = 353.6\ \text{N}$$

$$\text{Tension, } T_1 = T_3 \cos 30^\circ + T_2 \cos 45^\circ$$

$$= 500 \times \cos 30° + 353.6 \cos 45°$$
$$= 500 \times 0.866 + 353.6 \times 0.707$$
$$= 433 + 250 = 683 \text{ N.}$$

Solution : 26

Corner plate is subjected to a force of 500 N at point *B*, reactions at supports *C* and *A* will be developed to maintain equilibrium, *C* is a roller support, therefore reaction at *C* will be perpendicular to vertical surface of support *i.e.*, reaction R_C will be in horizontal direction. Line of application of reaction R_C and force 500 N meet at point *O*, 0.2 m away from *C*. These are concurrent forces, therefore line of action of reaction R_A will pass through O. Therefore



$$\tan\alpha = \frac{0.4}{0.6} = 0.667; \quad \alpha = 33.70°$$

Now, $$R_A \times \sin\alpha = R_C$$
$$R_A \cos\alpha = 500 \text{ N}$$
$$R_A = \frac{500}{\cos 33.7°} = \frac{500}{0.832} = 601 \text{ N}$$

As shown in the figure

$$\text{Reaction, } R_C = 601 \times \sin\alpha = 601 \times \sin 33.7°$$
$$= 601 \times 0.5548 = 333.5 \text{ N.}$$

Solution : 27

The beam is to remain in equilibrium and in horizontal position, net force and moment will be zero on the beam.

Load *W* is carried by two parts of the cable on pulley *C*, so tension in the cable is $\frac{W}{2}$ as shown.

Taking moments about *O*, $$Px = \frac{W}{2} \cdot L$$
$$x = \frac{WL}{2P} \quad ...(1)$$

Reaction at end *O*, $$R_0 = \left(P - \frac{W}{2}\right) \uparrow$$

Solution : 28

At the centre of the wheel there are 3 forces acting on the wheel *i.e.*, (*i*) Weight $W\downarrow$, (*ii*) Reaction R_B, (*iii*) Force P inclined at an angle α with radius OB of wheel.

When the wheel is to turn about the edge B, *reaction at contact point A will become zero*.

Taking moments about edge B

$$W \times DB - P \times BC = 0$$

or
$$P = W \times \frac{DB}{BC}$$

$$\cos\theta = \frac{40-10}{40} = 0.75$$

$$\theta = 41.41^\circ$$

$$\sin\theta = 0.66$$

$$DB = R\sin\theta = 40 \times 0.66 = 26.4 \text{ cm}$$

$$BC = R\sin\alpha$$

$$\text{Force, } P = \frac{W \times DB}{R\sin\alpha} = \frac{1000 \times 26.4}{40\sin\alpha} = \frac{660}{\sin\alpha}$$

P will be minimum when $\sin\alpha$ becomes maximum i.e., when

$$\alpha = 90^\circ$$

$$P_{\min} = 660 \text{ N}$$

Direction of $P_{\min}$ is shown in the figure. (direction of P_{min} is perpendicular to OB)

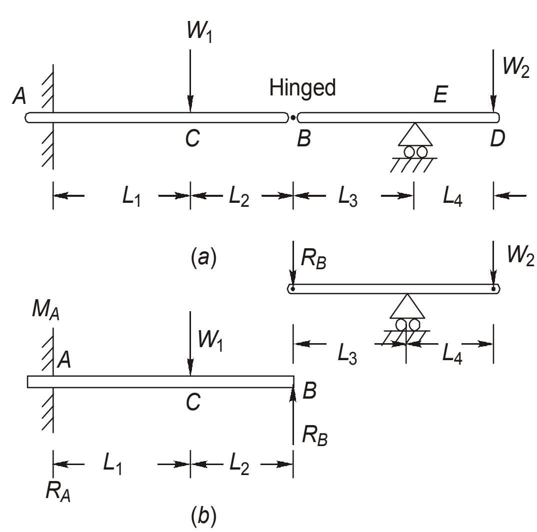
Solution : 29

Consider beam BED, roller supported at E, taking moments about E,

$$W_2 \, . \, L_4 = R_B \times L_3$$

or
$$\text{Reaction, } R_B = \frac{W_2 L_4}{L_3} \downarrow \qquad \text{...(1)}$$

We have not applied any force at B, reaction $R_B\downarrow$ has to be balanced by $R_B\uparrow$ for beam ACB.



Taking moments about A

$$M_A = R_B(L_1 + L_2) - W_1(L_1)$$

$$= +\frac{W_2 L_4}{L_3}(L_1 + L_2) - W_1 L_1 \quad \text{...(2)}$$

To balance M_A, fixed support will have fixing couple

$$M_A = -\frac{W_2 L_4}{L_3}(L_1 + L_2) + W_1 L_1$$

$$\text{Reaction, } R_A = W_1 - R_B = \left(W_1 - \frac{W_2 L_4}{L_3}\right)\uparrow$$

Solution : 30

Force in string BC is P, Note that ABC is an isosceles triangle because $AB = AC = 2$ m each.

$$\angle ACB = 90 - \frac{\theta}{2} = 90 - 20 = 70°$$

Because, $\theta = 40°$

Angle of BC with the vertical is $\frac{\theta}{2} = 20°$

So, Angle $\angle CBD = 180 - 20 = 160°$

$$\angle ABD = 180 - \angle ABO$$

$$\angle ABO = \angle ABC - \frac{\theta}{2} = \frac{180 - 40}{2} - 20 = 50°$$

$$\angle ABD = 180 - 50 = 130°$$

Using Lami's theorem

$$\frac{W}{\sin 70°} = \frac{P}{\sin 130°}$$

$$P = \frac{W \sin 130°}{\sin 70°} = W \times \frac{0.766}{0.9396} = 0.8152 \text{ W}$$

$$W = 300 \times 9.81 = 2943 \text{ N}$$

So, $P = 0.8152 \times 2943 = 2399 \text{ N} = 2.399 \text{ kN}.$

Solution : 31

Taking moments at end A, (R_{BH} passes through A)

$$W \times \frac{R}{2} = R_{BV} \times 2R$$

$$R_{BV} = \frac{W}{4}$$

$$R_{AV} = W - \frac{W}{4} = \frac{3W}{4} \quad \text{(for equilibrium)}$$

Consider portion *ACD* only,

$$\sum M_D = 0, \text{ moments about } D$$

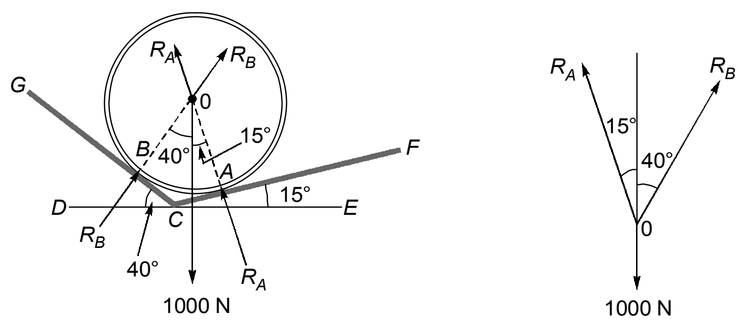
$$R_{AH} \times R + \frac{WR}{2} = \frac{3}{4}WR$$

$$R_{AH} = 0.25\, W(\rightarrow)$$

$$R_{BH} = \frac{W}{4}(\rightarrow) \text{ (for equilibrium)}$$

Solution : 32

Given, Weight of cylinder = 1000 N



(a) **(b)**

Let, R_A = Reaction at *A*, and

R_B = Reaction at *B*

The smooth cylinder lying in the groove is shown in figure (a). In order to keep the system in equilibrium, three forces i.e. R_A, R_B and weight of cylinder (1000 N) must pass through the centre of the cylinder. Moreover, as there is no friction, the reactions R_A and R_B must normal to the surfaces as shown in figure (a). The system of forces is shown in figure (b).

Applying Lami's equation, at 0,

$$\frac{R_A}{\sin(180° - 40°)} = \frac{R_B}{\sin(180° - 15°)} = \frac{1000}{\sin(15° + 40°)}$$

or,

$$\frac{R_A}{\sin 40°} = \frac{R_B}{\sin 15°} = \frac{1000}{\sin 55°}$$

∴

$$R_A = \frac{1000 \times \sin 40°}{\sin 55°} = \frac{1000 \times 0.6428}{0.8192} N = 784.7\ N$$

and

$$R_B = \frac{1000 \times \sin 15°}{\sin 55°} = \frac{1000 \times 0.2588}{0.8192} N$$

$$= 315.9\ N$$

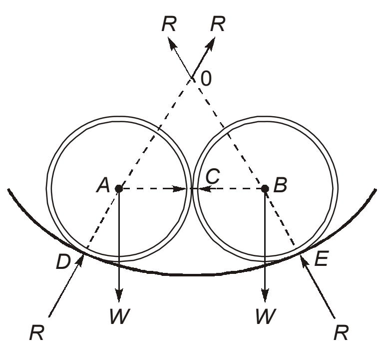
Solution : 33

Given,

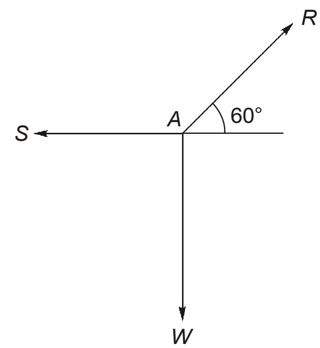
$$\text{Radius of spheres} = 50 \text{ mm}$$

$$\text{Radius of cup} = 150 \text{ mm}$$

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in figure (a). Let the two spheres touch each other at C, and touch the cup at D and E.



(a)



(b)

Let, R = Reaction between the spheres and cup, and

S = Reaction between the two spheres at C.

From the geometry of the figure, we find that,

$$OD = 150 \text{ mm}$$

and $AD = 50$ mm

Therefore, $OA = 100$ mm

Similarly $OB = 100$ mm

We also find that, $AB = 100$ mm

Therefore OAB is an equilateral triangle. The system of forces at A is shown in figure (b).

Applying Lami's equation at A,

$$\frac{R}{\sin 90°} = \frac{W}{\sin 120°} = \frac{S}{\sin 150°}$$

$$\frac{R}{1} = \frac{W}{\sin 60°} = \frac{S}{\sin 30°}$$

$$\therefore \quad R = \frac{S}{\sin 30°} = \frac{S}{0.5} = 2S$$

Hence the reaction between the cup and the sphere is double than that between the two spheres.

Solution : 34

Given,

Diameter of P = 100 mm

Weight of P = 200 N

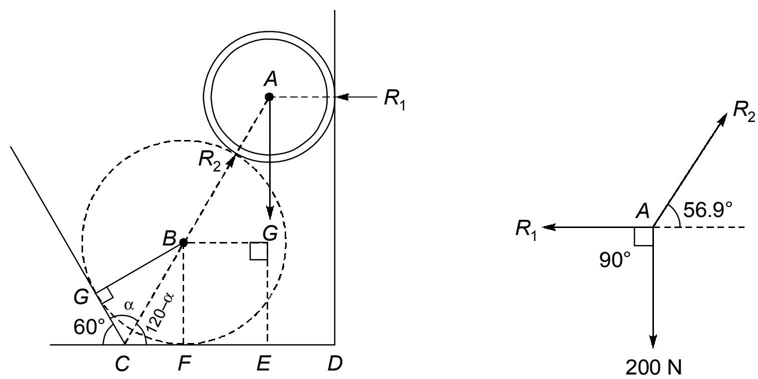
Diameter of Q = 180 mm;

Weight of Q = 500 N,

Width of channel = 180 mm

First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A i.e. the centre of the cylinder P as shown in figure.

1. Weight of the cylinder 200 N acting downwards.
2. Reaction (R_1) of the cylinder at the vertical side.
3. Reaction (R_2) of the cylinder at the point of contact with the cylinder Q.



(a) Free body diagram (b) Force diagram

From ΔBCF and ΔBGC

$$\sin\alpha = \frac{r}{BC}$$

$$\sin(120-\alpha) = \frac{r}{BC}$$

$$\sin\alpha = \sin(120-\alpha)$$

$$\sin\alpha = \sin 120\cdot\cos\alpha - \cos 120\sin\alpha$$

$$\alpha = 60^\circ$$

$$\angle BCF = 60^\circ$$

$\therefore$ $$CF = BF\cot 60^\circ = \left(\frac{180}{2}\right)0.577 = 52\text{ mm}$$

and $$ED = \frac{100}{2} = 50\text{ mm}$$

$\therefore$ $$FE = BG = 180-(52+50) = 78\text{ mm}$$

and $$AB = 50 + 90 = 140\text{ mm}$$

$\therefore \quad \cos\angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$

or $\quad \angle ABG = 56°\,9'$

The system of forces at *A* is shown in figure (b)

Applying Lami's equation at *A*,

$$\frac{R_1}{\sin(90° + 56°9')} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180° - 56°9')}$$

$$\frac{R_1}{\cos 56°9'} = \frac{R_2}{1} = \frac{200}{\sin 56°9'}$$

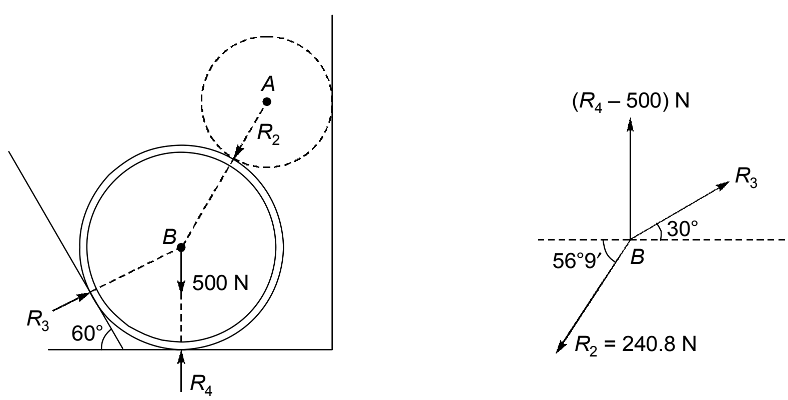
$$\therefore \quad R_1 = \frac{200\cos 56°9'}{\sin 56°9'} = \frac{200 \times 0.5571}{0.8305} N = 134.2\ N$$

and

$$R_2 = \frac{200}{\sin 56°9'} = \frac{200}{0.8305} N = 240.8\ N$$

Now consider the equilibrium of the cylinder *Q*. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in figure (a).

1. Weight of the cylinder 500 N acting downwards.
2. Reaction R_2 equal to 240.8 N of cylinder *P* on the cylinder *Q*.
3. Reaction R_3 of the cylinder *Q* on the inclined surface.
4. Reaction R_4 of the cylinder *Q* on the base of the channel.



(a) Free body diagram (b) Force diagram

A little consideration will show, that the weight of the cylinder *Q* is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$\therefore \quad$ Net upward force $= (R_4 - 500)$ N

The system of force is shown in figure (b).

Applying Lami's equation at point *B*,

$$\frac{R_3}{\sin(90° + 56°9')} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin(180° + 30° - 56°9')}$$

$$\frac{R_3}{\cos 56°9'} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin 26°9'}$$

$\therefore$ $$R_3 = \frac{240.8 \times \cos 56°9'}{\sin 60°} = \frac{240.8 \times 0.5571}{0.866} N = 154.9\ N$$

and $$R_4 - 500 = \frac{240.8 \times \sin 26°9'}{\sin 60°} = \frac{240.8 \times 0.4407}{0.866} N = 122.5\ N$$

$\therefore$ $$R_4 = 122.5 + 500 = 622.5\ N$$

■■■■

3 Kinematics of Point Mass and Rigid Bodies

LEVEL 1 Objective Questions

1.	(c)
2.	(b)
3.	(b)
4.	(c)
5.	(c)
6.	(a)
7.	(21.079)
8.	(14)
9.	(d)
10.	(680)
11.	(16)
12.	(a)
13.	(c)
14.	(b)
15.	(5)

LEVEL 2 Objective Questions

16.	(12)
17.	(a)
18.	(3.34)
19.	(b)
20.	(c)
21.	(c)
22.	(c)
23.	(a)
24.	(c)
25.	(72.11)
26.	(c)
27.	(10)
28.	(1400)
29.	(a)
30.	(c)
31.	(b)
32.	(a)
33.	(c)
34.	(c)
35.	(b)
36.	(b)
37.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 38

Say the time taken by body A to reach the point X, is t, then the time taken by body B will be $(t-4)$ seconds to reach the same destination.

$$a_A = 3 \text{ m/s}^2,\quad u_A = 0$$
$$a_B = 4 \text{ m/s}^2,\quad u_B = 0$$

So,
$$S = \frac{1}{2}a_A t^2 = \frac{3}{2}t^2 = 1.5t^2 \qquad \text{...(1)}$$
$$S = \frac{1}{2}(a_B)(t-4)^2 = \frac{1}{2}\times 4(t-4)^2 = 2(t-4)^2 = 2t^2 - 16t + 32 \qquad \text{...(2)}$$

From Equations (1) and (2),
$$1.5t^2 = 2t^2 - 16t + 32$$
or
$$0.5t^2 - 16t + 32 = 0$$
or
$$t^2 - 32t + 64 = 0$$
$$t = \frac{32+\sqrt{32^2 - 4\times 64}}{2} = \frac{32\pm\sqrt{1024-256}}{2}$$
$$= \frac{32\pm\sqrt{768}}{2} = \frac{32-27.713}{2} = 2.14 \text{ s} < 4 \text{ s}, \text{ not admissible}$$
$$t = \frac{32+27.713}{2} = 29.856 \text{ second}$$

Distance,
$$S = 1.5t^2 = 1.5\times 29.856^2 = 1337.1 \text{ m}$$

Solution : 39

$$x^2 = 200y$$
or
$$y = 0.005x^2 \qquad \text{...(1)}$$
or
$$\frac{dy}{dx} = 0.005\times 2x = 0.01x$$
$$\frac{d^2y}{dx^2} = 0.01$$

At point C,
$$\frac{dy}{dx} = 0 \text{ (slope is zero)}$$

If R is radius of curvature, then
$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1.5}}{\frac{d^2y}{dx^2}} = \frac{1}{\frac{d^2y}{dx^2}}$$

or Radius of curvature, $R = \dfrac{1}{0.01} = 100$ m

Constant velocity, $v = 20$ m/s

Normal acceleration,
$$a_n = \frac{v^2}{R} = \frac{20^2}{100} = 4 \text{ m/s}^2$$

Solution : 40

Figure shows the variation of angular velocity with respect to time as given in the problem.

Say maximum angular velocity $= \omega$ rad/s

Average velocity during acceleration $= \dfrac{\omega}{2}$

$$\text{Total angle travelled} = \frac{\pi}{2} \text{ radian}$$

So, $$\frac{\omega}{2}(30) + \omega\,(50) + \frac{\omega}{2} \times 20 = \frac{\pi}{2}$$

or $$75\omega = \frac{\pi}{2}$$

Maximum angular velocity, $$\omega = \frac{\pi}{150} = 2.09 \times 10^{-2} \text{ rad/s}$$

During Acceleration for 30 seconds (Initial velocity is zero)

Say $$\alpha_1 = \text{angular acceleration}$$

$$\omega = 0 + 30 \times \alpha_1$$

or $$\alpha_1 = \frac{\omega}{30} = \frac{2.09 \times 10^{-2}}{30} = 6.98 \times 10^{-4} \text{ rad/s}^2$$

During Deceleration for 20 seconds (Final velocity is zero)

$$\alpha_2 = \text{angular deceleration}$$

$$0 = \omega - \alpha_2 \times 20$$

$$\alpha_2 = \frac{2.09 \times 10^{-2}}{20} = 10.45 \times 10^{-4} \text{ rad/s}^2.$$

Solution : 41

Say initial velocity $= \omega_0$

Velocity after 8 seconds, $\omega_8 = \omega_0 + 8\alpha = 6$ rad/s

or $$\omega_0 = \omega_8 - 8\alpha, \text{ where } \alpha \text{ is the angular acceleration}$$

$$\omega_0 = (6 - 8\alpha)$$

Moreover, $$(\omega_{12}^2 - \omega_0^2) = 2.\alpha(\theta - \theta_0)$$

But $$\theta - \theta_0 = 80 \text{ radians}$$

$$\omega_{12}^2 - \omega_0^2 = 2.\alpha\,(80) = 160\alpha \quad \text{...(1)}$$

$$\omega_{12} = \omega_0 + 12\alpha, \text{ putting the value of } \omega_0$$

$$= 6 - 8\alpha + 12\alpha = 6 + 4\alpha$$

So $\omega_{12} = 6 + 4\alpha, \quad \omega_0 = 6 - 8\alpha$

Putting these values in Equation (1)

$$(6+4\alpha)^2 - (6-8\alpha)^2 = 160\alpha$$

$$36 + 48\alpha + 16\alpha^2 - (36 - 96\alpha + 64\alpha^2) = 160\alpha$$

$$144\alpha - 48\alpha^2 = 160\alpha$$

or $$-48\alpha^2 = 16\alpha$$

Angular acceleration, $\alpha = -0.333 \text{ rad/s}^2$

$$\omega_0 = 6 - 8\alpha = 6 - 8(-0.333) = 8.664 \text{ rad/s}$$

$$\alpha = -0.333 \text{ rad/s}^2$$

$$\omega_f = \text{final velocity} = 0$$

$$\omega_0 = 8.664 \text{ rad/s}$$

$$\omega_f = \omega_0 - \alpha.t$$

$$0 = 8.664 - 0.333 \,.\, t$$

Time, $t = 26$ seconds, *i.e.*, in 26 seconds shaft will come to rest

Angular displacement till the shaft comes to rest

$$\theta = \omega_0 t - \frac{1}{2}\alpha.t^2$$

$$= 8.664 \times 26 - \frac{1}{2} \times 0.333 \times 26^2$$

$$= 225.264 - 112.666 = 112.66 \text{ radians}$$

$$\theta = \omega_{av}.t$$

$$= \frac{1}{2} \times 8.664 \times 26 = 112.64 \text{ radians}$$

Solution : 42

$$\theta = \theta_0 \cos\omega t$$

Angular velocity, $\dot{\theta} = -\theta_0\, \omega \sin\omega t$ ...(1)

Time period, $T = \frac{1}{2}$ second, as the disc makes 2 oscillations per second

$$T = \frac{2\pi}{\omega} = \frac{1}{2}$$

Therefore, $\omega = 4\pi$ rad/s

Maximum angular velocity, $\dot{\theta} = \theta_0 . \omega = \theta_0 \times 4\pi = 3\pi$ rad/s (as given)

Therefore, $\theta_0 = 0.75$ rad

Angular velocity, $\dot{\theta} = -0.75 \times 4\pi \sin 4\pi t = -3\pi \sin 4\pi t$ ...(2)

Angular acceleration $\ddot{\theta} = -\theta_0 \omega^2 \cos\omega t$ ...(3)

$$\ddot{\theta} = -0.75 \times (4\pi)^2 \cos 4\pi t = -12\pi^2 \cos 4\pi t$$

Solution : 43

Say force $P = kt^2$

where k is a constant and t is in seconds

At $t = 6$ s, $P = 54$ N

$$54 = k \times 6^2$$

or constant, $k = \frac{54}{36} = 1.5$

So, $P = 1.5t^2$ ...(1)

Also, $P = 15 \times a$,

where 15 kg is mass and a is acceleration in m/s^2

$$1.5t^2 = 15a$$

or $a = 0.1t^2$

$$\frac{dV}{dt} = 0.1t^2$$

$$dV = 0.1t^2\, dt \qquad \text{...(2)}$$

Integrating both sides of Equation (2)

$$V = \left|\frac{0.1t^3}{3}\right|_0^6$$

$$V = \frac{0.1 \times 6^3}{3} = 7.2 \text{ m/s at } t = 6 \text{ s}$$

Again $V = \frac{0.1t^3}{3} = \frac{t^3}{30}$

or $\frac{dS}{dt} = \frac{t^3}{30}$

$$dS = \frac{t^3}{30}dt \qquad \text{...(3)}$$

Integrating Equation (3)

$$S = \left|\frac{t^4}{120}\right|_0^6, \text{ as graph is only upto 6 seconds}$$

$$S = \frac{6^4}{120} = 10.8 \text{ m}$$

Solution : 44

$$a = 3\sqrt{v}$$

or $$\frac{dv}{dt} = 3\sqrt{v}$$

or $$\frac{dv}{\sqrt{v}} = 3\,dt$$

$$\int \frac{dv}{\sqrt{v}} = 3\,dt$$

$$2\sqrt{v} = 3t + C_1$$ where C_1 is constant of integration

at $t = 2\text{s}$, $v = 6$ m/s,

So, $$2\sqrt{6} = 3 \times 2 + C_1$$

or constant, $C_1 = 4.899 - 6 = -1.101$

So, $$2\sqrt{v} = 3t - 1.101 \qquad ...(ii)$$

At $t = 4\text{s}$

$$2\sqrt{v_4} = 4 \times 3 - 1.101 = 10.899$$

Velocity, $$v_4 = \left(\frac{10.899}{2}\right)^2 = 29.697 \text{ m/s}$$

or $$\sqrt{v} = 1.5\,t - 0.5505$$

$$v = 2.25\,t^2 + 0.303 - 1.6515\,t$$

$$\frac{ds}{dt} = 2.25\,t^2 - 1.6515t + 0.303$$

$$S = \frac{2.25t^3}{3} - \frac{1.6515t^2}{2} + 0.303t + C_2$$

where, C_2 is another constant of integration.

At $t = 2s$, $s = 8$ m

So, $$S = 0.75 \times 2^3 - \frac{1.6515}{2} \times 4 + 0.303 \times 2 + C_2$$

$$S = 6 - 3.303 + 0.606 + C_2$$

$$S = 3.303 + C_2$$

$$C_2 = +4.697$$

or $$S = 0.75t^3 - \frac{1.6515}{2}t^2 + 0.303t + 4.697$$

$$t = 4\text{s}$$

$$S_4 = 0.75 \times 64 - \frac{1.6515}{2} \times 16 + 0.303 \times 4 + 4.697$$

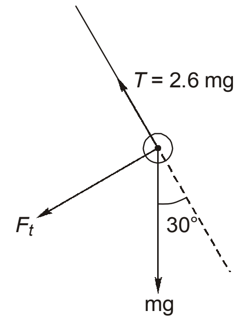
$$= 48 - 13.212 + 1.212 + 4.697 = 40.697 \text{ m}$$

Acceleration at $t = 4$ s

$$a_4 = 3\sqrt{v_4} = 3\sqrt{29.697} = 16.348 \text{ m/s}^2$$

Solution : 45

Figure shows the action of forces $T = 2.6$ mg and mg (weight of bob) in the position shown.



$$\text{Tangential force, } F_t = \text{mg}\sin 30^\circ$$
$$= 0.5\text{ mg} \quad \ldots(1)$$

Normal force towards the centre O

$$F_n = T - \text{mg}\cos 30^\circ$$
$$= 2.6\text{ mg} - 0.866\text{ mg}$$
$$= 1.734\text{ mg} \quad \ldots(2)$$

Tangential acceleration, a_t

$$m\,.\,a_t = 0.5m\text{g}$$
$$a_t = 0.5\text{ g} = 0.5 \times 9.81 = 4.905\text{ m/s}^2$$

Normal acceleration, a_n

$$m\,.\,a_n = 1.734\text{ mg}$$
$$a_n = 1.734 \times g = 1.734 \times 9.81 = 17.01\text{ m/s}^2$$

Now normal acceleration,

$$a_n = \frac{V^2}{R},$$

Where $R = 1.0$ m, length of cord of pendulum

$$V^2 = a_n \times R = 17.01 \times 1.0 = 17.01$$

Velocity, $V = 4.124$ m/s

$$\text{Acceleration } a = \sqrt{a_n^2 + a_t^2} = \sqrt{17.01^2 + 4.905^2} = 17.70\text{ m/s}^2$$

Solution : 46

Given: $s = 18t + 3t^2 - 2t^3$ ...(i)

(i) velocity and acceleration at start

Differentiating both sides equation (i) with respect to t,

$$\frac{ds}{dt} = 18 + 6t - 6t^2$$

i.e. Velocity, $v = 18 + 6t - 6t^2$ ...(ii)

Substituting, $t = 0$ in equation (iii),

$$v = 18 + 0 - 0 = 18\text{ m/s}$$

Differentiating both sides of equation (ii) with respect to t,

$$\frac{dv}{dt} = 6 - 12t$$

or Acceleration, $a = 6 - 12t$ ...(iii)

Now, substituting, $t = 0$ in equation (iii),

$$a = 6 \text{ m/s}^2$$

(ii) Time when the particle reaches maximum speed

For maximum speed, differentiating the relation for speed (ii) with respect to t i.e. equation (iii) and equating (iii) to zero.

$$6 - 12t = 0$$

or $$t = \frac{6}{12} = 0.5 \text{ s}$$

(iii) Maximum speed of the particle

The maximum speed of the particle may now be found out by substituting $t = 0.5$ second in equation (ii),

$$v_{max} = 18 + 6 \times 0.5 - 6\,(0.5)^2 = 19.5 \text{ m/s}$$

Solution : 47

Given, $v = 4.2$ m/s

Weight of each man, $w = 80$ kg-wt (in gravitational units),

or $m = 80$ kg (in absolute units),

Weight of boat $= 400$ kg-wt (in gravitational units)

or $M = 400$ kg (in absolute units)

Let $V =$ Velocity of the boat after the second man dives off the boat

A little consideration will show, that when the first man dives off the boat, it will give some momentum to the boat as well as the second man (who is still standing on the boat). When the second man also dives off the boat, it will also give some momentum to the boat. Therefore the total momentum gained by the boat is equal to the momentum given by the first man plus the momentum given by the second man to the boat.

Now final momentum of the boat=400 V kg m/s ...(i)

and momentum given by the first man to the boat

$$= 80 \times 4.2 = 336 \text{ kg-m/s}$$

Similarly, momentum given by the second man to the boat = 336 kgm/s

Total momentum gain = Momentum given by the 1st man + Momentum given by the 2nd man

$= 336 + 336 = 672$ kg-m/s ...(ii)

Now equating (i) and (ii),

By momentum conservation, $400\,V = 672$

$\therefore$ $$V = \frac{672}{400} = 1.68 \text{ m/s}$$

Solution : 48

Given, $m_A = 4$ kg

$m_B = 5$ kg

$m_C = 15$ kg

$\mu = 0.4$

(i) Acceleration of the masses

Let, a = Acceleration of the masses.

T_A = Tension in the string connected with body A

and T_B = Tension in the string connected with body B

We know that normal reaction on the horizontal surface due to body A,

$$R_A = m_A \cdot g = 4 \times 9.8 = 39.2 \text{ N}$$

$\therefore$ Frictional force, $F_A = \mu \cdot R_A = 0.4 \times 39.2 = 15.68$ N

Similarly, normal reaction on the inclined surface due to body B,

$$R_B = m_B \cdot g \cos\alpha = 5 \times 9.8 \cos 30° \text{ N}$$

$$= 49 \times 0.866 = 42.43 \text{ N}$$

$\therefore$ Frictional force, $F_B = \mu \cdot R_B = 0.4 \times 42.43 = 16.97$ N

First of all, consider the motion of body A which is moving horizontally. We know that forces acting on it are T_A Newtons (toward left) and frictional force of 15.68 Newtons (towards right). As the body is moving towards left, therefore resultant force

$$= T_A - 15.68 \text{ N} \quad \text{...(i)}$$

Since the body A is moving with an acceleration of (a) therefore force acting on it

$$= 4a \text{ N} \quad \text{...(ii)}$$

Equating the equations (i) and (ii),

$$T_A - 15.68 = 4a \quad \text{...(iii)}$$

or $$T_A = 4a + 15.68 \quad \text{..(iv)}$$

Now consider the motion of the body B, which is moving downwards on inclined surface. We know that the forces acting on it, along the plane, are T_B Newtons (downwards), $m_B\, g \sin\alpha$ (again downwards), frictional force equal to 16.97 Newtons (upwards, as the block is moving downwards). Therefore resultant force

$$= T_B + m_B\, g \sin\alpha - 16.97 \text{ N}$$

$$= T_B + 5 \times 9.8 \sin 30° - 16.67 \text{ N}$$

$$= T_B + 49 \times 0.5 - 16.97 = T_B + 7.53 \text{ N} \quad \text{..(v)}$$

Since the body is moving with an acceleration of (a), therefore force acting on it

$$= 5\,a \text{ N} \quad \text{...(vi)}$$

Equating the equations (v) and (vii),

$$T_B + 7.53 = 5a$$

or $$T_B = 5a - 7.53$$

Now consider the motion of the body C, which is coming down. We know that forces acting on it are $m_C \cdot g = 15 \times 9.8 = 147$ Newtons (downwards) and $(T_A + T_B)$ Newtons upwards. As the body is moving downwards, therefore resultant force

$$= 147 - (T_A + T_B)\text{ N}$$

Since the body is moving with an acceleration (a), there force acting on it

$$= 15a\text{ N}$$

Equating the equations, (viii) and (ix),

$$147 - (T_A + T_B) = 15a$$

Substituting the values of T_A and T_B from equation (iv) and (vii)

$$147 - [(4a + 15.68) + (5a - 7.53)] = 15a$$

$$147 - 4a - 15.68 - 5a + 7.53 = 15a$$

$$\therefore \quad 24a = 138.85$$

$$a = \frac{138.85}{24} = 5.8\text{ m/s}^2$$

(ii) Tensions in the two strings

Substituting the value of (a) in equation (iv),

$$T_A = 4a + 15.68 = (4 \times 5.8) + 15.68 = 38.88\text{ N}$$

Again substituting the values of (a) in equation (vii),

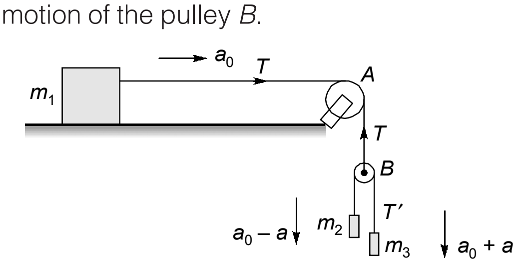
$$T_B = 5a - 7.53 = (5 \times 5.8) - 7.53 = 21.47\text{ N}$$

Solution : 49

Suppose the acceleration of m_1 is a_0 towards right. That will also be the downward acceleration of the pulley B because the string connecting m_1 and B is constant in length. Also the string connecting m_2 and m_3 has a constant length. This implies that the decrease in the separation between m_2 and B equals the increase in the separation between m_3 and B. So, the upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be a.

The acceleration of m_2 with respect to the ground $= a_0 - a$ (downward) and the acceleration of m_3 with respect to the ground $= a_0 + a$ (downward).

These accelerations will be used in Newton's laws. Let the tension be T in the upper strings and T' in the lower string. Consider the motion of the pulley B.



The forces on this light pulley are

(a) T upwards by the upper string and

(b) $2T'$ downwards by the lower string.

As the mass of the pulley is negligible,

$$2T' - T = 0$$

Giving, $$T' = \frac{T}{2} \qquad \text{...(i)}$$

Motion of m_1 :

The acceleration is a_0 in the horizontal direction. The forces on m_1 are

(a) T by the string (horizontal)

(b) $m_1 g$ by the earth (vertically downwards) and

(c) N by the table (vertically upwards)

In the horizontal direction, the equation is

$$T = m_1 a_0 \qquad \text{...(ii)}$$

Motion of m_2 : acceleration is $a_0 - a$ in the downward direction. The forces on m_2 are

(a) $m_2 g$ downward by the earth and

(b) $T' = \frac{T}{2}$ upward by the string

Thus, $$m_2 g - \frac{T}{2} = m_2 (a_0 - a) \qquad \text{...(iii)}$$

Motion of m_3 : The acceleration is $(a_0 + a)$ downward. The forces on m_3 are

(a) $m_3 g$ downward by the earth and

(b) $T' = \frac{T}{2}$ upward by the string. Thus,

$$m_3 g - \frac{T}{2} = m_3 (a_0 + a) \qquad \text{...(iv)}$$

We want to calculate a_0, so we shall eliminate T and a from (ii), (iii) and (iv).

Putting T from (ii) in (iii) and (iv),

$$a_0 - a = \frac{m_2 g - m_1 a_0 / 2}{m_2} = g - \frac{m_1 a_0}{2 m_2}$$

and $$a_0 + a = \frac{m_3 g - m_1 a_0 / 2}{m_3} = g - \frac{m_1 a_0}{2 m_3}$$

Adding, $$2a_0 = 2g - \frac{m_1 a_0}{2}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0 = g - \frac{m_1 a_0}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right] = g$$

or, $$a_0 = \frac{g}{1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)}$$

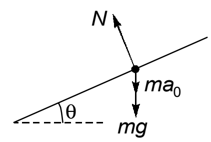
Solution : 50

Let us work in the elevator frame. A pseudo force ma_0 in the downward direction is to be applied on the particle of mass m together with the real forces. Thus, the forces of m are.

(i) N normal force,

(ii) mg downward (by the earth),

(iii) ma_0 downward (pseudo)



Let a be the acceleration of the particle with respect to the incline. Taking components of the forces parallel to the incline and applying Newton's law,

$$m\,g \sin\theta + m\,a_0 \sin\theta = m\,a$$

or $$a = (g + a_0)\sin\theta$$

This is the acceleration with respect to the elevator. In this frame, the distance travelled by the particle is $\frac{L}{\cos\theta}$. Hence,

$$\frac{L}{\cos\theta} = \frac{1}{2}(g + a_0)\sin\theta . t^2$$

or, $$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}$$

■■■■

4 Dynamics of Rigid Bodies, Momentum, Collision

LEVEL 1 Objective Questions

1.	(a)
2.	(c)
3.	(c)
4.	(b)
5.	(b)
6.	(a)
7.	(c)
8.	(b)
9.	(a)
10.	(d)
11.	(a)
12.	(b)
13.	(c)
14.	(c)

LEVEL 2 Objective Questions

15.	(b)
16.	(c)
17.	(a)
18.	(a)
19.	(1)
20.	(4)
21.	(b)
22.	(b)
23.	(a)
24.	(b)
25.	(a)
26.	(c)
27.	(a)
28.	(b)
29.	(a)
30.	(c)
31.	(b)

LEVEL 3 Conventional Questions

Solution : 32

(a) When the block tends to slide off the cart.

Due to friction between block and cart, the block moves alongwith the cart at an acceleration $\bar{a}$ m/s^2.

Inertia force $m\bar{a}$ at G of block will act in opposite direction. Block will slide off the cart if

$$m\bar{a} > \mu mg$$

or
$$\bar{a} > \mu g$$

$$\bar{a} > 0.35\,g$$

(b) When the block tends to tip about an edge.

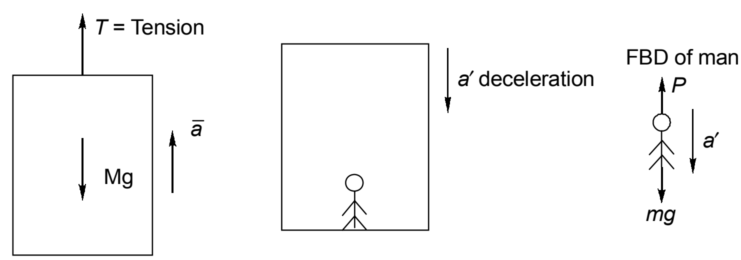
For the block to tip about an edge D as shown in figure

$$m\bar{a} \times 1\text{ m} > mg \times 0.3$$

$$\bar{a} > 0.3\text{ g}$$

As the dimension of the block are such that the block *will first tend to tip* about an edge than to slide off the cart.

Solution : 33



(a) (b)

(a) Initial velocity $= 0$ m/s

Final velocity, $V = 3$ m/s

Distance, $S = 3$ m

$$V^2 = 2\bar{a} \times S$$

$$\text{Acceleration, } \bar{a} = \frac{3^2}{2 \times 3} = 1.5\text{ m/s}^2$$

Cable Tension, $T = M.g + M\bar{a}$, where M is mass of elevator and Mg is weight of elevator

$$T = 4800 + \frac{4800}{9.81} \times 1.5 \text{ (figure a)}$$

Cable Tension, $T = 4800 + 733.95 = 5533.95$ N

(b) During stopping the elevator moves up with a deceleration a' m/s^2, as shown in figure (b).

Initial velocity, $V = 3$ m/s

Time, $t = 3$ s

$$\text{Deceleration, } a' = \frac{V}{t} = \frac{3}{3}$$

$$= 1\text{ m/s}^2$$

$$mg - P = ma', \text{ where } m \text{ is mass of man and } P \text{ is pressure exerted by man}$$

$$P = mg - ma' = 65 \times 9.81 - 65 \times 1 = 572.65 \text{ N}$$

(Note that apparent weight of man is reduced).

Solution : 34

Note μ for block B is less than μ for block A, therefore after releasing the masses, the block B will travel faster with more acceleration and block A will move slower with less acceleration.

Block A $\quad R_A = 6g\cos\theta = 6g\cos 30°$

Force of friction, $\quad F_A = 0.2 \times 6 \times g\cos 30° = 10.1945 \text{ N}$

$$\text{Effective force on } A = 6g\sin\theta - F_A$$

$$= 6 \times 9.81 \times \sin 30° - 10.1945 = 19.2355 \text{ N}$$

Acceleration of A, $\quad a_A = \dfrac{19.2355}{6} = 3.206 \text{ m/s}^2$

Block B Reaction, $\quad R_B = 10g\cos 30° = 10 \times 9.81 \times \cos 30° = 84.9546 \text{ N}$

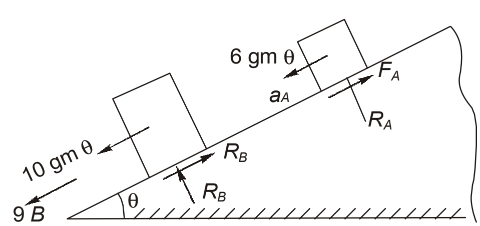
Force of friction $\quad F_B = 0.1 \times 84.9546 = 8.495 \text{ N}$

$$\text{Effective force on block } B = mg\sin\theta - F_B$$

$$= 10 \times 9.81 \times \sin 30° - 8.495 = 40.555 \text{ N}$$

Acceleration of block B, $\quad a_B = \dfrac{40.555}{10} = 4.0555 \text{ m/s}^2$

As shown in FBD diagram.



FBD diagram

Say after time t, distance between block is 6 m

$$S_A = \frac{1}{2} a_A t^2; \qquad S_B = \frac{1}{2} a_B \times t^2$$

$$S_B - S_A = \frac{t^2}{2}(4.0555 - 3.206) = 0.42475t^2 = 6 \text{ m}$$

Time, $t = 3.76$ second.

Solution : 35

$$\text{Velocity of collar, } V = 5 \text{ m/s}$$
$$\text{Time, } t = 2 \text{ s}$$

Acceleration of roller, $a = \dfrac{5}{2} = 2.5 \text{ m/s}^2$

For collar $F - 2T = 5a$ ...(1)

Where T is cable tension.

Note that if acceleration of collar B is 2.5 m/s^2, then acceleration of mass A will be $2.5 \times 2 = 5$ m/s^2 because if collar moves by distance x in horizontal direction, then during the same time, mass A goes up by $2x$ distance.

Mass A $T - 2g = 2 \times a'$

Where a' = acceleration of mass A

$= 5 \text{ m/s}^2$

$$T = 2g + 2a' = 2 \times 9.81 + 2 \times 5 = 29.62 \text{ N}$$

Collar: From equation (1)

$$F = 2T + 5a = 2 \times 29.62 + 5 \times 2.5 = 59.74 + 12.5 = 71.74 \text{ Newton.}$$

Solution : 36

(a) **Rolling without slipping:** Let us take xy co-ordinates along and perpendicular to the inclined plane as shown.

$$\sum F_x = mg\sin\theta - F = m\bar{a} \qquad ...(1)$$

Where $\bar{a}$ is linear acceleration of mass centre and F is the frictional force on cylinder

$\sum F_y = mg\cos\theta - N = 0$, where N is the normal reaction

$\sum M_G = \bar{I}.\alpha$ where I is moment of inertia of cylinder and α is the angular acceleration

$$M_G = F \times R$$

$$\bar{I} = \frac{mR^2}{2}$$

$\alpha = \dfrac{\bar{a}}{R}$ (for rolling without slipping), where $\bar{a}$ = mass centre acceleration

Therefore $F.R = \dfrac{mR^2}{2} \times \alpha = \dfrac{mR^2}{2} \times \dfrac{\bar{a}}{R} = \dfrac{mR\bar{a}}{2}$

or $F = \dfrac{m\bar{a}}{2}$ ...(2)

Putting the value of F in Equation (1)

$$mg\sin\theta - \frac{m\bar{a}}{2} = m\bar{a}$$

or Linear acceleration of mass centre of cylinder,

$$\bar{a} = \frac{2}{3}g\sin\theta$$

Angular acceleration $\alpha = \frac{\bar{a}}{R} = \frac{2}{3}\times\frac{g}{R}\times\sin\theta$ ...(3)

(*b*) Friction force $F = \mu N = \mu\, mg\cos\theta$

Say $\mu\, mg\cos\theta < \frac{m\bar{a}}{2}$, calculated in part (*a*).

Then rolling will occur with some slip and linear acceleration

$$a \neq \alpha.R$$

Rolling and slip will occur simultaneously for the cylinder.

Now $\sum F_x = mg\sin\theta - F = mg\sin\theta - \mu\, mg\cos\theta$

$$= m\bar{a}$$

or linear acceleration of mass centre,

$$\bar{a} = g\sin\theta - \mu g\cos\theta \quad ...(4)$$

$$\sum M_G = \bar{I}\alpha = FR$$

$$\frac{mR^2}{2}\times\alpha = \mu mg\cos\theta\times R$$

$$\alpha = \frac{2\mu g\cos\theta}{R} \quad ...(5)$$

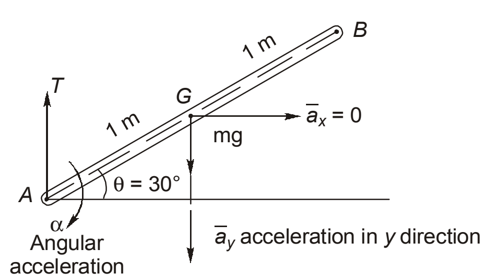
Please note that $\alpha R \neq \bar{a}$, because cylinder rolls down with some occasional slips.

Solution : 37

Say $\bar{a}_y$ = acceleration of bar at mass centre G in y direction

and T = tension in string at end A

Then $mg - T = m\bar{a}_y$ ...(1)



No force in x direction, so $\bar{a}_x = 0$, acceleration of mass centre in x direction.

Moment about G, $M_G = T\times\frac{l}{2}\cos\theta$

Where $l = 2$ m, length of bar

$\theta = 30^\circ$ as shown

Moment $M_G = T\times 1\times\cos 30° = 0.866T$ Nm

$$= I_G \cdot \alpha$$

I_G = moment of inertia of bar about axis of rotation through G

$$\frac{ml^2}{12} = \frac{9 \times 2^2}{12} = 3.00 \text{ kg m}^2$$

α = angular acceleration of the bar

Therefore, $M_G = I\alpha \Rightarrow 0.866T = 3\alpha$

or $T = 3.464\alpha$ ...(2)

Putting in equation (1)

$$mg - 3.464\alpha = m\bar{a}_y, \quad \text{...(3)}$$

Where $\bar{a}_y$ = Acceleration in y direction

$$= \frac{l}{2}\cos\theta.\alpha = 1 \times \cos 30° \times \alpha = 0.866\alpha$$

Putting in Equation (3)

$$mg - 3.464\alpha = m \times 0.866\alpha \quad \text{where } = m = 9 \text{ kg}$$

$$9 \times 9.81 - 3.464\alpha = 9 \times 0.866\alpha$$

or Angular acceleration, $\alpha = \dfrac{88.29}{11.258} = 7.842 \text{ rad/s}^2$

Tension in string, $T = 3.464\alpha = 3.464 \times 7.842 = 27.17$ N.

Solution : 38

Say of mass of cylinder = m

Radius of cylinder = R

Moment of inertia of cylinder,

$$I_{G1} = \frac{mR^2}{2}$$

Hoop

Then, Mass of hoop = m

Radius of hoop = R

Moment of inertia of hoop, $I_{G2} = mR^2$

Say acceleration of mass centre = $\bar{a}_2$

Then $mg\sin\theta - F_2 = m\bar{a}_2$ ...(1)

$$F_2 \times R = I_{G2} \times \alpha_2$$

Where α_2 = Angular acceleration of hoop

or $F_2 = \dfrac{mR^2 \times \alpha_2}{R} = mR\alpha_2$

$$= m\bar{a}_2, \text{ as } \bar{a}_2 = \alpha_2 R_2$$

Putting the value of F_2 in Equation (1)

$$mg\sin\theta - m\bar{a}_2 = m\bar{a}_2$$

or Acceleration, $$\bar{a}_2 = \frac{g\sin\theta}{2} \quad ...(2)$$

Cylinder $$mg\sin\theta - F_1 = m\bar{a}_1 \quad ...(3)$$

Where a_1 is the acceleration of mass centre of cylinder and F_1 is the force of friction at contact point.

Moreover, $F_1 \times R = I_{G1} \times \alpha_1$, where α_1 is the angular acceleration of cylinder

$$= \frac{mR^2}{2} \times \alpha_1$$

or $$F_1 = \frac{mR\alpha_1}{2} \text{ but } R\alpha_1 = \bar{a}_1 = \frac{m\bar{a}_1}{2}$$

Putting the value of F_1 in Equation (3)

$$mg\sin\theta - \frac{m\bar{a}_1}{2} = m\bar{a}_1$$

acceleration $$\bar{a}_1 = \frac{g\sin\theta}{1.5}$$

Now mass centre acceleration of cylinder is more than the mass centre acceleration of hoop

$$\bar{a}_1 = \frac{9.81 \times \sin 20^\circ}{1.5} = 2.237 \text{ m/s}^2$$

$$\bar{a}_2 = \frac{9.81 \times \sin 20^\circ}{2} = 1.68 \text{ m/s}^2$$

Distance covered by hoop and cylinder in 1 second

$$S_1 = \frac{1}{2} \times 2.237 \times 1^2 = 1.1185 \text{ m}$$

$$S_2 = \frac{1}{2} \times 1.68 \times 1^2 = 0.84$$

Gap between the two after 1 second

$$S_1 - S_2 = 1.1185 - 0.84 = 0.2785 \text{ m.}$$

Solution : 39

After the release of the rod from rest, rod will rotate about end A and therefore the point A becomes the instantaneous centre of rotation.

Moment of inertia of the rod about A,

$$I = \frac{ml^2}{3} \quad ...(1)$$

Moment of the force mg of bar about A

$$M = mg\frac{l}{2}\cos\theta = mg\frac{l}{2}\times\cos 75°$$

$$= 0.1294mgl \qquad ...(2)$$

Angular acceleration of the rod about axis through end A

$$\alpha = \frac{M}{I} = 0.1294mgl\times\frac{3}{ml^2} = \frac{0.3882g}{l} \qquad ...(3)$$

$$\text{Acceleration of } G \text{ of rod} = \alpha\cdot\frac{l}{2} = \frac{0.3882g}{l}\times\frac{l}{2} = 0.1941g$$

$$\bar{a} = 0.1941g \qquad ...(4)$$

Inertia force at $G = m\bar{a} = m\times 0.1941g$

Using D' Alembert's principle, inertia force, $m\bar{a}$ is applied in the opposite direction as shown in figure. So as to determine reactions at A.

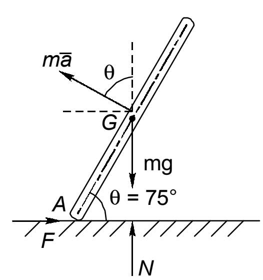
Vertical reaction, $N = mg - m\bar{a}\cos\theta$

$$= mg - m\bar{a}\cos 75° = mg - 0.2588m\bar{a}$$

$$= (mg - 0.2588m\times 0.1941g) \text{ putting the value of } \bar{a}$$

$$= 0.95mg$$

The bar will tend to slip towards left, therefore force of friction F will be towards right as shown in FBD diagram.



FBD diagram

$$F = m\bar{a}\sin\theta$$

$$= m\times 0.1941g\times\sin 75°$$

$$= 0.1941\times 0.9659mg = 0.1875mg$$

$$\text{Coefficient of friction, } \mu = \frac{F}{N} = \frac{0.1875mg}{0.95mg}$$

$$= 0.1974 \text{ (minimum value required)}$$

Solution : 40

$$\text{Mass of bar, } m = 12 \text{ kg}$$

$$\text{Length, } L = 3 \text{ m}$$

I_0 = moment inertia about pivot point, at one end

$$= \frac{mL^2}{3} = \frac{12 \times 3^2}{3} = 36 \text{ kg-m}^2$$

$$\text{Blow, } P = 250 \text{ N}$$

M_0 = moment about pivot $O = P \times 0.75 = 0.75 \times 250 = 187.5$ Nm

$$= I_0 . \alpha = 36 \times \alpha$$

α = angular acceleration $= \dfrac{187.5}{36} = 5.2083$ rad/s^2, angular acceleration

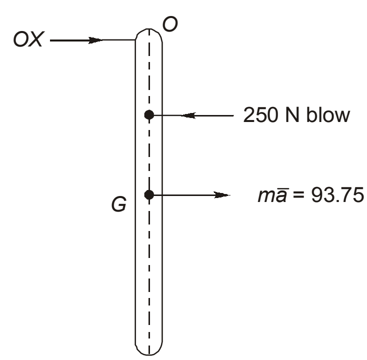
$\bar{a}$ = linear acceleration at mass centre

$$= \frac{L}{2} \times \alpha = 1.5 \times 5.2083 = 7.8125 \text{ m/s}^2$$

$$m\bar{a} = 12 \times 7.8125 = 93.75 \text{ N}$$

Inertia force at, G = 93.75 N in opposite direction

Reaction at O



$$O_X = 250 - 93.75 = \overrightarrow{156.25} \text{ N.}$$

Solution : 41

$m_1 = 1$ kg, $u_1 = 3$ m/s, $e = 0.75$, $m_2 = 5$ kg, $u_2 = 0.6$ m/s

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$1 \times 3 + 5 \times 0.6 = v_1 + 5v_2$$

$$v_1 + 5v_2 = 6 \qquad \text{...(1)}$$

$$e = \frac{V_2 - V_1}{u_1 - u_2} = 0.75$$

$$\frac{V_2 - V_1}{3 - 0.6} = 0.75$$

$$V_2 - V_1 = 1.8 \qquad \text{...(2)}$$

$$5V_2 + V_1 = 6$$

$$V_2 = 1.3 \text{ m/s}$$
$$V_1 = -0.5 \text{ m/s}$$
$$\text{Loss of KE} = \frac{1}{2}[1\times 3^2 + 5\times 0.6^2] - \frac{1}{2}[1\times(-0.5)^2 + 5\,(1.3)^2]$$
$$= 5.4 - 4.35 = 1.05 \text{ Nm.}$$

Solution : 42

The portion of the strings between the ceiling and the cylinder is at rest. Hence the points of the cylinder where the strings leave it are at rest. The cylinder is thus rolling without slipping on the strings. Suppose the centre of the cylinder falls with an acceleration *a*. The angular acceleration of the cylinder about its axis is $\alpha = a/r$, as the cylinder does not slip over the strings.

The equation of motion for the centre of mass of the cylinder is

$$mg - 2T = ma \quad \text{..(i)}$$

and for the motion about the centre of mass, it is

$$2\,Tr = \left(\frac{1}{2}mr^2\alpha\right) = \frac{1}{2}mra$$

or,
$$2T = \frac{1}{2}ma \quad \text{...(ii)}$$

From (i) and (ii),

$$a = \frac{2}{3}g$$

and
$$T = \frac{mg}{6}$$

As the centre of the cylinder starts moving from rest, the velocity after it has fallen through a distance *h* is given by

$$v^2 = 2\left(\frac{2}{3}g\right)h$$

or,
$$v = \sqrt{\frac{4gh}{3}}$$

Solution : 43

$$\text{Velocity of the centre} = v_0$$

and the angular velocity about the centre $= \dfrac{v_0}{2r}$.

Thus,
$$v_0 > \omega_0 r$$

The sphere slips forward and thus the friction by the plane on the sphere will act backward. As the friction is kinetic, its value is $\mu N = \mu Mg$ and the sphere will be decelerated by $a_{cm} = f/M$. Hence,

$$v(t) = v_0 - \frac{f}{M}t \quad \text{...(i)}$$

This friction will also have a torque $\Gamma = f\,r$ about the centre. This torque is clockwise and in the direction of ω_0. Hence the angular acceleration about the centre will be

$$\alpha = f\frac{r}{(2/5)Mr^2} = \frac{5f}{2Mr}$$

and the clockwise angular velocity at time t will be

$$\omega(t) = \omega_0 + \frac{5f}{2Mr}t = \frac{v_0}{2r} + \frac{5f}{2Mr}t$$

Pure rolling starts when $\quad v(t) = r\omega(t)$

i.e., $$v(t) = \frac{v_0}{2} + \frac{5f}{2M}t \quad \text{...(ii)}$$

Eliminating t from (i) and (ii),

$$\frac{5}{2}v(t) + v(t) = \frac{5}{2}v_0 + \frac{v_0}{2}$$

or, $$v(t) = \frac{2}{7}\times 3v_0 = \frac{6}{7}v_0$$

Thus, the sphere rolls with translational velocity $\frac{6v_0}{7}$ in the forward direction.

Alternative: Let us consider the torque about the initial point of contact A. The force of friction passes through this point and hence its torque is zero. The normal force and the weight balance each other. The net torque about A is zero. Hence the angular momentum about A is conserved.

Initial angular momentum is,

$$L = L_{cm} + Mrv_0 = I_{cm}\,\omega + Mrv_0$$

$$= \left(\frac{2}{5}Mr^2\right)\left(\frac{v_0}{2r}\right) + Mrv_0 = \frac{6}{5}Mrv_0$$

Suppose the translational velocity of the sphere, after it start pure rolling is v, The angular velocity is v/r. The angular momentum about A is,

$$L = L_{cm} + Mrv = \left(\frac{2}{5}Mr^2\right)\left(\frac{v}{r}\right) + Mrv = \frac{7}{5}Mrv$$

From equation (iii) and (iv)

Thus, $$\frac{6}{5}Mrv_0 = \frac{7}{5}Mrv$$

or, $$v = \frac{6}{7}v_0$$

Solution : 44

Take the particle plus the sphere as the system.

(a) Using conservation of linear momentum, the linear speed of the combined system v is given by

$$mv_0 = (M + m)\ v$$

or, $$v = \frac{mv_0}{M+m} \quad \text{...(i)}$$

(b) Next, we shall use conservation of angular momentum about the centre of mass, which is to be taken at the centre of the sphere ($M >> m$). Angular momentum of the particle before collision is $mv_0(h - R)$. If the system rotates with angular speed ω after collision, the angular momentum of the system becomes

$$\left(\frac{2}{5}MR^2 + mR^2\right)\omega$$

Hence, $$mv_0(h-R) = \left(\frac{2}{5}M + m\right)R^2\omega$$

or, $$\omega = \frac{mv_0(h-R)}{\left(\frac{2}{5}M+m\right)R^2}$$

(c) The sphere will start rolling just after the collision if

$$v = \omega R;$$

i.e., $$\frac{mv_0}{M+m} = \frac{mv_0(h-R)}{\left(\frac{2}{5}M+m\right)R}$$

Giving, $$h = \left(\frac{\frac{7}{5}M+2m}{M+m}\right)R \approx \frac{7}{5}R$$

■■■■

5 Work and Energy

LEVEL 1 Objective Questions

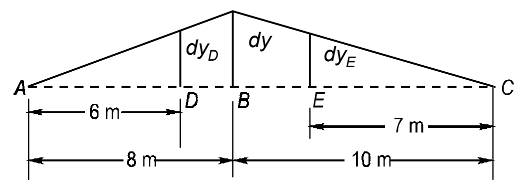
1.	(b)
2.	(a)
3.	(c)
4.	(b)
5.	(c)
6.	(b)
7.	(b)
8.	(b)
9.	(c)
10.	(b)
11.	(9.8)
12.	(d)
13.	(d)
14.	(0.0001)
15.	(b)
16.	(a)
17.	(50)
18.	(c)
19.	(c)
20.	(a)
21.	(b)
22.	(c)
23.	(c)
24.	(a)

■■■■

LEVEL 3 Conventional Questions

Solution : 25

Let us give virtual displacement dy to the point B from the horizontal position AC of the beam as shown in figure below,



Virtual displacements of points D and E are

$$dy_D = \frac{6}{8}dy = 0.75dy$$

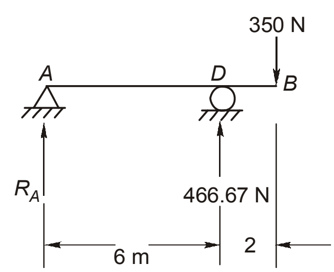
$$dy_E = \frac{7}{10}dy = 0.7dy$$

Reaction at D is upwards and load at E is downwards. Virtual displacements at ends A and C are zero. Using the principle of virtual work

$$R_A \times 0 + R_D.dy_D - W.dy_E + R_C \times 0 = 0$$

$$R_D \times 0.75dy - 500 \times 0.7dy = 0$$

Reaction at D, $\quad R_D = 350/0.75 = 466.66$ N.



Force transmitted by hinge B: Two determine this force F_B at hinge let us consider beam ADB only

$$R_D \times \delta y_D + F_B \times dy = 0 \qquad \text{(using the principle of virtual work)}$$

$$466.66 \times 0.75dy = -F_B.dy$$

or $\quad F_B = -350$ N = 350 N (downwards)

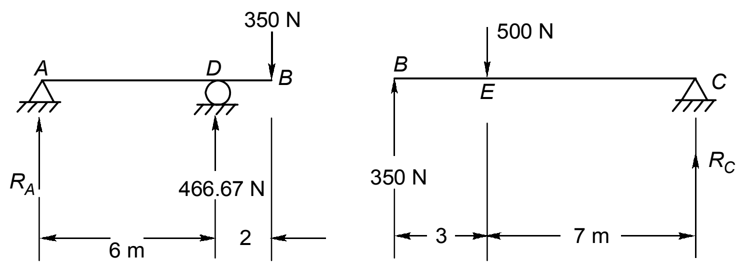
To verify this let us take beam BEC. Using the principle of virtual work for BEC

$$F_B \times dy - 500 \times dy_E = 0$$

$$F_B.dy = 500 \times 0.7dy$$

$$F_B = 350 \text{ N} \uparrow$$

Free body diagrams for beams ADB and BEC can be drawn as shown in the figure



Obviously the reactions will be

$R_A = 116.67 \text{ N}\downarrow$ and $R_C = 150 \text{ N}\uparrow$.

Solution : 26

Force P is applied at joint F. Let us take xy coordinate with origin at O as shown in figure, Then

$$x = 3a\cos\theta \text{ for the force } P$$

$$y = a\sin\theta \text{ for forces } P_1 \text{ each}$$

Then $$\delta x = -3a\sin\theta\,\delta\theta$$

$$\delta y = +a\cos\theta.\delta\theta$$

Using the principle of virtual work

$$P.\delta x + 2.P_1\delta y = 0$$

$$-P\times 3a\sin\theta.\delta\theta + 2P_1 a\cos\theta.\delta\theta = 0$$

or $$2P_1 = 3P\tan\theta$$

$$P_1 = 1.5P\tan\theta$$

If $$\theta = 45°,\ P_1 = 1.5P.$$

Solution : 27

Say the force applied is P Newton.

Horizontal component of P,

$$P_x = P\cos 30° = \overrightarrow{0.866P} \text{ N}$$

Vertical component of P, $$P_y = P\sin 30° = 0.5P \text{ N}\downarrow$$

Weight of the block, $$W = mg = 50\times 9.81 = 490.5 \text{ N}$$

Normal reaction of the plane on the body

$$N = W + P\sin 30° = (490.5 + 0.5P) \text{ N}$$

Force of friction, $$F = \mu_k \times N$$

$$= 0.25\,(490.5 + 0.5P) = 122.625 + 0.125P \quad ...(1)$$

Net force applied on block $= P_x - F$

$$= 0.866P - 122.625 - 0.125P = 0.741P - 122.625$$

Work done by the force on moving the block by 10 m

$$U = (0.741P - 122.625)\times 10 = (7.41P - 1226.25) \text{ Nm}$$

Kinetic energy of the block,

$$KE = \frac{1}{2}mV^2 = \frac{1}{2}\times 50\times 8^2 = 1600 \text{ Nm}$$

Using the work-energy principle

$$7.41P - 1226.25 = 1600 \text{ Nm}$$

Force, $P = 381.4$ N

Solution : 28

Mass of the flywheel, $m = 300$ kg

Radius of gyration of flywheel, $k = 0.8$ m

Moment of inertia of flywheel, $I = mk^2 = 300\times 0.8^2 = 192$ kg-m^2

Work done during one punching operation

$$U = 2100 \text{ Joules} = 2100 \text{ N-m}$$

Original speed of flywheel, $\omega_0 = \frac{2\pi\times 220}{60} = 23.04$ rad/s

Say final speed of flywheel after punching

$$= \omega_f \text{ rad/s}$$

then

$$-U = I\left(\frac{\omega_f^2}{2} - \frac{\omega_0^2}{2}\right)$$

$$2U = I(\omega_0^2 - \omega_f^2)$$

$$2\times 2100 = 192(23.04^2 - \omega_f^2)$$

or

$$\omega_f^2 = 23.04^2 - 21.875 = 530.841 - 21.875 = 508.96$$

$$\omega_f = 22.56 \text{ rad/s}$$

or Speed of flywheel after punching = 215.44 rpm

Solution : 29

Initial compression, $x_1 = 100$ mm

Final compression, $x_2 = 300 - 150 = 150$ mm

Spring constant, $k = 10$ N/mm

Additional work done in compressing the spring by further 50 mm

$$U_{1-2} = \int_{100}^{150} F\,dx = \int_{100}^{150} kx\,dx = \left|\frac{kx^2}{2}\right|_{100}^{150}$$

putting the value of k

$$= \frac{10}{2}[150^2 - 100^2] = 62500 \text{ Nmm} = 62.5 \text{ Nm}$$

Solution : 30

$$\text{Mass of wheel, } m = 15 \text{ kg}$$

$$\text{Weight of wheel, } mg = 15 \times 9.81 = 147.15 \text{ N}$$

Component along the plane,

$$mg \sin \theta = 147.15 \times \sin 30^\circ = 73.57 \text{ N}$$

$$\text{Normal reaction, } N = mg\cos\theta = 147.15 \times 0.866 = 127.43 \text{ N}$$

$$\text{Net force along the plane} = 100 - 73.57$$

or

$$F_{net} = 26.43 \text{ N} \qquad \ldots(1)$$

Please note that in this case force of friction will be equal to F_{net} and provides turning moment to wheel. This is the case of pure rolling without slipping. Force of friction will not be equal to,

$$\mu N = 0.25 \times 127.43 = 31.86 \text{ N}$$

But it will be only 26.43 N.

Work done in rolling the wheel up the plane by 3 m

$$U_{1-2} = (100 - mg\sin\theta)\, S = 26.43 \times 3 = 79.29 \text{ Nm}$$

Say the angular velocity is ω and linear velocity of wheel centre is V at the position 2.

$$U_{1-2} = \frac{1}{2} mV^2 + \frac{I}{2}\omega^2 \qquad \ldots(2)$$

where

$$V = \omega r = 0.1 \times \omega \text{ m/s}$$

$$I = \frac{mr^2}{2} = \frac{15 \times 0.1^2}{2} = 0.075 \text{ kg-m}^2$$

Substituting these values in Equation (2)

$$79.29 = \frac{1}{2} \times 15 \times (0.1\omega)^2 + \frac{0.075}{2} \times \omega^2$$

$$= 0.075\omega^2 + 0.0375\omega^2 = 0.1125\omega^2$$

Angular velocity,

$$\omega = \sqrt{\frac{79.29}{0.1125}} = \sqrt{704.8} = 26.54 \text{ rad/s.}$$

Solution : 31

Given:

$$\ell = 5 \text{ m};$$

$$W = 200 \text{ N}$$

Let,

T = Tension in the rope PQ,

x = Virtual vertical displacement of the rope PQ due to tension

From the geometry of the figure, we find that

$$\tan \theta = \frac{4}{3}$$

We also find that when the mid point P of ladder moves downwards (due to weight). it causes top of the ladder B to move downwards and bottom of the ladder A to move towards left. It causes tension (T) in the rope PQ. Moreover, when the virtual vertical displacement of the mid of the ladder P (or weight of the ladder) is x, then the virtual horizontal displacement of the mid of the ladder,

$$y = \frac{x}{\tan\theta} = \frac{x}{4/3} = \frac{3x}{4} = 0.75\,x$$

∴ Virtual work done by the tension in the rope

$$= +T \cdot x \qquad \text{...(Plus sign due to tension)}$$

and virtual work done by the 200 N weight of the ladder

$$= -200\,y \qquad \text{...(Minus sign due to downward movement of the weight)}$$

We know that as per principle of virtual work, the algebraic sum of the total virtual works done is zero.

$$T\,.\,x - 200\,y = 0$$

or,
$$T = \frac{200y}{x} = \frac{200 \times 0.75x}{x} = 150\text{ N}$$

Solution : 32

Given, $W = 1000$ N;

$\alpha_1 = 30°$

$\alpha_2 = 45°$

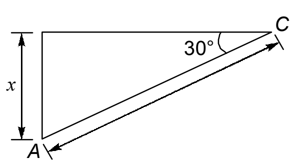
Let, x = Virtual vertical displacement of 1000 N weight, and

y = Virtual vertical displacement of the effort (P)

From the geometry of the system, we find that when the weight (1000 N) moves downwards, effort (P) will move upwards. We also find that the distance through which the 1000 N weight will move downwards on the inclined surface will be equal to the distance through which the load P will move upwards on the inclined surface.

∴ Distance through which the 1000 N weight moves on the inclined surface AC.

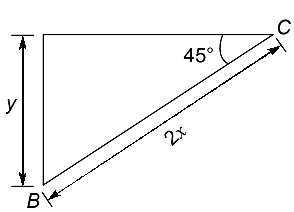
For surface AC,



$$AC = \frac{x}{\sin 30°} = \frac{x}{0.5} = 2x \qquad \text{...(i)}$$

Distance through which the load P will move on the inclined surface BC

For surface BC,



$$\sin 45° = \frac{y}{2x} \Rightarrow 2x = 1.414\,y \Rightarrow x = 0.707y$$

∴ Virtual work done by the effort (P)

$$= P \times y \qquad \text{...(Plus sign due to upward movement of the effort)}$$

and virtual work done by the 1000 N weight,

$$= -1000 \times x = -1000 \times 0.707y$$

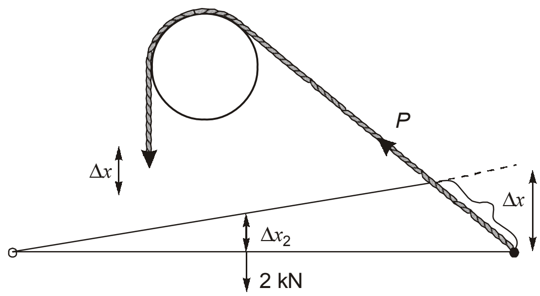
$$= -707y \qquad \text{...(Minus sign due to downward movement of the weight)}$$

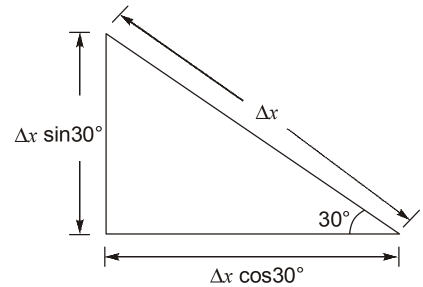
We know that as per principle of virtual work, algebraic sum of the virtual works done is zero.

∴ $(P\,.\,y) - 707\,y = 0$

or, $P = 707$ N

Solution : 33





$$\delta x_2 = \frac{\Delta x \sin 30°}{2}$$

$$F_2 = 2 \text{ kN}$$

$$\delta x_1 = \Delta x \sin 30°$$

$$F_1 = P \sin 30°$$

$$F_1 \times \Delta x_1 = F_2 \times \Delta x_2$$

$$P \sin 30° \times \Delta x \sin 30° = 2 \times \frac{\Delta x \sin 30°}{2}$$

$$\Rightarrow \quad P = 2 \text{ kN}$$

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6 Plane Trusses

LEVEL 1 Objective Questions

1. (d)
2. (a)
3. (b)
4. (a)
5. (d)
6. (b)
7. (b)
8. (b)
9. (a)
10. (b)

LEVEL 2 Objective Questions

11. (d)
12. (b)
13. (b)
14. (c)
15. (a)
16. (b)
17. (b)
18. (11.5)
19. (b)
20. (d)

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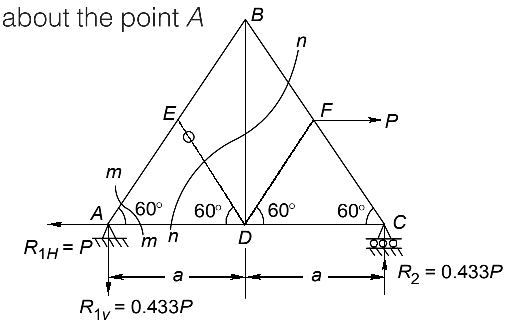
LEVEL 3 Conventional Questions

Solution : 21



Let us first determine the support reactions. Taking moments about the point A

$$P \times a \sin 60^{\circ} = 2a \,.\, R_2$$

$$R_2 = 0.433P \uparrow$$

Reaction at A

For equilibrium, $\quad R_{1V} = 0.433P \downarrow$

$$R_{1H} = \overleftarrow{P} \text{(to balance } P \text{ at } F)$$

Now at the joint E, members AE and EB are collinear and member DE is joined at E. Therefore force in DE

i.e., $\quad F_{DE} = 0$

Let us first take mm section as shown. Part I of the truss has forces (*i*) R_{1V} (*ii*) R_{1H} (*iii*) F_{EA} (*iv*) F_{AD} as shown in figure. Taking moments about the point E,

$$P \times a \times \sin 60^{\circ} = 0.433P \times a \sin 30^{\circ} + F_{AD} \times a \sin 60^{\circ}$$

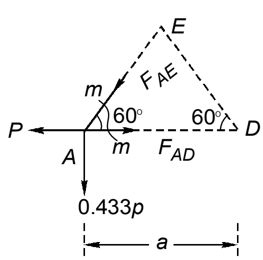
$$0.866P = 0.2165P + 0.866F_{AD}$$

$$F_{AD} = P - 0.25P = 0.75P \text{ (Tension)}$$

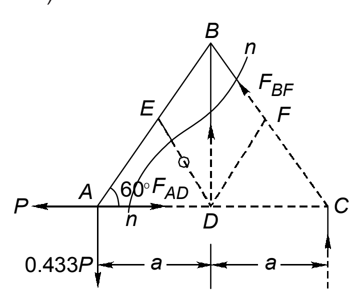
Now the truss is cut by another section nn, cutting the members, BF, BD, ED and AD as shown in figure. Now

$$F_{AD} = 0.75P$$

$$F_{ED} = 0 \text{ (as stated earlier)}$$



(*a*)



(*b*)

Taking moments of the forces about the point F,

$$F_{AD} \times a \sin 60^{\circ} + 0.433P \times 1.5a - P \times a \sin 60^{\circ} - F_{BD} \times \frac{a}{2} = 0$$

$$0.75 \times 0.866P + 0.433 \times 1.5P - P \times 0.866 = 0.5F_{BD}$$

$$F_{BD} = 0.866P \text{ (Compression)}$$

The same result follows, if the moment centre is chosen at C.

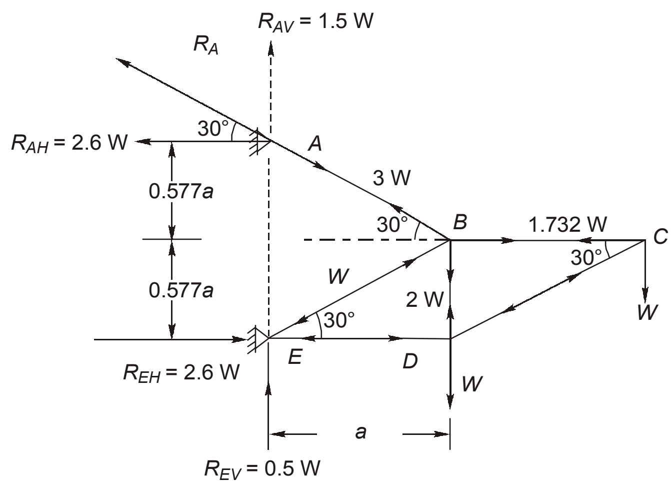
Solution : 22

$$\text{Distance, } AE = 2 \times a \times \tan 30^{\circ} = 1.154a$$

Reactions: Taking moments about E,

$$R_{AH} \times 1.154a = W \times a + 2W \times a \text{ (Note that } R_{AV} \text{ is passing through point } E)$$

$$R_{AH} = \frac{3\text{ W}}{1.154} = 2.6\text{ W}$$



$$R_{AV} = R_{AH} \tan 30^{\circ} = 2.6 \times 0.577 = 1.5\text{ W}$$

At E: Reaction $\quad R_{EV} = 2\text{ W} - 1.5\text{ W} = 0.5\text{ W}\uparrow$

$$R_{EH} = R_{AH} = 2.6\text{ W (for balancing)}$$

Joint C: $\quad F_{CD} \sin 30^{\circ} = W$

$$F_{CD} = \frac{W}{0.5} = 2W\ (C)$$

Joint D $\quad F_{BD} = F_{CD} \sin 30^{\circ} + W = 2\ W \times 0.5 + W = 2\ W \uparrow (T)$

$$F_{ED} = F_{CD} \cos 30^{\circ} = 2\ W \times 0.866 = 1.732\ W\ (C)$$

The reader can verify the correctness of the solution.

Solution : 23

Let us first determine the support reactions.

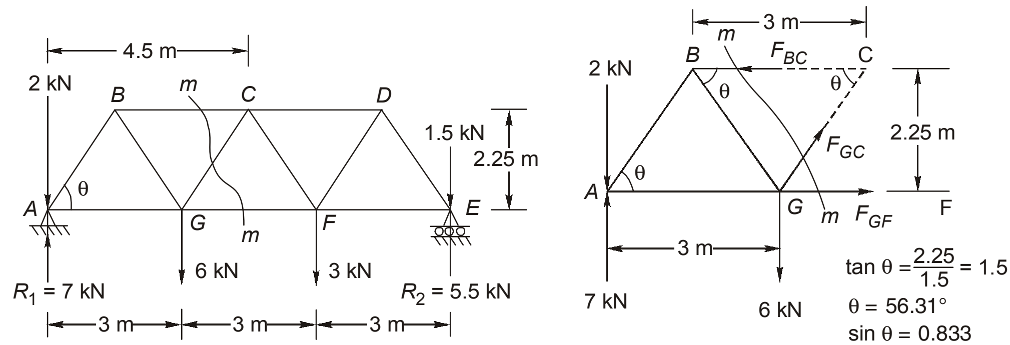
Taking moments of the forces about the joint A

$$6 \times 3 + 3 \times 6 + 1.5 \times 9 = 9 \times R_2$$

Reaction, $\quad R_2 = 5.5\text{ kN}$

$$R_1 = 2 + 6 + 3 + 1.5 - 5.5 = 7\text{ kN}$$

Now cutting the truss into two parts through the section $m - m$. This section is cutting the members BC, GC and GF. One part of the truss is shown in figure. Taking moments of the forces about the joint C.



(*a*) (*b*)

$$7\times(3+1.5)-2\times(3+1.5)-6\times1.5-F_{GF}\times2.25 = 0$$

$$31.5-9-9-2.25F_{GF} = 0$$

$$F_{GF} = 6 \text{ kN (Tension)}$$

Taking moments of the forces about the joint G

$$7\times3-2\times3-F_{BC}\times2.25 = 0$$

$$F_{BC} = \frac{15}{2.25} = 6.667 \text{ kN (Compression)}$$

Taking moments of the forces about the joint B

$$2\times1.5-7\times1.5-6\times1.5+F_{GF}\times2.25+F_{GC}\times3\sin\theta = 0, \text{ putting the value of } F_{GF}$$

$$-16.5+2.25\times6+F_{GC}\times3\times0.833 = 0$$

$$-3+2.5F_{GC} = 0$$

$$F_{GC} = 1.2 \text{ kN } (T).$$

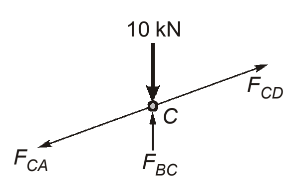
Solution : 24

$$\Sigma M_A = 0$$

$$\Rightarrow \quad -R_B\times l+10\times l+10\times 2l = 0$$

$$\Rightarrow \quad R_B = 30 \text{ kN } (\uparrow)$$

At joint C

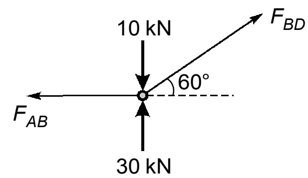


Since F_{CA} and F_{CD} are collinear

$\therefore \quad F_{CA} = F_{CD}$

So, $\quad F_{BC} = 10$ kN

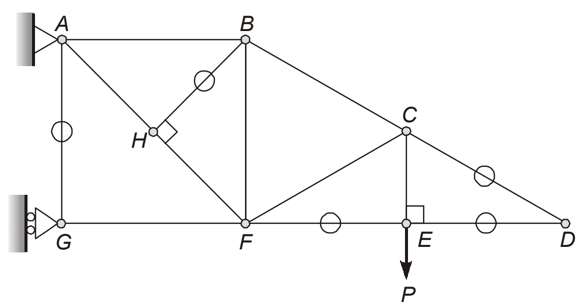
At joint B



$\Sigma F_y = 0 \qquad F_{BD} \sin 60° + 30 - 10 = 0$

$$\Rightarrow \qquad F_{BD} = -\frac{20\times 2}{\sqrt{3}} = -\frac{40}{\sqrt{3}} \text{ kN} = -23.09 \text{ kN} = 23.09 \text{ kN (compressive)}$$

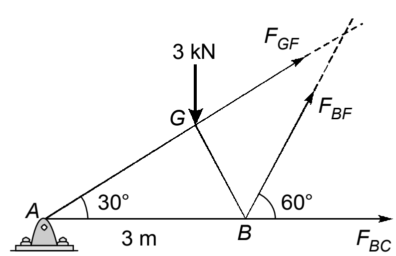
Solution : 25 $\Rightarrow$ 5



At joint D, there is no external force and members are not collinear, so forces in both members DE and CD will be 0.
At joint E, force in member EF will be 0 as force in member ED is 0.
At joint H, force in member HB will be 0.
At joint G, there is no vertical reaction so force in the member GA will be 0.

Solution : 26

Cut the truss by a section passing through GF, BF and BC



$$\Sigma M_A = 0$$

$$\Rightarrow \quad -F_{BF} \sin 60° \times 3 + 3 \times 3 \cos^2 30° = 0$$

$$\Rightarrow \quad \frac{3\times\sqrt{3}}{2} \times F_{BF} = 3 \times 2.25$$

$$\Rightarrow \quad F_{BF} = 2.6 \text{ kN}$$

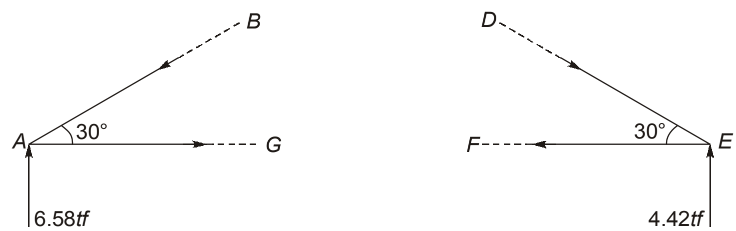
Solution : 27

From the geometry of the figure, we find that the load of 3 *tf* at B is acting at a distance of 1.5 m from the joint A. Taking moments about A and equating the same.

$$R_E \times 6 = (3 \times 1.5) + (5 \times 2) + (3 \times 4) = 26.5$$

$\therefore$ $$R_E = \frac{26.5}{6} = 4.42\ tf$$

and $$R_A = (3 + 5 + 3) - 4.42 = 6.58 \text{ tf}$$



The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints only as we have to find out the forces in all the members of the truss First of all, consider joint *A*. Let the direction of P_{AB} and P_{AG} be assumed as shown in above figure. Resolving the forces vertically and equating the same,

$$P_{AB} \sin 30° = 6.58$$

$\therefore$ $$P_{AB} = \frac{6.58}{\sin 30°} = \frac{6.58}{0.5} = 13.16\ tf \text{(Compression)}$$

and now resolving the forces horizontally and equating the same,

$$P_{AG} = P_{AB} \cos 30° = 13.16 \times 0.866 == 11.4\ tf \text{(Tension)}$$

Now consider joint *E*. Let the directions of P_{DE} and P_{FE} be assumed as shown in above . Resolving the force vertically and equating the same,

$$P_{DE} \sin 30° = 4.42$$

$\therefore$ $$P_{DE} = \frac{4.42}{\sin 30°} = \frac{4.42}{0.5} = 8.84\ tf \text{(Compression)}$$

and now resolving the force horizontally and equating the same,

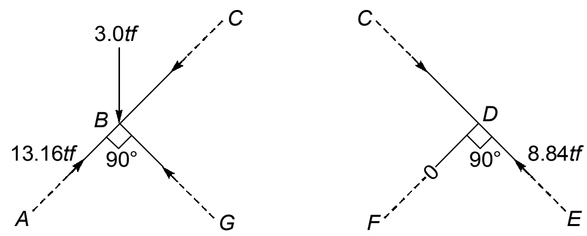
$$P_{FE} = P_{DE} \cos 30° = 8.84 \times 0.866 = 7.66\ tf \text{(Tension)}$$

Now consider joint *B*. Let the directions of P_{BC} and P_{BG} be assumed as shown in figure (a). We have already found out that the force in member *AB* or P_{AB} is 13.16 *tf* (Compression). Resolving the forces along the *ABC* and equating the same,

$$P_{BC} = P_{AB} - 3 \cos 60° = 13.16 - 3 \times 0.5\ tf$$
$$= 11.66\ tf \text{(Compression)}$$

and how resolving the forces at right angles to the rafter *ABC* and equating the same,

$$P_{BG} = 3 \sin 60° = 3 \times 0.866 = 2.6\ tf \text{(Compression)}$$



(a) Joint B **(b) Joint D**

Now consider the joint D. We have already found out that P_{DE} is 8.84 tf (Compression). A little consideration will show that the value of P_{CD} will be equal to P_{DE} i.e. 8.84 tf (Compression). Moreover, P_{DF} will be zero as there is no other member at join D to balance it.

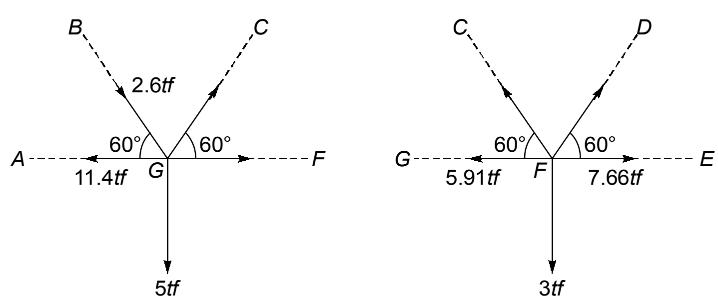
Now consider the joint G. We have already found out that P_{AG} is 11.4 tf (Tension) and P_{BG} is 2.6 tf (Compression). Let the direction of P_{GC} and P_{GF} be assumed as shown in figure (a). Resolving the forces vertically and equating the same.

$$P_{GC} \sin 60° = 5 + 2.6 \sin 60° = 5 + (2.6 \times 0.866) = 7.25$$

$$\therefore \qquad P_{GC} = \frac{7.25}{\sin 60°} = \frac{7.25}{0.866} = 8.37 \;\; tf \text{ (Tension)}$$

and how resolving the forces horizontally and equating the same.

$$\begin{aligned} P_{GF} &= P_{GA} - P_{BG} \cos 60° - P_{GC} \cos 60° \\ &= 11.4 - (2.6 \times 0.5) - (8.37 \times 0.5) \; tf \\ &= 5.91 \; tf \text{ (Tension)} \end{aligned}$$



(a) Joint G **(b) Joint F**

Now consider joint F. We have already found out that P_{FE} is 7.66 tf (Tension). P_{FD} is zero and P_{GF} is 5.91 tf (Tension). Let the direction of P_{FC} be assumed as shown in fig (b). Resolving the force vertically, and equating the same,

$$P_{FC} \sin 60° = 3$$

$$\therefore \qquad P_{FC} = \frac{3}{\sin 60°} = \frac{3}{0.866} = 3.46 \;\; tf \text{ (Tension)}$$

Now tabulate the results as given below:

S.No.	Member	Magnitude of force in *tf*	Nature of force
1	*AB*	13.16	Compression
2	*AG*	11.4	Tension
3	*DE*	8.84	Compression
4	*FE*	7.66	Tension
5	*BC*	11.66	Compression
6	*BG*	2.6	Compression
7	*DF*	0	-
8	*CD*	8.84	Compression
9	*GC*	8.37	Tension
10	*FG*	5.91	Tension
11	*FC*	3.46	Tension

Solution : 28

Since the structure is symmetrical in geometry and loading, therefore reaction at *A*,

$$R_A = R_B = 5 \text{ kN}$$

From the geometry of the structure, shown in figure, we find that

$$\tan\theta = \frac{3}{3} = 1.0 \text{ or } \theta = 45°$$

and

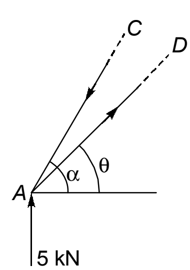
$$\tan\alpha = \frac{6}{3} = 2.0 \text{ or } \alpha = 63°\ 26'$$

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of joints only.

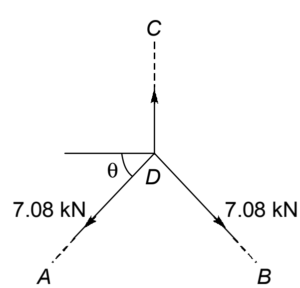
First of all, consider the joint *A*. Let the directions of the forces P_{AC} and P_{AD} be assumed as shown in figure (a). Resolving the forces horizontally and equating the same,

$$P_{AC}\cos 63°\ 26' = P_{AD}\cos 45°$$

$$\therefore \qquad P_{AC} = \frac{P_{AD}\cos 45°}{\cos 63°26'} = \frac{P_{AD}\times 0.707}{0.4473} = 1.58P_{AD}$$



(a) Joint A



(b) Joint D

and now resolving the forces vertically and equating the same,

$$P_{AC}\sin 63°\ 26' = 5 + P_{AD}\cos 45°$$

$$1.58\ P_{AD}\times 0.8945 = 5 + P_{AD}\times 0.707$$

$$0.7063\ P_{AD} = 5$$

$$P_{AD} = \frac{5}{0.7063} = 7.08 \text{ kN (Tension)}$$

and $\qquad P_{AC} = 1.58 \times 7.08 = 11.19 \text{ kN (Compression)}$

Now consider the joint D. Let the directions of the forces P_{CD} and P_{BD} be assumed as shown in figure (b). Resolving the forces vertically and equating the same,

$$P_{CD} = P_{AD} \sin 45° + P_{BD} \cos 45°$$

$$= 2 \times P_{AD} \cos 45° = 2 \times 7.08 \times 0.707 \text{ kN} \qquad (P_{BD} = P_{AD})$$

$$= 10.0 \text{ kN}$$

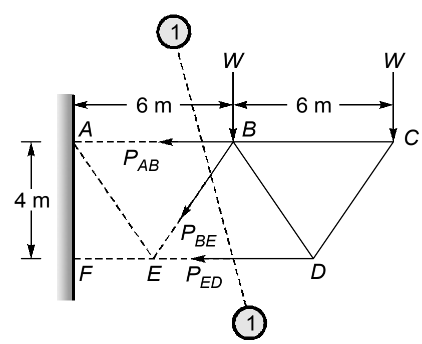
Now tabulate these results as given below:

S.No.	Member	Magnitude of force in kN	Nature of force
1	*AD*, *DB*	7.08	Tension
2	*AC*, *CB*	11.19	Compression
3	*CD*	10.0	Tension

Solution : 29

The example may be solved either by the method of joints or method of sections. But we shall solve it by the method of sections only as we have to find out the force in member AB only.

First of all, let us find out the force in the member AB of the truss in terms of W. Now pass section 1------1 cutting the truss through the members AB, BE and ED as shown in figure.



Now consider the equilibrium of the right part of the truss. Let the direction of P_{AB} be assumed as shown in figure.

Taking moments of the forces in the right part of the truss only, about the joint E and equating the same,

$$P_{AB} \times 4 = W \times 3 + W \times 9 = 12\ W$$

or $\qquad P_{AB} = 12\frac{W}{4} = 3\,W$

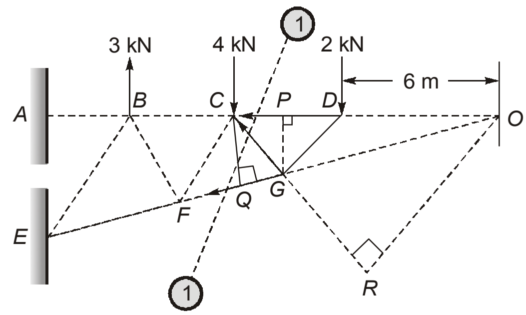
Hence the value of W, which would produce the force of 15 tones is the member AB

$$3\text{ W} = 15\ tf \Rightarrow W = 5\ tf$$

Solution : 30

First of all, extend the lines through the joints B, C and D as well as E, F and G meeting at O. Through G, draw GP perpendicular to CD. Similarly through C, draw CQ perpendicular to FG.

Now extend the line of action of the member CG, and through O, draw a perpendicular to this line meeting at R as shown in figure.



We know that in similar triangles OPG and OAE

$$\frac{AO}{AE} = \frac{AP}{AE - PG}$$

or $$\frac{AO}{4} = \frac{8}{2} = 4$$

∴ $$AO = 4 \times 4 = 16$$

or $$DO = 16 - 10 = 6 \text{ m}$$

Now in triangle CGP, we find that

$$\tan\angle GCP = \frac{2}{2} = 1 \text{ or } \angle GCP = 45°$$

∴ $$\angle COR = 90° - 45° = 45°$$

and $$OR = OC\cos 45° = 10 \times 0.707 = 7.07 \text{ m}$$

From the geometry of the triangle OPG, we find that

$$\tan\angle GOP = \frac{2}{8} = 0.25 \text{ or } \angle GOP = 14°\, 2'$$

Similarly, in triangle OCQ, we find that

$$CQ = CO\sin 14°\, 2' = 10 \times 0.2425 = 2.425 \text{ m}$$

Now pass section 1------1 cutting the frame through the members CD, CG and FG. Let the directions of the forces P_{CD}, P_{CG} and P_{FG} be assumed as shown in figure.

Taking moments of the forces acting on right part of the frame only, about the joint G and equating the same,

$$P_{CD} \times 2 = 2 \times 2$$

or $$P_{CD} = 2 \text{ kN (Tension)}$$

Similarly, taking moments of the forces acting in the right part of the truss only about the imaginary joint O and equating the same,

$$P_{CG} \times 7.07 = 2 \times 6 = 12$$

or $$P_{CG} = \frac{12}{7.07} = 1.7 \text{ kN (Tension)}$$

■■■■

7 Friction

LEVEL 1 Objective Questions

1.	(c)
2.	(a)
3.	(c)
4.	(b)
5.	(c)
6.	(c)
7.	(a)
8.	(b)
9.	(a)
10.	(b)
11.	(a)
12.	(b)

LEVEL 2 Objective Questions

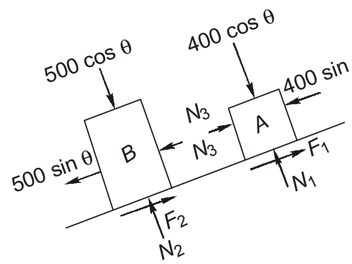
13.	(b)
14.	(81.17)
15.	(a)
16.	(1)
17.	(c)
18.	(330)
19.	(c)
20.	(0.5)
21.	(d)
22.	(380.2)
23.	(22.50)
24.	(a)
25.	(a

■■■■

LEVEL 3 Conventional Questions

Solution : 26

Say the blocks are at the point of sliding down the plane. Free body diagrams of blocks *A* and *B* are given in figure. Frictional force on block *A* is less than frictional force on block *B*, so block *A* will exert push on block *B*.



FBD diagrams of A and B blocks

Block *A*: Normal Reaction, $N_1 = 400\cos\theta$

Friction force, F_1 (upwards) $= 0.2 \times 400\cos\theta = 80\cos\theta$

Forces in the direction parallel to the plane,

$$N_3 = 400\sin\theta - F_1 = 400\sin\theta - 80\cos\theta \quad \text{...(1)}$$

N_3 is the reaction between blocks *A* and *B*

Block *B*: Normal reaction, $N_2 = 500\cos\theta$

Force of friction (upwards), $F_2 = 0.3 \times 500\cos\theta = 150\cos\theta$

Forces in the direction parallel to the plane

$$N_3 + 500\sin\theta = F_2$$

$$N_3 = 150\cos\theta - 500\sin\theta \quad \text{...(2)}$$

From Equations (1) and (2)

$$400\sin\theta - 80\cos\theta = 150\cos\theta - 500\sin\theta$$

$$900\sin\theta = 230\cos\theta$$

$$\tan\theta = \frac{230}{900} = 0.2555$$

$$\theta = \tan^{-1}0.2555 = 14.335^\circ = 14^\circ 20'.$$

Solution : 27

Pull exerted by boat $= 20\text{ kN} = 20{,}000\text{ N} = T_1$

Pull at the other end of hawser $= 250\text{ N} = T_2$

Ratio of hawser tensions $= \dfrac{T_1}{T_2} = \dfrac{20{,}000}{250} = 80 = e^{\mu\theta}$

$$\mu\theta = 4.382$$

Coefficient of friction, $\mu = 0.3$

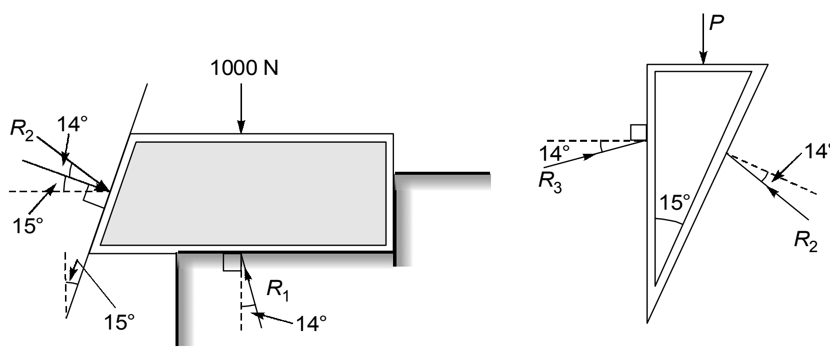
Therefore lap angle, $\theta = \dfrac{4.382}{0.3} = 14.60 \text{ radians } = 2\pi n$

where n = number of turns of hawser around capstan.

So number of turns, $n = \dfrac{14.60}{2\pi} = 2.32$ turns.

Solution : 28

Given: $\alpha = 15°$; $W = 1000$ N; $\phi = 14°$



(a) Body (B) **(b) Wedge (A)**

First of all, consider the equilibrium of the body. We know that it is a equilibrium under the action of the following forces as shown in figure (a).

1. Its own weight 1000 N acting downwards,
2. Reaction R_1 acting on the floor, and
3. Reaction R_2 of the wedge on the body

Resolving the forces horizontally,

$$R_1 \sin 14° = R_2 \cos(15° + 14°) = R_2 \cos 29°$$

$$R_1 \times 0.2419 = R_2 \times 0.8746$$

$$\therefore \quad R_1 = 3.616\, R_2$$

and now resolving the forces vertically,

$$R_2 \sin(15° + 14°) + 1000 = R_1 \cos 14°$$

$$R_2 \times 0.4848 + 1000 = R_1 \times 0.9703 = (3.616 R_2)\, 0.9703$$

$$= 3.51\, R_2 \qquad \ldots (\because R_1 = 3.616\, R_2)$$

$$\therefore \quad R_2 (3.51 - 0.4848) = 1000$$

or
$$R_2 = \frac{1000}{3.0252} = 330.6 \text{ N}$$

Now consider equilibrium of the wedge. We know that it is in equilibrium under the action of the following forces as shown in figure (b):

1. Reaction R_2 of the body on the wedge,
2. Force (P) acting vertically downwards, and
3. Reaction R_3 on the vertical surface.

Resolving the forces horizontally,

$$R_3\cos 14° = R_2\cos(14° + 15°) = R_2 \cos 29°$$

$$R_3 \times 0.9703 = R_2 \times 0.8746 = 330.6 \times 0.8746 = 289.1$$

$$\therefore \quad R_3 = \frac{289.1}{0.9703} = 297.9 \text{ N}$$

and now resolving the forces vertically,

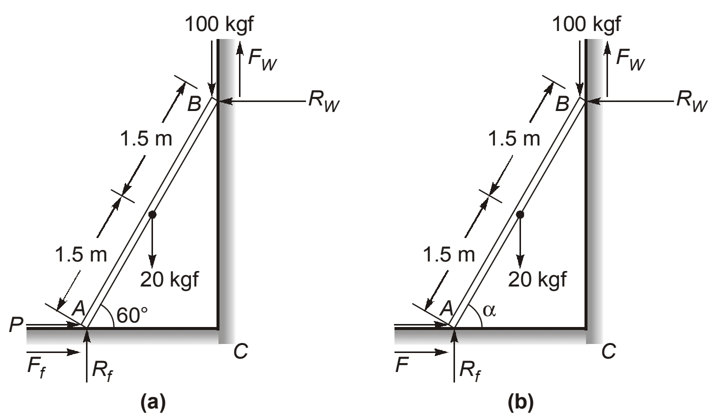
$$P = R_3 \sin(14°) + R_2 \sin(14° + 15°) \text{ N}$$

$$= (297.9 \times 0.2419) + (330.6 \times 0.4848) \text{ N}$$

$$= 232.3 \text{ N}$$

Solution : 29

Given: l = 3 m; W = 20 kgf; μ_w = 0.25; μ_f =- 0.35.



The force acting in both the cases are shown in figure (a) and (b).

(i) Horizontal force (P) applied to the ladder at floor level to prevent slipping.

First of all, consider the ladder inclined at an angle of 60° and subjected to a horizontal force (P) at the floor as shown in figure (a).

Resolving the forces horizontally,

$$P + F_f = R_w \quad \text{... (i)}$$

and now resolving the forces vertically,

$$R_f + F_w = 100 + 20 = 120 \text{ kgf} \quad \text{... (ii)}$$

Taking moments about A and equating the same,

$$20 \times 1.5 \cos 60° + 100 \times 3 \cos 60° = F_w \times 3 \cos 60° + R_w \times 3 \sin 60°$$

Dividing both sides by cos60°,

$$30 + 300 = 3 \times F_w + 3\, R_w \tan 60°$$

$\therefore$ $$110 = F_w + R_w \tan 60° \quad \text{... (iii)}$$

We know that, $$F_w = \mu_w \times R_w = 0.25\, R_w \quad \text{... } (\because \mu_w = 0.25)$$

Substituting this value of F_w in equation, (iii),

$$110 = 0.25\, R_w + R_w \tan 60°$$
$$= R_w (0.25 + 1.732) = R_w \times 1.982$$

$\therefore$ $$R_w = \frac{110}{1.982} = 55.5 \text{ kgf}$$

and $$F_w = 0.25\, Rw = 0.25 \times 55.5 = 13.87 \text{ kgf}$$

Now substituting the value of F_w in equation (ii),

$$R_f + 13.87 = 120$$

$\therefore$ $$R_f = 120 - 13.87 = 106.13 \text{ kgf}$$

and $$F_f = \mu_f R_f = 0.35 \times 106.13 = 37.15 \text{ kgf}$$

Now substituting the value of F_f in equation (i),

$$P + 37.15 = 55.5$$

$\therefore$ $$P = 55.5 - 37.15 = 18.35 \text{ kgf}$$

(ii) Inclination of the ladder with the horizontal for no slipping.

Now consider the ladder inclined at an angle (α) and without any horizontal force acting at the floor as shown in figure (b).

Resolving the forces horizontally,

$$R_w = F_f = \mu_f \times R_f = 0.35 \times R_f \quad \text{... (iv)}$$

and now resolving the forces vertically,

$$R_f + F_w = 100 + 20 = 120 \text{ kgf}$$

We know that, $$F_w = \mu_w \times R_w = 0.25(0.35\, R_f) = 0.09\, R_f \quad \text{... } (\because R_w = 0.35\, R_f)$$

or $$R_f + 0.09\, R_f = 120$$

$\therefore$ $$R_f = \frac{120}{1.09} = 110.1 \text{kgf}$$

and $$R_w = 0.35\, R_f = 0.35 \times 110.1 = 38.54 \text{ kgf}$$

Similarly, $$F_w = 0.09\, R_f = 0.09 \times 110.1 = 9.9 \text{ kgf}$$

Taking moments about A, and equating the same,

$$100 \times 3 \cos\alpha + 20 \times 1.5 \cos\alpha = F_w \times 3 \cos\alpha + R_w \times 3 \sin\alpha$$

Dividing both sides by, 3 cosα

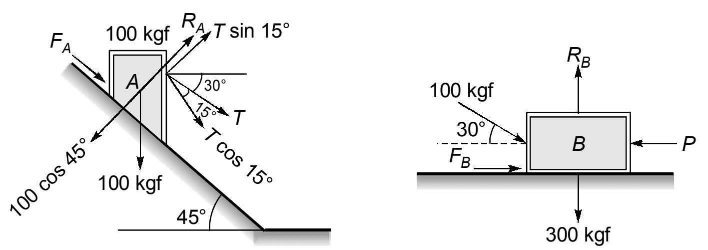
$$100 + 10 = F_w + R_w \tan\alpha$$
$$110 = 9.9 + 38.54 \tan\alpha$$
$$38.54 \tan\alpha = 110 - 9.9 = 100.1$$

$\therefore$ $$\tan\alpha = 2.5973 \text{ or } \alpha = 68° 57'$$

Solution : 30

Given: W_A = 100 kgf; W_B = 300 kgf; α = 45°; θ = 45° – 30° = 15°; ϕ = 15° or μ = tan15° = 0.2679.

The two blocks and the weightless rigid bar is shown in figure above. First of all, consider the equilibrium of the block (A), which is subjected to the forces as shown in figure (a).



(a) Load A **(b) Load B**

We know that the thrust or force in the rigid bar just to move the block (A) in the upward direction,

$$(100 \sin 45° + \mu\, 100 \cos 45°) = T\cos 15° - \mu T \sin 15° \Rightarrow T = \frac{100\sin(15+45)°}{\cos(15+15)°}$$

$$T = 100 \times \frac{\sin 60°}{\cos 30°} = 100 \times \frac{\sin 60°}{\cos 30°}\text{ kgf}$$

$$T = 100 \times \frac{0.866}{0.866} = 100\text{ kgf}$$

Now consider the equilibrium of the load B, which is subjected to the forces as shown in figure (b). We know that since the load is at the point of sliding towards left, therefore, the force of friction ($F_B = \mu.\ R_B$) will act towards right as shown in the figure. Now resolving the forces vertically,

$$R_B = 300 + 100 \sin 30° = 300 + 100 \times 0.5\text{ kgf}$$
$$= 350\text{ kgf}$$

and now resolving the forces horizontally,

$$P = 100 \cos 30° + F_B = 100 \times 0.866 + (0.2679 \times 350)$$
$$= 180.4\text{ kgf}$$

Solution : 31

Given: μ_A = 0.3; μ_B = 0.4; W_B = 100 N

Let W_A = Smallest weight of block *A*.

We know that force of friction of block A, which is acting horizontally on the block *B*,

$$P = \mu_A.W_A = 0.3 \times W_A = 0.3\ W_A$$

and angle of friction of block *B*

$$\tan\phi = \mu_B = 0.4 \text{ or } \phi = 21°48'$$

We also know that the smallest force, which will hold the system in equilibrium (or will prevent the block B from sliding downwards),

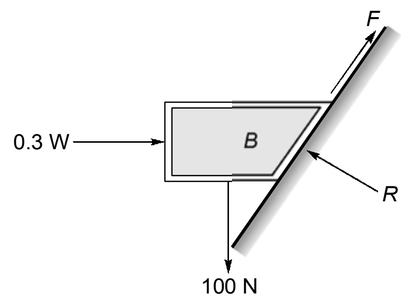
$$P = W_B \tan(\alpha - \phi) = 100 \tan(60° - 21°48') \text{ N}$$

$$0.3\ W_A = 100 \tan 38°12' = 100 \times 0.7869 = 78.69$$

$$\therefore \quad W_A = \frac{78.69}{0.3} = 262.3 \text{ N}$$

Alternative method

consider the equilibrium of the block B. We know that it is equilibrium under the action of the following four forces as shown in figure.



1. Its own weight 100 N
2. Normal reaction R,
3. Force of friction of block A (acting horizontally on B),

$$F_A = \mu_A \times W_A = 0.3\ W = 0.3\ W$$

4. Force of friction between the block B and inclined surface,

$$F = \mu_B \times R = 0.4R$$

Resolving the forces along the plane,

$$F = 100 \cos 30° - 0.3\ W \cos 60°$$

$$= 100 \times 0.866 - 0.3\ W \times 0.5$$

$$0.4R = 86.6 - 0.15\ W \quad \text{... (i)}$$

and now resolving the forces at right angles to the plane,

$$R = 0.3\ W \cos 30° + 100 \sin 30°$$

$$= 0.3\ W \times 0.866 + 100 \times 0.5$$

$$= 0.26\ W + 50 \quad \text{... (ii)}$$

Substituting the value of R in equation (i),

$$0.4(0.26\ W + 50) = 86.6 - 0.15\ W$$

$$0.104\ W + 20 = 86.6 - 0.15\ W$$

$$0.254\ W = 86.6 - 20 = 66.6$$

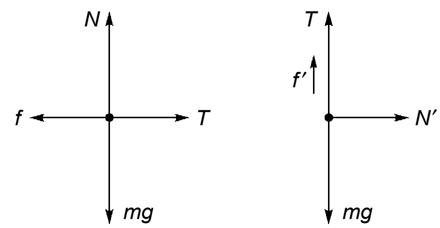
$$\therefore \quad W = \frac{66.6}{0.254} = 262.2 \text{ N}$$

Solution : 32

If no force is applied, the block A will slip on C towards right and the block B will move downward. Suppose the minimum force needed to prevent slipping is F. Taking $A + B + C$ as the system, the only external horizontal force on the system is F. Hence the acceleration of the system is

$$a = \frac{F}{M+2m} \quad \text{...(i)}$$

Now take the block A as the system. The forces on A are



(i) tension T by the string towards right,
(ii) friction f by the block C towards left,
(iii) weight mg downward and
(iv) normal force N upward

For vertical equilibrium $N = mg$

As the block moves towards right with an acceleration a,

$$T - f = ma$$

or, $$T - \mu\, mg = ma \quad \text{...(ii)}$$

Now take the block B as the system. The forces are
(i) tension T upward,
(ii) weight mg downward,
(iii) normal force N' towards right, and
(iv) friction f' upward

As the block moves towards right with an acceleration a,

$$N' = ma$$

As the friction is limiting, $f' = \mu N' = \mu\, ma$

For vertical equilibrium, $T + f' = mg$ ...(iii)

Eliminating T from (ii) and (iii),

$$a_{min} = \frac{1-\mu}{1+\mu} g$$

When a large force is applied the block A slips on C towards left and the block B slips on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is

$$a_{max} = \frac{1+\mu}{1-\mu} g$$

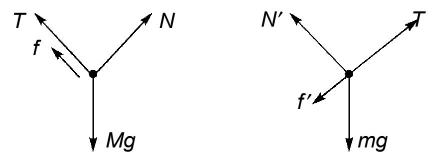
Thus, a lies between $\frac{1-\mu}{1+\mu} g$ and $\frac{1+\mu}{1-\mu} g$

From (i) the force F should be between

$\frac{1-\mu}{1+\mu}(M+2m)g$ and $\frac{1+\mu}{1-\mu}(M+2m)g$

Solution : 33

(a) Take the 2 kg block as the system. The forces on this block are shown in figure with $M = 2$ kg. It is assumed that m has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.



Taking components ⊥ to the incline

$$N = Mg\cos 45° = \frac{Mg}{\sqrt{2}}$$

Taking components || to the incline

$$T + f = Mg\sin 45° = \frac{Mg}{\sqrt{2}}$$

or,
$$T = \frac{Mg}{\sqrt{2}} - f$$

As it is a case of limiting equilibrium,

$$f = \mu_s N$$

or,
$$T = \frac{Mg}{\sqrt{2}} - \mu_s\frac{Mg}{\sqrt{2}} = \frac{Mg}{\sqrt{2}}(1-\mu_s) \quad \text{...(i)}$$

Now consider the other block as the system. The forces acting on this block are shown in figure. Taking components ⊥ to the incline,

$$N' = mg\cos 45° = \frac{mg}{\sqrt{2}}$$

Taking components || to the incline

$$T = mg\sin 45° + f' = \frac{mg}{\sqrt{2}} + f'$$

As it is the case of limiting equilibrium

$$f' = \mu_s N' = \mu_s\frac{mg}{\sqrt{2}}$$

Thus,
$$T = \frac{mg}{\sqrt{2}}(1+\mu_s) \quad \text{...(ii)}$$

From (i) and (ii),

$$m(1+\mu_s) = M(1-\mu_s) \quad \text{...(iii)}$$

or, $$m = \frac{(1-\mu_s)}{(1+\mu_s)}M = \frac{1-0.28}{1+0.28}\times 2\text{ kg} = \frac{9}{8}\text{ kg}$$

When maximum possible value of m is supplied, the directions of friction are reversed because m has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of m can be obtained from (iii) by putting $\mu = -0.28$. Thus, the maximum value of m is

$$m = \frac{1+0.28}{1-0.28}\times 2\text{ kg}$$

$$= \frac{32}{9}\text{ kg}$$

(b) If $m = \frac{9}{8}$ kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k \cdot \frac{Mg}{\sqrt{2}}$$

and $$f' = \frac{\mu_k mg}{\sqrt{2}}$$

where $\mu_k = 0.25$. If the acceleration is a, Newton's second law for M gives (figure)

$$Mg \sin 45° - T - f = Ma$$

or, $$\frac{Mg}{\sqrt{2}} - T - \frac{\mu_k Mg}{\sqrt{2}} = Ma \quad \text{...(iv)}$$

Applying Newton's second law for m gives (figure)

$$T - mg\sin 45° - f' = ma$$

or, $$T - \frac{mg}{\sqrt{2}} - \frac{\mu_k mg}{\sqrt{2}} = ma \quad \text{...(v)}$$

Adding (iv) and (v),

$$\frac{Mg}{\sqrt{2}}(1-\mu_k) - \frac{mg}{\sqrt{2}}(1+\mu_k) = (M+m)a$$

or, $$a = \left[\frac{M(1-\mu_k) - m(1+\mu_k)}{\sqrt{2}(M+m)}\right]g$$

$$= \left[\frac{2\times 0.75 - \left(\frac{9}{8}\right)\times 1.25}{\sqrt{2}\left(2+\frac{9}{8}\right)}\right]g = 0.208\text{ m/s}^2$$

■■■■