2020

RANK Improvement WORKBOOK

Answer key and Hint of Objective & Conventional *Questions*

Mechanical Engineering

Heat and Mass Transfer

MADE EASY Publications

1 Conduction

LEVEL 1 Objective Questions

1. (30)
2. (468)
3. (93.33)
4. (630)
5. (a)
6. (c)
7. (a)
8. (a)
9. (a)
10. (61.69)

LEVEL 2 Objective Questions

11. (a)
12. (82.18)
13. (140.42)
14. (15.94)
15. (0.044)
16. (57.36)
17. (b)
18. (d)
19. (b)
20. (a)
21. (b)
22. (a)
23. (0.188)
24. (42.76)
25. (a)
26. (c)
27. (209.77)
28. (a)
29. (a)
30. (12.5)
31. (12)

■■■■

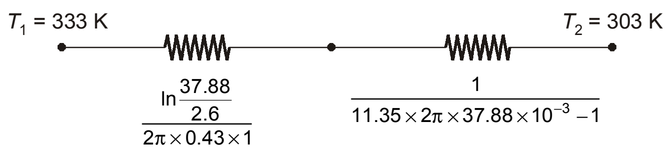
LEVEL 3 Conventional Questions

Solution : 32

$$r_c = \frac{k}{h} = \frac{0.43}{11.35} = 0.03788 \text{ m} = 37.88 \text{ mm}$$

So, Critical thickness $= 37.88 - 2.6 = 35.28$ mm

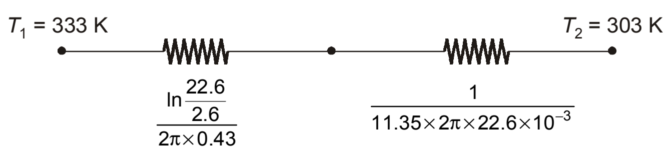
Heat loss with critical radius of insulation



$$\dot{q} = \frac{333 - 303}{\dfrac{\ln\dfrac{37.88}{2.6}}{2\pi \times 0.43 \times 1} + \dfrac{1}{11.35 \times 2\pi \times 37.88 \times 10^{-3}}}$$

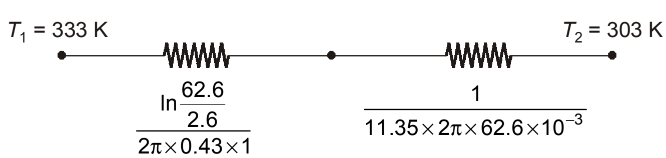
$$= \frac{30}{0.992 + 0.37} = 22.026 \text{ W}$$

Heat loss with 20 mm insulation



$$\dot{q} = \frac{30}{0.8004 + 0.6205} = 21.11 \text{ W}$$

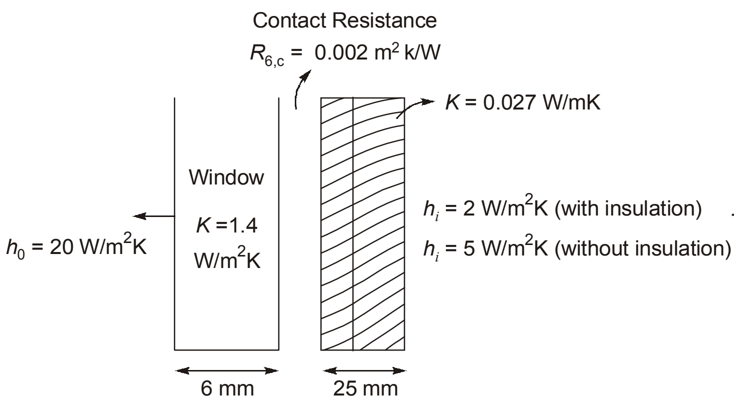
Heat loss with 60 mm insulation

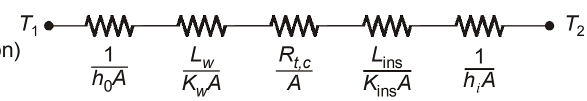


$$\dot{q} = \frac{30}{1.1775 + 0.224} = 21.41 \text{ W}$$

Solution : 33

The equivalent circuit can be drawn as follows



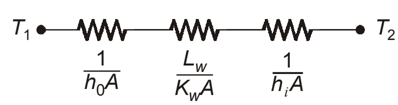


Now if insulation is used heat transfer per unit temperature change per unit area

$$\frac{Q}{A\Delta t} = \frac{1}{\left(\frac{1}{h_0}+\frac{L_w}{K_w}+R_{tc}+\frac{L_{ins}}{K_{ins}}+\frac{1}{h_i}\right)}$$

$$= \frac{1}{\left(\frac{1}{20}+\frac{0.006}{1.4}+0.002+\frac{25\times10^{-3}}{0.027}+\frac{1}{2}\right)} = 0.677 \text{ W/m}^2\text{K}$$

If there is no insulation



$$\frac{Q}{A\Delta t} = \frac{1}{\left(\frac{1}{h_0}+\frac{L_w}{K_w}+\frac{1}{h_i}\right)} = \frac{1}{\left(\frac{1}{20}+\frac{0.006}{1.4}+\frac{1}{5}\right)} = 3.933 \text{ W/m}^2\text{K}$$

(i) Reduction in heat loss

$$\Delta Q = 3.933 - 0.67 = 3.263 \text{ W/m}^2\text{K}$$

$$\% \text{ Reduction} = \frac{3.263}{3.933} = 82.96\ \%$$

(ii) For insulated, heat loss is

$$\frac{Q}{A\Delta T} = 0.67 \text{ W/m}^2\text{K}$$

If $A = 12 \text{ m}^2$

and $\Delta T = 20-(-12) = 32°\text{C}$

$\Rightarrow$ $Q = 0.67 \times 12 \times 32 = 257.28 \text{ W}$

Without insulation

$$\frac{Q}{A\Delta t} = 3.933$$

If $A = 12 \text{ m}^2$

and $\Delta T = 32$ K

$$Q = 3.933 \times 12 \times 32 = 1510.27 \text{ W}$$

(iii) Reduction in heat loss

$$= 3.263 \text{ W/m}^2\text{K}$$

For $A = 12 \text{ m}^2$

and $\Delta T = 32$ K

Reduction in heat loss $= 1.253 \times 10^{-3}$ MW

In 12 hours $\Rightarrow$ Energy lost due to no insulation $= 1.253 \times 10^{-3} \times 12 \times 60 \times 60 = 54.13$ MJ

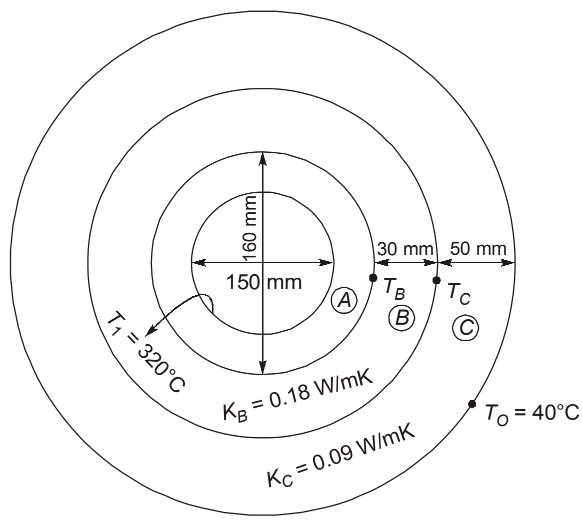
Efficiency of gas furnace = 0.8

$$\text{Heat energy required} = \frac{54.13}{0.8} = 67.66 \text{ MJ per day}$$

Daily savings = Rs. 67.66 per day

Solution : 34

Given, $r_{iA} = 75$ mm, $r_{oA} = 80$ mm, $r_{iB} = 80$ mm, $r_{oB} = 110$ mm, $r_{io} = 110$ mm, $r_{oC} = 160$ mm



Resistance circuit
$$R_B = \frac{\ln\left(\frac{r_{oB}}{r_{iB}}\right)}{2\pi k_B L}$$

$$R_A = \frac{\ln\left(\frac{r_{oA}}{r_{iA}}\right)}{2\pi k_{\text{pipe}} L}$$

$$R_C = \frac{\ln\left(\frac{r_{oC}}{r_{iC}}\right)}{2\pi k_C L}$$

$$R_{\text{total}} = R_A + R_B + R_C \text{ (Considering } L = 1 \text{ meter)}$$

$$= \frac{\ln(80/75)}{2\pi\times58\times1}+\frac{\ln(110/80)}{2\pi\times 0.18\times1}+\frac{\ln(160/110)}{2\pi\times0.09\times1} = 0.94435 \text{ K/Watt}$$

$$\dot{Q}_{\text{per meter}} = \frac{T_1-T_0}{R_{\text{total}}} = \frac{320-40}{0.94435} = 296.49 \text{ Watt /m}$$

Now to calculate T_B and T_C

$$296.49 = \frac{320-T_B}{R_A} \Rightarrow T_B = 319.94°C$$

Very small temperature gradient due to high conductivity of steel pipe.

Also,
$$Q = \frac{T_1-T_C}{R_A+R_B}$$

$$296.49 = \frac{320-T_C}{0.28175} \Rightarrow T_C = 236.46°\text{C}$$

(ii) Heat extracted by steam = 296.49 Watt

$$\dot{m}_{\text{steam}} = \frac{0.32}{60} = 5.333\times10^{-3} \text{ kg/sec}$$

Hence energy lost by 1 kg of steam $= \dfrac{296.49}{5.333\times10^{-3}} = 55.59$ kJ

So, the enthalpy of the steam coming out of one metre pipe is

$$h = 2703 - 55.59 = 2647.41 \text{ kJ/kg}$$

Now,
$$2647.71 = 1463 + x(1240) \Rightarrow x = 0.955$$

So the value of dryness fraction $x = 0.955$

Solution : 35

The temperature at a specified location at a given time can be determined from the Heisler charts.The half-thickness of the plate is $L = 0.02$ m. We have

$$\frac{1}{Bi} = \frac{k}{hL} = \frac{110}{120\times0.02} = 45.8$$

$$\text{Fo} = \frac{\alpha t}{L^2} = \frac{33.9\times10^{-6}\times7\times60}{(0.02)^2} = 35.6$$

When $\dfrac{1}{Bi} = 45.8$ and Fo = 35.6

$$\frac{T_c-T_\infty}{T_i-T_\infty} = 0.46$$

Also, $\dfrac{1}{Bi} = 45.8$ and $\xi = \dfrac{x}{L} = 1$

$$\frac{T_1-T_\infty}{T_c-T_\infty} = 0.99$$

where, T_1 is surface temperature of plates after a lapse of 7 min, T_c is temperature of plates at centre. T_i is surface temperature of plates when it leaves oven and T_∞ is an ambient temperature.

$$\frac{T_c - T_\infty}{T_i - T_\infty} \times \frac{T_1 - T_\infty}{T_c - T_\infty} = 0.46 \times 0.99 = 0.455$$

$\therefore$ $$T_1 = 0.455(20 - 500) + 500 = 281.6°\text{C}$$

Which is the surface temperature of the plates after a lapse of 7 min.

Solution : 36

Given that: Voltage, $V = 66$ kV = 66000 Volt, Current, $I = 900$ ampere, Wire dia, $d = 10$ mm = 0.01 m

$h = 10$ W/m^2-K, $t_\infty = 35°$C, Thermal conductivity, $k = 380$ W/m-K,

Resistivity, $\rho = 1.75 \times 10^{-6}$ Ω-cm = 1.75×10^{-8} Ω-m

So, as $$R = \rho \frac{l}{A_c}$$

or $$\frac{V}{I} = \rho \frac{l}{\frac{\pi}{4} d^2}$$

$\Rightarrow$ $$l = \frac{\pi V d^2}{4 I \rho}$$

$\Rightarrow$ $$l = 329119.23 \text{ m}$$

Heat generated, $Q = VI = 66000 \times 900 = 59.4$ MW

(i) Heat generation per unit volume, $$\dot{q}_g = \frac{Q}{\frac{\pi}{4} d^2 l}$$

$\Rightarrow$ $$\dot{q}_g = 2.298 \text{ MW/m}^3$$

(ii) Under steady state condition:

Heat generated in wire = Heat convected to surroundings

$\Rightarrow$ $$59.4 \times 10^6 = h \times \pi d l \times (t_s - t_\infty)$$
$$= 10 \times \pi \times 0.01 \times 329119.23 \times (t_s - 35)$$

Where t_s = Surface temperature of wire

$\Rightarrow$ $$t_s = 609.49°\text{C}$$

(iii) Maximum temperature in the line,

$$t_{max} = t_s + \frac{\dot{q}_g}{4k} R^2 = 609.49 + \frac{2.298 \times 10^6}{4 \times 380} \times \left(\frac{0.01}{2}\right)^2$$
$$= 609.49 + 0.038 = 609.528°\text{C}$$

Solution : 37

Given: Two insulation layers on a copper conductor:

$I = 1000$ A, $k = 390$ W/m.K, $d_1 = 25$ mm = 25×10^{-3} m, $r_1 = 12.5 \times 10^{-3}$ m, $\rho = 1.08 \times 10^{-8}$ Ω-m,

$r_2 = 17.5$ mm = 17.5×10^{-3} m, $k_1 = 0.058$ W/m.K, $k_2 = 0.42$ W/m.K, $T_\infty = 20°$C, $h = 20.5$ W/m^2.K

To find:

(i) thickness of plastic corresponds to minimum temperature in a cotton insulation.

(ii) temperature of copper rod and maximum temperature in plastic layer.

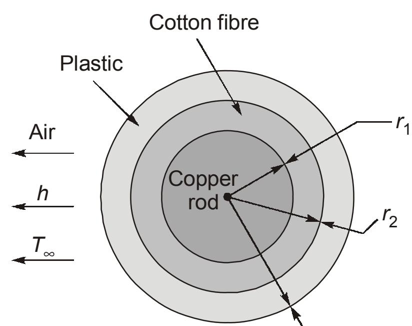
Assumptions:

(i) Steady state conditions.

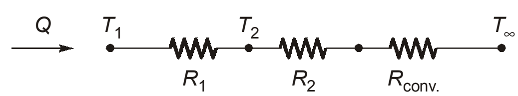
(ii) Heat transfer in radial direction only.

(iii) No contact resistance at interfaces.

(iv) 1 m length of copper conductor.



(a) Schematic



(b) Thermal network

Analysis:

(i) thickness of plastic layer for minimum temperature in cotton fibre insulation:

For given system the heat transfer rate is given as:

$$Q = \frac{\Delta T}{\Sigma R_{th}} = \frac{\Delta T}{\dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi L k_1} + \dfrac{\ln\left(\dfrac{r_3}{r_2}\right)}{2\pi L k_2} + \dfrac{1}{(2\pi r_3 L)h}}$$

For maximum heat transfer rate through plastic layer, which give minimum temperature of cotton insulation.

Differentiating above equation w.r.t. r_3

$$\frac{dQ}{dr_3} = 0 = \Delta T\left[\frac{0 + \dfrac{1}{2\pi L k_2}\left(\dfrac{r_2}{r_3}\right)\times\left(\dfrac{1}{r_2}\right) - \dfrac{1}{2\pi L h}\left(\dfrac{1}{r_3^2}\right)}{\left\{\dfrac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi L k_1} + \dfrac{\ln\left(\dfrac{r_3}{r_2}\right)}{2\pi L k_2} + \dfrac{1}{2\pi r_3 L h}\right\}^2}\right]$$

or $$\frac{1}{r_3 k_2} = \frac{1}{r_3^2 h} \text{ or } r_3 = \frac{k_2}{h}$$ (Condition of critical radius of insulation)

$$r_3 = \frac{0.42}{20.5} = 0.020488 \text{ m} = 20.488 \text{ mm}$$

Thickness of plastic layer $= r_3 - r_2 = 20.488 - 17.5 = 2.988$ mm

(ii) Temperature of copper rod and maximum temperature in plastic layer:

(a) Temperature of copper rod:

The resistance of 1 m copper conductor

$$R_e = \frac{\rho L}{A_c} = \frac{\rho L}{\left(\frac{\pi}{4}\right)d_1^2} = \frac{1.08\times10^{-8}\times1}{\left(\frac{\pi}{4}\right)\times(25\times10^{-3})^2}$$

$$= 2.2 \times 10^{-5}\ \Omega\text{/m}$$

Heat generation rate per metre, in copper conductor

$$Q = I^2R_e = (1000)^2 \times 2.2 \times 10^{-5}$$

$$= 22.0 \text{ W/m}$$

The individual resistance in thermal network

$$R_1 = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi L k_1} = \frac{\ln\left(\frac{17.5}{12.5}\right)}{2\pi\times1\times0.058} = 0.923 \text{ K/W}$$

$$R_2 = \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi L k_2} = \frac{\ln\left(\frac{20.488}{17.5}\right)}{2\pi\times1\times0.42} = 0.0597 \text{ K/W}$$

$$R_{\text{conv}} = \frac{1}{2\pi r_3 L h} = \frac{1}{2\pi\times20.488\times10^{-3}\times1\times20.5}$$

$$= 0.379 \text{ K/W}$$

The heat flow rate, $Q = \dfrac{\Delta T}{\Sigma R_{th}} = \dfrac{(T_1 - T_\infty)}{R_1 + R_2 + R_{conv.}}$

or $\quad T_1 - 20 = 22 \times (0.923 + 0.0597 + 0.379)$

or $\quad T_1 = 20 + 29.9574 = 49.9574°\text{C}$

(b) Maximum temperature in plastic layer:

It will occur at cotton fibre insulation and plastic layer interface, say it is T_2. From thermal network

$$Q = \frac{T_1 - T_2}{R_1} \text{ or } 49.95 - T_2 = 22 \times 0.923$$

or $\quad T_2 = 49.9574 - 20.306 = 29.6514°\text{C}$

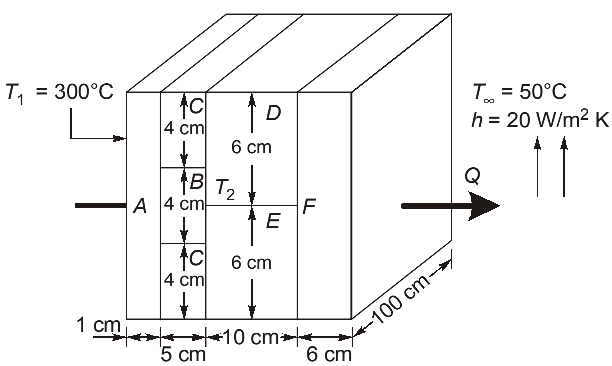
Solution : 38

Given: A composite wall

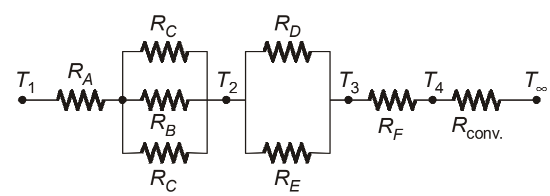
$$A_{wall} = 5 \text{ m} \times 8 \text{ m} = 40 \text{ m}^2$$

For representative cross-section of wall

$$k_A = k_F = 2 \text{ W/m.K}, \qquad k_B = 8 \text{ W/m.K},$$
$$k_C = 20 \text{ W/m.K}, \qquad k_D = 15 \text{ W/m.K},$$
$$k_E = 35 \text{ W/m.K}, \qquad L_A = 1 \text{ cm} = 0.01 \text{ m},$$
$$L_B = L_C = 5 \text{ cm} = 0.05 \text{ m}, \qquad L_D = L_E = 10 \text{ cm} = 0.1 \text{ m},$$
$$L_F = 6 \text{ cm} = 0.06 \text{ m},$$
$$z = 6 \text{ cm} + 6 \text{ cm} = 12 \text{ cm} = 0.12 \text{ m},$$
$$z_B = z_C = 4 \text{ cm} = 0.04 \text{ m} \qquad w = 100 \text{ cm} = 1 \text{ m},$$
$$z_D = z_E = 6 \text{ cm} = 0.06 \text{ m} \qquad T_1 = 300°\text{C}$$
$$T_\infty = 50°\text{C} \qquad h = 20 \text{ W/m}^2.\text{K}$$



(a) Schematic



(b) Equivalent thermal network

To find:

(i) Heat transfer rate through composite wall,

(ii) Temperature T_2 at the point where section B, D and E meet, and

(iii) Temperature drop across the slab F.

Assumptions:

(i) Steady state heat conduction.

(ii) No contact resistance at interfaces.

(iii) No radiation heat transfer in the system.

Analysis:

(i) Equivalent thermal network for given composite wall is shown in above figure (b).

The cross-section area of representative portion of wall

$$A_1 = w \times z = (1 \text{ m}) \times (0.12 \text{ m}) = 0.12 \text{ m}^2$$

Area for section B and C,

$$A_2 = w \times z_B = 1 \text{ m} \times 0.04 = 0.04 \text{ m}^2$$

Area for section D and E,

$$A_3 = w \times z_D = 1\text{ m} \times 0.06\text{ m} = 0.06\text{ m}^2$$

Individual thermal resistances of thermal network

$$R_A = \frac{L_A}{k_A A_1} = \frac{0.01}{2 \times 0.12} = 0.04167\text{ K/W}$$

$$R_B = \frac{L_B}{k_B A_2} = \frac{0.05}{8 \times 0.04} = 0.15625\text{ K/W}$$

$$R_C = \frac{L_C}{k_C A_2} = \frac{0.05}{20 \times 0.04} = 0.0625\text{ K/W}$$

$$R_D = \frac{L_E}{k_D A_3} = \frac{0.1}{15 \times 0.06} = 0.111\text{ K/W}$$

$$R_E = \frac{L_E}{k_E A_3} = \frac{0.1}{35 \times 0.06} = 0.0476\text{ K/W}$$

$$R_F = \frac{L_F}{k_F A_1} = \frac{0.06}{2 \times 0.12} = 0.25\text{ K/W}$$

$$R_{\text{Conv.}} = \frac{1}{hA_1} = \frac{1}{20 \times 0.12} = 0.4167\text{ K/W}$$

The resistance R_B and R_C are parallel as shown in thermal network figure (b), their equivalent resistance

$$\frac{1}{R_{eq1}} = \frac{1}{R_C} + \frac{1}{R_B} + \frac{1}{R_C} = \frac{1}{0.0625} + \frac{1}{0.15625} + \frac{1}{0.0625}$$

$$= \frac{1}{38.4} = 0.0260\text{ K/W}$$

Further the resistance R_D and R_E are parallel as shown and their equivalent resistance:

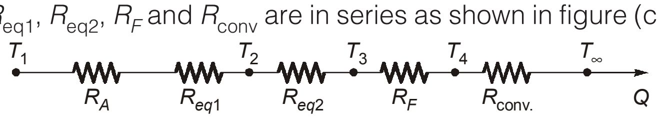
$$\frac{1}{R_{eq2}} = \frac{1}{R_D} + \frac{1}{R_E} = \frac{1}{0.111} + \frac{1}{0.0476}$$

$$= 30.017\text{ W/K}$$

$\therefore$

$$\frac{1}{R_{eq2}} = \frac{1}{30.017} = 0.0333\text{ K/W}$$

Now resistance R_A, R_{eq1}, R_{eq2}, R_F and R_{conv} are in series as shown in figure (c).



(Modified thermal network)

$$\Sigma R_{\text{th}} = R_A + R_{\text{eq1}} + R_{\text{eq2}} + R_F + R_{\text{conv.}}$$

$$= 0.04167 + 0.0260 + 0.0333 + 0.25 + 0.4167$$

$$= 0.76767\text{ K/W}$$

Heat flow rate in representative section of wall

$$Q = \frac{T_s - T_\infty}{\Sigma R_{th}} = \frac{300 - 50}{0.7677} = 325.66 \text{ W}$$

Heat transfer rate from total surface of the wall

$$= Q \times \frac{\text{Wall area}}{\text{Representative area}} = Q \times \frac{A_{\text{wall}}}{A_1}$$

$$= 325.66 \times \frac{40}{0.12}$$

$$= 108.55 \times 10^3 \text{ W} = 108.55 \text{ kW.}$$

(ii) Temperature T_2:

Considering the heat flow through resistance R_A and R_{eq1},

$$Q = \frac{T_s - T_2}{R_A + R_{eq1}}$$

or
$$T_2 = T_s - Q \times (R_A + R_{\text{eq1}})$$
$$= 300 - 325.66 \times (0.04167 + 0.0260)$$
$$= 300 - 22.03 = 278°\text{C}$$

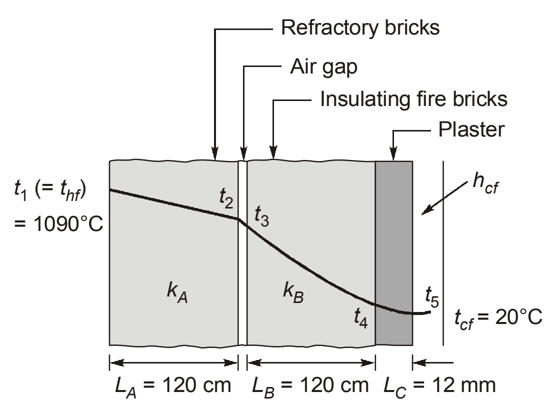
(iii) Temperature drop across slab F:

The heat flow through slab F can be expressed as:

$$Q = \frac{T_3 - T_4}{R_F}$$

or
$$T_3 - T_4 = QR_F$$
$$= 325.66 \times 0.25 = 81.415°\text{C}$$

Solution : 39



Thickness of refractory brick, $L_A = 1200 \text{ mm} = 0.12 \text{ m}$

Thickness of insulating fire brick

$$L_B = 120 \text{ mm} = 0.12 \text{ m}$$

Thickness of plaster, $L_C = 12 \text{ mm} = 0.012 \text{ m}$

Heat transfer coefficient from the outside wall surface to the air in the room,

$$h_{cf} = 18 \text{ W/m}^2\text{°C}$$

Resistance of air gap to heat flow = 0.16 K/W

Thermal conductivities:

$$\text{Refractory brick, } k_A = 1.6 \text{ W/m}^2\text{°C}$$

$$\text{Insulating fire brick, } k_B = 0.3 \text{ W/m}^2\text{°C}$$

$$\text{Plaster, } k_C = 0.14 \text{ W/m°C}$$

Temperature: $t_{hf} = 1090\text{°C}$; $t_{cf} = 20\text{°C}$

Consider 1 m^2 of surface area.

(i) Rate of heat loss per m^2 of surface area, q:

$$q = \frac{(t_{hf} - t_{cf})}{\frac{L_A}{k_A} + \text{air gap resistance} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

$$= \frac{(1090 - 20)}{\frac{0.12}{1.6} + 0.16 + \frac{0.12}{0.3} + \frac{0.012}{0.14} + \frac{1}{18}}$$

$$= \frac{1070}{0.075 + 0.16 + 0.4 + 0.0857 + 0.0555}$$

$$= 1378.5 \text{ W or } 1.3785 \text{ kW}$$

i.e., Rate of heat loss per m^2 of surface area

$$= 1.3785 \text{ kW}$$

(ii) Temperatures at interfaces, t_2, t_3, t_4:

$$Q = 1378.5 = \frac{1090 - t_2}{\frac{L_A}{k_A}} = \frac{1090 - t_2}{\frac{0.12}{1.6}} = \frac{1090 - t_2}{0.075}$$

$\therefore$ $$t_2 = 1090 - 1378.5 \times 0.075 = 986.6\text{°C}$$

$$Q = 1378.5 = \frac{t_2 - t_3}{\text{air gap resistance}} = \frac{986.6 - t_3}{0.16}$$

$\Rightarrow$ $$t_3 = 986.6 - 220.56 = 766.04\text{°C}$$

Also, $$Q = 1378.5 = \frac{t_3 - t_4}{\frac{L_B}{k_B}} = \frac{766.04 - t_4}{\frac{0.12}{0.3}} = \frac{766.04 - t_4}{0.4}$$

$\therefore$ $$t_4 = 766.04 - 1378.5 \times 0.4 = 214.64\text{°C}$$

(iii) Temperature of the outside surface of the wall, t_5:

$$Q = 1378.5 = \frac{t_4 - t_5}{\frac{L_C}{k_C}} = \frac{214.64 - t_5}{\frac{0.012}{0.14}} = \frac{214.64 - t_5}{0.0857}$$

$\therefore$ $$t_5 = 214.64 - 1378.5 \times 0.0857 = 96.5\text{°C}$$

Solution : 40

Critical radius of insulation

$$r_c = \frac{k}{h_a}$$

or,
$$r_c = \frac{0.20\times 100}{15} = 1.33 \text{ cm}$$

Critical thickness of insulation $= 1.33 - 1 = 0.33$ cm

Heat loss from the tube without insulation

$$Q_{\text{with}} = \frac{T_1 - T_\infty}{\frac{1}{2\pi kL}\ln\frac{r_o}{r_i} + \frac{1}{2\pi r_o L h_a}}$$

$$= \frac{2\pi r_o Lh(T_1 - T_\infty)}{1+\frac{r_o h_a}{k}\ln\frac{r_o}{r_i}}$$

Heat loss from the tube without insulation

$$Q_{\text{without}} = 2\pi r_i L h_a (T_1 - T_\infty)$$

∴ The ratio

$$\frac{Q_{\text{with}}}{Q_{\text{without}}} = \left[1+\frac{r_o h_a}{k}\ln\frac{r_o}{r_i}\right]^{-1} \quad \text{...(a)}$$

(a) If
$$r_o = r_c = \frac{k}{h_a}$$

$$\frac{Q_{\text{with}}}{Q_{\text{without}}} = \frac{r_o}{r_i}\left[1+\ln\frac{r_o}{r_i}\right]^{-1} \quad \text{...(b)}$$

$$= \frac{1.33}{1.0}[1+\ln 1.33]^{-1} = 1.035$$

Thus, heat loss is increased by 3.5% in spite of the fact that there is an insulation of thickness 0.33 cm.

(b) If another layer of 2 cm thickness of insulation is added, $r_o = 1.33 + 2 = 3.33$ cm. Substituting in equation (a).

$$\frac{Q_{\text{with}}}{Q_{\text{without}}} = \frac{3.33}{1.00}\left[1+\frac{0.0333\times 15}{0.20}\ln\frac{3.33}{1.0}\right]^{-1} = 0.8316$$

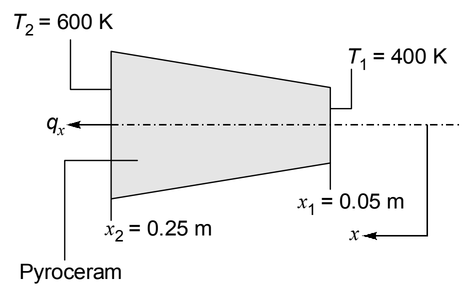
Insulation of 2.33 cm thickness reduces the heat loss by 16.84%.

Solution : 41

Known: Conduction in a circular conical section having diameter $D = ax$, where $a = 0.25$.]

Find:

1. Temperature distribution $T(x)$.
2. Heat transfer rate q_x.



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in the x-direction.
3. No internal heat generation
4. Constant properties.

Properties: Pyroceram (500 K): $k = 3.46$ W/m.K

Analysis

Since heat conduction occurs under steady-state, one-dimensional conditions with no internal heat generation, the heat transfer rate q_x is a constant independent of x. Accordingly, Fourier's law, Equation 2.1, may be used to determine the temperature distribution.

$$q_x = -kA\frac{dT}{dx}$$

where $A = \frac{\pi D^2}{4} = \frac{\pi a^2 x^2}{4}$. Separating variables,

$$\frac{4q_x dx}{\pi a^2 x^2} = -kdT$$

Integrating $q_x g$ from x_1 to any x within the cone, and recalling that q_x and k are constants, it follows that

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^2} = -\int_{T_1}^{T} dT$$

Hence $$\frac{4q_x}{\pi a^2}\left(-\frac{1}{x}+\frac{1}{x_1}\right) = -k(T - T_1)$$

or solving for T $$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x}\right)$$

Although q_x is a constant, it is as yet as unknown. However, it may be determined by evaluating the above expression at $x = x_2$, where $T(x_2) = T_2$. Hence

$$T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x_2}\right)$$

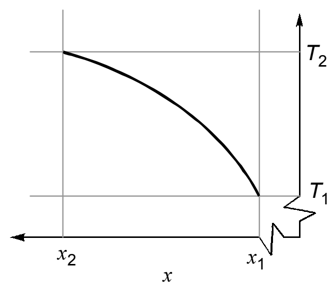
and solving for q_x

$$q_x = \frac{\pi a^2 k(T_1 - T_2)}{4\left[\left(\frac{1}{x_1}\right) - \left(\frac{1}{x_2}\right)\right]}$$

Substituting for q_x into the expression for $T(x)$, the temperature distribution becomes

$$T(x) = T_1 + (T_1 - T_2)\left[\frac{\left(\frac{1}{x}\right) - \left(\frac{1}{x_1}\right)}{\left(\frac{1}{x_1}\right) - \left(\frac{1}{x_2}\right)}\right]$$

From this result, temperature may be calculated as a function of x and the distribution is as shown.



Note that, since $\frac{dT}{dx} = \frac{-4q_x}{k\pi a^2 x^2}$ from Fourier's law, it follows that the temperature gradient and heat flux decrease with increasing x.

2. Substituting numerical values into the foregoing result for the heat transfer rate, it follows that

$$q_x = \frac{\pi(0.25)^2 \times 3.46 \text{W/m} \times \text{K}(400 - 600)\text{K}}{4\left(\frac{1}{0.05\text{ m}} - \frac{1}{0.25\text{ m}}\right)}$$

Comments: When the parameter a increases, the cross-sectional area changes more rapdily with distance, causing the one-dimensional assumption t become less appropriate.

■■■■

2 Heat Transfer from Extended Surface (FINS)

LEVEL 1 Objective Questions

1.	(b)
2.	(b)
3.	(d)
4.	(b)
5.	(c)
6.	(c)
7.	(25.66)
8.	(b)
9.	(d)
10.	(a)
11.	(c)

LEVEL 2 Objective Questions

12.	(c)
13.	(d)
14.	(b)
15.	(a)
16.	(a)
17.	(c)
18.	(4)
19.	(51.136)

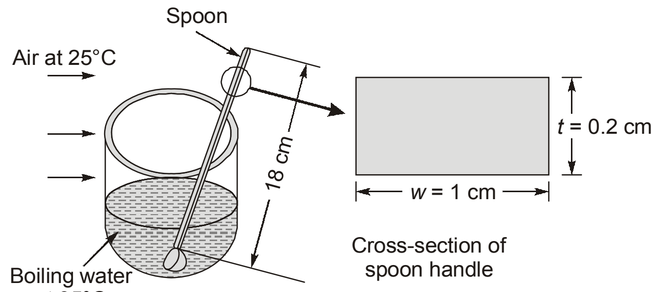
■■■■

LEVEL 3 Conventional Questions

Solution : 20

Given handle of a stainless steel spoon:

$$k = 15.1\text{ W/mK}, \qquad T_0 = 95°\text{C}$$
$$T_\infty = 25°\text{C} \qquad A_c = 0.2\text{ cm} \times 1\text{ cm}$$
$$L = 18\text{ cm} = 0.18\text{ m}, \qquad h = 15\text{ W/m}^2\text{K}$$



(Schematic)

To find: The temperature difference across exposed surface of handle.

Assumptions:
1. Steady state conditions.
2. The handle of spoon is thin and heat loss from its free end be negligible.
3. No heat generation.
4. Constant cross-section of handle.
5. Constant properties.

Analysis: The temperature distribution for insulated tip fin is given by equation

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

For temperature difference across the exposed surface of spoon handle i.e. $x = L$;

$$\frac{T_L - T_\infty}{T_0 - T_\infty} = \frac{1}{\cosh mL}$$

where,
$$m = \sqrt{\frac{hP}{kA_c}}$$

$$P = 2(w + t) = 2\,(1\text{ cm} + 0.2\text{ cm})$$
$$= 2.4\text{ cm} = 0.024\text{ m}$$
$$A_c = wt = 1\text{ cm} \times 0.2\text{ cm} = 0.2\text{ cm}^2 = 0.2 \times 10^{-4}\text{ m}^2$$

$\therefore$
$$m = \sqrt{\frac{15 \times 0.024}{15.1 \times 0.2 \times 10^{-4}}} = 34.526\text{ m}^{-1}$$
$$mL = 34.526 \times 0.18 = 6.2147$$
$$\cosh mL = 250.0$$

Then, $$T_L - T_\infty = \frac{95-25}{250} = 0.2799 = 0.28°C$$

or $$T_L = 25 + 0.28 = 25.28°C,$$

Thus, $$T_0 - T_L = 95 - 25.28 = 69.72°C$$

Solution : 21

Given: A cylinder with longitudinal fins

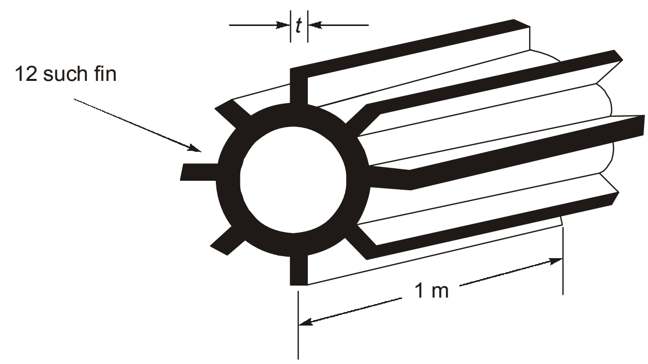
$w = 1$ $\quad$ $d = 5$ cm $= 0.05$ m,

$T_\infty = 40°C$ $\quad$ $N_{fin} = 12,$

$h = 23.3$ W/m^{2}K, $\quad$ $t = 0.75$ mm $= 0.75 \times 10^{-3}$ m,

$L = 2.5$ cm $= 2.5 \times 10^{-2}$ m,

$T_0 = 150°C,$ $\quad$ $k = 75$ W/mK



To find: The heat transfer rate from surface.

Analysis: The fin is of finite length, it will dissipate heat by convection from its tip. Therefore, using corrected length.

$$L_c = L + \frac{t}{2} = 2.5 \times 10^{-2} + \frac{0.75 \times 10^{-3}}{2} = 0.025375 \text{ m}$$

$$P = 2w = 2 \text{ m}$$

$$A_c = w \times t = 1 \times 0.75 \times 10^{-3} = 0.75 \times 10^{-3} \text{ m}^2$$

$$mL_c = L_c\sqrt{\frac{hP}{kA_c}} = 0.025375 \times \sqrt{\frac{23.3 \times 2}{75 \times 0.75 \times 10^{-3}}} = 0.73$$

The heat transfer rate from fins surface,

$$Q_{fin} = N_{fin}\sqrt{hPkA_c}(T_0 - T_\infty)\tanh(mL_c)$$

$$= 12 \times \sqrt{23.3 \times 2 \times 75 \times 0.75 \times 10^{-3}} \times (150 - 40) \times \tanh(0.73)$$

$$= 1332 \text{ W}$$

The heat transfer rate from unfinned (base) area

$$Q_{unfin} = hA_{unfin}(T_0 - T_\infty)$$

$$= h(\pi dw - 12 \times A_c)(T_0 - T_\infty)$$

$$= 23.3 \times (\pi \times 0.05 \times 1 - 12 \times 0.75 \times 10^{-3}) \times (150 - 40)$$

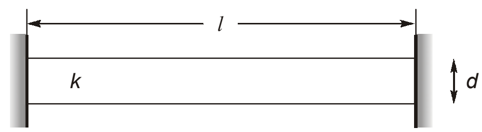
$$= 379.5 \text{ W}$$

Hence the total heat transfer $= Q_{unfin} + Q_{fin}$

$$= 379.5 + 1332 = 1711.5 \text{ W}$$

Solution : 22

Given: T_0 = 150°C, T_∞ = 36°C, k = 41.5 W/mk, d = 15 mm, L = 160 mm, h = 25 W/m^{2}k



Heat flow from insulated tip is given by

$$Q = \sqrt{hPkA} \cdot \theta_0 \tanh(mL)$$

where $P = \pi d$

and $A = \frac{\pi}{4} d^2$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 25}{41.5 \times 0.015}} = 12.67 \text{ m}^{-1}$$

$$Q = \sqrt{h \cdot P \cdot kA} \cdot (T_0 - T_\infty) \cdot \tanh(mL)$$

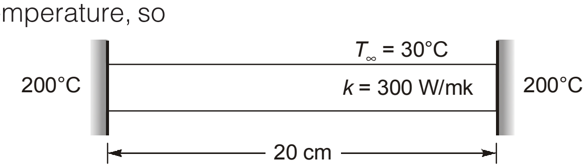
$$= \sqrt{25 \times (\pi \times 0.015) \times 41.5 \times \frac{\pi}{4} \times 0.015^2} \times (150 - 36) \times \tan(12.67 \times 0.16)$$

$$= 10.6 \times 0.965 = 10.235 \text{ Watts}$$

So the rate of heat flowing out of the hot surface through the rod is 10.235 W.

Solution : 23

Such rod can be considered equivalent to two pin fins attached to each other at $x = \frac{L}{2}$, but here both fins are attached to same temperature, so



$$Q = 2 \times \text{Heat transfer rate from one pin fin of 10 cm long} \left(\text{i.e., } \frac{L}{2}\right)$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{15 \times \pi \times 0.01}{300 \times \frac{\pi}{4} \times 0.01^2}} = 4.47 \text{ m}^{-1}$$

$\therefore$

$$Q = 2 \times \left\{ \sqrt{hPkA}(T_0 - T_\infty) \tanh\left(mf\left(\frac{L}{2}\right)\right)\right\}$$

$$= 2\times\left\{\sqrt{15\times\pi\times 0.01\times 300\times\frac{\pi}{4}\times 0.01^2}\times(200-30)\times\tanh(4.47\times 0.1)\right\}$$

$$= 2 \times 7.514 = 15.028 \text{ Watts}$$

Solution : 24

Given: A finite long fin with an equilateral triangle cross-section.

Side, a = 5 mm, L = 80 mm,

k = 54 W/m.K. T_0 = 400°C

T_∞ = 50°C, h = 90 W/m^{2}K

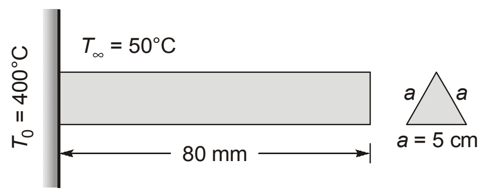


Fig. 5.16 (Schematic)

To find: The heat dissipation rate from the fin.

Assumptions:

1. Steady state conditions.
2. Length of fin is very large in comparison to its cross-section, thus treating fins as infinite long fin.

Analysis: For triangular fin

Cross-section area,

$$A_c = \frac{1}{2}\text{ base}\times\text{height} = \frac{1}{2}a\times\frac{\sqrt{3}}{2}a$$

$$= \frac{\sqrt{3}}{4}\times(5\times 10^{-3})^2 = 1.0825\times 10^{-5}\text{ m}^2$$

Perimeter, $P = 3a = 3\times 5\times 10^{-3} = 0.015$ m

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{90\times 0.015}{54\times 1.0825\times 10^{-5}}}$$

$$= 48.06\text{ m}^{-1}$$

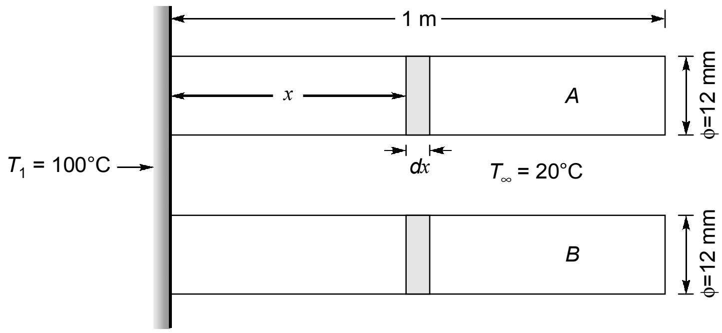
The heat transfer rate from infinite long fin

$$Q_{\text{fin}} = \sqrt{hPkA_c}(T_0 - T_\infty)$$

$$= \sqrt{90\times 0.015\times 54\times 1.0825\times 10^{-5}}(400-50)$$

$$= 9.82\text{ W}$$

Solution : 25



Base of both the rods is maintained at the same temperature as shown in figure.

Assumption:

(i) Since the length of the rods is very large 1 m as compared to their diameter, they can be assumed to be infinitely long with temperature at tip approaching surrounding temperature.

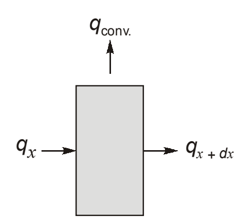
(ii) Heat transfer is purely one dimensional conduction in the rods and convection from rod surface.

Given: $h = 5\ \text{W/m}^2\text{K}$, $K_A = 60$ W/mK, $A_c = \dfrac{\pi d^2}{4} = 1.131\times 10^{-4}\text{m}^2$, $p = 2\pi r = 0.0377$ m

Consider an element of thickness dx at a distance x from the base on any rod.

Energy balance for this section

$$q_x - q_{x+dx} = q_{\text{conv}}$$



$$\Rightarrow \quad \frac{kA_c d^2T dx}{dx^2} - h(pdx)(T - T_\infty) = 0$$

$$\Rightarrow \quad \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where, $\theta = T - T_\infty$ and $m = \sqrt{\dfrac{hp}{kA_c}}$

Solution of this differential equation with following boundary conditions:

(i) $T\big|_{x=0} = T_1$

(ii) Infinitely long rod assumption due to which $T\big|_{x=L} = T_\infty$

Solution of this boundary condition

$$\theta = \theta_b e^{-mx}$$

For rod A:

$$m_A = \sqrt{\frac{hp}{k_A A_c}} = 5.271 \text{ as mL} = 5.271 \geq 2.65 \text{ so it is } \infty \text{ long fin.}$$

Temperature at 15 cm from base

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-m\times 0.15}$$

$$\Rightarrow \quad T = 329.29 \text{ K}$$

This temperature is equal to temperature of rod B at a distance 7.5 cm from base.

For rod b:

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-m_B \times 0.075}$$

$$\Rightarrow \quad \frac{329.29 - 293}{373 - 293} = e^{-m_B \times 0.075}$$

We know, $m_B = \sqrt{\dfrac{hp}{k_B A_C}} = 10.5398$

$$\Rightarrow \quad m_B = 10.5398$$

$$\Rightarrow \quad k_B = 15 \text{ W/mK}$$

Heat transfer for a rod in this case= $\sqrt{hpkA_C} \cdot (T_b - T_\infty)$

$\therefore$ Ratio of heat transfer for rod A and B

$$\frac{Q_A}{Q_B} = \frac{\sqrt{hpk_A A_c} \cdot (T_b - T_\infty)}{\sqrt{hpk_B A_c} \cdot (T_b - T_\infty)}$$

$$\Rightarrow \quad \frac{Q_A}{Q_B} = \sqrt{\frac{k_A}{k_B}} = 2$$

Solution : 26

$$\theta = \frac{\sin hm(L - x)\theta_1 + \sin hmx\theta_2}{\sin hmL}$$

$$Q_1 = -kA\left(\frac{d\theta}{dx}\right)_{x=0} = -kA\,(-58.22 \times 5.77e^{5.77x} - 238.22 \times 5.77e^{-5.77x})_{x=0}$$

$$= 0.06 \times 4 \times 10^{-4}\,(335.93 + 1374.53) = 0.0410 \text{ kW} = 41.0 \text{ W}$$

$$Q_2 = -kA\left(\frac{d\theta}{dx}\right)_{x=l} = -kA\,(-58.22 \times 5.77 \times 1.78 - 238.22 \times 5.77 \times 0.561)$$

$$= 0.06 \times 4 \times 10^{-4}\,(598 + 771.5) = 0.0328 \text{ kW} = 32.8 \text{ W}$$

Heat flow rate from the bar to the surroundings

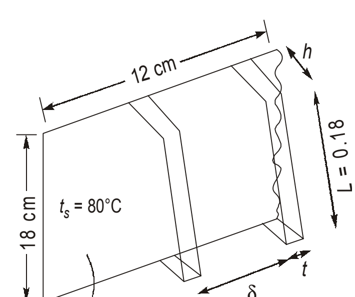
$$= Q_1 - Q_2 = 41.0 - 32.8 = 8.2 \text{ W}$$

Solution : 27

Given that:

Width 'B' = 12 cm = 0.12 m

Length 'L' = 18 cm = 0.18 m



Fins:

$$\text{Thickness } 't' = 0.1 \text{ cm} = 0.001 \text{ m}$$

$$\text{Length } l_f = 2.4 \text{ cm} = 0.024 \text{ m}$$

As for air at mean temperature, $t_m = 325.5$ K

$$k = 0.0279 \text{ W/mK}$$

$$\nu = 1.82 \times 10^{-5} \text{ m}^2\text{/s}$$

$$Pr = 0.709$$

$$\Rightarrow \quad Gr = \frac{g\beta\Delta t L^3}{\nu^2},$$

As $$T_{mean} = 325.5 \text{ K} = 52.5°\text{C} = \frac{t_s + t_\infty}{2}$$

$$t_s = 80°\text{C}$$

$$\Rightarrow \quad t_\infty = 25°\text{C}$$

$$\Rightarrow \quad Gr = \frac{9.81 \times \dfrac{1}{325.5} \times (80-25) \times (0.18)^3}{(1.82 \times 10^{-5})^2} = 2.9184 \times 10^7$$

$$\Rightarrow \quad Ra = Gr \times Pr = 2.0692 \times 10^7$$

(i) Optimum fin spacing, $$\delta = 2.714 \times \left(\frac{L}{Ra^{1/4}}\right) = 2.714 \times \left[\frac{0.18}{(2.0692 \times 10^7)^{1/4}}\right] = 7.243 \times 10^{-3} \text{ m}$$

or $$\delta = 7.243 \text{ mm}$$

Heat transfer coefficient for optimum spacing

$$h = 1.307 \times \frac{k}{\delta} = 1.307 \times \frac{0.0279}{0.007243}$$

$$= 5.0344 \text{ W/m}^2\text{K}$$

$$\text{Number of fins } 'n' = \frac{W}{\delta + t} = \frac{120}{7.243 + 1} = 14.55$$

$$\Rightarrow \quad n \simeq 15 \text{ fins}$$

$$\text{Overall surface area, } 'A' = \text{Unfinned area} + \text{Fin surface area}$$

$$= (B - nt) \times 0.18 + (0.18 \times 0.024 \times 2 \times 15)$$

$$= (0.12 - 15 \times 0.001) \times 0.18 + 0.1296$$

$$= 0.0189 + 0.1296$$

$$= 0.1485 \text{ m}^2$$ [Neglect tip area]

$$\text{Heat transfer, } 'Q' = hA(t_s - t_\infty)$$

$$= 5.0344 \times 0.1485 \times (80 - 25)$$

$$\Rightarrow \quad Q = 41.118 \text{ W}$$

■■■■

3 Convection

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(c)
4.	(1.2)
5.	(70.71)
6.	(a)
7.	(a)
8.	(6.80)
9.	(90)
10.	(5.943)
11.	(0.474)
12.	(b)
13.	(a)
14.	(c)
15.	(1.25)
16.	(b)
17.	(a)
18.	(b)
19.	(27.46)
20.	(a)
21.	(0.06)

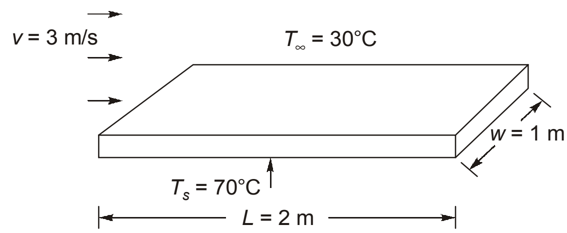
LEVEL 2 Objective Questions

22.	(a)
23.	(c)
24.	(c)
25.	(c)
26.	(d)
27.	(2)
28.	(a)
29.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 30



Consider the case of forced convection between heated plate and air is shown in figure.

Characteristic length for a flat plate,

$$L_c = 2 \text{ m (length of plate)}$$

Area of heat transfer, $A_s = 2 \text{ m} \times 1 \text{ m} = 2 \text{ m}^2$

Film temperature at which properties of fluid are to be evaluated,

$$T_f = \frac{T_s + T_\infty}{2} = \frac{70+30}{2} = 50°\text{C}$$

Reynolds number for flow $= \text{Re}_L = \dfrac{vL_c}{\nu} = \dfrac{3\times 2}{17.95\times 10^{-6}} = 334261.81 \simeq 33.4262 \times 10^4$

Prandtl number for fluid $= \dfrac{\nu}{\alpha} = \dfrac{\nu}{K}\cdot \rho C_p = \dfrac{17.95\times 10^{-6}\times 1.093\times 1005}{0.0283} = 0.6967$

Nusselt number $= \text{Nu} = 0.664\,\text{Re}^{1/2}\,\text{Pr}^{1/3} = 340.33$

We know, $\text{Nu} = \dfrac{hL}{K} = 340.33$

$\Rightarrow$ $h = \dfrac{340.33\times 0.0283}{2} = 4.816 \text{ W/m}^2\text{K}$

$\therefore$ Rate of heat transfer between the plate and air is

$$Q = h\,A_s\,(T_s - T_\infty) = 4.816 \times 2 \times 40 = 385.25 \text{ Watt}$$

Solution : 31

$$\text{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu} = \frac{80\times 1}{14.15\times 10^{-6}} = 5.65 \times 10^6$$

For turbulent boundary layer the boundary layer thickness is given by

$$\delta = \frac{0.371\,L}{(\text{Re}_L)^{1/5}} = \frac{0.371\times 1}{(5.65\times 10^6)^{0.2}} = 0.01655 \text{ m} = 16.55 \text{ mm}$$

Prandtl number, $\text{Pr} = \dfrac{\mu C_p}{k} = \dfrac{\rho\nu C_p}{k} = \dfrac{1.25\times 14.15\times 10^{-6}\times 1000}{0.022} = 0.804$

$$N\bar{u} = \frac{\bar{h}L}{k} = 0.036\,(\text{Ra}_L)^{0.8}\,(\text{Pr})^{1/3}$$

$$= 0.036\,(5.65 \times 106)^{0.8}\,(0.804)^{1/3} = 8437$$

$\therefore \quad \bar{h} = \dfrac{8437 \times 0.022}{1} = 185.61 \text{ W/m}^2\text{k}$

Thus, heat flow from the surface of the plate (only one side)

$$= \bar{h}A(T_w - T_\infty)$$

$$= 185.61 \times (1 \times 1)(50 - 10) = 7424.5 \text{ watts}$$

Solution : 32

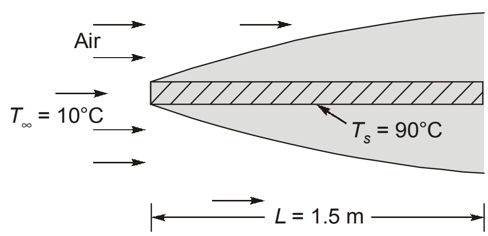
Given: Flow along a flat plate:

$L = 1.5,$ $\quad w = 1$ m,

$T_\infty = 10°\text{C},$ $\quad T_s = 90°\text{C},$

$Q = 3.75$ kW

To find: The velocity of air,



(Schematic)

Assumptions:

1. No radiation heat exchange.
2. Steady-state heat transfer conditions.
3. Air flow on both sides of the plate.

Analysis: The heat transfer rate by convection is given by,

$$Q = hA_s(\Delta T)$$

For two sides of the plate,

$$3.75 \times 10^3 = 2h \times (1.5 \times 1) \times (90 - 10) \qquad (\because \; A_s = 2 \text{ sides} \times 1.5 \times 1 \text{ m}^2)$$

or, $\quad h = 15.625 \text{ W/m}^2\text{ K}$

The Nusselt number, $\quad Nu_L = \dfrac{hL}{k_f} = \dfrac{15.625 \times 1.5}{0.028} = 837.05$

Assuming the laminar flow along the plate:

$$Nu_L = 0.664 \,\text{Re}^{1/2}\, Pr^{1/3}$$

or, $\quad 837.05 = 0.664\,\text{Re}^{1/2} \times (0.703)^{1/3}$

or, $\quad \text{Re}_L = 2.01 \times 10^6$

The Reynolds number Re_L is greater than critical Reynolds number 5×10^5, hence assumption made is wrong. The fluid flow is turbulent, using the relation,

$$Nu_L = [0.036\,\text{Re}^{0.8} - 836]\, Pr^{1/3}$$

Using the values, $\quad 837.05 = [0.036\,\text{Re}^{0.8} - 836] \times (0.703)^{1/3}$

or, $$Re_L^{0.8} = 49371.8$$

or, $$Re_L = 7.36 \times 10^5$$

Assumption made is correct. The velocity of air:

$$Re_L = \frac{\rho u_\infty L}{\mu}$$

$$u_\infty = \frac{\mu Re}{\rho L} = \frac{2.029 \times 10^{-5} \times 7.36 \times 10^5}{1.0877 \times 1.5} = 9.15 \text{ m/s}$$

Solution : 33

Given: $T_\infty = 275$ K, $u_\infty = 20$ m/s,
$L = 1.5$ m, $w = 1$ m,
$T_s = 325$ K, $Re_{cr} = 2 \times 10^5$

To find:

(a) The average h over the region of laminar boundary layer.

(b) The average h over the entire length of 1.5 m.

(c) The total heat transfer rate from the plate to the air over 1.5 m length and 1 m wide.

Assumptions:

1. No heat exchange by thermal radiation and heat conduction.
2. The stead-state heat transfer.
3. Air and surface temperatures are different, taking the properties at mean film temperature.

Properties of air: The film temperature,

$$T_f = \frac{T_s + T_\infty}{2} = \frac{275 + 325}{2} = 300 \text{ K}$$

The properties of air at 300 K,

$k_f = 0.026$ W/m-K, $Pr = 0.708$,
$\nu = 16.8 \times 10^{-6}$ m²/s, $\mu = 1.98 \times 10^{-5}$ kg/ms

Analysis: The Reynolds number,

$$Re_L = \frac{\mu_\infty L}{v} = \frac{20 \times 1.5}{16.8 \times 10^{-6}} = 1.785 \times 10^6$$

or, $Re_L > 2 \times 10^5$, hence flow is turbulent at

$$x = 1.5 \text{ m}$$

The critical length of flow for laminar boundary layer can be calculated by using critical Reynolds number.

$$Re_{cr} = \frac{u_\infty x_{cr}}{\nu}$$

or, $$x_{cr} = \frac{2 \times 10^5 \times 16.8 \times 10^{-6}}{20} = 0.168 \text{ m}$$

(a) The average heat transfer coefficient for the laminary boundary layer:

$$Nu_x = \frac{h x_{cr}}{k_f} = 0.664 \text{Re}_{cr}^{1/2} Pr^{1/3}$$

$$h = 0.664 \frac{k_f}{x_{cr}} \text{Re}_{cr}^{1/2} Pr^{1/3}$$

$$= \frac{0.664 \times 0.026}{0.168} \times (2 \times 10^5)^{1/2} \times (0.708)^{1/3}$$

$$= 40.96 \text{ W/m}^2\text{-K}$$

(c) The total heat transfer rate,

$$Q = h A_s(\Delta T) = (43.877 \text{ W/m}^2 \text{ K}) \times (1.5 \text{ m} \times 1 \text{ m}) \times (325 - 275)$$

$$= 3290.775 \text{ W}$$

Solution : 34

Given: Flow over a electric bulb:

$T_\infty = 27°\text{C}$, $u_\infty = 0.3$ m/s,

$P = 100$ W, $T_s = 127°\text{C}$,

$D = 60$ mm $= 0.06$ m

To find:

(i) The heat transfer rate.

(ii) Percentage of power lost due to convection.

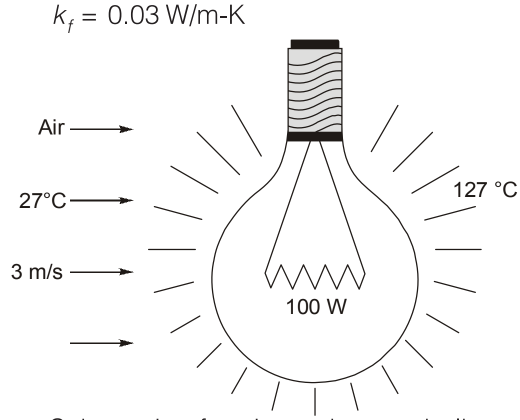
Properties of fluid: The film temperature,

$$T_f = \frac{T_s + T_\infty}{2} = \frac{127 + 27}{2} = 77°\text{C} = 350 \text{ K}$$

The properties of air at 77°C are

$$\nu = 2.09 \times 10^{-5} \text{ m}^2/\text{s}$$

$$k_f = 0.03 \text{ W/m-K}$$



Schematic of an incandescent bulb

Assumptions:

1. Spherical shape of the electric bulb.
2. No radiation heat exchange.
3. The steady-state heat transfer conditions.

Analysis: The Reynolds number,

$$Re_D = \frac{u_\infty D}{\nu} = \frac{0.3 \times 0.06}{2.09 \times 10^{-5}} = 861.24$$

$$Nu_D = 0.37\, Re_D^{0.6} = 0.37 \times (861.24)^{0.6} = 21.35$$

The heat transfer coefficient,

$$h = \frac{k_f}{D} Nu_D = \frac{0.03}{0.06} \times 21.35 = 10.675 \text{ W/m}^2\text{-K}$$

(i) The heat transfer rate,

$$Q = h A_s (T_s - T_\infty) = h(\pi D_2)(T_s - T_\infty)$$

$$= (10.675 \text{ W/m}^2\text{-K}) \times \pi \times (0.06 \text{ m})^2 \times (127 - 27) = 12.073 \text{ W}$$

(ii) The percentage of heat lost by forced convection

$$= \frac{Q}{P} \times 100 = \frac{12.073}{100} \times 100 = 12.073\%$$

Solution : 35

Velocity of air, $U_\infty = 30$ m/s

$$\text{Reynolds number, } Re_L = \frac{U_\infty L}{\nu} = \frac{30 \times L}{18.97 \times 10^{-6}} = 1.5814 \times 10^6 L \quad \text{...(i)}$$

As we know,

$$\text{Drag force, } F_D = C_D \times \frac{1}{2} \rho U_\infty^2 \times A$$

$$10.5 = \frac{0.0742}{(1.5814 \times 10^6 L)^{1/5}} \times \frac{1}{2} \times 1.06 \times 30^2 \times L^2$$

$$10.5 = 2.0376\, L^{(2-0.2)}$$

$$L^{1.8} = 5.1532$$

$\therefore$ $L = (5.1582)^{1/1.8}$

$\Rightarrow$ $L = 2.486$ m

From (i)

$$Re_L = 1.5814 \times 10^6 \times 2.486$$

$$Re_L = 3.932 \times 10^6$$

$$C_D = \frac{0.0742}{(3.932 \times 10^6)^{1/5}}$$

$\Rightarrow$ $C_D = 3.5602 \times 10^{-3}$

By Colburn analogy,

$$StPr^{2/3} = \frac{C_D}{2}$$

$$\frac{\bar{h}}{\rho U_\infty c_p}Pr^{2/3} = \frac{C_D}{2}$$

$$\Rightarrow \quad \bar{h} = \frac{3.5602\times10^{-3}}{2}\times1.06\times30\times1.005\times1000\times\frac{1}{(0.696)^{2/3}}$$

Heat transfer coefficient, $\bar{h} = 72.438$ W/m²K

Heat loss from plate surface,

$$Q = \bar{h}A(T - T_\infty)$$

$$= 72.438 \times (2.486)^2 \times (95 - 25) = 31337.55 \text{ W}$$

$$Q = 31.337 \text{ kW}$$

Solution : 36

Given: A circular disc in different configuration exposed to air:

$$D = 0.2 \text{ m},$$

$$T_s = 130°\text{C},$$

$$T_\infty = 25°\text{C}.$$

To find: The heat transfer rate from the disc when;

(i) horizontal with hot surface facing down,

(ii) horizontal with hot surface facing up and,

(iii) vertical,

Properties of fluid: The mean film temperature

$$T_f = \frac{T_s + T_\infty}{2} = \frac{130 + 25}{2} = 77.5°\text{C} = 350.5 \text{ K}$$

The physical properties of air:

$$\nu = 2.08 \times 10^{-5} \text{ m}^2/\text{s},$$

$$Pr = 0.697$$

$$k_{air} = 0.03 \text{ W/mK},$$

$$\beta = \frac{1}{T_f} = \frac{1}{350.5}\text{K}^{-1}$$

Analysis: The Grashoff number with characteristic length L_c:

$$Gr = \frac{g\beta\Delta TL_c^3}{v^3} = \frac{9.81\times\left(\frac{1}{350.5}\right)\times(130-25)\times L_c^3}{(2.08\times10^{-5})^2}$$

$$= 6.79 \times 10^9 L_c^3$$

The Rayleigh number, $\quad \text{Ra} = \text{Gr Pr} = (6.79 \times 10^9\, L_c^{\,3}) \times (0.697)$

$$= 4.734 \times 10^9\, L_c^{\,3}$$

(i) For horizontal disc facing down:

$$L_c = \frac{A_s}{P} = \frac{(\pi/4)D^2}{\pi D} = \frac{D}{4} = 0.05\,\text{m}$$

Hence, $\quad \text{Ra} = 4.734 \times 10^9 \times (0.05)^3 = 591.81 \times 10^3$

Thus the flow is laminar, and for horizontal disc facing down, the correlation

$$\text{Nu} = 0.27(\text{Ra})^{1/4} = 0.27 \times (591.81 \times 10^3)^{1/4} = 7.488$$

The average heat transfer coefficient of air,

$$h_1 = \frac{k_{air}}{L_c} Nu = \frac{0.03}{0.05} \times 14.977 = 8.986\ \text{W/m}^2\text{K}$$

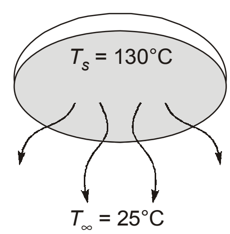


Fig. (i) (Horizontal disc facing down)

The heat transfer rate from the disc:

$$Q_1 = h_1 A_s(T_s - T_\infty) = 8.986 \times (\pi/4) \times (0.2)^2 \times (130 - 25)$$

$$= 29.64\ \text{W}$$

(ii) For horizontal disc from facing up:

The significant length remains same.

Hence, $\quad \text{Ra} = 4.734 \times 10^9 \times (0.05)^3 = 591.81 \times 10^3$

Thus the flow is laminar, and for horizontal disc facing up the correlation

$$\text{Nu} = 0.54(\text{Ra})^{1/4} = 0.54 \times (591.81 \times 10^3)^{1/4} = 14.977$$

The average heat transfer coefficient,

$$h_2 = \frac{k_{air}}{L_c} Nu = \frac{0.03}{0.05} \times 7.488 = 4.493\ \text{W/m}^2\text{K}$$

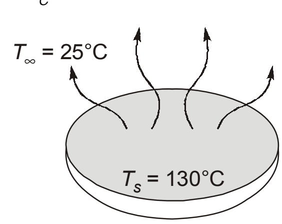


Fig. (ii) (Horizontal disc facing up)

The heat transfer rate from the disc:

$$Q_2 = h_2 A_s(T_s - T_\infty) = 4.493 \times (\pi/4) \times (0.2)^2 \times (130 - 25)$$

$$= 14.82\ \text{W}$$

(iii) For vertical disc:

$$L_c = D = 0.2 \text{ m}$$

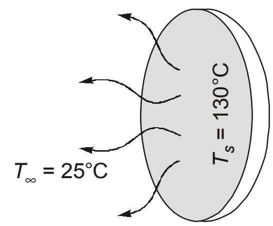


Fig. (iii) (Vertical disc)

Hence, $$\text{Ra}_D = 4.734 \times 10^9 \times (0.2)^3 = 37.872 \times 10^6$$

Thus the flow is laminar, and for vertical disc

$$\text{Nu} = 0.59(\text{Ra}_D)^{1/4} = 0.59 \times (37.852 \times 10^6)^{1/4} = 46.28$$

The average heat transfer coefficient,

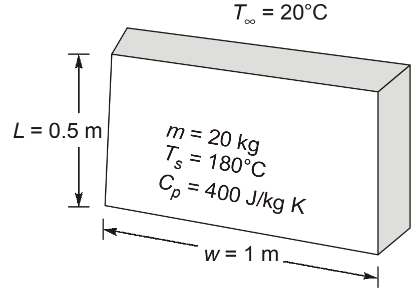
$$h_3 = \frac{k_{air}}{L_c} Nu_D = \frac{0.03}{0.2} \times 46.28 = 6.94 \text{ W/m}^2\text{K}$$

The heat transfer rate from the disc;

$$Q_3 = h_3 A_s(T_s - T_\infty) = 6.94 \times (\pi/4) \times (0.2)^2 \times (130 - 25)$$
$$= 22.9 \text{ W}$$

Solution : 37

Given: $L = 0.5$ m, $w = 1$ m, $T_s = 180$°C, $T_\infty = 20$°C, $m = 20$ kg, $C = 400$ J/kg K



To find:

(i) The heat transfer coefficient.

(ii) Initial rate of cooling of the plate in °C/min.

(iii) Time required to cool the plate from 80°C.

Properties of fluid: The mean film temperature

$$T_f = \frac{T_s - T_\infty}{2} = \frac{180 + 20}{2} = 100°\text{C} = 373 \text{ K}$$

The properties of air,

$$\beta = \frac{1}{373} = 2.68 \times 10^{-3} \text{ K}^{-1}$$

$$\nu = 23.18 \times 10^{-6} \text{ m}^2/\text{s}$$
$$k_f = 0.0321 \text{ W/mK}$$
$$\text{Pr} = 0.688$$

Assumption:

1. Radiation heat transfer is negligible.
2. Heat transfer from both sides of plate.
3. Transient heat conduction.
4. Constant properties.

Analysis:

(i) The characteristic length

$$L_c = L = 0.5 \text{ m}$$

The Grashoff number,

$$\text{Gr}_L = \frac{g\beta\Delta T L_c^3}{\nu^2} = \frac{9.81\times\left(2.68\times10^{-3}\right)\times(180-20)\times(0.5)^3}{(23.18\times10^{-6})^2}$$
$$= 978.95 \times 10^6$$

Rayleigh number, $\text{Ra}_L = \text{Gr}_L \text{ Pr} = (978.95 \times 10^6 \times 0.688)$

$$= 673.52 \times 10^6$$

The boundary layer is laminar, hence using the relation,

$$\text{Nu}_L = 0.59(\text{Ra}_L)^{1/4} = 0.59 \times (673.52 \times 10^6)^{1/4} = 95.047$$

The average value of heat transfer coefficient

$$h = Nu_L \frac{k_f}{L_c} = 95.047 \times \frac{0.0321}{0.5} = 6.1 \text{ W/m}^2\text{K}$$

(ii) The initial rate of cooling can be obtained by energy balance as

Rate of decrease of internal energy = Rate of heat convection from the plate

or $$-mC\frac{dT}{dt} = hA\,(T_s - T_\infty)$$

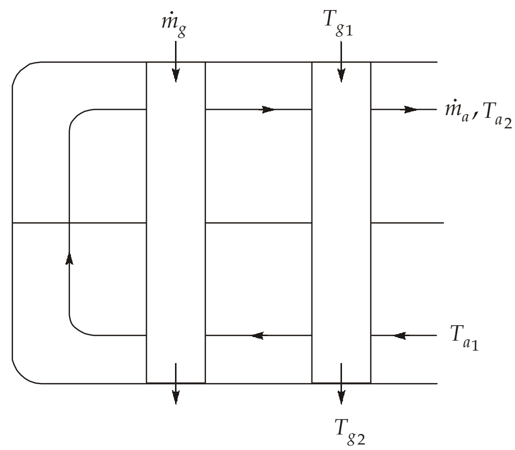
or $$\frac{dT}{dt} = \frac{6.1\times(2\times1\times0.5\text{ m}^2)\times(180-20)}{20\times400} = -0.122°\text{C/s.}$$
$$= -7.32°\text{C/min.}$$

(iii) The time taken by plate to cool to 80°C:

$$\frac{T_s - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{hA_s t}{\rho VC}\right] = \exp\left[-\frac{hA_s t}{mC}\right]$$

or $$t = \frac{-mC}{hA_s}\ln\left[\frac{T_s - T_\infty}{T_i - T_\infty}\right] = -\frac{20\times400}{6.1\times2\times1\times0.5}\times\ln\left[\frac{80-20}{180-20}\right]$$
$$= 1286 \text{ s} = 21.44 \text{ min}$$

Solution : 38



Energy balance,

$$Q = \dot{m}_g c_{p_g}(T_{g1} - T_{g2}) = \dot{m}_a c_{p_a}(T_{a2} - T_{a1})$$

$$= 15 \times 1.11(400 - T_{g2}) = 20 \times 1.01(250 - 30)$$

$$\Rightarrow \quad T_{g2} = 133.09°\text{C}$$

For gas,

$$\text{Re}_g = \left(\frac{u_m d_i}{\nu}\right)_g = \left(\frac{12 \times 0.050}{41.2 \times 10^{-6}}\right) = 14563.1068$$

For turbulent flow through pipe,

$$Nu_g = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3}$$

$$= 0.023(14563.1068)^{0.8}(0.66)^{0.3}$$

$$\Rightarrow \quad 43.4705 = \frac{h_i d_i}{k_g}$$

$$h_i = \frac{43.4705 \times 0.0454}{0.050} = 39.471 \text{ W/m}^2\text{K}$$

For air,

$$\text{Re}_q = \left(\frac{u_m d_o}{\nu}\right)_a = \left(\frac{10 \times 0.052}{28.3 \times 10^{-6}}\right) = 18374.558$$

For cross flow,

$$Nu_a = 0.41\,\text{Re}^{0.6}\,\text{Pr}^{0.33}$$

$$= 0.41(18374.558)^{0.6}(0.684)^{0.33}$$

$$\Rightarrow \quad 130.883 = \frac{h_o d_o}{k_a}$$

$$h_o = \frac{130.883 \times 0.0352}{0.052}$$

$$h_o = 88.598 \text{ W/m}^2\text{K}$$

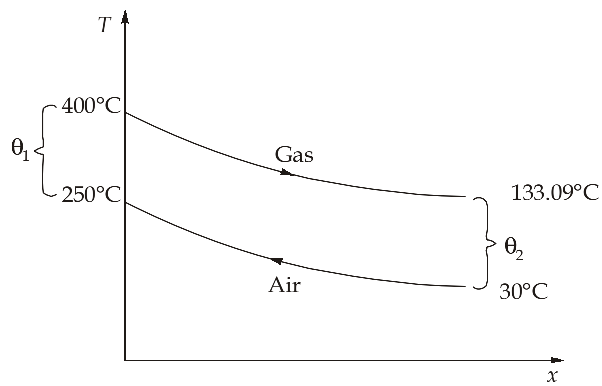
$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o}$$

$$\frac{1}{U_o} = \frac{1}{39.471} \times \frac{52}{50} + \frac{1}{88.598}$$

$$U_o = 26.571 \text{ W/m}^2\text{K}$$

$$\theta_1 = 400 - 250 = 150°\text{C}$$

$$\theta_2 = 133.09 - 30 = 103.09°\text{C}$$



Log mean temperature difference,

$$\Delta T_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{150 - 103.09}{\ln(150/103.09)} = 125.082°\text{C}$$

Now, $$Q = \dot{m}_a c_{p_a}(T_{a2} - T_{a1})$$

$$= 20 \times 1.01(250 - 30)$$

$\Rightarrow$ $$4444 \text{ kW} = U_o A_o \Delta T_m F$$

$\Rightarrow$ $$4444 \times 10^3 \text{ W} = 26.571 \times A_o \times 125.082 \times 0.88$$

$$A_o = 1519.458 \text{ m}^2$$

Solution : 39

$$\text{Re}_D = \frac{uD}{\nu} = \frac{12 \times 0.06}{0.805 \times 10^{-6}} = 0.894 \times 10^6$$

Since $\text{Re}_D > 2000$, so the flow is turbulent.

Using the Dittus-Boelter equation:

$$Nu_D = \frac{hd}{k} = 0.023(\text{Re}_D)^{0.8}\,\text{Pr}^{0.4}$$

$$\frac{\bar{h}D}{k} = 0.023\,(894000)^{0.8}\,(5.42)^{0.4}$$

$$\frac{\bar{h} \times 0.06}{0.61718} = 0.023 \times 57685.95 \times 1.966$$

$\therefore$ $$\bar{h} = 26832.32 \text{ W/m}^2\text{K}$$

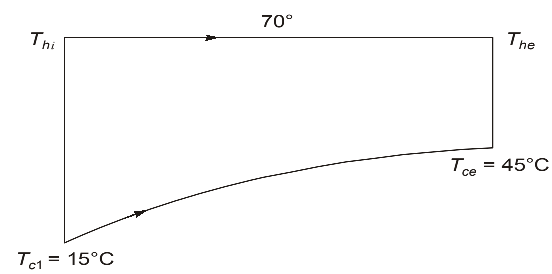
Heat transferred, $$Q = \dot{m}C_p(T_{b_2} - T_{b_1})$$

$$= \rho\left(\frac{\pi}{4}D^2 \times u\right)C_p(T_{b_2} - T_{b_1})$$

$$= 995.7 \times \frac{\pi}{4} \times 0.06^2 \times 12 \times 4.174 \times (45 - 15) = 4230355 \text{ W}$$

In order to find length of the tube, we know that temp. difference between hot and cold fluid is varying throughout the length of the tube, so we have

$$Q = \bar{h}(\pi DL)(\Delta T_m)$$



where, $$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

$$\Delta T_m = \frac{55 - 25}{\ln\left(\frac{55}{25}\right)} = 38.049°\text{C}$$

$\therefore$ $$L = \frac{4230{,}355}{(26832.32 \times \pi \times 0.06 \times 38.049)} = 21.98 \text{ m}$$

Solution : 40

The property values are taken at 20°C

ρ = 1000 kg/m^3, ν = 1.006 × 10^{-6} m^2/s, Pr = 7.02,

k = 0.5978 W/mK, c = 1178 J/kgK, P_{50} = 990 kg/m^3.

v_{50} = 5675 × 10^{-6} m^2/s

Friction factor is found from pressure drop and analogy is used to solve from in

$$\Delta P = f \cdot \rho \frac{L}{D}\frac{u^2}{2},\ u = 0.001 \times \frac{4}{\pi} \times 0.025^2 \text{ m/s} = 2.0372 \text{ m/s}$$

Substituting $$f = 7000 \times 0.025 \times \frac{2}{1000} \times 1.5\left(\frac{0.001 \times 4}{\pi \times 0.025^2}\right) = 0.05622$$

$$\text{Re} = uD/\nu = 2.0372 \times \frac{0.025}{1.006} \times 10^{-6} = 50626$$

If we assume smooth pipe, $f = (1.82 \log \text{Re} - 1.64)^{-2} = 0.02087$

So the pipe considered should be a rough pipe,

Using Colburn analogy

$$\therefore \qquad \text{StPr}^{2/3} = \frac{f}{8}$$

$$\text{Nu} = \left(\frac{f}{8}\right)\text{Re.Pr}^{1/3} = \frac{0.05622}{8} 50626 \times 7.02^{0.333}$$

$$= 681.22$$

Solution : 41

The Reynolds numberis

$$Re_D = \frac{VD\rho}{\pi} = \frac{4\dot{m}}{\pi D\mu} = \frac{41(1.26\text{ kg/s})}{p(0.012\text{m})(11.16\times10^{-4}(\text{Ns})/\text{m}^2)(\text{kgm})/(\text{Ns}^2)}$$

$$Re_D Pr = 1.2 \times 10^5(0.013) = 1557 > 100$$

Therefore, equation can be applied to calculate the Nusselt Number

$$\overline{Nu_D} = 0.82 + 0.0185\,(Re_D Pr)^{0.827} = 4.82 + 0.0185(1557)0.827 = 12.9$$

$$\bar{h}_e = \overline{Nu_D}\frac{k}{D} = 12.9\frac{(11.66\text{ W/(mK)})}{0.012\ m} = 1.25 \times 10^4 \text{ W/(m}^2\text{K)}$$

(b) The maximum allowable heat flux is determined byt he outlet conditions. the outlet wall temperature must not be higher than the mercury boiling point

$$\frac{q}{A} = (T_{\text{wall, max}} - T_{b,\ \text{out}})$$

$$\bar{h}_c = (355°\text{C} - 230°\text{C})(1.25 \times 10^4 \text{ W/(m}^2\text{K)}) = 1.57 \times 10^2 \text{ W/(m}^2\text{K)}$$

(a) The length of the tube required can be calculated from the following

$$q = \dot{m}c(T_{b,out} - T_{b,in}) = \frac{q}{A}(\pi DL)$$

Solving for the length

$$L = \frac{\dot{m}c(T_{b,out} - T_{b,in})}{\frac{q}{A}\pi D} = \frac{(1.26\text{kg/s})(140.6\text{ J/(kgK)})(230°\text{C} - 90°\text{C})}{(1.57\times10^6\text{ W/(m}^2\text{K))(J/(Ws))}\pi(0.012\text{m})}$$

Comments:

Note that L/D = 0.419 m/0.012 m = 35 > 30, therefore, the assumption of fully developed flow and use of equation is valid.

■■■■

4 Heat Exchanger

LEVEL 1 Objective Questions

1.	(d)
2.	(c)
3.	(a)
4.	(c)
5.	(c)
6.	(b)
7.	(d)
8.	(1.5)
9.	(a)
10.	(c)
11.	(0.56)
12.	(0.25)
13.	(3)
14.	(a)

LEVEL 2 Objective Questions

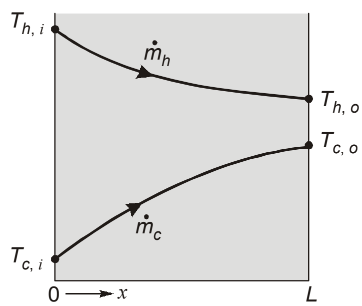
15.	(1.57)
16.	(1.62)
17.	(a)
18.	(c)
19.	(0.857)
20.	(40)
21.	(0.25)
22.	(19.7)
23.	(b)
24.	(15.7)
25.	(c)
26.	(a)
27.	(0.533)
28.	(a)
29.	(a)
30.	(c)

■■■■

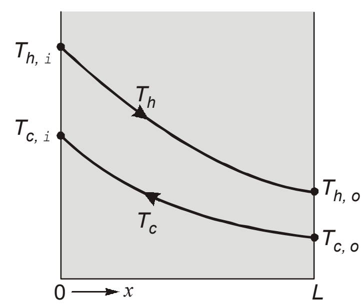
LEVEL 3 Conventional Questions

Solution : 31

Given: A double pipe heat exchanger, with $T_{h,i} = 150°C$, $T_{h,o} = 90°C$ and $T_{c,i} = 30°C$, $\dot{m}_h C_{p,h} = 0.5\dot{m}_c C_{p,c}$ and $T_{h,o\,\text{parallel}} = T_{h,o,\text{counter}}$.



(a) Parallel flow mode



(b) Counter flow mode

Analysis: The heat capacity rates for two fluids are related as:

$$\dot{m}_h C_{p,h} = 0.5\dot{m}_c C_{p,c}$$

which yields to, $$C_{\min} = \dot{m}_h C_{p,h} = C_h$$

$$C_{\max} = \dot{m}_c C_{p,c} = C_c$$

and capacity ratio, $$C = 0.5$$

The effectiveness of heat exchanger,

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})}$$

Using, $C_h = 0.5\,C_c$ or $C_c = 2C_h$, then

$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{2(T_{c,o} - T_{c,i})}{T_{h,i} - T_{c,i}}$$

or, $$T_{c,o} = T_{c,i} + 0.5(T_{h,i} - T_{h,o}) \quad \text{...(i)}$$

Let, A_{PF} = Area of parallel flow arrangement

A_{CF} = Area of counter flow arrangement

As, $T_{h,o,\text{ parallel}} = T_{h,o,\text{ counter}}$, therefore, the heat lost by hot fluid in both case be same.

Further, the heat transfer rate is also expressed as,

$$Q = UA_{PF}\Delta T_{lm,\text{ parallel}} = UA_{CF}\Delta T_{lm,\text{ counter}}$$

or, $$\frac{A_{CF}}{A_{PF}} = \frac{\Delta T_{lm},\text{parallel}}{\Delta T_{lm},\text{counter}}$$ ($\because$ U is constant in both cases)

(a) For counter flow arrangement,

$$\Delta T_1 = T_{h,i} - T_{c,o} = T_{h,i} - T_{c,i} - 0.5T(T_{h,i} - T_{h,o})$$

$$= 0.5T_{h,i} + 0.5T_{h,o} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

$$\Delta T_{lm,\,\text{counter}} = \frac{\Delta T_1 - \Delta T_2}{ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{0.5T_{h,i} + 0.5T_{h,o} - T_{c,i} - T_{h,o} + T_{c,i}}{ln\left\{\dfrac{0.5(T_{h,i} + T_{h,o}) - T_{c,i}}{T_{h,o} - T_{c,i}}\right\}}$$

$$= \frac{0.5(T_{h,i} - T_{h,o})}{ln\left\{\dfrac{0.5(T_{h,i} + T_{h,o}) - T_{c,i}}{T_{h,o} - T_{c,i}}\right\}} \qquad \text{...(iii)}$$

(b) For parallel flow arrangement,

$$\Delta T_1 = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,o} = T_{h,o} - T_{c,i} - 0.5(T_{h,i} - T_{h,o})$$

$$= 1.5T_{h,o} - 0.5T_{h,i} - T_{c,i}$$

$$\Delta T_{lm,\,\text{parallel}} = \frac{\Delta T_1 - \Delta T_2}{ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{T_{h,i} - T_{c,i} - 1.5T_{h,o} + 0.5T_{h,i} + T_{c,i}}{ln\left\{\dfrac{T_{h,i} - T_{c,i}}{1.5T_{h,o} - 0.5T_{h,i} - T_{c,i}}\right\}}$$

$$= \frac{1.5[T_{h,i} - T_{h,o}]}{ln\left[\dfrac{T_{h,i} - T_{c,i}}{1.5T_{h,o} - 0.5T_{h,i} - T_{c,i}}\right]} \qquad \text{...(iv)}$$

Substituting equations (iii) and (iv) in equation (ii),

$$\frac{A_{CF}}{A_{PF}} = \frac{1.5(T_{h,i} - T_{h,o})}{ln\left[\dfrac{T_{h,i} - T_{c,i}}{1.5T_{h,o} - 0.5T_{h,i} - T_{c,i}}\right]} \times \frac{ln\left[\dfrac{0.5(T_{h,i} + T_{h,o}) - T_{c,i}}{T_{h,o} - T_{c,i}}\right]}{0.5(T_{h,i} - T_{h,o})}$$

$$= 3\left[\frac{\ln\left\{\dfrac{0.5(T_{h,i} + T_{h,o}) - T_{c,i}}{T_{h,o} - T_{c,i}}\right\}}{\ln\left\{\dfrac{T_{h,o} - T_{c,i}}{1.5T_{h,o} - 0.5T_{h,i} - T_{c,i}}\right\}}\right]$$

For given data: $T_{h,i} = 150°C$,

$T_{c,i} = 30°C$

and $T_{h,o} = 90°C$

$$\frac{A_{CF}}{A_{PF}} = 3\left[\frac{\ln\left\{\dfrac{0.5(150 + 90) - 30}{90 - 30}\right\}}{\ln\left\{\dfrac{150 - 30}{1.5 \times 90 - 0.5 \times 150 - 30}\right\}}\right] = 3 \times \frac{\ln(1.5)}{\ln(4)} = 0.877$$

Solution : 32

Given: A tube in tube heat exchanger,

$$T_{c,i} = 15°C,\ T_{h,i} = 130°C$$

In parallel flow mode,

$$T_{c,o} = 50°C,\ T_{h,o} = 60°C$$

To find:

(i) Exit temperature of each of fluids, if the existing heat exchanger was switched to counter flow operation.

(ii) The minimum temperature to which the oil may be cooled by increasing the tube length with parallel flow operations.

(iii) The maximum possible effectiveness in parallel flow operation.

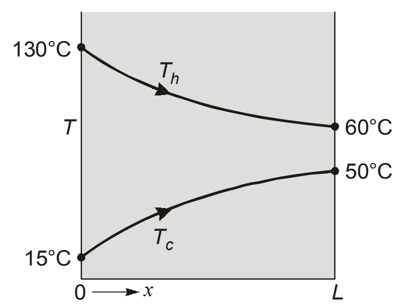
Analysis: Making the energy balance for two fluids.

$$\left(\dot{m}C_p\right)_{\text{water}}(\Delta T)_{\text{water}} = \left(\dot{m}C_p\right)_{\text{oil}}(\Delta T)_{\text{oil}}$$

$$C_{\text{water}} \times (50-15) = C_{\text{oil}} \times (130-60)$$

Hence, Capacity ratio, $C = \dfrac{C_{\text{oil}}}{C_{\text{water}}} = \dfrac{C_{\min}}{C_{\max}} = 0.5$

The temperature distribution in parallel flow heat exchanger, is shown in below figure.



(a) Existing parallel flow exchanger

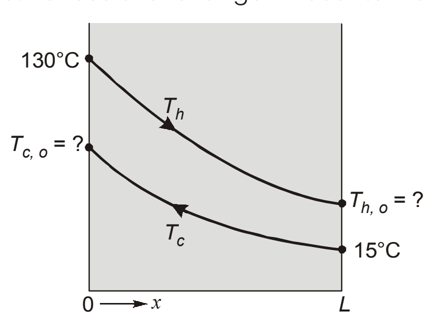
$$\varepsilon_{\text{parallel}} = \frac{130-60}{130-15} = 0.6086$$

Further, $$\varepsilon_{\text{parallel}} = \frac{1-\exp\{-NTU(1+C)\}}{1+C}$$

or $$0.6086 = \frac{1-\exp\{-NTU(1+0.5)\}}{1+0.5}$$

It gives, $\text{NTU} = 1.628$

(i) If the existing heat exchanger was switched to counter flow mode, its NTU and heat capacity ratio remain unchange. The effectiveness of exchanger in counter flow mode.



(b) Counter flow mode of exchanger

$$\varepsilon_{counter} = \frac{1-\exp\{-NTU(1-C)\}}{1-C\exp\{-NTU(1-C)\}}$$

or
$$\varepsilon_{counter} = \frac{1-\exp\{-1.628(1-0.5)\}}{1-0.5\exp\{-1.628(1-0.5)\}} = 0.7154$$

The exit temperatures:

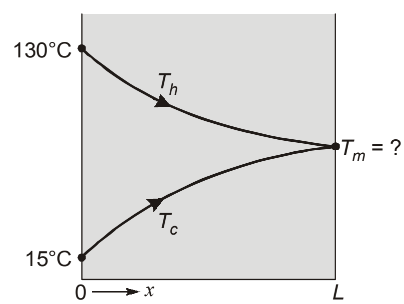
Hot fluid:

$$T_{h,o} = T_{h,i} - \varepsilon(T_{h,i} - T_{c,i}) = 130 - 0.7154 \times (130 - 15)$$
$$= 47.72°\text{C}$$

Cold fluid:

$$T_{c,o} = T_{c,i} - \varepsilon C(T_{h,i} - T_{c,i}) = 15 - 0.7154 \times 0.5 \times (130 - 15)$$
$$= 56.135°\text{C}$$

(ii) If the parallel flow heat exchanger is too long, then both fluids attain a common exit temperature (say T_m) as shown in figure (c)



(c) Very long parallel flow heat exchanger

Temperature profiles

$$(\dot{m}C_p)_{oil}(T_{h,i} - T_m) = (\dot{m}C_p)_{water}(T_m - T_{c,i})$$

$$130 - T_m = 2 \times (T_m - 15)$$

or
$$3T_m = 160$$

or
$$T_m = 53.33°\text{C}$$

It is minimum exit temperature of oil.

(iii) Maximum possible effectiveness in parallel flow mode:

For the hot fluid,
$$\varepsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}} = \frac{130 - 53.33}{130 - 15} = 0.667$$

Solution : 33

Given

Mass flow rate of steam = 25000 kg/hr [At pressure of 0.5 bar]

Temperature of condensing water = 25°C

Cooling water entry temperature = 15°C

Cooling water exit temperature = 25°C

Heat transfer coefficient = 10 kW/m^{2}K

No. of passes = 2

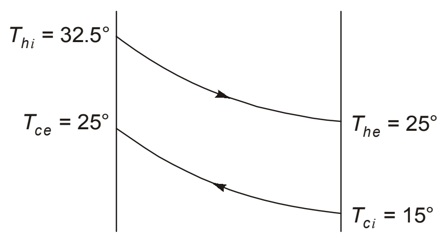
Outer dia of tube = 19 mm; thickness of tube = 1.2 mm

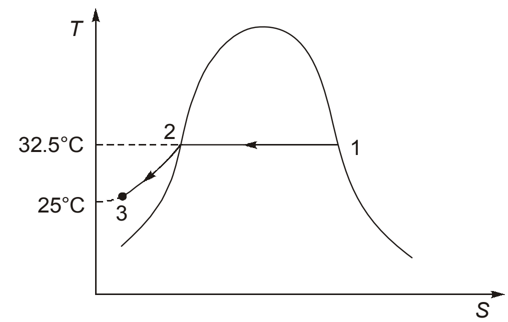
velocity of water = 1 m/s

correction factor = 0.86

Now at 0.5 bar saturation temperature is 32.55°C

So steam will condense at 32.55°C and then lose sensible heat so that the temperature of condensed water drops to 25°C.





Total heat lost by steam

$$= \dot{m}(h_{fg})_{32.55°C} + \dot{m}(h_2 - h_3) = \frac{25000}{3600}[2560 + 4.18(32.55 - 25)]$$

$$= 17.997 \times 10^3 \text{ kW}$$

Heat gained by cooling medium = $\dot{m}_w c_w (T_{C_e} - T_{C_i})$

Now $\qquad \dot{m}_w c_w (T_{Ce} - T_{Ci}) = 17.997 \times 10^3$ kW $\qquad$ [Energy balance]

$$\dot{m}_w = \frac{17.997 \times 10^3}{4.18 \times (25 - 15)} = 430.55 \text{ kg/s}$$

Now using LMTD method

$$\theta_1 = 32.55 - 25 = 7.55°C \qquad [\theta_1 = T_{Hi} - T_{Ce}]$$

$$\theta_2 = 25 - 15 = 10°C \qquad [\theta_2 = T_{He} - T_{Ci}]$$

Now

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{7.55 - 10}{\ln\left(\frac{7.55}{10}\right)} = 8.72°C$$

As we know

mass flow rate = velocity × cross-sectional area × density

$$m = V A_c \rho \Rightarrow A_c = \frac{m}{V\rho}$$

Total cross sectional area for water flow is

$$A_c = \frac{430.55}{1 \times 1000} = 0.43055 \text{ m}^2$$

$$\text{Cross sectional area per tube} = \frac{\pi}{4} \times [19 \times 10^{-3} - (2 \times 1.2 \times 10^{-3})]^2 = 2.16 \times 10^{-4} \text{ m}^2$$

$$\text{No. of tubes} = \frac{0.43055}{2.16 \times 10^{-4}} = 1989.379 \simeq 1990$$

Now $\quad Q = F\,A\,U\,\theta_m$

where $\quad F =$ Correction factor

$A =$ Area of contact

$U =$ Overall heat transfer coefficient

$\theta_m =$ log mean temperature difference

$$Q = 0.86 \times (1990 \times \pi \times 0.019 \times L) \times 10 \times 8.72$$

$$L = \frac{17.997 \times 10^3}{0.86 \times 1993 \times \pi \times 0.0166 \times 10 \times 8.72} = 2.02 \text{ m}$$

This is total length

$$\text{length per pass} = \frac{2.02}{2} = 1.01 \text{ m}$$

Solution : 34

$$\dot{m}_h = \frac{20{,}000}{3600} = 5.56 \text{ kg/s}, \; C_h = 3.3 \text{ kJ/kgK}$$

$\therefore \quad C_h = 5.56 \times 3.3 = 18.35$ kW/K

$$\dot{m}_c = \frac{50{,}000}{3600} = 13.89 \text{ kg/s}, \; C_c = 4.186 \text{ kJ/kgK}$$

$\therefore \quad C_c = 13.89 \times 4.186 = 58.14$ kW/K

Since $\quad C_n < C_c$

$\therefore \quad C_{\min} = 18.35$ kW/K

$$\text{NTU} = \frac{UA}{C_{\min}} = \frac{1050 \times 10}{18350} = 0.572$$

$$C = \frac{C_{\min}}{C_{\max}} = \frac{18.35}{58.14} = 0.3156$$

Effectiveness of H.E.,
$$\in = \frac{1 - \exp(-NTU(1+C))}{1+C} = \frac{1 - \exp[-0.572 \times 1.3156]}{1.3156} = \frac{1 - 0.471}{1.3156}$$

$$= 0.402$$

We know,
$$\in = \frac{T_{h_i} - T_{h_e}}{T_{h_i} - T_{c_i}} = 0.402$$

$\therefore \quad T_{h_e} = 120 - 0.402\,(120 - 20) = 79.8°\text{C}$

From energy balance, we have

$$\dot{m}_n C_h (T_{h_i} - T_{h_e}) = \dot{m}_c C_c (T_{c_e} - T_{c_i})$$

$$T_{ce} - T_{ci} = 0.3156 \times 40.2$$

$\therefore \quad T_{ce} = 20 + 12.69 = 32.69°\text{C}$

■■■■

5 Radiation

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(b)
4.	(162.313)
5.	(0.9)
6.	(c)
7.	(c)
8.	(d)
9.	(4)
10.	(c)
11.	(b)
12.	(b)
13.	(64.16)
14.	(0.8)
15.	(a)

LEVEL 2 Objective Questions

16.	(894)
17.	(0.927)
18.	(c)
19.	(d)
20.	(132.576)
21.	(2)
22.	(0.78)
23.	(51.176)
24.	(a)
25.	(a)
26.	(759.36)
27.	(c)
28.	(d)
29.	(1)
30.	(91.58)
31.	(0.75)
32.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 33

The rate of heat exchange between two bottle surfaces is given by

$$(Q_{12}) = (F_g)_{12} \cdot A_1\sigma_b(T_1^4 - T_2^4)$$

The gray body factor $$(F_g)_{12} = \frac{1}{\left(\frac{1-\epsilon_1}{\epsilon_1}\right) + \frac{1}{F_{12}} + \frac{A_1}{A_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

For infinite long parallel planes which see each other and nothing else $F_{12} = 1$ and $A_1 = A_2$,

$$\therefore \quad (F_g)_{12} = \frac{1}{\left(\frac{1-\epsilon_1}{\epsilon_1}\right) + 1 + \left(\frac{1-\epsilon_2}{\epsilon_2}\right)} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

$$= \frac{1}{\frac{1}{0.025} + \frac{1}{0.025} - 1} = 0.01266$$

$$\therefore \quad Q_{12} = 0.01266 \times 1 \times (5.67 \times 10^{-8})(375^4 - 300^4)$$

$$= 8.38 \text{ W}$$

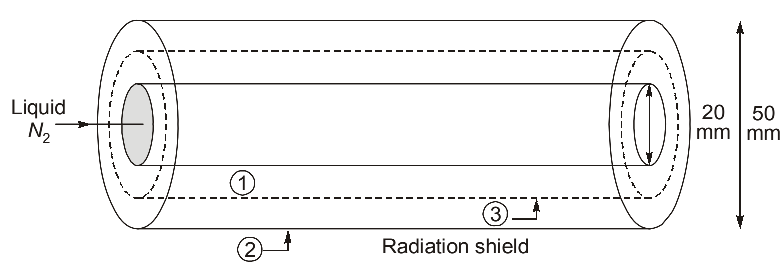
Let δ be the required thickness of cork

Then, $$Q = \frac{KA(T_1 - T_2)}{\delta}$$

$$8.38 = \frac{0.03 \times 1 \times (375 - 300)}{\delta}$$

$$\therefore \quad \delta = \frac{0.03 \times 75}{8.38} = 0.268 \text{ m} = 26.8 \text{ cm}$$

Solution : 34



For inner tube (1):

$$d_1 = 20 \text{ mm}$$

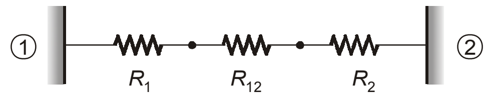
Since the tube is thin, its temperature will be equal to temperature of liquid N_2.

$$\therefore \quad T_1 = 77 \text{ K}, \quad \epsilon_1 = 0.02$$

For outer tube (2):

$$d_2 = 50 \text{ mm},\ T_2 = 300 \text{ K},\ \epsilon_2 = 0.05$$

Thermal network in this case will be,



$$R_{net} = R_1 + R_{12} + R_2$$

$$= \frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2} \qquad [\text{Since } F_{12} = 1]$$

$$= \frac{1}{A_1}\left[\frac{1-0.02}{0.02} + 1 + \left(\frac{1-0.05}{0.05}\right)\cdot\frac{A_1}{A_2}\right]$$

$$= \frac{1}{A_1}\left[\frac{1}{0.02} + \left(\frac{0.95}{0.05}\right)\cdot\frac{\pi\cdot 20\cdot L}{\pi\cdot 50\cdot L}\right] = \frac{1}{A_1}\cdot 57.6 \text{ m}^{-2}$$

Heat transfer to N_2 tube (1) from tube (2) per unit length,

$$= \frac{q_{21}}{L} = \frac{1}{L}\cdot\frac{E_{b_2} - E_{b_1}}{R_{net}} = \frac{\sigma\left(T_2^4 - T_1^4\right)}{L\cdot\dfrac{57.6}{A_1}}$$

$$= \frac{5.67\times 10^{-8}\times(300^4 - 77^4)}{L\cdot\dfrac{57.6}{\pi\cdot 20\times 10^{-3}\times L}} = 0.4988 \text{ watt} \simeq 0.5 \text{ watt}$$

If a radiation shield (3) is introduced midway between two tubes, for steady state condition,

$$q_{23} = q_{31}$$

$$\Rightarrow \quad \frac{E_{b_2} - E_{b_3}}{\dfrac{1-\epsilon_2}{\epsilon_2 A_2} + \dfrac{1}{A_2 F_{23}} + \dfrac{1-\epsilon_3}{\epsilon_3 A_3}} = \frac{E_{b_3} - E_{b_1}}{\dfrac{1-\epsilon_3}{\epsilon_3 A_3} + \dfrac{1}{A_3 F_{31}} + \dfrac{1-\epsilon_1}{\epsilon_1 A_1}}$$

$$\Rightarrow \quad \frac{\sigma\left(T_2^4 - T_3^4\right)}{\dfrac{1}{A_2}\left[\dfrac{1}{\epsilon_2} + \dfrac{A_2}{A_3}\left(\dfrac{1-\epsilon_3}{\epsilon_3}\right)\right]} = \frac{\sigma\left(T_3^4 - T_1^4\right)}{\dfrac{1}{A_3}\left[\dfrac{1}{\epsilon_3} + \dfrac{A_3}{A_1}\left(\dfrac{1-\epsilon_1}{\epsilon_1}\right)\right]}$$

$$\Rightarrow \quad \frac{300^4 - T_3^4}{\dfrac{1}{\pi\cdot 50\cdot L}\left[\dfrac{1}{0.05} + \dfrac{50}{35}\left(\dfrac{1-0.02}{0.02}\right)\right]} = \frac{T_3^4 - 77^4}{\dfrac{1}{\pi\cdot 35\cdot L}\left[\dfrac{1}{0.02} + \dfrac{35}{20}\left(\dfrac{1-0.02}{0.02}\right)\right]}$$

$$\Rightarrow \quad T_3 = 272.865 \text{ K}$$

$\therefore$ Heat transfer to liquid N_2 in this case per unit length,

$$\frac{q_{31}}{L} = \frac{1}{L}\cdot\frac{E_{b_3} - E_{b_1}}{\dfrac{1}{A_3}\left[\dfrac{1}{\epsilon_3} + \dfrac{A_3}{A_1}\left(\dfrac{1-\epsilon_1}{\epsilon_1}\right)\right]} = \frac{5.67\times 10^{-8}(272.86^4 - 77^4)}{\dfrac{1}{\pi\cdot 0.035}\left[\dfrac{1}{0.02} + \dfrac{35}{20}\left(\dfrac{1-0.02}{0.02}\right)\right]}$$

$$\Rightarrow \qquad \frac{q_{31}}{L} = 0.25298 = 0.253 \text{ watt}$$

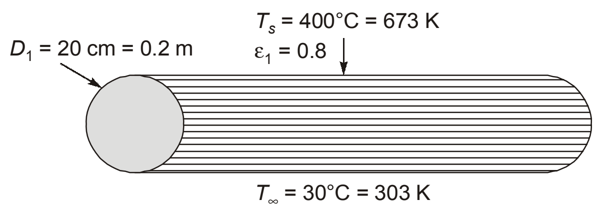
$\therefore$ % change in heat gained by liquid N_2 per unit length of tube

$$= \frac{0.5 - 0.253}{0.5} \times 100 = 49.4\%$$

$\therefore$ Heat transfer reduces by 49.4%.

Solution : 35

Given: A pipe carrying steam with



1. Brick conduit,

$$D_2 = 50 \text{ cm} = 0.5 \text{ m}, \qquad \varepsilon_2 = 0.9$$

2. Square conduit of side,

$$w = 0.5 \text{ m}, \qquad \varepsilon_3 = 0.9$$

To find:

(i) Net radiation heat transfer from pipe surface.

(ii) The radiation heat exchange when pipe is enclosed within a 50 cm diameter, brick conduit.

(iii) The radiation heat exchange, when pipe is enclosed within a square conduit.

Assumptions:

1. Surface are opaque, diffuse gray.
2. Space between two concentric pipes is evacuated.
3. No conduction and convection heat transfer.

Analysis:

(i) The net radiation heat exchange from pipe surface to room can expressed as:

$$\frac{Q}{L} = \varepsilon(\pi D_1)\,\sigma(T_s^4 - T_\infty^4)$$

$$= 0.8 \times \pi \times 0.2 \times 5.67 \times 10^{-8} \times (673^4 - 303^4)$$

$$= 5606.5 \text{ W/m}$$

(ii) The radiation heat exchange between pipe and a conduit can be calculated as:

$$Q = \frac{A_1 \sigma (T_s^4 - T_\infty^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\frac{A_1}{A_2}}$$

When pipe is enclosed within brick conduit:

$$\frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right) = \left(\frac{0.2}{0.5}\right) = 0.4$$

$$\frac{Q}{L} = \frac{\pi \times 0.2 \times 5.67 \times 10^{-8} \times (673^4 - 303^4)}{\frac{1}{0.8} + \left(\frac{1}{0.9} - 1\right) \times 0.4} = 5414 \text{ W/m}$$

The reduction in heat radiation = 5606.5 – 5414 = 192.5 W/m

(iii) When pipe is enclosed within a square conduit:

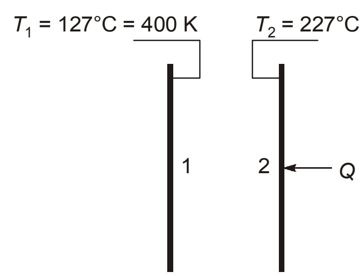
$$\frac{A_1}{A_2} = \left(\frac{\pi D_1 L}{4wL}\right) = \left(\frac{\pi \times 0.2}{4 \times 0.5}\right) = 0.314$$

$$\frac{Q}{L} = \frac{\pi \times 0.2 \times 5.67 \times 10^{-8} \times (673^4 - 303^4)}{\frac{1}{0.8} + \left(\frac{1}{0.9} - 1\right) \times 0.314} = 5454.2 \text{ W/m}$$

The reduction in heat radiation = 5606.5 – 5454.2 = 152.3 W/m

Solution : 36

Given: Two parallel infinite surfaces with, T_3 = 327°C = 600 K, ε_1 = 0.9, ε_2 = 0.7



Two parallel infinite gray surfaces

To find: Net radiation heat transfer.

Assumptions:

1. Surfaces are diffused and gray.
2. Heat is transferred by radiation only.

Analysis: The net radiation heat exchange between two parallel plates can be expressed as:

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)}$$

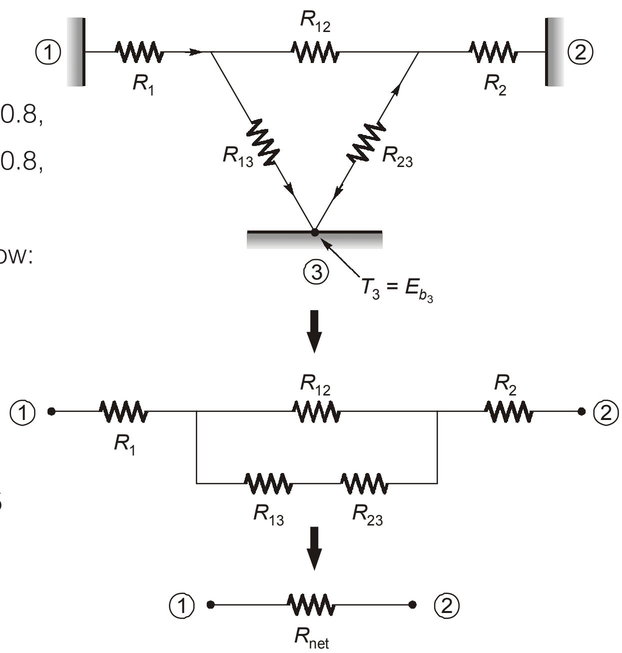
$$\frac{Q_1}{A} = \frac{5.67 \times 10^{-8} \times (500^4 - 400^4)}{\frac{1}{0.9} + \left(\frac{1}{0.7} - 1\right)} = 1358.87 \text{ W/m}^2$$

When the hot plate temperature is raised to 600 K, then,

$$\frac{Q_2}{A} = \frac{5.67 \times 10^{-8} \times (600^4 - 400^4)}{\frac{1}{0.9} + \left(\frac{1}{0.7} - 1\right)} = 3829.88 \text{ W/m}^2$$

Therefore, $$\frac{Q_2}{Q_1} = \frac{3829.88}{1358.87} = 2.82$$

Solution : 37



Given: For the walls (1) and (2), we have

$$T_1 = 900\text{ K},\quad \in_1 = 0.8,$$

$$T_2 = 400\text{ K},\quad \in_2 = 0.8,$$

For reradiating surface, $Q_3 = 0$

Thermal network in this case can be drawn as below:

Assume length of each side of the equilateral triangular furnace are 'l' m.

We know,

$$F_{12} = \frac{l_1 + l_2 - l_3}{2l_1}$$

$$= \frac{l + l - l}{2l} = 0.5$$

For triangular ducts

By symmetry $F_{13} = F_{23} = 0.5$

Calculating overall thermal resistance,

$$R_{net} = R_1 + R_2 + \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}} = R_1 + R_2 + \frac{R_{12} \cdot (R_{13} + R_{23})}{R_{12} + R_{13} + R_{23}}$$

$$= \frac{1 - \in_1}{\in_1 A} + \frac{1 - \in_2}{\in_2 A} + \frac{\frac{1}{AF_{12}} \cdot \left(\frac{1}{AF_{13}} + \frac{1}{AF_{23}}\right)}{\frac{1}{AF_{12}} + \frac{1}{AF_{13}} + \frac{1}{AF_{23}}}$$

$$= \frac{1}{A}\left[\frac{1 - 0.8}{0.8} + \frac{1 - 0.8}{0.8} + \frac{\frac{1}{0.5}\left(\frac{1}{0.5} + \frac{1}{0.5}\right)}{\frac{1}{0.5} + \frac{1}{0.5} + \frac{1}{0.5}}\right]$$

$\Rightarrow$ $$R_{net} = \frac{1}{A} \cdot \frac{11}{6}\text{ m}^{-2}$$

$\therefore$ Net radiation heat flux from hot wall,

$$\frac{q_{12}}{A} = \frac{E_{b_1} - E_{b_2}}{A \cdot R_{net}} = \frac{5.67 \times 10^{-8} \times (900^4 - 400^4)}{A \cdot \frac{11}{6A}} = 19.499\text{ kW} \simeq 19.5\text{ kW/m}^2$$

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