

# 2020

## **RANK** *Improvement* **WORKBOOK**



**Answer key and Hint of  
Objective & Conventional *Questions***

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**Electrical Engineering**  
Power Systems



**MADE EASY**  
Publications

# 1

## Power Generation and Economics of Generation

### LEVEL 1 Objective Questions

1. (200)

2. (60)

### LEVEL 2 Objective Questions

3. (48703.1)

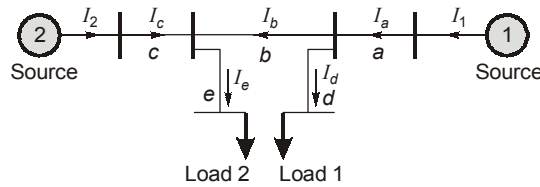
4. (136.36 MW)

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**LEVEL 3** Conventional Questions

**Solution : 1**



The currents of loads 1 and 2 are 0.4 and 0.6 p.u. of the total load current. Therefore, the current distribution factors are

$$\begin{aligned} N_{a1} &= 1, & N_{b1} &= 0.6N_{c1} = 0, & N_{d1} &= 0.4 \\ N_{a2} &= 0, & N_{b2} &= -0.4N_{c2} = 1, & N_{d2} &= 0.4 \\ N_{e1} &= 0.6 & N_{e2} &= 0.6 \end{aligned}$$

The bus voltage at the two sources are

$$\begin{aligned} V_1 &= 1.0 + (1.5 - j0.4)(0.03 + j0.09) \\ &= 1.081 + j0.123 = 1.088 \angle 6.49^\circ \\ V_2 &= 1.0 - (0.5 - j0.2)(0.1 + j0.3) \\ &\quad + (1 - j0.1)(0.03 + j0.09) \\ &= 0.929 - j0.043 = 0.93 \angle -2.65^\circ \end{aligned}$$

The phase angles of the source currents are

$$\begin{aligned} \theta_1 &= \tan^{-1} \frac{-0.4}{1.5} = -14.93^\circ \\ \theta_2 &= \tan^{-1} \frac{-0.1}{1.0} = -5.73^\circ \end{aligned}$$

$$\cos(\theta_1 - \theta_2) = \cos(9.2^\circ) = 0.987$$

Power factors at sources 1 and 2 are

$$\begin{aligned} \cos\phi_1 &= \cos(6.49 + 14.93^\circ) = 0.931 \\ \cos\phi_2 &= \cos(5.73 - 2.65^\circ) = 0.998 \end{aligned}$$

Loss coefficients are

$$B_{11} = \frac{1^2 \times 0.3 + 0.6^2 \times 0.1 + 0.4^2 \times 0.4 + 0.6^2 \times 0.04}{(1.088)^2 (0.931)} = 0.07876 \text{ p.u.}$$

$$B_{22} = \frac{0.4^2 \times 0.1 + 1^2 \times 0.03 + 0.4^2 \times 0.04 + 0.6^2 \times 0.04}{(0.93)^2 (0.998)} = 0.0774 \text{ p.u.}$$

$$B_{12} = \frac{-0.4 \times 0.6 \times 0.1 + 0.4 \times 0.4 \times 0.04 + 0.6 \times 0.6 \times 0.03}{1.088 \times 0.93 \times 0.931 \times 0.998} \times (0.987) = -0.00713 \text{ p.u.}$$

For converting the above per unit values, which are to a base of 100 MVA, to actual values the per unit values have to be divided by 100.

$$B_{11} = \frac{0.07876}{100} = 7.876 \times 10^{-4} \text{ MW}^{-1}$$

$$B_{22} = \frac{0.0774}{100} = 7.74 \times 10^{-4} \text{ MW}^{-1}$$

$$B_{12} = \frac{-0.00713}{100} = -0.713 \times 10^{-4} \text{ MW}^{-1}$$

**Solution : 2**

For 1200 MW load period it is necessary to run both units

$$\frac{dC_1}{dP_1} = 52.8 + 11 \times 10^{-3} P_1 = \lambda$$

$$\frac{dC_2}{dP_2} = 15 + 0.1P_2 = \lambda$$

Let the loads be  $P_1$  and  $1200 - P_1$ . For economic loading incremental costs should be equal

$$52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(1200 - P_1)$$

The solution is

$$P_1 = 740.54 \text{ MW,}$$

$$P_2 = 459.46 \text{ MW}$$

For 900 MW load also it is necessary to run both units. Let loads be  $P_1$  and  $(900 - P_1)$ .

$$\text{Then, } 52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(900 - P_1)$$

The solutions is

$$P_1 = 470.27 \text{ MW,}$$

$$P_2 = 429.73 \text{ MW}$$

For the 500 MW load, let both units be run. Then loads are  $P_1$  and  $(500 - P_1)$ . We get

$$52.8 + 11 \times 10^{-3} P_1 = 15 + 0.1(500 - P_1)$$

The result is

$$P_1 = 109.9 \text{ MW,}$$

$$P_2 = 390.1 \text{ MW}$$

However the minimum load on any unit is 200 MW. Therefore

$$P_1 = 200 \text{ MW,}$$

$$P_2 = 300 \text{ MW}$$

The operating cost for the 500 MW load period is

$$C_1 = 7700 + 52.8 \times 200 + 5.5 \times 10^{-3} \times 200^2 = ₹18480/\text{hour}$$

$$C_2 = 2500 + 15 \times 300 + 0.05(300)^2$$

$$= ₹11500/\text{hour}$$

Total operating cost for the 10 hour period of 500 MW is  $10(C_1 + C_2)$  and equals ₹299800.

The other option is to run only one unit during 500 MW load period.

It is easy to see that it is cheaper to run unit 2. Then the cost for 10 hour period is

$$₹10[2500 + 15(500) + (0.05)(500)^2] = ₹225000]$$

Add the cost of shutting down and starting unit 1 i.e. ₹1000. Therefore, total cost for 500 MW load duration is ₹22500 + ₹1000 or ₹226000. As compared to this, the cost of operation if both units are operating is ₹299800. Hence it is economical to run only unit 2 during 500 MW load. Hence the complete operating schedule is

1200 MW load:	$P_1 = 740.54 \text{ MW}$ , $P_2 = 459.46 \text{ MW}$
900 MW load:	$P_1 = 470.27 \text{ MW}$ , $P_2 = 429.73 \text{ MW}$
500 MW load:	$P_1 = \text{Shut-down unit 1}$ $P_2 = 500 \text{ MW}$

**Solution : 3**

Given,  $H = 5 \text{ MJ/MVA}$ ,  
 $M = 100 \text{ MVA}$ ,  
 $f_1 = 50 \text{ Hz}$

- Kinetic energy stored in rotating parts of generator and turbine =  $5 \times 100 = 500 \text{ MJ} (= \text{MH})$
- Excess power input to generator before the steam valve begins to close =  $65 \text{ MW}$
- Excess energy to rotating part in 0.5 seconds =  $(65)(0.5) = 32.5 \text{ MJ}$

Stored kinetic energy  $\propto (\text{frequency})^2$

With load throw off energy =  $(500 + 32.5)$

So, 
$$\frac{f_1^2}{f_2^2} = \frac{500}{500 + 32.5}$$

$$f_2 = \left( \frac{500 + 32.5}{500} \right)^{1/2} \times 50 = 51.60 \text{ Hz}$$

Change in frequency,  $f_2 - f_1 = 51.60 - 50 = 1.6 \text{ Hz}$   
 $\Delta f = 1.6 \text{ Hz}$

**Solution : 4**

The load is at the bus of plant 2. Evidently transmission loss is not affected by variation of  $P_2$ .

$\therefore B_{22} = B_{12} = 0$   
when  $P_1 = 100 \text{ MW}$ ,  $P_L = 15 \text{ MW}$

$\therefore 15 = B_{11}(100)^2$

or  $B_{11} = 0.0015 \text{ MW}^{-1}$

$$P_L = 0.0015P_1^2$$

$$\frac{\partial P_L}{\partial P_1} = 0.003P_1$$

$$\frac{\partial P_L}{\partial P_2} = 0$$

$$L_1 \text{ i.e. Penalty factor of plant 1} = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.003P_1}$$

$L_2 \text{ i.e. Penalty factor of plant 2} = 1$

The generation at each plant is required to be calculated for  $\lambda = 60$

$$\frac{dC_1}{dP_1} L_1 = \lambda$$

$$\frac{dC_2}{dP_2} L_2 = \lambda$$

$$\therefore \frac{0.2P_1 + 22}{1 - 0.003P_1} = 60 \text{ or } P_1 = 100 \text{ MW}$$

$$\text{and } 0.15 P_2 + 30 = 60$$

$$\text{or } P_2 = 200 \text{ MW}$$

$$\begin{aligned} \text{Total load} &= P_1 + P_2 - B_{11}P_1^2 \\ &= 100 + 200 - (0.0015) \times 100^2 = 285 \text{ MW} \end{aligned}$$

**Solution : 5**

The optimum generation found (transmission loss included) is

$$P_1 = 100 \text{ MW}, \quad P_2 = 200 \text{ MW}$$

If transmission loss is neglected in determining scheduling

$$0.2P_1 + 22 = 0.15 P_2 + 30$$

$$\text{and } P_1 + P_2 - 0.0015 P_1^2 = 285$$

Solution of these two equations of  $P_1$  and  $P_2$  gives

$$P_1 = 161.84 \text{ MW} \quad P_2 = 162.50 \text{ MW}$$

The load on plant 1 is increased from 100 MW to 161.80 MW. Additional cost is

$$\int_{100}^{161.84} (0.2P_1 + 22) dP_1 = \left[ 0.1P_1^2 + 22P_1 \right]_{100}^{161.84} = ₹ 2979.69$$

The load on plant 2 is decreased from 200 MW to 162.50 MW. The change in cost is

$$\int_{200}^{162.50} (0.15P_2 + 30) dP_2 = \left[ 0.075P_2^2 + 30P_2 \right]_{200}^{162.50} = ₹ -2144.53$$

$$\text{Net change in cost} = ₹ 835.16 \text{ per hour}$$

Thus scheduling the generation by taking transmission losses into account would mean a saving of ₹ 835.16 per hour in fuel cost.

**Solution : 6**

$$\text{Sum of individual maximum demands} = 31000 \text{ kW} = 31 \text{ MW}$$

$$\begin{aligned} \text{Maximum demand} &= \frac{\text{Sum of individual maximum demands}}{\text{Diversity factor}} = \frac{31}{1.5} \\ &= 20.667 \text{ MW} \end{aligned}$$

$$\text{Annual energy} = 20.667 \times 8760 \times 0.65 = 117677.9 \text{ MWh}$$

$$\text{Increase in maximum demand} = 20.667 \times 0.6 = 12.4 \text{ MW}$$

$$\text{Installed capacity} = 20.667 + 12.4 = 33.067 \text{ MW}$$

For the above value of installed capacity the power station can be either a diesel station or a gas turbine plant. The diesel units are manufactured in small sizes and therefore this station will need too many units and the cost would be very high. A gas turbine plant with 2 units of 10 MW each and two units of 15 MW each would be suitable. The station will have a total installed capacity of 50 MW and thus will have sufficient reserve capacity also.

**Solution : 7**

$$\begin{aligned}\text{Water used} &= \frac{1000 \times 10^6 \times 125 \times 0.8}{100} \text{ m}^3/\text{year} \\ &= \frac{1000 \times 10^6 \times 125 \times 0.8}{100 \times 365 \times 24 \times 3600} \text{ m}^3/\text{sec} = 31.71 \text{ m}^3/\text{sec}\end{aligned}$$

$$\text{Effective head} = 205 - 5 = 200 \text{ m}$$

$$\begin{aligned}P &= \frac{735.5}{75} (31.71)(200)(0.9)(0.95) \text{ kW} \\ &= 53175.8 \text{ kW} = 53.1758 \text{ MW}\end{aligned}$$

Since load factor is 75%

$$\text{Peak load on station} = \frac{53.1758}{0.75} = 70.9 \text{ MW}$$

Therefore the MW rating of station is 70.9 MW.

For heads above 200 m, Pelton turbine is suitable.

Francis turbine can also be used since the range of head for francis turbine is 30 m – 200 m.

However Pelton turbine is more suitable.



# 2

## Transmission Line Design and Performance

### LEVEL 1 Objective Questions

1. (b)
2. 22.9 (22.8 to 23.0)
3. 1.028 (1.01 to 1.05)
4. 1.398 (1.2 to 1.45)
5. (56.54 mm, 11.308 mm)
6. (a)
7. 0.57 (0.40 to 0.80)
8. (c)
9. (0.866, 12)
10. (a)
11. (0.7 kW/km/ph)
12. (d)
13. (a)

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### LEVEL 2 Objective Questions

14. (a)
15. (b)
16. (a)
17. (a)
18. (0.361 V/km)
19. (3.9)
20. (1.43)
21. (0.38 C)

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**LEVEL 3** Conventional Questions

**Solution : 1**

Horizontal spacing,  
Bundle conductor,  
For 4 conductors

$$d = 20 \text{ mm}, \quad s = 0.5 \text{ m}$$

$$r = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

GMR,

$$D_{sh}^b = (\sqrt{2}r's^3)^{1/4}$$

$$= (\sqrt{2} \times 0.7788 \times 20 \times 10^{-3} \times 0.5^3)^{1/4} = 0.2290 \text{ m}$$

Mutual GMD,

$$D_M = (D_{AB}D_{BC}D_{AC})^{1/3}$$

$$= (d \cdot d \cdot 2d)^{1/3} = (20 \times 20 \times 40)^{1/3} = 25.198 \text{ m}$$

Inductance of bundled conductor,

$$L_b = 2 \times 10^{-7} \ln \frac{D_M}{D_{SL}} = 2 \times 10^{-7} \ln \frac{25.198}{0.2290}$$

$$= 9.401 \times 10^{-7} \text{ H/m} = 9.401 \times 10^{-4} \text{ H/km}$$

Inductive reactance,

$$X_L = 2\pi f L_b = 2\pi \times 50 \times 9.401 \times 10^{-4}$$

$$X_L = 0.2953 \text{ } \Omega/\text{km}$$

For capacitive reactance,

$$D_M = \text{Same} = 25.198 \text{ m}$$

$$D_{SC} = (\sqrt{2}rs^3)^{1/4}$$

$$= [\sqrt{2} \times 20 \times 10^{-3} \times (0.5)^3]^{1/4} = 0.2438 \text{ m}$$

$$C_n = \frac{2\pi\epsilon}{\ln \frac{D_M}{D_{SC}}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \left[ \frac{25.198}{0.2438} \right]}$$

$$= 1.1988 \times 10^{-11} \text{ F/m}$$

$$= 11.988 \times 10^{-9} \text{ F/km}$$

Capacitive reactance ' $X_C$ ',

$$X_C = \frac{1}{2\pi f C_n} = \frac{1}{2\pi \times 50 \times 11.988 \times 10^{-9}}$$

$$X_C = 2.655 \times 10^5 \text{ } \Omega/\text{km}$$

**Solution : 2**

Given data:

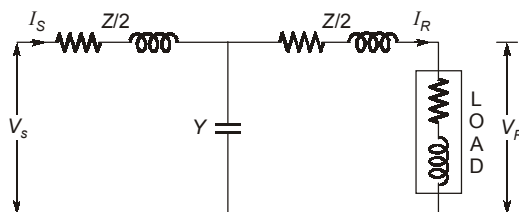
$$f = 50 \text{ Hz}, \quad R = 0.25 \text{ W/miles/phase}$$

$$L = 2 \text{ mH/miles/phase}, \quad C = 0.014 \text{ } \mu\text{F/miles/phase}$$

$$l = 60 \text{ miles}, \quad V_R = 132 \text{ kV}$$

$$P_R = 70 \text{ MW}, \quad \cos \phi = 0.8 \text{ lagging}$$

Using 'T' model:



$$\begin{aligned}
 Z &= (R + j\omega L) I \\
 &= [0.25 + j(2\pi \times 50 \times 2 \times 10^{-3})] \times 60 = 40.57 \angle 68.3^\circ \Omega \\
 Y &= j\omega C I = j(2\pi \times 50 \times 0.014 \times 10^{-6}) \times 60 = j0.264 \times 10^{-3} \text{ } \Omega^{-1}
 \end{aligned}$$

**ABCD parameters:**

$$\begin{aligned}
 A = D &= 1 + \frac{YZ}{2} \\
 &= 1 + \left( \frac{0.264 \times 10^{-3} \angle 90^\circ \times 40.57 \angle 68.3^\circ}{2} \right) = 0.995 \angle 0.114^\circ
 \end{aligned}$$

$$\begin{aligned}
 B &= Z \left( 1 + \frac{YZ}{4} \right) \\
 &= 40.57 \angle 68.3^\circ \left[ 1 + \frac{40.57 \angle 68.3^\circ \times 0.264 \times 10^{-3} \angle 90^\circ}{4} \right]
 \end{aligned}$$

$$B = 40.47 \angle 68.36^\circ \Omega$$

$$C = Y = 0.264 \times 10^{-3} \angle 90^\circ \text{ } \Omega^{-1}$$

From the given  $P_R$

$$|I_R| = \frac{P_R}{\sqrt{3} V_R \cos \phi_R} = \frac{70 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0.8} = 382.71 \text{ A}$$

$$\phi_R = \cos^{-1} 0.8 = 36.86^\circ$$

$$I_R = 382.71 \angle -36.86^\circ \text{ A}$$

(i) (a)

$$V_S = A V_R + B I_R$$

$$\begin{aligned}
 &= \left[ 0.995 \angle 0.114^\circ \times \frac{132 \times 10^3}{\sqrt{3}} + 40.47 \angle 68.36^\circ \times 382.71 \angle -36.86^\circ \right] \\
 &= 89.784 \angle 5.17^\circ \text{ kV}
 \end{aligned}$$

$$V_{sl} = \sqrt{3} V_S = 155.51 \angle 5.17^\circ \text{ kV}$$

(b)

$$I_S = C V_R + D I_R$$

$$\begin{aligned}
 &= \left[ \left( 0.264 \times 10^{-3} \angle 90^\circ \times \frac{132 \times 10^3}{\sqrt{3}} \right) + (0.995 \angle 0.114^\circ \times 382.71 \angle -36.86^\circ) \right]
 \end{aligned}$$

$$I_S = 369.11 \angle -34.25^\circ \text{ A}$$

(ii)

$$\% V_{REG} = \frac{\frac{|V_S|}{A} - |V_R|}{|V_R|} \times 100 = \frac{\frac{155.51}{0.995} - 132}{132} \times 100$$

$$\% V_{REG} = 18.403\%$$

(iii)

$$P_S = \sqrt{3} |V_S I_S| \cos \phi_s$$

$$= \sqrt{3} \times 155.51 \times 10^3 \times 369.11 \times \cos(34.25^\circ + 5.17^\circ)$$

$$P_S = 76.80 \text{ MW}$$

$$Q_S = P_S \tan \phi_s = \sqrt{3} |V_S I_S| \sin \phi_s$$

$$= \sqrt{3} \times 155.51 \times 10^3 \times 369.11 \times \sin(34.25^\circ + 5.17^\circ)$$

$$Q_S = 63.13 \text{ MVAR}$$

$$(iv) \quad \% \eta = \frac{P_R}{P_S} \times 100 = \frac{70}{76.80} \times 100 = 91.15\%$$

$$\% \eta = 91.15\%$$

**Solution : 3**

$$Z = 95 \angle 78^\circ \Omega, Y = 0.001 \angle 90^\circ S, P = 49 \text{ MW, p.f.} = 0.85$$

$$V_{\text{line}} = 138 \text{ kV}$$

(i) For nominal T-circuit of line

$$A = 1 + \frac{YZ}{2} = 1 + \frac{(95)(0.001) \angle 168^\circ}{2}$$

$$A = 0.9535 + j 9.875 \times 10^{-3}$$

$$A = 0.9536 \angle 0.59^\circ$$

$$B = Z \left( 1 + \frac{YZ}{4} \right) = 95 \angle 78^\circ \left[ 1 + \frac{(95)(0.001) \angle 168^\circ}{4} \right]$$

$$B = 95 \angle 78^\circ [0.9769 \angle 0.28^\circ] = 92.8 \angle 78.28^\circ \Omega$$

$$C = Y = 0.001 \angle 90^\circ$$

$$D = \left( 1 + \frac{YZ}{2} \right) = 0.9536 \angle 0.59^\circ$$

(ii)

$$V_s = AV_r + BI_r$$

$$V_r = \frac{138}{\sqrt{3}} \times 10^3 = 79.67 \text{ kV}$$

$$I_r = \frac{49 \times 10^6}{(\sqrt{3})(138 \times 10^3)(0.85)} = 241.18 \text{ A}$$

$$V_r = 79.67 \times 10^3 \angle 0^\circ$$

$$I_r = 241.18 \angle -\cos^{-1}(0.85) = 241.18 \angle -31.79^\circ$$

$$V_s = (0.9536 \angle 0.59^\circ)(79.67 \times 10^3) \angle 0^\circ + (92.8 \angle 78.28^\circ)(241.18 \angle -31.79^\circ)$$

$$V_s = 75969.284 + j 782.37 \times 10^{-3} + 15409.24 + j 16232.28$$

$$V_s = 91378.53 + j 17014.60 = 92.949 \times 10^3 \angle 10.547^\circ$$

$$V_{s \text{ line}} = 160.99 \angle 9.75^\circ \text{ kV}$$

(iii)

$$I_s = CV_r + DI_r$$

$$= (0.001 \angle 90^\circ)(79.67 \times 10^3 \angle 0^\circ) + (0.9536 \angle 0.59^\circ)(241.18 \angle -31.79^\circ)$$

$$= j 79.67 + 230 \angle -31.20^\circ = j 79 + 196.73 - j 119.15$$

$$I_s = (196.73 - j 39.38)$$

$$I_s = 200.6 \angle -11.31^\circ \text{ A}$$

(iv) Sending end power factor,

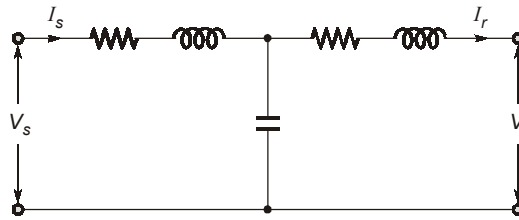
$$V_s = |V_s| \angle 10.547^\circ$$

$$I_s = |I_s| \angle -11.48^\circ$$

$$\phi = 10.547^\circ + 11.48^\circ = 22.027^\circ$$

$$\text{Power factor} = 0.927 \text{ lagging}$$

(v)



$$P_i = \sqrt{3} V_s I_s \cos \phi$$

$$= (\sqrt{3})(160.99 \times 10^3)(200.6)(0.927) = 51.852 \text{ MW}$$

$$\% \eta = \frac{P_o}{P_i} \times 100 = \frac{49}{51.852} \times 100 = 94.49\%$$

$$\% \eta = 94.49\%$$

**Solution : 4**

Given:  $d = 19.53 \text{ mm}$ ,  $r = 0.5d = 0.5 \times 19.53 \text{ mm} = 9.765 \times 10^{-3} \text{ m}$

Air density factor,  $\delta = \frac{0.392 p}{273 + t} = \frac{0.392 \times 750}{273 + 30} = 0.9703$

$$D = 3.8 \text{ m}, \quad m_0 = 0.85$$

$$g_0 = 21.1 \text{ kV (rms) per cm}$$

$$= 21.1 \times 1000 \times 100 \text{ V/m} = 2.11 \times 10^6 \text{ V/m}$$

The disruptive critical rms voltage per phase is given by

$$E_0 = g_0 m_0 r \delta \ln \frac{D}{r}$$

$$= 2.11 \times 10^6 \times 0.85 \times 9.765 \times 10^{-3} \times 0.9703 \ln \frac{3.8}{9.765 \times 10^{-3}}$$

$$= 101.456 \times 10^3 \text{ V} = 101.456 \text{ kV}$$

For local corona,  $m_v = 0.72$

The visual critical rms voltage per phase for local corona is given by

$$E_v = g_0 m_v r \delta \left( 1 + \frac{0.0301}{\sqrt{r \delta}} \right) \ln \frac{D}{r}$$

$$= 2.11 \times 10^6 \times 0.72 \times 9.765 \times 10^{-3} \times 0.9703$$

$$\left[ 1 + \frac{0.0301}{(9.765 \times 10^{-3} \times 0.9703)^{1/2}} \right] \times \ln \frac{3.8}{9.765 \times 10^{-3}} = 112.4 \text{ kV}$$

It is to be noted that there is no corona under normal working conditions since the actual operating voltage

to neutral is  $\frac{132}{\sqrt{3}}$ , that is, 76.21 kV.

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# 3

## Power System Control

### LEVEL 1 Objective Questions

1. (a)
2. (173.2)
3. (b)
4. (c)
5. (c)
6. 22.00 (21.50 to 22.50)
7. 0.268 (0.26 to 0.28)
8. (b)
9. (b)
10. (b)
11. (b)
12. (c)
13. (c)
14. (-2.53)
15. (4.8)

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### LEVEL 2 Objective Questions

16.  $(0.97 \angle 8.27^\circ)$
17.  $(0.504, j0.319)$
18.  $(1.5213 \angle 27.4, 0.435)$
19. (a)
21. (0.745 lag)
21. (0.212 lead)
23. (337 kVAR)
24. (4530)
26. (c)
27. (b)

■■■■

### LEVEL 3 Conventional Questions

**Solution : 1**

$$(a) \quad 50 \times 0.8 = \frac{132 \times 132}{110} \cos(\beta - \delta) - \frac{0.98(132)^2}{110} \cos(75^\circ - 3^\circ)$$

$$\text{or} \quad 40 = 158.4 \cos(\beta - \delta) - 47.97$$

$$\text{or} \quad (\beta - \delta) = 56.26^\circ$$

$$\begin{aligned} \text{Using equation,} \quad Q_r &= \frac{132 \times 132}{110} \sin(56.26^\circ) - \frac{0.98(132)^2}{110} \sin(75^\circ - 3^\circ) \\ &= -15.91 \text{ MVAR} \end{aligned}$$

Thus for the given operating conditions, a leading MVAR of 15.91 must be drawn from the line along with real power of 40 MW. Since the load requires  $50 \times 0.6$  i.e., 30 MVAR lagging, the static capacitors must deliver  $(30 + 15.91)$  i.e. 45.91 MVAR lagging (or absorb 45.91 MVAR leading). The capacity of shunt compensation equipment is, therefore, 45.91 MVAR.

$$(b) \text{ Using equation,} \quad 0 = \frac{132 \times 132}{110} \cos(\beta - \delta) - \frac{0.98(132)^2}{110} \cos(75^\circ - 3^\circ)$$

$$0 = 158.4 \cos(\beta - \delta) - 47.97$$

$$\text{or} \quad (\beta - \delta) = 72.37^\circ$$

$$\begin{aligned} \text{Using equation,} \quad Q_r &= \frac{132 \times 132}{110} \sin(72.37^\circ) - \frac{0.98(132)^2}{110} \sin(72^\circ) \\ &= 3.33 \text{ MVAR} \end{aligned}$$

Thus under no load condition the line delivers 3.33 MVAR at the receiving end. This reactive power must be absorbed by shunt reactor at the receiving end. Thus the capacity of shunt reactor, for no load condition, is 3.33 MVAR.

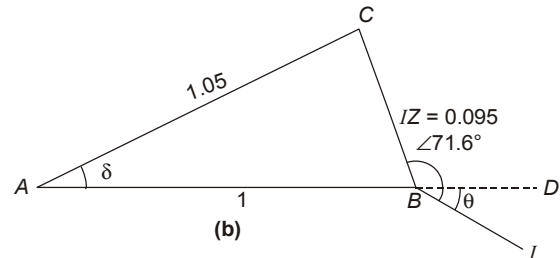
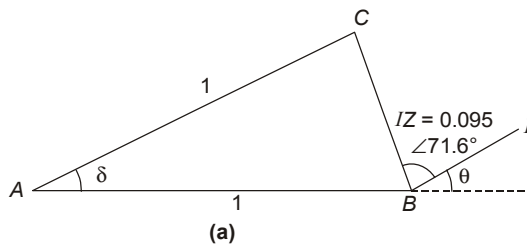
**Solution : 2**

$$Z = (3 + j9)\% = 0.03 + j0.09 \text{ p.u.}$$

$$I = 1 \text{ p.u.}$$

$$IZ = 0.03 + j0.09 = 0.095 \angle 71.60^\circ$$

(a) The phasor diagram is shown in figures (a):



$$1^2 + 1^2 - 2 \times 1 \times 1 \times \cos\delta = (0.095)^2$$

The solution is

$$\cos\delta = 5.445^\circ$$

Moreover,

$$1^2 = 1^2 + (0.095)^2 - 2 \times 1 \times 0.095 (\cos\angle ABC)$$

The solutions is

$$\angle ABC = 87.277^\circ$$

$$\angle \theta = 180 - 87.277^\circ - 71.6 = 21.1^\circ$$

$$\text{Real power transfer} = 1 \times 1 \times \cos\theta = 0.933 \text{ p.u.}$$

$$\text{Reactive power transfer} = 1 \times 1 \times \sin\theta = 0.36 \text{ p.u. leading}$$

(b) The phasor diagram is shown in figure (b).

$$1.05^2 + 1^2 - 2 \times 1.05 \times 1 \times \cos\delta = (0.095)^2$$

The solution is  $\delta = 4.52^\circ$ .

Moreover,

$$1.05^2 = 1^2 + (0.095)^2 - 2 \times 1 \times 0.095(\cos\angle DBC)$$

The solution is

$$\angle DBC = 60.53^\circ$$

$$\angle \theta = 71.6 - 60.53 = 11.07^\circ$$

$$\text{Real power} = 1 \times 1 \times \cos 11.07^\circ = 0.9814 \text{ p.u.}$$

$$\text{Reactive power} = 1 \times 1 \times \sin 11.07^\circ$$

$$= 0.192 \text{ p.u. leading}$$

### Solution : 3

(a)

$$R = 2 \text{ Hz/p.u. MW}$$

$$D = \frac{\partial P_L}{\partial f} = \frac{0.01 \times 5000}{0.01 \times 50} = 100 \text{ MW/Hz}$$

$$= \frac{100}{10000} = 0.01 \text{ p.u. MW/Hz}$$

$$\beta = D + \frac{1}{R} = 0.01 + \frac{1}{2} = 0.51 \text{ p.u. MW/Hz}$$

$$M = \frac{2}{100} \times 5000 = 100 \text{ MW} = 0.01 \text{ p.u. MW}$$

$$\Delta f_0 = \frac{0.01}{0.51} = -0.01961 \text{ Hz}$$

(b) Since speed governor loop is open,  $R$  is infinite.

$$\beta = D = 0.01 \text{ p.u. MW}$$

$$\Delta f_0 = -\frac{0.01}{0.01} = -1 \text{ Hz}$$

### Solution : 4

Choose 2000 MW base

$$D_A = \frac{0.01 \times 500}{0.01 \times 50} = 10 \text{ MW/Hz}$$

$$= \frac{10}{2000} = 0.005 \text{ p.u. MW/Hz}$$

$$D_B = \frac{0.01 \times 2000}{0.01 \times 50} = 40 \text{ MW/Hz}$$

$$= \frac{40}{2000} = 0.02 \text{ p.u. MW/Hz}$$

$$R_A = 2.5 \text{ Hz/p.u. MW on 500 MW base}$$

Therefore for 2000 MW base

$$R_A = 2.5 \times \frac{2000}{500} = 10 \text{ Hz/p.u. MW}$$

$$R_B = 2 \text{ Hz/p.u. MW}$$

$$\beta_A = D_A + \frac{1}{R_A} = 0.005 + \frac{1}{10} = 0.105 \text{ p.u. MW/Hz}$$

$$\beta_B = D_B + \frac{1}{R_B} = 0.02 + \frac{1}{2} = 0.52 \text{ p.u. MW/Hz}$$

$$(a) \quad M_A = \frac{20}{2000} = 0.01$$

$$M_B = 0$$

Using equation, 
$$\Delta f_0 = -\frac{0.01}{0.105 + 0.52} = -0.016 \text{ Hz}$$

$$\begin{aligned} \Delta P_{AB,0} &= \frac{-0.52 \times 0.01}{0.105 + 0.52} = -0.00832 \text{ p.u. MW} \\ &= (-0.00832)(2000) = -16.64 \text{ MW} \end{aligned}$$

The minus sign indicates that power transfer is from area *B* to area *A*

$$(b) \quad M_A = 0$$

$$M_B = \frac{20}{2000} = 0.01$$

Using equation, 
$$\Delta f_0 = -0.016 \text{ Hz}$$

$$\begin{aligned} \Delta P_{AB,0} &= \frac{0.105 \times 0.01}{0.105 + 0.52} = 0.00168 \text{ p.u. MW} \\ &= 0.00168 \times 2000 = 3.36 \text{ MW} \end{aligned}$$

The power transfer is from area *A* to area *B*.

### Solution : 5

(a) When switch *S* is open.

Transfer reactance, 
$$\begin{aligned} X &= X_{dg} + X_T + \frac{1}{2}X_I + \frac{1}{2}X_I \\ &= 0.8 + 0.1 + 0.3 + 0.3 = 1.5 \text{ pu} \end{aligned}$$

Therefore, the steady-state power limit,

$$P_{r\max} = \frac{EV}{X} = \frac{1.2 \times 1.0}{1.5} = 0.8 \text{ pu}$$

(b) When switch *S* is closed.

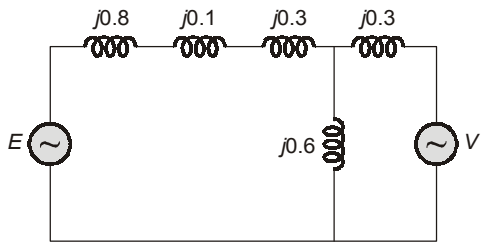
With switch *S* closed, the system is represented by the equivalent network shown in figure (b). The transfer impedance is found by using the relation,

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

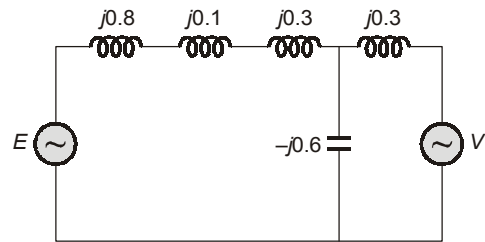
Here, 
$$Z_1 = j \left( X_{dg} + X_T + \frac{1}{2}X_I \right) = j(0.8 + 0.1 + 0.3) = j1.2 \text{ pu}$$



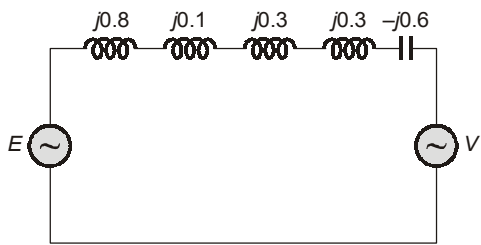
$$Z_2 = j\frac{1}{2}X_l = j0.3 \text{ pu}$$



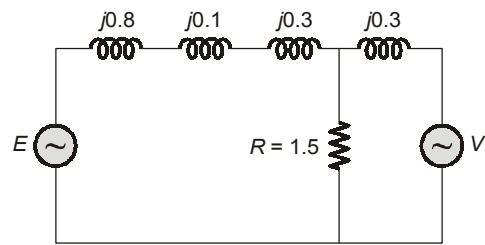
(b)



(c)



(d)



(e)

$$Z_3 = jX_r = j0.6 \text{ pu}$$

where,  $X_r$  = inductive reactance of the reactor,

$$\therefore B = jX = j1.2 + j0.3 + \frac{(j1.2) \times (j0.3)}{j0.6} = j2.1 \text{ pu}$$

$$|B| = X = 2.1 \text{ pu}, \beta = 90^\circ$$

$$P_{\max} = \frac{EV}{X} = \frac{1.2 \times 1.0}{2.1} = 0.5714 \text{ pu}$$

(c) When the shunt reactor is replaced by a shunt capacitor

Under this condition the equivalent network is shown in figure (c),

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} = j1.2 + j0.3 + \frac{(j1.2) \times (j0.3)}{(-j0.6)} = 0.9 \angle 90^\circ$$

$$|B| = X = 0.9, \beta = 90^\circ$$

$$P_{\max} = \frac{EV}{X} = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

(d) When the shunt capacitor is replaced by a series capacitor figure (d) shows this arrangement. The transfer reactance is given by

$$X = X_{dg} + X_T + X_l - X_c = 0.8 + 0.1 + 0.6 - 0.6 = 0.9$$

$$P_{\max} = \frac{EV}{X} = \frac{1.2 \times 1.0}{0.9} = 1.333 \text{ pu}$$

- (e) When the shunt inductor is replaced by a resistor figure (e) shows this arrangement. The generalized  $A$  and  $B$  constants of the T-network are given by

$$A = 1 + \frac{Z_1}{Z_3}$$

$$Z_1 = j0.8 + j0.1 + 0.3 = j1.2, \quad Z_3 = 1.5$$

$$\therefore A = 1 + \frac{j1.2}{1.5} = 1 + j0.8 = 1.2806 \angle 36.66^\circ$$

$$A = 1.2806, \quad \alpha = 36.66^\circ$$

$$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} = j1.2 + j0.3 + \frac{j(1.2) \times j0.3}{1.5}$$

$$= -0.24 + j1.5 = 1.519 \angle 99.1^\circ$$

$$B = 1.519, \quad \beta = 99.1^\circ$$

$$P_{\max} = \frac{EV}{B} - \frac{AV^2}{B} \cos(\beta - \alpha)$$

$$= \frac{1.2 \times 1}{1.519} - \frac{1.2806 \times 1}{1.519} \cos(99.1^\circ - 36.66^\circ) = 0.4 \text{ pu}$$

### Solution : 7

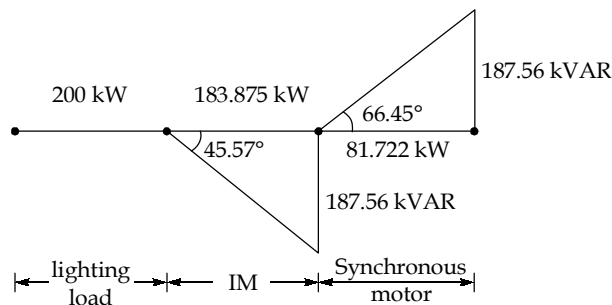
Active power consumed by induction motor,

$$= \frac{200 \times 735.5}{0.8} = 183.875 \text{ kW}$$

p.f.,  $\cos \phi_{IM} = 0.7$  lagging,  $\phi_{IM} = 45.57^\circ$

Active power consumed by synchronous motor,

$$= \frac{100 \times 735.5}{0.9} = 81.722 \text{ kW}$$



Reactive power consumed by IM

$$= 183.875 \times \tan(45.57^\circ) = 187.57 \text{ kVAR}$$

So reactive power supplies by synchronous motor = 187.57 kVAR

$$\phi = \tan^{-1} \left( \frac{187.57}{81.722} \right) = 66.458^\circ$$

The p.f. of synchronous motor =  $\cos(66.458^\circ)$

$$\cong 0.4 \text{ leading}$$



# 4

## Power System Studies

### LEVEL 1 Objective Questions

1. (14)
2. (d)
3. (72)
4. (a)
5. (d)
6. (a)

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### LEVEL 2 Objective Questions

7. (b)
8. (12.7°)
9. (1383.74)
10. (36.66)
11. (d)
12. (d)
13. (0.556)



## LEVEL 3 Conventional Questions

**Solution : 1**

Let the common MVA base chosen be 30 MVA and the voltage base in the generator circuit be 13.8 kV. The base voltages in other circuits are as follows:

Base voltage in transmission line

$$= 13.8 \times \frac{132}{13.2} = 138 \text{ kV}$$

$$\text{Base voltage in motor circuit} = 138 \times \frac{13.2}{132} = 13.8 \text{ kV}$$

The reactances of generator, transformers, line and motors are converted to pu values on appropriate bases as follows:

For generator:

$$\begin{aligned} G_1 &= \text{Base MVA} = 30, \\ &= \text{Base voltage} = 13.8 \text{ kV} \\ X_1 &= X_2 = X''_d = 0.15 \text{ pu} \\ X_0 &= 0.06 \text{ pu} \end{aligned}$$

For transformers  $T_1$  and  $T_2$ :

$$\begin{aligned} \text{Base MVA} &= 30 \\ \text{Base voltage on LT side} &= 13.8 \text{ kV} \\ \text{Base voltage on HT side} &= 138 \text{ kV} \end{aligned}$$

The reactances of transformers on appropriate bases are

$$\begin{aligned} X_1 &= X_2 = X_0 \\ &= 0.1 \times \frac{30}{50} \times \left( \frac{13.2}{13.8} \right)^2 = 0.0548 \text{ pu} \end{aligned}$$

For transmission line :

$$\text{Base MVA} = 30, \text{ Base voltage} = 138 \text{ kV}$$

The reactance are

$$X_1 = X_2 = \frac{90 \times 30}{(138)^2} = 0.142 \text{ pu}$$

$$X_0 = \frac{300 \times 30}{(138)^2} = 0.472 \text{ pu}$$

For motors  $M_1$  and  $M_2$ :

$$\text{Base MVA} = 30, \text{ Base voltage} = 13.8 \text{ kV}$$

The reactance of motor  $M_1$  are

$$\begin{aligned} X_1 &= X_2 = X''_d \\ &= 0.2 \times \frac{30}{15} \times \left( \frac{12.5}{13.8} \right)^2 = 0.328 \text{ pu} \end{aligned}$$

$$X_0 = 0.06 \times \frac{30}{15} \times \left( \frac{12.5}{13.8} \right)^2 = 0.098 \text{ pu}$$

The reactance of motor  $M_2$  are

$$X_1 = X_2 = X''_d$$

$$= 0.2 \times \frac{30}{10} \times \left( \frac{12.5}{13.8} \right)^2 = 0.492 \text{ pu}$$

$$X_0 = 0.06 \times \frac{30}{10} \times \left( \frac{12.5}{13.8} \right)^2 = 0.147 \text{ pu}$$

Reactance of current limiting reactor of generator

$$Z_n = X_n = 2 \times \frac{30}{(13.8)^2} = 0.315 \text{ pu}$$

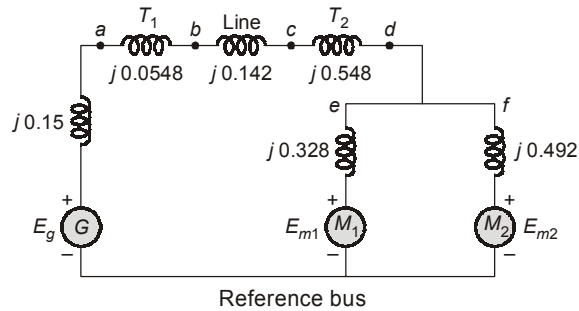
$$3Z_n = 3X_n = 3 \times 0.315 = 0.945 \text{ pu}$$

Reactance of current limiting reactor of motor  $M_1$

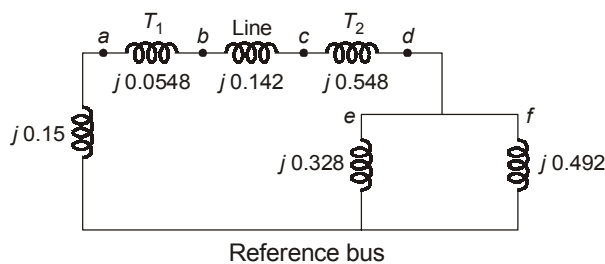
$$Z_n = X_n = 2 \times \frac{30}{(13.8)^2} = 0.315 \text{ pu}$$

$$3Z_n = 3X_n = 3 \times 0.315 = 0.945 \text{ pu}$$

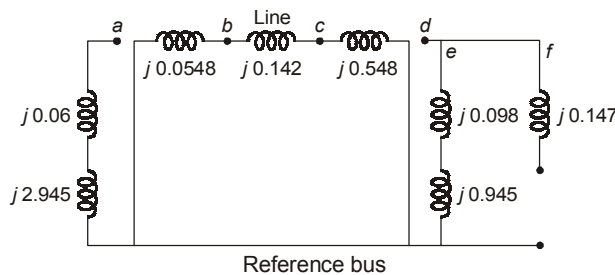
Using the values of the above reactances, the positive, negative and zero-sequence network are as shown in figure (a), (b) and (c) respectively.



**Fig. (a):** Positive sequence network



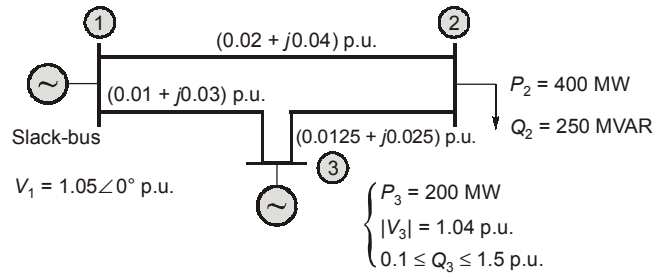
**Fig. (b):** Negative sequence network



**Fig. (c):** Zero sequence network for the system

**Solution : 2**

Single line diagram:



From given figure

Bus 1 : Slack bus

2 : PQ bus (load bus)

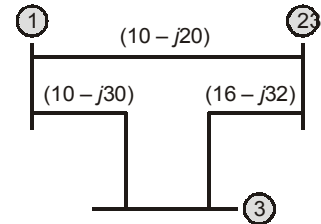
3 : PV bus

From the given line impedances convert in to admittances.

$$Y_{12} = \frac{1}{0.02 + j0.04} = (10 - j20) \text{ p.u.};$$

$$Y_{13} = \frac{1}{0.01 + j0.03} = (10 - j30) \text{ p.u.}$$

$$Y_{23} = \frac{1}{0.0125 + j0.025} = (16 - j32) \text{ p.u.}$$



$$[Y_{\text{BUS}}] = \begin{bmatrix} (20 - j50) & -(10 - j20) & -(10 - j30) \\ -(10 - j20) & (26 - j52) & -(16 - j32) \\ -(10 - j30) & -(16 - j32) & (26 - j62) \end{bmatrix} \text{ p.u.}$$

Initial voltages are

$$V_1 = 1.05 \angle 0^\circ$$

$$V_2 = 1 \angle 0^\circ$$

$$|V_3| = 1.04$$

$$V_3 = 1.04 \angle 0^\circ$$

(Assumed)

(Angle assumed)

Unknown parameters are  $|V_2| \angle \delta_2$  and  $\angle \delta_3$ 

By using fast decoupled method

$$\begin{bmatrix} \frac{\Delta P_2}{|V_2|} \\ \frac{\Delta P_3}{|V_3|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} \quad \dots(1)$$

$$\begin{bmatrix} \frac{\Delta Q_2}{|V_2|} \end{bmatrix} = [-B_{22}] [\Delta |V_2|] \quad \dots(2)$$

where,  $\left. \begin{matrix} B_{22} = -52, & B_{23} = 32 \\ B_{32} = 32, & B_{33} = -62 \end{matrix} \right\}$  from Y-Bus matrix

$$\Delta P_2 = P_{2S} - P_{2C}^{(0)}; \quad \Delta P_3 = P_{3S} - P_{3C}^{(0)}; \quad \Delta Q_2 = Q_{2S} - Q_{2C}^{(0)}$$

$$\begin{aligned} P_{2C}^{(0)} &= G_{22} |V_2|^2 + |V_1| |V_2| (G_{12} \cos \delta_{12} + B_{12} \sin \delta_{12}) + |V_2| |V_3| (G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23}) \\ &= 26 \times 1^2 + 1.05 \times 1 \times (-10 \cos(0) + 20 \sin(0)) + 1 \times 1.04 \times (-16 \cos(0) + 32 \sin(0)) \\ &= -1.14 \end{aligned}$$

$$\begin{aligned} P_{3C}^{(0)} &= G_{33} |V_3|^2 + |V_1| |V_3| (G_{13} \cos \delta_{13} + B_{13} \sin \delta_{13}) + |V_2| |V_3| (G_{23} \cos \delta_{23} + B_{23} \sin \delta_{23}) \\ &= 26 \times 1.04 + 1.05 \times 1.04(-10) + 1 \times 1.04 \times (-16) = -0.52 \text{ pu} \end{aligned}$$

$$\begin{aligned} Q_{2C}^{(0)} &= -B_{22} |V_2|^2 + |V_1| |V_2| (G_{12} \sin \delta_{12} - B_{12} \cos \delta_{12}) + |V_2| |V_3| (G_{23} \sin \delta_{23} + B_{23} \cos \delta_{23}) \\ &= -(-52) \times 1 + (1.05)(1)[-20] + 1 \times 1.04 \times (-32) \\ &= -2.28 \text{ pu} \end{aligned}$$

Assume base MVA =  $S_b = 100 \text{ MVA}$

$$P_{2S} = 400 \text{ MW}; \quad Q_{2S} = 250 \text{ MVAR}$$

$$S_{2S} = \frac{(P_{2S} + jQ_{2S})}{S_b} = \frac{(400 + j250)}{100} = (4 + j2.5) \text{ pu}$$

$$P_{3S} = 200 \text{ MW}, \quad P_{3S} = \frac{200}{100} = 2 \text{ pu}$$

$$\Delta P_2 = P_{2S} - P_{2C} = 4 + 1.14 = 5.14 \text{ pu.}$$

$$\Delta P_3 = P_{3S} - P_{3C}^{(0)} = 2 - (-0.52) = 2.52 \text{ pu}$$

$$\Delta Q_2 = Q_{2S} - Q_{2C}^{(0)} = 2.5 - (-2.28) = 4.78 \text{ pu}$$

By using fast decoupled matrix (1),

$$\begin{bmatrix} 5.14 \\ 1 \\ 2.52 \\ 1.04 \end{bmatrix} = \begin{bmatrix} -(-52) & -32 \\ -32 & -(-62) \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix}$$

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = \begin{bmatrix} 52 & -32 \\ -32 & 62 \end{bmatrix}^{-1} \begin{bmatrix} 5.14 \\ 2.42 \end{bmatrix} = \begin{bmatrix} 0.028 & 0.0145 \\ 0.0145 & 0.023 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 5.14 \\ 2.42 \end{bmatrix}_{2 \times 1}$$

$$\begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \end{bmatrix} = \begin{bmatrix} 0.18 \\ 0.13164 \end{bmatrix}$$

$$\Delta \delta_2 = 0.18^\circ$$

$$\Delta \delta_3 = 0.13164^\circ$$

From equation (2),  $\begin{bmatrix} \Delta Q_2 \\ |V_2| \end{bmatrix} = [-B_{22}] [\Delta |V_2|]$

$$\begin{bmatrix} 4.78 \\ 1 \end{bmatrix} = [-(-52)] [\Delta |V_2|]$$

$$\Delta |V_2| = 0.09192$$

At the end of first iteration,  $V_3 \angle \delta_3 = 1.04 \angle 0.13164^\circ$

$$V_2 \angle \delta_2 = 2.09192 \angle 0.18^\circ$$

**Solution : 3**

$$Y_{11} = y_{12} + y_{13} = (2 - j8) + (1 - j4) = (3 - j12)$$

$$Y_{21} = Y_{12} = -y_{12} = (-2 + j8)$$

$$Y_{31} = Y_{13} = -y_{13} = (-1 + j4)$$

$$Y_{41} = Y_{14} = 0$$

$$\begin{aligned} Y_{22} &= y_{12} + y_{23} + y_{24} \\ &= (2 - j8) + (0.666 - j2.664) + (1 - j4) \\ &= (3.666 - j14.664) \end{aligned}$$

$$Y_{32} = Y_{23} = -y_{23} = (-0.666 + j2.664)$$

$$Y_{42} = Y_{24} = -y_{24} = (-1 + j4)$$

$$\begin{aligned} Y_{33} &= y_{13} + y_{23} + y_{34} \\ &= (1 - j4) + (0.666 - j2.664) + (2 - j8) \\ &= (3.666 - j14.664) \end{aligned}$$

$$Y_{43} = Y_{34} = -y_{34} = (-2 + j8)$$

$$\begin{aligned} Y_{44} &= y_{42} + y_{43} \\ &= (1 - j4) + (2 - j8) = (3 - j12) \end{aligned}$$

Thus, the admittance matrix,

$$Y_{\text{BUS}} = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4 \\ -1 + j4 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8 \\ 0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

The power for generator bus are taken as positive while the power for load buses are taken as negative.

$$\text{Initial value, } V_1^{(0)} = 1.04 \angle 0^\circ, V_2^0 = V_3^0 = V_4^0 = 1.0$$

For the given system,

$$\begin{aligned} V_2^{(1)} &= \frac{1}{y_{22}} \left[ \frac{P_2 - jQ_2}{V_2^*} - Y_{21} V_1^{(0)} - Y_{23} V_3^{(0)} - Y_{24} V_4^{(0)} \right] \\ &= \frac{1}{(3.666 - j14.664)} \left[ \frac{-0.5 + j0.2}{1 - j0} - (-2 + j8) \times 1.04 - (-0.666 + j2.664)(1 + j0) - (-1 + j4)(1 + j0) \right] \\ &= \frac{1}{(3.666 - j14.664)} [-0.5 + j0.2 + 2.08 - j8.32 + 0.666 - j2.664 + 1 - j4] \end{aligned}$$

$$V_2^{(1)} = \frac{3.246 - j14.784}{3.666 - j14.664} = 1.00096 - j0.02888 = 1.0014 \angle -1.653^\circ$$

$$\begin{aligned} V_{2(\text{acce})} &= (1 + j0) + 1.6[(1.00096 - j0.02888) - (1 + j0)] \\ &= 1.00154 - j0.046208 = 1.0026 \angle -2.6416^\circ \end{aligned}$$

$$\begin{aligned} \text{Now, } V_3^{(1)} &= \frac{1}{Y_{33}} \left[ \frac{P_3 - jQ_3}{V_3^*} - Y_{31} V_1^{(0)} - Y_{32} V_2^{(1)} - Y_{34} V_4^{(0)} \right] \\ &= \frac{1}{(3.666 - j14.664)} \left[ \frac{-0.4 + j0.3}{(1 - j0)} - (-1 + j4) \times 1.04 - (-0.666 + j2.664)(1.00154 - j0.046208) - (-2 + j8)(1 + j0) \right] \\ &= \frac{1}{(3.666 - j14.664)} [-0.4 + j0.3 + 1.04 - j4.16 + 0.54393 - j2.69888 + 2 - j8] \end{aligned}$$



$$V_3^{(1)} = \frac{3.18393 - j14.55888}{3.666 - j14.664} = (0.99216 - j0.03114)$$

$$V_{3(\text{acce})}^1 = (1 + j0) + 1.6[(0.99216 - j0.03114) - (1 + j0)] \\ = (0.98746 - j0.049824) = 0.988712 \angle -2.8885^\circ$$

and 
$$V_4^1 = \frac{1}{Y_{44}} \left[ \frac{P_4 - jQ_4}{V_4^*} - Y_{42} V_2^{(1)} - Y_{43} V_3^{(1)} \right]$$

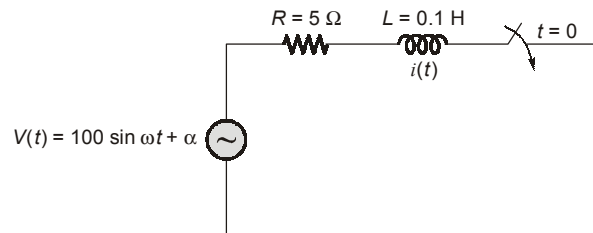
$$= \frac{1}{(3 - j12)} \left[ \frac{-0.3 + j0.1}{1 - j0} - (-1 + j4)(1.00154 - j0.046208) - (-2 + j8)(0.98746 - j0.049824) \right]$$

$$= \frac{1}{(3 - j12)} [-0.3 + j0.1 + 0.816708 - j4.05237 + 1.57633 - j7.99933]$$

$$V_4^{(1)} = \frac{2.09304 - j11.9517}{3 - j12} = 0.97843 - j0.070187$$

$$V_{4(\text{acce})}^{(1)} = (1 + j0) + 1.6[(0.97843 - j0.070187) - (1 + j0)] \\ = 0.965488 - j0.112992 = 0.971997 \angle -6.634456^\circ \text{ V}$$

**Solution : 4**



$$f = 50 \text{ Hz}$$

$$\omega = 2\pi f = 100\pi$$

$$X_L = \omega L = (100\pi)(0.1) = 10\pi$$

$$Ri(t) + \frac{L di(t)}{dt} = 100 \sin(\omega t + \alpha)$$

The solution of above equation, 
$$i(t) = I_m \left[ \sin\left(\omega t + \alpha - \tan^{-1} \frac{\omega L}{R}\right) - \sin\left(\alpha - \tan^{-1} \frac{\omega L}{R}\right) \right] e^{-\frac{R}{L}t}$$

$$I_m = \frac{100}{|Z|}, \quad Z = R + j\omega L = 5 + j10\pi; \quad |Z| = 31.80 \Omega$$

Here, 
$$\frac{R}{L} = \frac{5}{0.1} = 50$$

$$\frac{\omega L}{R} = \frac{100\pi(0.1)}{5} = \frac{10\pi}{5} = 2\pi = 6.28$$

$$\tan^{-1} \frac{\omega L}{R} = \tan^{-1} 6.28 = 80.95^\circ$$

$$I_m = \frac{100}{31.80} = 3.14 \text{ A}$$

$$i(t) = 3.14 \sin(\omega t + \alpha - 80.95^\circ) e^{-50t} - 3.14 e^{-50t} \sin(\alpha - 80.95^\circ)$$

The d.c. offset component  $i_{dc} = 3.14 e^{-50t} \sin(\alpha - 80.95^\circ)$

$i_{dc}(t)$  is maximum when  $(\alpha - 80.95^\circ) = \pm 90^\circ$

So,  $\alpha - 80.95^\circ = -90^\circ$

$$\alpha = -9.05^\circ$$

So, for no d.c. offset current, voltage should be applied when  $\alpha = 80.95^\circ$

For maximum d.c. offset current  $\alpha = -9.05^\circ$

### Solution : 5

Chose a system base MVA of 25 MVA.

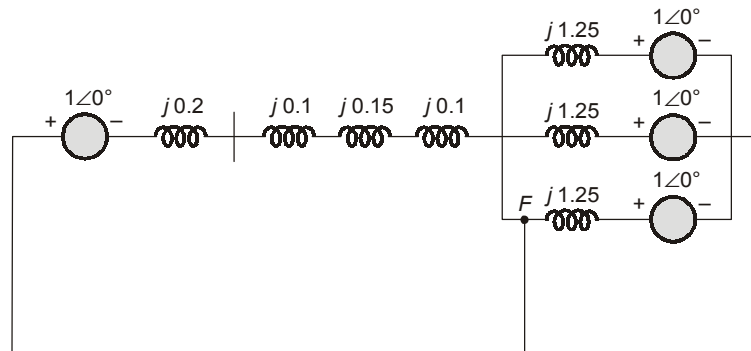
For a generator, voltage base is 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.

(i) For each motor,

$$X''_{dm} = j0.25 \times \frac{25}{5} = j1.25 \text{ pu}$$

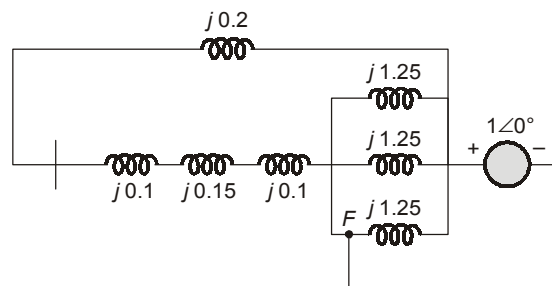
Line, transformers and generator reactances are already on the proper base.

The circuit model for fault condition is as under:

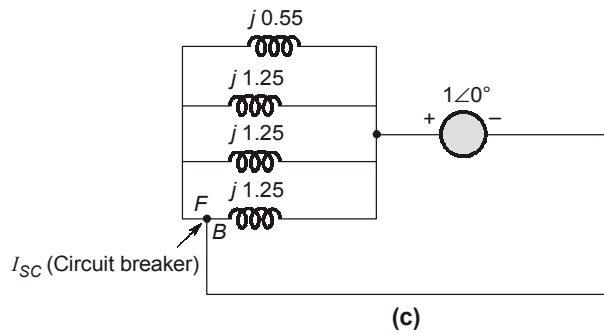


(a)

This system initially being on no load, the generator and motor induced emf are identical. The circuit can be reduced to (b) and then (c)



(b)



$$I_{SC} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j4.22 \text{ pu}$$

$$\text{Base current for 6.6 kV} = \frac{25 \times 1000}{(\sqrt{3})(6.6)} = 2187 \text{ A}$$

$$\therefore I_{SC} = (4.22)(2187) = 9229 \text{ A}$$

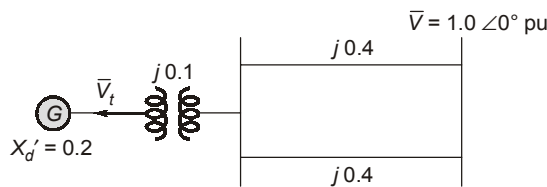
(ii) From (a) the current through C.B.

$$\begin{aligned} I_{SC}(B) &= 2 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j3.42 \text{ pu} \\ &= 3.42 \times 2187 = 7479.54 \text{ A} \end{aligned}$$

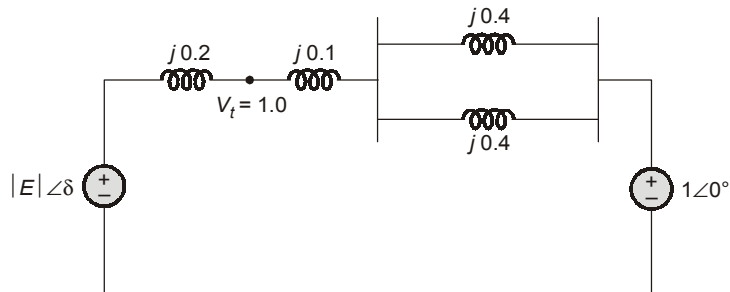
(iii) For finding momentary current through the breaker, we must add DC offset current to the symmetrical sub-transient current obtained in part (b). Generally momentary current through

$$\begin{aligned} \text{CB} &= (1.6) \times I_{SC}(B) \\ &= (1.6)(7479.54) = 11967 \text{ A} \end{aligned}$$

**Solution : 6**



Equivalent circuit,



Let,  $V_t = |V_t| \angle \alpha = 1 \angle \alpha$

Power angle equation,  $\frac{|V_t||V|}{X} \sin \alpha = P_e$

$$X = j0.1 + \frac{(j0.4)(j0.4)}{j0.8} = j0.3$$

$$\frac{(1 \times 1)}{0.3} \sin \alpha = 1$$

$$\sin \alpha = 0.3 \Rightarrow \alpha = 17.46^\circ$$

Current into infinite bus,  $I = \frac{|V_t| \angle \alpha - 1 \angle 0^\circ}{jX} = \frac{1 \angle 17.46^\circ - 1}{j0.3} = \frac{0.954 + j0.3 - 1}{j0.3}$

$$I = 1 + j0.153 = 1.011 \angle 8.70^\circ$$

Voltage behind transient reactance,

$$E = 1 \angle 0^\circ + jX' (1 + j0.153)$$

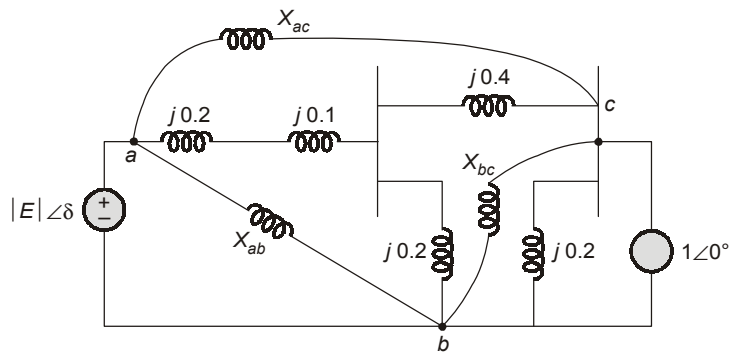
$$X' = j0.2 + j0.1 + j0.2 = j0.5$$

$$E = 1 \angle 0^\circ + j0.5 (1 + j0.153) = 0.9235 + j0.5 = 1.05 \angle 28.43^\circ$$

(i) When system is healthy,

$$P_{\max} = \frac{|V||E|}{X'} = \frac{(1)(1.05)}{0.5} = 2.1 \text{ pu}$$

(ii) When one line is shorted in middle



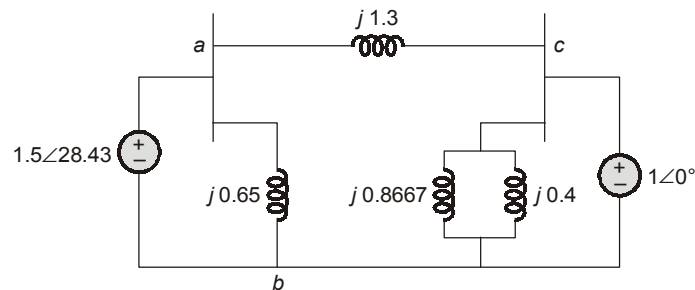
Converting star into delta,

$$X_{ac} = \frac{(0.3)(0.4) + (0.2)(0.3) + (0.2)(0.4)}{0.2} = \frac{0.12 + 0.06 + 0.08}{0.2}$$

$$X_{ac} = j1.3$$

$$X_{bc} = j0.8667$$

$$X_{ba} = j0.65$$



$$X_{ac} = 1.3$$

$$P_{\max} = \frac{(1)(1.05)}{1.3} = 0.8 \text{ pu}$$

(iii) One line open,

$$X_{ac} = j0.2 + j0.1 + j0.4 = j0.7$$

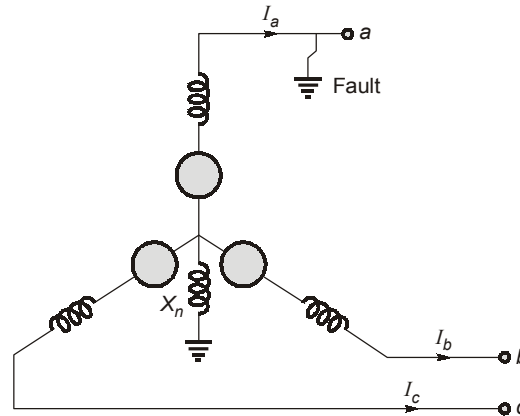
$$P_{\max} = \frac{|V||E|}{X_{ac}} = \frac{(1)(1.05)}{0.7} = 1.5 \text{ pu}$$

**Solution : 7**

3- $\phi$ , 11 kV, 20 MVA,

$$X_1 = j0.6 \text{ p.u.}, X_2 = j0.25 \text{ p.u.}, X_0 = j0.15 \text{ p.u.}$$

(i)



$X_{npu}$  = Reactance to limit neutral current in p.u.

For single line to ground fault

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3}$$

$$I_{a1} = \frac{E_a}{X_1 + X_2 + X_0 + 3X_{npu}}$$

$$E_a = 1 \text{ p.u.}$$

$$I_{a1} = \frac{1}{j(0.6 + 0.25 + 0.15 + 3X_{npu})} = -j \frac{1}{1 + 3X_n} = I_{a1}$$

$$I_a = \text{Fault current in p.u.} = 3I_{a1}$$

$$I_a = \frac{-j3}{1 + 3X_n} \text{ p.u.} \quad \dots(1)$$

Given that fault current should be restricted to rated current

$$\text{Rated current} = 1 \text{ p.u.} \quad \dots(2)$$

From (1) and (2),

$$\frac{3}{1 + 3X_{npu}} = 1$$

$$X_{npu} = 0.667 \text{ p.u. (magnitude in p.u.)}$$

So neutral reactance to limit fault current to 1 p.u. = 66.67% p.u

$$\text{Actual reactive impedance} = j(X_{npu})(Z_{\text{base}})$$

$$Z_{\text{base}} = \frac{(\text{kV})^2}{\text{MVA}} = \frac{(11)^2}{20} = 6.05 \Omega$$

$$X_n = j(0.667)(6.05) = 4.03 \Omega$$

$$X_n = j4.03 \Omega$$

(ii) Consider a resistance  $R_n$  in neutral

$R_{npu}$  = Resistance in p.u.

$$I_{a1} = \text{Positive sequence current} = \frac{1}{j(X_1 + X_2 + X_0) + 3R_{npu}}$$

$I_a$  = Fault current in p.u.

$$I_a = 3I_{a1} = \frac{3}{3R_{npu} + j(1)}$$

Given,

$$|I_a| = 1$$

$$\frac{3}{\sqrt{9R_{npu}^2 + 1}} = 1$$

$$9 = 9R_{npu}^2 + 1$$

$$R_{npu} = \sqrt{\frac{8}{9}} = 0.943 \Omega$$

$$R_n = (R_{npu})(Z_{base}) = (0.943)(6.05) = 5.70 \Omega$$

$$R_n = 5.70 \Omega$$

### Solution : 8

(a) Tie-bus system:

Figure shows the tie-bus system. Consider a symmetrical three-phase fault at point  $F$ . Let  $X$  be the per unit reactance of each reactor on a 20 MVA base. The equivalent circuit after the fault at  $F$  is shown in figure (b).

$X_T$  = Thevenin equivalent between the reactance at fault point  $F$   
= Total reactance between the zero potential bus and point  $F$ .

∴

$$\begin{aligned} X_T &= [(0.15 + X) \parallel (0.15 + X) + X] \parallel (0.15) \\ &= \left\{ \frac{1}{2}(0.15 + X) + X \right\} \parallel (0.15) = (0.075 + 1.5X) \parallel (0.15) \\ &= \frac{(0.075 + 1.5X)(0.15)}{0.075 + 1.5X + 0.15} = \frac{(0.05 + X)(0.15)}{0.15 + X} \text{ p.u.} \end{aligned}$$

Short-circuit MVA fed into the fault,

$$S_{sc} = \frac{\text{base MVA}}{\text{equivalent per unit reactance}} = \frac{20(X + 0.15)}{0.15(X + 0.05)}$$

This should not exceed 200 MVA.

$$\therefore \frac{20(X + 0.15)}{0.15(X + 0.05)} = 200$$

$$X + 0.15 = 1.5(X + 0.05); \quad X = 0.15 \text{ p.u.}$$

$$\text{Full load current, } I_{fl} = \frac{20 \times 10^6}{\sqrt{3} \times 11000} = 1049.7 \text{ A}$$

$$\text{Phase voltage, } V_p = \frac{11000}{\sqrt{3}} = 6350.8 \text{ V}$$

Ohmic reactance of each reactor,

$$X_{\Omega} = X_{pu} \times \frac{V_p}{I_{fl}} = 0.15 \times \frac{6350.8}{1049.7} = 0.9075 \Omega$$

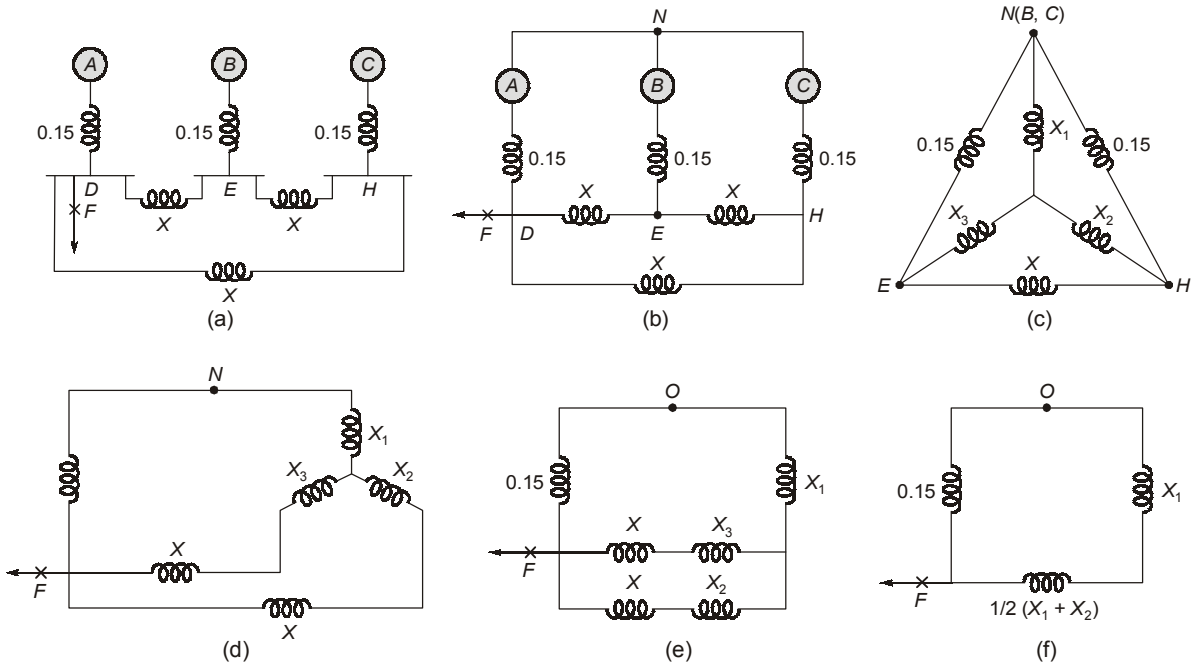
**(b) Ring system:**

Figure (a) shows the ring system of reactors. In order to determine Thevenin equivalent reactance at F we use the equivalent circuit after the fault at F as shown in figure (b). The delta-connected system between NEH is replaced by the star-connected system as shown in figure (c) where,

$$X_1 = \frac{0.15 \times 0.15}{0.15 + 0.15 + X} = \frac{0.0225}{0.3 + X}$$

$$X_2 = X_3 = \frac{0.15 X}{0.15 + 0.15 + X} = \frac{0.15 X}{0.3 + X}$$

Further simplification of network is shown in figure (d) to (f). The Thevenin equivalent reactance after fault at F is given from figure (f) as,



$$\begin{aligned} X_T &= 0.15 \parallel \left[ X_1 + \frac{1}{2}(X + X_2) \right] = 0.15 \parallel \left[ \frac{0.0225}{0.3 + X} + \frac{1}{2} \left( X + \frac{0.15 X}{0.3 + X} \right) \right] \\ &= 0.15 \parallel \left[ \frac{X}{2} + \frac{0.0225 + 0.075 X}{0.3 + X} \right] = 0.15 \parallel \left[ \frac{X}{2} + \frac{0.075(0.3 + X)}{0.3 + X} \right] \\ &= 0.15 \parallel \left( \frac{X}{2} + 0.075 \right) = \frac{0.15 \left( \frac{X}{2} + 0.075 \right)}{0.15 + \frac{X}{2} + 0.075} = \frac{0.15(X + 0.15)}{X + 0.45} \end{aligned}$$

Short-circuit MVA fed into the fault F,

$$S_{sc} = \frac{S_b}{X_T} = \frac{20(X + 0.45)}{0.15(X + 0.15)}$$

Since the short-circuit MVA is not to exceed 200 MVA, we have

$$\frac{20(X + 0.45)}{0.15(X + 0.15)} = 200$$

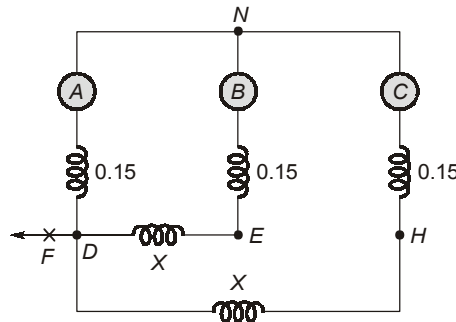
$$X + 0.45 = 1.5(X + 0.15); \quad X = 0.45$$

Reactance of each reactor,

$$X_{\Omega} = X_{pu} \frac{V_p}{I_{fl}} = 0.45 \times \frac{6350.8}{1049.7} = 2.722 \Omega$$

Alternative method for finding  $X_T$ :

In figure (b), the two paths  $NBEDF$  and  $NCHDF$  between  $N$  and  $F$  are symmetrical because the reactance of  $B$  and  $C$  are equal and the reactances between  $E$  and  $D$  and between  $H$  and  $D$  are also equal. Therefore  $E$  and  $H$  are at the same potential and there is no current in the reactor  $EH$ . The circuit of figure (b) can, therefore, be reduced to figure.



Equivalent per unit reactance between,

$$N \text{ and } D = \frac{1}{2}(X + 0.15)$$

Equivalent reactance between  $N$  and  $F$  is

$$X_T = (0.15) \parallel \left[ \frac{1}{2}(X + 0.15) \right] = \frac{0.15 \times \frac{1}{2}(X + 0.15)}{0.15 + \frac{1}{2}(X + 0.15)} = \frac{0.15(X + 0.15)}{X + 0.45}$$

as obtained before.

### Solution : 9

Base MVA for the complete system = 30 MVA

Base kV for generator side = 11 kV

Base kV for the line circuit = 66 kV

The per-unit reactance of various components on the common TVA base of 30 MVA are calculated below:

Generators  $G_1$  and  $G_2$  :  $0.15 \times \frac{30}{15} = 0.30 \text{ pu}$

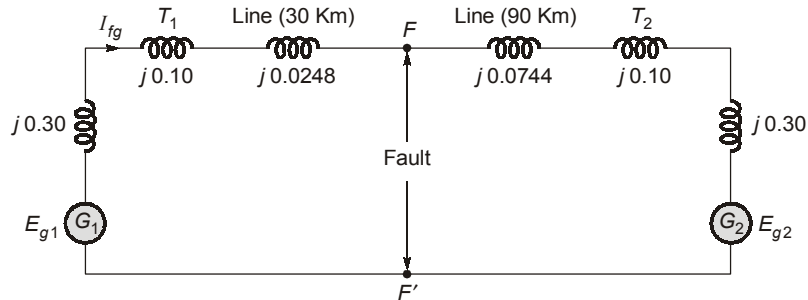
Transformers  $T_1$  and  $T_2$  :  $0.10 \times \frac{30}{30} = 0.10 \text{ pu}$

Line of 30-km length :  $\frac{(30 \times 0.12) \times 30}{(66^2)} = 0.0248 \text{ pu}$



Line of 90-km length : 
$$\frac{(90 \times 0.12) \times 30}{(66^2)} = 0.0744 \text{ pu}$$

From these values, the per-unit reactance figure (b) is drawn.



**Fig. (b):** Per-unit reactance diagram of the system

For fault calculation the Thevenin's equivalent circuit is drawn with respect to the fault point  $F$ . The pre-fault voltage  $V_f$  is the open-circuit voltage across  $FF'$ .

Hence, Thevenin's per unit voltage  $V_{f, pu} = 1.0 \text{ pu}$

The Thevenin's equivalent reactance  $X_{eq, pu}$  as viewed from the fault point  $F$  with voltage sources short-circuited is the equivalent reactance of two parallel branches whose reactances are

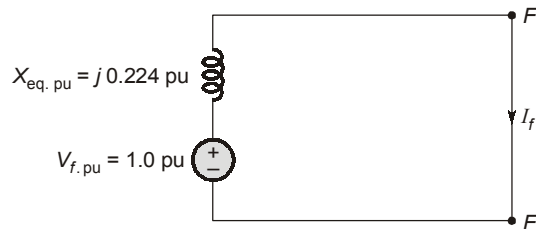
$$j0.30 + j0.10 + j0.0248 = j0.4248 \text{ pu}$$

and  $j0.30 + j0.10 + j0.0744 = j0.4744 \text{ pu}$

Hence, the Thevenin's equivalent reactance  $X_{eq, pu}$  is given by

$$X_{eq, pu} = \frac{j0.4248 \times j0.4744}{j0.4248 + j0.4744} = j0.224 \text{ pu}$$

The Thevenin's equivalent circuit with fault path  $FF'$  showing the fault current  $I_f$  is shown in figure (c).



**Fig. (c):** Thevenin's equivalent circuit

Per unit fault current,

$$I_{f, pu} = \frac{V_{f, pu}}{X_{eq, pu}} = \frac{1.0}{j0.224} = -j4.46 \text{ pu}$$

Base current in the line circuit is given by

$$I_b = \frac{(MVA)_b \times 10^3}{\sqrt{3} (kV)_b} \text{ A} = \frac{30 \times 10^3}{\sqrt{3} \times 66} = 262.43 \text{ A}$$

The actual fault current,

$$I_f = I_{f, pu} \times I_b = 4.46 \times 262.43 = 1170.44 \text{ A}$$

Fault MVA,

$$\begin{aligned} \sqrt{3} \times V_f \times I_f \times 10^{-3} &= \sqrt{3} \times 66 \times 1170.44 \times 10^{-3} \\ &= 133.8 \text{ MVA} \end{aligned}$$

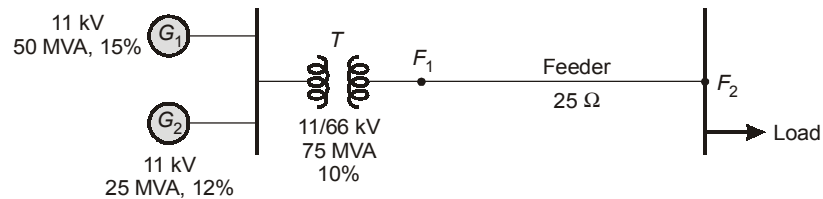
**Alternative Method:**

$$\text{Fault MVA (MVA)}_f = \frac{(\text{MVA})_b}{X_{\text{eq. pu}}} = \frac{30}{0.224} = 133.8 \text{ MVA}$$

$$\text{Fault current } I_f = \frac{(\text{MVA})_f \times 10^3}{\sqrt{3} \times (\text{kV})_b} = \frac{133.8 \times 10^3}{\sqrt{3} \times 66} = 1172 \text{ A}$$

**Solution : 10**

The single-line diagram of the system is shown in figure (a).



**Fig. (a):** One-line diagram for the system

Let us choose a common base MVA of 75 MVA and base voltage of 11 kV on LT side and 66 kV on the HT side.

The per unit reactances of the various equipment on the chosen base values are:

$$\text{Generator } G_1 : X''_{G1, \text{ pu}} = 0.15 \times \frac{75}{50} = 0.225 \text{ pu}$$

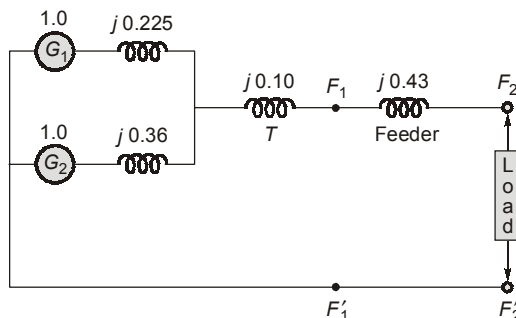
$$\text{Generator } G_2 : X''_{G2, \text{ pu}} = 0.12 \times \frac{75}{25} = 0.36 \text{ pu}$$

$$\text{Transformer } T : X_T, \text{ pu} = 0.10 \text{ pu}$$

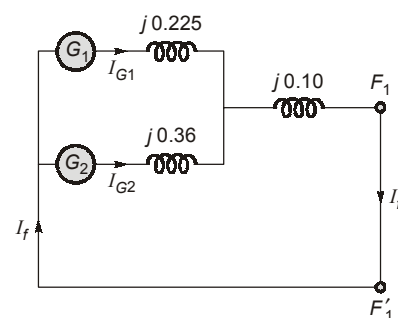
$$\text{Feeder} : X_F, \text{ pu} = X_F \times \frac{(\text{MVA})_b}{(\text{kV})_b^2} = 25 \times \frac{75}{(66)^2} = 0.43 \text{ pu}$$

The per unit reactance diagram of the system is shown in figure (b).

**(a)** The circuit model of the system for fault at the high voltage terminals of the transformer is shown in figure (c).  $F_1$  is the fault point and  $F_1F'_1$  is the fault path.



**Fig. (b):** Per unit reactance diagram



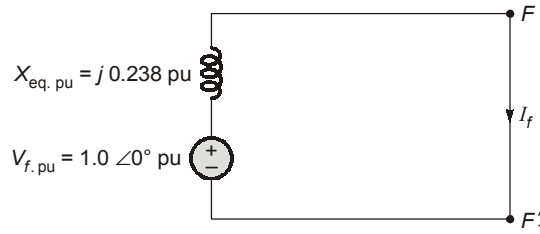
**Fig. (c):** Circuit model of the system for the fault at high-voltage terminals

The pre fault voltage at the fault point  $F_1$  is the rated voltage of the HT side of the transformer, (i.e., 66 kV), which is the base voltage of that side.

Hence, the per unit pre fault voltage  $V_{f, pu} = 1.0 \angle 0^\circ$  pu and Thevenin's per unit voltage  $V_{f, pu} = 1.0 \angle 0^\circ$  pu. The Thevenin's equivalent reactance  $X_{eq, pu}$  of the system as viewed from the fault point  $F_1$  with the voltage sources shorted is computed as follows:

$$X_{eq, pu} = j0.10 + \frac{j0.225 \times j0.36}{(j0.225 + j0.36)} = j0.238 \text{ pu}$$

The Thevenin's equivalent circuit with the fault (short-circuit) path  $F_1 F'_1$  showing the fault current  $I_f$  is shown in figure (d).



**Fig. (d):** Thevenin's equivalent circuit

The per unit fault current  $I_{f, pu}$  is given by.

$$I_{f, pu} = \frac{V_{f, pu}}{X_{eq, pu}} = \frac{1.0}{j0.238} = -j4.20 \text{ pu}$$

The base current  $I_b$  is given by

$$I_b = \frac{(MVA)_b \times 10^3}{\sqrt{3} \times (kV)_b} \text{ A} = \frac{75 \times 10^3}{\sqrt{3} \times 66} = 656.08 \text{ A}$$

The actual fault current,

$$I_f = I_{f, pu} \times I_b = 4.20 \times 656.857 = 2755.536 \text{ A}$$

Fault MVA,

$$(MVA)_f = \frac{(MVA)_b}{X_{eq, pu}} = \frac{75}{0.238} = 315.126 \text{ MVA}$$

$$\text{Fault current on LT side} = 2755.536 \times \frac{66}{11} = 16533.216 \text{ A}$$

The fault current shared by the generators can be found from the circuit shown in figure (c).

$$I_{G1, pu} = I_{f, pu} \times \frac{X_{G2, pu}}{X_{G1, pu} + X_{G2, pu}} = 4.20 \times \frac{0.36}{0.225 + 0.36} = 2.585 \text{ pu}$$

$$I_{G2, pu} = I_{f, pu} \times \frac{X_{G1, pu}}{X_{G1, pu} + X_{G2, pu}} = 4.20 \times \frac{0.225}{0.225 + 0.36} = 1.615 \text{ pu}$$

Base current on LT side,

$$I_b = \frac{(MVA)_b \times 10^3}{\sqrt{3} \times (kV)_b} = \frac{75 \times 10^3}{\sqrt{3} \times 11} = 3941.145 \text{ A}$$

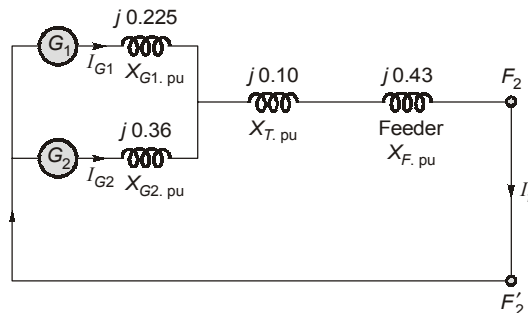
Hence,  $I_{G1}$  and  $I_{G2}$  in amperes are given by

$$I_{G1} = I_{G1, pu} \times I_b = 2.585 \times 3941.145 = 10187.86 \text{ A}$$

$$I_{G2} = I_{G2, pu} \times I_b = 1.615 \times 3941.145 = 6364.94 \text{ A}$$

- (b) The circuit model of the system for fault at the load end of the feeder is shown in figure (c).  
The per unit open circuit voltage across

$$F_2 F'_2 = V_{f, \text{ pu}} = 1.0 \angle 0^\circ \text{ pu}$$

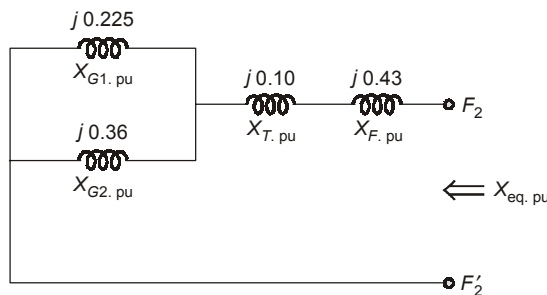


**Fig. (e):** Circuit model of the system for fault at load end of feeder

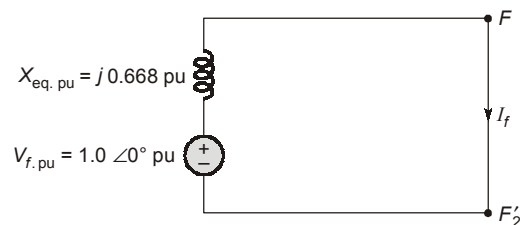
The Thevenin's equivalent reactance  $X_{\text{eq, pu}}$  is compared from the following circuit, figure (f).

$$\begin{aligned} X_{\text{eq, pu}} &= X_{F, \text{ pu}} + X_{T, \text{ pu}} + \frac{X_{G1, \text{ pu}} X_{G2, \text{ pu}}}{X_{G1, \text{ pu}} + X_{G2, \text{ pu}}} \\ &= j0.43 + j0.10 + \frac{j0.225 \times j0.36}{j0.225 + j0.36} = j0.668 \text{ pu} \end{aligned}$$

The Thevenin's equivalent circuit with the fault path  $F_2 F'_2$  showing the fault current is shown in figure (g).



**Fig. (f):** Thevenin's equivalent reactance



**Fig. (g):** Thevenin's equivalent circuit

If fault at the load end of the feeder that is point  $F_2$ ,

$$X_{\text{eq}} = j0.238 + j0.43 = j0.608$$

The per-unit fault current,  $I_{f, \text{ pu}} = \frac{1.0}{j0.668} = -j1.497 \text{ pu}$ .

Fault current,  $I_f \text{ (on HT side)} = I_{f, \text{ pu}} \times I_b = 1.497 \times 655.08 = 980.655 \text{ A}$

Fault MVA,  $(\text{MVA})_f = \frac{(\text{MVA})_b}{X_{\text{eq, pu}}} = \frac{75}{0.668} = 112.275 \text{ MVA}$

The fault current shared by generators is given by

$$I_{G1, \text{ pu}} = 1.497 \times \frac{0.36}{0.225 + 0.36} = 0.9212 \text{ pu}$$

$$I_{G2, \text{ pu}} = 1.497 \times \frac{0.225}{0.225 + 0.36} = 0.5758 \text{ pu}$$

$$\frac{75 \times 10^3}{\sqrt{3} \times 11} = 3936.48 \text{ A}$$

Hence, current  $I_{G1}$  and  $I_{G2}$  in amperes are given by

$$I_{G1} = I_{G2, \text{ pu}} \times I_b = 0.9212 \times 3936.48 = 3626.285 \text{ A}$$

$$I_{G2} = I_{G2, \text{ pu}} \times I_b = 0.5758 \times 3936.48 = 2266.62 \text{ A}$$

$$\text{Total fault current } I_f \text{ on LT side} = (I_{G1} + I_{G2}) = I_{f, \text{ pu}} \times I_b = 1.497 \times 3936.48 = 5892.91 \text{ A}$$

**Note:** From the calculated values of the fault current and fault MVA, it is clear that these values reduce drastically as the fault point moves further and further away from the source. This reduction is mainly due to more impedance coming in the path of the fault current. The MVA rating of the circuit breaker to be placed at any location should be more than the fault MVA (fault level) at that location.

**Solution : 11**

Let us choose the base MVA of 25. For a generator voltage base of 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.

The per unit reactance of various components on the chosen base are the following:

$$\text{Generator } G : X''_{dg, \text{ pu}} = j0.20 \text{ pu}$$

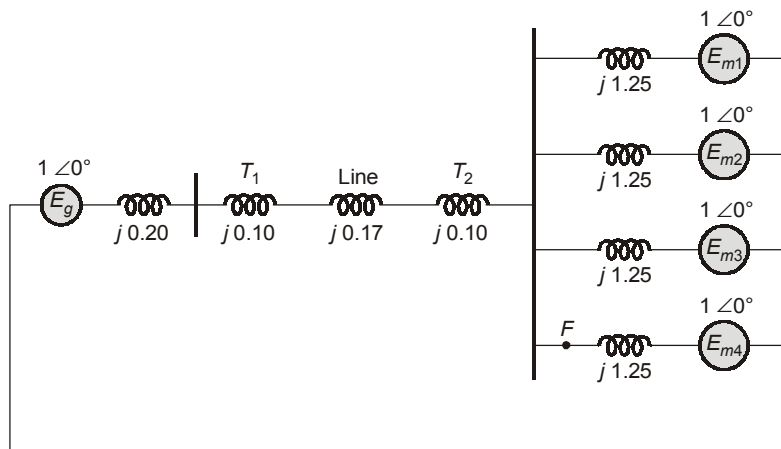
$$\text{Each motor : } X''_{dm, \text{ pu}} = j0.25 \times \frac{25}{5} = j1.25 \text{ pu}$$

$$X'_{dm, \text{ pu}} = j0.30 \times \frac{25}{5} = j1.5 \text{ pu}$$

$$\text{Each transformer : } X_{T, \text{ pu}} = j0.10 \text{ pu}$$

$$\text{Line : } X_{L, \text{ pu}} = j30 \times \frac{25}{(66)^2} = j0.17 \text{ pu}$$

(a) The reactance diagram of the system is shown in figure (a). The reactance diagram of the system for fault at point  $F$  is shown in figure (b). Since the system is initially on no load, the generator and motor induced emfs are identical. The circuit shown in figure (b) can therefore be reduced to that of figure (c), (d) and then to figure (e).



**Fig. (a):** The reactance diagram

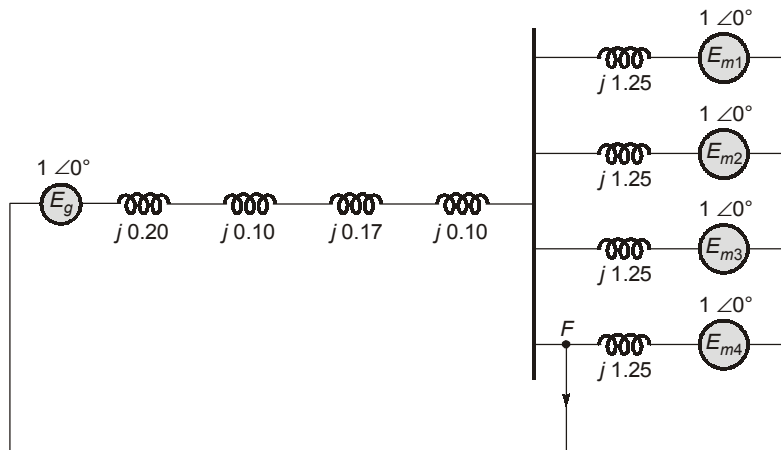


Fig. (b): Reactance diagram for fault at F

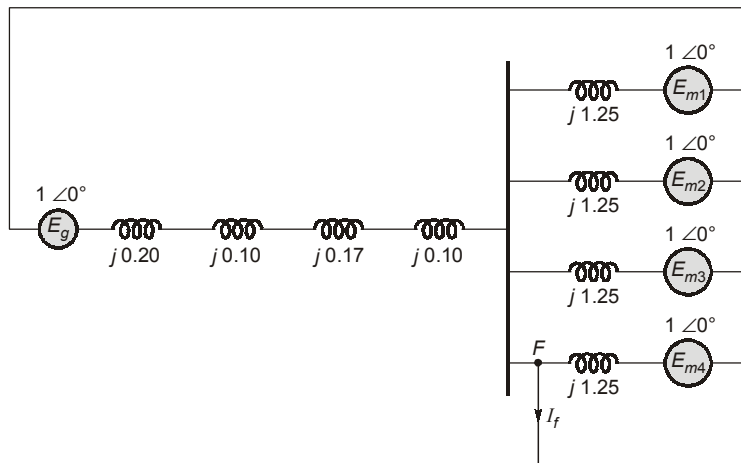


Fig. (c): Reduced circuit

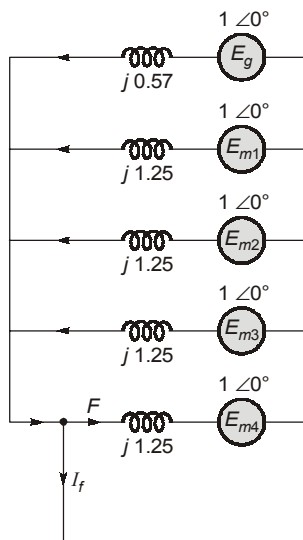


Fig. (d): Reduced circuit

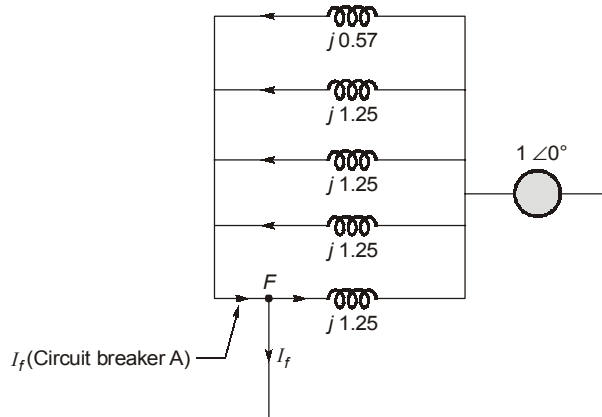
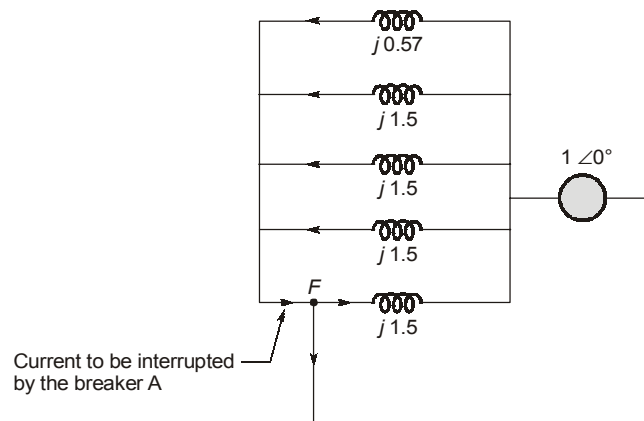


Fig. (e): Reduced circuit



**Fig. (f): Modified reactances**

From circuit of figure (e), the equivalent reactance is given by

$$X_{eq. pu} = \frac{\left( j \frac{1.25}{4} \times j0.57 \right)}{\left( j \frac{1.25}{4} + j0.57 \right)} = j0.202 \text{ pu}$$

The per unit subtransient fault current,

$$I_{f. pu} = \frac{1.0}{j0.202} = -j4.95 \text{ pu}$$

Base current,  $I_b$  in 6.6 kV circuit =  $\frac{(MVA)_b \times 10^3}{\sqrt{3} \times (kV)_b} = \frac{25 \times 10^3}{\sqrt{3} \times 6.6} = 2187 \text{ A}$

Therefore,

$$I_f = I_{f. pu} \times I_b = 4.95 \times 2187 = 10826 \text{ A}$$

**Alternative method:**

From figure (e), 
$$I_{f. pu} = 4 \times \frac{1}{j1.25} + \frac{1}{j0.57} = -j3.20 - j1.75 = -j4.95 \text{ pu}$$

$$I_f = I_{f. pu} \times I_b = 4.95 \times 2187 = 10826 \text{ A}$$

(b) From figure (e), current through circuit breaker A is

$$I_{f. pu} (A) = 3 \times \frac{1}{j1.25} + \frac{1}{j0.57} = -j2.40 - j1.75 = -j4.15 \text{ pu}$$

and

$$I_f (A) = 4.15 \times 2187 = 10826 \text{ A}$$

(c) For finding the momentary current through the breaker, the presence of dc offset current is taken into account by multiplying the symmetrical subtransient current by a factor of 1.6.

Hence, momentary current through the breaker A.

$$= 1.6 \times I_f (A) = 1.6 \times 10826 = 17321.6 \text{ A}$$

(d) To calculate the current to be interrupted by the breaker, motor subtransient reactance ( $X''_{dm} = j0.25$ ) is replaced by transient reactance ( $X'_{dm} = j0.30$ ).

Hence, the reactance of the circuit of figure (e) modify to that of figure (f).

Current (symmetrical) to be interrupted by the breaker (as shown by arrow)

$$= 3 \times \frac{1}{j1.5} + \frac{1}{j0.57} = -j3.75 \text{ pu}$$

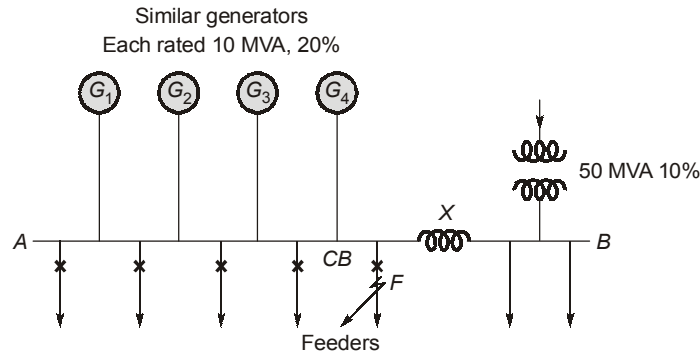
Allowance for the dc offset value is made by multiplying the symmetrical current by a factor of 1.1.

Therefore, the current to be interrupted by breaker A is

$$= 1.1 \times 3.75 \times 2187 = 9021.375 \text{ A}$$

**Solution : 12**

The single-line diagram of the system is shown in figure (a). Selecting a base voltage of 33 kV and base MVA of 50, the per unit reactance of generators and transformer are



**Fig. (a)**

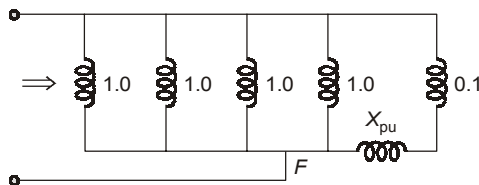
$$\text{Generators : } X_{g, \text{ pu}} = \frac{50}{10} \times 0.2 = 1.0 \text{ pu}$$

$$\text{Transformer : } X_f = \frac{50}{50} \times 0.1 = 0.1 \text{ pu}$$

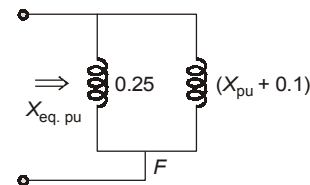
$$\text{Reactor : } X_{\text{pu}} = \frac{X}{X_b}$$

Where  $X$  is the actual reactance of the reactor in ohm and  $X_b$  is the base reactance.

For a symmetrical three phase fault at point  $F$  on an outgoing feeder connected to  $A$ , the reactance diagram with voltage sources short circuited will be as shown as figure (b). The circuit of figure (b) reduces to that shown in figure (c).



**Fig. (b):** Reactance diagram



**Fig. (c):** Reduced circuit

From figure (c), the equivalent reactance of the network as viewed from the fault point  $F$  is given by

$$X_{\text{eq. pu}} = \frac{0.25(X_{\text{pu}} + 0.1)}{[0.25 + (X_{\text{pu}} + 0.1)]} = \frac{0.25(X_{\text{pu}} + 0.1)}{(X_{\text{pu}} + 0.35)}$$

Per unit fault MVA is

$$(\text{MVA})_{f, \text{ pu}} = \frac{500}{50} = 10$$



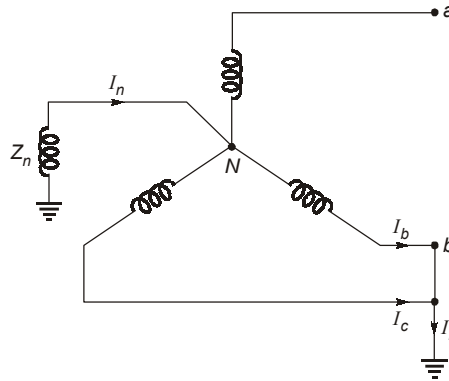
Hence, 
$$10 = \frac{(X_{pu} + 0.35)}{0.25(X_{pu} + 0.1)}$$

or, 
$$X_{pu} = 0.066 \text{ pu}$$

Actual value, 
$$X = X_{pu} \times X_b = 0.066 \times \frac{(33)^2}{50} = 1.437 \Omega$$

**Solution : 13**

Let us choose the rating of the generator as base, i.e. base MVA of 15 MVA and base voltage of 6.6 kV. The generator with fault between lines 'b' and 'c' is shown in figure.



(a) When the fault does not involve ground per unit impedances of the generator are

$$Z_1 = j0.20 \text{ pu}$$

$$Z_2 = j0.20 \text{ pu}$$

$$Z_0 = Z_{g0} + 3 Z_n$$

$$= j0.10 + 3 \times j0.05 = j0.25 \text{ pu}$$

Assuming line to line fault between phases 'b' and 'c', the currents are

$$I_a = 0$$

$$I_b + I_c = 0$$

The symmetrical components of the current are

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} = \frac{1.0}{j0.20 + j0.20} = \frac{1.0}{j0.40} = -j2.5 \text{ pu}$$

$$I_{a2} = -I_{a1} = j2.5 \text{ pu}$$

$$I_{a0} = 0$$

Line currents are

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

$$= a^2 I_{a1} - a I_{a1} = (a^2 - a) I_{a1}$$

$$= -j\sqrt{3} I_{a1} = (-j1.732) \times (-j2.5)$$

$$= -4.33 \text{ pu} = 4.33 \angle 180^\circ \text{ pu}$$

$$I_c = -I_b = 4.33 \text{ pu} = 4.33 \angle 0^\circ \text{ pu}$$

$$\text{Base current} = \frac{15 \times 10^3}{\sqrt{3} \times 6.6} = 1312 \text{ A}$$

Line currents in amperes are:

$$I_b = 4.33 \times 1312 \angle 180^\circ = 5680 \angle 180^\circ \text{ A}$$

$$I_c = 4.33 \times 1312 \angle 0^\circ = 5680 \angle 0^\circ \text{ A}$$

(b) When the fault is solidly grounded figure.

For fault between phases 'b' and 'c' involving ground, we have

$$I_{a1} = \frac{E_a}{\left[ Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0} \right]}$$

$$= \frac{1.0}{\left[ j0.20 + \frac{j0.20 \times j0.25}{(j0.20 + j0.25)} \right]} = \frac{1.0}{j0.20 + j0.11}$$

$$= \frac{1.0}{j0.31} = -j3.22 \text{ pu}$$

$$I_{a2} = -I_{a1} \frac{Z_0}{Z_2 + Z_0} = j3.22 \times \frac{j0.25}{j0.45} = j1.78 \text{ pu}$$

$$I_{a0} = -I_{a1} \frac{Z_2}{Z_2 + Z_0} = j3.22 \times \frac{j0.20}{j0.45} = j1.43 \text{ pu}$$

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0}$$

$$= (-0.5 - j0.866)(-j3.22) + (-0.5 + j0.866)(j1.78) + j1.43$$

$$= j1.61 - 2.79 - j0.89 - 1.54 + j1.43$$

$$= (-4.33 + j2.15) \text{ pu} = 4.83 \angle 153.6^\circ \text{ pu}$$

Actual value in amperes,

$$I_b = 4.83 \times 1312 = 6337 \text{ A}$$

$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0}$$

$$= (-0.5 + j0.866)(-j3.22) + (-0.5 - j0.866)(j1.78) + j1.43$$

$$= j1.61 + 2.79 - j0.89 + 1.54 + j1.43$$

$$= 4.33 + j2.15 = 4.83 \angle 26.4^\circ \text{ pu}$$

Actual value in amperes,

$$I_c = 4.83 \times 1312 = 6337 \text{ A}$$

$$I_n = (I_b + I_c)$$

$$= 3I_{a0} = 3 \times j1.43 = j4.29 \text{ pu}$$

Actual value in amperes,

$$I_n = 4.29 \times 1312 = 5628.5 \text{ A}$$

Therefore line current in phases b and c will be 6337 A and neutral current 5628.5 A.

#### Solution : 14

$$I_a = 12 + j6,$$

$$I_b = 12 - j12 = 16.97 \angle -45^\circ$$

$$I_c = -15 + j10$$

$$= 18.028 \angle 146.31^\circ$$

(a) Calculation of zero sequence currents,

$$I_{a0} = I_{b0} = I_{c0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}(12 + j6 + 12 - j12 - 15 + j10)$$

$$= 3 + j1.333 = 3.283 \angle 23.96^\circ$$

(b) Calculation of positive sequence currents,

$$\begin{aligned}
 I_{a1} &= \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) \\
 &= \frac{1}{3}[(12 + j6) + (1\angle 120^\circ)(16.97\angle -45^\circ) + (1\angle 240^\circ)(18.028\angle 146.31^\circ)] \\
 &= \frac{1}{3}[(12 + j6) + 16.97\angle 75^\circ + 18.028\angle 386.31^\circ] \\
 &= \frac{1}{3}(12 + j6 + 4.392 + j16.392 + 16.16 + j7.99) \\
 &= \frac{1}{3}(32.55 + j30.382) = 14.842\angle 43.02^\circ \\
 I_{b1} &= \alpha^2 I_{a1} = (1\angle 240^\circ)(14.842\angle 43.02^\circ) \\
 &= 14.842\angle 283.02^\circ = 3.343 - j14.46 \\
 I_{c1} &= \alpha I_{a1} = (1\angle 120^\circ)(14.842\angle 43.02^\circ) \\
 &= 14.842\angle 163.02^\circ = -14.19 + j4.33
 \end{aligned}$$

(c) Calculation of negative sequence currents,

$$\begin{aligned}
 I_{a2} &= \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) \\
 &= \frac{1}{3}[(12 + j6) + (1\angle 240^\circ)(16.97\angle -45^\circ) + (1\angle 120^\circ)(18.028\angle 146.31^\circ)] \\
 &= [(12 + j6) 16.97\angle 195^\circ + 18.028\angle 266.31^\circ] \\
 &= \frac{1}{3}(12 + j6 - 16.392 - j4.392 - 1.16 - j17.99) \\
 &= -1.85 - j5.45 = 5.766\angle -108.72^\circ \\
 I_{b2} &= \alpha I_{a2} = (1\angle 120^\circ)(5.766\angle -108.72^\circ) \\
 &= 5.766\angle 11.28^\circ = 5.65 + j1.127 \\
 I_{c2} &= \alpha^2 I_{a2} = (1\angle 240^\circ)(5.766\angle -108.72^\circ) \\
 &= 5.766\angle 131.28^\circ = -3.80 + j4.33
 \end{aligned}$$

**Solution : 15**

Let us assume base quantities to be the rating of the generator. So, base MVA = 10 and base voltage = 6.6 kV (line to line).

$$\begin{aligned}
 \text{Base current} &= \frac{(\text{MVA})_b \times 10^3}{\sqrt{3}(\text{kV})_b} \\
 &= \frac{10 \times 10^3}{\sqrt{3} \times 6.6} = 875 \text{ A}
 \end{aligned}$$

(a) All the generator neutrals are solidly grounded and a ground fault occurs on one busbar i.e. on one phase (say phase 'a') as shown in figure (a).

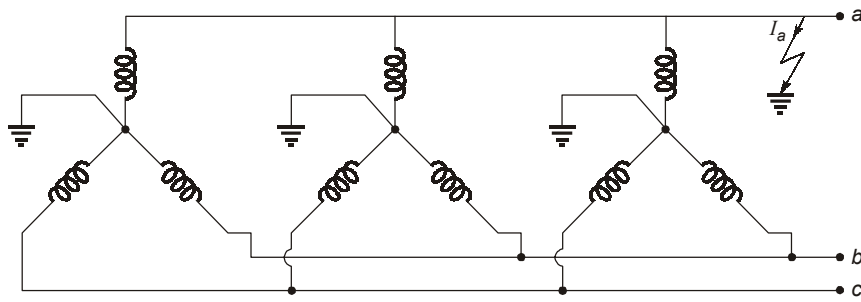


Fig. (a): Ground fault on one busbar

The sequence impedances of the individual generators are

$$Z_{g1} = j0.18 \text{ pu}$$

$$Z_{g2} = j0.18 \times 0.75 = j0.135 \text{ pu}$$

$$Z_{g0} = j0.18 \times 0.30 = j0.054 \text{ pu}$$

When the three generators are operating in parallel, the resultant impedances will be  $\frac{1}{3}$ rd, i.e.,

$$Z_1 = \frac{j0.18}{3} = j0.06 \text{ pu}$$

$$Z_2 = \frac{j0.135}{3} = j0.045 \text{ pu}$$

$$Z_0 = \frac{j0.054}{3} = j0.018 \text{ pu}$$

Therefore for an L-G fault, we get

$$\begin{aligned} \text{Fault current, } I_f = I_a = 3I_{a1} &= \frac{3E_a}{Z_1 + Z_2 + Z_0} \\ &= \frac{3 \times 1}{j(0.06 + 0.045 + 0.018)} = \frac{3}{j0.126} = -j24.39 \text{ pu} \end{aligned}$$

$$\text{Actual fault current in amperers} = -j24.39 \times 875 = -j21341 \text{ A}$$

- (b) Refer figure (b). In this case the neutral of only one generator is solidly grounded and that of others are isolated. Since the neutrals of the second and third generators are isolated, their zero sequence impedances do not come into picture. The positive and negative sequence impedances will be the same as in case (a).

Therefore,

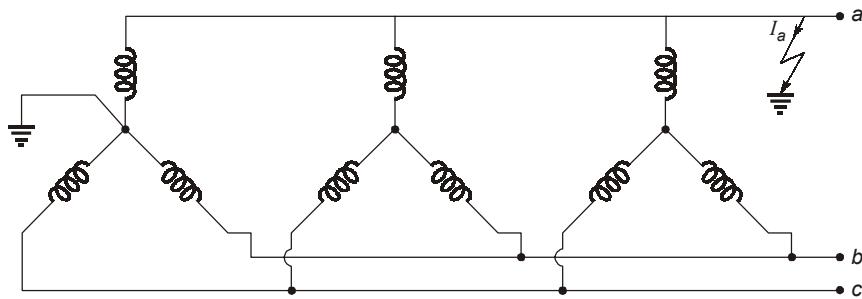
$$\begin{aligned} Z_1 &= j0.06 \text{ pu} \\ Z_2 &= j0.045 \text{ pu and} \\ Z_0 &= Z_{g0} = j0.054 \text{ pu} \end{aligned}$$

Therefore, fault current,

$$\begin{aligned} I_f = I_a &= \frac{3E_a}{Z_1 + Z_2 + Z_0} \\ &= \frac{3 \times 1}{j0.06 + j0.045 + j0.054} = \frac{3}{j0.159} = -j18.87 \text{ pu} \end{aligned}$$

Actual value of fault current in amperers

$$= -j18.87 \times 875 = -j16511 \text{ A}$$



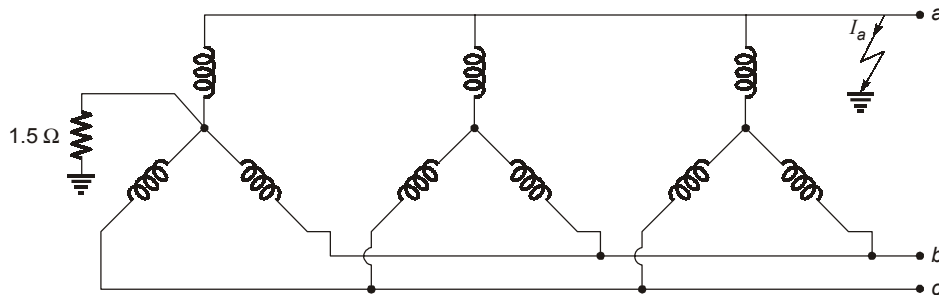
**Fig. (b):** One generator's neutral grounded, other isolated

(c) Refer figure (c). In this case  
The pu value of the grounding resistance

$$= 1.5 \times \frac{10}{(6.6)^2} = 0.344 \text{ pu}$$

Hence,

$$Z_n = (0.344 + j0) \text{ pu}$$



**Fig. (c):** One generator's neutral grounded through 1.5 Ω, other isolated

$$\begin{aligned} Z_0 &= Z_{g0} + 3Z_n \\ &= j0.054 + 3(0.344 + j0) \text{ pu} \\ &= (1.032 + j0.054) \end{aligned}$$

Fault current,

$$\begin{aligned} I_f = I_a &= \frac{3 \times 1}{j0.06 + j0.045 + (1.032 + j0.054)} \\ &= \frac{3}{1.032 + j0.159} = (2.84 - j0.437) \text{ pu} \end{aligned}$$

$$\begin{aligned} \text{Actual fault current in amperes} &= (2.84 - j0.437) \times 875 \text{ A} \\ &= (2485 - j382) \text{ A} \end{aligned}$$

$$\text{Magnitude of the fault current} = \sqrt{(2485)^2 + (382)^2} = 2514 \text{ A}$$



# 5

## Power System Protection

### LEVEL 1 Objective Questions

1. (c)
2. (519.6, 43 k $\Omega$ )
3. (17.96 kV)
4. (12.628 k $\Omega$ )
5. (0.453 kV/ $\mu$ s)
9. (84.26)
10. (c)
11. (d)
12. (d)

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### LEVEL 2 Objective Questions

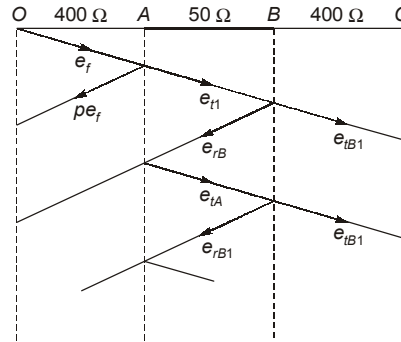
13. (a)
14. (b)
20. (a)
21. (a)
22. (c)

■■■■

**LEVEL 3** Conventional Questions

**Solution : 2**

Here it shall be solved with the help of Bewley's lattice diagram shown in figure.



Let the two lines  $OA$  and  $BC$  be joined by the cable  $AB$ . The surge impedances of  $OA$ ,  $AB$  and  $BC$  are 400, 50 and 400 ohms respectively.

Transmission coefficient for voltage at junction A,

$$\tau_1 = \frac{2Z_2}{Z_1 + Z_2} = \frac{2 \times 50}{400 + 50} = \frac{2}{9}$$

Reflection coefficient for voltage at junction A,

$$\rho_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{50 - 400}{50 + 400} = -\frac{7}{9}$$

Transmission coefficient for voltage at junction B,

$$\tau_2 = \frac{2Z_3}{Z_3 + Z_2} = \frac{2 \times 400}{400 + 50} = \frac{16}{9}$$

Reflection coefficient for voltage at junction B,

$$\rho_2 = \frac{Z_3 - Z_2}{Z_3 + Z_2} = \frac{400 - 50}{400 + 50} = \frac{7}{9}$$

At junction A, the incident wave  $e_f$  is partially reflected in the transmission line  $OA$ , and partially it is transmitted in the cable  $AB$ .

The voltage transmitted in  $AB = e_{t1} = \tau_1 e_f$

$e_{t1}$  now becomes the incident voltage in the cable towards the junction B. At B the voltage  $e_{t1}$  is partially transmitted in the line  $BC$  and partially reflected in the cable.

The transmitted part which is the first pulse entering the line  $BC$  is  $e_{tB}$ .

$$\begin{aligned} e_{tB} &= \tau_2 e_{t1} = \tau_2 \tau_1 e_f \\ &= \frac{16}{9} \times \frac{2}{9} \times 10 = 3.95 \text{ kV} \end{aligned}$$

The reflected component of  $e_{t1}$  is  $e_{rB}$ . It is given by

$$e_{rB} = \rho_2 e_{t1}$$

$e_{rB}$  now becomes the incident voltage in the cable towards the junction A. At A the voltage  $e_{rB}$  is partially transmitted in the line  $OA$  and partially it is reflected in the cable. The reflected voltage  $e_{tA}$  is given by

$$e_{tA} = \rho_1 e_{rB} = \rho_1 \rho_2 e_{t1} = \rho_1 \rho_2 \tau_1 e_f$$

$e_{tA}$  now becomes the incident voltage at the junction  $B$  where it is partially reflected and partially transmitted. The voltage transmitted in the line  $BC$  is given by

$$\begin{aligned} e_{tB1} &= \tau_2 e_{tA} = \rho_1 \rho_2 \tau_1 \tau_2 e_f \\ &= \frac{7}{9} \times \frac{7}{9} \times \frac{2}{9} \times \frac{16}{9} \times 10 = 2.39 \text{ kV} \end{aligned}$$

The second pulse  $e_{tB1}$  entering the line  $BC$  is, therefore 2.39 kV.

**Assumptions:**

1. The lines and cable are loss-free, hence  $V/I$  is purely resistive and equal to  $Z_0$  at all the points throughout the lengths of the lines and the cable.
2. The generator supplying the initial 10 kV pulse is matched to the line, so that there are no reflections from the sending end.
3. The second transmission line is infinitely long, or it is correctly terminated, thus, there are no reflections from the transmission line towards the junction  $B$ .

**Solution : 3**

$$L = \frac{8}{2\pi 50} = \frac{8}{100\pi} = 0.02544 \text{ H}$$

$$(a) \text{ Natural frequency of oscillation} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} = \frac{1}{2\pi} \sqrt{\frac{1}{0.0254 \times 0.025 \times 10^{-6}}} = 6.316 \text{ kHz}$$

(b) Frequency of damped oscillation is given by

$$\begin{aligned} f &= \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{4C^2R^2}} = \frac{1}{2\pi} \sqrt{\frac{1}{0.0254 \times 0.025 \times 10^{-6}} - \frac{1}{4(0.025 \times 10^{-6})^2 \times (600)^2}} \\ &= \frac{1}{2\pi} \sqrt{\frac{10^{10}}{6.35} - \frac{10^{10}}{9}} = 3.423 \text{ kHz} \end{aligned}$$

(c) The value of critical resistance

$$R = \frac{1}{2} \sqrt{\frac{L}{C}} = \frac{1}{2} \sqrt{\frac{0.0254}{0.025 \times 10^{-6}}} = 504.35 \Omega$$

(d) The damped frequency of oscillation is

$$\frac{1}{4} \times 6.316 \text{ kHz} = 1579 \text{ Hz}$$

$$1579 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{4C^2R^2}} = \frac{1}{2\pi} \sqrt{\frac{1}{0.0254 \times 0.025 \times 10^{-6}} - \frac{1}{4(0.025 \times 10^{-6})^2 \times R^2}}$$

or,

$$1579 = \frac{1}{2\pi} \sqrt{\frac{10^{10}}{6.36} - \frac{10^{16}}{25R^2}}$$

Therefore,

$$R = 520.5 \Omega$$





# 6

## Power Distribution

### LEVEL 1 Objective Questions

1. (b)
2. (234.95)
3. (b)
4. 188.49 (188.00 to 189.00)
5. (c)
6. (c)
8.  $(18.87 \angle -36.5^\circ)$
9.  $(239.8 \angle -2.54^\circ)$

### LEVEL 2 Objective Questions

10. (b)
11. (a)
12. (a)
13. (a)
14. (b)
15. (b)
17. (261.78)
18.  $(3.78^\circ)$

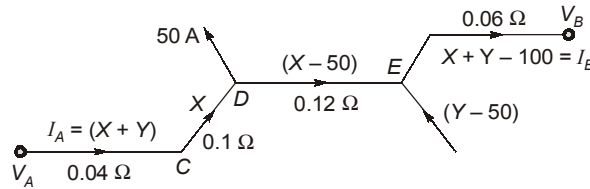
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### LEVEL 3 Conventional Questions

**Solution : 1**

**Path-1:**

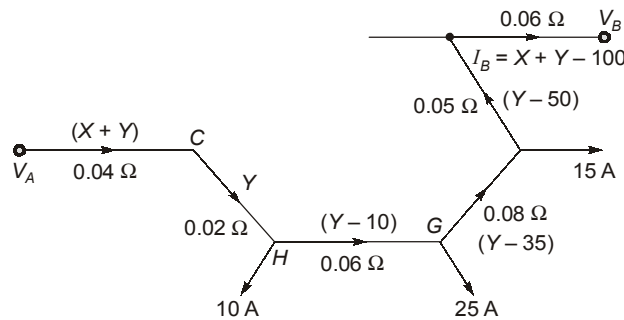


$$V_A - V_B = (X + Y) 0.04 + (0.1X) + (X - 50) 0.12 + (X + Y - 100) 0.06$$

$$V_A - V_B = 0.32X + 0.1Y - 12$$

...(i)

**Path-2:**



$$V_A - V_B = (X + Y) 0.04 + 0.02Y + (Y - 10) 0.06 + (Y - 35) 0.08 + (Y - 50) 0.05 + (X + Y - 100) 0.06$$

$$V_A - V_B = 0.1X + 0.31Y - 11.9$$

...(ii)

**Case-I:** If,

$$V_A = V_B$$

$$0.32X + 0.1Y = 12$$

$$0.1X + 0.31Y = 11.9$$

$$X = 28.3632$$

$$Y = 29.2376$$

$$I_A = (X + Y)$$

$$= 28.3632 + 29.2376 = 57.6 \text{ A}$$

$$I_B = X + Y - 100$$

$$I_B = 57.6 - 100 = -42.4 \text{ A}$$

**Case-II:** If,

$$V_B = V_A + 6$$

$$V_A - V_A - 6 = 0.32X + 0.1Y - 12$$

$$V_A - V_A - 6 = 0.1X + 0.31Y - 11.9$$

$$0.32X + 0.1Y = 6$$

$$0.1X + 0.31Y = 5.9$$

$$X = 14.2376$$

$$Y = 14.4394$$

$$I_A = X + Y = 14.2376 + 14.4394 = 28.677 \text{ A}$$

$$I_B = X + Y - 100 = -71.323 \text{ A}$$

**Solution : 2**

Let the current flowing in section  $AB = x$ ; then currents in various sections will be as follows:

$$\text{Current in section } AB = x$$

$$\text{Current in section } BC = x - 20$$

$$\text{Current in section } CD = x - 30$$

$$\text{Current in section } DE = x - 80$$

$$\text{Current in section } EA = x - 100$$

Applying Kirchhoff's second law along the closed loop  $ABCDEA$ , we get

$$\frac{2\rho}{a_1} [100x + 200(x - 20) + 200(x - 30) + 150(x - 80) + 150(x - 100)] = 0$$

$$\text{or } 800x = 37000$$

$$\text{or } x = 46.25 \text{ amperes}$$

Voltage drop from  $A$  to  $D$ ,

$$= \frac{2\rho}{a_1} [46.25 \times 100 + 26.25 \times 200 + 16.25 \times 200]$$

$$= 26250 \frac{\rho}{a_1} \quad \dots(i)$$

where,  $a_1$  is the cross-section of various sections.

Addition of interconnector  $AD$  will change the current distribution.

Let the cross-sections of all conductors including interconnectors be  $a$

$$\text{Resistance of interconnector } AD = \frac{\rho}{a} \times 250 \times 2 = \frac{500\rho}{a}$$

Resistance of network viewed from points  $A$  and  $D$ ,

$$\frac{2\rho}{a} \left[ \frac{(100 + 200 + 200)(150 + 150)}{100 + 200 + 200 + 150 + 150} \right] = \frac{2\rho}{a} \times \frac{5}{8} \times 300 = \frac{375\rho}{a}$$

Now according to Thevenin's theorem current flowing in the interconnector  $AD$

$$= \frac{\text{Potential difference between terminals } A \text{ and } D}{\text{Resistance of distribution network} + \text{Resistance of interconnector}}$$

$$= \frac{26,250\rho/a}{(375 + 500)\rho/a} = \frac{26,250}{875} = 30 \text{ A}$$

In the second case i.e., with interconnector let the cross-section of the conductors including interconnector be  $a_2$ . Then voltage drop between  $A$  and  $D$

$$= 30 \times \frac{\rho}{a_2} \times 250 \times 2 = 15,000 \frac{\rho}{a_2} \quad \dots(ii)$$

For same voltage drop between terminals  $A$  and  $D$ , equating equations (i) and (ii), we get

$$26,250 \frac{\rho}{a_1} = 15,000 \frac{\rho}{a_2}$$

$$\text{or } \frac{a_2}{a_1} = \frac{15,000}{26,250} = \frac{4}{7}$$

i.e., the conductor cross-section with interconnector is 4/7 times the corresponding cross-section without interconnector. However, total lengths of conductor with and without interconnector are 1050 and 800 meters respectively so that ratio of volumes (or weights) in the two cases

$$= \frac{4}{7} \times \frac{1050}{800} = 0.75$$

**Solution : 3**

$$(i) \quad C = \frac{\epsilon_r}{18 \ln \frac{R}{r}} \mu\text{F/km} = \frac{2.4}{18 \ln \frac{15}{10}} = 0.329 \mu\text{F/km}$$

$$\text{Total cable capacitance} = 0.329 \times 2.5 = 0.8225 \mu\text{F}$$

$$(ii) \quad \text{Charging current, } I_c = \omega CV = 2\pi f CV \\ = 2\pi \times 50 \times 0.8225 \times 10^{-6} \times 11 \times 10^3 \\ = 2.84 \text{ A}$$

The no-load current may approximately be taken as charging current, i.e.,

$$I_0 = I_c$$

(iii) Generated reactive voltamperes,

$$VI_c = 11 \times 10^3 \times 2.84 \text{ VAr} \\ = 11 \times 2.84 \text{ kVAr} = 31.24 \text{ kVAr}$$

$$(iv) \quad \text{Dielectric loss, } P_d = VI_0 \cos \phi_0 \cong VI_c \delta \\ = 11 \times 10^3 \times 2.84 \times 0.031 \\ = 968 \text{ W}$$

(v) If  $R_i$  is the equivalent insulation resistance,

$$P_d = \frac{V^2}{R_i}$$

$$R_i = \frac{V^2}{P_d} = \frac{(11 \times 10^3)^2}{968} \Omega = 0.125 \text{ M}\Omega$$



# 7

## Recent Trends in Power System

LEVEL 2 Objective Questions

7. (14.78)

### LEVEL 3 Conventional Questions

#### Solution : 1

Following are the types of static VAR systems (SVS):

1. Static VAR compensators (SVC)
2. Mechanically switched capacitor or reactors

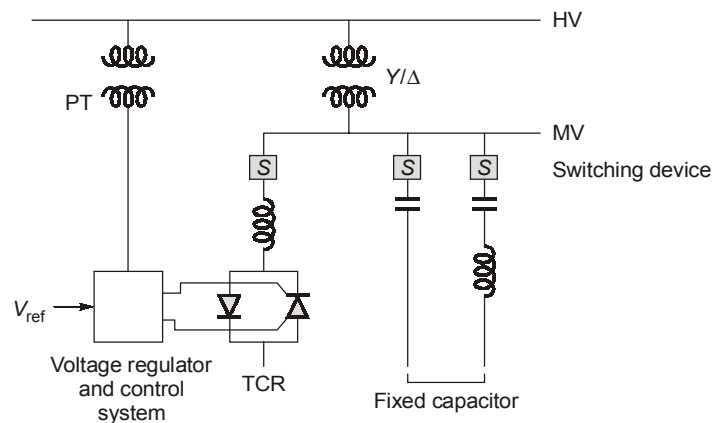
#### 1. SVC

- (a) Thyristor controlled reactor (TCR)
- (b) Thyristor switched capacitor (TSC)
- (c) Saturated reactors

Common features of these SVC's are:

- (i) A fixed shunt capacitor in parallel with controlled susceptance. The fixed capacitors are usually tuned with smaller reactors to harmonic frequencies to absorb harmonics generated by controlled susceptance (TCR or SR) to avoid harmful resonances.
- (ii) A step down transformer.

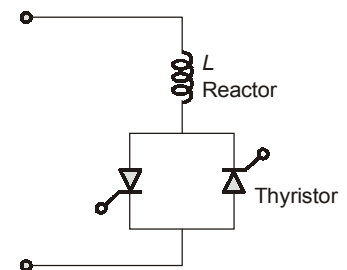
#### (a) Thyristor controlled reactor (TCR):



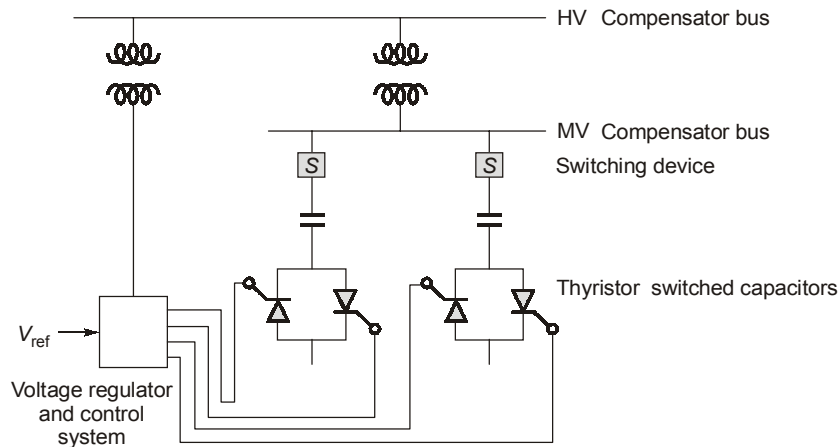
*Line diagram of TCR*

The basic TCR is as under. The controlled element is reactor and controlling element is the thyristor controller consisting of two oppositely poled thyristor which conducts every alternate half cycle.

The reactive power absorbed by the reactor will depend upon the instant of switching on the voltage wave. If voltage is passing through its peak value at the instant of switching of thyristor, maximum reactive power will be absorbed and as the delay angle increases between  $90^\circ$  to  $180^\circ$ , the reactive power absorbed decreases.

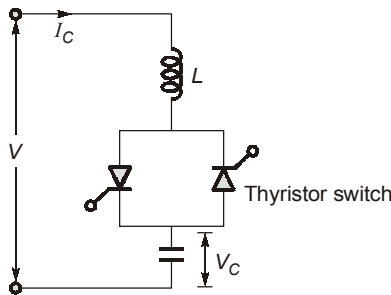


**(b) Thyristor switched capacitor (TSC):**



*Thyristor switched capacitors (TSC)*

A TSC scheme consists of a capacitor bank split up into appropriately sized units, each of which is switched on and off by using thyristor switches. Each single phase unit consists of a capacitor in series with a bidirectional thyristor switch and a small inductor as shown below. The purpose of inductor is to limit switching transients, to damp in rush currents and to prevent resonance with network.

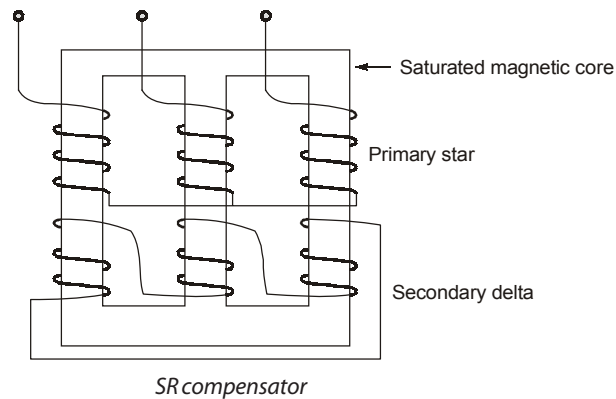


*Single phase TSC*

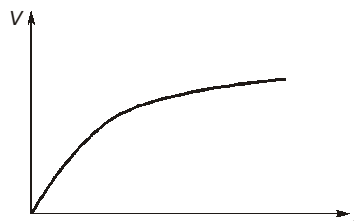
The susceptance control principle used by a TSC is known as the integral cycle control. The susceptance is switched in for integral number of exact half cycles. The susceptance is divided into several parallel units and the susceptance is varied by controlling the number of units in conduction. A change can be made every half cycle.

**(c) Saturated reactor compensator:**

The plain saturated reactor is unsuitable for use in transmission system as the voltage or the current contains lot of harmonics. The three phase Saturated Reactor (SR) having a short circuited delta winding which eliminates 3<sup>rd</sup> harmonic current from primary winding as these currents are supplied by the delta winding. The 3- $\phi$  SR is shown below. The secondary current is predominately 3<sup>rd</sup> harmonic and the current is thus an elementary frequency tripler.



Volt-amp characteristic of SR is as below:



Slope varies between 5% – 15%. The characteristic is linear below 10% of rated current. A lower slope characteristic can be obtained by connecting a capacitor in series with the saturated reactor.

2. **Mechanically switched capacitor (MSC):** MSC scheme consists of one or more capacitor unit connected to the power system by circuit-breaker. A small inductor may be connected in series damp out transients. VI characteristics is similar to TSC.

### Solution : 2

Point to point dc transmission link refers to injecting power at one point and drawing power from another point. In this system tapping of power is not possible. This system is mostly used for bulk transfer of power. The point to point transmission via HVDC can use different configuration like

- Monopolar link
- Bipolar link
- Homopolar link

### Point to point HVDC link in India:

1.  $\pm 500$  kV, 1500 MW Rihand Dadri Bipole HVDC of about 820 Km length. It is used to carry bulk power from thermal power plant of Rihand (eastern part of northern grid) to Dadri (western part of northern grid). Each pole carry 750 MW with 10%, 2 hrs overload and 33%, 05 seconds overload capacity.
2.  $\pm 500$  kV, 2000 MW, HVDC Talchar-Kolar link of length about 1370 Km.
3.  $\pm 800$  kV, Champa-Kurukshetra HVDC link is under construction stage with 3000 MW capacity.

**Back to back HVDC:** Two independent neighbouring system with different and incompatible electrical parameters are connected via a DC link. In this rectifier and inverter are located in the same station. Back-to-back HVDC is used:

- (a) To connect asynchronous high voltage power system or systems with different frequencies.
- (b) To stabilize weak AC links.
- (c) To supply more active power where AC system is at the limit of its short circuit capability.
- (d) For grid power flow control within synchronous system.



**Back to back HVDC link in India:**

1.  $2 \times 250$  MW, HVDC Vindhyachal back to back station with AC of 400 kV and DC voltage of  $\pm 70$  kV. It connects Vindhyachal thermal power station to Singrauli TPS. Bidirectional power flow is possible. Achieve load diversity in Northern and Western region.
2.  $2 \times 500$  MW Chandrapur back to back station.  
AC = 400 kV  
DC =  $\pm 205$  kV  
Connects Chandrapur TPS in western region to Ramagundum TPS in southern region. Achieve load diversity in Southern and Western region.

**Solution : 3**

$$\begin{aligned} X_c &= 0.3 \Omega, & E_{L-L} &= 440 \text{ V} \\ I_d &= 220 \text{ A}, & \alpha &= 15^\circ \\ V_{d0} &= \frac{3\sqrt{2}}{\pi}, & E_{L-L} &= \frac{3\sqrt{2}}{\pi} \times 440 \end{aligned}$$

Equivalent commutation resistance,

$$R_c = \frac{3X_c}{\pi} = \frac{3 \times 0.3}{\pi} = 0.286 \Omega$$

$$I_d = \frac{\sqrt{3}E_m}{2\omega L_c} (\cos \alpha - \cos \delta)$$

But, 
$$V_{d0} = \frac{3\sqrt{3}E_m}{\pi}$$

and 
$$\omega L_c = \frac{\pi R_c}{3}$$

So, substituting the value, we have

$$220 = \frac{594.2}{2 \times 0.286} [\cos 15^\circ - \cos \delta]$$

which gives,

$$\cos \delta = 0.7541$$

or,

$$\delta = 41.05^\circ$$

overlap angle,

$$\mu = (\delta - \alpha) = 41.05^\circ - 15^\circ$$

or,

$$\mu = 26.05^\circ$$

The average direct voltage is given by

$$V_d = \frac{V_{d0}}{2} [\cos \alpha + \cos \delta]$$

or,

$$V_d = \frac{594.2}{2} [\cos 15^\circ + \cos 41.05^\circ] = 511.02 \text{ V}$$

**Solution : 4**

General converter relationship,

$$I_{L1} = \frac{\sqrt{6}}{\pi} I_d$$

$$V_{d0} = \frac{3\sqrt{2}}{\pi} E_{L-L}$$

$$V_{d0} = \frac{3\sqrt{3} E_m}{\pi} = \frac{3\sqrt{2} E_{L-L}}{\pi}$$

Given,

$$X_{Ci} = 100 \text{ } \Omega$$

$$V_{di} = 245 \text{ kV}$$

$$I_{di} = 950 \text{ A}$$

For the inverter,

$$V_{Li} = 325 \text{ kV}$$

$$V_{d0i} = \frac{3\sqrt{2} E_{(L-Li)}}{\pi} = \frac{3\sqrt{2}}{\pi} \times 325 = 438.90 \text{ kV}$$

From the equivalent circuit,

$$V_{di} = V_{d0i} \cos \gamma - R_{Ci} I_{di}$$

$$245 \times 10^3 = 438.90 \times 10^3 \cos \gamma - \frac{3}{\pi} \times 100 \times 950$$

Solving we get,

$$\cos \gamma = 0.7649 \text{ or } \gamma = 40.1^\circ$$

or,

$$V_{di} = \frac{V_{d0i}}{2} (\cos \beta + \cos \gamma)$$

$$V_{di} = \frac{V_{d0i}}{2} (\cos(\gamma + \mu) + 0.7649)$$

Substituting values,  $245 \times 10^3 = \frac{438.9 \times 10^3}{2} (\cos(\gamma + \mu) + 0.7649)$

$$\cos(\gamma + \mu) = 0.3515$$

$$\gamma + \mu = 69.4^\circ$$

∴

$$\mu = 69.4^\circ - 40.1^\circ = 29.3^\circ \text{ (overlap angle)}$$

