2020

RANK Improvement WORKBOOK

Answer key and Hint of
Objective & Conventional *Questions*

Electrical Engineering

Electromagnetic Fields

MADE EASY Publications

1 Static Electric Fields

LEVEL 1 Objective Solutions

1.	(d)
2.	(c)
3.	(c)
4.	(a)
5.	(c)
6.	(a)
7.	(a)
8.	(c)
9.	(b)
10.	(c)
11.	(b)
12.	(d)
13.	(c)

LEVEL 2 Objective Solutions

14.	(d)
15.	(b)
16.	(a)
17.	(–3)
18.	(d)
19.	(a)
20.	(a)
21.	(c)
22.	(c)
23.	(b)
24.	(–3)

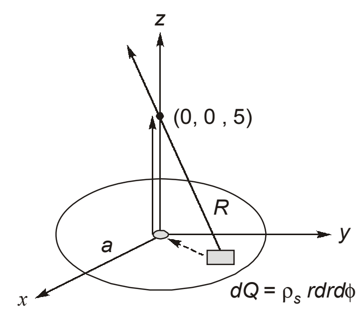
■■■■

LEVEL 3 Conventional Solutions

Solution: 1

Now, the charge density can be given as

$$\rho_s = \frac{Q}{A} = \frac{500\pi \times 10^{-6}}{25\pi} = 20\mu C/m^2$$



In cylindrical coordinate system

$$R = -r\hat{a}_r + 5\hat{a}_z$$

Then each differential charge contributes to differential force

$$dF = \frac{\left(50\times10^{-6}\right)\left(\rho_s\, rdr\, d\phi\right)}{\frac{10^{-9}}{9}\left(r^2+25\right)}\left[\frac{-r\hat{a}_r + 5\hat{a}_z}{\sqrt{r^2+5^2}}\right]$$

From the symmetry, radial components will cancel and only z-components will contribute to net force.

$$\therefore \qquad F = \int_0^{2\pi}\int_0^{5} \frac{\left(50\times10^{-6}\right)\left(20\times10^{-6}\right)5rdr\,d\phi}{\frac{10^{-9}}{9}(r^2+25)^{3/2}}\cdot\hat{a}_z$$

$$= 90\pi\int_0^5 \frac{rdr}{\left(r^2+25\right)^{3/2}}\cdot\hat{a}_z = 90\pi\left[\frac{-1}{\left(r^2+25\right)^{1/2}}\right]_0^5\cdot\hat{a}_z = 16.56\,\hat{a}_z\,N$$

Solution: 2

Laplace's and Poisson's equation

We know, $\qquad \nabla\cdot\vec{D} = \rho_v$

$\Rightarrow \qquad \vec{D} = \in\vec{E}$

If the homogeneous medium is assumed

$\therefore \qquad \in\nabla\vec{E} = \rho_v$

$\Rightarrow \qquad \in\nabla(-\nabla V) = \rho_v$

$$\Rightarrow \qquad \nabla^2 V = -\frac{\rho_v}{\in} \qquad \text{...(i)}$$

The above equation (i) is called Poisson's equation.

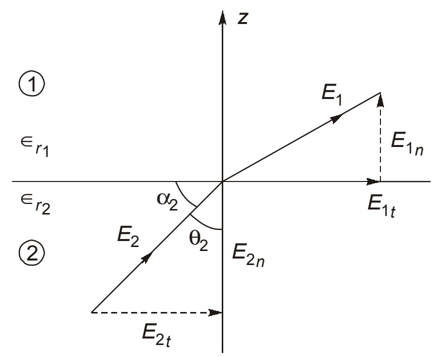
If in a charge free region $\rho_v = 0$, then

$$\nabla^2 V = 0 \qquad \text{...(ii)}$$

The above equation (ii) is called Laplace equation.

$\Rightarrow$ Poisson's equation and Laplace's equation are used to develop potential function. They are also used to analyze the junction characteristics of *P-N* junction.

Solution: 3



(a) Since $\hat{a}_z$ is normal to the boundary plane, we obtain the normal components as

$$E_{1n} = E_a \cdot \hat{a}_z = E_a \cdot \hat{a}_z = 3$$

$$E_{1n} = 3\hat{a}_z$$

$$E_{1t} = E_1 - E_{1n} = 5\hat{a}_x - 2\hat{a}_y$$

$$E_{2t} = E_{1t} = 5\hat{a}_x - 2\hat{a}_y$$

Also, $$D_{2n} = D_{1n} \Rightarrow \in_{r2} E_{2n} = \in_{r1} E_{1n}$$

$$E_{2n} = \frac{\in_{r1}}{\in_{r2}} E_{1n} = \frac{4}{3} \times (3\hat{a}_z) = 4\hat{a}_z$$

$$E_2 = 5\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z$$

(b) Let α_2 and α_2 be the angles E_1 and E_2 make with the interface while θ_1 and θ_2 are the angle they make with the normal to the interface as shown in above figure.

$$\alpha_1 = 90 - \theta_1$$

$$\alpha_2 = 90 - \theta_2$$

Since $$E_{1n} = 3, \text{ and } E_{1t} = \sqrt{25+4} = \sqrt{29}$$

$$\tan\theta_1 = \frac{E_{1t}}{E_{1n}} = \frac{\sqrt{29}}{3} = 1.795 \quad \Rightarrow \theta_1 = 60.9°$$

Hence, $$\alpha_1 = 29.1°$$

Similarly, $$E_{2n} = 4, \quad E_{2t} = E_{1t} = \sqrt{29}$$

$$\tan\theta_2 = \frac{E_{2t}}{E_{2n}} = \frac{\sqrt{29}}{4} = 1.346 \quad \Rightarrow \theta_2 = 53.4°$$

Hence, $$\alpha_2 = 36.6°$$

(c) The energy densities are given by

$$W_{E_1} = \frac{1}{2}\in_1 |E_1|^2 = \frac{1}{2} \times 4 \times \frac{10^{-9}}{36\pi} \times (25+4+9) \times 10^6$$

$$= 672 \text{ μJm/}^3$$

$$W_{E_2} = \frac{1}{2}\in_2 |E_2|^2 = \frac{1}{2} \times 3 \times \frac{10^{-9}}{36\pi} \times (25+4+16) \times 10^6$$

$$= 597 \text{ μJ/m}^3$$

(d) At the centre (3, 4, –5) of the cube of side 2 m, $z = -5 < 0$; that is, the cube is in region 2 with $2 \le x \le 4$, $3 \le y \le 5$, $-6 \le z \le -4$. Hence,

$$W_E = \int W_{E_2} dv = \int_{x=2}^{4} \int_{y=3}^{5} \int_{z=-6}^{-4} W_{E_2}\, dx\, dy\, dz$$

$$= W_{E_2}(2)\times(2)\times(2)$$

$$W_E = 597 \times 8\ \mu\text{J} = 4.776\ \text{mJ}$$

Solution: 4

(a) Since D and E are normal to the dielectric interface, the capacitor in Figure (a) can be treated as consisting of two capacitors C_1 and C_2 in series.

$$C_1 = \frac{\epsilon_o \epsilon_{r1}\, s}{\frac{d}{2}} = \frac{2\,\epsilon_o \epsilon_{r1}\, s}{d},\quad C_2 = \frac{2\,\epsilon_o \epsilon_{r2}\, s}{d}$$

Total capacitor C is given by

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{2\epsilon_o\, s}{d}\,\frac{\epsilon_{r1}\epsilon_{r2}}{\epsilon_{r1}+\epsilon_{r2}} = \frac{2\times 10^{-9}}{36\pi}\times\frac{30\times 10^{-4}}{5\times 10^{-3}}\times\frac{4\times 6}{10}$$

$$C = 25.46\ \text{pF}$$

(b) In this case, D and E are parallel to the dielectric interface we may treat the capacitor as consisting of two capacitor C_1 and C_2 in parallel.

$$C_1 = \frac{\epsilon_o \epsilon_{r1} \frac{s}{2}}{d} = \frac{\epsilon_o \epsilon_{r1} s}{2d},\quad C_2 = \frac{\epsilon \epsilon_{r2}\, s}{3d}$$

The total capacitance is

$$C = C_1 + C_2 = \frac{\epsilon_o\, s}{2d}(\epsilon_{r1} + \epsilon_{r2}) = \frac{10^{-9}}{36\pi}\cdot\frac{30\times 10^{-4}}{2\times(2\times 10^{-3})}\times 10$$

$$C = 26.53\ \text{pF}$$

Solution: 5

Since V depends only on ϕ, Laplace's equation in cylindrical co-ordinate becomes

$$\nabla^2 V = \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} = 0$$

Since $\rho = 0$ is excluded owing to the insulating gap, we can multiply by ρ^2 to obtain

$$\frac{d^2V}{d\phi^2} = 0$$

Which is integrated twice to give

$$V = A\phi + B$$

We apply the boundary condition to determine constant A and B when $\phi = 0$, $V = 0$.

$$0 = 0 + B \quad\Rightarrow\quad B = 0$$

When $\phi = \phi_0$,

$$V = V_0$$

$$V_0 = A\phi_0 \quad\Rightarrow A = \frac{V_0}{\phi_0}$$

Hence,

$$V = \frac{V_0}{\phi_0}\phi$$

Also, $$E = -\nabla V = -\frac{1}{\rho}\frac{dV}{d\phi}\hat{a}_\phi = -\frac{V_0}{\rho\phi_0}\hat{a}_\phi$$

$$E = -\frac{V_0}{\rho\phi_0}\hat{a}_\phi$$

Substituting $V_0 = 100$ and $\phi_0 = \nabla_0$ gives

$$V = \frac{600}{\pi}\phi \quad \text{and} \quad E = -\frac{600}{\pi\rho}\hat{a}_\phi$$

Solution: 6

(a) Throughout the problem, assume $2L$ as the length of all the cylinder i.e.

$$\int_{-L}^{L} dz = 2L$$

Consider a cylinder of radius ρ such that $\rho < 2$ as shown in Figure (a)

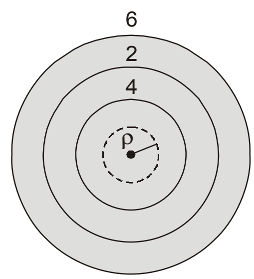


Figure (a)

This cylinder does not enclose any charge.
Hence, flux leaving through this cylinder is zero.

$\therefore$ $\vec{D} = 0$ when $\rho < 2$ m

Consider a cylinder of radius ρ such that $2 < \rho < 4$ as shown in Figure (b)

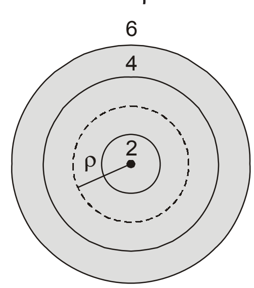


Figure (b)

This cylinder encloses the surface charge density present on $\rho = 2$ m.

Hence, Q = Charge enclosed by the cylinder = charge present on $\rho = 2$ m

$Q = (\rho_s \text{ on } \rho = 2)$ (area of $\rho = 2$ cylinder)

$= (20 \times 2\pi \times 2 \times 2L)$nC

Area of the cylinder $= (2\pi\rho)(2L)$

$$\therefore \quad \vec{D} = \frac{Q}{\text{Area}}\hat{a}_\rho = \frac{(20)(4\pi)(2L)}{(2\pi\rho)(2L)}\hat{a}_\rho = \frac{40}{\rho}\hat{a}_\rho \text{ nC/m}^2$$

Consider a cylinder of radius ρ such that $4 < \rho < 6$ m as shown in Figure (c).

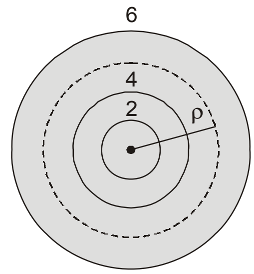


Figure (c)

This cylinder encloses the surface densities of 20 nC/m^2 and –4 nC/m^2 which are present on $\rho = 2$ m and $\rho = 4$ m respectively. Hence,

$$Q = \text{Charge present on } \rho = 2 + \text{charge present on } \rho = 4$$
$$= 20 \times (2\pi \times 2 \times 2L) - 4 \times (4\pi \times 4 \times 2L)$$
$$\text{Area of cylinder} = (2\pi\,\rho)\,(2L) = 4\pi\,\rho L$$

$$\therefore \qquad \vec{D} = \frac{Q}{\text{Area}}\hat{a}_\rho = \frac{96\pi L}{4\pi\,\rho L}\hat{a}_\rho = \frac{2a}{\rho}\hat{a}_\rho \text{ nC/m}^2$$

Consider a cylinder of radius ρ such that $\rho > 6$ m as shown in Figure (d).

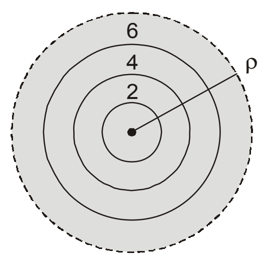


Figure (d)

This cylinder enclose all the three surface charge densities.

Hence, $\quad Q = \text{Charge present on } \rho = 2 + \text{charge present on } \rho = 4 + \text{charge present on } \rho = 6$

$$Q = 96\pi L + \rho_{S_0}(2\pi \times 6) \times 2L = (96\pi L + 24\rho_{S_0}\pi L)\text{nC}$$
$$\text{Area of cylinder} = (2\pi\rho)\,(2L) = 4\pi\,\rho L$$

$$\therefore \qquad \vec{D} = \left(\frac{96\pi L + 24\rho_{S_0}\pi L}{4\pi\rho L}\right)\hat{a}_\rho = \left(\frac{24 + 6\rho_{S_0}}{\rho}\right)\hat{a}_\rho \text{ nC/m}^2$$

$$\vec{D} = \begin{cases} 0 & \rho < 2 \quad ...(i) \\ \left(\dfrac{40}{\rho}\right)\hat{a}_\rho & 2 < \rho < 4 \quad ...(ii) \\ \left(\dfrac{24}{\rho}\right)\hat{a}_\rho & 4 < \rho < 6 \quad ...(iii) \\ \left(\dfrac{24 + 6\rho_{S_0}}{\rho}\right)\hat{a}_\rho & \rho > 0 \quad ...(iv) \end{cases}$$

Where ρ is in meter and $\vec{D}$ is in nC/m^2

From equation (i), for $\rho = 1$ m $\qquad \vec{D} = 0$

From equation (ii), for $\rho = 3$ m $\quad \vec{D} = \frac{40}{3}\hat{a}_\rho = 13.333\hat{a}_\rho \text{ nC/m}^2$

From equation (iii), for $\rho = 5$ m $\quad \vec{D} = \frac{24}{5}\hat{a}_\rho = 4.8\hat{a}_\rho \text{ nC/m}^2$

(b) From equation (iv) for $\rho = 7$ m

$$\vec{D} = \left(\frac{24 + 6\rho_{S_0}}{7}\right)\hat{a}_\rho \text{ nC/m}^2$$

$$\frac{24 + 6\rho_{S_0}}{7} = 0$$

$$\rho_{S_0} = -4 \text{ nC/m}^2$$

Solution: 7

The two charge configuration are parallel to the x-axis. Hence the figure is given below, looking the yz plane from positive x.

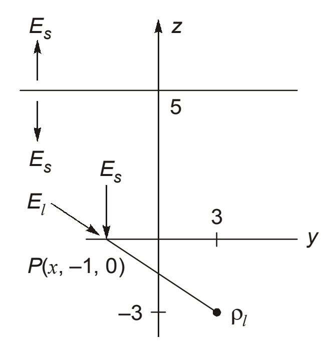
Due to the charge sheet, $\quad E_s = \frac{\rho_z}{2\epsilon_o}\hat{a}_n$

At P, $\quad \hat{a}_n = -\hat{a}_z$ and $E_s = -6\hat{a}_z$ V/m

Due to the line charge, $\quad E_l = \frac{\rho_L}{2\pi\epsilon_o r}\hat{a}_r$

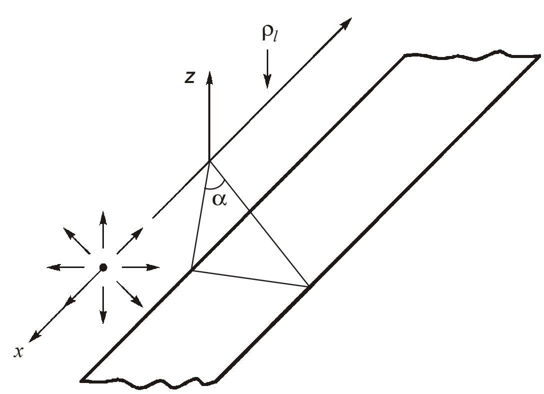
and at P, $\quad E_l = 8\hat{a}_y - 6\hat{a}_z$ V/m

Total electric field is the sum, $\quad E = E_l + E_s = 8\hat{a}_y - 12\hat{a}_z$ V/m



Solution: 8

The flux is uniformly distributed around the line charge. Thus the amount crossing the strip is obtained from the angle subtained by 2π.



$$\alpha = 2\arctan\left(\frac{2}{3}\right) = 1.176 \text{ radian}$$

Then,
$$\frac{\Psi}{L} = 50\left(\frac{1.176}{2\pi}\right)$$
$$\frac{\Psi}{L} = 9.36\ \mu\text{C/m}$$

Solution: 9

$$E = \frac{K}{r}\hat{a}_r$$

Since the field has only a radial component

$$dW = -QE \cdot dl = -QE_r\, d_r = -\frac{KQ}{r}dr$$

For the limit of integration use r_1 and $2r_1$.

$$W = -KQ\int_{r_1}^{2r_1}\frac{d_r}{r} = -KQ\ln 2$$
$$W = -KQ\ln 2, \text{ independent of } r_1$$

Solution: 10

To find energy, W_E, stored in a limited region of space, one must integrate the energy density through the region.

Between the half plane,
$$E = -\nabla V = -\frac{1}{r}\frac{\partial}{\partial\phi}\left(-\frac{60\phi}{\pi}\right)\hat{a}_\phi \text{ V/m}$$
$$E = \frac{60}{\pi r}\hat{a}_\phi \text{ (V/m)}$$
$$dW = \frac{1}{2}\epsilon_o E^2$$

Energy stored,
$$W = \frac{\epsilon_o}{2}\int_0^1\int_0^{\pi/6}\int_{-0.1}^{-0.6}\left(\frac{60}{\pi r}\right)^2 r\,dr\,d\phi\,dz$$
$$W = \frac{300\epsilon_o}{\pi}\ln 6 = 1.51\text{nJ}$$

Solution: 11

For aluminium,

The conductivity $\sigma = 3.82 \times 10^7$ S/m

Mobility $\mu = 0.0014\ \text{m}^2/\text{Vs}$

$$J = \rho v = \frac{\sigma}{\mu}v = \frac{3.83\times 10^7}{0.0014}\times 5.3\times 10^{-4}$$
$$J = 1.45 \times 10^7\ \text{A/m}^2$$
$$E = \frac{J}{\sigma} = \frac{v}{\mu} = 3.79\times 10^{-1}\ \text{V/m}$$

■■■■

2 Static Magnetic Fields

LEVEL 1 Objective Solutions

1. (c)

2. (d)

3. (c)

4. (a)

5. (b)

6. (a)

7. (314.15)

8. (d)

9. (c)

LEVEL 3 Conventional Questions

Solution: 1

The force on the conductor is

$$\vec{F} = I\vec{L} \times \vec{B} = IL\hat{a}_z \times B_o\hat{a}_r$$

$$\vec{F} = B_o IL\,\hat{a}_\phi$$

So that applied force is

$$\vec{F}_a = B_o IL(-\hat{a}_\phi)$$

The conductor is to be turned in the $\hat{a}_\phi$ direction.

The work required for one full revolution is

$$\omega = \int_0^{2\pi} B_o IL(-\hat{a}_\phi) \cdot rd\phi\,\hat{a}_\phi$$

$$= -2\pi r B_0 I_L$$

Since N revolution per minute is $\frac{N}{60}$ per second.

The power,

$$P = -\frac{2\pi r B_o ILN}{60}$$

[Negative indicates power supplied]

Solution: 2

Choosing the unit normal

$$\hat{a}_n = \left(\frac{\hat{a}_y + \hat{a}_z}{\sqrt{2}}\right)$$

$$B_{n_1} = \frac{(2\hat{a}_x + \hat{a}_y)\cdot(\hat{a}_y + \hat{a}_z)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$B_{n_1} = \frac{1}{\sqrt{2}} \cdot \hat{a}_n = 0.5\hat{a}_y + 0.5\hat{a}_x$$

$$\because \quad B_{n_1} = B_{n_2}$$

$$B_{n_2} = 0.5\hat{a}_y + 0.5\hat{a}_z$$

$$B_{t_1} = 2\hat{a}_x + 0.5\hat{a}_y - 0.5\hat{a}_z$$

$$H_{t_1} =$$

$$\frac{1}{\mu_o}(0.5\hat{a}_x + 0.125\hat{a}_y - 0.125\hat{a}_z) = H_{t_2}$$

$$B_{t_2} = \mu_o \mu_{r_2} H_{t_2}$$

$$= (3\hat{a}_x + 0.75\hat{a}_y - 0.75\hat{a}_z)\text{ T}$$

$$B_2 = B_{t_1} + B_{t_2} = 3\hat{a}_x + 1.25\hat{a}_y - 0.25\hat{a}_z\text{T}$$

$$H_2 = \frac{1}{\mu_o\mu_{r_2}} B_2$$

$$= \frac{1}{\mu_o}\left[0.50\hat{a}_x + 0.21\hat{a}_y - 0.04\hat{a}_z\right]\text{ A/m}$$