2020

RANK improvement WORKBOOK

Answer key and Hint of
Objective & Conventional *Questions*

Electrical Engineering

Electrical Machines

MADE EASY Publications

1 DC Machines

LEVEL 1 Objective Questions

1.	(a)
2.	(b)
3.	(a)
4.	(d)
5.	(b)
6.	1.255
7.	(d)
8.	(404.13)
9.	0.298

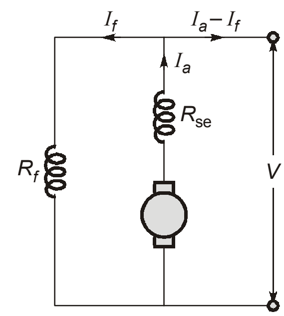
LEVEL 2 Objective Questions

10.	(b)
11.	(a)
12.	(c)
13.	(b)
14.	(a)
15.	(1.17)
16.	(3.6)
17.	(57)
18.	(63.85)
19.	(a)
20.	(80.95)
21.	(a)
22.	(c)

LEVEL 3 Conventional Questions

Solution : 1

(a)
$$V = 220\text{ V}$$
$$R_a = 0.1\ \Omega$$
$$R_f = 50\ \Omega$$
$$R_{se} = 0.06\ \Omega \text{ (long shunt)}$$



$$I_f = \frac{220}{50} = 4.4\text{ A}$$

Load current, $I_l = I_a - I_f$

$$I_a = I_l + I_f = 100 + 4.4 = 104.4\text{ A}$$

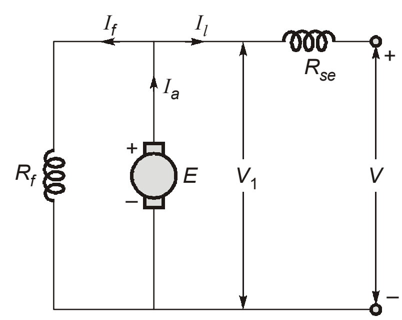
This current flows through armature as well as series field winding hence,

$$I = I_{se} = I_a$$

Induced imf in armature, $E = V + I(R_a + R_{se}) = 220 + 104.4\,(0.1 + 0.06)$

$$= 236.7\text{ V}$$

(b) When R_{se} connected as short shunt



Here, $I_l = I_{se} = 100$ A

Voltage across shunt field and armature,

$$V_1 = V + I_{se} \times R_{se}$$
$$= 220 + 100 \times 0.06 = 226\text{ V}$$

$$I_f = \frac{V_1}{R_f} = \frac{226}{50} = 4.52\text{ A}$$

Armature current,

$$I_a = I_l + I_f = 100 + 4.52 = 104.52\text{ A}$$
$$E = V_1 + I_a R_a = 226 + 104.52 \times 0.1$$
$$= 236.452\text{ V}$$

(c) Series field ampere turns are directly proportional to series field current.

Series field current with diverter $= \dfrac{0.1}{0.1 + 0.06},\quad I = 0.625I$

% change in series field AT $= \dfrac{I - 0.625I}{I} = 37.5\%$

Solution : 2

When run individually $\Rightarrow E_{a1} = E_{a2}$ (as identical run)

Identical run means back emf generated are equal

$$E_{a1} = E_{a2} = V - R_a I_a = 500 - (0.36 \times 50) = 482\text{ V}$$

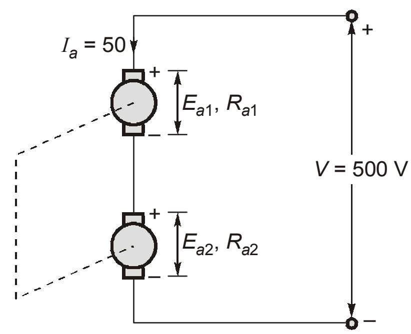
Now,
$$\frac{E_{a1}}{E_{a2}} = \frac{k\phi_1 N_1}{k\phi_2 N_2}$$

$\Rightarrow$
$$k\phi_1 N_1 = k\phi_2 N_2 \qquad (\text{as } E_{a1} = E_{a2})$$

$$\frac{\phi_1}{\phi_2} = \frac{N_1}{N_2} = \frac{750}{700} = \frac{15}{14} \qquad \text{... (i)}$$

When run mechanically coupled ($N_1 = N_2$):

From diagram,



$$E_{a1} + E_{a2} + I_a(R_{a1} + R_{a2}) = V$$

$$E_{a1} + E_{a2} + 50(0.36 + 0.36) = 500$$

$$E_{a1} + E_{a2} = 500 - 50(0.72) = 464 \text{ V} \quad \text{...(ii)}$$

Further, $$\frac{E_{a1}}{E_{a2}} = \frac{k\phi_1 N_1}{k\phi_2 N_2} = \frac{\phi_1}{\phi_2} = \frac{14}{15} \quad \text{...(iii)}$$

Putting in equation (ii),

$$E_{a1} + \frac{14}{15}E_{a2} = 464$$

$$E_{a1} = 240 \text{ V}$$

$$E = k\phi N$$

Comparing machine 1 when individually run and run in coupled condition

$$E_1' = k\phi_1' 700 = 482$$

$$E_1'' = k\phi_1'' N_1'' = 240$$

But $\phi_1' = \phi_1''$ as ϕ of E_1 is constant in both conditions

Hence, $$\frac{N_1''}{700} = \frac{240}{482}; \quad N_1'' = \frac{240}{482} \times 700 = 348.55 \text{ rpm}$$

Solution : 3

We know the formula is given below,

$$E_b = k\phi\omega_m \quad \text{...(i)}$$

$$E_b = V - I_a R_a \quad \text{...(ii)}$$

$$T = k\phi I_a \quad \text{...(iii)}$$

From (i) and (ii), $$k\phi\,\omega_m = V - I_a R_a$$

$$\omega_m = \frac{V}{k\phi} - \frac{I_a R_a}{k\phi} \quad \text{...(iv)}$$

Putting (iii) in (iv), $$\omega_m = \frac{V}{k\phi} - \frac{TR_a}{k^2\phi^2} \quad \text{...(v)}$$

Differentiating ω_m with respect to ϕ we get

$$\frac{d\omega_m}{d\phi} = -\frac{V}{k\phi} + \frac{2TR_a}{k^2\phi^3} = -\frac{V}{k\phi^2} + \frac{2I_a R_a}{k\phi^2} \quad \text{[From equation (iii)]}$$

$$= -\frac{1}{k\phi^2}\left[V - 2I_a R_a\right] \quad \text{...(vi)}$$

Case-1:
$$E_b < \frac{V}{2}$$
$$V - I_a R_a < \frac{V}{2}$$
$$\frac{V}{2} < I_a R_a$$
$$V < 2I_a R_a$$

Putting in (vi), we get $$\frac{d\omega_m}{d\phi} > 0$$

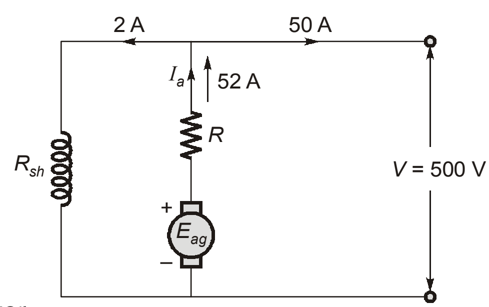
Case-2:
$$E_b > \frac{V}{2}$$
$$V - I_a R_a > \frac{V}{2}$$
$$\frac{V}{2} > I_a R_a$$
$$V > 2I_a R_a$$

Putting in (vi), we get $$\frac{d\omega_m}{d\phi} < 0$$

Solution : 4

When the machine is working as a generator.

It is given that, voltage drop across R is 5% of 500 V



$\Rightarrow$ $$52 \times R = 25$$
$$R = 0.48\ \Omega$$
$$E_{ag} = V + I_a R_a$$
$$= 500 + 52 \times 0.48$$
$$E_{ag} = 525\text{ V}$$

Let, $$P_{Rg} = \text{Rotational losses in the generator}$$

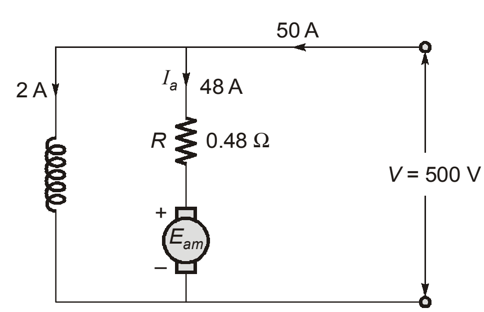
Efficiency of generator, $$\eta_G = \frac{\text{Output power}}{\text{Input power}}$$

$$0.8 = \frac{500 \times 50}{P_{in}} \Rightarrow P_{in} = 31250\ \Omega$$

Rotational losses in the generator,
$$P_{Rg} = P_{in} - \text{Power delivered by armature}$$
$$= 31250 - 52 \times 525$$
$$= 3950\text{ W}$$

Given, Speed of generator $= N_g = 1500$ rpm

When the machine is working as a motor,



$$E_{am} = V - I_a R$$
$$E_{am} = 500 - 48 \times 0.48$$
$$E_{am} = 476.96\text{ V}$$

Now, let the speed of the motor be N_m.

We know that for a dc machine,

$$E_a \propto \phi N$$

As ϕ is equal in this case for motor and generator, we can write,

$$\frac{E_{ag}}{E_{am}} = \frac{N_g}{N_m}$$

$$N_m = \frac{E_{am}}{E_{ag}} N_g = \frac{476.96}{525} \times 1500$$

$$= 1362.74 \text{ rpm}$$

Let the rotational losses for the motor = P_{Rm}.

It is given that, $P_R \propto N^2$

$$\frac{P_{Rg}}{P_{Rm}} = \left(\frac{N_g}{N_m}\right)^2$$

$$P_{Rm} = 3950 \times \left(\frac{1362.74}{1500}\right)^2 = 3260.186 \text{ W}$$

Total losses in motor = Losses in armature resistor + Losses in field resistance + P_{Rm}

$$= (48)^2 (0.48) + 2 \times 500 + 3260.186$$

$$= 5366.106 \text{ W}$$

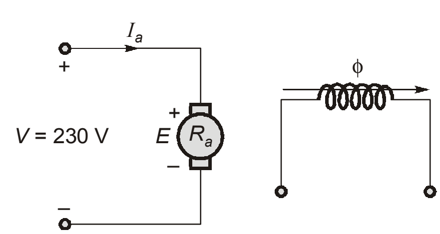
$$\text{Efficiency of motor} = \frac{\text{Power output}}{\text{Power input}}$$

$$= \frac{\text{Power input} - \text{Total losses}}{\text{Power input}} = \frac{500 \times 50 - 5366.106}{500 \times 50}$$

$$\eta_M = \frac{19633.894}{25000} = 0.7854$$

Solution : 5

In separately excited dc motor flux per pole, ϕ = constant,



As rotational losses are neglegible,

Torque developed = Load torque

$$T = T_L$$

$$K_a \phi I_a = 500 - 100\omega \qquad \text{...(i)}$$

$R_a = 0.5\ \Omega$, $V = 230$ V, $N = 250$ rpm, $I_a = 100$ A

Back emf, $E = V - I_a R_a$

and $= 230 - 100 \times 0.5 = 180$ V

$$E = K_a \phi \omega$$

Speed, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi}{60} \times 250 = 26.18$ rad/sec.

The constant, $K_a\phi = \dfrac{E}{\omega} = \dfrac{180}{26.18} = 6.88$ V/(rad/sec.)

Using equation (i), $6.88\, I_a = 500 - 10\omega$ ...(ii)

$E = V - I_a R_a = 230 - 0.5 I_a$

As, $E = K_a \phi \omega = 6.88\omega$

So, $6.88\omega = 230 - 0.5 I_a$ ...(iii)

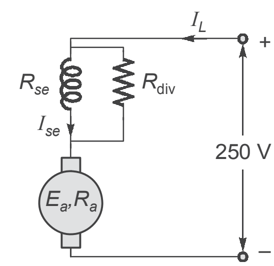
By solving equation (i) and (ii), we get

$I_a = 38.73$ A

and $\omega = 30.62$ rad/sec.

Solution : 6

Given that :



$$I_L = 100 \text{ A} = I_{se1} = I_{a1}$$

Supply voltage = 250 V

Back emf, $E_{a1} = V - I_{a1}(R_a + R_{se})$

$= 250 - 100\,(0.15 + 0.05)$

$E_{a1} = 230$ V

After connecting a diverter of resistance 0.09 Ω in parallel with the field winding then speed of motor be N_2 and current I_{a2} respectively.

$$I_{se2} = I_{a2}\frac{R_{div}}{R_{div} + R_{se}} = \frac{0.09}{0.09 + 0.05} I_{a2} = 0.64286\, I_{a2}$$

and Torque, $T_2 = \dfrac{3}{4} T_1$

$$I_{a2} \cdot \phi_2 = \frac{3}{4} I_{a1} \phi_1 \,(\phi \propto I_{se})$$

$$I_{a2} I_{se2} = \frac{3 \cdot I_{a1} I_{se1}}{4} \qquad (I_{se1} = I_{a1})$$

$$I_{a2} \times 0.64286 \cdot I_{a2} = \frac{3}{4} I_{a1}^2$$

$$I_{a2}^2 = \frac{3}{4 \times 0.64286} I_{a1}^2$$

$$I_{a2} = 1.08\, I_{a1} = 1.08 \times 100 = 108 \text{ A}$$

$$I_{se2} = 0.64286 \times 108 = 69.4289 \text{ A}$$

$$E_{a2} = V - I_{a2} R_a - I_{se2} R_{se}$$

$$= 250 - 108 \times 0.15 - 69.4289 \times 0.05$$

$$= 250 - 19.6714 = 230.3286 \text{ volts}$$

and $\dfrac{N_2}{N_1} = \dfrac{E_{a2}}{E_{a1}} \times \dfrac{\phi_1}{\phi_2}$

$$N_2 = N_1 \times \frac{E_{a2}}{E_{a1}} \times \frac{I_{se1}}{I_{se2}} = 900 \times \frac{230.3286}{230} \times \frac{100}{69.4289}$$

$$N_2 = 1298.142 \text{ rpm}$$

Speed of motor is 1298.142 rpm if torque is 3/4 of the full load value.

Solution : 7

$$\text{No load input} = 400 \times 5 = 2000 \text{ W}$$

The input goes to meet all kinds of no-load losses i.e. armature copper losses and constant losses.

$$I_{sh} = \frac{400}{200} = 2 \text{ A}$$

No load armature current, $I_a = 5 - 2 = 3$ A

$$\text{No load armature copper loss} = 32 \times 0.5 = 4.5 \text{ W}$$

$$\text{Constant losses} = 2000 - 4.5 = 1995.5 \text{ W}$$

When line current is 50 A,

$$I_a = 50 - 2 = 48 \text{ A}$$

$$\text{Armature copper losses} = 48^2 \times 0.5 = 1152 \text{ W}$$

$$\text{Total loss on full load} = 1152 + 1995.5$$

$$= 3147.5 \text{ W}$$

$$\text{Input} = 50 \times 400 = 20000 \text{ W}$$

$$\text{Output} = 20000 - 3147.5 = 16852.5 \text{ W}$$

$$\text{Full load efficiency} = \frac{16852.5}{20000} = 84.26\%$$

Now,

$$E_{b1} = 400 - (3 \times 0.5) = 398.5 \text{ V}$$

$$E_{b2} = 400 - (48 \times 0.5) = 376 \text{ V}$$

$$\frac{N_1 - N_2}{N_2} = \frac{E_{b1} - E_{b2}}{E_{b2}} = \frac{398.5}{376} = \frac{22.5}{376} = 5.98\%$$

Solution : 8

Given,

$$I_L = \frac{10 \times 10^3}{250} = 40 \text{ A}$$

$$I_a \approx I_L = 40 \text{ A}$$

$$I_1 = 2I_{f1} = 80 \text{ A}$$

$$I_2 = I_{f1} = 40 \text{ A}$$

$$\gamma = \frac{I_1}{I_2} = 2$$

Now,

$$R_1 = \frac{V_1}{I_1} = \frac{250}{80} = 3.125\ \Omega$$

$$R_{starter}\text{ (total)} = 3.125 - 0.15 = 2.975\ \Omega$$

Also,

$$\gamma^{k-1} = \frac{R_1}{R_a} \quad \text{or} \quad 2^{k-1} = \frac{3.125}{0.15}$$

which gives, $k = 5 \Rightarrow 4$ sections are required.

Resistance of various sections are computed as shown below,

$$R_1 = 3.125\ \Omega$$

$$R_2 = \frac{3.125}{2} = 1.56\ \Omega, \qquad r_1 = 1.56\ \Omega$$

$$R_3 = \frac{1.56}{2} = 0.78\ \Omega, \qquad r_2 = 0.78\ \Omega$$

$$R_4 = \frac{0.78}{2} = 0.39\ \Omega, \qquad r_3 = 0.39\ \Omega$$

$$R_5 = \frac{0.39}{2} = 0.195\ \Omega, \qquad r_4 = 0.195\ \Omega$$

■■■■

2 Transformers

LEVEL 1 Objective Questions

1.	(a)
2.	(a)
3.	(a)
4.	(150)
5.	235.66
6	(30)
7.	(a)
8.	(5.68)
9.	(c)
10.	(d)
11.	(b)

LEVEL 2 Objective Questions

12.	(c)
13.	(d)
14.	(b)
15.	(c)
16.	(96.77)
17.	(333.33)
18.	(67.92)
19.	(75.58)
20.	(7.52)
21.	(a)
22.	(d)
23.	(a)

■■■■

LEVEL 3 Conventional Questions

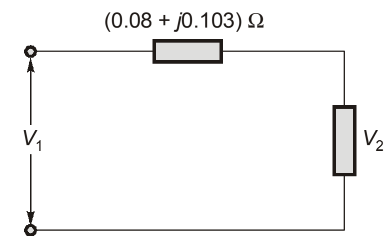
Solution : 1

We will refer transformer impedance to the LV side.

$$R_{LV} = 0.05 + \frac{3}{100} = 0.08\ \Omega$$

$$X_{LV} = 0.05 + \frac{5.3}{100} = 0.103\ \Omega$$

The circuit model is drawn in figure,



$$I_2 = \frac{20 \times 1000}{200} = 100\text{ A}$$

(a) (i) $\text{Voltage drop} = 100(0.08 \times 0.8 + 0.103 \times 0.6) = 12.58\text{ V}$

$$\text{Voltage regulation} = \frac{12.58}{200} \times 100 = 6.29\%$$

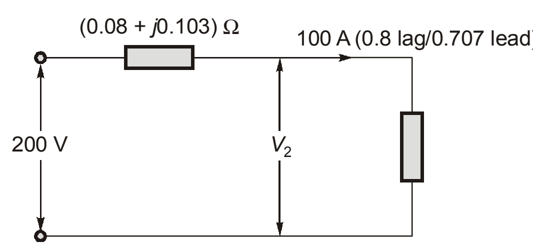
(ii) $\text{Voltage drop} = 100(0.08 \times 1 + 0.103 \times 0) = 8\text{ V}$

$$\text{Voltage regulation} = \frac{8}{200} \times 100 = 4\%$$

(iii) $\text{Voltage drop} = 100(0.08 \times 0.707 - 0.103 \times 0.707) = -1.63\text{ V}$

$$\text{Voltage regulation} = \frac{-1.63}{200} \times 100 = -0.815\%$$

(b) The circuit model is shown in figure,



(i) $\text{Voltage drop} = 12.58\text{ V}$

$$V_2 = 200 - 12.58 = 187.4\text{ V}$$

(ii) $\text{Voltage drop} = 8\text{ V}$

$$V_2 = 200 - 8 = 192\text{ V}$$

(iii) $\text{Voltage drop} = -1.63\text{ V}$

$$V_2 = 200 - (-1.63) = 201.63\text{ V}$$

Solution : 2

Consider the figure (a) and figure (b).

Furnace A : 500 kW at 0.71 pf lag,

Furnace B : 800 kW at 0.71 pf lag.

$$\frac{N_1}{N_2} = \frac{6600}{110} = 60$$

and
$$\frac{\sqrt{3}}{2}\frac{N_1}{N_2} = 51.96$$

$$I_a = \frac{500\times1000}{110\times0.71} = 6402\text{ A};\quad \phi_a = \cos^{1}(0.71) = 45° \quad \text{(with } V_b \text{ as reference)}$$

$$I_b = \frac{800\times1000}{110\times0.71} = 10243\text{ A};\quad \phi_b = \cos^{1}(0.71) = 45° \quad \text{(with } V_b \text{ as reference)}$$

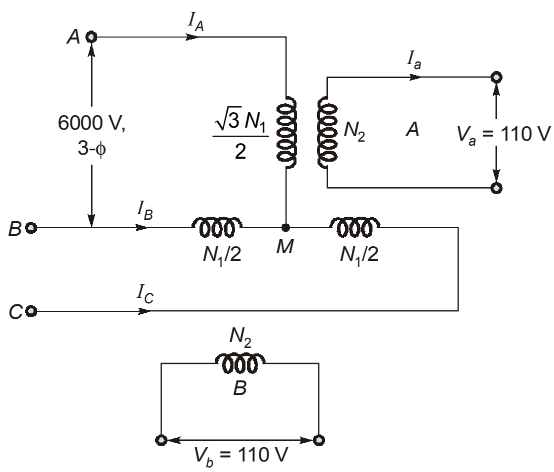


Fig. (a)

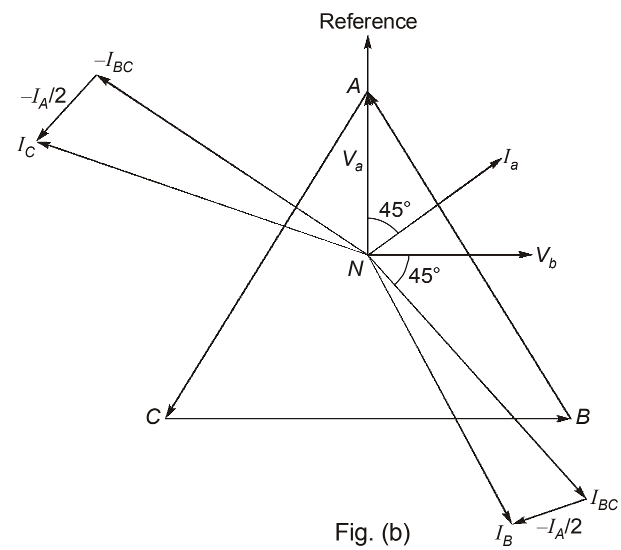


Fig. (b)

Now,
$$\vec{I}_A = \frac{2}{\sqrt{3}}\frac{N_2}{N_1}\vec{I}_a = \frac{6402}{51.96} = 123.2\angle 45°\text{ A}$$

$$\vec{I}_B = \vec{I}_{BC} - \frac{\vec{I}_A}{2} = 170.7(0.71 - j0.71) - \frac{123.2}{2}(0.71 + j0.71)$$

$$= (77.46 - j164.93)\text{ A}$$

$$|I_B| = 182.2\text{ A}$$

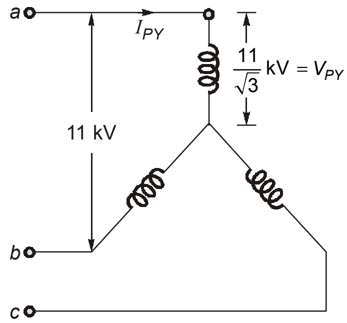
$$\vec{I}_C = -\left(I_{BC} + \frac{\vec{I}_A}{2}\right) = -170.7(0.71 - j0.71) - \frac{123.2}{2}(0.71 + j0.71)$$

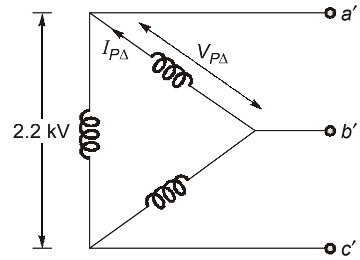
$$= -164.93 + j77.46\text{ A}$$

∴
$$|I_C| = 182.2\text{ A}$$

Solution : 3

(a) 11 kV/2.2 kV, Y/Δ, 1000 kVA load:





$$\text{Rating of each transformer} = \frac{1000}{3} = 333.33 \text{ kVA in each case}$$

$$\text{Transformer ratio} = \frac{11/\sqrt{3}}{2.2} = 2.89$$

$$3\,V_{P\Delta} I_{P\Delta} = 1000 \text{ kVA} \quad \text{...(i)}$$

$$V_{P\Delta} = 2.2 \text{ kV from figure}$$

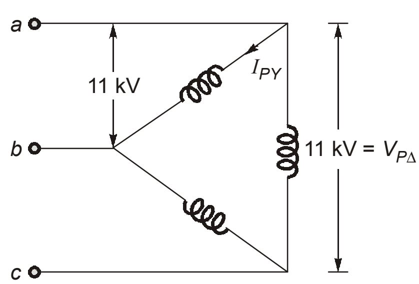
Putting $V_{P\Delta}$ in equation (i),

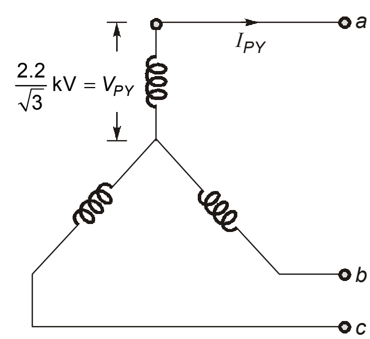
$$I_{P\Delta} = \frac{1000}{3 \times 2.2} = 151.51 \text{ A}$$

$$I_{PY} = \frac{151.51}{2.88} = 52.60 \text{ A}$$

$$V_{PY} = \frac{11}{\sqrt{3}} = 6.35 \text{ kV}$$

(b) 11 kV/2.2 kV, Δ/Y:





$$\text{Turn ratio} = \frac{11}{2.2/\sqrt{3}} = 8.66$$

$$\sqrt{3} I_{PY} V_{LY} = 1000$$

$$I_{PY} = \frac{1000}{\sqrt{3} \times 2.2} = 262.4 \text{ A}$$

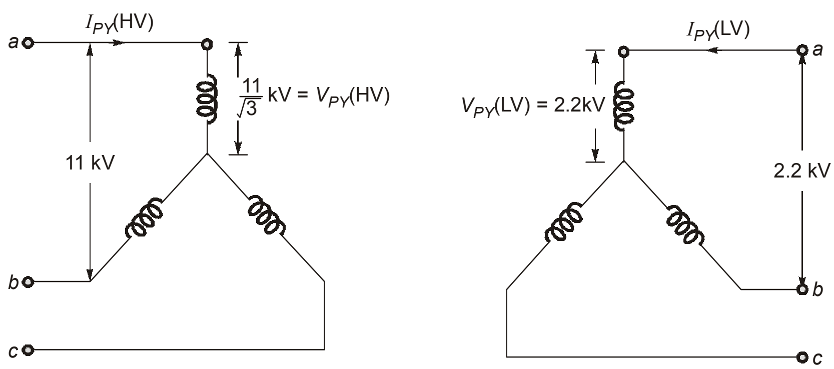
$$V_{PY} = \frac{2.2}{\sqrt{3}} = 1.27 \text{ kV}$$

$$3\,I_{P\Delta}V_{P\Delta} = 1000\text{ kVA} \quad \ldots(i)$$

$$I_{P\Delta} = \frac{1000}{3\times 11} = 30.30\text{ A}$$

$$V_{P\Delta} = 11\text{ kV}$$

(c) 11 kV/2.2 kV, Y/Y:



$$\text{Turn ratio} = \frac{11}{2.2} = 5$$

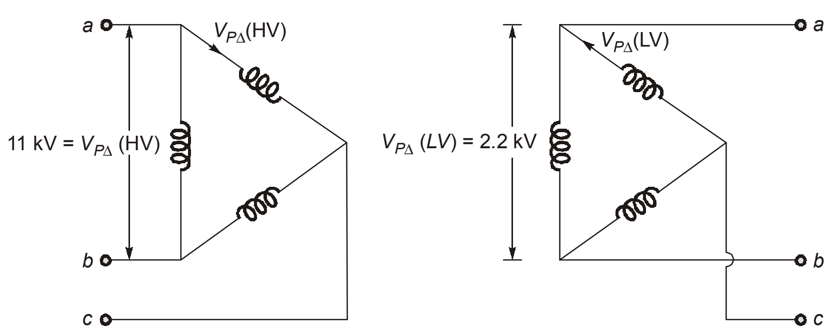
$$I_{P\Delta}(LV) = \frac{1000}{\sqrt{3}\times 2.2} = 262.4\text{ A}$$

$$V_{P\Delta}(LV) = 1.27\text{ kV}$$

$$I_{PY}(HV) = \frac{262.4}{5} = 52.5\text{ A}$$

$$V_{PY}(HV) = \frac{11}{\sqrt{3}} = 6.35\text{ kV}$$

(d) 11 kV/2.2 kV, Δ/Δ:



$$\text{Turn ratio} = \frac{11}{2.2} = 5$$

$$I_{P\Delta}(LV) = \frac{1000}{3\times 2.2} = 151.51\text{ A}$$

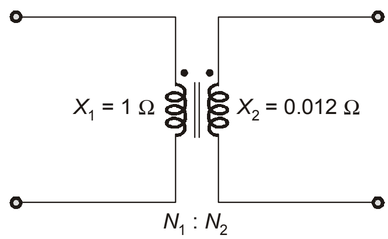
$$V_{P\Delta}(LV) = 2.2\text{ kV}$$

$$I_{P\Delta}(HV) = \frac{151.5}{5} = 30.3\text{ A}$$

$$V_{P\Delta}(HV) = 11\text{ kV}$$

Solution : 4

The turn ratio given as, $\dfrac{N_2}{N_1} = \dfrac{12}{99} = \dfrac{4}{33}$



The secondary reactance referred to primary,

$$X_2' = \frac{X_2}{(N_2/N_1)^2} = \frac{0.012}{(4/33)^2} = 0.82\ \Omega$$

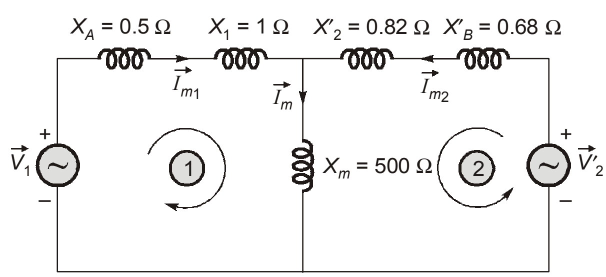
Secondary reactance $X_B = 0.01\ \Omega$ referred to primary,

$$X_B' = \frac{0.01}{(4/33)^2} = 0.68\ \Omega$$

Secondary voltage $V_2 = 400$ V referred to primary,

$$V_2' = \frac{V_2}{(N_2/N_1)} = \frac{400}{(4/33)} = 3300\text{ V}$$

Magnetizing reactance $X_m = 500\ \Omega$ on primary. The equivalent circuit,



As, $\vec{V}_1 = \vec{V}_2' = 3300\angle 0°$ V

The magnetizing currents from primary and secondary,

$$\vec{I}_{m_1} + \vec{I}_{m_2} = \vec{I}_m$$

KVL in loop-1,

$$\vec{V}_1 = j\vec{I}_{m_1}(X_A + X_1) + j\vec{I}_m X_m$$

$$= j[\vec{I}_{m_1}(X_A + X_1 + X_m) + \vec{I}_{m_2} X_m]$$

$$501.5\vec{I}_{m_1} + 500\vec{I}_{m_2} = -j3300 \qquad \text{...(i)}$$

KVL in loop-2,

$$\vec{V}_2' = j[\vec{I}_{m_2}(X_B' + X_2') + \vec{I}_m X_m]$$

$$= j[\vec{I}_{m_1} X_m + \vec{I}_{m_2}(X_B' + X_2' + X_m)]$$

$$500\vec{I}_{m_1} + 501.5\vec{I}_{m_2} = -j3300 \qquad \text{...(ii)}$$

By solving equations (i) and (ii), we get

$$\vec{I}_{m_1} = -j3.26$$

$$\vec{I}_{m_2} = -j3.33$$

Resultant magnetizing current referred to primary side,

$$\vec{I}_m = \vec{I}_{m_1} + \vec{I}_{m_2} = -j6.59$$

So magnetizing mmf or ampere turns,

$$F_m = N_1 I_m = 99 \times 6.59$$
$$= 652.41\ AT^s$$

As in the given circuit currents lag behind the voltage by 90°, power factor = 0, so power $P = 0$ hence no power is transferred.

Solution : 5

(a)
$$V_1 = 2000\text{ V}$$
$$V_2 = 2000 + 200 = 2200\text{ V}$$

(b)
$$I_2 = \frac{20 \times 1000}{200} = 100\text{ A}$$
$$I_1 - I_2 = 10\text{ A},\ I_1 = 110\text{ A}$$
$$\text{kVA rating} = \frac{2200 \times 100}{1000} = 220\text{ kVA}$$

It is therefore seen that a 20 kVA two winding transformer has a rating of 220 kVA as autotransformer, an 11 times increase.

(c) kVA transferred inductively $= \dfrac{V_1(I_1 - I_2)}{1000} = \dfrac{2000 \times 10}{1000} = 20$ kVA

kVA transferred conductively = 220 – 20 = 200 kVA

(d) Core loss (excitation voltage 2000 V) = 120 W

Full load copper loss = 300 W

Total losses = 420 W

Since, $V_2 = 2200$ V, $I_2 = 100$ A

Full load output = 2200 × 100 × 0.8 = 176 kW

$$\%\eta = \left(1 - \frac{420}{176000}\right) \times 100 = 99.76\%$$

This transformer as a two winding transformer has a full load efficiency of 97.44%. The reason for such high efficiency (99.76%) for the autotransformer is its higher output for the same excitation voltage and winding currents i.e., for the same losses.

Solution : 6

On the basis of 500 kVA,

$$\%Z_A = (2 + j3)\ \%$$

$$\%Z_B = \frac{500}{200}(1.5 + j4) = (3 + j8)\%$$

$$\frac{Z_A}{Z_A + Z_B} = \frac{2 + j3}{5 + j10} = 0.3\angle -9.3^\circ$$

$$\frac{Z_B}{Z_A+Z_B} = \frac{3+j8}{5+j11} = 0.711\angle 3.8^\circ$$

Now, $S = 750\ \angle{-45^\circ}$ kVA

(a) $$S_A = S\left(\frac{Z_B}{Z_A+Z_B}\right) = 750\angle -45^\circ \times 0.711\angle 3.8^\circ$$

$$= 533\ \angle{-41.2^\circ} = (400 - j351) \text{ kVA}$$

$$S_B = S\left(\frac{Z_A}{Z_A+Z_B}\right) = (750\angle -45^\circ)\times(0.3\angle -9.3^\circ) = 225\ \angle{-54.3^\circ}$$

(b) $$\cos\phi_A = \cos 41.2^\circ = 0.752 \text{ (lag)}$$

$$\cos\phi_B = \cos 54.3^\circ = 0.5835 \text{ (lag)}$$

(c) Since voltage drop of each transformer is same its value in the case of transformer A would only be calculated. Now for transformer A, kW = 400 for active component of current and kVAR = 351 for reactive component.

$$\therefore \quad \text{% resistive drop} = 2\times\frac{400}{500} = 1.6\%$$

$$\therefore \quad \text{% reactive drop} = 3\times\frac{351}{500} = 2.1\%$$

$$\text{Total % drop} = \sqrt{(2.1)^2+(1.6)^2} = 2.64\%$$

$$\text{Secondary line voltage} = 400 - \left(2.64\times\frac{400}{100}\right) = 389.4 \text{ V}$$

Solution : 7

$$\text{kVA} = \frac{VI_{fl}}{1000}, \qquad I_{fl} = \frac{1000\times \text{kVA}}{V}$$

Full load currents of transformer A and B are

$$I_A = \frac{1000\times 100}{925} = 108.1 \text{ A}$$

$$I_B = \frac{1000\times 50}{925} = 54.05 \text{ A}$$

It is more convenient to work with ohmic impedances. Therefore percentage impedances are converted into ohmic values. Let us assume that the secondary terminal voltage is 925 V. This arbitrarily chosen value is less than either of the two no-load emfs.

$$I_A R_A = 1.5\% \text{ of } V = \frac{1.5}{100}\times 925$$

$$R_A = \frac{1.5}{100}\times\frac{925}{108.1} = 0.1284\ \Omega$$

$$I_A X_A = 8\% \text{ of } V = \frac{8}{100}\times 925$$

$$X_A = \frac{8 \times 925}{100 \times 108.1} = 0.6846\ \Omega$$

$$I_B R_B = 2\% \text{ of } V = \frac{2}{100} \times 925$$

$$R_B = \frac{2 \times 925}{100 \times 54.05} = 0.3423\ \Omega$$

$$I_B X_B = 6\% \text{ of } V = \frac{6 \times 925}{100}$$

$$X_B = \frac{6 \times 925}{100 \times 54.05} = 1.0268\ \Omega$$

$$Z_A = R_A + jX_A = (0.1284 + j0.6846)\ \Omega$$

$$Z_B = R_B + jX_B = (0.3423 + j1.0268)\ \Omega$$

$$Z_A + Z_B = 0.1284 + j0.6846 + 0.3423 + j1.0268$$

$$= 0.4707 + j1.7114 = 1.775\angle 74.62°\ \Omega$$

Circulating current,
$$I_C = \frac{E_{2A} - E_{2B}}{Z_A + Z_B} = \frac{1000 - 950}{1.775\angle 74.62°} = 28.17\angle{-74.62°}\text{ A}$$

Solution : 8

$$S = 100 \text{ kVA} = 100 \times 10^3 \text{ VA}$$

$$\eta = \frac{mS\cos\phi_2}{mS\cos\phi_2 + P_i + m^2 P_{cfl}}$$

At full load,
$$m = 1,\ \cos\phi_2 = 0.8$$

$$\eta_{fl} = \frac{1 \times 100 \times 10^3 \times 0.8}{1 \times 100 \times 10^3 \times 0.8 + P_i + P_{cfl}} = 0.985$$

$$\frac{100 \times 10^3 \times 0.8}{0.985} = 100 \times 10^3 \times 0.8 + P_i + P_{cfl}$$

$$P_i + P_{cfl} = 100 \times 10^3 \times 0.8\left(\frac{1}{0.985} - 1\right)$$

or,
$$P_i + P_{cfl} = 1218 \quad \text{...(i)}$$

At half-full load,
$$m = \frac{1}{2},\ \cos\phi_2 = 1$$

$$\eta_{1/2fl} = \frac{\left(\frac{1}{2}\right) \times 100 \times 10^3 \times 1}{\frac{1}{2} \times 100 \times 10^3 + P_i + \left(\frac{1}{2}\right)^2 P_{cfl}} = 0.99$$

$$P_i + \frac{1}{4}P_{cfl} = 50 \times 10^3\left(\frac{1}{0.99} - 1\right) = 505 \text{ W} \quad \text{...(ii)}$$

Subtracting equations (i) and (ii), we get

$$\frac{3}{4}P_{cfl} = 713, \quad P_{cfl} = 950.7 \text{ W}$$

and $$P_i = 1218 - 950.7 = 267.3 \text{ W}$$

Full load current on the secondary side

$$= \frac{100 \times 1000}{11000} = 9.09 \text{ A}$$

At maximum efficiency, $$I_{2M} = I_{2fl}\sqrt{\frac{P}{P_{cfl}}} = 9.09 \times \sqrt{\frac{267.3}{950.7}} = 4.82 \text{ A}$$

■■■■

3 Induction Machines

LEVEL 1 Objective Questions

1.	(d)
2.	(b)
3.	(0.177)
4.	(a)
5.	(b)
6.	(381.97)
7.	(b)
8.	(22.1)
9.	(b)
10.	(a)
11.	(c)

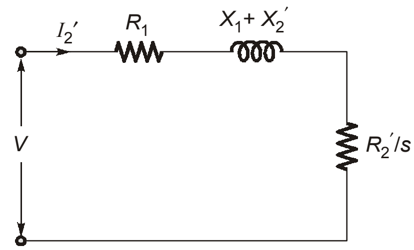
LEVEL 2 Objective Questions

12.	(d)
13.	(b)
14.	(0.34)
15.	(583.8)
16.	(53.33)
17.	(0.03)
18.	(39.42)
19.	(42)
20.	(1)
21.	(b)

LEVEL 3 Conventional Questions

Solution : 1

Given: R_1 = 0.6 Ω, R'_2 = 0.3, $X_1 = X_2'$ = 1 Ω
The magnetizing branch is neglected, $I_0 \simeq 0$
The per-phase equivalent circuit,



Per phase rated voltage, $\vec{V} = \dfrac{440}{\sqrt{3}}\angle 0° = 254.03\angle 0°$ V

f = 50 Hz, P = 6

Synchronous speed, $N_s = \dfrac{120f}{P} = 1000$ rpm

or, $\omega_s = \dfrac{4\pi f}{P} = 104.72$ rad/ s

At rated speed, N_r = 960 rpm

$$\text{Slip, } s = \frac{N_S - N_r}{N_S} = 0.04$$

$$\text{Rated current} = \frac{\vec{V}}{\sqrt{\left(R_1 - \dfrac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}} \qquad \text{...(i)}$$

$$= \frac{254.03}{\sqrt{\left(0.6 + \dfrac{0.3}{0.04}\right)^2 + (1+1)^2}} = 30.45 \text{ A}$$

(i) Speed at which machine is running,

Applied voltage = 330 V

Given current is same as rated current as per phase voltage,

$$V = \frac{330}{\sqrt{3}} = 190.52 \text{ V}$$

Putting values in equation (i),

$$\frac{190.52}{\sqrt{\left(0.6 + \dfrac{0.3}{s}\right)^2 + (2)^2}} = 30.45$$

$$\Rightarrow \quad \frac{190.52}{30.45} = \sqrt{\left(0.6+\frac{0.3}{s}\right)^2 + 4}$$

$$\Rightarrow \quad 39.16 - 4 = \left(0.6 + \frac{0.3}{s}\right)$$

$$\Rightarrow \quad 35.16 - 0.6 = \frac{0.3}{s}$$

$$\Rightarrow \quad s = 8.67 \times 10^{-3}$$

We know, $\quad \frac{N_s - N_r}{N_s} = s$

$$\frac{1000 - N_r}{1000} = 8.67 \times 10^{-3}$$

$$1000 - N_r = 8.67$$

$$N_r = 991.33 \text{ rpm}$$

(ii) Torque developed at new speed,

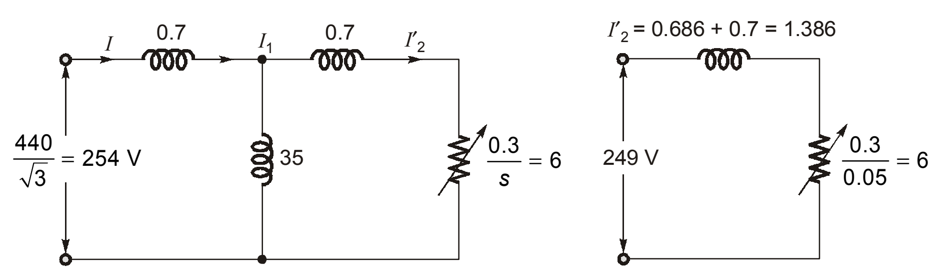
$$T_{\text{new}} = \frac{3}{\omega_s} I_2'^2 \frac{R_2'}{s}$$

$$= \frac{3}{104.72} \times (30.45)^2 \times \frac{0.3}{8.67 \times 10^{-3}}$$

$$= 919.11 \text{ Nm}$$

Solution : 2

The circuit model of the motor is drawn in figure (a) and its Thevenin equivalent is given in figure (b).



(a) (b)

$$s = \frac{1000 - 950}{1000} = 0.05$$

$$V_{\text{Th}} = \frac{254 \times 35}{35 + 0.7} = 249 \text{ V}$$

$$x_{\text{Th}} = \frac{35 \times 0.7}{35 + 0.7} = 0.686\ \Omega$$

$$I_2' = \frac{249}{\sqrt{(1.386)^2 + (6)^2}} = 40.44 \text{ A}$$

$$P_m(\text{gross}) = \left(\frac{1}{s} - 1\right) \times 3 I_2'^2 r_2'$$

$$= \left(\frac{1}{0.05} - 1\right) \times 3 \times (40.44)^2 \times 0.3 = 27.96 \text{ kW}$$

$$P_m(\text{net}) = 27.96 - 0.75 = 27.21 \text{ kW}$$

For input current, $\vec{Z}_f = \dfrac{j35(6 + j0.7)}{6 + j35.7} = 5.84\angle 16.2° = (5.61 + j1.63)\ \Omega$

$$\vec{Z}(\text{total}) = 5.61 + j(0.7 + 1.63) = 5.61 + j2.33 = 6.07\angle 22.6°\ \Omega$$

$$I_1 = \frac{254}{6.07} = 41.8 \text{ A}$$

$$\text{p.f.} = \cos 22.6° = 0.923 \text{ lagging}$$

Solution : 3

Given: $r_2 = 4.5\ \Omega,\quad x_2 = 8.5\ \Omega$

(a) $T_\text{start} = \dfrac{3}{\omega_s} \dfrac{V^2 r_2'}{r_2'^2 + x_2'^2}$ $\quad [\because R_\text{ext} = 0]$

But, $\omega_s = \dfrac{120 \times 50}{4} \times \dfrac{2\pi}{60} = 157.1 \text{ rad/s}$

$$85 = \frac{3}{157.1} \frac{V'^2 \times 4.5}{(4.5)^2 + (8.5)^2}$$

Standstill rotor voltage, $V' = 302.5 \text{ V}$ or 523.9 V (line)

(b) With external resistance, $R_\text{ext} = 3\ \Omega$

$$T_\text{start} = \frac{3}{\omega_s} \times \frac{V^2 (r_2' + R_\text{ext}')}{(r_2' + R_\text{ext}')^2 + x_2'^2}$$

$$T_\text{start} = \frac{3}{157.1} \frac{(302.5)^2 (4.5 + 3)}{(4.5 + 3)^2 + (8.5)^2} = 102 \text{ Nm}$$

(c) Induced rotor voltage at slip, $s = s \times$ standstill voltage

$= 0.03 \times 302.5$

$= 9.1 \text{ V}$ or 15.7 (line)

$$T = \frac{3}{\omega_s} \times \frac{V^2 (r_2'/s)}{(r_2'/s)^2 + x_2'^2} = \frac{3}{157.1} \frac{(302.5)^2 (4.5/0.03)}{(4.5/0.03)^2 + (8.5)^2} = 11.6 \text{ Nm}$$

Solution : 4

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

(i) Frequency of stator current = 50 Hz

(ii) Speed of rotor magnetic field w.r.t. rotor = $(N_s - N) = 1500 - 1440$

$= 60$ rpm

Rotor magnetic field is in same direction as rotor.

(iii) Speed of stator magnetic field w.r.t. rotor = $(N_s - N) = 1500 - 1440$
$= 60$ rpm

Stator magnetic field is in same direction as rotor.

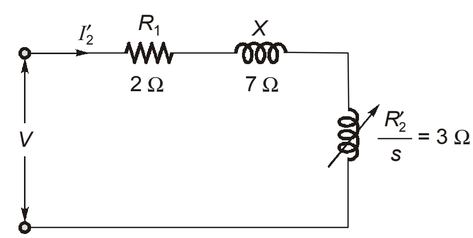
(iv) Speed of stator magnetic field with respect to rotor magnetic field = Zero

Solution : 5

Given, constant V/f,

Voltage, $V = \dfrac{400}{\sqrt{3}}$;

Frequency, $f = 50$ Hz



$$\frac{V}{f} = \frac{400}{\sqrt{3} \times 50}$$

or $\dfrac{V}{f} = 4.6$

and frequency 10% to rated means $5 \to 50$ Hz

At $V = \dfrac{400}{\sqrt{3}}$; $f = 50$ Hz

$\because$ $N_r = 1380$ rpm;

$$N_s = \frac{120 \times 50}{P} = 1500 \text{ rpm}$$

$$p = 4$$

$$\omega_s = \frac{2\pi N_s}{60} = \frac{2 \times \pi \times 1500}{60} = 157 \text{ rad/sec}$$

$$S_{fl} = \frac{N_s - N_r}{N_s} = \frac{1500 - 1380}{1500} = 0.08$$

Full load torque,

$$T_{fl} = \frac{3}{\omega_s} \cdot \frac{V^2 \cdot R_2' / S_{fl}}{\left(R_1 + \dfrac{R_2'}{S_{fl}}\right) + x^2} = \frac{3}{157} \cdot \frac{\left(\dfrac{400}{\sqrt{3}}\right)^2 \times \dfrac{3}{0.08}}{\left(2 + \dfrac{3}{0.08}\right)^2 + (7)^2}$$

$$T_{fl} = 23.75 \text{ Nm}$$

(a) $f = ?$ $\dfrac{V}{f} = 4.6$

$$N_r = 1000 \text{ rpm}$$

$$T_{fl} = 23.75$$

$$\omega_s = \frac{2\pi \times N_s}{60} = \frac{2\pi}{60} \times \frac{120f}{4}$$

$$\omega_s = \pi f$$

$$X = 2\pi f L = \frac{f}{50} \times (2\pi \times 50 \times L)$$

$$X_{50\text{ Hz}} = 2\pi \times 50 \times L = 7$$

or, $$X = \frac{7f}{50}$$

Slip, $$s_{fl} = \frac{\omega_s - \omega_r}{\omega_s} = 1 - \frac{104.7}{\pi f} = 1 - \frac{33.3}{f}$$

$$23.5 = \frac{3}{\pi f} \times \frac{(4.6f)^2 \dfrac{3}{\left(1 - \dfrac{33.3}{f}\right)}}{\left[2 + \left(\dfrac{3}{1 - \dfrac{33.3}{f}}\right)\right]^2 + \left(\dfrac{7f}{50}\right)^2}$$

For full load i.e. low slip region.

$$\left(R_1 + \frac{R_2'}{S}\right)^2 + X^2 \cong \left(\frac{R_2'}{S}\right)^2$$

i.e., $$23.5 \cong \frac{3}{\pi f} \cdot \frac{(4.6f)^2 \left(\dfrac{3}{1 - \dfrac{33.3}{f}}\right)}{\left(\dfrac{3}{1 - \dfrac{33.3}{f}}\right)^2} \quad \text{or} \quad 23.5 = \frac{3}{\pi} \cdot (4.6)^2 \left(\frac{f - 33.3}{3}\right)$$

$$f = 36.78 \text{ Hz}$$

(b) Torque for a frequency of 35 Hz and speed of 950 rpm.

$$N_r = 950; \quad N_s = 1050$$

Slip, $$s = \frac{1050 - 950}{1050} = 0.09$$

$$T = \frac{3}{\omega_s} \cdot \frac{V^2}{\left(R_1 + \dfrac{R_2'}{S}\right)^2 + (X_1 + X_2')^2} \cdot \frac{R_2'}{S_{f1}}$$

$$\omega_s = \frac{2\pi N_s}{60} = \frac{2\pi \times 1050}{60} = 109.95 \text{ rad/sec.}$$

$$\frac{V}{f} = 4.6 \text{ constant}$$

Voltage, $$V = 4.6 \times 35 = 161 \text{ V}$$

$$X = \frac{35}{50} \times 7 = 4.9 \qquad \left(\because X = \frac{7f}{50}\right)$$

$$T = \frac{3}{109.25} \frac{(161)^2 \times \dfrac{3}{0.09}}{\left(2 + \dfrac{3}{0.09}\right)^2 + (4.9)^2} = 19.39 \text{ Nm}$$

Solution : 6

Given:

Supply voltage, $V = 400\text{ V}$

Output power, $P_0 = 20\text{ kW}$

Rated speed, $N_1 = 1440\text{ rpm}$

Initial frequency, $f_1 = 50\text{ Hz}$

Initial rotor leakage impedance,

$$Z_1 = \left(\underbrace{0.4}_{R} + j\underbrace{1.6}_{R}\right)\Omega/\text{phase}$$

Final frequency, $f_2 = 120\text{ Hz}$

Now change in frequency will change rotor leakage impedance and synchronous speed as $X \propto f$

New rotor leakage impedance,

$$Z_2 = 0.4 + j1.6\left(\frac{120}{50}\right) = \underbrace{0.4}_{R} + j\underbrace{3.84}_{X_2}$$

Let us assume that the motor has 4 poles,

Initial synchronous speed, $N_{s1} = \dfrac{120f_1}{4} = 1500\text{ rpm}$

Final synchronous speed, $N_{s2} = \dfrac{120f_2}{4} = 3600\text{ rpm}$

(i) The torque for induction motor is given by; where k is a constant

$$\tau = \frac{ks}{N_s(R^2 + s^2X^2)}$$

Now, rated slip in initial condition,

$$s_1 = \frac{N_{s1} - N_1}{N_{s1}} = \frac{60}{1500} = 0.04$$

∴ Rated torque in initial condition,

$$\tau_1 = \frac{ks_1}{N_{s1}(R^2 + s_1^2X_1^2)} = \frac{k(0.04)}{1500(0.16 + 0.0041)} = k(0.00016251)$$

Rated torque in final condition,

$$\tau_2 = \frac{ks_2}{N_{s2}(R^2 + s_2^2X_2^2)} = \frac{ks_2}{3600(0.16 + s_2^2 14.7450)}$$

$$\tau_1 = \tau_2$$

$$k(0.00016251) = \frac{ks_2}{3600(0.16 + s_2^2 14.7456)}$$

$$8.6267s_2^2 - s_2 + 0.0936 = 0$$

The solutions for this equation are imaginary. This means that when supplied at 120 Hz, the motor cannot generate the rated load torque of 50 Hz frequency.

(ii) The slip at which maximum torque occurs is given by

$$s_{max} = \frac{R}{X} = \frac{0.4}{3.84} = 0.1042$$

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4 Synchronous Machines

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(24.00)
4.	(a)
5.	(0.9253)
6.	(b)
7.	(c)
8.	(c)
9.	(d)
10.	(b)
11.	(d)
12.	(d)

LEVEL 2 Objective Questions

13.	(b)
14.	(d)
15.	(d)
16.	(d)
17.	(d)
18.	(157.5)
19.	(2158.74)
20.	(a)
21.	(180)
22.	(c)

LEVEL 3 Conventional Questions

Solution : 1

Two identical generators (1) and (2) each rated 100 MVA and $X_d = X_q = 0.8$ pu as $X_d = X_q$ it means cylindrical rotor with synchronous reactance $X_s = 0.8$ p.u.

Total load = 100 MVA at 0.8 p.f. lag

Let, Base MVA, $(\text{MVA})_B$ = 100 and rated kV as base voltage

$S_L = 1$ p.u. p.f. $\cos\theta = 0.8$ lag

As, $\vec{S}_L = P_L + jQ_L$

$P_L = S_L \cos\theta = 0.8$ pu

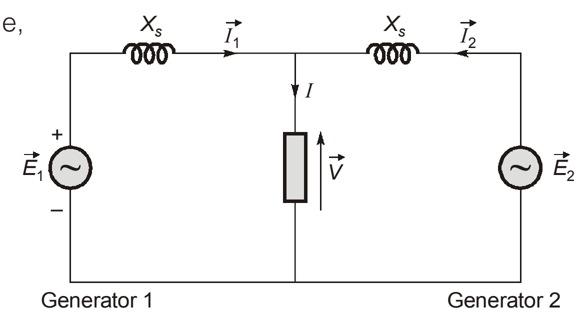
$Q_L = S_L \sin\theta = +0.6$ pu, +ve as p.f. lagging

Load shared by each generator,

$$\vec{S}_1 = \vec{S}_2 = \frac{\vec{S}_L}{2} = 0.4 + j0.3$$

or, $S_1 = S_2 = 0.5$ pu

Both are operating at rated voltage i.e., terminal voltage,



$V_1 = V_2 = 1$ pu

Let, $\vec{V}_1 = \vec{V}_2 = \vec{V} = 1\angle 0°$ pu

$S_L = VI$ (in pu)

$\Rightarrow$ $1 = 1 \times I$

$\Rightarrow$ $I = 1$ pu

As, p.f. $\cos\theta = 0.8$ lagging

$\Rightarrow$ $\theta = 36.9°$

So, $\vec{V} = 1\angle 0°$ pu, $\vec{I} = 1\angle -36.9°$ pu

Both are identical so, $$\vec{I}_1 = \vec{I}_2 = \frac{\vec{I}_L}{2} = 0.5\angle -36.9° \text{ pu}$$

$$\vec{E}_1 = \vec{E}_2 = \vec{V} + j\vec{I}_1 X_s$$

$= 1\angle 0° + j0.8 \times 0.5\angle -36.9°$

$= 1.28\angle 14.47°$ pu

Excitation emfs, $E_1 = E_2 = 1.28$ pu

If field current I_{f1} of 1st generator is reduced by 5%.

Because saturation is neglected, i.e., $E \propto I_f$

$E_1' = 0.95\, E_1 = 1.216$ pu

The field current I_f of 2nd generator is increased by 5%.

i.e., $E_2' = 1.05\, E_1 = 1.344$ pu

In case of changing field excitation the active power output remains same only operating p.f. or reactive power changes.

So, $P_1 = P_2 = 0.4$ p.u. remains same

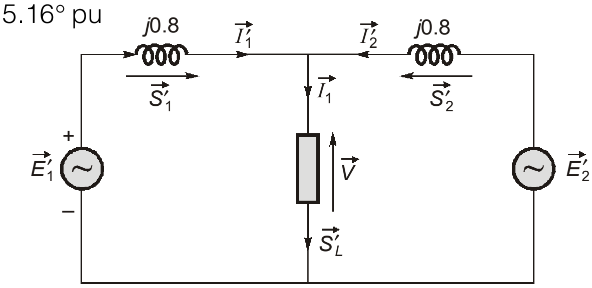
Terminal voltage $V = 1$ p.u. is to be maintained constant.

$$P_1 = \frac{E_1'V}{X_s}\sin\delta_1'$$

$\Rightarrow$ $$0.4 = \frac{1.216\times 1}{0.8}\sin\delta_1'$$

$\Rightarrow$ $$\delta_1' = 15.26^\circ$$

So, $$\vec{E}_1' = E_1'\angle\delta_1' = 1.216\angle 15.16^\circ \text{ pu}$$



$$P_2 = \frac{E_2'V}{X_s}\sin\delta_2'$$

$$0.4 = \frac{1.344\times 1}{0.8}\sin\delta_2'$$

$\Rightarrow$ $$\delta_2' = 13.77^\circ$$

$$\vec{E}_2' = E_2'\angle\delta_2 = 1.344\angle 13.77^\circ \text{ pu}$$

The generator currents, $$\vec{I}_1' = \frac{\vec{E}_1'-\vec{V}}{j0.8} = \frac{1.216\angle 15.16^\circ - 1\angle 0^\circ}{j0.8}$$

$$= 0.45\angle{-28.64^\circ} \text{ pu}$$

Complex power, $$\vec{S}_1' = \vec{V}\vec{I}_1'^{*} = 1\angle 0^\circ \times 0.45\angle 28.54^\circ = 0.4 + j0.22 \text{ pu}$$

Load power and power factor are same,

$$\vec{S}_L = (0.8 + j0.6) \text{ pu}$$

From equation, $$\vec{S}_1+\vec{S}_2 = \vec{S}_L$$

$$\vec{S}_2 = \vec{S}_L - \vec{S}_1 = (0.4 + j0.38) \text{ pu}$$

From, $$\vec{S} = P + jQ \text{ and } (\text{MVA})_B = 100$$

$$P_1 = 0.4 = 40 \text{ MW}$$
$$Q_1 = 0.22 = 22 \text{ MVAR}$$
$$P_2 = 0.4 = 40 \text{ MW}$$
$$Q_2 = 0.38 = 38 \text{ MVAR}$$

The P_1 and P_2 share can be changed by changing mechanical input, according to speed governer characteristics.

Solution : 2

The synchronous motor rated at 50 kW i.e. rated power output = 50 kW.

At 50% of load

The power output, $P_0 = 25$ kW

This motor is driven by another motor which is lossless, i.e. the power input of driving motor is completely transferred to the synchronous motor. When field excitation is not switched on, there will not be any induced emf or current in armature so only losses are rotational losses, i.e. Friction and windage (F and W) losses.

Hence, Rotational loss = 800 W

In short-circuit condition, the motor takes 2500 W at rated current 10 A. As induced emf in this condition is very small (as terminal voltage $V = 0$) the Iron losses are neglegible so only rotational loss and armature Cu losses are supplied. Hence Cu loss at 10 A (rated)

$$= 2500 - 800 = 1700 \text{ W}$$

Full load Cu loss, $P_{CF} \propto I_{afl}^2$

Cu loss at $x = 0.5$ of load, i.e, $I_a = 0.5\, I_{fl}$

$$P_C' = x^2 P_{CF} = \frac{1}{4} P_{CF} = 425 \text{ W}$$

When armature is open circuited, i.e. $I_a = 0$ or Cu loss $= 0$, but excitation is given hence there induced emf and iron losses are present so power in this condition is iron loss and rotational loss. Iron loss and rotational loss = 2500 W.

Hence Iron loss, $P_i = 2500 - 800 = 1700$ W

Total losses at half-full load $= 800 + 425 + 1700 = 2925$ W $= 2.925$ kW

The efficiency,
$$\%\eta = \frac{\text{Output}}{\text{Output + Losses}} \times 100 = \frac{25 \times 100}{25 + 2.925} \times 100 = 89.53\%$$

Solution : 3

Given: $x_d = 9.6\ \Omega$, $x_q = 6\ \Omega$

$$V_t = \frac{6.6}{\sqrt{3}} = 3.81 \text{kV}$$

$$I_a(\text{rated}) = \frac{3.5 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 306.17 \text{ A}$$

I_a at 2.5 MW at 0.8 pf is
$$= \frac{2.5 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3 \times 0.8} = 273.4 \text{ A}$$

$$\phi = \cos^{-1}(0.8) = 36.9° \text{ lag}$$

As we know,
$$\tan\Psi = \frac{V_t \sin\phi + I_a x_q}{V_t \cos\phi + I_a r_a} = \frac{3810 \times 0.6 + 273.4 \times 6}{3810 \times 0.8} = 1.29$$

or, $\Psi = 52.2°$

$$\delta = \Psi - \phi$$
$$= 52.2° - 36.9° = 15.3°$$

Using equation,
$$E_f = V_t \cos\delta + I_d x_d$$
$$= 3810 \cos 15.3° + (I_a \sin\Psi)\, x_d$$
$$= 3810 \cos 15.3° + 273.4 \times \sin 52.2 \times 9.6$$
$$= 5749 \text{ V or } 9.96 \text{ kV (line)}$$

$$\text{Regulation} = \frac{5749 - 3810}{3810} = 50.9\%$$

Power output,
$$P_e = \frac{E_f V_t}{x_d} \sin\delta + V_t^2 \left(\frac{x_d - x_q}{2 x_d x_q} \right) \sin 2\delta$$

Given, $E_f = 0$

$$P_e = V_t^2\left(\frac{x_d - x_q}{2x_d x_q}\right)\sin 2\delta$$

$$P_{e,\max} = V_t^2\left(\frac{x_d - x_q}{2x_d x_q}\right) = (3.81)^2 \times \left(\frac{9.6-6}{2\times 9.6\times 6}\right)\times 10^6 \simeq 0.454 \text{ MW}$$

Solution : 4

$$V_t = \frac{500}{\sqrt{3}} = 288.7 \text{ V}$$

$$E_f = \frac{600}{\sqrt{3}} = 346.4 \text{ V}$$

$$z_s \text{(eqv. star)} = \frac{1}{3}(0.4 + j5) = (0.133 + j1.67)\ \Omega$$

$$= 1.675\angle 85.44°\ \Omega$$

Using equation for maximum output power,

$$P_m(\text{gross})|_{\max} = -\frac{E_f^2 r_a}{z_s^2} + \frac{E_f V_t}{z_s}$$

$$= -\frac{(346.4)^2 \times 0.133}{(1.675)^2} + \frac{346.4 \times 288.7}{1.675}$$

$$= 54 \text{ kW (per phase)}$$

$$P_m(\text{out,net})|_{\max} = 54 \times 3 - 1.2$$

$$= 160.8 \text{ kW} \quad \text{(3-phase)}$$

For maximum power output, $\delta = \theta = 85.44°$

$$I_a = \frac{V_t\angle 0 - E_f\angle -\delta}{z_s\angle\theta}$$

$$= \frac{288.7 - 346.4\angle -85.44°}{1.675\angle 85.44°}$$

$$= 172.4\angle -85.44° - 206.8\angle -170.9°$$

$$= 217.9 - j139.14$$

or, $I_a = 258.5\angle -32.56°$ A

$$\text{p.f.} = \cos 32.56° = 0.842 \text{ lagging}$$

$$P_e(\text{in}) = \sqrt{3} \times 500 \times 258.5 \times 0.842$$

$$= 188.49 \text{ kW}$$

Motor efficiency, $\%\eta = \dfrac{160.8 \times 100}{188.49} = 85.3\%$

Solution : 5

Generator 1,

$$\text{No load frequency} = 51.5 \text{ Hz}$$

$$(S_{p_1}) = \frac{1\text{ MW}}{\text{Hz}}$$

Generator 2,

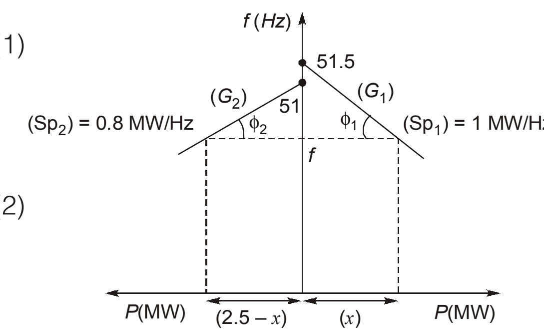
$$\text{No load frequency} = 51 \text{ Hz}$$

$$(S_{p_2}) = \frac{0.8\text{ MW}}{\text{Hz}}$$

(i) Generator 1, $\tan\phi_1 = \dfrac{51.5 - f}{x} = 1$

$\Rightarrow$ $51.5 - f = x$

$\Rightarrow$ $x + f = 51.5$ ...(1)



Generator 2, $\tan\phi_2 = \dfrac{51 - f}{2.5 - x} = \dfrac{1}{0.8}$

$\Rightarrow$ $51 \times 0.8 - 0.8 \times f = 2.5 - x$

$\Rightarrow$ $x - 0.8f = -38.3$ ...(2)

Solving (1) and (2), we get

$$x = 1.61 \text{ MW}$$

$$f = 49.89 \text{ Hz}$$

Frequency of operation = 49.89 Hz

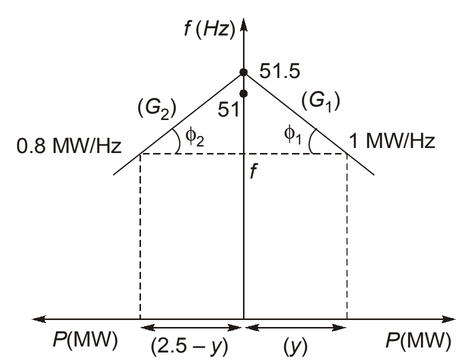
Power supplied by G_1 = 1.61 MW

Power supplied by G_2 = (2.5 – 1.61) = 0.89 MW

(ii) After increased governor set point to 0.5 Hz

Generator 1, $\tan\phi_1 = \dfrac{51.5 - f}{y} = 1$

$\Rightarrow$ $y + f = 51.5$ ...(3)



Generator 2,

$$\tan\phi_2 = \frac{51.5 - f}{2.5 - y} = \frac{1}{0.8}$$

$\Rightarrow$ $51.5 \times 0.8 - 0.8f = 2.5 - y$

$\Rightarrow$ $y - 0.8f = -38.7$ ...(4)

Solving equation (3) and (4), we get

$$y = 1.39 \text{ MW}$$

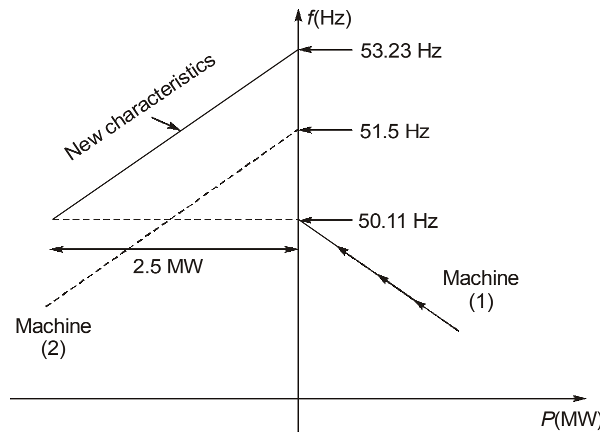
$$f = 50.11 \text{ Hz}$$

System frequency = 50.11 Hz

$$P_{G1} = 1.39 \text{ MW}$$

$$P_{G2} = 1.11 \text{ MW}$$

(iii) For the load transfer from generator '1' to generator '2' without changing system frequency, generator '1' must be operating at no load at frequency $f = 50.11$ Hz as shown in characteristics.



Whereas for generator '2' the total load = 2.5 MW operating at $f = 50.11$ Hz

Now the no load frequency of generator '2'; f_{0_2}

$$\because \qquad 0.8 = \frac{2.5}{f_{0_2} - 50.11} = f_{0_2} = 53.235\text{ Hz}$$

Therefore, generator '2' no load frequency increases from 51.50 Hz to 53.23 Hz by 1.73 Hz and generator '1' no load frequency lower from 51.50 Hz to 50.11 Hz by 1.39 Hz.

Solution : 6

For cylindrical rotor hydro-generator,
Assuming $R_a = 0$,

$$P_{out} = \frac{E_f V_t}{X_s}\sin\delta$$

$$Q_{out} = \frac{(E_f\cos\delta - V_t)V_t}{X_s}$$

$V_t = 1$ p.u., $X_s = j0.725 + j0.11 = j0.835$ p.u.
$E_{f_1} = 1.5\text{ p.u.},\ P_{out_1} = 0.25\text{ p.u.}$

$$P_{out1} = \frac{E_{f_1} V_t}{X_s}\sin\delta_1$$

$$\Rightarrow \qquad 0.25 = \frac{1.5\times 1}{0.835}\times\sin\delta_1 \quad \Rightarrow \quad \delta_1 \simeq 8^\circ$$

$$Q_{out1} = \frac{(E_{f_1}\cos\delta_1 - V_t)V_t}{X_s} = \frac{(1.5\cos 8^\circ - 1)\times 1}{0.835} = 0.5813\text{ p.u.}$$

(a) 100% increase in torque means 100% increase in real power

$$P_{out2} = 0.5\text{ p.u.} = \frac{E_{f_1} V_t}{X_s}\sin\delta_2$$

$$\Rightarrow \qquad 0.5 = \frac{1.5\times 1}{0.835}\sin\delta_2 \quad \Rightarrow \quad \delta_2 = 16.16^\circ$$

$$Q_{out2} = \frac{(E_{f_1}\cos\delta_2 - V_t)V_t}{X_s} = \frac{(1.5\cos 16.16 - 1)\times(1)}{0.835} = 0.5278$$

$$\%\text{ change in } Q_{out} = \frac{Q_{out_2} - Q_{out_1}}{Q_{out_1}} \times 100$$

$$= \frac{0.5278 - 0.5813}{0.5813} \times 100 = -9.20\%$$

(b) $P_{out3} = 0.25$ {Turbine torque is constant}

$$E_{f3} = 1.2\times E_{f_1} = 1.2\times 1.5 = 1.8$$

$$P_{out3} = \frac{E_{f_3}V_t}{X_s}\sin\delta_3$$

$$\Rightarrow \quad 0.25 = \frac{1.8\times 1}{0.835}\sin\delta_3 \quad\Rightarrow\quad \delta_3 = 6.66°$$

$$Q_{out3} = \frac{(E_{f_3}\cos\delta_1 - V_t)V_t}{X_s} = \frac{(1.8\cos 6.66 - 1)\times(1)}{0.835} = 0.9435$$

$$\%\text{ change in } Q_{out} = \frac{Q_{out_3} - Q_{out_1}}{Q_{out_1}} \times 100$$

$$= \frac{0.9435 - 0.5813}{0.5813} \times 100 = 62.30\%$$

Solution : 7

Given: $X_d = 0.1\ \Omega,\quad X_q = 0.07\ \Omega$

Line voltage, $V_L = 400$ V

For delta-connected alternator, phase voltage = line voltage

$\therefore$ $V_p = 400$ V

For delta connection phase current $= \frac{1}{\sqrt{3}} \times$ line current

$$I_a = \frac{1}{\sqrt{3}}\times 1000 = 577.4 \text{ A}$$

Taking V_p as reference phasor.

$$\therefore \quad V_p = V_p\angle 0° = 400\angle 0° \text{ V} = 400 + j0$$

$$I_a = 577.4\angle{-\cos^{-1} 0.8}$$

$$= 577.4\angle{-36.9°} \text{ A}$$

(a) Saliency neglected,

$$E_{fp} = V_p + I_a Z_s$$

$$= V_p + jI_a X_s = V_p + jI_a X_d$$

$$= 400 + (1\angle 90°)(577.4\angle{-36.9°}) \times 0.1$$

$$= 400 + 57.74\angle 90° - 36.9°$$

$$= 400 + 57.74\angle 53.1°$$

$$= 400 + 34.7 + j46.2 = 437.15\angle 6.07° \text{ V}$$

$$E_{fL} = E_{fp} = 437.15 \text{ V}$$

(b) Saliency taken into account,

Since ϕ is taken positive for lagging pf,

$$\phi = \cos^{-1} 0.8 = 36.9°$$

From equation, $$\tan \Psi = \frac{V_p \sin\phi + X_q I_a}{V_p \cos\phi + R_a I_a}$$

If the armature resistance is neglected, $R_a = 0$.

$$\tan \Psi = \frac{V_p \sin\phi + X_q I_a}{V_p \cos\phi}$$

$$= \frac{400 \times 0.6 + 0.07 \times 577.4}{400 \times 0.8} = 0.8763$$

$$\Psi = 41.2°$$

$$\delta = \Psi - \phi = 41.2° - 36.9 = 4.3°$$

$$I_d = I_a \sin\Psi = 577.4 \sin 41.2° = 380.3 \text{ A}$$

$$E_{fp} = V_p \cos\delta + X_d I_d$$

$$= 400 \cos 4.3° + 0.1 \times 380.3 = 436.9 \text{ V}$$

$$E_{fl} = E_{fp} = 436.9 \text{ V}$$

Solution : 8

$$V_L = 6600 \text{ V}$$

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{6600}{\sqrt{3}} = 3810.5 \text{ V}$$

$$\text{Rated kVA} = 1200 \text{ kVA}$$

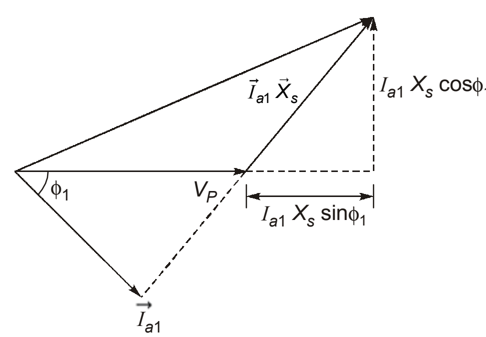
For rated current I_a,

$$\frac{\sqrt{3} \times 6600 \times I_a}{1000} = 1200$$

$$I_a = \frac{1200 \times 1000}{\sqrt{3} \times 6600} = 105 \text{ A}$$

For synchronous reactance, $X_S = 0.25$ pu

$$\frac{I_a X_S}{V_P} \times 100 = 25$$



$$X_S = 9.073 \, \Omega$$

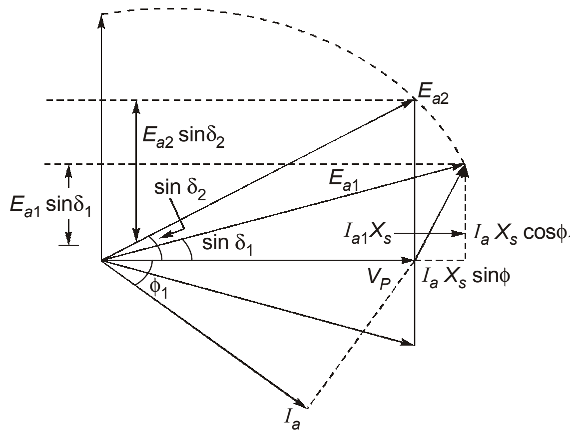
$$E_{a_1}^2 = (V_P + I_{a1} X_S \sin\phi_1)^2 + (I_{a1} X_S \cos\phi_1)^2$$

$$= (3810.5 + 105 \times 9.073 \times 0.6)^2 + (105 \times 9.073 \times 0.8)^2$$

$$E_{ai} = 4448 \text{ V}$$

(a) When power factor is unity and supply gradually increased

$$P = \frac{VE_a \sin\delta}{X_S} = VI_a \cos\phi$$



As P is increasing, $E_a \sin\delta = I_a \cos\phi$ will increase.

As E_a is constant, locus of E_a will be circle.

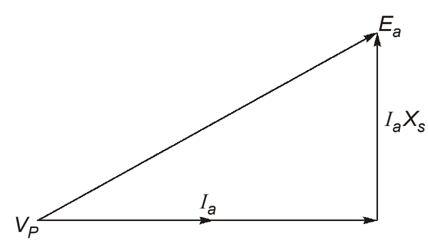
At unity power factor:

$$E_a^2 = V_P^2 + I_a X_S^2$$

$$I_a^2 X_S^2 = (4488)^2 - (3810.5)^2$$

$$I_a X_S = 2371.12 \text{ V}$$

$$I_a = \frac{2371.12}{9.083} = 261.34 \text{ A}$$

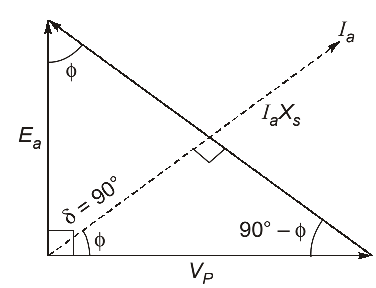


Power output at unity power factor

$$= \frac{\sqrt{3} V_L I \cos\phi}{1000} = \frac{\sqrt{3} \times 6600 \times 261.34 \times 1}{1000}$$

$$= 2987.5 \text{ kW}$$

(b) For maximum load without losing synchronism



Since, $$P = \frac{E_a V_t \sin\delta}{X_S}$$

$$\sin\delta = 1$$

Maximum load, $$\delta = 90°$$

$$E_a^2 + V_P^2 = (I_a X_S)^2$$

$$I_a X_S = 5887.44$$

$$I_a = \frac{5887.4}{9.073} = 648.89 \text{ A}$$

$$\tan\phi = \frac{V_P}{E_a} = \frac{3810.5}{4488} = 0.8490$$

$$\phi = 40.33°$$

$$\cos\phi = 0.762 \quad \text{(leading) as can be seen from phaser}$$

$$\text{Maximum power output} = \frac{\sqrt{3} \times 6600 \times 648.89 \times 0.762}{1000} = 5652.37 \text{ kW}$$

■■■■

5 Single-phase Motors & Speical Machines and Energy Conversion System

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(d)
4.	(c)
5.	(c)
6.	(a)

LEVEL 2 Objective Questions

7.	(188.56)
8.	(130.50)
9.	(c)
10.	(12)
11.	(295.5)

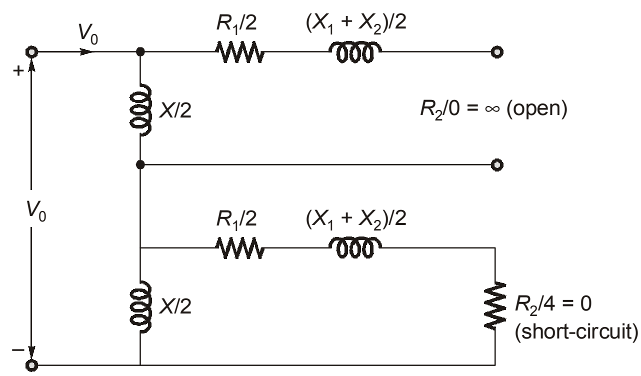
■■■■

LEVEL 3 Conventional Questions

Solution : 1

Parameters of the circuit model are calculated using both no-load as well as rotor-blocked tests.

(i) **No-load test:** Assuming the slip to be zero, the circuit model on no-load is drawn in figure with magnetizing reactance at input terminals.



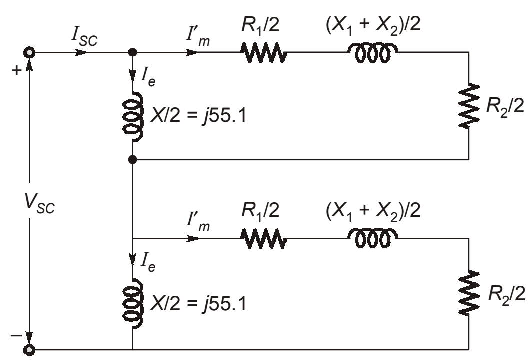
$(R_2 << 4)$

Since the backward circuit is short-circuited for practical purposes, as X being magnetizing reactance is much larger,

$$\frac{X}{2} = \frac{215}{3.9} = 55.1\,\Omega$$

Rotational loss, $P_0 = 185$ W

(ii) **Rotor-blocked test (s = 1):** The circuit model on rotor-blocked test in shown in figure,



$$390 = 85 \times 9.8 \times \cos\phi_{SC}$$

or, $\phi_{SC} = 62°$ lagging

with reference to figure,

$$\vec{I}_e = \frac{V_{SC}}{jX} = \frac{85}{j2 \times 55.1} = -j0.77 \text{ A}$$

$$\vec{I}'_m = \vec{I}_{SC} - \vec{I}_e$$

$$= 9.8\angle{-62°} - (-j0.77)$$

$$= 4.6 - j7.88 = 9.13\angle{-59.7°} \text{ A}$$

$$\vec{Z}'_f = (R_1 + R_2) + j(X_1 + X_2)$$

$$\frac{V_{SC}}{I'_m} = \frac{85}{9.13\angle -59.7^\circ}$$

$$= 9.31\angle 59.7^\circ = (4.7 + j8.04)\ \Omega$$

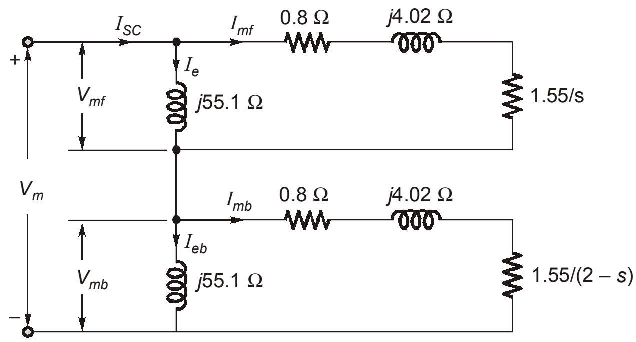
$$R_1 + R_2 = 4.7\ \Omega$$

$$R_1 = 1.6\ \Omega \quad \text{(Given)}$$

$$R_2 = 3.1\ \Omega$$

$$X_1 + X_2 = 8.04\ \Omega$$

The circuit model with parameter values is drawn in figure.



Solution : 2

Given that :

Main winding impedance,

$$\vec{Z}_m = (4.8 + j\,8.6)\ \Omega = 9.849\ \angle\ 60.8324^\circ\ \Omega$$

Main winding current, I_m lags behind the applied voltage V by 60.8324°.

Auxiliary winding impedance,

$$\vec{Z}_a = (22.5 + j\,16.2)\ \Omega = 27.725\ \angle\ 35.754^\circ\ \Omega$$

Since phase angle between auxiliary winding current I_a and main winding current I_m is 90°; so auxiliary winding current I_a must lead the applied voltage by (90° – 60.8324°) or 29.1676°

If X_c is the capacitive reactance of the capacitor connected in series with the auxiliary winding then impedance of the auxiliary winding will be given as

$$\vec{Z}_a = 22.5 + j\,(16.2 - X_c)$$

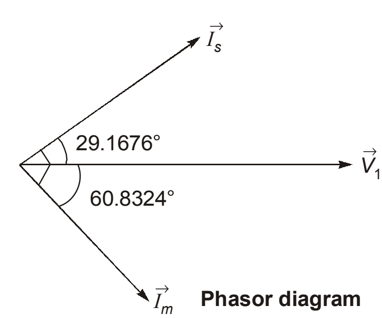
For auxiliary winding,

$$\tan\phi_a = \frac{16.2 - X_c}{22.5}$$

$$X_c = 16.2 - 22.5\tan\phi_a$$

$$= 16.2 - 22.5\tan(-29.1676^\circ)$$

$$X_c = 28.758\ \Omega$$



Phasor diagram

Capacitance of capacitor,

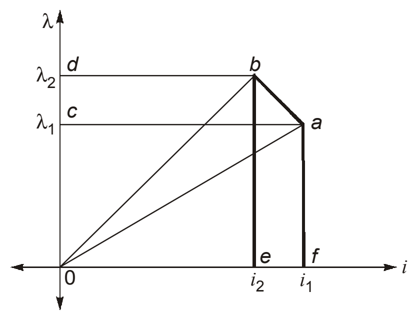
$$C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi \times 50 \times 28.758} = 110.685\ \mu\text{F}$$

Solution : 3

Electrical energy input, $\Delta W_e = \text{Area of } cabd = (\lambda_2 - \lambda_1)i_2 + \frac{1}{2}(\lambda_2 - \lambda_1) \times (i_1 - i_2)$

Increase in field energy, $\Delta W_f = \text{Area of } obd - \text{Area of } oca$

$$= \frac{1}{2}\lambda_2 i_2 - \frac{1}{2}\lambda_1 i_1$$



Mechanical output, $\Delta W_m = \Delta W_e - \Delta W_f$

$$= (\lambda_2 - \lambda_1)\, i_2 + \frac{1}{2}(\lambda_2 - \lambda_1) \times (i_1 - i_2) - \frac{1}{2}\lambda_2 i_2 - \frac{1}{2}\lambda_1 i_1 = \frac{1}{2}(\lambda_2 i_1 - \lambda_1 i_2)$$

We know, $L(x) = \dfrac{\mu_0 N^2 A}{2x}$

where, $R = \dfrac{2x}{\mu A}$

$$L(x) = \frac{4\pi \times 10^{-7} \times (1000)^2 \times (0.05)^2}{2x} = \frac{\pi \times 10^{-3}}{2x}$$

$$L(0.01) = 0.157 \text{ H}$$

$$L(0.005) = 0.314 \text{ H}$$

$$\lambda_1 = L(0.01)i_1 = 0.157 \times 2 = 0.314 \text{ Wb turns}$$

$$\lambda_1 = L(0.005)i_2 = 0.314 \times 0.15 = 0.471 \text{ Wb turns}$$

$$\text{Mechanical output} = \frac{1}{2}(0.471 \times 2 - 0.314 \times 1.5) = 0.2355\text{J}$$

Solution : 4

$$\mathfrak{R} = \frac{x}{\mu_0 A}$$

$$\lambda = N\phi = \frac{N^2 i}{R}\left(\frac{N^2 \mu_0 A}{x}\right) i$$

$$L\,(x) = \frac{N^2 \mu_0 A}{x} = \frac{a}{x}$$

(a) $W_f'(i, x) = \dfrac{1}{2}L(x)i^2$

$$F_f = \frac{\partial W_f'}{\partial x} = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = -\frac{1}{2} i^2 \left(\frac{a}{x^2}\right)$$

$$F_f = -\frac{1}{2} I^2 \left(\frac{a}{x^2}\right) \cos^2 \omega t$$

$$F_f(av) = -\frac{1}{4} I^2 \left(\frac{a}{x^2}\right) \quad \text{(in a direction to reduce } x\text{)}$$

(b)
$$v = ri + L\frac{di}{dt}$$

$$V(j\omega) = (r + j\omega L)\, I(j\omega)$$

or,
$$I(j\omega) = \frac{V(j\omega)}{(r + j\omega L)}$$

$\therefore$
$$i = -\frac{1}{4} \frac{V}{\sqrt{r^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{r}\right)$$

$$F_f(av) = -\frac{1}{4}\left(\frac{V}{r^2 + \omega^2 L^2}\right)\left(\frac{a}{x^2}\right)$$

Substituting $L(x) = a/x$, we get

$$F_f(av) = -\frac{aV^2}{(r^2 x^2 + a^2 \omega^2)}$$

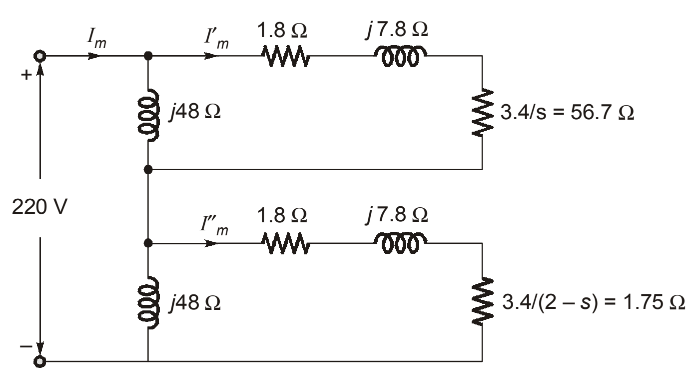
Solution : 5

Slip,
$$s = \frac{1000 - 940}{1000} = 0.06$$

The circuit model is drawn in figure,

$$\vec{Z}_f\,(\text{total}) = j48 \,||\, (1.8 + 56.7 + j7.8) = j48 \,||\, (58.5 + j7.8)$$
$$= 35\angle 54° = 20.6 + j28.3\ \Omega$$

$$\vec{Z}_b\,(\text{total}) = j48 \,||\, (1.8 + 1.75 + j7.8) = j48 \,||\, (3.55 + j7.8)$$
$$= 7.36\angle 69.1° = 2.63 + j6.88\ \Omega$$



$$\vec{Z}\,(\text{total}) = (20.6 + j28.3) + (2.63 + j6.88)$$
$$= 23.23 + j35.18 = 42.16\angle 56.6°\ \Omega$$

$$\vec{I}_m = \frac{220}{42.16\angle 56.6°} = 5.22\angle -56.6° \text{ A}$$

$$I_L = I_m = 5.22 \text{ A},$$

$$\text{p.f.} = \cos 56.6° = 0.55 \text{ lagging}$$

$$I'_m = 5.22 - \angle 56.6° \times \frac{j48}{58.5 + j55.8} = 3.1\angle -10° \text{ A}$$

$$I''_m = 5.22\angle -5.66° \times \frac{j48}{3.55 + j55.8} = 4.48\angle -53° \text{ A}$$

$$n_s = 1000 \text{ rpm}, \quad \omega_s = 104.7 \text{ rad/sec.}$$

$$T = \frac{1}{104.7}[(3.1)^2 \times 56.7 - (4.48)^2 \times 1.75]$$

$$= 4.87 \text{ Nm}$$

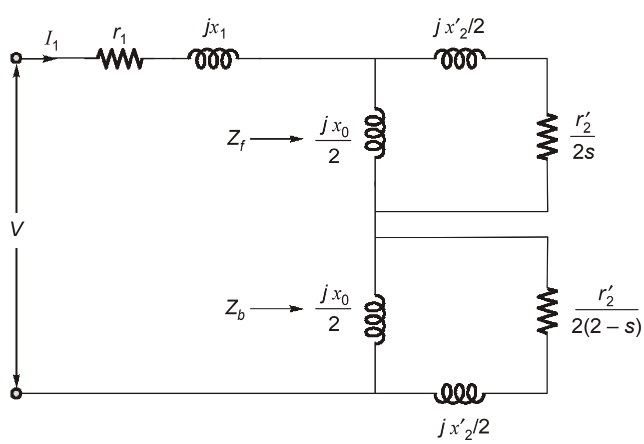
$$P_m = 104.7\,(1 - 0.06) \times 4.87 = 479.3 \text{ W}$$

$$P_{out} = 479.3 - 75 = 404.3 \text{ W}$$

$$P_{in} = 220 \times 5.22 \times 0.55 = 631.6 \text{ W}$$

Efficiency, $$\%\eta = \frac{404.3}{631.6} = 64\%$$

Solution : 6



From circuit diagram we can observe that,

$$Z_f = \left(\frac{jX_0}{2}\right) \Bigg|\Bigg| \left(\frac{r'_2}{2s} + j\frac{x'_2}{2}\right) = \left(j\frac{275}{2}\right) \Bigg|\Bigg| \left(\frac{13.8}{2 \times 0.06} + j\frac{14.3}{2}\right)$$

$\Rightarrow$ $$Z_f = 85.75\angle 42.04\,\Omega$$

$$Z_b = \left(\frac{jX_0}{2}\right) \Bigg|\Bigg| \left(\frac{r'_2}{2(2-s)} + j\frac{x'_2}{2}\right) = \left(j\frac{275}{2}\right) \Bigg|\Bigg| \left(\frac{13.8}{2 \times (2 - 0.06)} + j\frac{14.3}{2}\right)$$

$$= 7.58\angle 64.96°\,\Omega$$

Taking supply voltage as the reference,

$$V = 230\angle 0°\text{ V}$$

$$\text{Input current} = I = \frac{V}{(r_1 + jX_1) + Z_f + Z_b}$$

$$\Rightarrow \quad I = \frac{230\angle 0°}{(11.4 + j14.3) + 85.75\angle 42.04° + 7.58\angle 64.96°} = 2.07\angle{-45.10°} \text{ Amp.}$$

Input power factor

$$\Rightarrow \quad \cos\phi = \cos(45.10°) = 0.7058 \quad \text{(lagging)}$$

$$\text{Input power} = VI\cos\phi = 230 \times 2.07 \times 0.7058 = 336.03 \text{ Watts}$$

Solution : 7

Method-1:

$$N = 1440 \text{ rpm}, \; s = \frac{1500 - 1440}{1500} = 0.04$$

$$\omega_m = 150.79 \text{ rad/sec}$$

Air gap power for forward field = P_{gf} = 200 watt

Air gap power for backward field = P_{gb} = 21 watt

Rotational loss = 41 watt

Rotor copper-loss corresponding to forward field = sP_{gf}

Rotor copper-loss corresponding to backward field = $(2 - s)P_{gb}$

Total rotor copper loss = $sP_{gf} + (2 - s)P_{gb}$

Electrical power converted to gross mechanical form is

$$P_m = (1 - s)\omega_s T = \omega_m T = (1 - s)(P_{gf} - P_{gb})$$

it can be also written as, $P_m = (1 - s)P_{gf} + [1 - (2 - s)]P_{gb}$

or, $P_m = (1 - 0.04) \times 200 + [1 - (2 - 0.04)] \times 21$

$$P_m = 192 + (-20.16) = 171.84 \text{ watt}$$

$$P_{\text{shaft}} = P_m - P_{\text{rot.loss}} = (171.84 - 41) \text{ watt} = 130.84 \text{ watt.}$$

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_m}$$

$$T_{\text{shaft}} = \frac{130.84}{150.79} = 0.8676 \text{ Nm}$$

Method-2:

$$\omega_m = 150.79 \text{ rad/sec}$$

$$T_{fwd} = \frac{P_{gf}}{\omega_s} = \frac{200}{157.08} = 1.2732$$

$$T_{bwd} = \frac{P_{gb}}{\omega_s} = \frac{21}{157.08} = 0.1337$$

$$\text{Net developed torque} = T_{fwd} - T_{bwd}$$

$$= 1.2732 - 0.1337 = 1.1395 \text{ Nm}$$

$$\text{Mech. power developed} = T_{\text{net}} \times \omega_m$$

$$P_{dev} = 1.1395 \times 150.79 = 171.825 \text{ Watt}$$

$$P_{\text{shaft}} = P_{dev} - P_{\text{rotational loss}}$$

$$= 171.825 - 41 = 130.825 \text{ Watt}$$

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_m} = \frac{130.825}{150.79} = 0.8676 \text{ Nm}$$

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