2020

RANK Improvement WORKBOOK

Answer key and Hint of Objective & Conventional *Questions*

Electrical Engineering

Electrical & Electronic Measurements

MADE EASY Publications

1 Basics of Measurement Systems, Error Analysis

LEVEL 1 Objective Questions

1.	(d)
2.	(a)
3.	(c)
4.	(a)
5.	(b)
6.	(a)
7.	(c)
8.	(b)

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LEVEL 2 Objective Questions

11.	(a)
12.	(c)
13.	(c)
14.	(b)
15.	(a)
16.	(–3.9)
17.	(d)

LEVEL 3 Conventional Questions

Solution : 1

Given, $P_i = 3650$ W, $P_0 = 3385$ W

Uncertainties, $W_{Pi} = \pm 10$ W

$W_{P0} = \pm 10$ W

Losses in transformer, $P_L = P_i - P_0$

$P_L = 3650 - 3385 = 265$ W

$$\therefore \quad \frac{\partial P_L}{\partial P_i} = 1,\ \frac{\partial P_L}{\partial P_0} = -1$$

$$\therefore \quad \text{Uncertainty in loss} = \pm\sqrt{\left(\frac{\partial P_L}{\partial P_i}\right)^2 W_{Pi}^2 + \left(\frac{\partial P_L}{\partial P_0}\right)^2 W_{P0}^2} = \pm\sqrt{(1)^2 10^2 + (-1)^2 10^2} = \pm 10\sqrt{2}\,\text{W}$$

$$\%\ \text{Uncertainty in loss} = \frac{\pm 10\sqrt{2}}{265} \times 100 = \pm 5.34\%$$

Efficiency, $\eta = \dfrac{P_0}{P_i}$

$$\frac{\partial \eta}{\partial P_i} = -\frac{P_0}{P_i^2} = -\frac{3385}{(3650)^2} = -2.54 \times 10^{-4}$$

$$\frac{\partial \eta}{\partial P_0} = \frac{1}{P_i} = 2.74 \times 10^{-4}$$

$$\text{Uncertainty in } \eta = \pm\sqrt{\left(\frac{\partial \eta}{\partial P_i}\right)^2 W_{Pi}^2 + \left(\frac{\partial \eta}{\partial P_0}\right)^2 W_{P0}^2}$$

$$= \pm\sqrt{(2.54 \times 10^{-4})^2 (10)^2 + (2.74 \times 10^{-4})^2 (10)^2}$$

$$= \pm\sqrt{645.2 \times 10^{-8} + 750.8 \times 10^{-8}} = \pm 37.36 \times 10^{-4}$$

$$\eta = \frac{P_0}{P_i} = \frac{3385}{3650} = 0.927$$

$$\%\ \text{Uncertainty in efficiency} = \pm\frac{37.36 \times 10^{-4}}{0.927} \times 100\% = \pm 40.30 \times 10^{-2}\ \%$$

$$\%\ \text{Uncertainty in efficiency} = \pm 0.4030\%$$

Solution : 2

The voltage drop across R_b without the voltmeter connected is calculated using the voltage equation.

$$VR_b = \frac{R_b}{R_a + R_b} \times V = \frac{5k}{45k + 5k} \times 50 = \frac{50 \times 5k}{50k} = 5\ \text{V}$$

On the 5 V range $R_m = S \times \text{range} = 20\ \text{k}\Omega \times 5\ \text{V} = 100\ \text{k}\Omega$

$$\therefore \quad R_{eq} = \frac{R_m \times R_b}{R_m + R_b} = \frac{100k \times 5k}{100k + 5k} = \frac{500k^2}{105k} = 4.76\ \text{k}\Omega$$

The voltmeter reading is $VR_b = \dfrac{R_{eq}}{R_a + R_{eq}} \times V = \dfrac{4.76k}{45k + 4.76k} \times 50 = 4.782\text{ V}$

The % error on the 5 V range is

$$\%\text{Error} = \frac{\text{Actual voltage} - \text{Voltage reading in meter}}{\text{Actual voltage}} = \frac{\text{Measured} - \text{True}}{\text{True}} \times 100$$

$$= \frac{4.782\text{ V} - 5\text{ V}}{5\text{ V}} \times 100 = -\frac{0.218\text{ V}}{5\text{ V}} \times 100 = -4.36\%$$

on 10 V range, $R_m = S \times \text{range} = 20\text{ k}\Omega/\text{V} \times 10\text{ V} = 200\text{ k}\Omega$

$\therefore$ $R_{eq} = \dfrac{R_m \times R_b}{R_m + R_b} = \dfrac{200k \times 5k}{200k + 5k} = 4.87\text{ k}\Omega$

The voltmeter reading is, $VR_b = \dfrac{R_{eq}}{R_{eq} + R_a} \times V = \dfrac{4.87k}{4.87\text{ k} + 45\text{ k}} \times 50 = 4.88\text{ V}$

The % error on the 10 V range $= \dfrac{4.88\text{ V} - 5\text{ V}}{5\text{ V}} \times 100 = -2.4\%$

On 30 V range, $R_m = S \times \text{range} = 20\text{ k}\Omega/\text{V} \times 30\text{ V} = 600\text{ k}\Omega$

$\therefore$ $R_{eq} = \dfrac{R_m \times R_b}{R_m + R_b} = \dfrac{600\text{ k} \times 5\text{ k}}{600\text{ k} + 5\text{ k}} = 4.95\text{ k}\Omega$

The voltmeter reading on the 30 V range

$$VR_b = \frac{R_{eq}}{R_{eq} + R_a} \times V = \frac{4.95\text{ k}}{45\text{ k} + 4.95\text{ k}} \times 50 = 4.95\text{ V}$$

The % error on the 30 V range $= \dfrac{4.95\text{ V} - 5\text{ V}}{5\text{ V}} \times 100 = \dfrac{-0.05}{5\text{ V}} \times 100 = -1\%$

In the above example, the 30 V range introduces the least error due to loading. However, the voltage being measured causes only a 10% full scale deflection, whereas on the 10 V range the applied voltage causes approximately a one third of the full scale deflection.

Solution : 3

We have, $T \propto M^p E^q Z^t$

Therefore, we can write, $T = kM^p E^q Z^t$

where, K = a constant (a dimensionless quantity).

Writing the dimensions of various quantities:

$$\text{Torque} = \frac{\text{Power}}{\text{Angular velocity}}$$

$\therefore$ $$\text{Dimension of torque} = \frac{[ML^2T^{-3}]}{[T^{-1}]} = [ML^2T^{-2}]$$

$$[\text{Mutual inductance}] = [\mu L]$$

$$[\text{Emf}] = [E] = [\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]$$

$$[\text{Impedance}] = [Z] = [\mu L T^{-1}]$$

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Substituting the dimensions of various quantities in the expression for torque, we have:

$$[ML^2T^{-2}] = [\mu L]^p[\mu^{1/2} M^{1/2} L^{3/2} T^{-2}]^q [\mu LT^{-1}]^t$$
$$= \left[\mu^{p+q/2+t} + M^{q/2} + L^{p+(3/2)q+t} + T^{-2q-t}\right]$$

In order that the two sides should balance dimensionally,

$$p+\frac{q}{2}+t = 0, \qquad \frac{q}{2}=1$$
$$p+\frac{3}{2}q+t = 2, \quad \text{and} \quad -2q-t=-2$$

From above, $p = 1,\ q = 2$ and $t = -2$

Therefore, the equation is $T \propto ME^2Z^{-2} \propto \dfrac{ME^2}{Z^2}$.

Solution : 4

$$R = 0.0105\ \Omega$$

∵ True value of resistor is 0.3% lower than 0.0105 Ω

∴ True value of resistance,

$$R' = R\left(1-\frac{0.3}{100}\right) = 0.0105\left(1-\frac{0.3}{100}\right) = 0.0104685\ \Omega$$

Current, $I = 30.4$ A

∵ True current is 1.2% lower than I.

∴ True current, $I' = 30.4\left(1-\dfrac{1.2}{100}\right) = 30.0352$ A

True power, $P' = I'^2R' = (30.0352)^2 \times 0.0104685$
$= 9.44377\ \Omega$

But power calculated by freshman,

$$P = I^2 R = (30.4)^2 \times 0.0105 = 9.70368\ \Omega$$

$$\frac{P'}{P}\times 100 = \frac{9.44377}{9.70368}\times 100 = 97.32\%$$

Solution : 5

(i) Arithmetic mean,

$$\bar{R} = \frac{147.2+147.4+147.9+148.1+147.1+147.5+147.6+147.4+147.6+147.5}{10}$$
$$= \frac{1475.3}{10} = 147.53\ \Omega$$

(ii)
$$d_1 = R_1 - \bar{R} = 147.2 - 147.53 = -0.33\ \Omega$$
$$d_2 = R_2 - \bar{R} = 147.4 - 147.53 = -0.13\ \Omega$$
$$d_3 = R_3 - \bar{R} = 147.9 - 147.53 = 0.37\ \Omega$$
$$d_4 = R_4 - \bar{R} = 148.1 - 147.53 = 0.57\ \Omega$$
$$d_5 = R_5 - \bar{R} = 147.1 - 147.53 = -0.43\ \Omega$$
$$d_6 = R_5 - \bar{R} = 147.5 - 147.53 = -0.03\ \Omega$$

$$d_7 = R_7 - \bar{R} = 147.6 - 147.53 = 0.07\ \Omega$$
$$d_8 = R_8 - \bar{R} = 147.4 - 147.53 = -0.13\ \Omega$$
$$d_9 = R_9 - \bar{R} = 147.6 - 147.53 = 0.07\ \Omega$$
$$d_{10} = R_{10} - \bar{R} = 147.5 - 147.53 = -0.03\ \Omega$$

Average deviation,

$$\bar{D} = \frac{|d_1| + |d_2| + \cdots + |d_{10}|}{10} = 0.216\ \Omega$$

(iii) Standard deviation,

$$\sigma = \sqrt{\frac{\Sigma d^2}{n-1}} \qquad (n < 10)$$

$$\sigma = \sqrt{\frac{(0.33)^2 + (0.13)^2 + (0.37)^2 + (0.57)^2 + (0.43)^2 + (0.03)^2 + (0.07)^2 + (0.13)^2 + (0.07)^2 + (0.03)^2}{10-1}}$$

$$= \sqrt{\frac{0.801}{9}} = 0.2983\ \Omega$$

Solution : 6

Energy registered by the energy meter in 40 revolutions

$$= \frac{40}{1200}\ \text{kW} = \left(\frac{40}{1200}\right) \times 1000 \times 3600$$
$$= 120 \times 10^3\ \text{Ws.}$$

Power registered by wattmeter at 240 V and 5 A

$$= 240 \times 5 = 1200\ \text{W}$$

Range of wattmeter = 1200 W

Scale division = 500

$$\therefore \quad 1\ \text{scale division} = \frac{1200}{50} = 2.4\ \text{W}$$

Observational error of wattmeter = $\pm 0.1 \times 2.4 = \pm 0.24$ W

$$\text{Constructional error of wattmeter} = \left(\frac{0.05}{100} \times 1200\right) \pm 0.6\ \text{W}$$

Total error of wattmeter = $\pm 0.24 \pm 0.6 = \pm 0.84$ W

Observational error of stop watch = $\pm$ 0.05s

Construction error of stop watch = $\pm$0.01s

Total error of stop watch = $99.8 \pm \times 0.06$s

Energy obtained from readings of stop watch and wattmeter = $(1200 \pm 0.84)\,(99.8 \pm 0.06)$ Ws.

$$\%\ \text{Limiting error} = \frac{A_m - A_T}{A_T} \times 100$$

$$A_T = \text{Power} \times \text{Time} = 1200 \times 100 = 120 \times 10^3$$
$$A_m = (1200 \pm 0.84)\,(99.8 \pm 0.06)$$

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$$\text{+ve limiting error} = \frac{(1200+0.84)\,(99.8+0.06)-120\times10^3}{120\times10^3}\times100$$
$$= -0.07\%$$
$$\text{–ve limiting error} = \frac{(1200-0.84)\,(99.8-0.06)-120\times10^3}{120\times10^3}\times100$$
$$= -0.3298\%$$

Solution : 7

(a) When all the components have zero error,

$$L = 160\ \mu\text{H} = 160\times10^{-6}\ \text{H}$$

and $$C = 160\ \text{pF} = 160\times10^{-12}\ \text{F}$$

∴ Resonant frequency,
$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{160\times10^{-6}\times160\times10^{-12}}}$$
$$= 1\ \text{MHz}$$

(b) When the components are +10%,

$$C = 160+0.1\times160 = 176\ \text{pF},$$

and $$L = 160+0.1\times160 = 176\ \mu\text{H}$$

∴ $$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{176\times10^{-6}\times176\times10^{-12}}} = 0.9\ \text{MHz}$$

Hence, $$\text{Error} = \frac{0.9-1.0}{1.0} = -10\%$$

This is a case of known errors and can be solved by

$$f_r = \frac{1}{2\pi}L^{-1/2}C^{-1/2}$$

∴ Relative error in f_r is
$$\frac{\delta f_r}{f_r} = \left(-\frac{1}{2}\frac{\delta L}{L}-\frac{1}{2}\frac{\delta C}{C}\right) = -\frac{1}{2}(0.1+0.1) = -0.1 = -10\%$$

(c) When the components are –10%,

$$C = 160-0.1\times160 = 144\ \text{pF},$$

and $$L = 160-0.1\times160 = 144\ \mu\text{H}$$

∴ $$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{144\times10^{-6}\times144\times10^{-12}}} = 1.1\ \text{MHz}$$

Hence, $$\text{Error} = \frac{1.1-1.0}{1.0}\times100 = +10\%$$

■■■■

2 Indicating Instrument, Power & Energy Measurement

LEVEL 1 Objective Questions

1. (c)
2. (d)
3. (b)
4. (d)
5. (d)
6. (b)
7. (b)
8. (b)
9. (c)

LEVEL 2 Objective Questions

10. (3.20)
11. (c)
12. (c)
13. (d)
14. (125.33)
15. (1.20)
16. (a)
17. (71.2)
18. (21.17)
19. (224)
20. (c)
21. (c)
22. (b)
23. (b)

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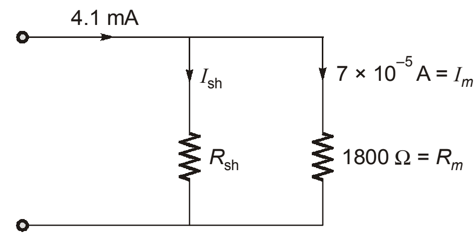
LEVEL 3 Conventional Questions

Solution : 1

(a) 5 mA corresponds to 100 μA

For 3.5 mA

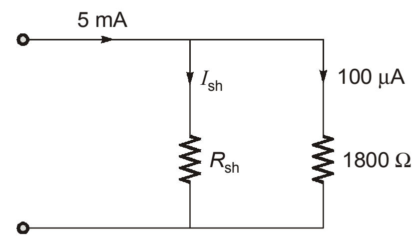
$$\Rightarrow \quad \frac{100\times10^{-6}\times3.5}{5} = 7\times10^{-5}\text{ A} = I_m$$



Actual current is 4.1 mA

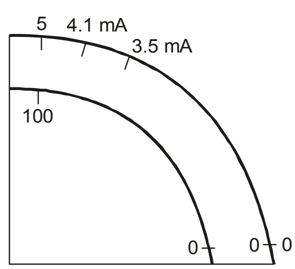
$$I_{sh} = (4.1\times10^{-3} - 7\times10^{-5})$$
$$= 4.03\times10^{-3}$$
$$R_{sh}\, I_{sh} = R_m\cdot I_m$$
$$R_{sh} = \frac{1800\times7\times10^{-5}}{4.03\times10^{-3}} = 31.26\ \Omega$$

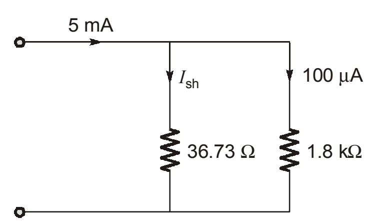
(b) For 5 mA range:



$$R_{sh} = \frac{R_m}{(m-1)} = \frac{R_m}{\left(\frac{I}{I_m}-1\right)} = \frac{1800}{\left(\frac{5\times10^{-3}}{100\times10^{-6}}-1\right)} = 36.73\ \Omega$$

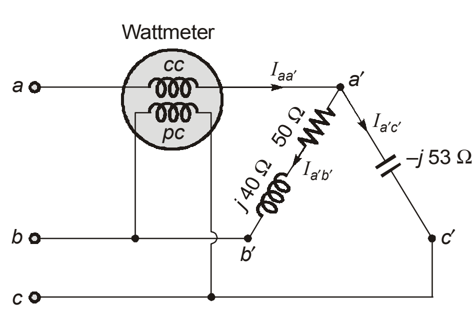
Now the actual value of reading is 4.1 mA.





$$I_m = 100\ \mu\text{A},\ I = 5\text{ mA}$$
$$m = \frac{I}{I_m} = \frac{5\times10^{-3}}{100\times10^{-6}} = 50$$
$$R_m = 1.8\text{ k}\Omega$$
$$R_{sh} = \frac{R_m}{(m-1)} = \frac{1800}{(50-1)} = 36.73\ \Omega$$

Solution : 2



Given that, $V_{ab} = 400\angle 0° \text{ V}$

$V_{bc} = 400\angle -120° \text{ V}$

and $V_{ca} = 400\angle 120° \text{ V}$

or, $V_{ac} = -400\angle 120° \text{ V}$

Also, impedance, $Z_{a'b'} = 50 + j40 = 64.03\angle 38.66° \ \Omega$

and impedance, $Z_{a'c'} = -j53 = 53\angle -90° \ \Omega$

Then, $$I_{a'b'} = \frac{V_{ab}}{Z_{a'b'}} = \frac{400\angle 0°}{64.03\angle 38.66°} = 6.247\angle -38.66° \text{ A}$$

and $$I_{a'c'} = \frac{V_{ac}}{Z_{a'c'}} = \frac{-400\angle 120°}{53\angle -90°} = -7.547\angle 210° \text{ A}$$

Then, $$\begin{aligned} I_{aa'} &= I_{a'b'} + I_{a'c'} \\ &= 6.247\angle -38.66° - 7.547\angle 210° \\ &= 11.415\angle -0.647° \text{ A} \end{aligned}$$

Thus, the current through current coil (cc) of the wattmeter is 11.415 A.

The potential coil of wattmeter is connected between *b* and *c*.

So, the voltage across the wattmeter is

$$V_{bc} = 400\angle -120°$$

Reading of the wattmeter, $$\begin{aligned} W &= V_{bc} \times I_{aa'} \times \cos\phi \\ &= 400 \times 11.415 \cos[-120° - (-0.647°)] = 2238.203 \text{ W} \end{aligned}$$

Solution : 3

Phantom or fictious loading: When the current rating of a meter is high, a test with actual loading arrangements would involve a considerable waste of power. In order to avoid this 'Phantom' or 'Fictitious' loading is done.

Phantom loading consists of supplying the pressure coil circuit from a circuit of required normal voltage, and the current coil circuit from a separate low voltage supply. It is possible to circulate the rated current through the current circuit with a low voltage supply as the impedance of this circuit is very low. With this arrangement the total power supplied for the test is that due to small pressure coil current at normal. Voltage, plus that due to the current circuit therefore required for testing the meter with phantom loading is comparatively very small.

$$\begin{aligned} \text{Actual kWh consumed} &= VI\cos\phi \times t \times 10^{-3} \\ &= 230 \times 12 \times 0.8 \times 1/2 \times 10^{-3} = 1.104 \text{ kWh} \end{aligned}$$

$$\text{Registered kWh} = \frac{\text{No. of revolutions done}}{\text{Meter constant in revolution/kWh}} = \frac{1150}{1200} = 0.958 \text{ kWh}$$

$$\therefore \quad \text{Error registration} = \frac{1.104 - 0.958}{1.104} \times 100\% = 13.22\%$$

$$\begin{aligned} \text{Speed of the disc} &= \text{Energy supplied per minute} \times \text{Meter constant} \\ &= \frac{1.104}{60} \times 1200 = 22.08 \text{ rev/min.} \end{aligned}$$

$$\text{Revolution registered/min} = \frac{1150}{30} = 38.33$$

$$\%\text{ error} = \frac{38.33 - 22.08}{22.08} \times 100 = 73.6\% \text{ (fast)}$$

It can be rectified by bringing braking magnet near to the centre of disc.

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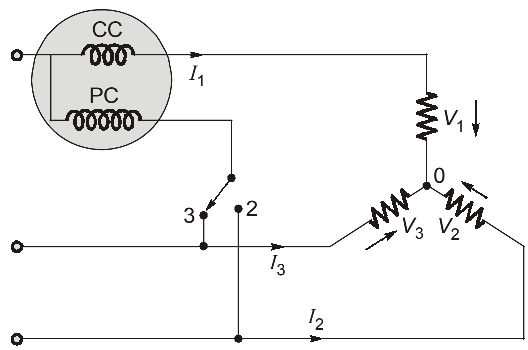
Solution : 4

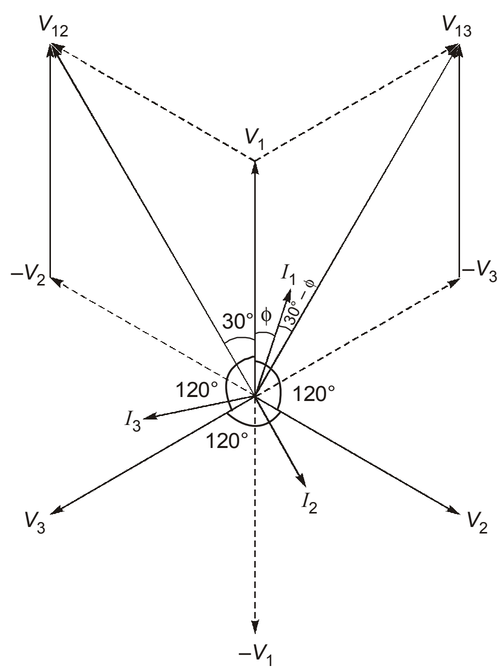
One wattmeter method can be used only when load is balanced. The connection are as under:

Measurement of active power:



One wattmeter method

The current coil is connected in one of the line and one end of pressure coil to the same line, other end being connected alternatively to other two lines. The phasor diagram is as under.



Phasor diagram

$$V_1 = V_2 = V_3 = V \quad (V \rightarrow \text{phase voltage})$$
$$I_1 = I_2 = I_3 = 1 \quad (I \rightarrow \text{phase current})$$
$$V_{13} = V_{12} = \sqrt{3}V$$

Reading of wattmeter switch at 3:

$$P_1 = V_{13} I_1 \cos(30 - \phi)$$
$$= \sqrt{3} V I \cos(30 - \phi)$$

Reading when switch at 2:

$$P_2 = V_{12} I_1 \cos(30 + \phi)$$
$$= \sqrt{3} VI \cos(30 + \phi)$$

Sum of 2 reading, $P_1 + P_2 = \sqrt{3} VI(\cos(30-\phi) + \cos(30+\phi))$

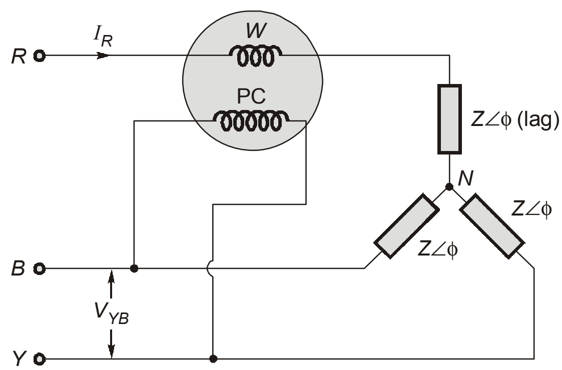
We know, $\cos(A + B) + \cos(A - B) = 2\cos A \cdot \cos B$

$$= 3\,VI\cos\phi$$

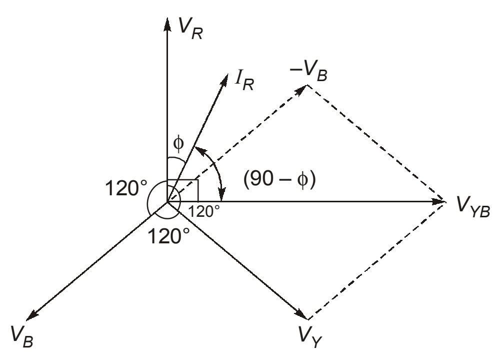
= Power consumed by load

However an drawback/limitation of this method is that it is suitable for balanced load only.

Measurement of reactive power:



Current coil is connected to one phase and potential coil connected between remaining two phases.



Wattmeter reading, $W = V_{YB} I_R \cos$ (Angle between V_{YB} and I_R)

$$= V_L I_L \cos(90 - \phi)$$
$$= V_L I_L \sin\phi \text{ (1-}\phi\text{ reactive power)}$$

∴ Three phase reactive power is

$$Q = \sqrt{3} W = \sqrt{3} V_L I_L \sin\phi$$

= Reactive power consumed by 3-ϕ load

Solution : 5

The value of R_s which will limit current to full scale deflection current can be calculated as

$$R_s = \frac{E}{I_m} - R_m = \frac{3}{1\,\text{mA}} - 100\Omega = 3\,\text{k}\Omega - 100\,\Omega = 2.9\,\text{k}\Omega$$

The value of R_x with a 20% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where $$P = \frac{I}{I_m} = 20\% = \frac{20}{100} = 0.2$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{0.2} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{0.2} - 3\,\text{k}\Omega = 15\,\text{k}\Omega - 3\,\text{k}\Omega = 12\,\text{k}\Omega$$

The value of R_x with a 40% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where $$P = \frac{I}{I_m} = 40\% = \frac{40}{100} = 0.4$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{0.4} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{0.4} - 3\,\text{k}\Omega = 7.5\,\text{k}\Omega - 3\,\text{k}\Omega = 4.5\,\text{k}\Omega$$

The value of R_x with a 50% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where $$P = \frac{I}{I_m} = 50\% = \frac{50}{100} = 0.5$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{0.5} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{0.5} - 3\,\text{k}\Omega = 6\,\text{k}\Omega - 3\,\text{k}\Omega = 3\,\text{k}\Omega$$

The value of R_x with a 75% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where $$P = \frac{I}{I_m} = 75\% = \frac{75}{100} = 0.75$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{0.75} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{0.75} - 3\,\text{k}\Omega = 4\,\text{k}\Omega - 3\,\text{k}\Omega = 1\,\text{k}\Omega$$

The value of R_x with a 90% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where $$P = \frac{I}{I_m} = 90\% = \frac{90}{100} = 0.90$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{0.90} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{0.90} - 3\,\text{k}\Omega = 3.333\,\text{k}\Omega - 3\,\text{k}\Omega = 0.333\,\text{k}\Omega$$

The value of R_x with a 100% deflection is

$$R_x = \frac{(R_s + R_m)}{P} - (R_s + R_m)$$

where, $$P = \frac{I}{I_m} = 100\% = \frac{100}{100} = 1$$

$$R_x = \frac{(2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)}{1} - (2.9\,\text{k}\Omega + 0.1\,\text{k}\Omega)$$

$$R_x = \frac{3\,\text{k}\Omega}{1} - 3\,\text{k}\Omega = 3\,\text{k}\Omega - 3\,\text{k}\Omega = 0$$

Solution : 6

Suppose that the instrument is reflecting type. Therefore, if the final steady deflection is θ_F, the spot moves through an angle $2\theta_F$.

The spot moves a distance of 1 mm at 1 m with a current of 0.001 μA or in other words, a current of 0.001 μA deflects the instrument through

$$\theta = \frac{1}{2} \times \frac{1}{r} = \frac{1}{2} \times \frac{1}{1000} = 0.5 \times 10^{-3} \text{ rad.}$$

∴ Control constant, $$K = \frac{Gi}{\theta_F} = \frac{5 \times 10^{-3} \times 0.001 \times 10^{-6}}{0.5 \times 10^{-3}} = 10 \times 10^{-9} \text{ Nm/rad}$$

Undamped period, $$T_0 = 2\pi\sqrt{\frac{J}{K}}$$

or Inertia constant, $$J = \frac{KT_0^2}{4\pi^2} = \frac{10 \times 10^{-9} \times (6)^2}{4\pi^2} = 9.12 \times 10^{-9} \text{ kgm}^2$$

Total circuit resistance for critical damping or dead beat

$$R = \frac{G^2}{2\sqrt{KJ}} = \frac{(5 \times 10^{-3})^2}{2\sqrt{10 \times 10^{-9} \times 9.12 \times 10^{-9}}} = 1309\ \Omega$$

Solution : 7

Total resistance of instrument circuit $= \dfrac{150}{0.05} = 3000\ \Omega$

Resistance of coil, $R = 400\ \Omega$

∴ Series resistance, $R_s = 3000 - 400 = 2600\ \Omega$

(a) Change in resistance of coil/°C $= 0.004 \times 400 \times 1 = 1.60\ \Omega$

Change in swamping resistance/°C $= 0.00015 \times 2600 \times 1 = 0.39\ \Omega$

Total change in resistance of instrument circuit/°C

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$$= 1.60 + 0.36 = 1.99\ \Omega$$

$\therefore$ Resistance temperature coefficient of instrument

$$= \frac{\text{Change is resistance/}\,^\circ\text{C}}{\text{Total resistance}} = \frac{1.99}{3000} = 0.00066/^\circ\text{C}$$

(b) Reactance of coil at 100 Hz $= 2\pi \times 100 \times 0.75 = 471.2\ \Omega$

Impedance of instrument at 100 Hz $= \sqrt{(3000)^2 + (471.2)^2} = 3037\ \Omega$

Current drawn by instrument at 100 Hz $= \dfrac{150}{3037} = 0.0494$ A

$\therefore$ Reading of instrument at 100 Hz $= \left(\dfrac{0.0494}{0.05}\right) \times 150 = 148.2$ V.

$$\text{Error} = \frac{148.2 - 150}{150} \times 100 = 1.2\%\ \text{low.}$$

(c) In order that there is no frequency error, value of capacitance to be connected across R_s

$$C = 0.41\frac{L}{R_s^2} = 0.41 \times \frac{0.75}{(2600)} = 0.0455\ \mu\text{F}$$

Solution : 8

With DC:

Power, $P = 250$ W.

Current coil current, $I = 1$ A

Voltage across pressure coil circuit, $V = \dfrac{250}{1} = 250$ V

Pressure coil current, $I_p = 0.05$ A

$\therefore$ Resistance of potential coil circuit,

$$R_p = \frac{250}{0.05} = 5000\ \Omega$$

With AC:

Instantaneous current, $i = 10\sin(377t + 15°) = 5\sin(1131t)$

Putting, $317t = \theta$, we have

$$i = 10\sin(\theta + 15°) = 5\sin(3\theta)$$

Instantaneous voltage, $v = 500\sin(377t - 30°) + 800\sin(754t + 45°)$

$$= 500\cos(\theta - 30°) + 800\sin(2\theta + 45°)$$

Instantaneous power, $p = vi = 500 \times 10\cos(\theta + 15°)\cos(\theta - 30°) + 00 \times 5\sin(3\theta)\cos(\theta - 30°)$
$+ 800\sin(754t + 45°) + 800 \times 5\sin(30°)\sin(2\theta + 45°)$

Average power over a cycle, $P = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} vi\,d\theta$

We have the following identities:

(i) $\dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} A\sin(\theta + \alpha)\cdot B\sin(\theta + \beta)\,d\theta = \dfrac{1}{2}AB\cos(\alpha - \beta)$

(ii) $\frac{1}{2\pi}\int_0^{2\pi} A\sin(\theta+\alpha)\cdot B\sin(\theta+\beta)\, d\theta \;=\; \frac{1}{2}AB\sin(\alpha-\beta)$

(iii) $\frac{1}{2\pi}\int_0^{2\pi} A\sin(m\theta+\alpha)\cdot B\sin(n\theta+\beta)\, d\theta \;=\; 0$

(iv) $\frac{1}{2\pi}\int_0^{2\pi} A\sin(m\theta+\alpha)\cdot B\cos(n\theta+\beta)\, d\theta \;=\; 0$

Therefore,

$$\frac{1}{2\pi}\int_0^{2\pi} 500\times 10\sin(\theta+15^\circ)\cos(\theta-30^\circ)d\theta \;=\; \frac{1}{2}\times 500\times 10\sin 45^\circ = 1768$$

Now, $$\frac{1}{2\pi}\int_0^{2\pi} 800\times 10\sin(\theta+15^\circ)\sin(2\theta+45^\circ) \;=\; 0$$

and $$\frac{1}{2\pi}\int_0^{2\pi} 800\times 5\sin(3\theta)\sin(2\theta+45^\circ) \;=\; 0$$

Hence, Average power, $P = 1768$ W

■■■■

3 Power Factor Meter, Potentiometer, Bridge Measurement & Instrument Transformer

LEVEL 1 Objective Questions

1. (b)
2. (c)
3. (b)
4. (a)
5. (c)
6. (b)
7. (a)
8. (d)

LEVEL 2 Objective Questions

9. (b)
10. (b)
11. (a)
12. (0)
13. (d)
14. (0.004)
15. (1.68)
16. (0.08275)
17. (0.00734)
18. (c)
20. (6.53)
21. (d)

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LEVEL 3 Conventional Questions

Solution : 1

Given: $R_m = 100\ \Omega$, $I_m = 50\ \mu A$.

For 0 – 1 mA range

$$I_{sh} R_{sh} = I_m R_m$$

$$\therefore \quad 950\text{ mA } (R_1 + R_2 + R_3 + R_4) = 50\ \mu A \times 10$$

$$\therefore \quad R_1 + R_2 + R_3 + R_4 = \frac{50\ \mu A \times 100}{950\ \mu A} = \frac{5000}{950} = 5.26\ \Omega \qquad \text{... (i)}$$

From 0 – 10 mA

$$9950\ \mu A\ (R_1 + R_2 + R_3) = 50\ \mu A\ (100 + R_4) \qquad \text{... (ii)}$$

From 0 – 50 mA

$$49950\ \mu A\ (R_1 + R_2) = 50\ \mu A\ (100 + R_3 + R_4) \qquad \text{... (iii)}$$

From 0 – 100 mA

$$99950\ \mu A\ (R_1) = 50\ \mu A\ (100 + R_2 + R_3 + R_4) \qquad \text{... (iv)}$$

But $\quad R_1 + R_2 + R_3 = 5.26 - R_4$.

Substituting in equation (ii), we have

$$99950\ \mu A\ (5.26 - R_4) = 50\ \mu A\ (100 + R_4)$$

$$9950\ \mu A \times 5.26 - 9950\ \mu A \times R_4 = 5000\ \mu A + 50\ \mu A\ R_4$$

$$(9950\ \mu A \times 5.26 - 5000\ \mu A) = 9950\ \mu A\ R_4 + 50\ \mu A\ R_4$$

Therefore,

$$R_4 = \frac{9950\ \mu A \times 5.26 - 5000\ \mu A}{10\ \mu A} = \frac{47377\ \mu A}{10\text{ mA}} = 4.737\ \Omega$$

$$R_4 = 4.74\ \Omega$$

In equation (i), substituting for R_4, we get

$$R_1 + R_2 + R_3 = 5.26 - 4.74 = 0.52$$

$$\therefore \quad R_1 + R_2 = 0.52 - R_3$$

Substituting in equation (iii), we have

$$49950\ \mu A\ (0.52 - R_3) = 50\ \mu A\ (R_3 + 4.74 + 100)$$

$$49950\ \mu A \times 0.52 - 49950\ \mu A \times R_3 = 50\ \mu A \times R_3 + 50\ \mu A \times 4.74 + 50\ \mu A \times 100$$

$$49950\ \mu A \times 0.52 - 50\ \mu A \times 4.74 = 49950\ \mu A \times R_3 + 50\ \mu A \times R_3 + 5000\ \mu A$$

$$(25974 - 237)\ \mu A = 50\text{ mA} \times R_3 + 5000\ \mu A$$

$$25737\ \mu A = 50\text{ mA} \times R_3 + 5000\ \mu A$$

$$R_3 = \frac{25737\ \mu A - 5000\ \mu A}{50\text{ mA}} = \frac{20737\ \mu A}{50\text{ mA}}$$

$$R_3 = 0.4147 = 0.42\ \Omega$$

But, $\quad R_1 + R_2 = 0.52 - R_3$

$$\therefore \quad R_1 + R_2 = 0.52 - 0.4147 = 0.10526$$

Therefore

$$R_2 = 0.10526 - R_1 \qquad \text{... (v)}$$

$$99950\ \mu A\ (R_1) = 50\ \mu A \times (100 + R_2 + R_3 + R_4)$$

But $\quad R_2 + R_3 + R_4 = 5.26 - R_1 \qquad$ [From equation (i)]

Substituting in equation (iv),

$$99950\ \mu A \times R_1 = 50\ \mu A \times (100 + 5.26\ \ R_1)$$

$$= 5000\ \mu A + (50\ \mu A \times 5.26) - (R_1 + 50\ \mu A)$$

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$$99950\ \mu A \times R_1 + 50\ \mu A \times R_1 = 5000\ \mu A + 50\ \mu A \times 5.26$$
$$(99950\ \mu A \times R_1 + 50\ \mu A)R_1 = 5000\ \mu A + 263\ \mu A$$
$$100\text{ mA} \times R_1 = 5263\ \mu A$$
$$R_1 = \frac{5263\ \mu A}{100\text{ mA}} = 0.05263\ \Omega$$

Therefore, From equation (v), we have

$$R_2 = 0.10526,\ R_1 = 0.10526\ \ 0.05263 = 0.05263\ \Omega$$

Hence the value of shunts are

$$R_1 = 0.05263\ \Omega;$$
$$R_2 = 0.05263\ \Omega$$
$$R_3 = 0.4147\ \Omega;$$
$$R_4 = 4.74\ \Omega$$

Solution : 2

The total shunt resistance R_{sh} is determined by

$$R_{sh} = \frac{R_m}{(n-1)} \qquad \text{where, } n = \frac{I}{I_m}$$

Given: $I_m = 100\ \mu A$ and $R_m = 1000\ \Omega$

For 10 mA range:

$$n = \frac{I}{I_m} = \frac{10\text{ mA}}{100\ \mu A} = 100$$

$$R_{sh} = \frac{R_m}{(n-1)} = \frac{1000\ \Omega}{100-1} = \frac{1000}{99} = 10.1\Omega$$

When the meter is set on the 100 mA range, the resistance R_b and R_c provides the shunt. The shunt can be found from the equation

$$R_{sh2} = (R_b + R_c) = \frac{I_m(R_m + R_{sh})}{I} = \frac{100\ \mu A(10.1 + 1000)}{100\text{ mA}} = 1.01\Omega$$

The resistor which provides the shunt resistance on the 1 A range can be found from the equation

$$R_c = \frac{I_m(R_m + R_{sh})}{I} = \frac{100\ \mu A(10.1 + 1000)}{1000\text{ mA}} = 1.101\Omega$$

But,

$$R_b + R_c = 1.01\ \Omega$$
$$R_b = 1.01 - R_c = 1.01 - 0.101\ \Omega = 0.909\ \Omega$$

Resistor R_a is found by

$$R_a = R_{sh} - (R_b + R_c) = 10.1 - (0.909 + 0.101)\ \Omega$$
$$= 10.1 - 1.01\ \Omega = 9.09\ \Omega$$

Hence $R_a = 9.09\ \Omega$, $R_b = 0.909\ \Omega$ and $R_c = 0.101\ \Omega$

Solution : 3

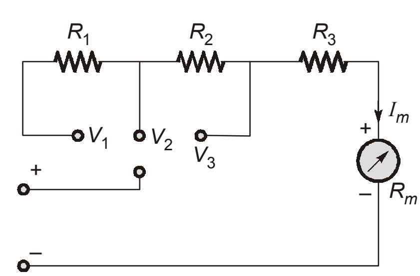
Given: $I_m = 10$ mA, $R_m = 100\ \Omega$

Step-1 : For a 5 V (V_3) the total circuit resistance is

$$R_t = \frac{V}{I_{fsd}} = \frac{5}{10\text{ mA}} = 0.5\ \Omega$$

Therefore,

$$R_3 = R_t - R_m = 500\ \Omega - 100\ \Omega$$
$$= 400\ \Omega$$



Step-2: For a 50 V (V_3) position,

$$R_f = \frac{V}{I_{fsd}} = \frac{50}{10\text{ mA}} = 5\ \Omega$$

Therefore,
$$R_2 = R_t - (R_3 + R_m) = 5\text{ k}\Omega - (400\ \Omega + 100\ \Omega)$$
$$= 5\text{ k}\Omega - 500\text{ W} = 4.5\text{ k}\Omega$$

Step-3: For a 100 V range (V_1) position

$$R_f = \frac{V}{I_{fsd}} = \frac{100}{10\text{ mA}} = 10\ \Omega$$

Therefore,
$$R_1 = R_t - (R_2 + R_3 + R_m)$$
$$= 10\text{ k}\Omega\ (4.5\text{ k}\Omega + 400\ \Omega + 100\ \Omega)$$
$$= 10\text{ k}\Omega - 5\ \Omega = 5\text{ k}\Omega$$

Hence it can be seen that R_3 has a non-standard value.

Solution : 4

Given: L = 25 cm, R_p = 2500 Ω, P = 4 W. Therefore, current in the circuit is

$$I = \sqrt{\frac{P}{R_p}} = \sqrt{\frac{4}{2500}} = 0.04\text{ A}$$

and excitation voltage is
$$V = 2500 \times 0.04 = 100\text{ V}$$

(a)
$$\text{Sensitivity} = \frac{100}{25} = 4\text{ V/cm}$$

(b) Actual input displacement, x = 15 cm

Therefore, resistance across x is

$$R_x = \frac{15}{25} \times 2500 = 1500\ \Omega$$

and actual voltage across R_x is $V_x = 15 \times 4 = 60$ V

The recorder has been connected parallel to V_x. Their combined resistance is

$$\frac{1500 \times 5000}{1500 + 5000} = 1153.85\ \Omega$$

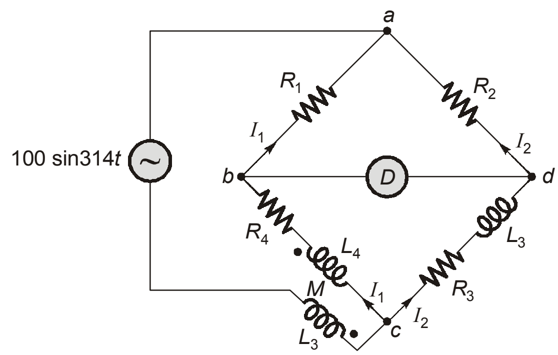
Hence the total resistance of the circuit is now

$$(1153.85 + 1000) = 2153.85\ \Omega$$

Therefore, voltage across,
$$R_x = \frac{60}{2153.85} \times 1153.85 = 32.14\text{ V}$$

$$\text{Loading error} = \frac{60 - 16.07}{60} \times 100 = 46.4\%$$

Solution : 5



Let, M = Unknown mutual inductance

L_4 = Self inductance of secondary of mutual inductance

L_3 = Known self inductance

R_1, R_2, R_3 and R_4 = Non-inductive resistors

At balance condition voltage drop between c and d must be equal to the voltage drop between a and d.
Also voltage drop across c-b-a must be equal to the voltage drop across c-d-a.

Thus, $$I_1R_1 = I_2R_2 \quad \text{...(i)}$$

and $$(I_1 + I_2)(j\omega M) + I_1(R_1 + R_4 + j\omega L_4) = I_2(R_2 + R_3 + j\omega L_3) \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$I_2\left(\frac{R_2}{R_1}+1\right)(j\omega M) + I_2\left(\frac{R_2}{R_1}\right)(R_1 + R_4 + j\omega L_4) = I_2(R_2 + R_3 + j\omega L_3)$$

or, $$(j\omega M)\left(1+\frac{R_2}{R_1}\right) + R_2 + \frac{R_2R_4}{R_1} + j\omega L_4 \cdot \frac{R_2}{R_1} = R_3 + R_2 + j\omega L_3$$

Equating real and imaginary part, we get.

Thus, $$R_4 = \frac{R_3R_1}{R_2}$$

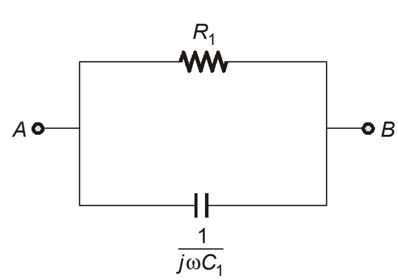
and $$M = \frac{L_3 - L_4R_2/R_1}{\left(1+\dfrac{R_2}{R_1}\right)} = \frac{R_1L_3 - R_2L_4}{R_1 + R_2}$$

If $R_1 = R_2$, $$M = \frac{L_3 - L_4}{2}$$

and $$R_3 = R_4$$

Solution : 6

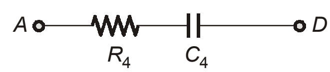
Arm AB:



$$Z_{AB} = \frac{R_1 \cdot \frac{1}{j\omega C_1}}{R + \frac{1}{j\omega C_1}} - \frac{R_1}{1 + j\omega R_1 C_1} \quad ...(1)$$

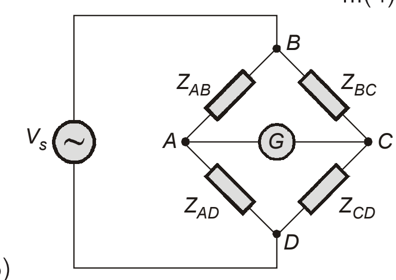
Arm BC: $Z_{BC} = R_2$ ...(2)

Arm AD:



$$Z_{AD} = \left(R_4 - \frac{j}{\omega C_4}\right) \quad ...(3)$$

Arm CD: $= R_x$ ...(4)



For a balance bridge,

For balance, $Z_{AB} . Z_{CD} = Z_{BC} . Z_{AD}$

From (1), (2), (3) and (4)

$$\frac{R_1}{1 + j\omega R_1 C_1} \cdot R_x = R_2 \cdot \left(R_4 - \frac{j}{\omega C_4}\right)$$

$$R_x = \frac{R_2}{R_1}\left[R_4 - \frac{j}{\omega C_4} + j\omega R_4 R_1 C_1 + \frac{\omega R_1 C_1}{\omega C_4}\right] \quad ...(5)$$

Comparing real and imaginary parts,

$$R_x = \frac{R_2}{R_1}\left(R_4 + \frac{R_1 C_1}{C_4}\right) = \frac{R_2 R_4}{R_1} + \frac{R_2 C_1}{C_4}$$

and $$\frac{1}{\omega C_4} = \omega R_4 R_1 C_1$$

$$\omega^2 = \frac{1}{R_1 R_4 C_1 C_4}$$

$$\omega = \frac{1}{\sqrt{R_1 R_4 C_1 C_4}}$$

(i) $$R_x = \frac{R_2}{R_1}\left(R_4 + \frac{R_1 C_1}{C_4}\right) \quad ...(6)$$

(ii) $$\omega = \frac{1}{\sqrt{R_1 C_1 R_4 C_4}} \quad ...(7)$$

(iii) Given, $R_1 = 200\ \Omega$, $C_1 = 1\ \mu F$, $R_2 = 400\ \Omega$

$$\frac{C_1}{C_4} = 0.5 \quad ...(8)$$

$$\frac{R_4}{R_1} = 2 \quad ...(9)$$

From equation (8), $C_4 = 2\ \mu F$

From equation (9), $R_4 = 400\ \Omega$

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Putting values in equation (6),

$$R_x = \frac{400}{200}\left(400 + \frac{(200)}{0.5}\right) = 2\,(800) = 1600\ \Omega$$

$$\omega = \frac{1}{\sqrt{R_1R_4C_1C_4}} = \frac{1}{\sqrt{(200)(400)(2)(1)\times 10^{-12}}}$$

$$\omega = \frac{1}{\sqrt{(80000)(2)\times 10^{-6}}} = 2.5 \times 10^3 \text{ rad/sec}$$

$$\omega = 2500 \text{ rad/sec}$$

Solution : 7

We need to find R_x and L_x, we have

$$R_x = \frac{\omega^2 R_1R_2R_3C_1^2}{1+\omega^2R_1^2C_1^2} = \frac{(3000)^2 \times 10\text{ k}\times 2\text{ k}\times 1\text{k}\times(1\times 10^{-6})^2}{1+(3000)^2\times(2k)^2\times(1\times 10^{-6})^2}$$

$$= \frac{180\times 10^3}{1+36} = \frac{180}{37}\times 10^3 = 4.86\text{ k}\Omega$$

and we have,

$$L_x = \frac{R_2R_3C_1}{1+\omega^2R_1^2C_1^2} = \frac{10\text{ k}\times 2\text{ k}\times 1\text{k}\times(1\times 10^{-6})^2}{1+(3000)^2\times(2k)^2\times(1\times 10^{-6})^2}$$

$$= \frac{10}{1+36} = \frac{10}{37} = 0.27 = 270\text{ mH}$$

Therefore, $R_x = 4.86\text{ k}\Omega$ and $L_x = 270$ mH

Solution : 8

(a) At balance, the value of unknown resistance,

$$R = \frac{P}{Q}\cdot S = \frac{1000}{1000}\times 0.001 = 0.001\,\Omega$$

(b) If we examine the Kelvin bridge circuit, we find the resistors P, Q and p, q are in parallel with the resistance of link, r. Since r is negligible and P, Q, p and q have large values, the effect of ratios arms can be neglected for the purpose of calculation of current.

$\therefore$ Current under balance conditions,

$$I = \frac{E}{R_b + R + S} = \frac{100}{5+0.01+0.0001} \approx 20\text{ A}$$

where, R_b = Resistance in the battery circuit.

(c) The value R is changed by 0.1 percent.

$\therefore$ New value of $R = 1.001 \times 0.001 = 0.001001\ \Omega$

Voltage between points a and c

$$E_{ad} = \frac{R+S+r}{R_b+R+S+r}$$

$$E = \frac{R+S}{R_b+R+S}E \text{ as } r = 0$$

$$= \frac{R+S}{R_b} = \frac{0.001+0.001001}{5}\times 100 = 40\times 10^{-3}$$

$$E_0 = E_{ab} - E_{amd} = \left(\frac{P}{P+Q} - \frac{R}{R+S}\right)E_{ac}$$

$$= \left(\frac{1000}{1000+1000} - \frac{0.001001}{0.001+0.001001}\right) \times 40 \times 10^{-3}$$

$$= 0.01 \times 10^{-3}\text{ V} = 0.01\text{ mV} \qquad \text{(Considering } r = 0)$$

Since R, S, r and R_b are quite small as compared to P, Q, p and q, we can use circuit. For the calculation of internal resistance, R_0 as viewed from terminals d and b.

$$R_0 = \frac{PQ}{P+Q} + \frac{pq}{p+q} = \frac{1000 \times 1000}{1000+1000} + \frac{1000 \times 1000}{1000+1000}$$

$$= 1000\ \Omega$$

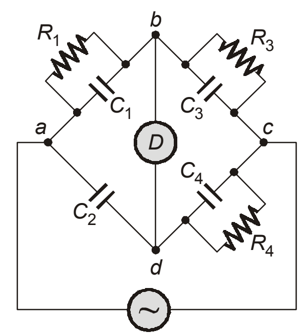
Galvanometer current, $$I_g = \frac{E_0}{R_0 + G} = \frac{0.01 \times 10^{-3}}{1000+500} = 0.0067 \times 10^{-6}\text{ A}$$

$$= 0.0067\ \mu\text{A}$$

$\therefore$ Deflection of galvanometer, $\theta = S_i I_g = 200 \times 0.0067 = 1.34$ mm.

Solution : 9

The voltage circuit is shown in figure.



For balance, $$Y_1 Y_4 = Y_2 Y_3$$

or $$\left(\frac{1}{R_1} + j\omega C_1\right)\left(\frac{1}{R_4} + j\omega C_4\right) = (j\omega C_2)\left(\frac{1}{R_3} + j\omega C_3\right)$$

or $$\left(\frac{1}{R_1 R_4} - \omega^2 C_1 C_4\right) + j\omega\left(\frac{C_4}{R_1} + \frac{C_1}{R_4}\right) = j\omega\frac{C_2}{R_3} - \omega^2 C_2 C_3$$

Equating the real and imaginary parts, we have

$$\frac{1}{R_1 R_4} - \omega^2 C_1 C_4 = -\omega^2 C_2 C_3 \qquad \text{... (i)}$$

and $$\frac{C_4}{R_1} + \frac{C_1}{R_4} = \frac{C_2}{R_3} \qquad \text{... (ii)}$$

From (i) and (ii), we have

$$C_1 = \frac{\dfrac{C_2 R_4}{R_3} + \omega^2 C_2 C_3 C_4 R_4^2}{1 + \omega^2 C_4^2 R_4^2}$$

Now, $$\omega^2 C_2 C_3 C_4 R_4^2 << \frac{C_2 R_4}{R_3} \text{ and } \omega^2 C_4^2 R_4^2 << 1$$

Hence we can write $$C_1 = C_2 \frac{R_4}{R_3}$$

When the capacitor C_1 is without specimen dielectric, let its capacitance be C_0.

$\therefore$ $$C_0 = C_2 \frac{R_4}{R_3} = 150 \times \frac{5000}{5000} = 150\text{ pF}$$

When the specimen is inserted as dielectric, let the capacitance be C_s.

$$\therefore \quad C_s = C_2 \frac{R_4}{R_3} = 900 \times \frac{5000}{5000} = 900 \text{ pF.}$$

Now,
$$C_0 = \frac{\varepsilon_0 A}{d} \text{ and } C_s = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Hence relative permittivity of specimen,

$$\varepsilon_r = \frac{C_s}{C_0} = \frac{900}{150} = 6$$

Solution : 10

(a) Value of resistance for which the errors are equal for the two types of connections

$$R = \sqrt{R_A R_v} = \sqrt{2 \times 5000} = 100\ \Omega$$

Since the resistance to be measured has a value less than 100 Ω, the method should be used as it results in smaller error. Use (ii) method to reduce error.

Measured value of resistance,

$$R_{m2} = \frac{V}{I} = \frac{35.5}{0.42} = 84.52\ \Omega$$

True value of resistance,
$$R = R_{m2}\left(\frac{1}{1 - R_{m2}/R_v}\right) = 84.52 \times \left(\frac{1}{1 - (84.52/5000)}\right)$$
$$= 85.97\ \Omega \approx 86\ \Omega = \text{true value}$$

(b) Error in ammeter reading $= \left(\frac{0.5}{100}\right) \times 1 = 0.005$ A

$\therefore$ Percentage error at 0.42 A reading $= \left(\frac{0.005}{0.42}\right) \times 100 = 1.19\%$

Error in voltmeter reading $= 0.5 \times \left(\frac{50}{100}\right) = 0.25$ V

$\therefore$ Percentage error at 35.5 V reading $= \left(\frac{0.25}{35.5}\right) \times 100 = 0.704\%$

Since the error correspond to standard deviations, error due to ammeter and voltmeter

$$= \sqrt{(1.19)^2 + (0.704)^2} = \pm 1.38\%$$

Absolute error due to ammeter and voltmeter

$$= \left(\frac{1.38}{100}\right) \times 86 \approx \pm 1.2\ \Omega$$

$\therefore$ The resistance is specified as 85.97 ± 1.2 Ω.

Because of % error in voltmeter reading reading is less hence this method is used.

■■■■

4 CRO, Q-meter, Frequency Measurement Data Acquistion System

LEVEL 1 Objective Questions

1.	(a)
2.	(b)
3.	(d)
4.	(a)
5.	(d)
6.	(b)
7.	(b)

LEVEL 2 Objective Questions

8.	(a)
9.	(a)
10	(a)
11.	(100)
12.	(a)
13.	(48.71)
14.	(c)
15.	(d)
16.	(a)

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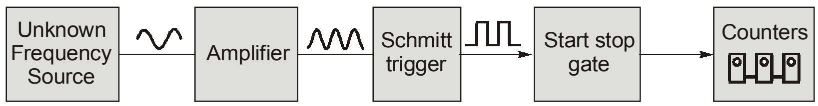
LEVEL 3 Conventional Questions

Solution : 1

Functional Block diagram of Digital Frequency Meter:



Principle of Operation:

1. The unknown frequency signal is fed to the schmitt trigger. The signal may be amplified before being applied to schmitt trigger.
2. In a schmitt trigger, the signal is converted into a square wave with very fast rise and fall times, then differentiated and clipped.
3. As a result, the output from a schmitt trigger is a train of pulses, one pulse, for each cycle of the signal.
4. The output pulses from the schmitt trigger are fed to start stop gate when this gate opens (start), the input pulses pass through this gate and are fed to an electronic counter which starts registering the input pulses.
5. When the gate is closed (stop), the input of pulses to counter ceases and it stops counting.
6. The counter displays the number of pulses that have passed through it in the time interval between start and stop. If this interval is known, the pulse rate and hence the frequency of the input signal can be known. Frequency of unknown signal,

$$f = \frac{N}{t}$$

where, f = frequency of unknown signal
N = number of counts displayed by counter
t = time interval between start and stop of gate
f = frequency of the signal = 3.5 kHz = 3500 Hz

Assuming 5 digit display

(i) $t = 0.1$ second

$$f = \frac{N}{t}$$

$$N = ft = 3500 \times 0.1 = 350$$

display = 00350

(ii) $t = 1$ second,

$$f = \frac{N}{t}$$

$$N = ft = 3500 \times 1 = 3500$$

display = 03500

(iii) $t = 10$ second,

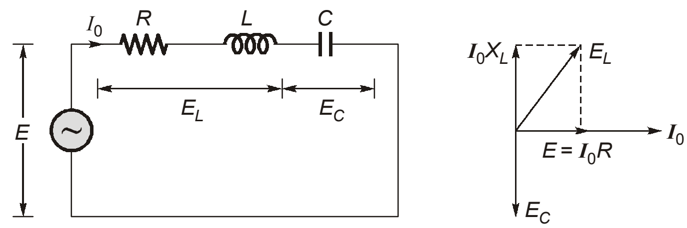
$$N = ft = 3500 \times 10 = 35000$$

display = 35000

Solution : 2

Q-meter is an instrument which is designed to measure the value of Q directly and as such is very useful in measuring characteristic of coils.

Working principle: It is based on the characteristic of a resonant series R, L, C circuit



(a) **(b)**

At resonant frequency f_0, we have

$$X_C = X_L$$

$$X_C = \frac{1}{2\pi f_0 C} = \frac{1}{\omega_0 C}$$

$$X_L = 2\pi f_0 L = \omega_0 L$$

So,
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At resonance,
$$I_0 = \frac{E}{R}$$

$$E_C = |I_0 X_C| = |I_0 X_L| = I_0 \omega L = E_L$$

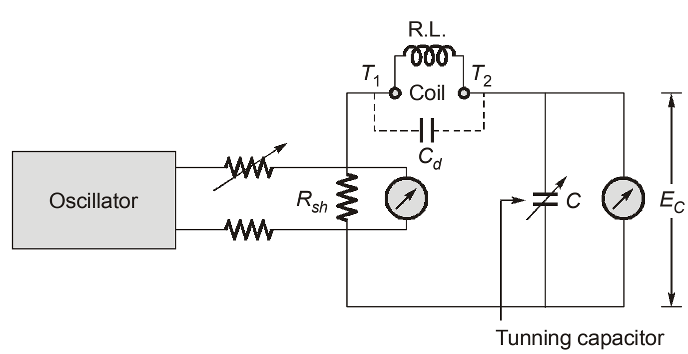
$$E = I_0 R$$

$$\frac{E_C}{E} = \frac{I_0 \omega_0 L}{I_0 R} = \frac{\omega_0 L}{R} = Q$$

So Q is measured by measuring supply voltage and voltage across capacitor.

Error:

- $Q = \dfrac{E_C}{E}$ and if supply voltage is kept constant, voltmeter across the capacitor may be calibrated to read value of Q of the coil. However there is an error. Measured value of Q is the Q of whole circuit and not the coil.
- **Circuit of a Q-meter:**



- Oscillator deliver current to R_{sh}. Further there is a distributed capacitance of the coil. There is an error on account of R_{sh} and distributed capacitance (C_d).

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Correction for R_{sh} gives $Q_{true} = Q_{measured}\left(1+\frac{R_{sh}}{R}\right)$. If $R >> R_{sh}$ error will be lesser. If measurement is carried out for high Q coil i.e. R is smaller then large error may occur due to R_{sh}.

- Given,

$$f_1 = 2\text{ MHz},\ C_1 = 450\text{ pF}$$
$$f_2 = 5\text{ MHz},\ C_2 = 60\text{ pF}$$
$$f_1 = \frac{1}{2\pi\sqrt{L(C_1+C_d)}} \qquad \text{...(i)}$$
$$f_2 = \frac{1}{2\pi\sqrt{L(C_2+C_d)}}$$
$$\frac{f_2}{f_1} = \frac{\sqrt{C_1+C_d}}{\sqrt{C_2+C_d}}$$
$$\frac{5\times10^6}{2\times10^6} = \frac{\sqrt{C_1+C_d}}{\sqrt{C_2+C_d}}$$
$$\frac{C_1+C_d}{C_2+C_d} = 6.25$$
$$C_d = \frac{C_1-6.25C_2}{5.25}$$
$$= \frac{450\times10^{-12}-(6.25)(60\times10^{-12})}{5.25} = 10^{-12}\frac{[450-(6.25\times60)]}{5.25} = 14.29\text{ pF}$$

Put C_d in equation (i),

$$2\times10^6 = \frac{1}{2\pi\sqrt{L(14.29+450)\times10^{-12}}}$$
$$(4\times10^{12})(4\pi^2) = \frac{1}{L(464.29\times10^{-12})}$$
$$L = \frac{1}{(4\pi^2)(4)(464.29)} = 13.57\ \mu\text{H}$$

Solution : 3

Let, $X \rightarrow$ Horizontal plate deflection

$Y \rightarrow$ Vertical plate deflection

$$X = V_1\sin(\omega t+\theta_1)$$
$$X = V_1(\sin\omega t\cos\theta_1 + \cos\omega t\sin\theta_1)$$

Similarly, $Y = V_2(\sin\omega t\cos\theta_2 + \cos\omega t\sin\theta_2)$

$$\frac{X}{V_1} = \sin\omega t\cos\theta_1 + \cos\omega t\sin\theta_1 \qquad \text{...(1)}$$
$$\frac{Y}{V_2} = \sin\omega t\cos\theta_2 + \cos\omega t\sin\theta_2 \qquad \text{...(2)}$$

From (1) and (2), $$\frac{X}{V_1}+\frac{Y}{V_2} = \sin\omega t(\cos\theta_1+\cos\theta_2) + \cos\omega t(\sin\theta_1+\sin\theta_2)$$

Let, $\cos\theta_1 + \cos\theta_2 = K_1;\ \sin\theta_1 + \sin\theta_2 = K_2$

$$\frac{X}{V_1} + \frac{Y}{V_2} = K_1 \sin \omega t + K_2 \cos \omega t \quad ...(3)$$

Similarly,

$$\frac{X}{V_1} - \frac{Y}{V_2} = K_1 \sin \omega t - K_2 \cos \omega t \quad ...(4)$$

Adding (3) and (4) gives,

$$\frac{X}{V_1} = K_1 \sin \omega t \quad ...(5)$$

Substracting (3) and (4) gives,

$$\frac{Y}{V_2} = K_2 \cos \omega t \quad ...(6)$$

From (5) and (6) we gets,

$$\frac{X}{K_1 V_1} = \sin \omega t \quad ...(7)$$

$$\frac{Y}{K_2 V_2} = \cos \omega t \quad ...(8)$$

Squaring and adding (7) and (8),

$$\frac{X^2}{(K_1V_1)^2} + \frac{Y^2}{(K_2V_2)^2} = 1$$

Let, $K_1V_1 = a,\ K_2V_2 = b$ (Constants)

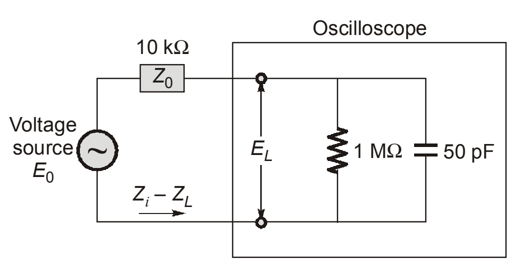
$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

Its an equation of ellipse with

$$a = V_1(\cos\theta_1 + \cos\theta_2)$$
$$b = V_2(\sin\theta_1 + \sin\theta_2)$$

Solution : 4

The equivalent circuit for the measurement system is shown in below figure.



When, Frequency = 100 kHz;

The value of capacitive reactance at 100 kHz is

$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 1000 \times 50 \times 10^{-12}} = 32000\ \Omega$$

The input impedance of the oscilloscope is

$$Z_L = \frac{R(-jX_c)}{R - jX_c} = \frac{10^6 \times (-j32 \times 10^3)}{10^6 - j32 \times 10^3}$$

$$= -j32 + 10^3\ \Omega = 32 \times 10^3 \angle -90°\ \Omega$$

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The voltage across the load, $E_L = \dfrac{E_0}{1+Z_0/Z_L} = 10\angle 0° \times \dfrac{1}{1+\dfrac{10\times 10^3\angle 0°}{32\times 10^3\angle -90°}}$

$= 0.954\angle{-17.4°}$ V(peak)

This means that the magnitude of voltage indicated by the oscilloscope is 0.954 of its original value.

$\therefore$ The error is $(1 - 0.954) \times 100 = 4.6$ percent. Also the voltage under loaded conditions lags the voltage under open circuit conditions by an angle of 17.4°.

When Frequency = 1 MHz

The value of capacitive reactance of oscilloscope is,

$$X_c = \frac{1}{2\pi\times 10^6 \times 50\times 10^{-12}} = 3200\ \Omega$$

The input impedance of the oscilloscope is,

$$Z_L = \frac{R(-jX_c)}{R-jX_c} = \frac{10^6\times(-j3.2\times 10^3)}{10^6\times -j3.2\times 10^3} = 3.2\times 10^3\angle{-90°}\ \Omega$$

$\therefore$ The voltage across the load is $E_L = \dfrac{E_0}{1+Z_0/Z_L} = 10\angle 0° \times \dfrac{1}{1+\dfrac{10\times 10^3\angle -0°}{3.2\times 10^3\angle -90°}}$

$= 0.304\angle{-72.3°}$ V (Peak)

In this case the measured value is only 0.304 of its original value and the phase shift is 72.3°. Thus the output is considerably attenuated and is less than one third of its original value. The value of phase angle between voltages under no load and load conditions is 72.3°.

This indicates the effect of distortion of signal on account of increased shunting effect due to increase in frequency.

■■■■

5 Electronic Measuring Instrument and Transducer

LEVEL 1 Objective Questions

1.	(b)
2.	(c)
3.	(a)
4.	(a)
5.	(a)
6.	(b)
7.	(c)
8.	(c)
9.	(b)

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LEVEL 2 Objective Questions

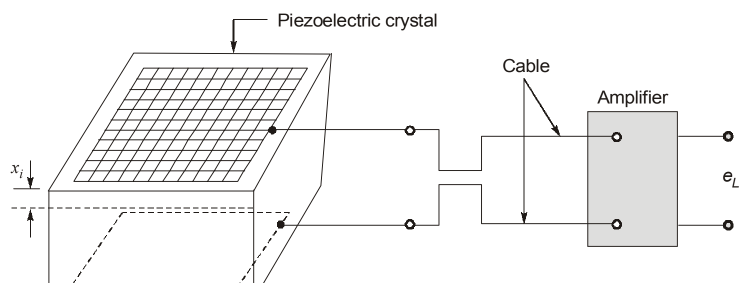
10.	(c)
11.	(b)
12.	(d)
13.	(2.5)
14.	(0.2)
15.	(d)
16.	(677.4)
17.	(c)
18.	(c)

LEVEL 3 Conventional Questions

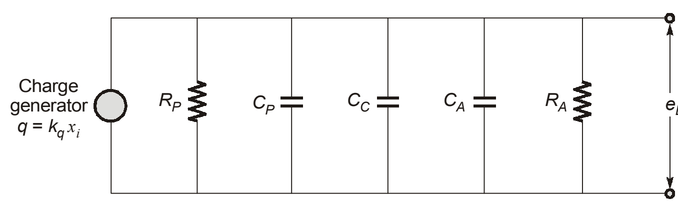
Solution : 1

A piezo-electric material is one in which an electric potential appears across certain surfaces of a crystal if the dimensions of the crystal are changed by the application of a mechanical force. This potential is produced by the displacement of charges. The effect is reversible, i.e. conversely, if a varying potential is applied to the proper axis of the crystal, it will change the dimensions of the crystal thereby deforming it. This effect is known as piezo-electric effect. Elements exhibiting piezo-electric quantities are called as electro-resistive elements. The materials that exhibit a significant and useful piezo-electric effect are divided into two categories: (a) Natural group and (b) Synthetic group.

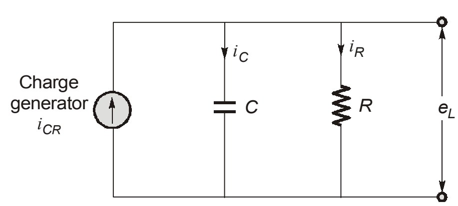
Quartz and Rochelle salt belong to natural group while materials like lithium sulphate, ethylene diamine tartarate belong to the synthetic group.



(a)



(b)



(c)

Consider the transducer, the connecting cable and the amplifier as a unit. The impedance of the transducer is very high and hence an amplifier with a high input impedance has to be used in order to avoid loading errors.

Charge produced, $q = K_q x_i$ coulomb

where, K_q = sensitivity; C/m

and x_i = displacement; m

Figure (b) shows the equivalent circuit:

R_P = Leakage resistance of transducer; Ω

C_P = Capacitance of transducer; F

C_C = Capacitance of cable; F

C_A = Capacitance of amplifier; F

R_A = Resistance of amplifier; Ω

The charge generator is converted into a constant current generator as shown in figure (c). The capacitance connected across the current generator is C where:

$$C = C_P + C_C + C_A$$

Resistance, $$R = \frac{R_A R_P}{R_A + R_P}$$

Since the leakage resistance of transducer is very large (of the order of $0.1 \times 10^{12}\ \Omega$) and therefore,

$$R = R_A$$

Converting the charge generator into a current generator

$$i_{CR} = \frac{dq}{dt} = K_q\left(\frac{dx_i}{dt}\right)$$

where i_{CR} is the current of the constant current generator.

Now, $$i_{CR} = i_C + i_R$$

$\therefore$ Output voltage at load, $$e_L = e_C = \frac{1}{C}\int i_C\,dt = \frac{1}{C}\int (i_{CR} - i_R)dt$$

or $$\frac{d(e_L)}{dt} = \frac{1}{C}(i_{CR} - i_R)$$

or $$C\frac{d(e_L)}{dt} = i_{CR} - i_R = K_q\frac{d(x_i)}{dt} - \frac{e_L}{R}$$

or $$RC\frac{d(e_L)}{dt} + e_L = K_q R\frac{d(x_i)}{dt}$$

$$\tau\frac{d(e_L)}{dt} + e_L = K\tau\frac{d(x_i)}{dt}$$

where, $$K = \text{sensitivity} = \frac{K_q}{C}\ \text{V/m}$$

Taking Laplace transform:

$$(\tau s + 1)\,E_L(s) = K\tau s\,X_i(s)$$

$\therefore$ Transfer function:

$$\frac{E_L(s)}{X_i(s)} = \frac{K\tau s}{1 + \tau s}$$

Sinusoidal transfer function:

$$\frac{E_L}{X_i}(j\omega) = \frac{j\omega K\tau}{1 + j\omega\tau}$$

Solution : 2

Digital to Analog Converter: In D/A conversion, the input to the converter is in the form of bits, 0 or 1 (binary digit) or as BCD and is converted to a voltage or current proportional to the digital value, A general block diagram is,

$a_0, a_1, a_2, \ldots, a_{n-2}, a_{n-1}$ is the word length for n bits. V_R is the reference voltage and it is the maximum voltage possible at the analog output.

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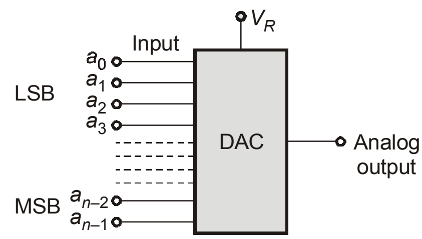
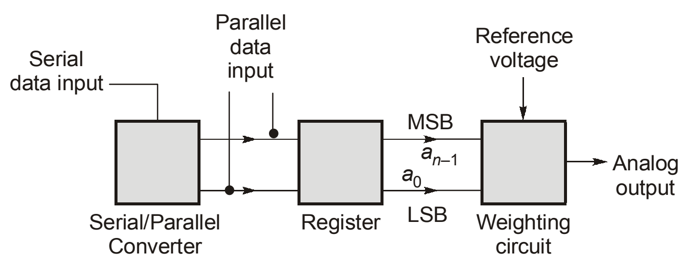
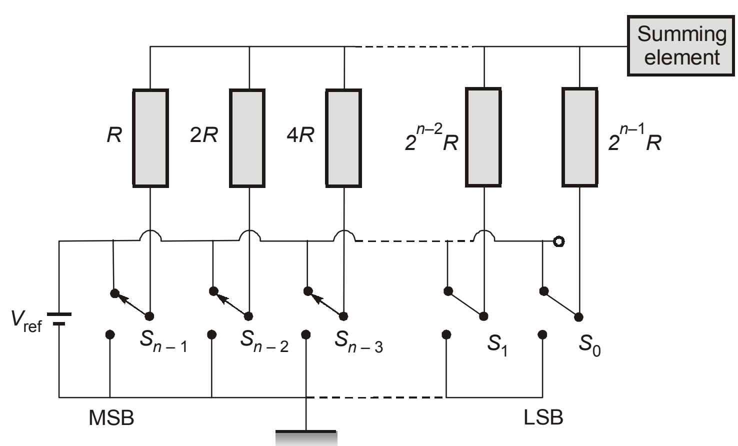


In general, analog output = $K \times$ digital input
where, K is the proportionality factor and a constant for given DAC.

Binary-Weighted resistance D/A Converter: A general structure of a digital to analog converter is



We will consider the weighting circuit only and is for a 4 bit as follows:



The major components of the binary weighted resistance DAC are

1. a weighted resistor network, R to $2^{n-1}R$.
2. n switches one for each bit applied to input.
3. reference voltage V_{ref}.
4. Summing element which sums up the currents flowing in the resistors to give an output proportional to input, usually an op-amp.

When a particular switch is closed, current flows through the connected resistor and the circuit is complete through the summing element. So is the LSB switch and when it is closed current is

$$I(\text{LSB}) = \frac{V_{ref}}{2^{n-1}R}$$

This current is the least.

When the MSB switch S_{n-1} is closed.

$$\text{Current } I\,(\text{MSB}) = \frac{V_{\text{ref}}}{R}$$

and is the highest. So the currents are in multiples of 2, 2^{n-1}. When the switch is connected to supply the bit $a_i = 1$ and when grounded $a_i = 0$. The resistors need not be in multiples of 2. It can be in submultiples of 2, $2^{-(n-1)}$. If all the switches are closed, all the resistors come in parallel. If, say, two switches are closed these two resistors are in parallel while the grounded resistors are in parallel with the load (input resistance of summing element). A binary number N of n-bits is

$$N = a_0\, 2^0 + a_1\, 2^1 + a_2\, 2^2 + \ldots + a_{n-2}\, 2^{n-2} + a_{n-1}\, 2^{n-1}$$

$$N = \sum_{i=0}^{n-1} a_i\, 2^i \qquad \ldots(v)$$

The current flowing into the summing element assuming to be shorted is

$$I_{SC} = S_{n-1}\frac{V_R}{R} + S_{n-2}\frac{V_R}{2R} + \ldots + \frac{S_1 V_R}{2^{n-2} R} + \frac{S_0 V_R}{2^{n-1} R} \qquad \ldots(vi)$$

If each switch position corresponds to one bit

$S_{n-1} = a_{n-1}$; $S_{n-2} = a_{n-2}$; ... $S_1 = a_1$; $S_0 = a_0$

then equation (vi) can be written as

$$I_{SC} = \frac{V_R}{2^{n-1} R}\left(a_{n-1}\, 2^{n-1} + a_{n-2}\, 2^{n-2} + \ldots + a_1\, 2^1 + a_0\, 2^0\right)$$

$$I_{SC} = \frac{V_R}{2^{n-1} R}\sum_{i=0}^{n-1} a_i\, 2^i \qquad \ldots(vii)$$

The output voltage $I_{SC} R$ of DAC is proportional to the bit corresponding to switches connected to V_R i.e., $a_i = 1$.

Maximum output current will flow when all switches are closed i.e., $a_i = 1$.

$$I_{\max} = \frac{V_R}{R} \cdot \frac{2^n - 1}{2^{n-1}} \qquad \ldots(viii)$$

Similarly, when all switches are closed, all the resistors come in parallel so that total resistance

$$R_T = \frac{V_R}{I_{\max}} = \frac{2^{n-1} R}{2^n - 1} \qquad \ldots(ix)$$

$I_{\min}$ will be only when one resistor is through,

$$I_{\min} = \frac{V_R}{R} \cdot \frac{1}{2^{n-1}}$$

Solution : 3

DVM stands for Digital Voltmeter

Following are the type of DVM

1. Ramp type DVM
2. Integrating type DVM
3. Potentiometric type DVM
4. Successive approximation type DVM
5. Flash or parallel type DVM

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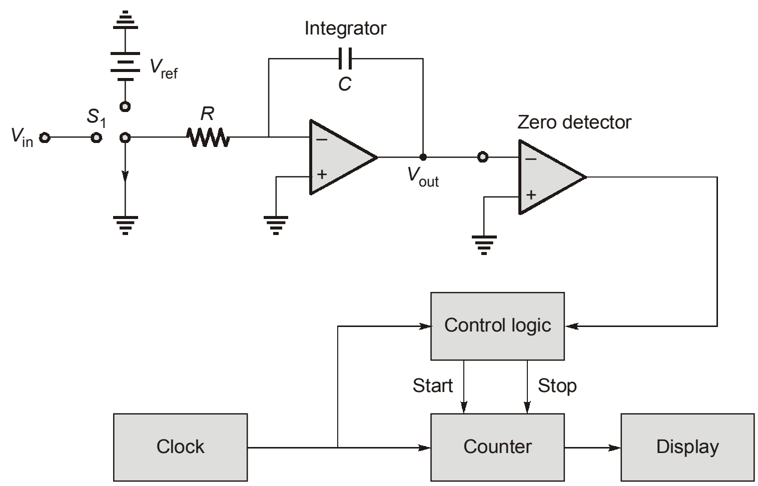
Integrating type DVM: Integrating type DVM has following components:

- Integrator
- Electronically controlled switches
- Counter
- Clock
- Control logic
- Zero comparator

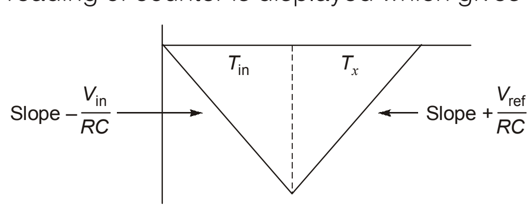
Working: Block diagram is given as:



Integrating type Digital Voltmeter

A dual slope integrator based DVM integrates an unknown input voltage V_{in} for fixed time T_{in} and then de-integrates in time T_x using a known reference voltage V_{ref}.

- At $t = 0$, S_1 is set so as to integrates the input V_{in} for T_{in}. So at $t = T_{in}$, $V_{out} = -\frac{V_{in}T_{in}}{RC}$.
- When S_1 is set counter began to count clock pulse. The counter is reset to zero at $t = T_{in}$.
- At $t = T_{in}$, S_1 is set so $-V_{ref}$ is the input to the integrator with initial voltage $\frac{V_{in}T_{in}}{RC}$.
- The integrator voltage drops to zero with time T_x and slope $+\frac{V_{ref}}{RC}$.
- A comparator used to determine when the output voltage of integrator cross zero.
- When it is zero, the reading of counter is displayed which gives V_{in}.



Let N_1 = Number of pulses counted during T_{in}

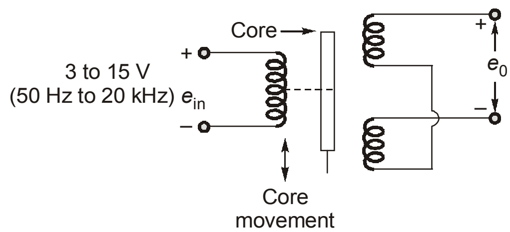
N_2 = Number of pulses counted during T_x

$$\frac{V_{in}}{V_{ref}} = \frac{N_1}{N_2} = \frac{T_{in}}{T_x}$$

Solution : 4

LVDT works on the principle of change of mutual inductance. For LVDT frequency limit is 50 Hz - 20 kHz. Voltage range of LVDT is 3 V to +15 V. Lower voltage is due to the fact that insulation requirement is reduced which reduces the size of LVDT. Also higher frequency implies more sensitivity and lesser power requirement. Range of frequency is limited by the eddy current loss.

In LVDT there are two secondaries identical in nature which are connected in opposition as shown below.

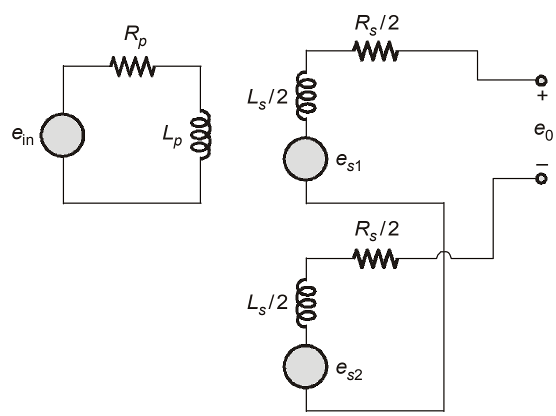


Advantage of using this connection is that when core is at geometric null position, the output voltage 'e_0' will be zero.

Let $i_p \rightarrow$ Current in primary

R_p, $L_p \rightarrow$ Resistance and inductance of primary

M_1 $M_2 \rightarrow$ Mutual inductance between primary and secondary winding.



Equivalent circuit

Applying KVL on primary side,

$$i_pR_p + L_p\frac{di_p}{dt} - e_i = 0$$

$$\therefore \qquad i_p = \frac{e_i}{R_p + L_pD} \qquad \text{...(1)}$$

$$D = \frac{d}{dt}$$

$$e_{s1} = M_1\frac{di_p}{dt}$$

$$e_{s2} = M_2\frac{di_p}{dt}$$

$$e_0 = e_{s1} - e_{s2} = (M_1 - M_2)\frac{di_p}{dt} \qquad \text{...(2)}$$

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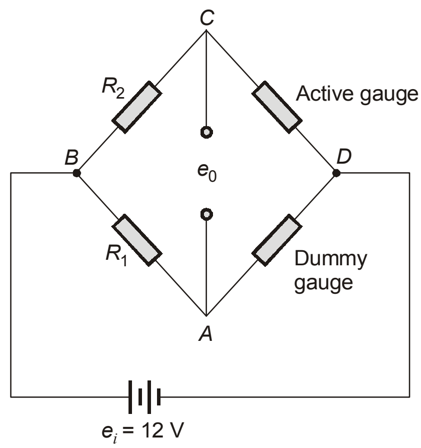
$(M_1 - M_2)$ varies linearly with the core motion.

From equation (1) and (2), $$e_0 = \frac{[(M_1 - M_2)] \cdot D}{L_p D + R_p} \cdot e_i = \frac{\left[(M_1 - M_2)/R_p\right] D}{\tau_p D + 1} e_i$$

$$\tau_p = \frac{L_p}{R_p}$$



Bridge has equal arms when no strain. It has only one active gauge.

$$e_0 = \text{Offset voltage}$$

$$= \frac{G_f \in e_i}{2}, \text{ for one active gauge}$$

$$G_f = \text{Gauge factor} = 2.0$$

$$\in \rightarrow \text{Strain}$$

$$= 1250 \times 10^{-6} \text{ m/m}$$

$$e_i = \text{Input voltage} = 12 \text{ V}$$

$$e_0 = \frac{G_f \in e_i}{2} = \frac{(2)(1250 \times 10^{-6})(12)}{2} = 15 \text{ mV}$$

$$e_0 = 15 \text{ mV}$$

Solution : 5

(a) Uses of capacitive transducer:

1. Capacitance transducer can be used for measurement of linear and angular displacement. They can measure extremely small displacements e.g. 0.1×10^{-6} mm. They can also be used for distance measurement like 30 m.
2. They can be used to measure force and pressure by converting them to displacement.
3. They can be used to measure pressure directly in cases where dielectric constant of a medium changes with pressure e.g. benzene.
4. They can be used to measure humidity in gases as dielectric constant of gas changes with humidity.

(b) Maximum acceleration that is to be measured,

$$a_m = 100 \text{ m/s}^2$$

$$a_m = \omega^2 x_m$$

$$\omega = 2\pi f = 20\pi = 62.8 \text{ rad/sec}^2$$

$$x_m = \frac{100}{(62.8)^2} = 2.53 \text{ cm}$$

For $a = 0, \quad x = 0$

$\therefore$ range of displacement for transducer = 0 to 2.53 cm

Solution : 6

The sensitivity with a DC voltmeter is calculated as follows:

$$S_{DC} = \frac{I}{I_{fsd}} = \frac{1}{100}\,\mu\text{A} = 10\,\text{k}\Omega/\text{V}$$

The multiplier resistor R_s can be calculated as follows:

$$R_s = S_{DC} \times \text{range} = 10 \text{ k}\Omega/\text{V} \times 10 \text{ V} = 100 \text{ k}\Omega$$

The voltage across resistor R_2 read by the voltmeter can be obtained as follows:

$$ER_2 = \frac{R_2 \parallel R_s}{R_1 + R_2 \parallel R_s} \times e = \frac{10\text{ k} \parallel 100\text{ k}}{10\text{ k} + 10\text{ k} \parallel 100\text{ k}} \times 10\,V$$

$$= \frac{9.09\text{ k}}{10\text{ k} + 9.09\text{ k}} \times 10\text{ V} = \frac{9.09\text{ k}}{19.09\text{ k}} \times 100\text{ V} = 4.76\text{ V}$$

The reading obtained with AC voltmeter using half-wave rectifier is calculated as follows:

$$S_{hw} = 0.45 \times S_{DC} = 0.45 \times 10 \text{ k}\Omega/\text{V} = 4.5 \text{ k}\Omega/\text{V}$$

$$R_s = S_{DC} \times \text{range} = 4.5 \text{ k}\Omega/\text{V} \times 10 \text{ V} = 45 \text{ k}\Omega$$

The voltage read by the AC voltmeter is calculated as follows:

$$E = \frac{R_2 \parallel R_s}{R_1 + R_2 \parallel R_s} \times E = \frac{45\text{ k} \parallel 10\text{ k}}{10\text{ k} + 10\text{ k} \parallel 45\text{ k}} \times 10\text{ V}$$

$$= \frac{8.18\text{ k}}{10\text{ k} + 8.18\text{ k}} \times 10\text{ V} = \frac{8.18\text{ k}}{18.18\text{ k}} \times 10\text{ V} = 4.499\text{ V}$$

And finally the reading obtained with AC voltmeter using full-wave rectifier is calculated as follows:

$$S_{fw} = \frac{0.90 \times S_{DC}}{V} = 0.90 \times 10 \text{ k}\Omega/\text{V} = 9.0 \text{ k}\Omega/\text{V}$$

$$R_s = S_{fw} \times \text{range} = 9.0 \text{ k}\Omega/\text{V} \times 10 \text{ V} = 90 \text{ k}\Omega$$

The voltage read by the AC voltmeter is calculated as follows:

$$E = \frac{R_2 \parallel R_s}{R_1 + R_2 \parallel R_s} \times E = \frac{90\text{ k} \parallel 10\text{ k}}{10\text{ k} + 10\text{ k} \parallel 90\text{ k}} \times 10\text{ V}$$

$$= \frac{9\text{ k}}{10\text{ k} + 9\text{ k}} \times 10\text{ V} = \frac{9\text{ k}}{19\text{ k}} \times 10\text{ V} = 4.73\text{ V}$$

As can be seen an AC voltmeter using half wave or full wave rectifier has more loading effect than DC voltmeter.

Solution : 7

Given:

Area of the piezoelectric transducer,

$$A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

Thickness, $t = 1 \text{ mm} = 10^{-3} \text{ m}$

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Young's modulus, $Y = 9 \times 10^{10}$ Pa,
Charge sensitivity, $d = 20$ pC/N $= 20 \times 10^{-12}$ C/N
Relative permittivity, $\varepsilon_r = 5, \varepsilon = \varepsilon_0\varepsilon_r = 4.405 \times 10^{-11}$ F/m
Resistivity, $\rho = 10^{14}$ Ω-cm $= 10^{10}$ Ω-m
Parallel resistance, $R = 100$ MΩ-cm $= 10^{10}$ Ω-m
Parallel capacitance, $C = 20$ pF $= 20 \times 10^{-12}$ F
Therefore,
Resistance of the piezoelectric transducer,

$$R_p = \frac{\rho A}{t} = 10^{11}\ \Omega$$

Capacitance of the piezoelectric transducer,

$$C_p = \frac{\varepsilon A}{t} = 4.425 \times 10^{-12}\ \text{F}$$

Equivalent resistance, $R_{eq} = R_p \parallel R \simeq 10^8\ \Omega$
Equivalent capacitance, $C_{eq} = C_p + C = 24.425 \times 10^{-12}$ F
Time constant, $\tau = R_{eq}C_{eq} = 24.425 \times 10^{-4}$ s

The applied force is sinusoidal with an amplitude of 0.02 N, i.e. with a peak-to-peak value, $(F)_{p-p} = 0.04$ N and its angular frequency, $\omega = 10^3$ rad.

(a) Therefore, we get,

$$(e_0)_{p-p} = \frac{d}{C_{eq}} \frac{1}{\sqrt{1+\left(\frac{1}{\omega\tau}\right)^2}} (F)_{p-p}$$

$$= \frac{2 \times 10^{-12}}{24.425 \times 10^{-12}} \times \frac{1}{\sqrt{1+\frac{1}{(10^3 \times 24.425 \times 10^{-4})^2}}} \times 0.04\ \text{V} \simeq 2.8\ \text{mV}$$

(b) Since, Young's modulus, $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta t / t}$, we have,

$$(\Delta t)_{p-p} = \frac{(F)_{p-p} t}{AY} = \frac{0.04 \times 10^{-3}}{10^{-4} \times 9 \times 10^{10}}\ \text{m} \simeq 4.4 \times 10^{-12}\ \text{m} = 4.4\ \text{pm}$$

Solution : 8

(i) The noise voltage is:

$$E_n = 2\sqrt{kTR\Delta f} = 2\sqrt{1.38 \times 10^{-23} \times 300 \times 120 \times 100 \times 10^3}\ \text{Volt}$$
$$= 0.446\ \mu\text{V}$$

Ratio of signal voltage to noise voltage is,

$$\frac{0.12 \times 10^{-3}}{0.446 \times 10^{-6}} = 269$$

In this case the noise voltage is negligible as compared with the signal voltage and therefore the interference duty to noise is insignificant and thus doesn't distort the signal. Hence, the output is faithfully reproduced.

(ii) Assume a linear relationship between the output voltage of the bridge and the applied pressure.

∴ Output (signal) voltage, when the applied pressure is 7 kN/m^2

$$= \frac{7\times10^3}{7000\times10^3}\times0.12\times10^{-3}\text{ Volt} = 0.12\times10^{-6} = 0.12\ \mu\text{V}$$

∴ Ratio of signal voltage to noise voltage $= \dfrac{0.12\times10^{-6}}{0.446\times10^{-6}} = 0.27$

This indicates that the noise has a magnitude which is about 3.7 times that of signal, hence the signal will be completely lost in the noise. Amplification of the signal provides no solution to the problem as the signal and noise voltages will be equally amplified i.e., the ratio of signal voltage to noise voltage will remain the same at the output.

Solution : 9

(a) We have gauge factor, $G_f = \dfrac{\Delta R/R}{\Delta L/L}$

and therefore change in length, $\Delta L = \dfrac{\Delta R/RL}{G_f} = \dfrac{(0.013/240)}{2.2}\times0.1 = 2.46\times10^{-6}$ m

Strain, $s = E_s = \dfrac{207\times10^9\times2.46\times10^{-6}}{0.1} = 5.092\times10^6$ N/m^2

Force, $F = sA = 5.092\times10^6\times4\times10^{-4}$

$= 2.037\times10^3$ N

(b) Difference mode gain, $A_d = \dfrac{v_0}{v_d} = \dfrac{3}{30\times10^{-3}} = 100$

Common mode gain, $A_c = \dfrac{v_0}{v_c} = \dfrac{5\times10^{-3}}{500\times10^{-3}} = 0.01$

∴ Common mode rejection ratio CMRR

$$= \frac{A_d}{A_c} = \frac{100}{0.01} = 10000 = 80\text{ dB}$$

Solution : 10

(a) The reading is 05 00 000 μs or

Reading $= 5000\,000\ \mu\text{s} = 5\times10^6\ \mu\text{s}$

0.005 percent of reading $= \pm\dfrac{0.005}{100}\times5\times10^6 = \pm\,250\ \mu\text{s}$

The digit in LSD have a value of 1 μs

∴ Maximum error $= \pm250 \pm1 = \pm251\ \mu\text{s}$

(b) The reading is 00 000 500 s or Reading = 500 s

0.005 percent of reading $= \pm\dfrac{0.005}{100}\times500 = \pm0.025$ s

The digit in LSD has now a value of 1 s

∴ Maximum error $= \pm0.025 \pm1 = \pm1.025$ s

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(c) Maximum accuracy means minimum error.

Minimum error is obtained when the time is read on the μs readout.

$$500s = 500 \times 10^6\ \mu s = 500\ 000\ 000\ \mu s$$

This readout requires 9 digit positions and hence when the meter is put on the readout will show an overflow as the meter has 8 digit display. Hence a reading of 500s cannot be taken the μs range.

Let us try the *ms* readout.

$$500s = 500 \times 10^3\ \text{ms} = 500\ 000\ \text{ms}$$

∴ The *ms* readout will display the reading as 00500 000

$$0.005 \text{ percent of reading} = \pm\frac{0.005 \times 500 \times 10^3}{100} = \pm 25\ \text{ms}$$

The LSD has a value of 1 ms.

∴ Maximum possible accuracy with which a reading of 500s can be read by this meter is ±25±1

$$= \pm 26\ \text{ms}$$

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