2020

# RANK Improvement WORKBOOK

## Answer key and Hint of Objective & Conventional *Questions*

## Electrical Engineering

### Signals and Systems

MADE EASY Publications

# 1 Continuous Time Signals & Systems

## LEVEL 1 Objective Solutions

1. (d)
2. (d)
3. (2)
4. (8)
5. (0.303)
6. (d)
7. (c)
8. (a)
9. (b)
10. (d)
11. (b)
12. (c)

## LEVEL 2 Objective Solutions

13. (c)
14. (a)
15. (d)
16. (1)
17. (b)
18. (c)
19. (c)
20. (b)
21. (d)
22. (a)
23. (b)
24. (a)

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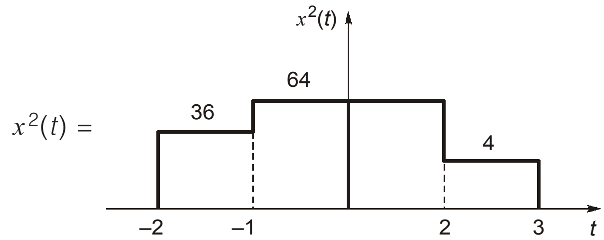
## LEVEL 3 Conventional Solutions

**Solution : 1**




$$\text{Energy of } x(t) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-2}^{-1} 36dt + \int_{1}^{2} 64dt + \int_{2}^{3} 4dt$$

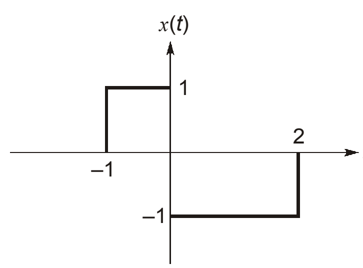
$$= 36 \times 1 + 64 \times 3 + 4 \times 1 = 232 \text{ J}$$

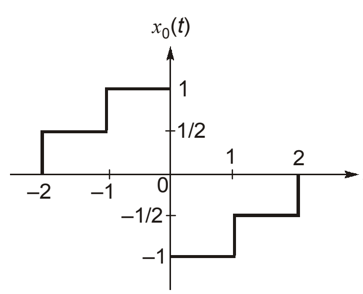
**Solution : 2**

(i) $h(t) = e^{-2(t-1)}\,u(t-1)$

(ii) Yes

**Solution : 3**



**Solution : 4**

$$x(t) = \delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t)$$

$\therefore$
$$h(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(t)dt$$

$\Rightarrow$
$$h(t) = \left[u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)\right] \times \frac{1}{T}$$

The system is not causal as we can clearly see from equation (i) that for calculation of $y(t)$ at any time $t$, we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any causal system.

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# 2 Discrete Time Signals & Sysems

## LEVEL 1 Objective Solutions

| | |
|---|---|
| 1. | (b) |
| 2. | (12) |
| 3. | (b) |
| 4. | (c) |
| 5. | (3) |
| 6. | (c) |
| 7. | (1) |
| 8. | (d) |
| 9. | (d) |
| 10. | (b) |
| 11. | (b) |



## LEVEL 2 Objective Solutions

| | |
|---|---|
| 12. | (a) |
| 13. | (0.5) |
| 14. | (1) |
| 15. | (a) |
| 16. | (b) |
| 17. | (c) |
| 18. | (d) |
| 19. | (b) |
| 20. | (b) |
| 21. | (d) |
| 22. | (a) |
| 23. | (c) |
| 24. | (c) |

## LEVEL 3 Conventional Solutions

**Solution : 1**

(i) Non-linear

(ii) Time variant

**Solution : 2**

$$h[n] = \frac{3}{2}\delta[n]$$

**Solution : 3**

(i) No

(b) Yes

**Solution : 4**

So,
$$\sum_{n=-\infty}^{\infty} h(k) = \sum_{n=0}^{\infty} |a^n| + \sum_{n=-\infty}^{-1} |b^n|$$

$$\sum_{n=-\infty}^{-1} |b^n| = \sum_{n=-\infty}^{-1} |b|^n = \sum_{n=1}^{\infty} |b|^{-n}$$

**Solution : 5**

$$x[n] = \sin\left[\frac{3\pi}{7}n + \frac{\pi}{4}\right] + \cos\frac{\pi}{4}n$$

$$\Rightarrow \quad x[n + n_0] = \sin\left[\frac{3\pi}{7}(n + n_0) + \frac{\pi}{4}\right] + \cos\left(\frac{\pi}{4}(n + n_0)\right)$$

$$= \sin\left[\frac{3\pi}{7}n + \frac{3\pi}{7}n_0 + \frac{\pi}{4}\right] + \cos\left[\frac{\pi}{4}n + \frac{\pi}{4}n_0\right]$$

The sine function here has a period of $n_0 = 14$, as $n_0$ can only be an integer. The cosine function here has a period of $n_0 = 8$, as $n_0$ can only be an integer.

Thus, the fundamental period of $x[n]$ is LCM of 14 and 8, which is 56.

**Solution : 6**

$$\Rightarrow \quad y(n) = \sum_{k=-\infty}^{\infty} h_2(k)x(n-k)$$

$$\Rightarrow \quad y(n) = \sum_{k=1}^{\infty} x(n-k)$$

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# 3 Continuous Time Fourier Series

## LEVEL 1 Objective Solutions

| | |
|---|---|
| 1. | (d) |
| 2. | (d) |
| 3. | (c) |
| 4. | (a) |
| 5. | (0.5) |
| 6. | (a) |
| 7. | (a) |
| 8. | (c) |
| 9. | (b) |
| 10. | (a) |
| 11. | (d) |
| 12. | (a) |
| 13. | (b) |

## LEVEL 2 Objective Solutions

| | |
|---|---|
| 14. | (d) |
| 15. | (a) |
| 16. | (c) |
| 17. | (a) |
| 18. | (b) |
| 19. | (d) |
| 20. | (b) |

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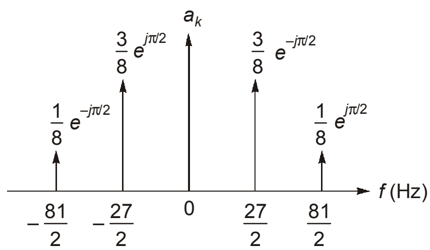
## LEVEL 3 Conventional Solutions

**Solution : 1**

(i)




(ii) $T = \frac{2}{27}$ sec

**Solution : 2**

$$x(t) = 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t$$

**Solution : 3**

(a) Half wave symmetry.

(b) $x(t) = -4\sin 10\pi t + 6\cos 30\pi t + 8\cos 50\pi t + 2\sin 70\pi t$

**Solution : 4**

Given that
$$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t} \qquad \text{...(iii)}$$

$\therefore$
$$f_1(t) + f_2(t) = 2\cdot\frac{3}{4+(3\pi)^2}\cos 3\pi t = \frac{6}{4+(3\pi)^2}\cos 3\pi t$$

Comparing with $A\cos 3\pi t$, we have, $A = \frac{6}{4+9\pi^2} = 6.464\times 10^{-2}$

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# 4 Fourier Transform and Sampling Theorem

## LEVEL 1 Objective Solutions

1. (a)
2. (d)
3. (1)
4. (–12.56)
5. (b)
6. (c)
7. (7)
8. (520)
9. (b)
10. (c)
11. (a)
12. (b)
13. (c)
14. (c)

## LEVEL 2 Objective Solutions

15. (d)
16. (2.41)
17. (0)
18. (b)
19. (a)
20. (a)
21. (b)
22. (c)
23. (c)
24. (a)
25. (a)

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## LEVEL 3 Conventional Solutions

**Solution : 1**

(i) $E_y = \dfrac{1000}{3\pi}$ J

(ii) $\omega_s = 20$ rad/sec

**Solution : 2**

$$x(t) = \frac{1}{\pi t}[\cos t - 1]$$

**Solution : 3**

(i) Fourier transform of $x_e(t) = \text{Re}\{X(\omega)\}$

(ii) Fourier transform of $x_0(t) = j\,\text{img}\{X(\omega)\}$

**Solution : 4**

$$T_s = \frac{1}{f_s} = \frac{1}{5000}\text{sec}$$

$$T_s = 2 \times 10^{-4} \text{ sec}$$

The sampling period given in question is greater than the maximum allowable sampling period. Thus, it cannot be recovered using a low pass filter.

**Solution : 5**

$$\left.\begin{aligned} C_2 &= \frac{1}{8j - 8\pi}, & C_{-2} &= \frac{1}{-8j - 8\pi} \\ C_3 &= \frac{1+j}{2\sqrt{2}(4 + j6\pi)} & C_{-3} &= \frac{1-j}{2\sqrt{2}(4 - j6\pi)} \end{aligned}\right\}$$

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# 5 Laplace Transform

## LEVEL 1 Objective Solutions

1. (a)
2. (d)
3. (1)
4. (c)
5. (c)
6. (−2)
7. (b)
8. (d)
9. (b)
10. (a)
11. (d)

## LEVEL 2 Objective Solutions

12. (c)
13. (b)
14. (a)
15. (c)
16. (d)
17. (a)
18. (1)
19. (a)
20. (b)
21. (a)
22. (c)
23. (b)

## LEVEL 3 Conventional Solutions

**Solution : 1**

$$\frac{2}{s(s+4)}$$

**Solution : 2**

$$X(s)=\frac{2}{s}+\frac{1}{s^2}-e^{-s}-\frac{e^{-s}}{s}-\frac{2e^{-3s}}{s^2}+\frac{2e^{-4s}}{s^2}$$

**Solution : 3**

$m = 1$, $\gamma = 2$ and $k = 2$.

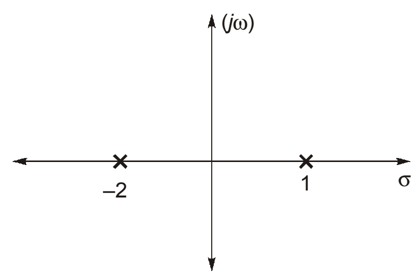
**Solution : 4**

(i)

$$H(s) = \frac{-1/3}{(s+2)}+\frac{1/3}{(s-1)}$$

**(ii)**

pole zero plot of the system is as shown in the figure




thus

1. for system to be causal ROC of $H(s)$ is $Re(s) > 1$

$\therefore$ $$h(t) = \frac{-1}{3}(e^{-2t}-e^{t})u(t)$$

$\because$ $$\frac{1}{s+a} \xleftrightarrow{\ell} e^{-at}\,u(t)$$

2. For system to be stable ROC must contain ($j\omega$) axis

$\therefore$ $$-2 < Re(s) < 1$$

$\therefore$ $$h(t) = -\frac{1}{3}e^{-2t}u(t)-\frac{1}{3}e^{t}u(-t)$$

3. For system to be neither causal nor stable

$$Re(s) < -2$$

$\therefore$ $$h(t) = \frac{1}{3}e^{-2t}u(-t)-\frac{1}{3}e^{t}u(-t)$$

**Solution : 5**

$$y(t) = L^{-1}[Y(s)] = L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = L^{-1}\left[\frac{1}{(s+1)}-\frac{1}{(s+2)}\right]$$

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# 6 Discrete Time Fourier Transform

## LEVEL 1 Objective Solutions

1. (a)
2. (c)
3. (c)
4. (b)
5. (2)
6. (d)
7. (0)
8. (b)
9. (b)
10. (a)
11. (c)
12. (b)

## LEVEL 2 Objective Solutions

13. (1759.292)
14. (a)
15. (0.25)
16. (0.25)
17. (b)
18. (b)
19. (1.333)
20. (−77.64)
21. (b)
22. (c)
23. (b)
24. (a)

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## LEVEL 3 Conventional Solutions

**Solution : 1**

$$y[n] = \cos\left(\frac{3\pi}{2}n + \frac{11}{12}\pi\right)$$

**Solution : 2**

(i) $h[n] = \delta[n] + 2\,\delta[n-1] + \delta[n-2]$

(ii) Yes

**Solution : 3**

(a) 6

(b) 2

(c) $4\pi$

**Solution : 4**

(i) $$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$\therefore$ $$X(0) = \sum_{n=-\infty}^{\infty} x[n] = 7$$

(ii) Using central ordinate theorem;

$$\int_{-\pi}^{\pi} X(\Omega)d\Omega = 2\pi x[0] = 0$$

(iii) $$X(\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\pi} = \sum_{n=-\infty}^{\infty} x[n](-1)^n$$

$$= 2 - 1 - 3 - 3 + 2 - 1 + 1 = -3$$

(iv) Using parseval's theorem

$$\int_{\pi}^{\pi} |X(\Omega)|^2 d\Omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= 2\pi\,[4 + 1 + 9 + 9 + 4 + 1 + 1]$$

$$= 58\,\pi$$

**Solution : 5**

$\therefore$ $$y[n] = \frac{\sin(n\pi/2)}{n\pi} * x_1[n] = \frac{\sin(n\pi/2)}{n\pi} * [\delta[n] - \delta[n-1]]$$

$\therefore$ $$h[n] = \frac{\sin(n\pi/2)}{n\pi} - \frac{\sin[(n-1)\pi/2]}{(n-1)\pi}$$

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# 7 z-Transform

## LEVEL 1 Objective Solutions

| | |
|---|---|
| 1. | (a) |
| 2. | (c) |
| 3. | (a) |
| 4. | (b) |
| 5. | (a) |
| 6. | (d) |
| 7. | (2) |
| 8. | (c) |
| 9. | (d) |
| 10. | (b) |
| 11. | (d) |
| 12. | (a) |
| 13. | (a) |

## LEVEL 2 Objective Solutions

| | |
|---|---|
| 14. | (5) |
| 15. | (d) |
| 16. | (b) |
| 17. | (c) |
| 18. | (11) |
| 19. | (b) |
| 20. | (7) |
| 21. | (d) |
| 22. | (c) |
| 23. | (a) |
| 24. | (a) |
| 25. | (b) |
| 26. | (b) |

## LEVEL 3 Conventional Solutions

**Solution : 1**

$$X(z) = \frac{2z^2}{\left(z-\frac{1}{2}e^{j\pi/3}\right)\left(z-\frac{1}{2}e^{-j\pi/3}\right)}; \quad \text{ROC} = |z| > \frac{1}{2}$$

**Solution : 2**

$$R_x[n] = \frac{4}{3}\cdot\left(\frac{1}{2}\right)^{|n|}$$

**Solution : 3**

$$h[n] = \frac{1}{3}[\delta[n-1]+\delta[n]+\delta[n+1]]$$

**Solution : 4**

(i) $$h(n) = 81\left(\frac{1}{3}\right)^{n+2} u(n+2) - 81\left(\frac{1}{3}\right)^{n+1} u(n+1)$$

(ii) Because system is LTI $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

Hence system is stable

For $n < -2 \Rightarrow$ $h(n) = 0$

Hence system is right sided and non-causal.

**Solution : 5**

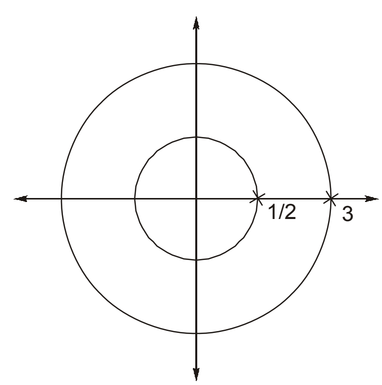
$$H(z) = -\frac{2}{(z^{-1}-2)} - \frac{2}{3\left(z^{-1}-\frac{1}{3}\right)} = -2\left[\frac{1}{(z^{-1}-2)} + \frac{1}{3\left(z^{-1}-\frac{1}{3}\right)}\right]$$

So poles are at $z = 1/2$ and $z = 3$.

**(i)** When the system is stable

ROC must include unit circle

$\therefore$ ROC will be $\frac{1}{2} < |z| < 3$




$\therefore$ $$H(z) = -2\left[\frac{1}{(z^{-1}-2)} + \frac{1/3}{\left(z^{-1}-\frac{1}{3}\right)}\right] \qquad \frac{1}{2} < |z| < 3$$

$$H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{2}{(1-3z^{-1})} \qquad \frac{1}{2} < |z| < 3$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 2.3^n u[-n-1]$$

(ii) When the system is causal

ROC must be outside the outermost pole and include ∞ in ROC

So ROC will be $|z| > 3$

$$\therefore \quad H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{2}{\left(1-3z^{-1}\right)} \qquad |z| > 3$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + 2.3^n u[n]$$

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# 8 Discrete Fourier Transform

## LEVEL 1 Objective Solutions

1. (a)
2. (4)
3. (c)
4. (a)
5. (b)
6. (b)
7. (c)
8. (b)



## LEVEL 2 Objective Solutions

9. (a)
10. (700)
11. (0.2)
12. (c)
13. (c)
14. (b)
15. (c)
16. (b)
17. (a)

## LEVEL 3 Conventional Solutions

**Solution : 1**

$$X_2[k] = (-1)^k X_2[k]$$

**Solution : 2**

(i) $y[n] = \{-3, -6, 3, 6\}$

(ii) $X[k] = 1 - e^{-j\pi k};\ 0 \le k \le 3$ and $H[k] = 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k}$

**Solution : 3**

$y[n] = x[n-2]_5$

So, $y[n] = \{2, 0, 2, 1, 1\}$

**Solution : 4**

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = \begin{bmatrix} p+q+r+s \\ p-jq-r+js \\ p-q+r-s \\ p+jq-r-js \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 \\ \beta^2 \\ \gamma^2 \\ \delta^2 \end{bmatrix}$$

**Solution : 5**

By definition

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}k\cdot n} = \frac{1}{4}\sum_{k=0}^{3} X[k] e^{j\frac{2\pi}{4}kn}$$

$$= \frac{1}{4}\left[1 + 0 + e^{j\frac{2\pi}{4}\times 2n} + 0\right] = \frac{1}{4}[1 + (-1)^n]$$

**Solution : 6**

By defintion

$$X[k] = \sum_{n=0}^{9} x[n] e^{-j\frac{2\pi}{10}k\cdot n}$$

(i)

$$X[0] = \sum_{n=0}^{9} x[n] = 2 + 1 + 1 + 0 + 3 + 2 + 0 + 3 + 4 + 6 = 22$$

(ii) $X[5]$

Here $N = 10$,

So

$$X[5] = X\left[\frac{N}{2}\right] = X\left[\frac{10}{2}\right]$$

$$X[5] = \sum_{n=0}^{9} x[n]e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot n} = \sum_{n=0}^{9} x[n](-1)^n$$

$$X[5] = 2 - 1 + 1 - 0 + 3 - 2 + 0 - 3 + 4 - 6 = -2$$

(iii) $\sum_{k=0}^{9} X[k]$

$\because$
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{-j\frac{2\pi}{N}\cdot kn}$$

Here $N = 10$

So
$$x[0] = \frac{1}{10}\sum_{k=0}^{9} X[k]$$

$$\sum_{k=0}^{9} X[k] = 10\,x[0] = 10 \times 2 = 20$$

(iv) $|X[k]|^2$

According to Parseval's theorem

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$

$\Rightarrow$
$$\sum_{k=0}^{N-1} |X[k]|^2 = N\sum_{n=0}^{N-1} |x[n]|^2$$

$$= 10\,[2^2 + 1^2 + 1^2 + 0^2 + 3^2 + 2^2 + 0^2 + 3^2 + 4^2 + 6^2] = 800 \text{ Watt}$$

**Solution : 7**

$$x(n) = \sum_{k=<N>} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

$\Rightarrow$
$$x(n) = \left(\frac{-1}{2}\right) + \frac{1}{2}e^{j\frac{2\pi}{8}n} + \frac{1}{2}e^{j\frac{14\pi}{8}n}$$

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# 9 Digital Filters

## LEVEL 1 Objective Solutions

| | |
|---|---|
| 1. | (4) |
| 2. | (0.25) |
| 3. | (a) |
| 4. | (a) |
| 5. | (b) |
| 6. | (a) |
| 7. | (c) |
| 8. | (b) |

## LEVEL 2 Objective Solutions

| | |
|---|---|
| 9. | (c) |
| 10. | (b) |
| 11. | (d) |
| 12. | (a) |
| 13. | (b) |
| 14. | (a) |

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## LEVEL 3 Conventional Solutions

**Solution : 1**

(a) $b = -1,\ b = 0.1$

(b) $\omega_c = 2.92$ rad/sec

**Solution : 2**

$$H(z) = \frac{2(1+z^{-1})(1+z^{-1})}{63+62z^{-1}+15z^{-2}} = \frac{1+2z^{-1}+z^{-2}}{31.5+31z^{-1}+7.5z^{-2}}$$

**Solution : 3**

$$H(z) = \frac{0.675z^{-1}}{1-1.347z^{-1}+0.45z^{-2}} = \frac{0.675z}{z^2-1.347z+0.45}$$

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