2020

RANK improvement WORKBOOK

Answer key and Hint of Objective & Conventional *Questions*

Electrical Engineering

Power Electronics

MADE EASY Publications

1 Power Semiconductor Devices

LEVEL 1 Objective Questions

1.	(a)
2.	(b)
3.	(b)
4.	(a)
5.	(d)
6.	(d)
7.	(c)
8.	(b)
9.	(b)
10.	(b)
11.	(b)
12.	(a)
13.	(c)

LEVEL 2 Objective Questions

14.	(d)
15.	(1.51)
16.	(c)
17.	(d)
18.	(b)
19.	(0.67)
20.	(2.93)
21.	(c)
22.	(34.17)
23.	(7)

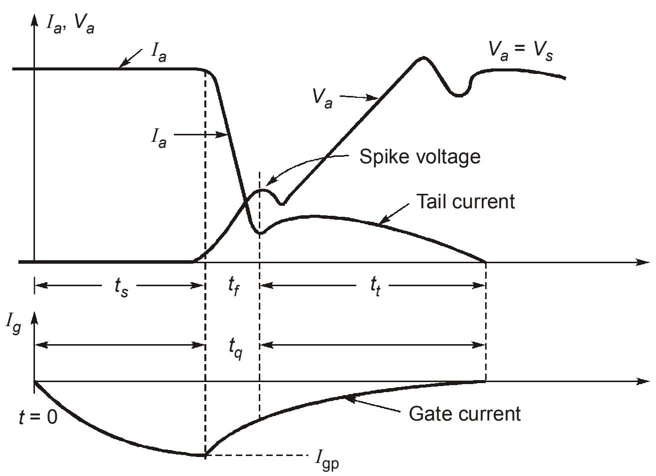
LEVEL 3 Conventional Questions

Solution : 1

Before the initiation of turn-off process, a GTO carries a steady current I_a. This figure shows a typical turn-off dynamic characteristic for a GTO. The total turn off time t_q is subdivided into three different periods; namely the storage period (t_s), the fall period (t_f) and the tail period (t_t). In other words,

$$t_q = t_s + t_f + t_t$$

Initiation of turn-off process starts as soon as negative gate current begins to flow after $t = 0$. The rate of rise of this gate current depends upon the gate circuit inductance and the gate voltage applied. During the storage period, anode current I_a and anode voltage (equal to on-state voltage drop) remain constant. Termination of the storage period is indicated by a fall in I_a and rise in V_a.



During t_s, excess charges, i.e. holes in p-base are removed by negative gate current and the centre junction comes out of saturation. In other words, during storage time t_s, the negative gate current rises to a particular value and prepares the GTO for turning-off (or commutation) by flushing out of the stored carriers. After t_s, anode current begins to fall rapidly and anode voltage starts rising. As shown in figure, the anode current falls to a certain value and then abruptly changes its rate of fall. Interval during which anode current falls rapidly is the fall time t_f, figure and is of the order of 1 μsec [4]. The fall period t_f is measured from the instant gate current is maximum negative to the instant anode current falls to its tail current.

At the time $t = t_s + t_f$, there is a spike in voltage due to abrupt current change. After t_f, anode current I_a and anode voltage V_a keep moving towards their turn-off values for a time t_t called tail time. After t_t, anode current reaches zero value.

A GTO has the following disadvantage as compared to a conventional thyristor:

(i) Magnitude of latching and holding currents is more in a GTO.

(ii) On state voltage drop and the associated loss is more in a GTO.

(iii) Due to the multicathode structure of GTO, triggering gate current is higher than that required for a conventional SCR.

(iv) Gate drive circuit losses are more

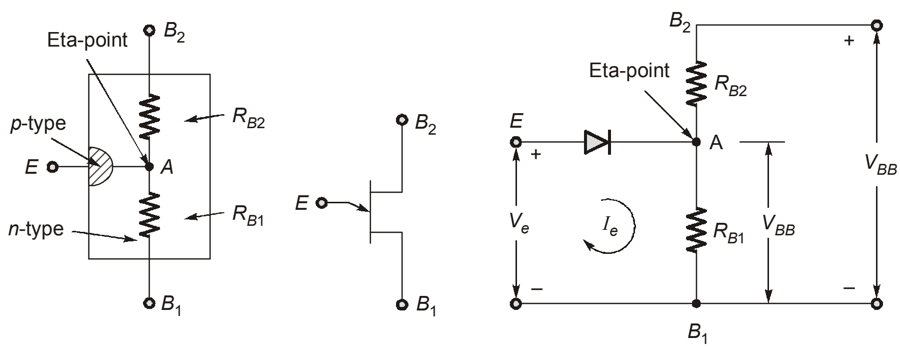
(v) Its reverse-voltage blocking capability is less than its forward-voltage blocking capability. But this is no disadvantage so far as inverter circuits are concerned.

Inspite of all these demerits, GTO has the following advantages over an SCR:

(i) GTO has faster switching speed.
(ii) Its surge current capability is comparable with an-SCR.
(iii) It has more *di*/*dt* rating at turn-on.
(iv) GTO circuit configuration has lower size and weight as compared to SCR circuit unit.
(v) GTO unit has higher efficiency because an increase gate-drive power loss and on-state loss is more than compensated by the elimination of forced commutation losses.
(vi) GTO unit has reduced acoustical electromagnetic noise due to elimination of commutation chokes.

Solution : 2

An UJT is made up of an n-type silicon (*Si*) base to which p-type emitter is embedded. It has three terminals the emitter (*E*), base one B_1 and base two B_2 between bases B_1 and B_2, the Unijunction behave like ordinary resistance.



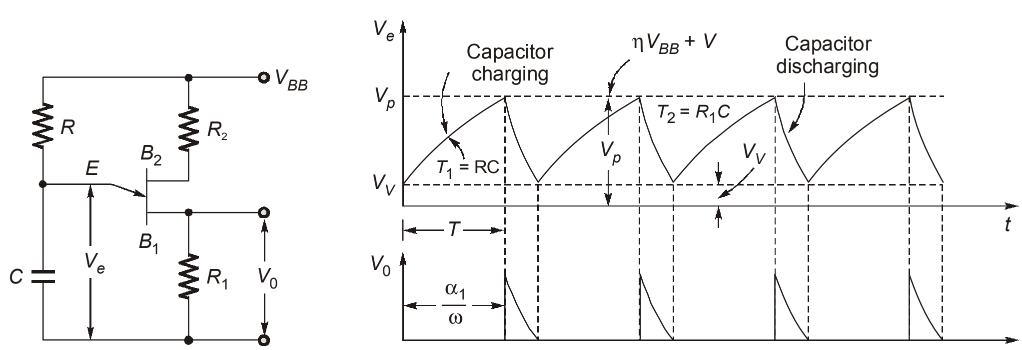
Basic structure of UJT   Symbolic representation   Equivalent circuit

In figure (a), when source voltage V_{BB} is applied, capacitor *C* begins to charge through *R* exponentially towards V_{BB}. During this charging, emitter circuit of UJT is an open circuit.

The capacitor voltage v_c, equal to emitter voltage v_e, is given by

$$v_C = v_e = V_{BB}(1 - e^{-t/RC})$$

The time constant of the charge circuit is τ_1 = RC.



(a)   (b)

Fig. (2) UJT oscillator (a) Connection diagram and (b) Voltage waveforms

When this emitter voltage v_e (or v_c) reaches the peak-point voltage V_p (= η V_{BB} + V_D), the Unijunction between E–B_1 breaks down. As a result, UJT turns on and capacitor *C* rapidly discharge through low resistance R_1 with a time constant $\tau_2 = R_1$ *C*. Here τ_2 is much smaller than τ_1. When the emitter voltage decays to the valley-point voltage V_v, UJT turns off. The time *T* required for capacitor *C* to charge from initial voltage V_v to peak-point voltage V_p, through large resistance *R*, can be obtained as under:

$$V_p = \eta V_{BB} + V_D = V_v + V_{BB}(1 - e^{-T/RC})$$

Assuming, $$V_D = V_v,\ \eta = (1 - e^{-T/RC})$$

$$T = \frac{1}{f} = RC\ln\left(\frac{1}{1-\eta}\right)$$

In case T is taken as the time period of output pulse duration (neglecting small discharge time), then the value of firing angle α_1 is given by

$$\alpha_1 = \omega T = \omega RC\ln\frac{1}{1-\eta}$$

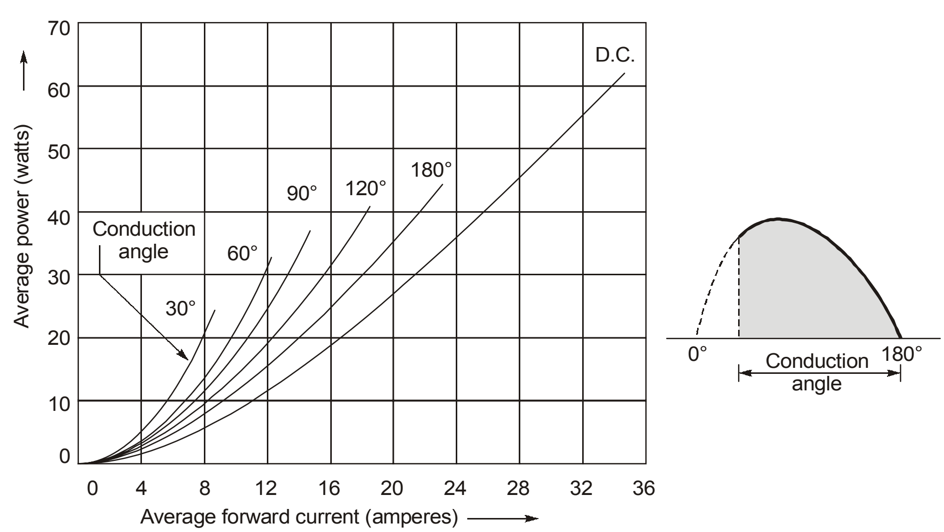
where 'ω' is the angular frequency of UJT oscillator.

Solution : 3

The power-generated in the junction region of a thyristor in a normal operation consists of the following components of dissipation:

1. Forward-conduction
2. Turn-off or commutation
3. Turn-on switching
4. Forward and Reverse blocking
5. Gate pulse triggering

1. **Forward Conduction Loss:** The average anode current multiplied by the forward voltage drop across the SCR is the average power dissipated in the thyristor. ON state conduction losses are the major source of junction heating for normal duty cycle and power-frequencies. figure shown below illustrates the variation of the ON state average conduction loss in watts with the average current in amperes for various conduction angles for operation on from 50 Hz to 400 Hz. This type of information is generally supplied by the manufacturer. The curve marked d.c. is applicable for continuous direct-current These curves are based on the current waveform which is the remainder of the half-sine wave which results when delayed angle triggering is used in a single phase resistive load circuit. These power curves are integrated product of the instantaneous anode current and ON state voltage drop. This integration can be performed graphically or analytically for conduction angles other than those listed, using the ON state voltage-current characteristic curves for the specific device. In the line commutated converters and a.c. regulator circuits, the forward conduction loss is the major source of junction heating.



Average forward power dissipation for sinusoidal current waveform

2. **Turn-on Losses:** Since the switching process takes a finite time, there is a relatively high voltage across the thyristor while a current flows. Therefore, this loss is rather higher than the turn-off loss. For example, by the time the current has reached 90 per cent of its final value, there may still be 10 percent of the supply voltage across the device. Accordingly, appreciable power may be dissipated during this turn-on interval. Above 400 Hz switching, additional circuitry is used to reduce the switching losses or else some derating of the normal forward current is made to allow for the extra dissipation.
3. **Turn-off Losses:** The turn-off power losses arises during the time of decay of reverse current, according to the product of the instantaneous values of reverse current and reverse voltage, may reach high peak values up to several kilowatts. It is possible during rapid turn-off for the reverse current to rise to a value comparable to the forward current. When the thyristor impedance starts to increase, dissipation occurs as the current falls and the reverse voltage builds up. To limit the rate of change of current at turn-off and hence the energy to be dissipated, circuit inductance is used. This also limits the rate of rise of forward current which is an advantage but the inductance can give rise to high reverse voltage transients during turn-off. In high frequency inverters, where the thyristors arc switched ON and OFF several times in each supply cycle, the turn-on and turn-off losses may also have to be taken into consideration while selecting the device ratings since the switching loss may constitute a significant portion of the total loss.
4. **Forward and Reverse Blocking Losses:** As mentioned in the previous section, a thyristor has different regions of operation. In the forward blocking region, anode is made positive with respect to cathode and the anode current is the small forward leakage current Therefore, the forward blocking power loss is the integration of product of the forward blocking voltage and forward leakage current. Similarly, reverse power loss occurs in reverse blocking region. The forward power loss is generally small compared to the conduction-loss.
5. **Gate Power Loss:** The gate power loss is the mean power loss due to gate current between the gate and main terminals. Gate losses arc negligible for pulse types of triggering signals. Losses may become more significant for gate signals with a high duty cycle.

Solution : 4

Average energy loss in the transistor = Average energy loss during turn-off + Average energy loss during turn on

$$\text{Average energy loss during turn on} = \int_0^{t_{on}} i_c\, v_{CE}\, dt$$

$$i_C = \frac{100}{50\times10^{-6}}(t) = m_1 t$$

$$v_{CE} = m_2 t + C = -\frac{200}{40\times10^{-6}}t + 200$$

$$= \int_0^{t_{on}} \left[\frac{100}{50\times10^{-6}}t\right]\left[200 - \frac{200t}{40\times10^{-6}}\right] dt$$

$$= \int_0^{t_{on}} [2\times10^6\, t]\,[200 - 5\times10^6\, t]\, dt$$

$$= \left[\frac{400\times10^{6}\,t^{2}}{2}\right]_0^{t_{on}=40\,\mu s} - \left[\frac{10\times10^{12}\,t^{3}}{3}\right]_0^{t_{on}}$$

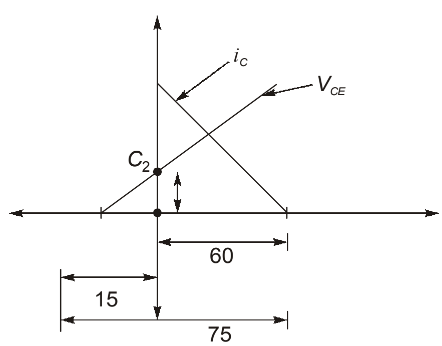
$$= \frac{400\times10^{6}\times(40)^{2}\times10^{-12}}{2} - \left[\frac{10\times10^{12}\times(40)^{3}\times(10^{-6})^{3}}{3}\right]$$

$$= 40\times8000\times10^{-6} - \left[\frac{10\times64\times10^{3}\times10^{-6}}{3}\right]$$

$$= 320000\times10^{-6} - \frac{64}{3}\times10^{-2}$$

$$= \left[32-\frac{64}{3}\right]\times10^{-2} = \frac{32}{3}\times10^{-2} = 0.1067 \text{ Watt-sec}$$

Average power loss during turn-off $= \int_0^{t_{off}} i_C V_{CE}\, dt$



$$C_2 = m_2\times15\times10^{-6}$$

$$= \frac{200\times15\times10^{-6}}{75\times10^{-6}} = 40$$

$$i_C = m_1 t + C = \left[-\frac{100}{60\times10^{-6}}\,t + 100\right]$$

$$v_{CE} = m_2 t + C_2 = \left[\frac{200}{75}\times10^{6}\,t + 40\right]$$

$$= \int_0^{t_{off}=60\,\mu s}\left[100-\frac{100t\times10^{6}}{60}\right]\left[40+\frac{200\times10^{6}t}{75}\right]dt$$

$$= [4000t]_0^{t_{off}} + \left[\frac{200\times10^{6}t^{2}}{75\times2}\times100\right]_0^{t_{off}} - \left[\frac{40\times100\times10^{6}\,t^{2}}{60\times2}\right]_0^{t_{off}} - \left[\frac{100\times10^{6}\times200\times10^{6}}{60\times75}\,\frac{t^{3}}{3}\right]_0^{t_{off}}$$

$= 0.24 + 0.48 - 0.12 - 0.32$

$= 0.28$ Watt-sec

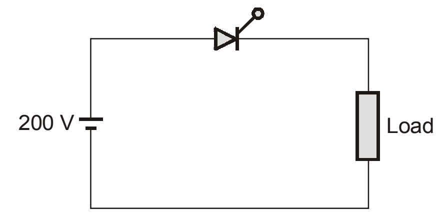
Total energy loss in one cycle $= 0.1067 + 0.2800$

$= 0.3867$ Watt-sec

Average power loss in transistor = Switching frequency × Average energy loss in one cycle

$$f = \frac{300}{0.3867} = 775.79 \text{ Hz}$$

Solution : 5



(a) $$E = L\frac{di}{dt} \Rightarrow 200 = 0.2\frac{di}{dt}$$

$$\Rightarrow \quad 200 = 0.2\frac{I_{\text{latch}}}{t_{\text{gate width}}}$$

$$\Rightarrow \quad t_{\text{gate width}} = \frac{0.2\times 100\times 10^{-3}}{200} = 100\,\mu\text{sec}$$

(b) $$E = Ri + L\frac{di}{dt}$$

$$i = \frac{E}{R}(1-e^{\frac{-R}{L}t})$$

$$\Rightarrow \quad 0.100 = \frac{200}{20}(1-e^{-100t}) \Rightarrow 0.01 = 1 - e^{-100t}$$

$$\Rightarrow \quad e^{-100t} = 0.99 \Rightarrow -100t = -0.010050$$

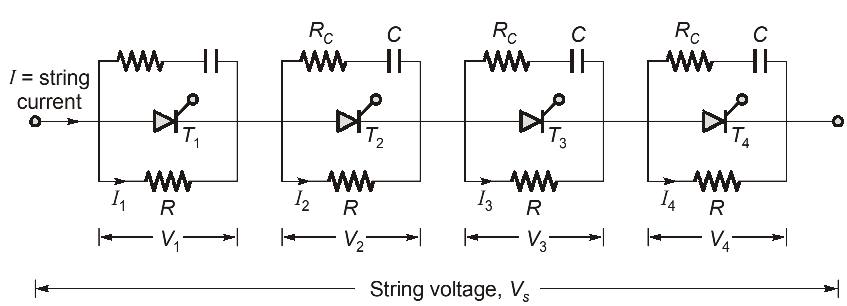
$$t = 100.503\,\mu\text{s}$$

(c) $$i = \frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]$$

$$0.100 = \frac{200}{20}(1-e^{-10t})$$

$$t = 1005.03\,\mu\text{-sec}$$

Solution : 6



I = string current in off state
R = state equalizing resistance
R_C = Dynamic equalizing resistance
C = Dynamic equalizing capacitance
I_1, I_2, I_3, I_4 = Currents through R

(a) During static conditions:
- Capacitances will be out of circuit and hence no current will flow through R_C and C.
- Leakage currents will flow through SCRs.

$$\text{Voltage across } T_1 = (I - 0.022) \times 25000 = V_1$$
$$\text{Voltage across } T_2 = (I - 0.026) \times 25000 = V_2$$
$$\text{Voltage across } T_3 = (I - 0.020) \times 25000 = V_3$$
$$\text{Voltage across } T_4 = (I - 0.018) \times 25000 = V_4$$
$$V_1 + V_2 + V_3 + V_4 = V_5$$
$$25000(4I - 0.085) = 10000$$
$$4I - 0.085 = 0.4$$
$$I = 0.121 \text{ A}$$
$$V_1 = 25000 \times (0.121 - 0.022) = 2475 \text{ V}$$
$$V_2 = 25000 \times (0.121 - 0.026) = 2375 \text{ V}$$
$$V_3 = 25000 \times (0.121 - 0.020) = 2525 \text{ V}$$
$$V_4 = 25000 \times (0.121 - 0.018) = 2575 \text{ V}$$

(b) **During dynamic conditions:**

At time of turn-on capacitors will short-circuit.

$$I_{C_1} = \text{Discharge current through capacitor } 1 = \frac{V_1}{R_1} = \frac{2475}{50} = 49.5 \text{ A}$$

$$I_{C_2} = \text{Discharge current through capacitor } 2 = \frac{V_2}{R_2} = \frac{2375}{50} = 47.5 \text{ A}$$

$$I_{C_3} = \text{Discharge current through capacitor } 3 = \frac{V_3}{R_3} = \frac{2525}{50} = 50.5 \text{ A}$$

$$I_{C_4} = \text{Discharge current through capacitor } 4 = \frac{V_4}{R_4} = \frac{2575}{50} = 51.5 \text{ A}$$

(c)
$$\text{Derating factor} = 1 - \text{string efficiency}$$
$$= 1 - \frac{(\text{Actual voltage of whole string})}{(\text{Number of SCR} \times \text{Individual rating})}$$
$$= 1 - \frac{10000}{4 \times 3000} = 1 - \frac{10000}{12000}$$
$$= 1 - 0.833 = 0.16667 \text{ or } 16.67\%$$

■■■■

2 Controlled & Uncontrolled Rectifiers

LEVEL 1 Objective Questions

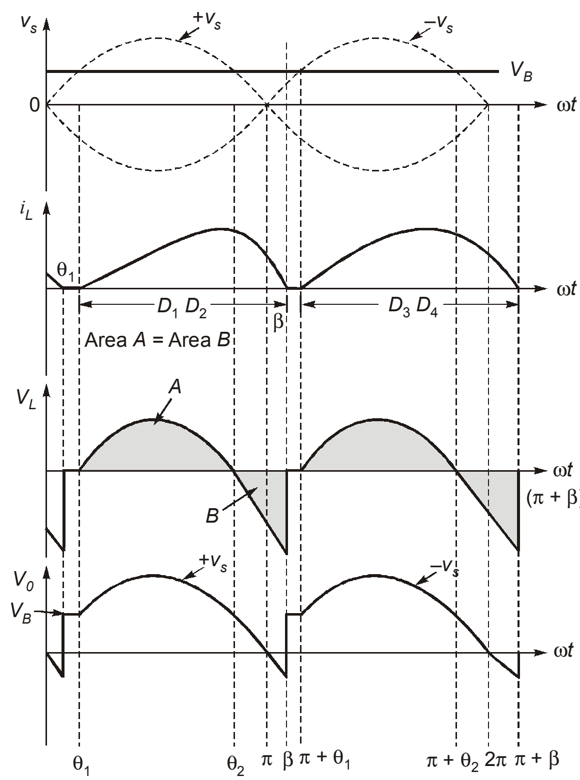
1.	(c)
2.	(c)
3.	(b)
4.	(0.70)
5.	(a)
6.	(147.51)
7.	(1.66)
8.	(0.0170)
9.	(a)
10.	(b)
11.	(b)
12.	(a)
13.	(11.99)
14.	(170)
15.	(b)
16.	(c)
17.	(c)

LEVEL 2 Objective Questions

18.	(b)
19.	(19.96)
20.	(9.28)
21.	(0.955)
22.	(b)
23.	(b)
24.	(d)
25.	(d)
26.	(b)
27.	(10)
28.	(c)
29.	(5.03)
30.	(17.25)

LEVEL 3 Conventional Questions

Solution : 1



$\theta_1 \le \omega t \le \beta$

KVL : $D_1\ D_2 \to$ ON

$$-V_S + V_L + V_B = 0$$

$$V_L = V_S - V_B$$

$$V_L = V_m \,.\, \sin \omega t - V_B$$

$$L\frac{di_L}{dt} = V_m \sin\omega t - V_B$$

$$\int di_L = \frac{1}{L}\int [V_m \sin\omega t - V_B].dt$$

$$i_L = \frac{-V_m}{\omega L}\cos\omega t - \frac{V_B \cdot t}{L} + K$$

Use initiall conditions to find K.

At $\omega t = \theta_1$, $\quad i_L = 0$

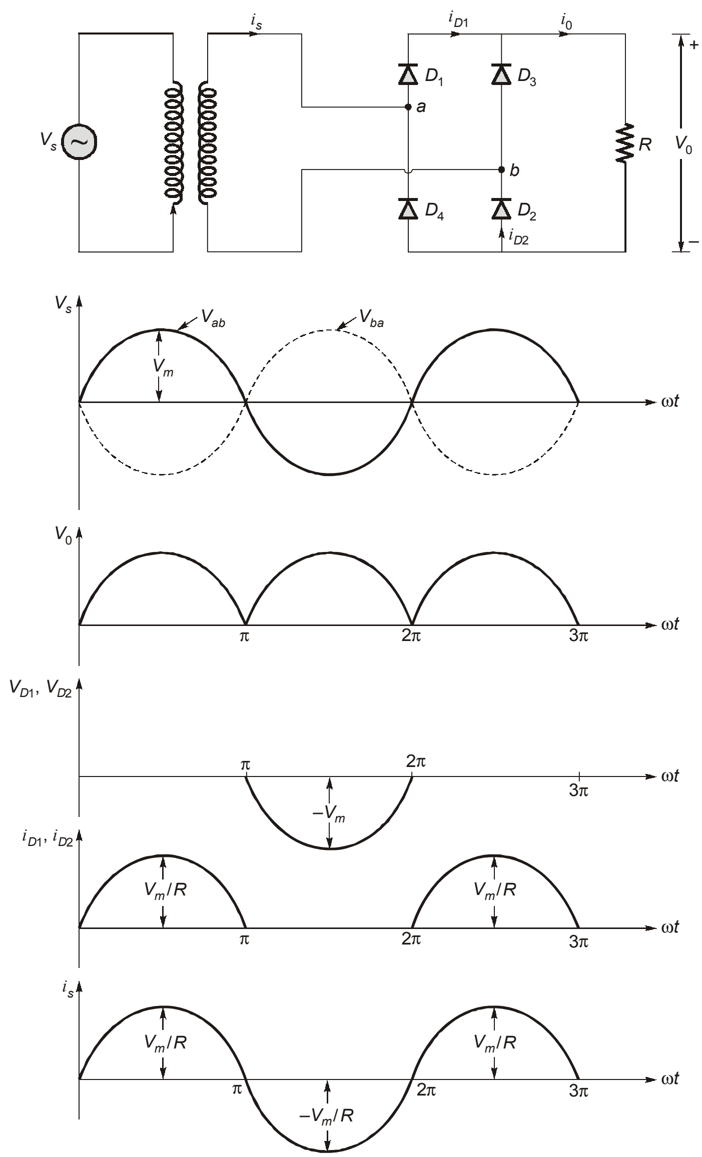
$$0 = \frac{-V_m}{\omega L}\cos\theta_1 - \frac{V_B \cdot \theta_1}{\omega L} + K$$

$$K = \frac{1}{\omega L}\left[V_m \cos\theta_1 + V_B\theta_1\right]$$

$$I_{L\,avg} = \frac{1}{\pi}\int_{\theta_1}^{\beta} i_L \,.d(\omega t)$$

Solution : 2

Single-phase diode bridge rectifier:



When 'a' is positive with respect to 'b' diode D_1, D_2 conduct together so that output voltage is V_{ab}. Each of the diodes D_3 and D_4 is subjected to a reverse voltage of V_s.

When 'b' is positive with respect to 'a', diodes D_3, D_4 conduct together and output voltage is V_{ba}. Each of the two diodes D_1 and D_2 experience a reverse voltage of V_s.

Average value of output voltage,

$$V_0 = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d(\omega t) = \frac{2V_m}{\pi}$$

Average output current, $\quad I_0 = \dfrac{V_0}{R}$

Rms value of output voltage,

$$V_{or} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t\, d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}} = V_s$$

Rms value of load current, $I_{or} = \dfrac{V_s}{R}$

Form factor, $FF = \dfrac{V_{or}}{V_0} = \dfrac{V_m/\sqrt{2}}{2V_m/\pi} = \dfrac{\pi}{2\sqrt{2}} = 1.11$

Ripple voltage, $V_r = \sqrt{V_{or}^2 - V_0^2} = \left[\left(\dfrac{V_m}{\sqrt{2}}\right)^2 - \left(\dfrac{2V_m}{\pi}\right)^2\right]^{1/2} = 0.30776\ V_m$

Voltage ripple factor, $VRF = \dfrac{V_r}{V_0} = \dfrac{0.30776\ V_m}{2V_m/\pi} = 0.4834$

Transformer utilization factor, $TUF = \dfrac{P_{dc}}{VA \text{ rating of transformer}}$

Rms value of source voltage, $V_s = \dfrac{V_m}{\sqrt{2}}$

Rms value of source current, $I_s = \dfrac{I_m}{\sqrt{2}}$

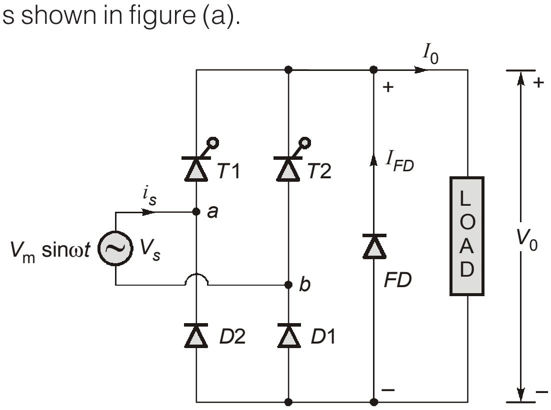
VA rating of transformer $= V_s I_s = \dfrac{V_m}{\sqrt{2}} \cdot \dfrac{I_m}{\sqrt{2}} = \dfrac{V_m I_m}{2}$

$$P_{dc} = V_0 I_0 = \frac{2V_m}{\pi} \cdot \frac{2I_m}{\pi} = \left(\frac{2}{\pi}\right)^2 \cdot V_m I_m$$

So, $TUF = \dfrac{P_{dc}}{VA \text{ rating of transformer}} = \dfrac{\frac{4}{\pi^2} V_m I_m}{V_m I_m / 2} = \dfrac{8}{\pi^2} = 0.81057$

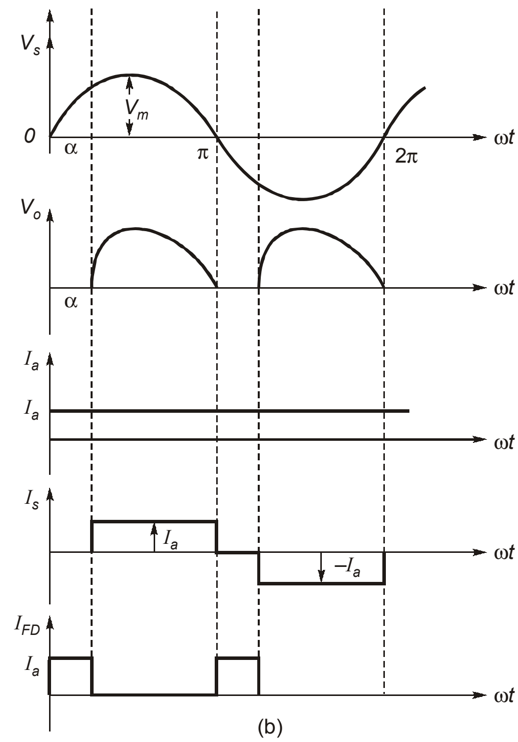
Solution : 3

1-ϕ semiconverter circuit is shown in figure (a).



(a)

- The load current is assumed to be continous and ripple free.
- The various waveforms V_s, V_0, I_a and I_s are shown in figure (b).



(b)

- The instantaneous input current $i_s(t)$ can be expressed in Fourier series as:

$$i_s(t) = a_0 + \sum_{n=1,2,3}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

where,
$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} i_s(t)\,d(\omega t)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cdot\cos n\omega t\cdot d(\omega t)$$

and
$$b_n = \frac{1}{\pi}\int_0^{2\pi} i_s(t)\cdot\sin n\omega t\cdot d(\omega t)$$

$$a_0 = \frac{1}{2\pi}\left[\int_\alpha^{\pi} i_a\cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_a\cdot d(\omega t)\right] = 0$$

$$a_n = \frac{1}{\pi}\left[\int_\alpha^{\pi} i_a \cos n\omega t\cdot d(\omega t) - \int_{\pi+\alpha}^{2\pi} I_a \cos n\omega t\cdot d(\omega t)\right]$$

$$= \frac{I_a}{n\pi}\left[\left|\sin n\omega t\right|_\alpha^\pi\right] - \left[\left|\sin n\omega t\right|_{n+\alpha}^{2\pi}\right]$$

$$= -\frac{2I_a}{n\pi}\sin n\alpha \text{ .... for } n = 1, 3, 5 \text{ ....}$$

$$= 0 \text{ .... for } n = 2, 4, 6 \text{ ....}$$

$$b_n = \frac{1}{\pi}\left[\int_{\alpha}^{\pi} I_a \sin n\omega t \cdot d(\omega t) - \int_{n+\alpha}^{2\pi} I_a \sin n\omega t \cdot d(\omega t)\right]$$

$$= \frac{2I_a}{n\pi}(1+\cos n\alpha) \text{ .... for } \quad n = 1, 3, 5 \text{ ....}$$

$$= 0 \text{ .... for } n = 2, 4, 6$$

$$C_n = \left[\left(-\frac{2I_a}{n\pi}\sin n\alpha\right)^2 + \left(\frac{2I_a}{n\pi}(1+\cos n\alpha)\right)^2\right]^{1/2} = \frac{2\sqrt{2}I_a}{n\pi}(1+\cos n\alpha)^{1/2}$$

as we know, $1 + \cos\theta = 2\cos^2\frac{n\theta}{2}$

$$C_n = \frac{2\sqrt{2}I_a}{n\pi}\left[2\cos^2\frac{n\alpha}{2}\right]^{1/2} = \frac{4I_a}{n\pi}\cos\frac{n\alpha}{2}$$

$$\theta_n = \tan^{-1}\left(\frac{a_n}{b_n}\right) = \tan^{-1}\left[-\frac{\sin n\alpha}{1+\cos n\alpha}\right] = \tan^{-1}\left[-\frac{2\sin\frac{n\alpha}{2}\cdot\cos\frac{n\alpha}{2}}{2\cos^2\frac{n\alpha}{2}}\right] = -\frac{n\alpha}{2}$$

Hence, $$i_s(t) = \sum_{n=1,3,5}^{\infty}\frac{4I_a}{n\pi}\cos\frac{n\alpha}{2}\cdot\sin\left(n\omega t - \frac{n\alpha}{2}\right)$$

Rms value of nth harmonic input current,

$$I_{sn} = \frac{4I_a}{\sqrt{2}\,n\pi}\cos\frac{n\alpha}{2} = \frac{2\sqrt{2}}{n\pi}I_a\cdot\cos\frac{n\alpha}{2}$$

Hence, Rms value of the fundamental current,

$$I_{sI} = \frac{2\sqrt{2}}{n\pi}I_a\cdot\cos\frac{\alpha}{2} \quad \text{for} \quad n = 1$$

Rms value of total input current,

$$I_s = \left[\frac{I_a^2(\pi-\alpha)}{\pi}\right]^{1/2} = I_a\left[\frac{\pi-\alpha}{\pi}\right]^{1/2}$$

$$\text{CDF} = \frac{I_{sI}}{I_s} = \left(\frac{2\sqrt{2}}{\pi}I_a\cos\frac{\alpha}{2}\right)\times\left(\frac{\sqrt{\pi}}{I_a\sqrt{\pi-\alpha}}\right) = \frac{2\sqrt{2}\cos\frac{\alpha}{2}}{\sqrt{\pi(\pi-\alpha)}}$$

$$\text{Harmonic factor} = \left[\frac{1}{CDF^2}-1\right]^{1/2} = \left[\frac{\pi(\pi-\alpha)}{8\cos^2\frac{\alpha}{2}}-1\right]^{1/2} = \left[\frac{\pi(\pi-\alpha)}{4(1+\cos\alpha)}-1\right]^{1/2}$$

Hence, harmonic factor of input current for $\alpha = \pi/2$

$$= \left[\frac{\pi(\pi-\pi/2)}{4(1+\cos\pi/2)}-1\right]^{1/2} = 0.4834$$

Solution : 4

The given circuit is a three-phase three-pulse converter.

Case-I: $\alpha \le \frac{\pi}{6}$, free wheeling diode will not conduct

$$i_R = I_0 \qquad \text{when } T_1 \text{ is ON}$$
$$i_R = 0 \qquad \text{when } T_1 \text{ OFF}$$
$$i_R = a_1 \cos\omega t + b_1 \sin\omega t$$
$$a_1 = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)\cdot\cos\omega t\cdot d(\omega t)$$
$$= \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_0 \cos\omega t\cdot d(\omega t) \qquad \left[\because T_{ON} \text{ from } \left(\frac{\pi}{6}+\alpha\right) \text{ to } \left(\frac{5\pi}{6}+\alpha\right)\right]$$
$$a_1 = \frac{\sqrt{3}I_0}{\pi}\sin\alpha$$
$$b_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} I_0 \sin\omega t\cdot d(\omega t) = \frac{\sqrt{3}I_0}{\pi}\cos\alpha$$
$$c_1 = \sqrt{a_1^2+b_1^2} = \frac{\sqrt{3}I_0}{\pi}$$
$$\phi_1 = \tan^{-1}\left(\frac{a_1}{b_1}\right) = \alpha$$
$$i_{R_1} = c_1\sin(\omega t+\phi_1) = \frac{\sqrt{3}I_0}{\pi}\sin(\omega t+\alpha)$$

Fundamental displacement factor = FDF = $\cos\phi_1 = \cos\alpha$

$$\text{Distortion factor} = g = \frac{I_{R_1\,\text{rms}}}{I_{R\text{rms}}} = \frac{\frac{\sqrt{3}I_0}{\sqrt{2}\pi}}{I_0/\sqrt{3}} = \frac{3}{\sqrt{2}\pi}$$

$$\text{Power factor} = g\times\text{FDF} = \frac{3}{\sqrt{2}\pi}\cos\alpha$$

Case-II: $\alpha > \frac{\pi}{6}$ freewheeling diode will conduct

T_1 will conduct from $\left(\frac{\pi}{6}+\alpha\right)$ to π

$$i_{R_1} = a_1\cos\omega t + b_1\sin\omega t$$
$$a_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} I_0\cos\omega t\cdot d(\omega t) = -\frac{I_0}{\pi}\sin\left(\frac{\pi}{6}+\alpha\right)$$

$$b_1 = \frac{1}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\pi} I_0 \sin\omega t\cdot d(\omega t) = \frac{I_0}{\pi}\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]$$

$$c_1 = \sqrt{a_1^2 + b_1^2}$$

$$= \frac{I_0}{\pi}\sqrt{\sin^2\left(\frac{\pi}{6}+\alpha\right)+\left[1+\cos\left(\frac{\pi}{6}+\alpha\right)\right]^2} = \frac{2I_0}{\pi}\cos\left(\frac{\pi}{12}+\frac{\alpha}{2}\right)$$

$$I_{R_{1(\text{rms})}} = \frac{\sqrt{2}I_0}{\pi}\cos\left(\frac{\pi}{12}+\frac{\alpha}{2}\right)$$

$$\phi_1 = \tan^{-1}\left(\frac{a_1}{b_1}\right) = \tan^{-1}\left(-\tan\left(\frac{\frac{\pi}{6}+\alpha}{2}\right)\right) = -\left(\frac{\frac{\pi}{6}+\alpha}{2}\right)$$

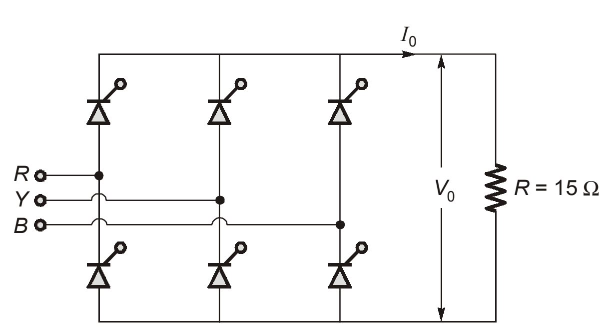
$$\text{FDF} = \cos\phi_1 = \cos\left(\frac{\pi}{12}+\frac{\alpha}{2}\right)$$

$$\text{Distortion factor} = \frac{I_{S_1}}{I_S} = \frac{\frac{\sqrt{2}I_0}{\pi}\cos\left(\frac{\pi}{12}+\frac{\alpha}{2}\right)}{I_{OX}\sqrt{\frac{\frac{5\pi}{6}-\alpha}{2\pi}}}$$

$$\text{Digital factor} = \frac{2\cos\left(\frac{\alpha}{2}+\frac{\pi}{12}\right)}{\sqrt{\pi\left(\frac{5\pi}{6}-\alpha\right)}}$$

$$\text{Power factor} = \frac{2\cos^2\left(\frac{\alpha}{2}+\frac{\pi}{12}\right)}{\sqrt{\pi\left(\frac{5\pi}{6}-\alpha\right)}}$$

Solution: 5



$$V_0 = \frac{1}{\frac{\pi}{3}} \int_{-\left(\frac{\pi}{6}-\alpha\right)}^{\left(\frac{\pi}{6}+\alpha\right)} V_{ml} \cos\omega t \, d(\omega t) = \frac{3V_{ml}}{\pi} \cos\alpha$$

$\alpha = 75°$,
$$= \frac{3\sqrt{3}V_m}{\pi} \cos 75° = 0.428\, V_m$$

$$\text{Power consumed by load} = 6\text{ kW} = \frac{V_0^2}{R}$$

$$6 \times 10^3 = \frac{(0.428V_m)^2}{15}$$

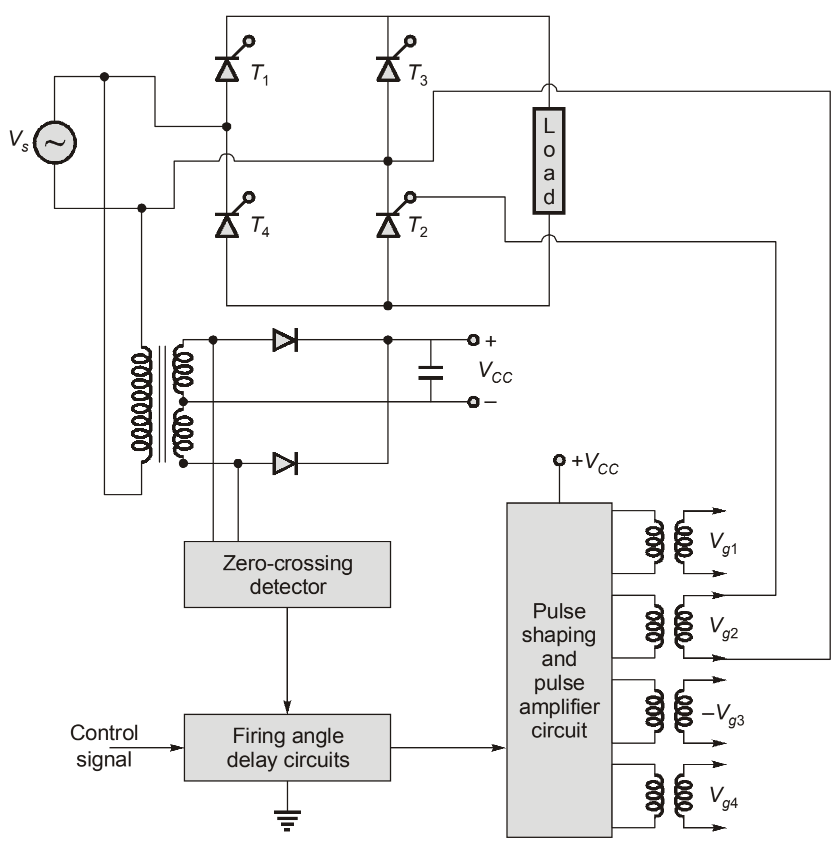
$$V_m = \frac{\sqrt{6 \times 10^3 \times 15}}{0.428} = 700.9345\text{ V}$$

Solution : 6

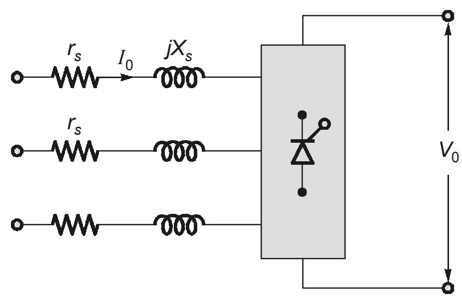
The generation of gating signals for thyristors of ac to dc converters involves the following steps:

1. Zero crossing detection of input voltage.
2. Phase shifting of the signals to the required value.
3. Pulse shaping to generate pulses of short duration.
4. Pulse isolation through pulse transformers.

Block diagrams:



Solution : 7



Power circuit diagram above shows that source resistance r_s will lead to a voltage drop of $2I_0r_s$. Two thyristors, one from positive and other from negative group will conduct together. Thus there would be a constant voltage drop of 2 V_T. The source reactance leads to overlap and it will lead to a drop of = $\frac{3\omega L_s}{\pi} I_0$.

Hence, $$V_0 = \frac{3V_{ml}}{\pi}\cos\alpha - 2I_0 r_s - 2V_T - \frac{3\omega L_s I_0}{\pi}$$

As converter is working in converter mode polarities of output voltage would be reversed,

$$-V_0 = \frac{3V_{ml}\cos\alpha}{\pi} - 2I_0 r_s - 2V_T - \frac{3\omega L_s I_0}{\pi}$$

$$V_0 = \frac{3V_{ml}\cos\alpha}{\pi} + 2I_0 r_s + 2V_T + \frac{3\omega L_s I_0}{\pi}$$

$$= \frac{3(440\sqrt{2})\cos 35^\circ}{\pi} + (2\times 60\times 0.05) + (2\times 2) + \frac{3\times 0.3\times 60}{\pi}$$

$$= 486.75 + 6 + 4 + 17.18 = 513.9 \text{ V}$$

Solution : 8

For a fully controlled three-phase bridge with source inductance,.

Change in output voltage, $$\Delta V = 6fL_sI_0 = \frac{Vd_0}{2}(\cos\alpha - \cos(\alpha+\mu)) \quad ...(1)$$

where,
f = 50 Hz
L_s = 0.32 mH
I_0 = Maximum current
μ = Overlap angle or recovery angle = 5°
α = Firing angle
= 180° – 30° = 150° ($\therefore$ as it is firing advance angle)

$$V_{d0} = \frac{3V_{ml}}{\pi} = \frac{3\times 220\sqrt{6}}{\pi} = 514.59 \text{ V}$$

By equation (i),

$$6\times 50\times 0.32\times 10^{-3}\times I_0 = 257.295(\cos 150^\circ - \cos 155^\circ)$$

$$I_0 = \frac{10.364\times 10^3}{6\times 50\times 0.32} = 107.9 \text{ A}$$

■■■■

3 Choppers

LEVEL 1 Objective Questions

1. (a)

2. (b)

3. (d)

4. (d)

5. (d)

6. (c)

7. (c)

8. (54)

9. (4.9)

10. (a)

11. (a)

12. (a)

13. (d)

14. (a)

LEVEL 2 Objective Questions

15. (c)

16. (b)

17. (38.47)

18. (c)

19. (c)

20. (a)

21. (40.00)

22. (0.13)

23. (171.52)

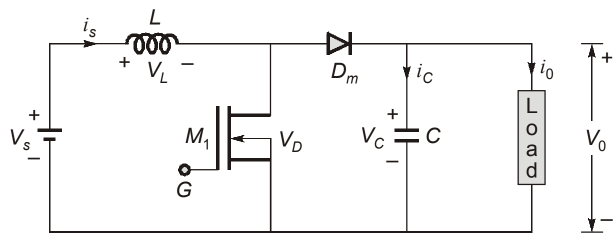
■■■■

LEVEL 3 Conventional Questions

Solution : 1

Boost regulator:

In boost regulator the output voltage is greater than input voltage hence the name 'boost'.

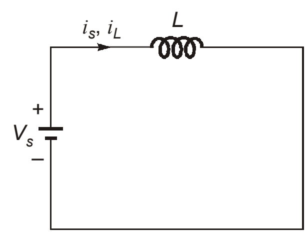


(a)

The circuit operation is divided into two modes.

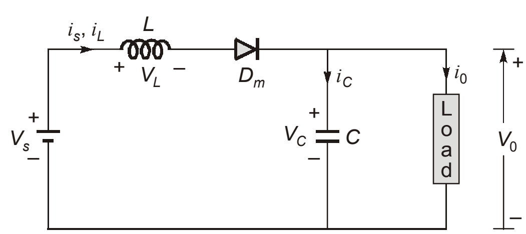
Mode-1:

Begin when MOSFET M_1 is switched on at $t = 0$



(b)

The input current, which rises flows through inductor L and MOSFET M_1. Mode 2 begins when MOSFET M_1 is switched-off at $t = t_1$.



The current that was flowing through the MOSFET would now flow through L, C load and diode D_m. The inductor current will flow until MOSFET M_1 is turned-on again in the next cycle. Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1.

$$V_s = \frac{L(I_2 - I_1)}{t_1} = L\frac{\Delta I}{t_1} \qquad ...(1)$$

$$t_1 = T_{ON} = \frac{\Delta I_L}{V_s} \qquad ...1\text{ (a)}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_s - V_0 = \frac{-L\Delta I}{t_2}$$

$$T_{OFF} = t_2 = \frac{\Delta I\, L}{V_0 - V_s} \quad ...(2)$$

where, $\Delta I = I_2 - I_1$ is peak to peak ripple current of inductor L.
From equation (1) and (2),

$$\Delta I = \frac{V_s T_{ON}}{L} = \frac{(V_0 - V_s)T_{OFF}}{L} \quad ...(3)$$

Substituting,
$$T_{ON} = \alpha T$$
$$T_{OFF} = (1-\alpha)\, T \qquad \text{[in equation (3)]}$$
$$\frac{V_s \alpha T}{L} = \frac{(V_0 - V_s)(1-\alpha)T}{k}$$
$$\alpha V_s = V_0 - \alpha V_0 - V_s + \alpha V_s$$
$$V_s = V_0(1-\alpha)$$
$$V_0 = \frac{V_s}{1-\alpha} = \text{Average output voltage} \qquad \textbf{[Ans. (i)]}$$

For voltage ripple:
When switch 's' is on, capacitor supplies the load current for $t = T_{on}$. The average capacitor current during time T_{ON} is $I_C = I_0$ and peak to peak ripple voltage of capacitor is

$$\Delta V_C = V_C(\text{at } t = T_{ON}) - V_C(\text{at } t = 0)$$
$$= \frac{1}{C}\int_0^{T_{ON}} I_C\, dt = \frac{1}{C} I_0\, T_{ON} \quad ...(4)$$
$$= \frac{1}{C} I_0\, \alpha T = \frac{I_0\, \alpha}{fC} \qquad \textbf{[Ans. (iii)]}$$

For minimum inductance for continous current:
If I_L is the average inductor current, the inductor ripple current is

$$\Delta I = 2I_L \quad ...(5)$$

From equation (3),

$$\Delta I = \frac{V_s\, \alpha}{fL} \qquad \boxed{\text{as } T_{ON} = \frac{\alpha}{f}} \quad ...(6)$$

$$I_L = I_0 = \frac{V_0}{R} = \frac{V_s}{(1-\alpha)R} \quad ...(7)$$

Putting equation (6) and (7) in (5),

$$\frac{V_s\, \alpha}{fL} = \frac{2V_s}{(1-\alpha)R}$$
$$L = \frac{\alpha(1-\alpha)R}{2f} \qquad \textbf{[Ans. (ii)]}$$

For critical capacitance:
If V_C = average capacitor voltage, the capacitor voltage ripple,

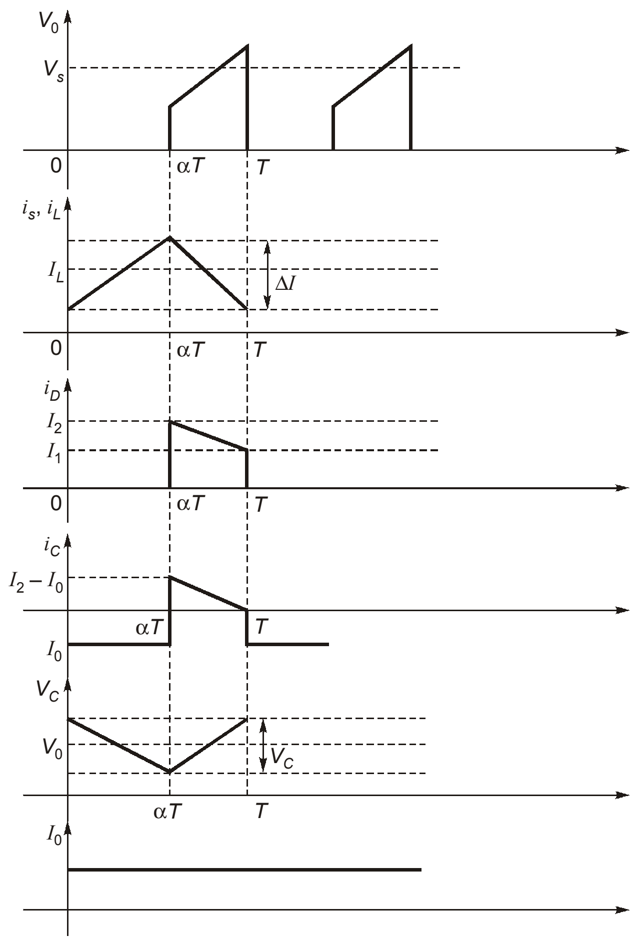
$$\Delta V_C = 2\, V_0 \quad ...(8)$$
$$\Delta V_C = \frac{I_0\, \alpha}{fC}$$

Putting ΔV_C and V_0 in equation (8),

$$\frac{I_0 \alpha}{fC} = 2I_0 R$$

$$C = \frac{\alpha}{2fR} \qquad \text{[Ans. (iv)]}$$

Output waveforms:



Solution : 2

(a) Limit of continuous conduction is reached when I_{mn} goes to zero. The value of duty cycle ∝ at the limit of continuous conduction is obtained by equating I_{mn} to zero. Therefore,

$$I_{mn} = \frac{V_s}{R}\left[\frac{e^{T_{ON}/T_a} - 1}{e^{T/T_a} - 1}\right] - \frac{E}{R} = 0$$

$$\Rightarrow \qquad \frac{e^{T_{ON}/T_a} - 1}{e^{T/T_a} - 1} = \frac{E}{V_s} = m \;\Rightarrow\; e^{T_{on}/T_a} = 1 + m(e^{T/T_a} - 1)$$

$$\Rightarrow \qquad \alpha' = \frac{T_{ON}}{T} = \frac{T_a}{T}\, ln[1 + m(e^{T/T_a} - 1)]$$

$$T_a = \frac{L}{R} = 5 \times 10^{-3} \text{ sec}$$

$$\frac{T_a}{T} = \frac{5 \times 10^{-3}}{2000 \times 10^{-6}} = 2.5$$

$$\frac{T}{T_a} = 0.4$$

$$m = \frac{E}{V_s} = \frac{24}{220} = 0.11$$

$$\alpha = \frac{600}{2000} = 0.3$$

$$\frac{T_{ON}}{T_a} = \frac{600}{5000} = 0.12$$

$$\alpha' = \frac{T_{ON}}{T} = \frac{T_a}{T} \times ln[1 + m(e^{T/T_a} - 1)]$$

$$= 2.5\, ln[1 + 0.11(e^{0.4} - 1)] = 0.13172$$

As actual value of duty cycle $\alpha = 0.3$ is greater than α', load current is continuous.

(b)
$$I_0 = \frac{\alpha V_s - E}{R} = \frac{0.3 \times 220 - 24}{1} = 42 \text{ A}$$

(c)
$$I_{mx} = \frac{V_s}{R}\left[\frac{1 - e^{-T_{ON}/T_a}}{1 - e^{-T/T_a}}\right] - \frac{E}{R}$$

$$= \frac{220}{1}\left[\frac{1 - e^{-0.12}}{1 - e^{-0.4}}\right] - \frac{24}{1} = 51.46 \text{ A}$$

$$I_{mn} = \frac{V_s}{R}\left[\frac{e^{T_{ON}/T} - 1}{e^{T/T_a} - 1}\right] - \frac{E}{R} = \frac{220}{1}\left[\frac{e^{0.12} - 1}{e^{0.4} - 1}\right] - 24 = 33.031 \text{ A}$$

(d)
$$V_n = \frac{2V_s}{n\pi} \sin n\pi d \sin(n\omega t + \theta_n)$$

Rms value of 1st harmonic,

$$V_1 = \frac{2V_s}{\sqrt{2}\pi} \sin(\pi \times 0.3) = \frac{2 \times 220}{\sqrt{2}\,\pi} \sin 54° = 80.121 \text{ V}$$

$$Z_1 = \sqrt{R^2 + (\omega L)^2} = \sqrt{1^2 + (2\pi 500 \times 5 \times 10^{-3})^2} = 15.739\ \Omega$$

$$I_1 = \frac{V_1}{Z_1} = \frac{80.121}{15.739} = 5.0903 \text{ A}$$

$$I_2 = \frac{2 \times 200}{2 \times \sqrt{2}\,\pi} \sin 108° \times \frac{1}{\sqrt{1 + (2\pi \times 500 \times 2 \times 5 \times 10^{-3})^2}}$$

$$= 1.498 \text{ A}$$

$$I_3 = \frac{2\times 200}{3\sqrt{2}\,\pi}\sin 162^\circ \times \frac{1}{\sqrt{1+(2\pi\times 500\times 3\times 5\times 10^{-3})^2}} = 0.21643 \text{ A}$$

$$I_{av} = \frac{\alpha(V_s - E)}{R} - \frac{L}{RT}(I_{mx} - I_{mn}) = 12.7275 \text{ A}$$

(e) Input power $= E \times$ average supply current

$= 220 \times 12.7275 = 2800.05$ Watts

Power absorbed by load emf $= E \times$ Average load current

$= 24 \times 42 = 1008$ Watts

Power loss in resistor $= 2800.05 - 1008 = 1792.05$ Watts

(f)
$$I_{or} = \sqrt{I_{av}^2 + I_1^2 + I_2^2 + I_3^2}$$

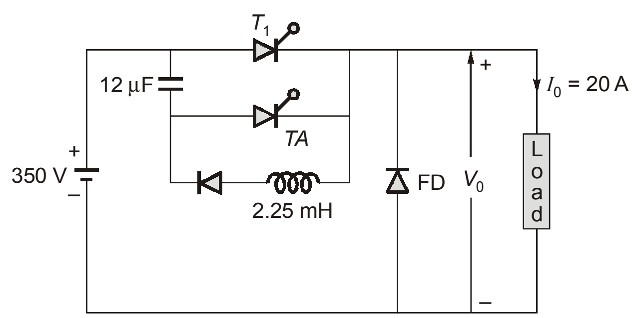
$$= \sqrt{42^2 + (5.0903)^2 + (1.4983)^2 + (0.2164)^2} = 42.31 \text{ A}$$

Power loss in resistor $= I^2R = 1792.05$ W

$$I_{or} = \sqrt{\frac{1792.05}{1}} = 42.33 \text{ A}$$

Solution : 3

Voltage commutated chopper circuit,



Circuit turn-off time:

$$t_C = \frac{CV_s}{I_0} = \frac{12\times 10^{-6}\times 350}{30} = 140 \times 10^{-6} = 1.4 \times 10^{-4} \text{ s}$$

Minimum on-period for this chopper is

$$t_1 = \frac{\pi}{\omega_0} = \pi\sqrt{LC}$$

$$= \pi\sqrt{2.25\times 10^{-3}\times 12\times 10^{-6}}$$

$$= \pi\sqrt{27\times 10^{-9}} = \pi\times 10^{-4}\sqrt{2.7}$$

$$= 5.162 \times 10^{-4} \text{ s}$$

The minimum average output voltage of the circuit is

$$V_{0,\min} = f\,V_s[t_1 + 2t_c]$$

$$= 250 \times 350[5.162 \times 10^{-4} + 2 \times 1.4 \times 10^{-4}]$$

$$= 8.75 \times 7.962 = 69.67 \text{ V}$$

Solution : 4

$$C = \frac{t_C I_0}{V_s}$$

By the question, $$t_C = \frac{CV_s}{I_0} = \frac{30\times10^{-6}\times150}{I_0}$$

(i) Peak current through main thyristor $= I_0 + V_s\sqrt{\frac{C}{L}}$

$$2I_0 = I_0 + V_s\sqrt{\frac{C}{L}}$$

$$I_0 = V_s\sqrt{\frac{C}{L}} = 150\sqrt{\frac{30}{35}} = 138.87 \text{ A}$$

(ii) $$t_C = \frac{CV_s}{I_0} = \frac{30\times10^{-6}\times150}{138.87} = 32.40\ \mu\text{s}$$

(iii) Minimum value of duty cycle:

$$\alpha_{mn} = \pi f\sqrt{LC} = \pi\times300\sqrt{30\times35\times10^{-6}\times10^{-6}} = 0.0305$$

Minimum value of output voltage

$$= V_s(\alpha_{mn} + 2f\,t_c)$$
$$= 150(0.0305 + 2\times300\times32.40\times10^{-6})$$
$$= 150(0.0305 + 194.4\times10^{-4})$$
$$= 150(0.0305 + 0.0194)$$
$$= 7.485 \text{ V}$$

Maximum value of output voltage = 150 V

Solution : 5

(i) $$L = \frac{V_s t_c}{xI_0\left[\pi - 2\sin^{-1}\left(\frac{1}{x}\right)\right]} = \frac{230\times60\times10^{-6}}{2\times200\left[\pi-2\sin^{-1}\left(\frac{1}{2}\right)\right]} = 16.473\ \mu\text{H}$$

$$C = \frac{x I_0 t_c}{V_s\left[\pi - 2\sin^{-1}\left(\frac{1}{x}\right)\right]} = \frac{2\times200\times60\times10^{-6}}{230\left[\pi-2\sin^{-1}\left(\frac{1}{2}\right)\right]} = 49.822\ \mu\text{F}$$

(ii) Peak capacitor voltage, $V_{cp} = V_s + I_0\sqrt{\frac{L}{C}}$

$$= 230 + 200\sqrt{\frac{16.473}{49.822}} = 345 \text{ Volts}$$

(iii) Peak commutating current $= xI_0 = 2\times200 = 400$ A

Solution : 6

(a) Peak commutating current, is

$$I_{CP} = C_s\sqrt{\frac{C}{L}} = 230\sqrt{\frac{50}{20}} = 363.66 \text{ A}$$

$$x = \frac{I_{CP}}{I_0} = \frac{363.66}{200} = 1.8183$$

Turn-off time of main SCR, is

$$t_c = \left[\pi - 2\sin^{-1}\left(\frac{1}{x}\right)\right]\sqrt{LC}$$

$$= \left[\pi - 2\sin^{-1}\left(\frac{1}{1.8183}\right)\right]\sqrt{20\times 50\times (10^{-6})^2}$$

$$= 62.52\ \mu\text{s}$$

(b) $$\theta_1 = \sin^{-1}\left(\frac{I_0}{I_{CP}}\right) = \sin^{-1}\left(\frac{200}{363.66}\right) = 33.656°$$

Total commutation interval is

$$= \left(\frac{5\pi}{2} - \theta_1\right)\sqrt{LC} + CV_s\frac{(1-\cos\theta_1)}{I_0}$$

$$= \left(\frac{5\pi}{2} - \frac{33.365\times\pi}{180°}\right)\sqrt{1000}\times 10^{-6} + 50\times 10^{-6}\times 230\left(\frac{1-\cos 33.6}{200}\right)$$

$$= 229.95 \times 10^{-6} + 9.477 \times 10^{-6}$$

$$= 239.427\ \mu\text{s}$$

(c) Turn-off time of auxiliary thyristor

$$= (\pi - \theta_1)\sqrt{LC}$$

$$= \left[\pi - \frac{33.365\times\pi}{180°}\right]\sqrt{1000}\times 10^{-6} = 80.931\ \mu\text{s}$$

Solution : 7

V_s = 18 V, V_0 = 45 V, I_a = 1.5 A, f = 20 kHz, L = 300 μH, C = 450 μF

(a) $$V_a = \frac{V_s\alpha}{(1-\alpha)}$$

$$\Rightarrow \quad 45 = \frac{18\alpha}{1-\alpha}$$

$$\Rightarrow \quad 45 - 45\alpha = 18\alpha$$

$$\Rightarrow \quad \alpha = \frac{45}{63} = 0.714$$

(c) Ripple voltage, $$\Delta V_C = \frac{\alpha I_0}{fC} = \frac{0.714\times 1.5}{20\times 10^3\times 450\times 10^{-6}} = 119 \text{ mV}$$

(b) Ripple current, $$\Delta I = \frac{\alpha V_s}{fL} = \frac{0.714\times 18}{20\times 10^3\times 300\times 10^{-6}} = 2.142 \text{ A}$$

(d) Peak current of transistor,

$$I_s = \frac{I_a \alpha}{1-\alpha} = \frac{1.5\times 0.714}{(1-0.714)} = 3.74 \text{A}$$

$$I_P = \frac{I_s}{\alpha}+\frac{\Delta I}{2} = \frac{3.74}{0.714}+\frac{2.142}{2} = 6.309 \text{ A}$$

(e) $$R = \left|\frac{V_a}{I_a}\right| = \frac{45}{1.5} = 30\ \Omega$$

$$L_C = \frac{(1-\alpha)\alpha R}{2f}$$

$$= \frac{(1-0.714)\,(0.714)\times 30}{2\times 20\times 10^3} = 0.153\times 10^{-3} = 153\ \mu\text{H}$$

$$C_C = \frac{\alpha}{2fR} = \frac{0.714}{2\times 20\times 10^3\times 30} = 0.595\ \mu\text{H}$$

Solution : 8

(a) $$V_a = kV_s$$

$$k = \frac{V_a}{V_s} = \frac{5}{12} = 0.4167$$

(b) $$\Delta I = \frac{V_a(V_s - V_a)}{fLV_s}$$

$$L = \frac{5(12-5)}{0.8\times 25000\times 12} = 145.83\ \mu\text{H}$$

(c) $$\Delta V_C = \frac{\Delta I}{8fC}$$

$$C = \frac{0.8}{8\times 20\times 10^{-3}\times 25000} = 200\ \mu\text{F}$$

(d) $$L_C = \frac{(1-K)R}{2f} = \frac{(1-0.4167)\times 500}{2\times 25\times 10^3} = 5.83 \text{ mH}$$

$$C_C = \frac{1-K}{16Lf^2} = \frac{(1-0.4167)}{16\times 5.83\times 10^{-3}(25\times 10^{3^2})} = 1500\ \mu\text{F}$$

Solution : 9

$$\frac{T_{OFF}}{T_{ON}} = \frac{1}{4}$$

$$\frac{T_{\text{OFF}}}{T_{\text{ON}}}+1 = \frac{5}{4}$$

$$\frac{T}{T_{\text{ON}}} = \frac{5}{4} \Rightarrow \frac{T_{\text{ON}}}{\text{T}} = \alpha = \frac{4}{5}$$

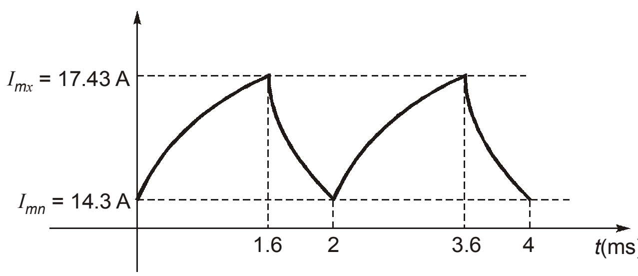
Average value of load voltage = V_0

$$V_0 = \alpha V_s = \frac{4}{5}\times 100 = 80 \text{ V}$$

Chopping period, $T = 2$ m-sec

$$T_{\text{ON}} = \frac{4}{5}\times 2 = 1.6 \text{ ms}$$

Hence,



During T_{ON}:

$$V_s = L\frac{di}{dt}+iR$$

Taking Laplace transform:

$$\frac{V_s}{s} = L(sI(s) - I_{mn}) + I(s)\,R$$

$$\frac{V_s}{s}+LI_{mn} = I(s) + (sL + R)$$

$$I(s) = \frac{V_s}{s(sL+R)}+\frac{LI_{mn}}{sL+R}$$

$$I(s) = \frac{V_s}{Ls\left(s+\frac{R}{L}\right)}+\frac{I_{mn}}{s+\frac{R}{L}} = \frac{V_s}{R}\left[\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right]+\frac{I_{mn}}{s+\frac{R}{L}}$$

$$i(t) = \frac{V_s}{R}(1-e^{-\frac{Rt}{L}})+I_{mn}\,e^{-Rt/L}$$

$$= 20[1 - e^{500t}] + I_{mn}\,e^{-500t}$$

During T_{OFF}:

$$0 = L\frac{di}{dt}+iR$$

$$\frac{di}{dt} = -\frac{R}{L}(i)$$

$$\int_{i_{mn}}^{i} \frac{di}{i} = -\int_{0}^{t} \frac{R}{L} dt$$

$$ln \frac{i}{i_{mx}} = -\frac{Rt}{L}$$

$$i = i_{mx} e^{-Rt/L}$$

$$i = i_{mx} e^{-500t}$$

at $t = T_{ON}$:

$$I_{mx} = 20(1 - e^{-0.8}) + I_{mn} e^{-0.8}$$

$$I_{mx} - 0.45 I_{mn} = 11.01 \quad ...(1)$$

at $t = T_{OFF}$:

$$I_{mn} = I_{mx} e^{-0.2}$$

$$I_{mn} = 0.82 I_{mx} \quad ...(2)$$

Solving,

$$I_{mx} = 17.43 \text{ A}$$

$$I_{mn} = 14.3 \text{ A}$$

■■■■

4 Inverters

LEVEL 1 Objective Questions

1.	(b)
2.	(d)
3.	(18.08)
4.	(b)
5.	(b)
6.	(c)
7.	(b)
8.	(a)
9.	(c)
10.	(a)
11.	(b)
12.	(a)

LEVEL 2 Objective Questions

13.	(b)
14.	(60)
15.	(d)
16.	(2.78)
17.	(2.25)
18.	(b)
19.	(0.122)
20.	(72.92)
21.	(b)
22.	(c)
23.	(a)
24.	(b)
25.	(121.54)
26.	(710.46)
27.	(c)

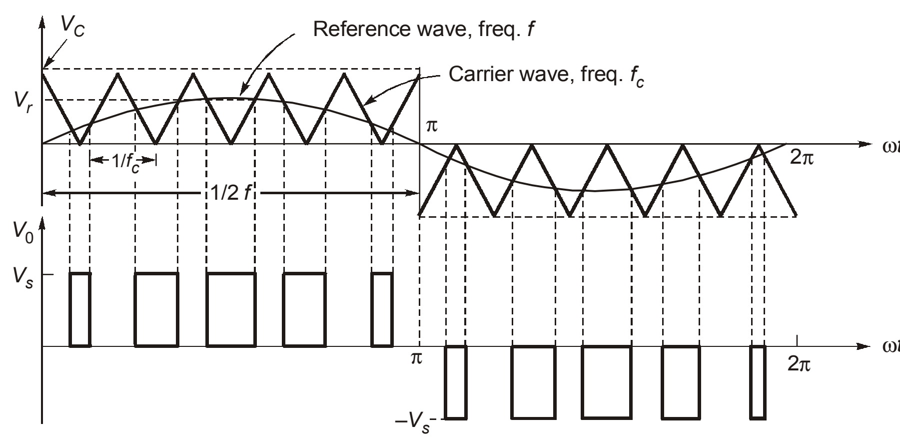
LEVEL 3 Conventional Questions

Solution : 1

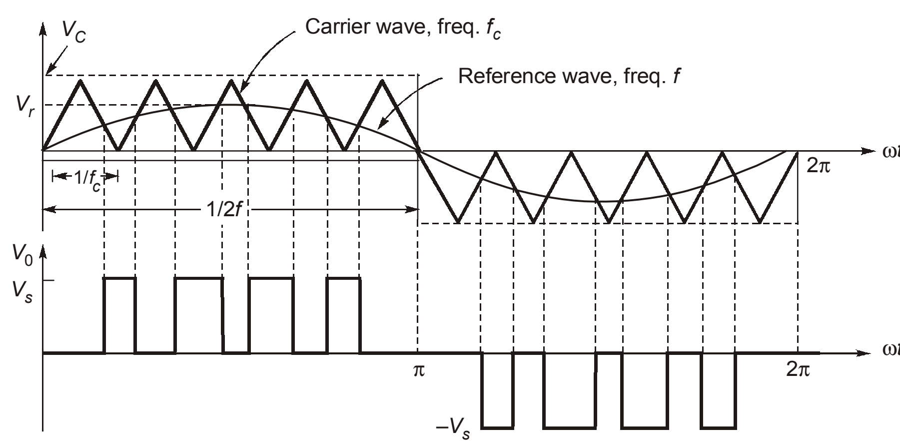
PWM (pulse width modulation) is a internal voltage control of inverter. In this method, a fixed dc input voltage is given to the inverter and a controlled output voltage is obtained by, adjusting the on and off periods of the inverter components. In Sine modulation [Sin *M*] the pulse width is a sinusoidal function of the angular position of the pulse in a cycle as shown in figure.

For realizing sin *M*, a high-frequency triangular carrier wave V_c is compared with a sinusoidal reference wave V_r of the desired frequency. The intersection of V_c and V_r waves determines the switching instants and commutation of the modulated pulse. In figure V_C is the peak value of triangular carrier wave and V_r that of the reference, or modulating signal.

The carrier and reference waves are mixed in a comparator. When sinusoidal wave has magnitude higher than the triangular wave, the comparator output is high, otherwise it is low. The comparator output is processed in a trigger pulse generator in such a manner that the output voltage wave of the inverter has a pulse width in agreement with the comparator output pulse width.



(a)



(b)

Output voltage waveforms with sinusoidal pulse modulation

When triangular carrier wave has its peak coincident with zero of the reference sinusoid, there are $N = f_c/2f = \frac{f_c}{2f}$ pulses per half cycle ; Fig. (a) has five pulses. In case zero of the triangular wave coincides with zero of the reference sinusoid, there are $(N - 1)$ pulses per half cycle ; Fig. (b) has $(f_c/2f - 1)$, i.e. four, pulses per cycle.

The ratio of V_r/V_c is called the modulation index (MI) and it controls the harmonic content of the output voltage waveform. The magnitude of fundamental component of output voltage is proportional to MI, but MI can never be more than unity. Thus the output voltage is controlled by varying MI.

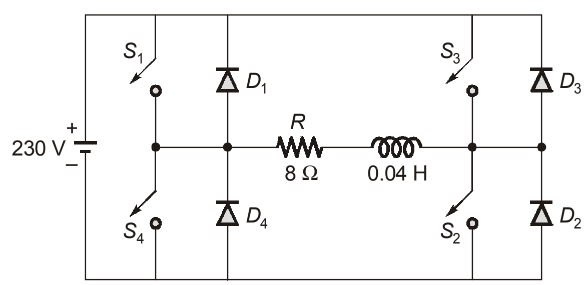
Harmonic analysis of the output modulated voltage wave reveals that sin M has the following important features:

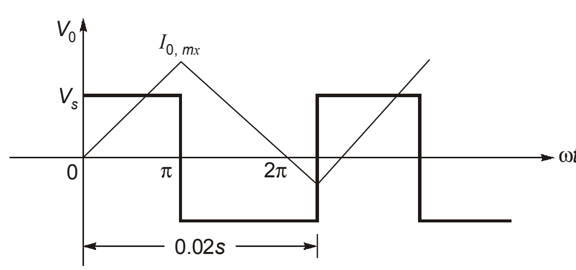
(i) For MI less than one, largest harmonic amplitudes in the output voltage are associated with harmonics of order $f_c/f \pm 1$ or $2N \pm 1$, where N is the number of pulses per half cycle. Thus, by increasing the number of pulses per half cycle, the order of dominant harmonic frequency can be raised, which can then be filtered out easily.

It is observed from above that as N is increased, the order of significant harmonic increases and the filtering requirements are accordingly minimised. But higher value of N entails higher switching frequency of thyristors. This amounts to more switching losses and therefore an impaired inverter efficiency. Thus a compromise between the filtering requirements and inverter efficiency should be made.

(ii) For MI greater than one, lower order harmonics appear, since for MI > 1, pulse width is no longer a sinusoidal function of the angular position of the pulse.

Solution : 2





For the first-half cycle:

$$0 < t < \frac{T}{2}$$

The voltage equation for RL load is

$$V_s = Ri_0 + L\frac{di_0}{dt}$$

$$i(t) = \frac{V_s}{R}\left(1-e^{\frac{-t}{\tau}}\right) = \frac{220}{8}(1-e^{-200t}) \qquad \left[\text{where, } \tau = \frac{L}{R} = 0.005\right]$$

$$= 27.5\,(1 - e^{-200t})$$

During second half cycle:

$$I_0 = I_{\text{initial}} = \frac{V_s}{R}$$

$$I_\infty = I_{\text{field}} = -\frac{V_s}{R}$$

$$I(t) = (I_\infty - (I_\infty - I_0)e^{-t/\tau})$$

$$= \left[-\frac{V_s}{R} - \left(-\frac{V_s}{R} - \frac{V_s}{R}\right)e^{-t/\tau}\right] = -\frac{V_s}{R} + \frac{2V_s}{R}\,e^{-t/\tau}$$

$$= -\frac{220}{8} + \frac{2\times 220}{8}\,e^{-200t} = -27.5 + 55e^{-200t}$$

Solution : 3

Sinusoidal Pulse-Width Modulation:

$$m_A = \frac{A_{\text{Reference}}}{A_{\text{carrier}}}$$

$$m_F = \frac{f_{\text{carrier}}}{f_{\text{Reference}}}$$

Instead of maintaining the width of all pulses the same as in the case of multiple-pulse modulation, the width of each pulse is varied in proportion to the amplitude of a sine wave evaluated at the center of the same pulse. The DF and LOH are reduced significantly. The gating signals are generated by comparing a sinusoidal reference signal with a triangular carrier wave of frequency f_c. This sinusoidal pulse-width modulation (SPWM) is commonly used in industrial applications. The frequency of reference signal f_r determines the inverter output frequency f_0; and its peak amplitude A_r controls the modulation index M, and then in turn the rms output voltage V_0. The rms output voltage can be varied by varying the modulation index M_A. It can be observed that the area of each pulse corresponds approximately to the area under the sine wave between the adjacent midpoints of off periods on the gating signals. The harmonic profile is shown in figure for five pulses per half-cycle. The DF is significantly reduced compared with that of multiple-pulse modulation.

The output voltage of an inverter contains harmonics. The PWM pushes the harmonics into a high-frequency range around the switching frequency f_c and its multiples, that is around harmonics m_f, $2m_f$, $3m_f$, and so on. The frequncies at which the voltage harmonics occur can be related by

$$f_n = (jm_f \pm k)\, f_c$$

where the n^{th} harmonic equals the k^{th} sideband of j^{th} times the frequency to modulation ratio m_f.

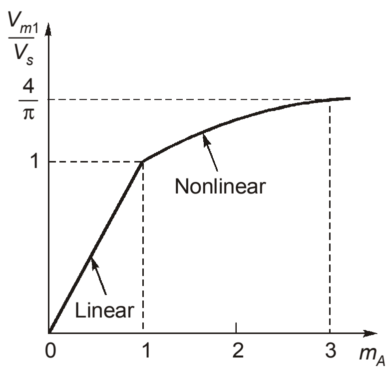
$$n = jm_f \pm k$$
$$= 2\,jp \pm k$$

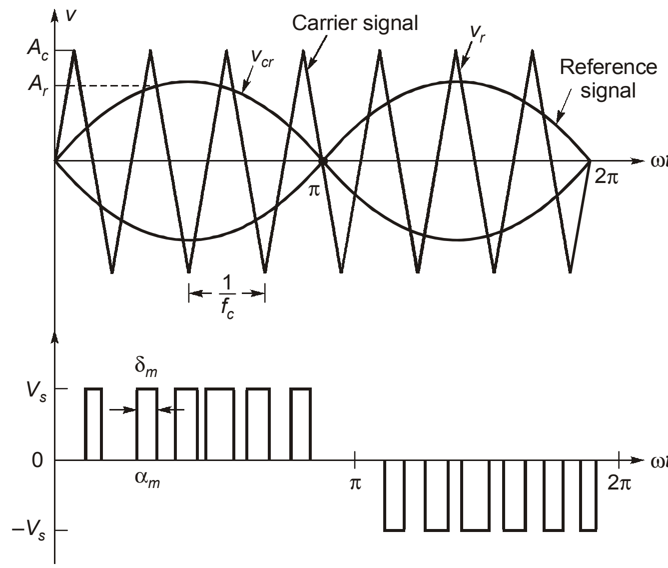
For $j = 1, 2, 3, \ldots$ and $k = 1, 3, 5, \ldots$

The peak fundamental output voltage for PWM and SPWM control can be found approximately from

$$V_{m1} = m_A V_s \text{ for } 0 \le m_A \le 1.0$$

For $m_A = 1$, equation gives the maximum peak amplitude of the fundamental output voltage as $V_{m1(max)} = V_s$. According to equation, $V_{m1(max)}$ could be as high as $4V_s/\pi = 1.273\ V_s$ for a square-wave output. To increase the fundamental output voltage, m_A must be increased beyond 1.0. The operation beyond $m_A = 1.0$ is called overmodulation. The value of m_A at which $V_{m1(max)}$ equals 1.273 V_s is dependent on the number of pulses per half-cycle p and is approximately 3 for $p = 7$, as shown in figure. Overmodulation basically leads to a square-wave operation and adds more harmonics as compared with operation in the linear range (with $m_A \le 1.0$). Overmodulation is normally avoided in applications requiring low distortion (e.g. uninterruptible power supplies [UPS]).





Solution : 4

The voltage waveform of the given figure is symmetrical about π as well as $\pi/2$.

As this voltage waveform has quarter wave symmetry,

$$a_n = 0$$

and

$$b_n = \frac{4V_s}{\pi}\left[\int_0^{\alpha_1} \sin\omega t \cdot d\omega(t) - \int_{\alpha_1}^{\alpha_2} \sin n\omega t \cdot d\omega(t) + \int_{\alpha_2}^{\pi/2} \sin n\omega t \cdot d\omega(t)\right]$$

$$b_n = \frac{4V_s}{\pi}\left[\left(-\frac{\cos n\omega t}{n}\right)\Bigg|_0^{\alpha_1} - \left(-\frac{\cos n\omega t}{n}\right)\Bigg|_{\alpha_1}^{\alpha_2} + \left(-\frac{\cos n\omega t}{n}\right)\Bigg|_{\alpha_2}^{\pi/2}\right]$$

$$= \frac{4V_s}{\pi}\left[\frac{1-\cos n\alpha_1}{n} + \frac{\cos\alpha_2 - \cos\alpha_1}{n} + \frac{\cos n\alpha_2}{n}\right]$$

$$b_n = \frac{4V_s}{\pi}\left[\frac{1-2\cos n\alpha_1 + 2\cos n\alpha_2}{n}\right]$$

Now, for the values of $\alpha_1 = 23.62°$ and $\alpha_2 = 33.30°$

The amplitude of 7th harmonics

$$b_7 = \frac{4V_s}{\pi}\left[\frac{1-2\cos(7\times 23.62°) + 2\cos(7\times 33.30°)}{7}\right] = 0.3154\ V_s$$

The amplitude of 9th harmonics

$$b_9 = \frac{4V_s}{\pi}\left[\frac{1-2\cos(9\times 23.62°)+2\cos(9\times 33.30°)}{9}\right] = 0.5201\,V_s$$

and the amplitude of 11th harmonics,

$$b_{11} = \frac{4V_s}{\pi}\left[\frac{1-2\cos(11\times 23.62°)+2\cos(11\times 33.30°)}{11}\right] = 0.3867\,V_s$$

Solution : 5

Rms value of fundamental component at the output of inverter,

$$V_1 = \frac{4\times 12}{\pi}\times\frac{1}{\sqrt{2}} = \frac{48}{\sqrt{2}\pi} = 10.8\text{ V}$$

(i) $$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

$\Rightarrow$ $$N_2 = 230\times\frac{10}{10.8} = 212.96\text{ V}$$

(ii) Rms value of secondary current,

$$I_2 = \frac{230}{100} = 2.3\text{ A}$$

$$I_1 N_1 = I_2 N_2$$

$$I_1 = \frac{2.3\times 212.96}{10} = 48.98\text{ A}$$

(iii) Rms value of thyristor current $= \dfrac{\text{Rms value of primary current}}{\sqrt{2}} = \dfrac{48.98}{\sqrt{2}} = 34.634\text{ A}$

■■■■

5 Resonant Converters and Power Electronics Applications (Drives, SMPS)

LEVEL 1 Objective Questions

1.	(d)
2.	(a)
3.	(d)
4	Sol.
5.	(b)
6.	(b)
7.	(c)
8.	(a)
9.	(d)

LEVEL 2 Objective Questions

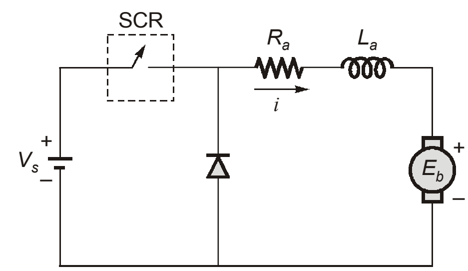
10.	(a)
11.	(b)
12.	(d)
13.	(180)
14.	(b)
15.	(d)
16.	(c)
17.	(c)
18.	(b)
19.	(98.72)
20.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 1

From the question:



$$i(0) = 10 \text{ A}$$

To calculate $i(10^{-3})$

During ON condition:

$$V_s = iR_a + L\frac{di}{dt} + E_b$$

$$180 = 0.4i + 8\times10^{-3}\frac{di}{dt} + 80$$

$$100 = 0.4i + 8\times10^{-3}\frac{di}{dt}$$

$$\int_0^t dt = 8\times10^{-3}\int_0^i \frac{di}{100-0.4i}$$

$$t = \frac{8\times10^{-3}}{-0.4}\, ln\left(\frac{100-0.4i}{96}\right)$$

$$-50t = ln\left(\frac{100-0.4i}{96}\right)$$

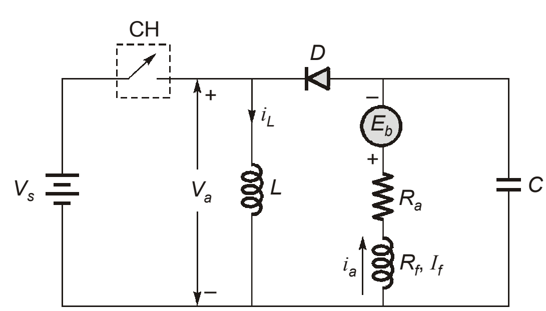
$$100 - 0.4i = 96\, e^{-50t}$$

$$i = \frac{1}{0.4}[100 - 96e^{-50t}] = 5[50 - 48\, e^{-50t}]$$

$$= 250 - 240\, e^{-50t}$$

At $t = 10^{-3}$ sec., $$i = 250 - 240e^{-\frac{50}{1000}} = 21.705 \text{ A}$$

Solution : 2



$$V_a = \frac{\alpha V_s}{1-\alpha} \qquad [\alpha = \text{Duty cycle} = T_{ON}/T]$$

Now, $$V_a = E_b + I_a(R_a + R_f)$$

$$= E_b + I_aR$$

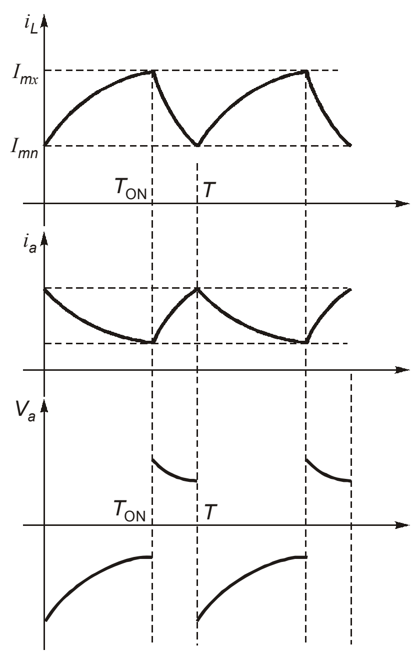
$$\Rightarrow \quad V_a = k_1 N I_a + I_a R \quad [E_b \propto N\phi \propto N I_a]$$

$$\Rightarrow \quad N = \frac{V_a - I_a R}{k_1 I_a}$$

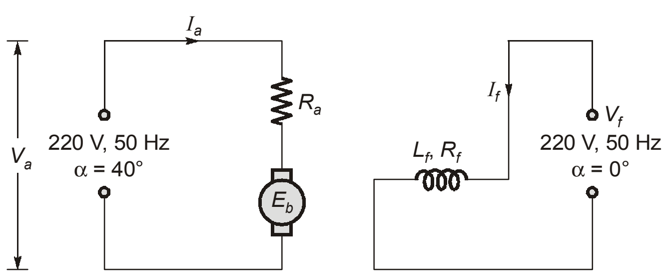
$$\Rightarrow \quad N = \frac{V_a}{k_1 I_a} - \frac{R}{k_1}$$

For constant I_a [i.e. constant torque as torque a I_a^2],

$$N \propto V_a$$



Solution : 3



(i) $$E_b = K\omega I_f \quad \text{...(1)}$$

$$I_f = \frac{V_f}{R_f}$$

$$V_f = \frac{2V_m}{\pi}\cos 0 = \frac{2 \times 220 \times \sqrt{2}}{\pi} = 198.069 \text{ V}$$

$$= \frac{198.069}{150} = 1.32 \text{ A}$$

Further,
$$V_a = I_a R_a + E_b$$
$$E_b = V_a - I_a R_a$$
$$V_a = \frac{2V_m}{\pi}\cos 40° = 151.73 \text{ V}$$
$$= 151.73 - (40 \times 0.6) = 127.73 \text{ V}$$

Putting in equation (1),
$$\omega = \frac{E_b}{kI_f} = \frac{127.73}{0.6 \times 1.32} = 161.275 \text{ rad/sec.}$$
$$N = \omega \times \frac{60}{2\pi} = 1540.06 \text{ rpm}$$

(ii) Power factor of armature current
$$= \frac{I_a^2 R_a + E_b I_a}{220 I_a} = \frac{I_a R_a + E_b}{220}$$
$$= \frac{151.73}{220} = 0.6869 \text{ lag}$$

(iii) Power factor of driving scheme
$$= \frac{I_a^2 R_a + E_b I_a + I_f^2 R_f}{220\sqrt{I_a^2 + I_f^2}}$$
$$= \frac{40^2(0.6) + 127.73 \times 40 + 1.32^2 + 150}{220\sqrt{(40)^2 + (1.32)^2}} = 0.7189 \text{ lag}$$

Solution : 4

At rated operation:
$$E = V_a - I_a R_a$$
$$= 200 - 150 \times \frac{6}{100} = 191 \text{ V}$$

(i) E at 750 rpm:
$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$
$$\Rightarrow \quad E_2 = \frac{E_1}{N_1} \times N_2 = 191 \times \frac{750}{875} = 163.7 \text{ V}$$
$$V_a = E + I_a R_a$$
$$= 163.7 + (150 \times 0.06) = 172.7 \text{ V}$$
$$\frac{2V_m}{\pi}\cos\alpha = V_a$$
$$\cos\alpha = \frac{\pi V_a}{2V_m} = \frac{\pi \times 172.7}{2 \times \sqrt{2} \times 220} = 0.872$$
$$\alpha = 29.3°$$

(ii) E at –500 rpm:

$$E = -\frac{500}{875} \times 191 = -109 \text{ V}$$

$$V_a = E + I_a R_a$$

$$= -109 + (150 \times 0.06) = -100 \text{ V}$$

$$\frac{2V_m}{\pi}\cos\alpha = V_a$$

$$\cos\alpha = -\frac{100\pi}{2 \times \sqrt{2} \times 220} = -0.504$$

$$\alpha = 120°$$

(iii) $\alpha = 160°$:

$$V_a = \frac{2V_m}{\pi}\cos\alpha = \frac{2 \times 220\sqrt{2}\cos 160°}{\pi} = -186 \text{ V}$$

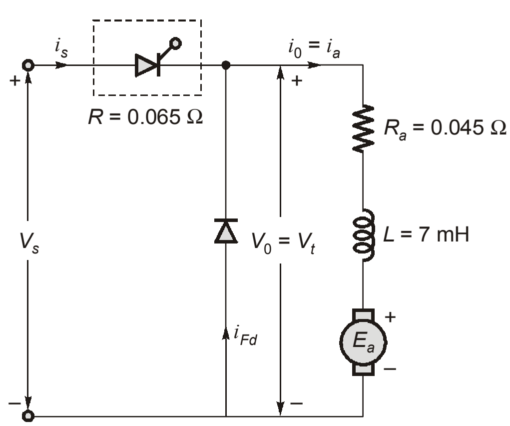
$$V_a = E + I_a R_a$$

$$-186 = E + 150 \times 0.06$$

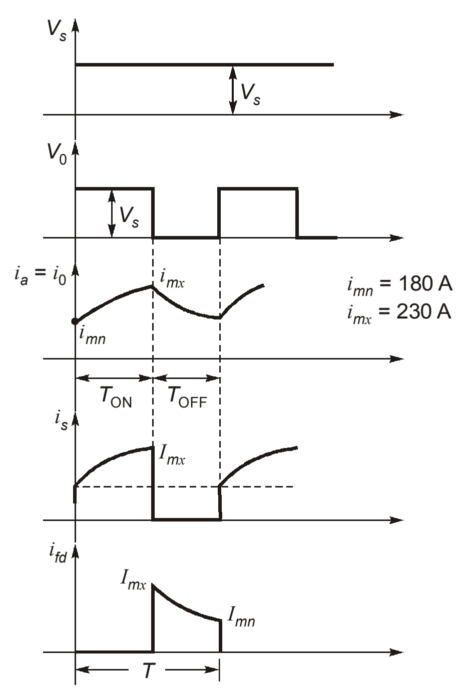
$$E = -195 \text{ V}$$

$$N_2 = -\frac{195}{191} \times 875 = -893.2 \text{ rpm}$$

Solution : 5



Circuit arrangement



Performance of circuit during one-cycle

For a separately-excited DC motor:

$$V_t = V_0 = E_a + I_a r_a$$
$$72 = \text{kN} + (200 \times 0.045)$$
$$k = \frac{72-9}{2500} = \frac{63}{2500}\ \text{V/rpm}$$

At 1000 rpm, counter emf of motor:

$$E_a = k \times 1000$$
$$= \frac{63}{2500} \times 1000 = 25.2\ \text{V}$$
$$I_{mx} = \frac{V_s - E}{R}(1 - e^{-T_{ON}/T_a}) + I_{mn}\, e^{-T_{ON}/T_a} \quad ...(1)$$
$$T_a = \frac{L}{R} = \frac{7 \times 10^{-3}}{0.045 + 0.065} = 0.064 s$$

Putting in equation (1),

$$230 = \frac{72 - 25.2}{0.11}[1 - e^{-T_{ON}/0.064}] + 180 - e^{T_{ON}/0.064}$$
$$\Rightarrow \quad 245.45 e^{-T_{ON}/0.064} = 195.45$$
$$T_{ON} = 0.01458\text{s}$$

During free wheeling period:

$$I_{mn} = -\frac{E}{R}(1 - e^{-T_{OFF}/T_a}) + I_{mx}\, e^{-T_{OFF}/T_a}$$
$$T_a = \frac{L}{R} = \frac{7 \times 10^{-3}}{0.045} = 0.1556 s$$
$$180^\circ = -\frac{25.2}{0.045}(1 - e^{-T_{OFF}/0.1556}) + 230 e^{-T_{OFF}/0.1556}$$
$$T_{OFF} = 0.1556\ In \frac{790}{740} = 0.01017 s$$

$$\text{Chopping period} = T = T_{ON} + T_{OFF}$$
$$= 0.01458 + 0.01017$$
$$= 0.02475 s$$

Chopping frequency, $$f = \frac{1}{T} = \frac{1}{0.02475} = 40.40\ \text{Hz}$$

$$\text{Duty cycle} = \frac{T_{ON}}{T} = \frac{0.01458}{0.024754} = 0.589 = 58.9\%$$

Solution : 6

(a) Motor constant = 0.5 V-sec/rad = 0.5 Nm/A = K_m

Motor torque, $T = K_m I_a$

$$I_a = \frac{15}{0.5} = 30\ \text{A}$$

Motor emf, $$E_a = K_m\,\omega_m = 0.5 \times \frac{2\pi}{60} \times 1000 = 52.36 \text{ V}$$

For a 1-phase, half wave converter feeding a dc motor,

$$V_t = \frac{V_m}{2\pi}(1+\cos\alpha) = E_a + I_a r_a$$

$$= \frac{\sqrt{2}\,230}{2\pi}(1+\cos\alpha) = 52.36 + (30 \times 0.7) = 73.36 \text{ V}$$

Firing angle of armature converter,

$$\alpha = 65.336°$$

(b) Rms value of thyristor current,

$$I_{\text{Thyristor}} = I_a\sqrt{\frac{\pi-\alpha}{2\pi}} = 30\sqrt{\frac{180-65.336}{360}} = 16.931$$

Rms of free wheeling diode current,

$$I_{fd} = I_a\sqrt{\frac{\pi+\alpha}{2\pi}} = 30\sqrt{\frac{180+65.336}{360}} = 24.766 \text{ A}$$

(c) Input power factor of armature converter $= \dfrac{V_t\, I_a}{V_s\, I_{sr}}$

$$I_{sr} = I_{\text{Thyristor}}$$

$$= \frac{73.36 \times 30}{230 \times 16.931} = 0.5651 \text{ lag}$$

Solution : 7

(a) Motor terminal voltage $= \dfrac{3V_{ml}}{\pi}\cos 30° = \dfrac{3 \times 400 \times \sqrt{2}}{\pi}\dfrac{\sqrt{3}}{2} = 467.75 \text{ V}$

$$V_t = E_a + I_a r_a + 2$$

$$467.75 = K_m \omega_m + (21 \times 0.1) + 2$$

No-load motor speed $= \dfrac{467.75 - 4.1}{1.6} = 289.78$ rad/sec.

(b) At rated armature current and at 2000 rpm

$$V_0 = K_\omega\, \omega_m + I_a r_a + 2$$

$$\frac{3\sqrt{2} \times 400}{\pi}\cos\alpha = \left(1.6 \times \frac{2\pi}{60} \times 2000\right) + (210 \times 0.1) + 2 = 358.1 \text{ V}$$

$$\cos\alpha = \frac{358.1 \times \pi}{3\sqrt{2} \times 400} = 0.663$$

$$\alpha = 48.47°$$

Rms value of source current,

$$I_{sr} = I_a\sqrt{\frac{2}{3}} = 210\sqrt{\frac{2}{3}} = 171.46$$

Supply p.f. $= \dfrac{V_t\, I_a}{\sqrt{3}\, V_s I_{sr}} = \dfrac{358.10 \times 210}{\sqrt{3} \times 400 \times 171.46} = 0.633$ lag

(c) At rated load, speed is 2000 rpm, armature voltage V_t = 358.1 V and firing angle is 48.47°.
If load reduced to zero then,

$$V_t = K_m\omega_m + I_a r_a + 2$$

$$358.1 = K_m\omega_m + (21 \times 0.1) + 2$$

$$\omega_m = \frac{358.1 - 4.1}{1.6} = 221.25 \text{ rad/sec. or } 2112.8 \text{ rpm}$$

$$\text{Speed regulation} = \frac{2112.8 - 2000}{2000} \times 100 = 5.64\%$$

Solution : 8

Rotor induced emf at stand still = 700 V (line)

$$E_2 = \frac{700}{\sqrt{3}} \text{ V}$$

$$s = \frac{1500 - 1200}{1500} = 0.2$$

DC voltage across diode rectifier is

$$V_d = \frac{3\sqrt{6}\, sE_2}{\pi} - 2 \times 0.7 = \frac{3\sqrt{2} \times 0.2 \times 700}{\pi} - 1.4$$

Inverter dc voltage is given by

$$V_{dc} = -\left[\frac{3\sqrt{2} \times 415}{\pi}\cos\alpha - 2 \times 1.5\right]$$

With no voltage drop in inductor,

$$V_{dc} = V_d$$

$$-\frac{3\sqrt{2} \times 415}{\pi}\cos\alpha + 3 = \frac{3\sqrt{2} \times 0.2 \times 700}{\pi} - 1.4$$

$$\alpha = \cos^{-1}\left[\frac{-184.6379 \times \pi}{3\sqrt{2} \times 415}\right] = 109.24°$$

$$\text{Firing advance angle of inverter} = 180° - \alpha = 70.76°$$

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