2020

RANK improvement WORKBOOK

Answer key and Hint of
Objective & Conventional *Questions*

Electrical Engineering

Electric Circuits

MADE EASY Publications

1 Basics of Network Analysis

LEVEL 1 Objective Solutions

1.	(b)
2.	(b)
3.	(a)
4.	(a)
5.	(80)
6.	(c)
7.	(b)
8.	(25)

LEVEL 2 Objective Solutions

9.	(100)
10.	(c)
11.	(1.71)
12.	(a)
13.	(a)
14.	(a)
15.	(c)
16.	(0.196)

LEVEL 3 Conventional Solutions

Solution : 1

$$I_{in} = \frac{V_{in}}{R_{eq}} = 0.8 \text{ mA}$$

Since $\quad R_{eq} = 3 + 47 = 50\ \Omega$

$\Rightarrow \quad V_1 = \frac{47}{50} \times 40 \times 10^{-3} = 37.6 \times 10^{-3}$ V

So, using current division

$$I_{out} = \frac{0.125}{0.125 + 0.0625}\left(79.8 \times 37.6 \times 10^{-3}\right) = 2\text{A}$$

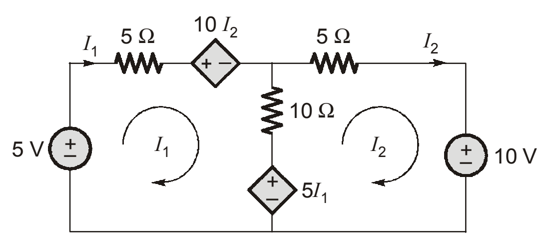
$\Rightarrow \quad V_{out} = 2 \times 8 = 16$ V

$\Rightarrow \quad P_{in} = V_1 I_{in} = 47\, I^2_{in} = 30.08\ \mu$w

$\Rightarrow \quad P_{out} = V_{out} \times I_{out} = 16 \times 2 = 32$ W

$\therefore \quad$ Power gain $= \frac{32}{30.08} \times 10^6 = 1.064 \times 10^6$ W

Solution : 2



By Mesh Analysis

Loop - 1

$5I_1 + 10I_2 + 10(I_1 - I_2) + 5I_1 - 5 = 0$

$$20I_1 = 5$$

$$I_1 = \frac{5}{20} = 0.25 \text{ A}$$

Loop - 2

$5I_2 + 10 - 5I_1 + 10(I_2 - I_1) = 0$

$$15I_2 - 15I_1 = -10$$

since $\quad I_1 = 0.25$ A

$$15I_2 = 15 \times 0.25 - 10$$

$$I_2 = -0.417 \text{ A}$$

5 V independent source = delivering [current I_1 leaving positive terminal]

$10I_2$ dependent source = absorbing [current I_1 entering positive terminal]

$5\,I_1$ dependent source = absorbing [current $(I_1 - I_2)$ entering positive terminal]

10 V independent source = delivering [current I_2 leaving positive terminal]

Solution : 3

Applying KVL in loop *a-b-c-d*,

$$V_L = V_S - \left(\frac{r_1 r_2}{r_1 + r_2}\right) i_S = 10 - \frac{2\times 2}{2+2} i_S$$

or, $$V_L = 10 - i_s \quad \text{... (1)}$$

Again,

$$\beta_1 i_1 + \beta_2 i_2 = i$$

since $$\beta_1 = \beta_2 = \beta \text{ (say)},$$

$$i = \beta(i_1 + i_2) = \beta i_s \quad \text{... (2)}$$

However in figure

$$V_s = \frac{r_1 r_2}{r_1 + r_2} \cdot i_S + (i_S + i) r_L \quad \text{[applying KVL in outer loop]}$$

or, $$V_s = i_s + i_s r_L + \beta i_s r_L \quad \text{[using (2)]}$$

$\therefore$ R_{in} (net resistance of the original circuit across *a-d*)

$$= \frac{V_S}{i_S} = (1 + r_L + \beta r_L)\Omega = (1 + 5 + 2 \times 5) = 16\ \Omega$$

$\therefore$ $$i_s = \frac{V_S}{R_{in}} = \frac{10}{16} = 0.625 \text{ A.}$$

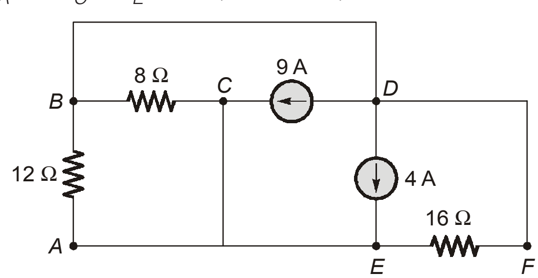
Then from (1), $$V_L = 10 - i_s = 10 - 0.625 = 9.375 \text{ V.}$$

Then, $$\frac{V_L}{V_S} = \frac{9.375}{10} = 0.9375$$

Solution : 4

Let, $$V_B = V_D = V_F = V_1$$

and $$V_A = V_C = V_E = 0 \text{ V (reference)}$$



Using KCL at node *B*,

$$\frac{V_B - V_A}{12} + \frac{V_B - V_C}{8} + \frac{V_F - V_E}{16} + 9 + 4 = 0$$

$$\frac{V_1}{3} + \frac{V_1}{2} + \frac{V_1}{4} + 52 = 0$$

$$\frac{13 V_1}{12} = -52$$

$\Rightarrow$ $$V_1 = -48 \text{ V}$$

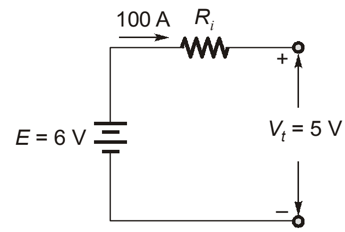
∵ Resistors always dissipates the power irrespective of direction of current flowing through them.

∴ Power dissipated across 12 Ω resistor = $\dfrac{(48)^2}{12} = 192$ W

Power dissipated across 8 Ω resistor = $\dfrac{(48)^2}{8} = 288$ W

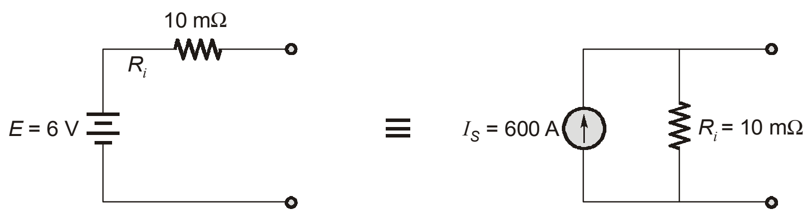
Power dissipated across 16 Ω resistor = $\dfrac{(48)^2}{16} = 144$ W

Solution : 5



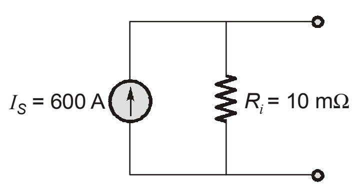
E = Open voltage of battery = 6 V with a current of 100 A

$$V_t = E - (100)\,R_i$$
$$V_t = 5\text{ V} = 6 - 100\,R_i$$
$$R_i = 10\text{ m}\Omega$$

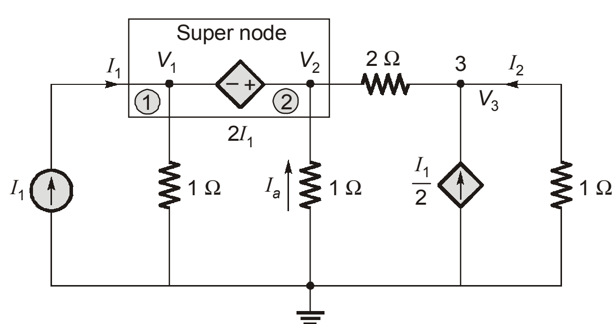


$$I_s = \frac{E}{R_i} = \frac{6}{10\times10^{-3}} = 600\text{ A}$$

Constant current source



Solution : 6



Consider nodes (1) and (2). These two nodes constitutes a super node.

$$V_2 - V_1 = 2I_1 \quad \text{...(1)}$$

Super node equation

$$\frac{V_1}{1} - I_1 + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0 \quad \text{...(2)}$$

$$V_1 + V_2 + 0.5V_2 - 0.5V_3 = I_1$$
$$V_1 + 1.5V_2 - 0.5V_3 = I_1$$

Node 3:

$$\frac{V_3 - V_2}{2} - I_2 - \frac{I_1}{2} = 0 \quad \text{...(3)}$$

Put $\frac{V_3 - V_2}{2} = 0.5I_1 + I_2$, in equation (2)

$$V_1 - I_1 + V_2 - 0.5I_1 - I_2 = 0$$
$$V_1 + V_2 = 1.5I_1 + I_2 \quad \text{...(4)}$$

From (1) and (4)

$$V_2 = 1.75I_1 + 0.5I_2$$

Also

$$V_3 - V_2 = I_1 + 2I_2$$
$$V_3 = V_2 + I_1 + 2I_2$$
$$V_3 = 2.75I_1 + 2.5I_2 \quad \text{...(5)}$$

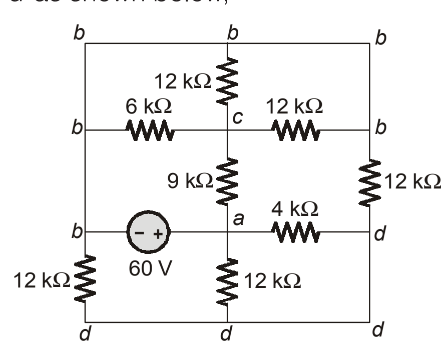
Also $I_2 = -V_3$

From (5)

$$-I_2 = 2.75I_1 + 2.5I_2$$
$$-3.5I_2 = 2.75I_1$$
$$\alpha = \frac{I_2}{I_1} = \frac{-2.75}{3.5} = -0.786$$
$$\alpha = \frac{I_2}{I_1} = -0.786$$

Solution: 7

We assign nodes *a*, *b*, *c* and *d* as shown below,



Simplifying the circuit in following steps,

Between nodes *c* and *b*

$$6\text{ k}\Omega \,||\, 12\text{ k}\Omega \,|\, 12\text{ k}\Omega = 3\text{ k}\Omega$$

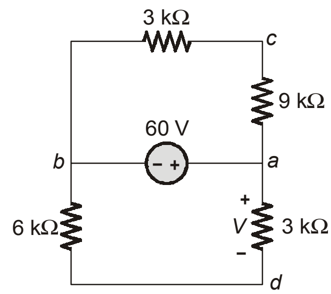
Between nodes *b* and *d*

$$12\text{ k}\Omega \,||\, 12\text{ k}\Omega = 6\text{ k}\Omega$$

Between nodes *a* and *d*

$$4\text{ k}\Omega \,||\, 12\text{ k}\Omega = 3\text{ k}\Omega$$

Between node *c* and *a*, 9 kΩ resistor is connected.



Using voltage division rule,

$$V = \frac{3}{3+6}(60) = 20\text{ V}$$

∴ Power absorbed by 4 kΩ resistor,

$$P = \frac{V^2}{4\times10^3} = \frac{(20)^2}{4\times10^3} = 100\text{ mW}$$

■■■■

2 Steady State Sinusoidal Analysis

LEVEL 1 Objective Solutions

1.	**(d)**
2.	**(1)**
3.	**(a)**
4.	**(a)**
5.	**(c)**
6.	**(b)**
7.	**(b)**

LEVEL 2 Objective Solutions

8.	**(a)**
9.	**(c)**
10.	**(c)**
11.	**(d)**
12.	**(c)**
13.	**(89.44)**
14.	**(b)**
15.	**(b)**
16.	**(1)**

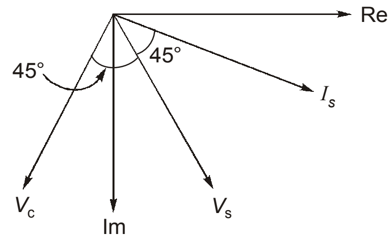
LEVEL 3 Conventional Solutions

Solution : 1

Voltage across capacitor is

$$V_C = I_S \frac{1}{j\omega C} = \frac{I_S}{500 \times 2 \times 10^{-6}} \angle -90^\circ = \frac{I_S}{10^{-3}} \angle -90^\circ$$

Voltage across capacitor is lagging by 90° from I_s. In the circuit V_C plus unknown element voltage will result in V_s.



$$\therefore \qquad V_s = V_C + V_x$$

where V_x = unknown element voltage.

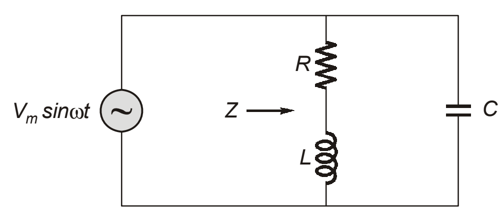
So, the unknown voltage should be in same phase with I_s. This is a resistors.

$$|V_C| = |V_x|$$

$$\left| I_S \left(\frac{1}{\omega C} \right) \right| = |I_S R| \quad ; \quad R = \frac{1}{\omega C} = \frac{1}{2 \times 10^{-6} \times 500} = 1\text{k}\Omega$$

Solution : 2

For unity power factor Z should be purely real



$$Z = (R + j\omega L) \| \frac{1}{j\omega C}$$

$$= \frac{(R + j\omega L) \times \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{\frac{R}{j\omega C} + \frac{L}{C}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \times \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R - j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\left(\frac{R}{j\omega C} + \frac{L}{C}\right)\left[R - j\left(\omega L - \frac{1}{\omega C}\right)\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \frac{\frac{R^2}{j\omega C} + \frac{RL}{C} - \frac{R}{\omega C}\left(\omega L - \frac{1}{\omega C}\right) - j\frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

equating imaginary part to zero

$$\frac{R^2}{j\omega C} - j\frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right) = 0$$

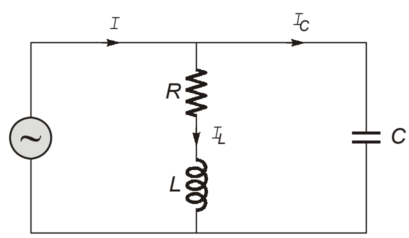
$$\frac{-jR^2}{\omega C} - j\frac{L}{C}\left(\omega L - \frac{1}{\omega C}\right) = 0$$

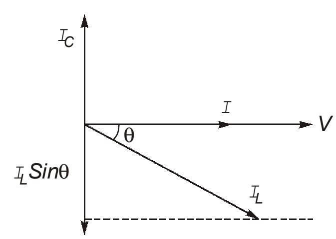
$$\frac{-R^2}{\omega} - L\left(\omega L - \frac{1}{\omega C}\right) = 0$$

$$\frac{L}{\omega C} = \frac{R^2}{\omega} + \omega L^2$$

$$C = \frac{L}{R^2 + (\omega^2 L^2)}$$

Alternate solution:





where $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$I_c = \omega C \cdot V$$

$$I_L = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega L^2}}$$

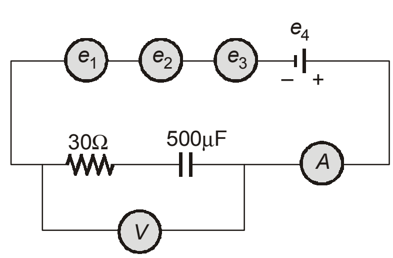
For power factor to be unity

$$I_C = I_L \sin\theta$$

$$\omega C V = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \times \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$C = \frac{L}{R^2 + (\omega L)^2}$$

Solution : 3



Given,

$$e_1 = 100 \sin 30t \text{ V}$$
$$e_2 = 50 \sin 80t \text{ V}$$
$$e_3 = 70 \sin 100t \text{ V}$$

The rms values of various voltage sources are:

$$e_4 = 120\text{ V}$$

$\because$
$$E_1 = \frac{100}{\sqrt{2}} = 70.71\text{ V}$$

$$E_2 = \frac{50}{\sqrt{2}} \approx 35.36\text{ V}$$

$$E_3 = \frac{70}{\sqrt{2}} \approx 49.50\text{ V}$$

$$E_4 = 120\text{ V (for dc, } V_{rms} = V_{dc})$$

Now, Voltmeter reading $V = \sqrt{E_1^2 + E_2^2 + E_3^2 + E_4^2}$

$$= \sqrt{(70.71)^2 + (35.36)^2 + (49.50)^2 + (120)^2} \approx 152\text{V}$$

Solution : 4

The circuit shown in figure is first reduced as shown in figure(a) using the concept of series parallel equivalent.

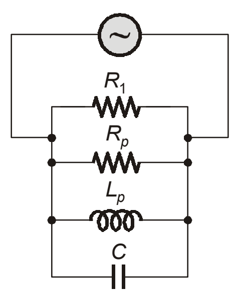


Fig. (a)

Here, in figure (a)

$$R_p = \frac{R_C^2 + X_{LC}^2}{R_C} = R_C + R_C\frac{X_{LC}^2}{R_C^2} = R_C(1+Q_C^2)$$

$$\left[\because Q_C = \frac{X_{LC}}{R_C} \text{ at resonance; } Q_C \text{ being the } Q \text{ factor of the coil}\right]$$

Also,
$$X_p = \frac{R_C^2 + X_{LC}^2}{X_{LC}} = X_{LC} + X_{LC}\frac{R_C^2}{X_{LC}^2}$$

$$= X_{LC}\left[1 + 1/X_{LC}^2/R_C^2\right] = X_{LC}\left[1 + \frac{1}{Q_C^2}\right]$$

Finally, following figure (b)

$$R_{eq} = \frac{R_1 \ R_p}{R_1 + R_p}$$

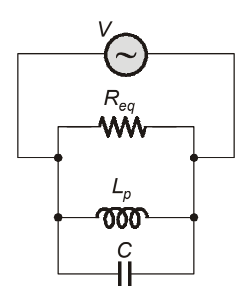


Fig. (b)

Also, in figure (b)

The current magnification [i.e., the net Q factor (Q_p)] is given by

$$\frac{I_C}{I_{R_{eq}}} = \frac{V/X_C}{V/R_{eq}} = \frac{R_{eq}}{X_C} = \omega_0\, C\, R_{eq}$$

[ω_0 being the frequency of resonance]
Thus, for the given circuit,

$$R_{eq} = \frac{R_1 R_p}{R_1 + R_p};\ R_p = R_C\left(1 + Q_{coil}^2\right)$$

$$X_p = X_{LC}\left(1 + \frac{1}{Q_{coil}^2}\right)$$

Q_p, quality factor of the parallel circuit $= \omega_0 C R_{eq}$

The frequency of resonance $\omega_0 = \dfrac{1}{L_C}\sqrt{\dfrac{L_C}{C} - R_C^2}$

Here, $$Q_{coil} = \frac{\omega_0 L_C}{R_C} = \frac{1414 \times 0.05}{1} = 70.7$$

$$\left[\because \omega_0 = \frac{1}{0.05}\sqrt{\frac{0.05}{10 \times 10^{-6}} - 1^2} = 1414 \text{ rad/sec.}\right]$$

$\therefore$ $$R_p = 1\,(1 + 70.7^2) = 5000\ \Omega$$

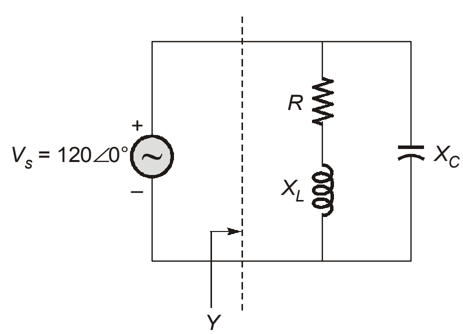
This gives $$R_{eq} = \frac{5000 \times 2}{5000 + 2} = 2\Omega$$

Then Q_p, the Q factor of the parallel circuit is

$$Q_p = \omega_0 C R_{eq} = 1414 \times 10 \times 10^{-6} \times 2 = 0.03$$

Thus for the given circuit, the Q factor is 0.03 and ω_0 (the frequency of resonance) = 1414 rad/sec.

Solution : 5



Y = equivalent admittance of the parallel circuit

$$= \frac{1}{R + jX_L} + \frac{1}{(-jX_C)} = \frac{(R - jX_L)}{(R + jX_L)(R - jX_L)} + \frac{j}{X_C}$$

$$= \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C} = \frac{R}{R^2 + X_L^2} + j\left(\frac{1}{X_L} - \frac{X_L}{R^2 + X_L^2}\right)$$

For unity power factor resonance, imaginary term must be zero

$$\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} = 0$$

$\Rightarrow$ $$X_L^2 - X_C X_L + R^2 = 0 \qquad \ldots(i)$$

Roots of the equation (*i*),

$$X_L = \frac{X_C}{2} \pm \sqrt{\frac{X_C^2}{4} - R^2} \qquad ...(ii)$$

Putting values of R and X_C in equation (*ii*),

$$X_L = \frac{20}{2} \pm \sqrt{\frac{20^2}{4} - 8^2}$$

$$\Rightarrow \qquad X_{L_1} = 16\ \Omega \text{ and } X_{L_2} = 4\ \Omega$$

$$Y = \frac{R}{R^2 + X_L^2} \qquad \text{(Imaginary part is zero)}$$

Current drawn from the supply,

$$I_s = V_s Y = 120 \frac{R}{(R^2 + X_L^2)}$$

For $\quad X_L = X_{L_1} = 16\Omega$

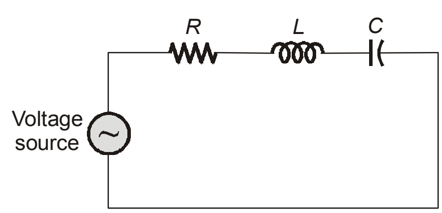
$$I_{s_1} = 120 \times \left(\frac{8}{8^2 + 16^2}\right) = 3\text{ A}$$

For $\quad X_L = X_{L_2} = 4\ \Omega$

$$I_{s_1} = 120 \times \left(\frac{8}{8^2 + 4^2}\right) = 12\text{ A}$$

Minimum value of the current drawn from the supply $I_{s_1} = 3\text{ A}$.

Solution : 6



where, $\quad R = 25\ \Omega,\ L = 2\text{ H}$

and $\quad C = 30\ \mu\text{F}$

Impedance of the circuit,

$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)\Omega$$

$$\theta = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$$

$$\Rightarrow \qquad \tan\theta = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

(i) For phase angle leading or $\theta = -45°$ [current leads voltage by 45°]

$$\tan(-45°) = \frac{\left(2\omega - \frac{1}{\omega \times 30 \times 10^{-6}}\right)}{25}$$

$$\Rightarrow \quad -25 = 2\omega - \frac{10^6}{30\,\omega}$$

$$\Rightarrow \quad -25 \times 30\,\omega = 2 \times 30\,\omega^2 - 10^6$$

$$\Rightarrow \quad 60\,\omega^2 + 750\,\omega - 10^6 = 0 \quad \ldots(i)$$

Roots of the equation (*i*),

$$\omega_1 = 123 \text{ rad/s and } \omega_2 = -135.5 \text{ rad/sec.}$$

$\omega_2 = -135.5$ rad/s is rejected as ω can be negative

$$f = \frac{123}{2\pi} = 19.57 \text{ Hz}$$

For frequency of 19.57 Hz, phase angle of the circuit is 45° leading.

(ii) For phase angle 45° lagging, $\theta = 45°$ [current lages voltage by 45°]

$$\tan(45°) = \frac{\left(\omega L - \frac{1}{\omega L}\right)}{R} = \frac{\left(2\omega - \frac{10^6}{30\,\omega}\right)}{25}$$

$$\Rightarrow \quad 25 = 2\omega - \frac{10^6}{30\,\omega}$$

$$\Rightarrow \quad 60\,\omega^2 - 750\,\omega - 10^6 = 0 \quad \ldots(ii)$$

Roots of the equation (*ii*),

$\omega_1 = -123$ rad/sec and $\omega_2 = 135.5$ rad/sec.

$\omega_1 = -123$ rad/sec is rejected as ω can not be negative

$$f = \frac{135.5}{2\pi} = 21.56 \text{Hz}$$

For frequency of 21.56 Hz, phase angle of the circuit is 45° lagging.

Solution : 7

$$Z(s) = \frac{20((s+1)^2 + 10^2)}{s} = \frac{20(s^2 + 2s + 101)}{s}$$

$$Z(s) = 20s + 40 + \frac{2020}{s} \quad \ldots(1)$$

For series RLC circuit

$$Z(s) = R + SL + \frac{1}{Cs} \quad \ldots(2)$$

Comparing (1) with (2)

$$R = 40\ \Omega$$
$$L = 20 \text{ H}$$
$$C = 4.95 \times 10^{-4} \text{ F}$$

(i) Resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{20\times 4.95\times 10^{-4}}} = 10.05 \text{ rad/sec}$$

(ii) Q factor
$$\frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
$$Q = \frac{1}{40}\times\sqrt{\frac{20}{4.95\times 10^{-4}}} = 5.02$$

(iii) Bandwidth
$$\beta = \frac{R}{L} = \frac{40}{20} = 2$$

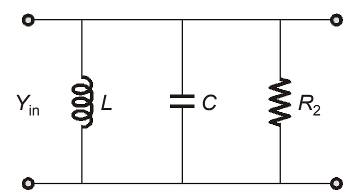
(iv) Impedance under resonance $= Z_0 = R = 40\ \Omega$

At resonance capacitive reactance is equal to inductive reactive

So $\quad Z_0 = R = 40\ \Omega$

Solution : 8

(i) Z_{xy} is maximum when impedance of parallel R_2, L, C circuit is maximum as R_1 is real and positive. Consider the parallel circuit



$$Y_{in}(\omega) = \frac{1}{R_2} + \frac{1}{j\omega L} + j\omega C = \frac{1}{R_2} + j\left(\omega C - \frac{1}{\omega L}\right)$$

The network is parallel resonance (or high impedance) when $Y_{in}(\omega)$ and thus $Z_{in}(\omega)$ is real. In this case $|Y_{in}(\omega)|$ will be minimum and $Z_{in}(\omega) = \dfrac{1}{Y_{in}(\omega)}$ will be maximum.

So for real $\quad Y_{in}(\omega) = -\dfrac{1}{\omega L} + \omega C = 0$

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

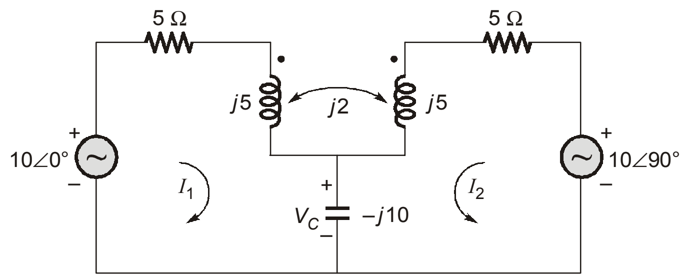
(ii)
$$Z_{xy} = R_1 + Z_{in}(\omega_0)$$

For
$$\omega = \omega_0$$
$$Z_{in}(\omega_0) = R_2$$
$$Z_{xy} = R_1 + R_2$$

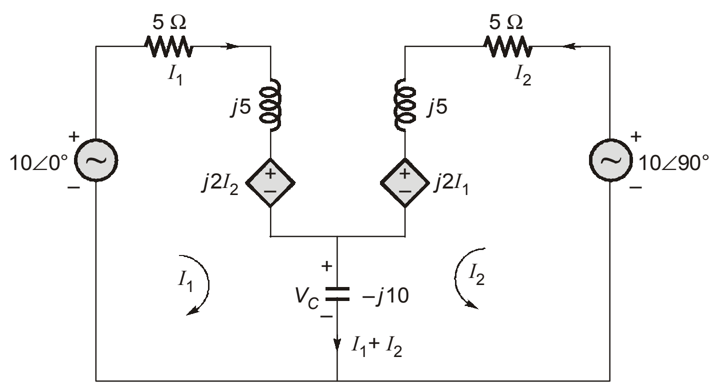
Current
$$I = \frac{V}{R_1 + R_2}$$
$$i(t) = \frac{V}{R_1 + R_2}\sin\omega t$$

Solution : 9

Since the current going into the windings from both sides should support the flux in the same direction, hence dotted equivalent circuit can be drawn as,



Using decoupled technique,



Applying KVL in loop (1) and (2),

$$10\angle 0 = I_1[5 + j5 - j10] + I_2[j2 - j10]$$

$$10\angle 0 = I_1[5 - j5] + I_2[-j8] \quad \text{...(1)}$$

$$10\angle 90 = I_2[5 + j5 - j10] + I_1[j2 - j10]$$

$$10\angle 90 = I_2[5 - j5] + I_1[-j8] \quad \text{...(2)}$$

$$V_C = -j10(I_1 + I_2) \quad \text{...(3)}$$

Now adding equation (1) and (2),

$$10 + j0 = I_1[5 - j13] + I_2[5 - j13]$$

$$\therefore \quad I_1 + I_2 = \frac{10 + j10}{5 - j13}$$

$$\therefore \quad V_C = -j10[I_1 + I_2]$$

$$\therefore \quad V_C = -j10 \times \frac{10 + j10}{5 - j13}$$

$$\therefore \quad V_C = 10.15\angle 24^\circ \text{ Volts}$$

■■■■

3 Network Theorems

LEVEL 1 Objective Solutions

1.	(b)
2.	(a)
3.	(160)
4.	(5)
5.	(b)
6.	(d)

LEVEL 2 Objective Solutions

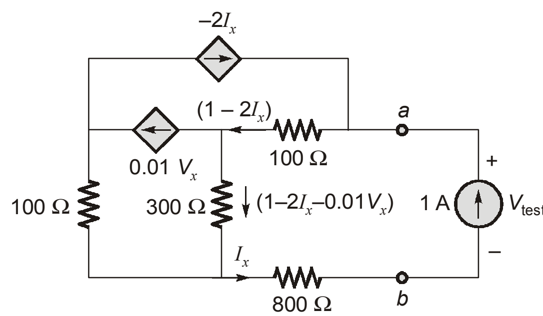
7.	(257.99)
8.	(1)
9.	(c)
10.	(b)
11.	(1.797)
12.	(c)
13.	(c)
14.	(0.8)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

Writing currents into 100 Ω and 300 Ω resistors by using KCL as shown in figure below,



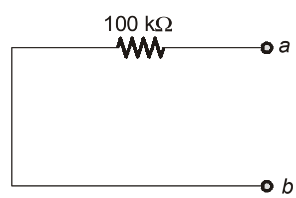
$$I_s = 1\text{ A}$$
$$V_x = V_{test}$$

Writing mesh equation for bottom right mesh,

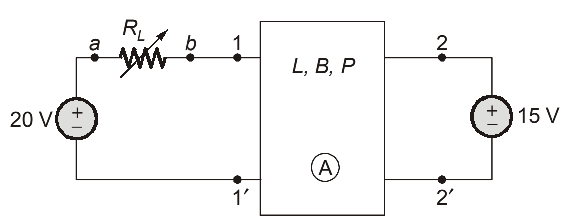
$$V_{test} = 100(1-2I_x) + 300(1-2I_x-0.01V_x) + 800 = 100\text{ V}$$

$\therefore$
$$R_{Th} = \frac{V_{Test}}{1} = 100\ \Omega$$

Thevenin's equivalent circuit,

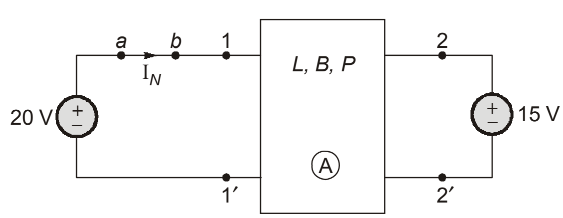


Solution : 2



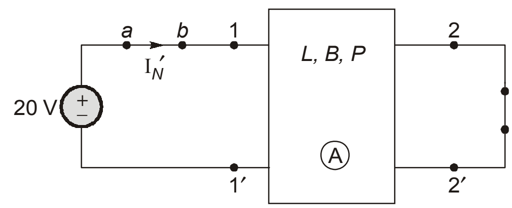
We can determine the norton's equivalent across the terminals *a* and *b*.

To determine I_N:



We can use superposition theorem to determine I_N.

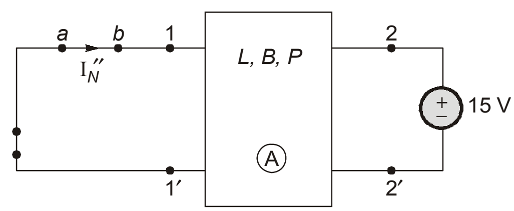
When only 20 V source is acting alone.



From figure A in the question, we can calculate I_N'.

$$I_N' = 8 \text{ A}$$

When only 15 V source is acting alone:

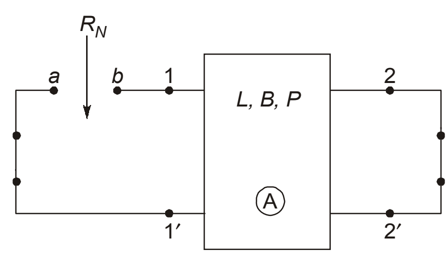


By applying the Reciprocity principle to the figure. A in the given question, we can calculate I_N''.

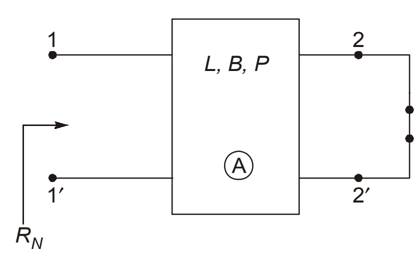
$$I_N'' = -\frac{2}{10} \times 15 = -3\text{A} \qquad \left(\frac{15}{I_N''} = \frac{10}{2}\right)$$

Thus, $\quad I_N = I_N' + I_N'' = 8 - 3 = 5\text{A}$ (Using superposition theorem)

To determine R_N:



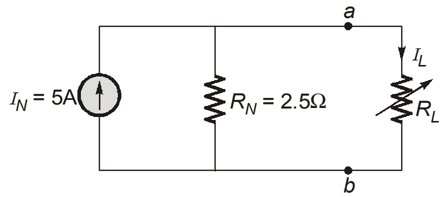
Equivalently, we can write the above circuit as show below.



From figure A in the given question, we can calculate R_N as

$$R_N = \frac{10\text{V}}{4\text{A}} = 2.5\Omega$$

So, the Norton's equivalent across the terminals *a* and *b* is will be as shown below.



Maximum power will be delivered to R_L, when R_L will be equal to $R_N = 2.5$ W

When $$R_L = R_N = 2.5\ \Omega,\ I_L = \frac{5}{2} = 2.5\ \text{A}$$

Hence, $$P_{max} = I_L^2 R_L = I_L^2 R_N = (2.5)^2 \times 2.5 = 15.625\ \text{W}$$

Solution : 3

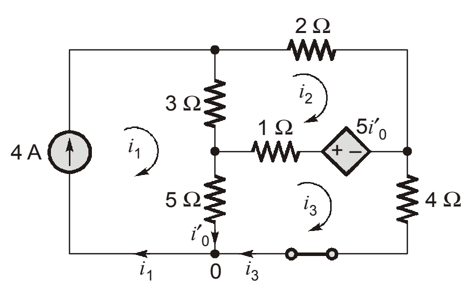
The superposition theorem states that the voltage across (or current through) an element in a linear and bidirectional circuit is the algebraic sum of the voltages (or currents through) that element due to each independent source acting alone. The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

- While we are considering one independent source at a time, all other independent sources must be turned-off. This implies that we replace every ideal independent voltage source by 0 V (or a short circuit) and every ideal independent current source by 0 A (or an open circuit).
- Dependent sources are left intact because they are controlled by circuit variables.

The circuit in Figure involves a dependent source, which must be left intact.

Let, $$i_0 = i'_0 + i''_0 \qquad ...(i)$$

where i'_0 and i''_0 are due to the 4 A current source and 20 V voltage source respectively. To obtain i'_0, we turn off the 20 V source so that we have the circuit in Figure (a). We apply mesh analysis in order to obtain i'_0.



(a)

For loop 1,

$$i_1 = 4\ \text{A} \qquad ...(ii)$$

For loop 2, $$-3i_1 + 6i_2 - 1i_3 - 5i'_0 = 0 \qquad ...(iii)$$

For loop 3, $$-5i_1 - i_2 + 10i_3 + 5i'_0 = 0 \qquad ...(iv)$$

But at node 0, $$i_3 = i_1 - i'_0 = 4 - i'_0 \qquad ...(v)$$

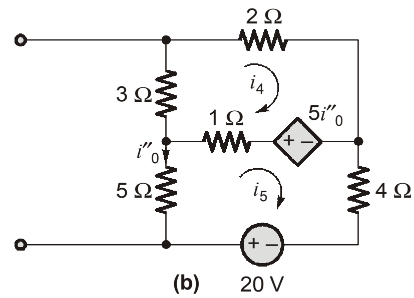
Substituting Equations (*ii*) and (*v*) into Equations (*iii*) and (*iv*) gives two simultaneous equations.

$$3i_2 - 2i'_0 = 8 \qquad ...(vi)$$

$$i_2 + 5i'_0 = 20 \qquad ...(vii)$$

which can be solved to get, $i'_0 = 52/17$ A ...(*viii*)

To obtain i''_0, we turn off the 4 A current source so that the circuit becomes that shown in Figure (b).



(b)

For loop 4, KVL gives

$$6i_4 - i_5 - 5i''_0 = 0 \quad \text{...(ix)}$$

and for loop 5, $-i_4 + 10i_5 - 20 + 5i''_0 = 0$ ...(*x*)

But $i_5 = -i''_0$, substituting this in Equations (*ix*) and (*x*) gives

$$6i_4 - 4i''_0 = 0 \quad \text{...(xi)}$$

$$i_4 + 5i''_0 = -20 \quad \text{...(xii)}$$

which we solve to get

$$i''_0 = -60/17 \text{ A} \quad \text{...(xiii)}$$

Now substituting Equations (*viii*) and (*xiii*) into Equation (*i*) gives

$$i_0 = -\frac{8}{17} = -0.4706 \text{ A}$$

Solution : 4

Let us first remove R_L (Figure (a))

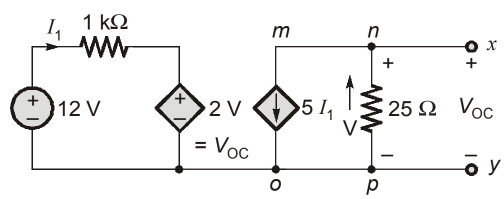


Fig. (a)

Here $V_{OC} = V = V_{xy} = (-5I_1) \times 25$

[∵ The direction of 5 I_1, the current in the loop *m-n-o-p*, is opposite to the polarity of assumed voltage drop *V* in 25 Ω resistor]

i.e., $V_{OC} = -125\, I_1$

But $I_1 = \dfrac{12-2V}{1000} = \dfrac{12-2V_{OC}}{1000} [\because V_{OC} = V]$

Thus finally, $V_{OC} = -125\left(\dfrac{12-2V_{OC}}{1000}\right)$

or, $0.75\, V_{OC} = -1.5$ i.e., $V_{OC} = -2$ V

To determine R_{Th}, let us short the terminals *x - y* (figure (b)). Due to short circuit between

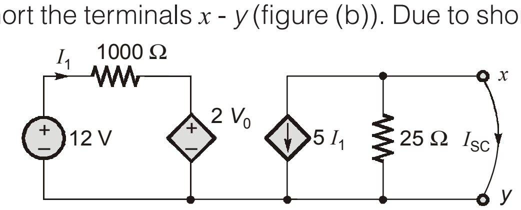


Fig. (b)

x - y, the voltage V diminishes to zero. Thus the dependent source (2 V) in the left loop vanishes (Figure (c))

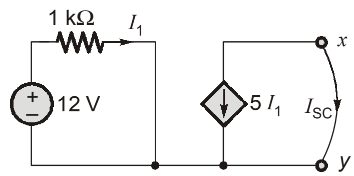


Fig. (c)

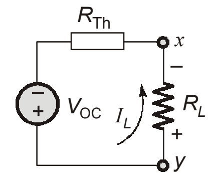


Fig. (d)

Obviously, $$I_{SC} = -5\,I_1 = -5\left(\frac{12}{1000}\right) = -0.06\text{ A}$$

Thus, $$R_{Th} = \frac{V_{0.c}}{I_{s.c}} = \frac{-2}{-0.06} = 33.33\ \Omega$$

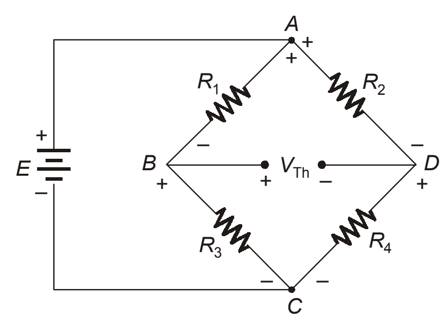
Figure (d) being representing the Thevenin's equivalent circuit, I_L (current through R_L)

$$= \frac{V_{0.c}}{R_{Th} + R_L}$$

or, $$I_L = \frac{-2}{33.33+10} = -46.15\text{ mA} \qquad \text{(i.e., 46.15 mA anticlockwise)}$$

Solution : 5

Finding Thevenin's equivalent across terminal B-D. So open circuit the resistor R_5



So $$V_{AB} = \frac{ER_1}{R_1 + R_3}$$

$$V_{AD} = \frac{ER_2}{R_2 + R_4}$$

So $$V_{Th} = -V_{AB} + V_{AD}$$

$$V_{Th} = \frac{ER_2}{R_2 + R_4} - \frac{ER_1}{R_1 + R_3}$$

Given $E = 12$ V

$R_1 = 1.2$ kΩ

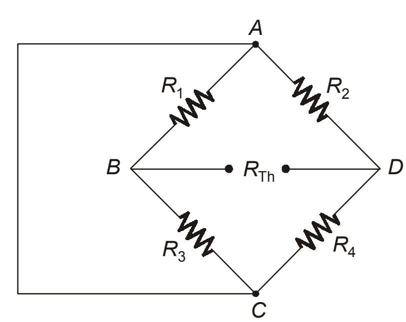
$R_2 = 1.5$ kΩ

$R_3 = 4$ kΩ

$R_4 = 3.6$ kΩ

$$V_{Th} = \frac{12\times1.5}{1.5+3.6} - \frac{12\times1.2}{1.2+4}$$

$$= 3.53 - 2.77 = 0.76\text{ V}$$



To find R_{Th} for this:

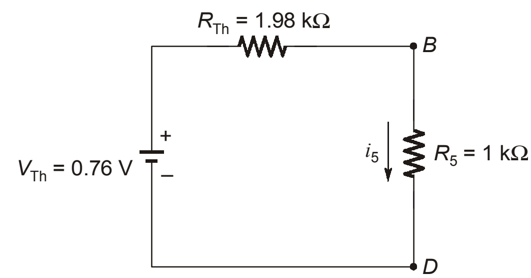
by short circuit the voltage source

$$R_{Th} = (R_1 \parallel R_3) + (R_4 \parallel R_2)$$

$$= \frac{R_1R_3}{R_1+R_3} + \frac{R_2R_4}{R_2+R_4}$$

$$= \frac{1.2\times 4}{1.2+4}+\frac{1.5\times 3.6}{1.5+3.6}$$
$$= 0.92 + 1.06 = 1.98\ \text{k}\Omega$$

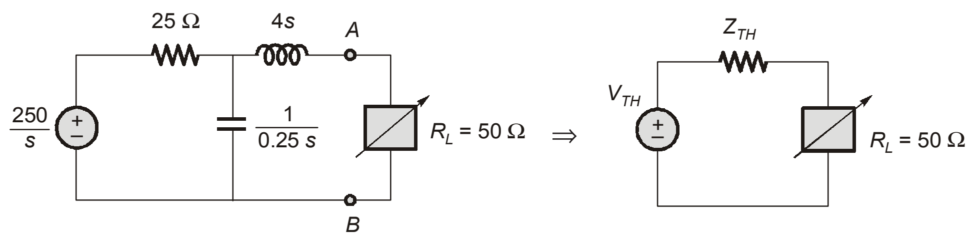
Thevenin's equivalent



$$i_5 = \frac{+V_{Th}}{R_{Th}+R_5} = \frac{0.76\ V}{(1.98+1)k\Omega}$$
$$i_5 = 0.255\ \text{mA}$$

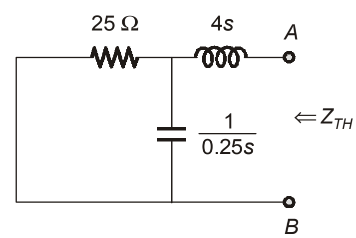
Solution : 6

After closing of the switch 'S', the circuit in s domain is



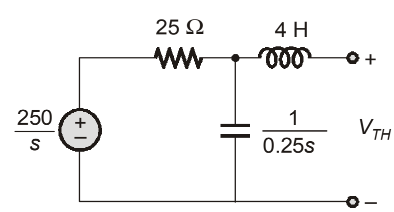
Calculating Z_{TH}:

All independent sources are replaced by their internal impedances.



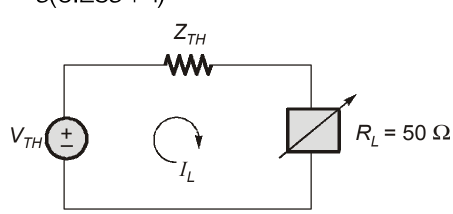
$$\frac{25\left(\frac{1}{0.25s}\right)\times 0.25s}{25\times 0.25s+1}+4s = Z_{TH}$$
$$Z_{TH} = \frac{25}{6.25s+1}+4s \quad \text{...(i)}$$

Calculating V_{TH}:



$$V_{TH} = \frac{250/s}{25+\frac{1}{0.25s}}\times\frac{1}{0.25s} \quad \text{...(ii)}$$
$$V_{TH} = \frac{250/s\times 0.25s}{25\times 0.25s+1}\times\frac{4}{s} = \frac{62.5}{6.25s+1}\times\frac{4}{s}$$
$$V_{TH} = \frac{250}{s(6.25s+1)} \quad \text{...(iii)}$$

$\therefore$



The current through load resistance R_L

$$I_L(s) = \frac{V_{TH}}{Z_{TH}+R_L} = \frac{250/s(6.25s+1)}{\dfrac{25+25s^2+4s+50+312.5s}{(6.25s+1)}}$$
$$= \frac{250(6.25s+1)}{s(6.25s+1)(25s^2+316.5s+75)} = \frac{250}{s(25s^2+316.5s+75)} = \frac{250}{s(s+0.243)(s+12.418)}$$

Using partial fraction

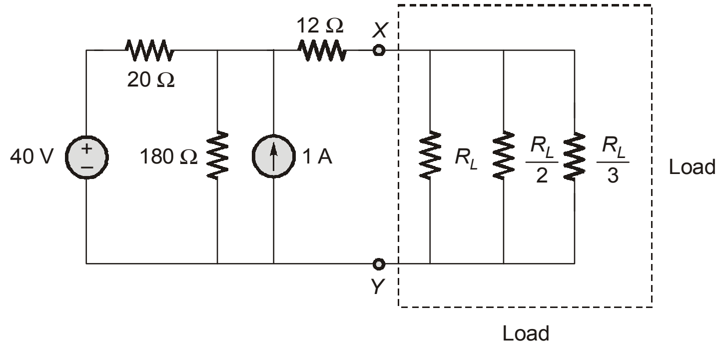
$$I_L(s) = \frac{82.848}{s} + \frac{-84.50}{(s+0.243)} + \frac{1.654}{(s+12.418)}$$

Taking inverse Laplace we get

$$i_L(t) = 82.848u(t) - 84.50e^{-0.243t}u(t) + 1.654\, e^{-12.418}u(t) \text{ A}$$

$$\boxed{i_L(t) = (82.848 - 84.50e^{-0.243t} + 1.654e^{-12.418})u(t)} \text{ A}$$

Solution : 7



Net load resistance Z_L

$$Z_L = \frac{1}{\frac{1}{R_L} + \frac{2}{R_L} + \frac{3}{R_L}}$$

$$= \frac{R_L}{6} \quad \text{...(a)}$$

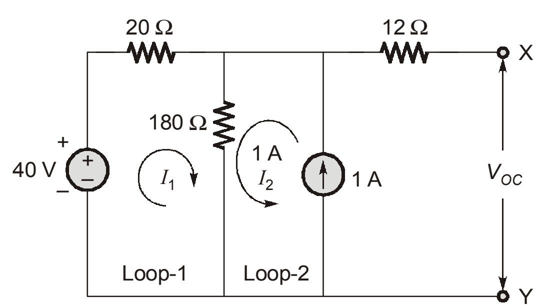
Now we will find Thevenin equivalent with respect to terminals *X*-*Y*.

Now first we will find Thevenin voltage between terminals *X*-*Y*.

i.e. open circuit voltage between terminals *X*-*Y*.

i.e. $V_{th} = V_{OC}$

Now circuit diagram will be



Applying KVL in Loop-1, we have

$$40 - 200I_1 - 180I_2 = 0 \quad \text{...(i)}$$

But from Loop-2, we have

$$I_2 = 1\text{A} \quad \text{...(ii)}$$

∴ From equation (i)

$$40 - 200\, I_1 - 180 = 0$$

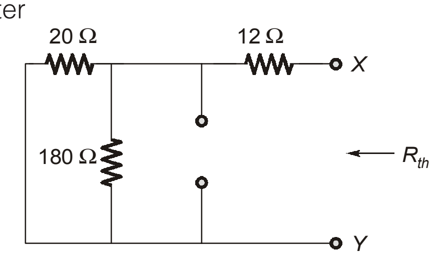
$$200\, I_1 = -140$$

$$\therefore \qquad I_1 = -\frac{140}{200} = -0.7 \text{ A}$$

Net current flowing through 180 Ω register is

$$I = I_1 + I_2$$
$$I = -0.7 + 1$$
$$I = 0.3 \text{ A}$$

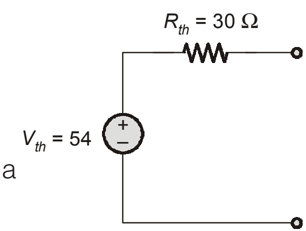
$$\therefore \qquad V_{OC} = \text{Voltage drop across 180 }\Omega\text{ register}$$
$$V_{OC} = 180 \times 0.3 \quad (RI)$$
$$V_{th} = V_{OC} = 54 \text{ V}$$



Now we will find R_{th} w.r.t. terminals X and Y for this circuit will be

$$R_{th} = 12 + 180 \| 20$$
$$R_{th} = 12 + \frac{180 \times 20}{200}$$
$$R_{th} = 12 + 18$$
$$R_{th} = 30 \ \Omega$$

∴ Equivalent circuit with respect to terminal X-Y



According maximum power transfer theorem power delivered by source V_{th} to a load across X-Y will be maximum if load will be equal to R_{th}.

i.e. $\qquad Z_L = R_{th}$

$\Rightarrow \qquad Z_L = 30 \ \Omega \qquad$ ...(iii)

From (iii) and (a), we have

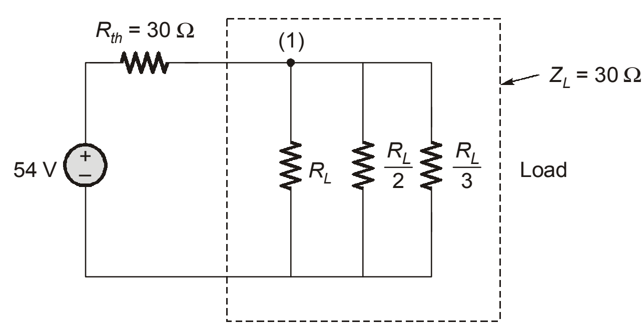
$$\frac{R_L}{6} = 30 \ \Omega$$
$$R_L = 180 \ \Omega$$

Now circuit will be

$$R_{th} = 30 \ \Omega$$

$$\because \qquad Z_L = R_{th} = 30 \ \Omega$$

∴ So voltage across load Z_L



$$V = \frac{54}{(R_{th} + Z_L)} \times Z_L$$
$$V = \frac{54}{60} \times 30$$
$$V = 27 \text{ V}$$

∵ In parallel voltage will be equal

∴ Voltage at node 1 → $\quad V_1 = V = 27$ V

∴ Power delivered to load R_L

$$P_1 = \frac{V_1^2}{R_L} = \frac{(27)^2}{180} = 4.05 \text{ W}$$

Power delivered to $R_L/2$ $\qquad P_2 = \dfrac{V_1^2}{(R_L/2)} = \dfrac{2V_1^2}{R_L} = 2 \times 4.05 = 8.1$ W

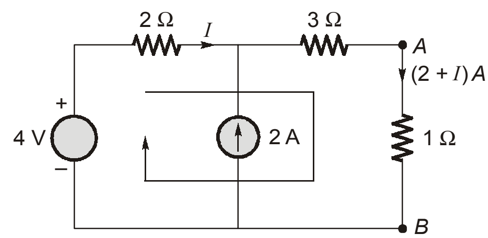
$$P_2 = 8.1 \text{ W}$$

Power delivered to load $R_L/3$

$$P_3 = \frac{V_1^2}{(R_L/3)} = \frac{3.V_1^2}{R_L} = 3\times\frac{(27)^2}{180}$$

$$P_3 = 3 \times 4.05 = 12.15 \text{ W}$$

Solution : 8



KVL in loop 1

$$4 = 2I + 3(2 + I) + 1(2 + I) = 2I + 6 + 3I + 2 + I$$

$$4 = 6I + 8$$

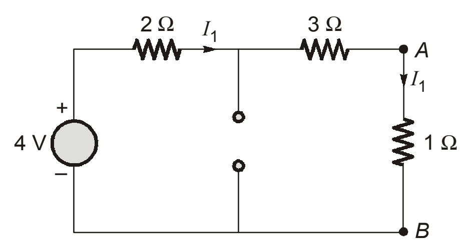
$$6I = -4$$

$\Rightarrow$ $$I = -\frac{2}{3}\text{A}$$

Current in 1 Ω resistor $$(2 + I) = 2-\frac{2}{3} = \frac{4}{3}\text{A}$$

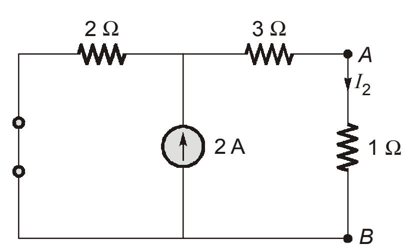
Verification by superposition:

Only 4 V source:



$$I_1 = \frac{4}{6} = \frac{2}{3}\text{A}$$

Only 2 A current source



$$I_2 = \frac{(2)(2)}{2+4} = \frac{2}{3}\text{A}$$

$$I = I_1 + I_2 = \frac{2}{3}+\frac{2}{3} = \frac{4}{3}\text{A}$$

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4 Transient State Analysis

LEVEL 1 Objective Solutions

1.	(b)
2.	(b)
3.	(a)
4.	(b)
5.	(c)
6.	(d)
7.	(b)
8.	(d)
9.	(b)
10.	(6.99)

LEVEL 2 Objective Solutions

11.	(2.54)
12.	(a)
13.	(5.41)
14.	(54.14)
15.	(b)
16.	(b)
17.	(c)
18.	(c)
19.	(d)
20.	(1)
21.	(0.25)
22.	(c)

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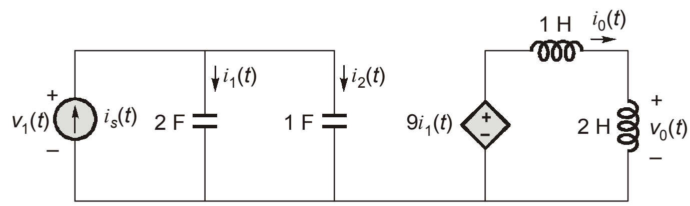
LEVEL 3 Conventional Solutions

Solution : 1

Redrawing the given circuit, we get,



$$i_s(t) = i_1(t) + i_2(t) \qquad \text{(Using KCL)}$$

hence, $$i_1(t) = 2\frac{dv_1(t)}{dt}, \qquad i_2(t) = 1\frac{dv_1(t)}{dt}$$

So, $$i_s(t) = 2\frac{dv_1(t)}{dt} + 1\frac{dv_1(t)}{dt} = 3\frac{dv_1(t)}{dt}$$

Also, $$i_s(t) = 3\frac{i_1(t)}{2} \quad \text{and} \quad \frac{dv_1(t)}{dt} = \frac{1}{2}i_1(t)$$

or, $$i_1(t) = \frac{2}{3}i_s(t) \qquad \text{...(1)}$$

Now writing KVL in right mesh, we get,

$$9i_1(t) = 1\frac{d\,i_0(t)}{dt} + 2\frac{di_0}{dt}$$

$$9i_1(t) = 3\frac{d\,i_0(t)}{dt}$$

$$9i_1(t) = 3\frac{v_0(t)}{2}$$

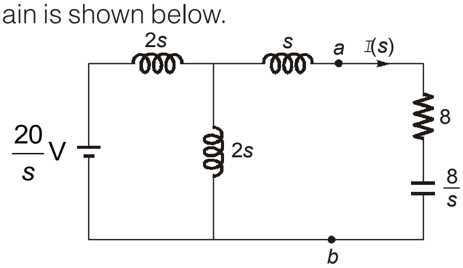
$$v_0(t) = 6i_1(t) \text{ and } v_0(t) = \frac{2di_0(t)}{dt}$$

Substituting $i_1(t)$ from equation (i), we get

$$v_0(t) = 6\times\frac{2}{3}i_s(t) = 4i_s(t) = 80\sin(2t) \text{ V}$$

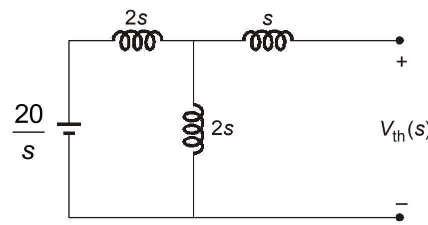
Solution : 2

The given circuit in Laplace domain is shown below.



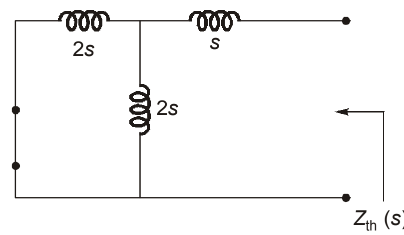
Thevenin's equivalent across terminals *a* and *b* can be found as follows:

To find $V_{Th}(s)$:



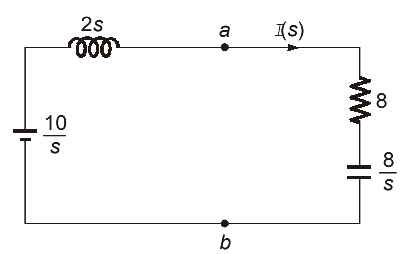
$$V_{Th}(s) = \frac{10}{s}$$

To find $Z_{Th}(s)$:



$$Z_{Th}(s) = 2s$$

The Thevenin's equivalent circuit is shown below.

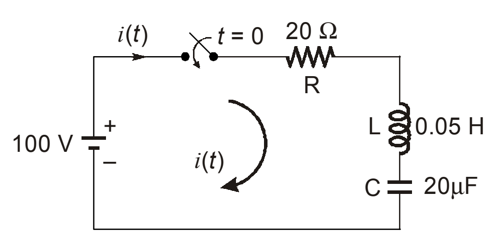


Now,
$$I(s) = \frac{\frac{10}{s}}{8 + 2s + \frac{8}{s}} = \frac{5}{s^2 + 4s + 4} = \frac{5}{(s+2)^2}$$

By applying Inverse Laplace transform,

$$i(t) = 5te^{-2t}\, u(t)\ A$$

Solution : 3



Writing differential equation for the circuit with current *i*.

$$100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20 \times 10^{-6}}\int i\,dt$$

Differentiating the equation,

$$\frac{d^2 i}{dt^2} + 400\frac{di}{dt} + 10^6 i = 0$$

$$\Rightarrow \quad (D^2 + 400\,D + 10^6)i = 0$$

$$D_1, D_2 = \frac{-400 \pm \sqrt{(400)^2 - 4\times 10^6}}{2}$$

$$= -200 \pm \sqrt{(200)^2 - 10^6}$$

So, $i = e^{-200t}[C_1 \cos 979.8\, t + C_2 \sin 979.8\, t\,]$

At $t = 0; \quad i = 0;$

$$0 = 1[C_1 \cos 0 + C_2 \sin 0]$$

$\therefore$ $C_1 = 0$

Now, $i = e^{-200t}[C_2 \sin 979.8\, t]$

Differentiating above equation,

$$\frac{di}{dt} = C_2[e^{-200t}(979.8) \cos 979.8\, t + e^{-200t}(-200) \sin 97938\, t]$$

At $t = 0$, voltage across inductor is 100 V

or, $L\frac{di}{dt} = 100$

or, $\frac{di}{dt} = 2000$

So, $2000 = C_2\,(979.8) \cos 0$

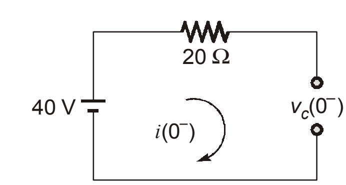
$\therefore$ $C_2 = 2.04$

Hence, current in the circuit is:

$$i = e^{-200t}\,(2.04 \sin 979.8\, t) \text{ Amp}$$

Solution : 4

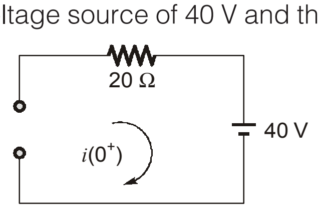
At $t = 0^-$, the network attains steady state. Hence, the capacitor acts as an open circuit.



$$v_C(0^-) = 40 \text{ V}$$

$$i(0^-) = 0$$

At $t = 0^+$, the capacitor acts as a voltage source of 40 V and the inductor acts as an open circuit.



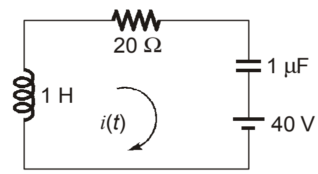
$\therefore$ $v_C(0^+) = 40 \text{ V}$

$$i(0^+) = 0$$

Writing KVL equation for $t > 0$,

$$-1\frac{di(t)}{dt} - 20i(t) - \frac{1}{1\times10^{-6}}\int_{0^+}^{t} i(t)\,dt - 40 = 0 \quad ...(i)$$

At $t = 0^+$,



$$-\frac{di(0^+)}{dt} - 20i(0^+) - 0 - 40 = 0$$

$$\frac{di(0^+)}{dt} = -40 \text{ A/s} \qquad \because i(0^+) = 0$$

Differentiating the equation (i), we get

$$-\frac{d^2i(t)}{dt^2} - 20\frac{di(t)}{dt} - 10^6 i(t) - 0 = 0$$

At $t = 0^+$,

$$-\frac{d^2i(0^+)}{dt^2} - 20\frac{di(0^+)}{dt} - 10^6 i(0^+) = 0$$

$$\frac{d^2i(0^+)}{dt^2} = -20\frac{di(0^+)}{dt} = 800 \text{ A/s}^2 \qquad \because i(0^+) = 0$$

Solution : 5

Let the selected node be (x) in the circuit of figure.

Using nodal analysis,

$$\frac{V_C(t) - V_0(t)}{R} + C\frac{dV_C(t)}{dt} = 0$$

or,
$$\frac{V_C(t) - [u(t) - u(t-1)]}{R} + \frac{dV_C(t)}{dt} = 0$$

Here $R = C = 1$

$\therefore$
$$V_C(t) + \frac{dV_C(t)}{dt} = u(t) - u(t-1)$$

Taking Laplace transform for both the sides of the above equation,

$$V_C(s) + [s\,V_C(s) - V_C(0)] = \frac{1}{s} - \frac{1}{s}e^{-s}$$

However,
$$V_C(0^-) = V_C(0^+) \quad \text{[following application of switching]}$$
$$= 2 \text{ volts}$$

$\therefore$
$$V_C(s) + [s\,V_C(s) - 2] = \frac{1}{s}(1 - e^{-s})$$

or,
$$V_C(s)\,[1 + s] = 2 + \frac{1}{s}(1 - e^{-s})$$

or $$V_C(s) = \frac{2}{s+1} + \frac{1}{s(s+1)} - \frac{e^{-s}}{s(s+1)}$$

Using the concept of partial fraction,

$$\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{(s+1)}$$

$\therefore$ $$V_C(s) = \left[\frac{1}{s} - \frac{1}{s+1}\right] - e^{-s}\left[\frac{1}{s} - \frac{1}{s+1}\right] + \frac{2}{s+1}$$

Taking the inverse Laplace transform,

$$V_C(t) = [1 - e^{-t}]\, u(t) - (1 - e^{-(t-1)})u(t-1) + 2e^{-t}u(t)$$
$$= (1 + e^{-t})\, u(t) - (1 - e^{-(t-1)})\, u(t-1)$$

However, $u(t) = 0$ if $t < 0$ and $= 1$ if $t > 0$

Also, $u(t-1) = 0$ if $t < 1 = 1$ if $t > 1$;

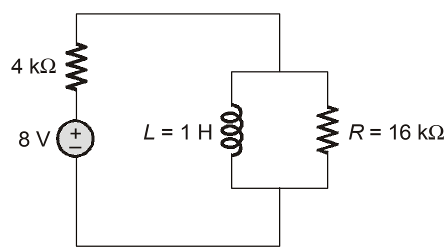
$\therefore$ for $1 > t > 0$, $V_C(t) = (1 + e^{-t})$ V

and for $t > 1$, $V_C(t) = (e^{-(t-1)} + e^{-t})$ V

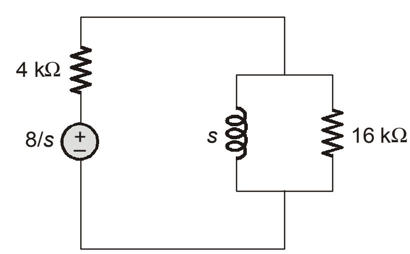
It may be noted here that figure represents the circuit at $t = 0^+$.

Solution : 6

Method-I:



After converting into s-domain,



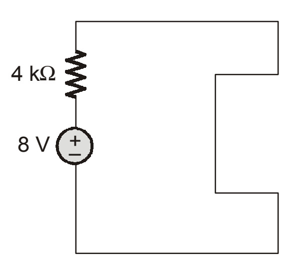
$$I(s) = \frac{8/s}{\frac{16k \times s}{16k + s} + 4k}$$

at steady state, $$I(s)_{ss} = \lim_{s\to 0} sI(s) = \lim_{s\to 0} s \cdot \frac{8/s}{\frac{16k \times s}{16k + s} + 4k}$$

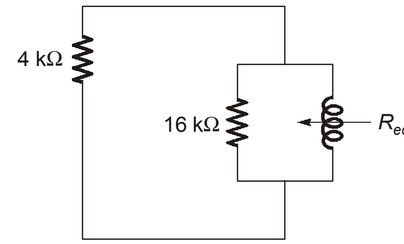
$$= \frac{8}{4\,\text{k}\Omega} = 2\,\text{mA}$$

Method-II:

At steady state, inductor behaves as short circuit. So,



$$I_{ss} = \frac{8}{4\text{ k}\Omega} = 2\text{ mA}$$



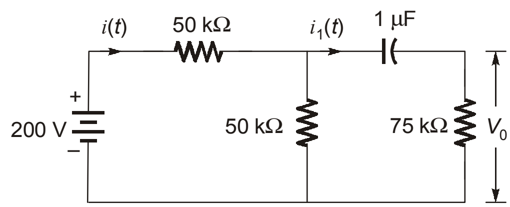
$$R_{eq} = \frac{4\text{ k}\Omega \times 16\text{ k}\Omega}{20\text{ k}\Omega} = 32\text{ k}\Omega$$

$$L_{eq} = 1\text{ H}$$

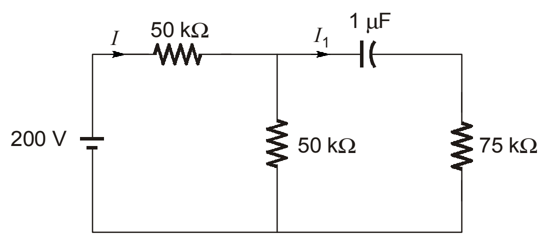
$$\tau = \frac{L_{eq}}{R_{eq}} = \left(\frac{1}{3.2\text{ k}\Omega}\right) = 0.3125\text{ msec.}$$

Solution : 7

The circuit for $t > t_0$ is shown below (in this case switch 'S' is closed).



At $t \geq 0$ circuit will be



So in s-domain

$$I_1(s) = \left[\frac{50\times10^3}{50\times10^3 + 75\times10^3 + \frac{1}{10^{-6}s}}\right] I(s) \qquad \text{...(i)}$$

and

$$\frac{200}{s} = 50\times10^3 I(s) + 50\times10^3 [I(s) - I_1(s)]$$

$$\frac{200}{s} = 2\times50\times10^3 I(s) - 50\times10^3 I_1(s) \qquad \text{...(ii)}$$

$$\frac{200}{s} = 100\times10^3\left[\frac{50\times10^3 + 75\times10^3 + \frac{1}{10^{-6}s}}{50\times10^3}\right] I_1(s) - 50\times10^3 I_1(s)$$

$$\frac{200}{s} = \left[50\times10^3 + 150\times10^3 + 2\times\frac{10^6}{s}\right] I_1(s)$$

$$I_1(s) = \frac{200}{s\left[2\times10^5 + \frac{2\times10^6}{s}\right]}$$

$$I_1(s) = \frac{200}{2\times10^5 s + 2\times10^6}$$

$$I_1(s) = \frac{10^{-3}}{s+10} \quad \text{...(iii)}$$

Taking inverse Laplace transform we get

$$i_1(t) = 10^{-3}\, e^{-10t}\, u(t) \text{ A}$$

Now, taking inverse Laplace transform of equation (iii),

$$\therefore \quad i_1(t) = 10^{-3}\, e^{-10t} \text{ A} \quad \text{...(iv)}$$

Since, the switch is closed at $t = t_0$ so equation (iv) becomes,

$$i_1(t) = 10^{-3}\, e^{-10(t-t_0)} \quad \text{...(v)}$$

$$\text{Voltage } V_0(t) = 75\times10^3 \cdot i_1(t)$$

$$\Rightarrow \quad V_0(t) = 75\times10^3 \times 10^{-3} e^{-10(t-t_0)}$$

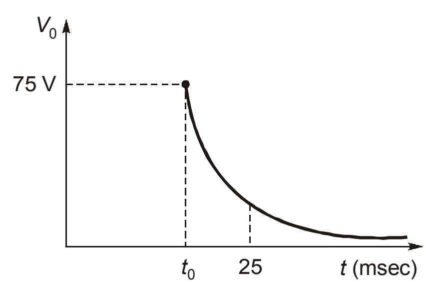
$$\Rightarrow \quad V_0(t) = 75e^{-10(t-t_0)}\, V$$

At $t = t_0$, $\quad V_0 = 75\, e^{-10\,(t_0 - t_0)} = 75\, e^{-0} = 75$ V

At $t = 25$m sec, $\quad V_0 = 75e^{-10\left(25\times10^{-3}-t_0\right)}\, V$

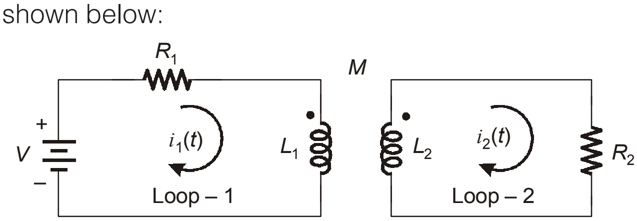
At $t \to \infty$, $\quad V_0 = 75e^{-10(\infty-10)} = 75\, e^{-\infty} = 0$ V

The corresponding transient is shown below:



Solution : 8

The circuit for $t > 0$ is shown below:



Apply KVL in Loop-1,

$$V = R_1 i_1(t) + L_1\frac{di_1(t)}{dt} - M\frac{di_2(t)}{dt} \quad \text{...(i)}$$

Apply KVL in Loop-2,

$$L_2\frac{di_2(t)}{dt} - M\frac{di_1(t)}{dt} + R_2 i_2(t) = 0 \quad \text{...(ii)}$$

Taking Laplace transform of the equations (i) and (ii),

$$\Rightarrow \quad \frac{V}{s} = R_1 I_1(s) + sL_1 I_1(s) - sMI_2(s) \quad \text{...(iii)}$$

$$\Rightarrow \qquad sL_2I_2(s) - sMI_1(s) + R_2I_2(s) = 0 \qquad \text{...(iv)}$$

Putting the values of given parameters in the above equations (iii) and (iv) we get,

$$\Rightarrow \qquad \frac{5}{s} = I_1(s) + sI_1(s) - 2sI_2(s) \qquad \text{...(v)}$$

$$\Rightarrow \qquad 4sI_2(s) - 2sI_1(s) + I_2(s) = 0 \qquad \text{...(vi)}$$

Putting value of I_2 (s) from equation (vi) into equation (v) we have,

$$\Rightarrow \qquad \frac{5}{s} = (s+1)I_1(s) - 2s\left(\frac{2s}{4s+1}\right)I_1(s)$$

$$\Rightarrow \qquad I_1(s)\left[\frac{4s^2+5s+1-4s^2}{4s+1}\right] = \frac{5}{s}$$

$$\Rightarrow \qquad I_1(s) = \frac{5(4s+1)}{s(5s+1)}$$

$$\Rightarrow \qquad I_1(s) = \frac{5}{s} + \frac{5\left(\frac{-4}{5}+1\right)}{\left(\frac{-1}{5}\right)(5s+1)} = \frac{5}{s} - \frac{5}{5s+1}$$

$$\Rightarrow \qquad I_1(s) = \frac{5}{s} - \frac{1}{s+1/5} \qquad \text{...(vii)}$$

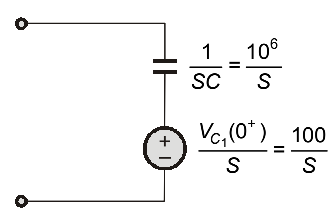
Taking inverse Laplace transform of equation (vii),

$$i_1(t) = (5 - e^{-t/5})\, u(t)\ A$$

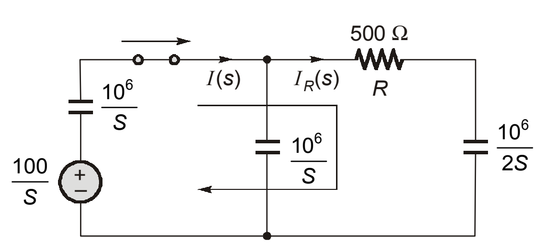
Solution : 9

$$V_{C_1}(0^-) = 100\text{ V} = V_{C_1}(0^+)$$

Laplace equivalent of changed capacitor



Laplace equivalent of complete circuit



$$I_R(s) = \frac{\left(\frac{10^6}{S}\right) I(s)}{\frac{10^6}{S} + 500 + \frac{0.5\times10^6}{S}} = \frac{10^6 I(s)}{500S + 1.5\times10^6}$$

$$I(s) = \frac{(500S+1.5\times10^6)}{10^6} I_R(s) \qquad \ldots(1)$$

KVL for loop (1),

$$\frac{100}{s} = \frac{10^6}{S}\cdot I(s) + \left(500 + \frac{0.5\times10^6}{s}\right) I_R(s)$$

$$100 = 10^6\, I(s) + (500S + 0.5\times 10^6)\, I_R(s)$$

Put $I(s)$ from (1),

$$100 = 10^6 \cdot \frac{(500S+1.5\times10^6)}{10^6} I_R(s) + (500S + 0.5\times10^6) I_R(s)$$

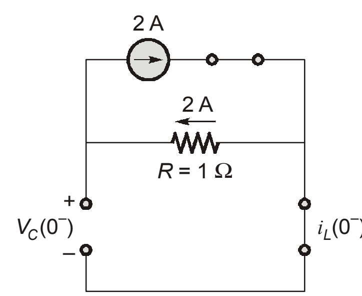
$$100 = (1000S + 2\times 10^6)\, I_R(s)$$

$$I_R(s) = \frac{100}{1000(S+2\times10^3)} = \frac{0.1}{S+2\times10^3}$$

Taking Laplace inverse, $\quad i_R(t) = 0.1\, e^{-2000t}$ for $t > 0$

Solution : 10

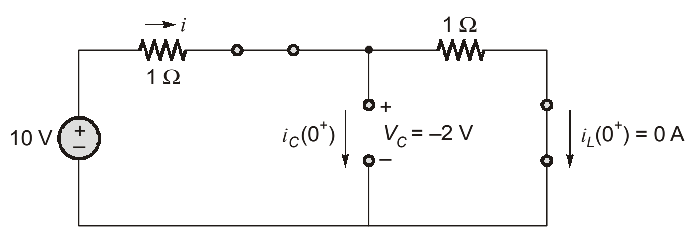
Given: $\quad C = \frac{1}{2}$ F, $L = 1$ H



Steady state condition at $t = 0^-$

$$V_C(0^-) = 2\times 1 = -2\text{ V}$$
$$i_L(0^-) = 0\text{ A}$$
$$V_C(0^+) = V_C(0^-) = -2\text{ V}$$
$$i_L(0^-) = i_L(0^+) = 0\text{ A}$$

Circuit for $t > 0$



$$i = \frac{10-(-2)}{1} = 12\text{ A} = i_C(0^+)$$

For capacitor

$$i_C(0^+) = \frac{C\,dV_C(0^+)}{dt}$$

$$\frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{12}{\frac{1}{2}} = 24\text{ V/s}$$

$$\frac{dV_C(0^+)}{dt} = 24 \text{ V/s}$$

As current can't change instantaneously in inductors,

$$i_L(0^+) = 0 \text{ A}$$

$$V_L(0^+) = V_C(0^+) = -2 \text{ V}$$

$$V_L = L\frac{di_L}{dt}$$

$$V_L(0^+) = L\frac{di_L(0^+)}{dt}$$

$$\frac{di_L(0^+)}{dt} = \frac{1}{L}V_L(0^+) = \frac{-2}{1} = -2 \text{ A/s}$$

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5 Two-Port Network Parameters

LEVEL 1 Objective Solutions

1.	(a)
2.	(4)
3.	(85)
4.	(0.5)
5.	(d)
6.	(c)
7.	(d)
8.	(c)

LEVEL 2 Objective Solutions

9.	(d)
10.	(c)
11.	(d)
12.	(11.11)
13.	(d)
14.	(b)
15.	(c)
16.	(b)
17.	(3)
18.	(c)

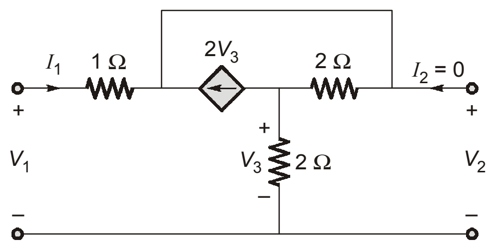
■■■■

LEVEL 3 Conventional Solutions

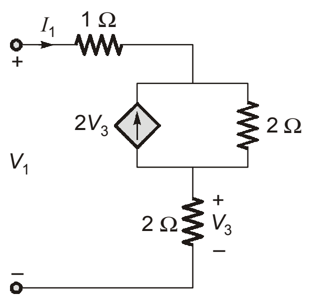
Solution : 1

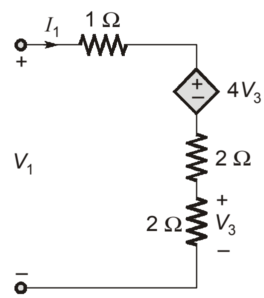
Let us first open circuited the output terminal, we get,

$I_2 = 0$, then dependent voltage source $3I_2 = 0$ and it acts as short circuit.



We can see that dependent current source and the right most 2 Ω resistor (Horizontal one) are in parallel, so simplified circuit is shown below.





Applying KVL in the loop, we get,

$$V_1 - 1I_1 - 4V_3 - 2I_1 - 2I_1 = 0$$

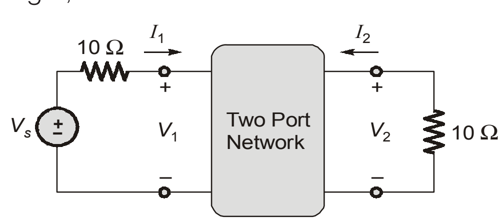
$$V_1 - 1I_1 - 4(2I_1) - 2I_1 - 2I_1 = 0 \qquad (V_3 = 2I_1)$$

$$V_1 - 13I_1 = 0$$

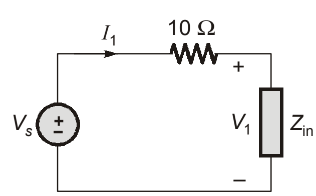
or,
$$R_{11} = \frac{V_1}{I_1} = 13\ \Omega$$

Solution : 2

Redrawing the given circuit, we get,



Let the input impedance be Z_{in}



$$Z_{in} = \frac{V_1}{I_1}$$

$$V_1 = \frac{V_s}{Z_{in}+10} Z_{in}$$

$$\frac{V_1}{V_s} = \frac{Z_{in}}{Z_{in}+10} = \frac{1}{2}$$

$$2Z_{in} = Z_{in} + 10$$

$$Z_{in} = 10$$

or Input admittance

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{10} = 0.1\,\text{S}$$

The input admittance is given as,

$$Y_{in} = y_{11} - \frac{y_{12}\,y_{21}}{y_{22}+Y_L}$$

$$0.1 = y_{11} - \frac{(0.02)(2)}{0.2+0.1} \qquad \left(Y_L = \frac{1}{10} = 0.1\,\text{S}\right)$$

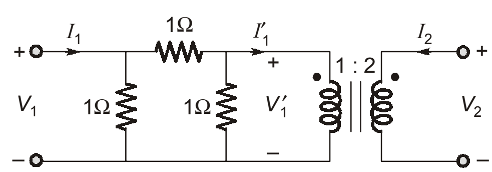
$$0.1 = y_{11} - 0.133$$

or,

$$y_{11} = 0.233\ \text{S}$$

Solution : 3

Reciprocal network : A network is reciprocal when the ratio of response at port 2 to the excitation at port '1' is same as the ratio of response at port 1 to the excitation at port 2.



Using nodal analysis in conjunction with the ideal transformer equations.

$$I_1 = V_1 + (V_1 - V_1') .1$$

$$= 2V_1 - V_1' = 2V_1 - \frac{1}{2}V_2 \qquad \text{...(1)}$$

and

$$I_1' = -V_1' + (V_1 - V_1') \qquad \text{...(2)}$$

$$\frac{V_1'}{V_2} = \frac{N_1}{N_2} = \frac{1}{2}$$

and

$$\frac{I_2}{I_1'} = -\frac{N_1}{N_2} = -\frac{1}{2} \qquad \text{...(3)}$$

From equation (2) and (3)

$$I_2 = -\frac{1}{2}I_1' = -\frac{1}{2}[-V_1' + (V_1 - V_1')] \qquad \text{...(4)}$$

$$I_2 = +\frac{2V_1'}{2} - \frac{V_1}{2} = -\frac{V_1}{2} + \frac{1}{(2)}V_2 \qquad \text{...(5)}$$

From equations (1) and (5)

$$I_1 = 2V_1 - \frac{V_2}{2}$$

$$I_2 = -\frac{V_1}{2} + \frac{V_2}{2}$$

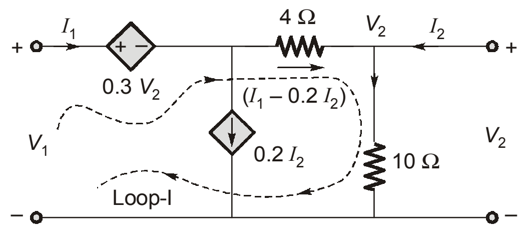
$\therefore$
$$Y = \begin{bmatrix} 2 & -\frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} \end{bmatrix}$$

Here, $Y_{12} = Y_{21}$

Hence, the given network is reciprocal.

Solution : 4

Given two-port network is as,



Apply KVL in loop-I,

$$0.3V_2 + 4(I_1 - 0.2\,I_2) + V_2 = V_1$$

$\Rightarrow$ $$1.3\,V_2 + 4I_1 - 0.8\,I_2 = V_1 \quad \text{...(i)}$$

Also, $$V_2 = 10[I_1 - 0.2\,I_2 + I_2]$$

$$V_2 = 10\,I_1 + 8I_2 \quad \text{...(ii)}$$

Putting this value of V_2 from equation (ii) in equation (i) we get,

$\Rightarrow$ $$V_1 = 1.3 \times 10\,I_1 + 1.3 \times 8\,I_2 + 4I_1 - 0.8\,I_2$$

$\therefore$ $$V_1 = 17\,I_1 + 9.6\,I_2 \quad \text{...(iii)}$$

Z-parameter is, $$V_1 = Z_{11}\,I_1 + Z_{12}\,I_2 \quad \text{...(iv)}$$

Comparing (iii) and (iv) we get,

$$Z_{12} = 9.6\ \Omega$$

Since, Y-Parameter is,

$$I_1 = Y_{11}\,V_1 + Y_{12}\,V_2 \quad \text{...(v)}$$

KCL at node 2,

$$I_2 = \frac{V_2}{10} + \{-(I_1 - 0.2I_2)\}$$

$$I_2 = 0.1\,V_2 - I_1 + 0.2\,I_2$$

$\Rightarrow$ $$I_2 = \frac{1}{8}V_2 - \frac{5}{4}I_1$$

$\Rightarrow$ $$I_2 = 0.125\,V_2 - 1.25\,I_1 \quad \text{...(vi)}$$

From equation (i) and (vi) we get,

$$V_1 = 1.3\,V_2 + 4\,I_1 - 0.8\,[0.125\,V_2 - 1.25\,I_1]$$

$$V_1 = 1.2\,V_2 + 5\,I_1 \quad \text{...(vii)}$$

$\Rightarrow$ $$I_1 = 0.2\,V_1 - 0.24\,V_2 \quad \text{...(viii)}$$

Comparing equation (v) and (viii) we get,

$$Y_{12} = -0.24 \ \mho$$

For h-parameter,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \text{...(ix)}$$

Comparing equation (vii) and (ix) we get,

$$h_{12} = 1.2$$

Solution : 5

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

(i) $I_1 = 0$ A, $I_2 = 1$ A, $V_1 = 4.5$ V, $V_2 = 1.5$ V

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{4.5}{1.5} = 3$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{1.5} = \frac{2}{3} \ \mho$$

(ii) $I_1 = 4$ A, $I_2 = 0$, $V_1 = 6$ V, $V_2 = 1.5$ V

$$I_2 = h_{21}I_1 + h_{22}V_2 = 0$$

$$h_{21} \times 4 + \frac{2}{3} \times 1.5 = 0$$

$\Rightarrow$
$$h_{21} = -0.25$$

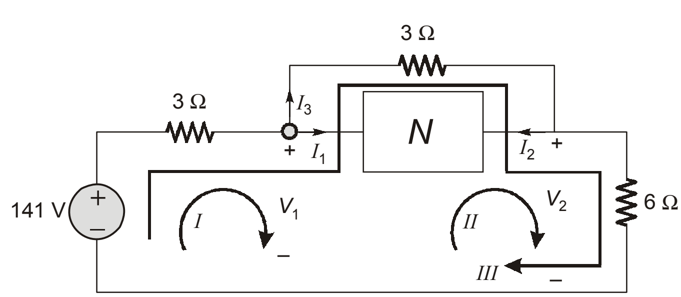
$$V_1 = h_{11}I_1 + h_{12}V_2 = 6$$

$$h_{11} \times 4 + 3 \times 1.5 = 6$$

$$h_{11} = \frac{1.5}{4} = 0.375 \ \Omega$$

$\Rightarrow$ h parameter matrix is $\begin{bmatrix} 0.375 & 3 \\ -0.25 & 0.667 \end{bmatrix}$

Solution : 6



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} V_1 &= 2I_1 + I_2 \\ V_2 &= I_1 + 4I_2 \end{aligned}$$

Applying KVL

Loop I

$$3(I_1 + I_3) + V_1 = 141$$
$$3I_1 + 3I_3 + 2I_1 + I_2 = 141$$
$$5I_1 + I_2 + 3I_3 = 141 \quad \ldots(i)$$

Loop II

$$V_2 = (I_3 - I_2) \cdot 6$$
$$V_2 = I_1 + 4I_2$$
$$I_1 + 4I_2 = 6I_3 - 6I_2$$
$$I_1 + 10I_2 - 6I_3 = 0 \quad \ldots(ii)$$

Loop III

$$3(I_1 + I_3) + 3I_3 + 6(I_3 - I_2) = 141$$
$$3I_1 - 6I_2 + 12I_3 = 141 \quad \ldots(iii)$$

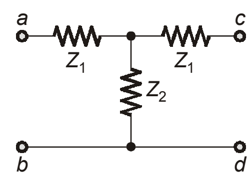
By equation (i), (ii) and (iii)

$I_1 = 24$ A $\quad I_2 = 1.5$ A

$I_3 = 6.5$ A

Solution : 7

A symmetrical T-section network figure shown below:



Given $\quad Z_{OC} = Z_1 + Z_2 = 800\ \Omega \quad \ldots(i)$

$\Rightarrow \quad Z_2 = (800 - Z_1)\ \Omega$

And under SC condition,

$$Z_{SC} = Z_1 + \frac{Z_1 Z_2}{Z_1 + Z_2} = 600\ \Omega \quad \ldots(ii)$$

or $\quad Z_1 + \dfrac{Z_1(800 - Z_1)}{800} = 600 \quad$ [utilising equation(i)]

or $\quad Z_1^2 - 1600\ Z_1 + 48 \times 10^4 = 0$

or $\quad (Z_1 - 800)^2 = (\pm 400)^2$

or $\quad Z_1 = 400$ or 1200 ohms

If Z_1 is 1200 ohms Z_2 becomes negative; therefore,

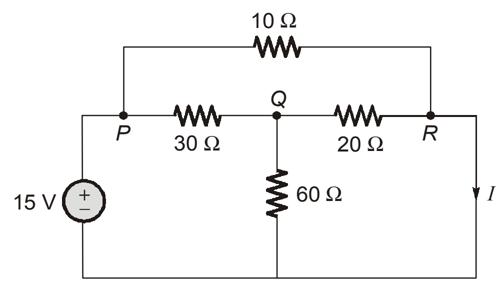
$$Z_1 = 400 \text{ ohms and } Z_2 = 400 \text{ ohms.}$$

Solution : 8

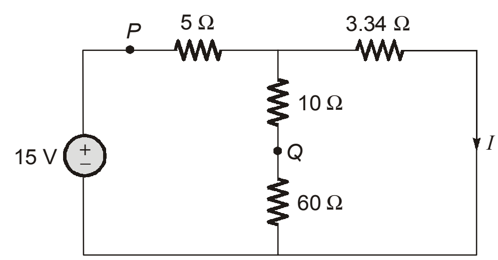
A two port network is termed to be reciprocal if the ratio of the response variable to the excitation variable remains identical even if the positions of the response and excitation in the network are interchanged. Conditions for reciprocal network in various parameters.

Parameter	Condition for reciprocal
Z	$Z_{12} = Z_{21}$
Y	$y_{12} = y_{21}$
h	$h_{12} = -h_{21}$
$ABCD$	$AD - BC = 1$

15 V voltage source is connected at terminals 1 – 1′ and terminals 2 – 2′ are shorted.

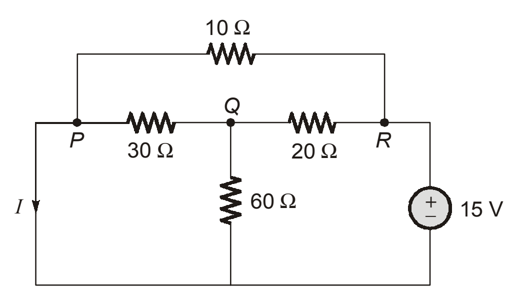


Converting Δ into Y-circuit.

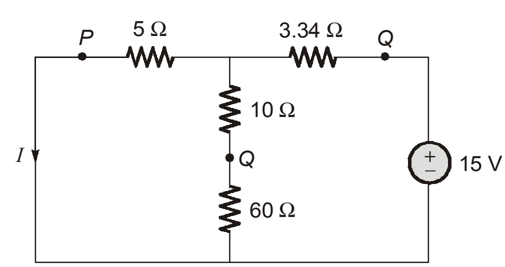


$$I = \frac{15}{5+(10+60)\|3.34} \times \frac{(10+60)}{(10+60)+3.34} = 1.748 \text{ A}$$

Now 15 V voltage source is at 2 – 2′ terminals at terminals 1 – 1′ are shorted.



Converting Δ into Y-circuit.



$$I = \frac{15}{(10+60)\|5+3.34} \times \frac{(10+60)}{(10+60)+5} = 1.748 \text{ A}$$

For the same excitation (15 V voltage source) placed at both terminals the response (I = 1.748 A) remains identical. Hence the given network is reciprocal.

Solution : 9

$$v_1 = Z_{11}i_1 + Z_{12}\, i_2 \quad \text{...(i)}$$

$$v_2 = Z_{21}i_1 + Z_{22}i_2 \quad \text{...(ii)}$$

$$v_s = R_s i_1 + v_1 \quad \text{...(iii)}$$

$$\Rightarrow \quad v_s = R_s i_1 + Z_{11}i_1 + Z_{12}i_2 \quad \text{...(iv)}$$

Taking Laplace transform of equation (iv) is,

$$V_s(s) = R_s I_1(s) + Z_{11}I_1(s) + Z_{12}I_2(s)$$

Putting values as given we get,

$$\Rightarrow \quad \frac{1}{s} = 2I_1(s) + \frac{2}{s+1}I_1(s) + \frac{1}{s+1}I_2(s)$$

$$\Rightarrow \quad (2s+4)\, I_1(s) + I_2(s) = \frac{s+1}{s} \quad \text{...(v)}$$

Now, $\quad v_2 = -R_L i_2 \quad$ ...(vi)

Putting value of v_2 from equation (ii) into equation (vi),

$$Z_{21}i_1 + Z_{22}i_2 = -R_L i_2$$

Taking Laplace transform,

$$Z_{21}I_1(s) + Z_{22}I_2(s) + R_L I_2(s) = 0$$

Again putting values,

$$\Rightarrow \frac{1}{s+1}I_1(s) + \left(\frac{6}{s+1}+1\right)I_2(s) = 0$$

$$\Rightarrow \quad I_1(s) + (s+7)\, I_2(s) = 0$$

$$\Rightarrow \quad I_1(s) = -(s+7)I_2(s) \quad \text{...(vii)}$$

Putting value of $I_1(s)$ in equation (v)

$$\Rightarrow -(2s+4)\,(s+7)\, I_2(s) + I_2(s) = \frac{s+1}{s}$$

$$\Rightarrow I_2(s)\, [-2s^2 - 18s - 28 + 1] = \frac{s+1}{s}$$

$$\Rightarrow \quad I_2(s) = \frac{-(s+1)}{s(2s^2+18s+27)}$$

$$\because \quad V_2(s) = -R_L I_2(s)$$

$$\therefore \quad V_2(s) = \frac{(s+1)}{s(2s^2+18s+27)}$$

$$\Rightarrow \quad V_2(s) = \frac{s+1}{2s\,(s+1.902)\,(s+7.098)}$$

$$\Rightarrow \qquad V_2(s) = \frac{0.037}{s} + \frac{0.0456}{s+1.902} - \frac{0.083}{s+7.098} \qquad \text{...(viii)}$$

Taking inverse Laplace transform of equation (viii),

$$\Rightarrow \qquad v_2(t) = (0.037 + 0.0456\, e^{-1.902t} - 0.083\, e^{-7.098t})u(t)$$

Solution : 10

For parallel network $\quad [Y] = [Y_1] + [Y_2]$

$$[Y_{n_1}] = [Z_{n_1}]^{-1} \quad \text{or} \quad [Z_n] = [Y_n]^{-1}$$

$$[Y_{n_1}] = [Z_{n_1}]^{-1} = \frac{1}{|Z_{n_1}|}\text{adj}\,[Z_n] = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 0.45 & -0.27 \\ -0.27 & 0.36 \end{bmatrix}$$

$$[Y_{n_2}] = [Z_{n_2}]^{-1}$$

$$= \frac{1}{8}\begin{bmatrix} 4 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.25 \\ -0.25 & 0.375 \end{bmatrix}$$

$$[Y_n] = [Y_{n_1}] + [Y_{n_2}]$$

$$= \begin{bmatrix} 0.95 & -0.52 \\ -0.52 & 0.735 \end{bmatrix}$$

$$[Z_n] = [Y_n]^{-1}$$

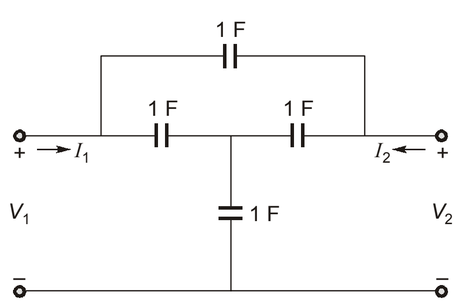
$$\left|[Y_n]^{-1}\right| = (0.95)(0.735) - (0.52)^2 = 0.698 - 0.270 = 0.427$$

$$[Z_n] = \frac{1}{0.427}\begin{bmatrix} 0.735 & 0.52 \\ 0.52 & 0.95 \end{bmatrix}$$

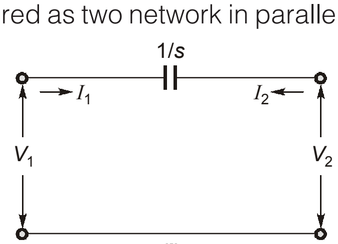
$$[Z_n] = \begin{bmatrix} 1.72 & 1.22 \\ 1.22 & 2.22 \end{bmatrix}$$

Solution : 11

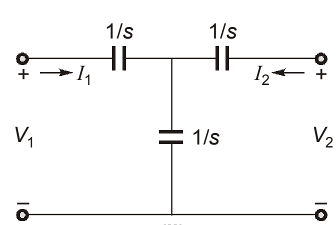
Bridge *T*-section



It can be considered as two network in parallel



(I)



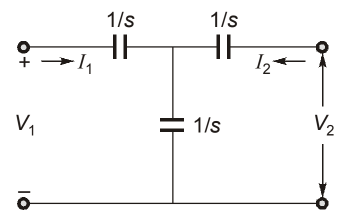
(II)

y-parameter of (I) network:

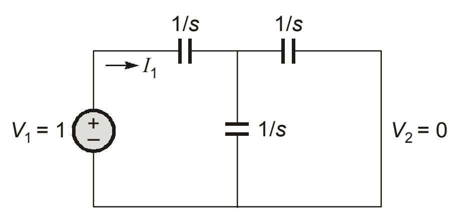
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = s = y_{22}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -s = y_{21}$$

y-parameter of network (II):



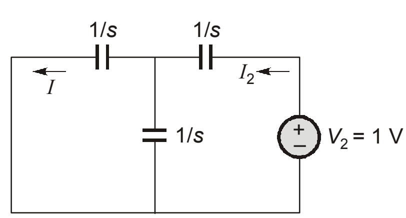
$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



$$I_1 = \frac{1}{\frac{1}{s} + \frac{\frac{1}{s}\cdot\frac{1}{s}}{\frac{1}{s}+\frac{1}{s}}} = \frac{2s}{3}$$

$$y_{11} = \frac{2s}{3} = y_{22}\,(\text{symmetrical})$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



$$I_2 = \frac{2s}{3}$$

$$I = \frac{\frac{2s}{3}\times\frac{1}{s}}{\frac{1}{s}+\frac{1}{s}} = \frac{s}{3}$$

$$I_1 = -I$$

$$y_{12} = -\frac{s}{3} = y_{21}\,(\text{reciprocal})$$

$$[y_{\text{II}}] = \begin{bmatrix} \frac{2s}{3} & -\frac{s}{3} \\ -\frac{s}{3} & \frac{2s}{3} \end{bmatrix}, \quad y_{\text{I}} = \begin{bmatrix} s & -s \\ -s & s \end{bmatrix}$$

$$[y] \text{ for Bridge-T} = [y_{\text{I}}] + [y_{\text{II}}]$$

$$[y] = \begin{bmatrix} \frac{2s}{3}+s & -s-\frac{s}{3} \\ -s-\frac{s}{3} & \frac{2s}{3}+s \end{bmatrix} = \begin{bmatrix} \frac{5s}{3} & -\frac{4s}{3} \\ -\frac{4s}{3} & \frac{5s}{3} \end{bmatrix}$$

■■■■

6 Network Synthesis and Graph Theory

LEVEL 1 Objective Solutions

1. (c)
2. (b)
3. (d)
4. (c)
5. (c)

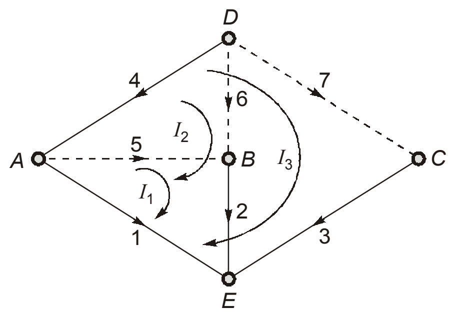
LEVEL 2 Objective Solutions

6. (c)
7. (a)
8. (c)
9. (b)
10. (c)

LEVEL 3 Conventional Solutions

Solution : 1

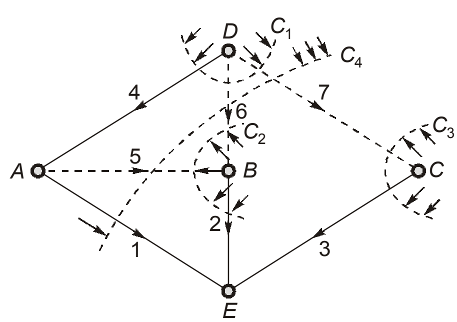
The tree is arbitrarily selected, which is shown in figure below with branches 4-1-2-3. Thus the twigs are these branches while the links are dotted lines.



Tie-set-1 (loop current I_1): Formed by twigs 1 and 2 with link 5.
Tie-set-2 (loop current I_2): Formed by twigs 1, 2 and 4 with link 6.
Tie-set-3 (loop current I_3): Formed by twigs 3, 1, 4 and link 7.
[Each fundamental loop contains only one link]
The Tie-set matrix is shown below:

Loop currents	Branches						
	1	2	3	4	5	6	7
I_1	–1	1	0	0	1	0	0
I_2	–1	1	0	–1	0	1	0
I_3	–1	0	1	–1	0	0	1

To obtain the cut-set matrix, the graph is redrawn with twigs in bold and links in dotted lines as shown in figure below.



Cut-sets are formed by taking one twig at a time.
C_1, Cut-set-1: Twig 4, links 6 and 7
C_2, Cut-set-2: Twig 2, links 5 and 6
C_3, Cut-set-3: Twig 3, links 7
C_4, Cut-set-4: Twig 1, links 5, 6, 7.
[Note that total number of fundamental cut-sets = Number of nodes –1 = 5 – 1 = 4]

The necessary cut-set matrix is shown below:

Cut-Sets	Branches						
	1	2	3	4	5	6	7
C_1	0	0	0	1	0	1	1
C_2	0	1	0	0	–1	–1	0
C_3	0	0	1	0	0	0	–1
C_4	1	0	0	0	1	1	1

Solution : 2

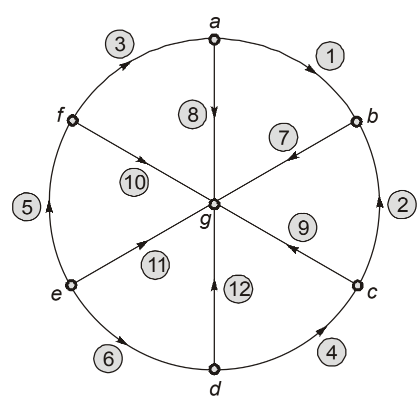
There are 6 loops and 12 branches in graph whose tie set matrix is given.

Let us construct the graph using tie set matrix since there are 6 loops corresponding to 6 branches which are not there in tree (any tree of the graph). So there are 6 branches in tree and in a tree there are (n + 1) nodes if there are n branches.

$\therefore$ Number of nodes in graph = 6 + 1 = 7

So now we will find out the graph having 7 nodes and 12 branches.

So graph of the above network will be



Where a, b, c, d, e, f, g are nodes

(1), (2) ....(12) are branches in the graph.

Now we will find fundamental cut set for above graph

Corresponding to each branch of the tree there will exist a f-cut set.

So f-cut set 1 → [1, 3, 8]

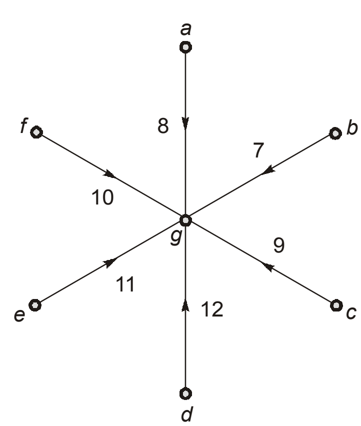
f-cut set 2 → [1, 2,7]

f-cut set 3 → [2, 4, 9]

f-cut set 4 → [4, 6, 12]

f-cut set 5 → [5, 6, 11]

f-cut set 6 → [3, 5, 10]



$\therefore$ fundamental cut set matrix.

	Branches											
f cut set	1	2	3	4	5	6	7	8	9	10	11	12
f - 1	1	0	–1	0	0	0	0	1	0	0	0	0
f - 2	–1	–1	0	0	0	0	1	0	0	0	0	0
f - 3	0	1	0	–1	0	0	0	0	1	0	0	0
f - 4	0	0	0	1	0	–1	0	0	0	0	0	1
f - 5	0	0	0	0	1	1	0	0	0	0	1	0
f - 6	0	0	1	0	–1	0	0	0	0	1	0	0

Solution : 3

$$Y(s) = \frac{12(s+1)}{s(s+2)(s+3)}$$

$$Y(s) = \frac{I(s)}{V(s)}$$

$$V(s) = L[V(t)]$$

$$V(t) = \delta(t)$$

$$V(s) = 1$$

$$I(s) = \frac{12(s+1)}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = s\,I(s)\big|_{s=0} = 2$$

$$B = (s+2)\,I(s)\big|_{s=-2} = 6$$

$$C = s+3\,I(s)\big|_{s=-3} = -8$$

$$I(s) = \frac{2}{s} + \frac{6}{s+2} - \frac{8}{s+3}$$

Taking Laplace inverse

$$I(t) = 2 + 6e^{-2t} - 8e^{-3t}$$

For unit step voltage

$$V(s) = \frac{1}{s}$$

$$I(s) = \frac{12(s+1)}{s^2(s+2)(s+3)} = \frac{A}{s^2} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = s^2 I(s)\big|_{s=0} = 2$$

$$B = s+2\,I(s)\big|_{s=-2} = -12$$

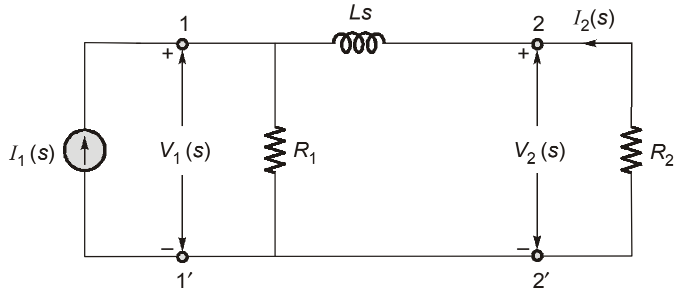
$$C = (s+3)\,I(s)\big|_{s=-3} = 24$$

$$I(s) = \frac{2}{s^2} + \frac{(-12)}{s+2} + \frac{24}{s+3}$$

Laplace inverse

$$I(t) = 2t - 12e^{-2t} + 24e^{-3t}$$

Solution : 4



$$Y_{21}(s) = \frac{I_1(s)}{V_2(s)} \quad \text{...(1)}$$

$$V_2(s) = -I_2(s)\,R_2$$

$$-I_2(s) = \frac{I_1(s)\cdot R_1}{R_1+R_2+Ls} \quad \text{...(2)}$$

$$V_2(s) = \frac{-I_1(s)R_1R_2}{R_1+R_2+Ls}$$

Putting in (1)

$$Y_{12}(s) = -\left(\frac{R_1+R_2+Ls}{R_1R_2}\right)$$

$$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$$

$$V_1(s) = (I_1+I_2)\,R_1$$

$$Z_{12}(s) = \left(\frac{I_1}{I_2}+1\right)R_1$$

Taking $\quad \dfrac{I_1}{I_2} = -\dfrac{R_1+R_2+Ls}{R_1}$ from (2)

$$Z_{12}(s) = -\frac{(R_2+Ls)}{R_1}$$

$$Z_{11}(s) = \text{Driving point function} = \frac{V_1}{I_1}$$

$$Z_{11}(s) = (R_1)\,||\,(R_2+Ls)$$

$$Z_{11}(s) = \frac{R_1(R_2+Ls)}{R_1+R_2+Ls}$$

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$$

Here $\quad V_2(s) = -I_2(s)\,R_2$

$$Z_{22}(s) = -R_2$$

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