# 2020

## RANK Improvement WORKBOOK

**Detailed Explanations of
Objective & Conventional *Questions***

## Mechanical Engineering
### Thermodynamics

# 1 Basic Concepts and Zeroth Law of Thermodynamics

**LEVEL 1** Objective Questions

| | |
|---|---|
| 1. | (c) |
| 2. | (b) |
| 3. | (a) |
| 4. | (b) |
| 5. | (a, b & c) |
| 6. | (c) |
| 7. | (a) |
| 8. | (d) |
| 9. | (a) |
| 10. | (d) |
| 11. | (b) |
| 12. | (a) |

**LEVEL 2** Objective Questions

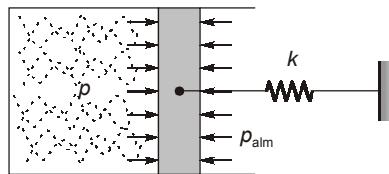| | |
|---|---|
| 13. | (b) |
| 14. | (d) |
| 15. | (b) |
| 16. | (b) |
| 17. | (d) |
| 18. | (a) |
| 19. | (c) |
| 20. | (c) |
| 21. | (4) |
| 22. | (c) |
| 23. | (d) |

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**LEVEL 3** Conventional Questions

**Solution: 24**

First we convert the pressure reading into pascals. We have

$$p_{gauge} = (\rho g h)_{Hg}$$
$$= 13600 \times 9.81 \times 0.020$$
$$= 2668.32 \, Pa = 2.668 \, kPa$$
$$p_{atm} = 79.50 \, kPa$$

We know that

$$p_{abs} = p_{gauge} + p_{atm}$$
$$= 2.668 + 79.50$$
$$= 82.168 \, kPa$$

**Solution : 25**



A force balance on the piston provides

$$pA = p_{atm}A + k\Delta x$$

$$p \times \frac{\pi}{4}d^2 = p_{atm} \times \frac{\pi}{4}d^2 + k\Delta x$$

$$600 \times \frac{\pi}{4} \times (0.1)^2 = 101.3 \times \frac{\pi}{4} \times (0.1)^2 + 4.8 \times \Delta x$$

or
$$\Delta x = 0.816 \, m$$

**Solution : 26**

Given data:

At NTP,

$$T = 0°C = 273 \, K$$
$$p = 101.325 \, kPa$$

Applying equation of state,

$$pV = mRT$$
$$101.325 \times 0.773 = 1 \times R \times 273$$
$$R = 0.2869 \, kJ/kgK$$

**Solution : 27**

Given data:

$$M = 27.62, \, V = 150 \, litres = 0.15 \, m^3$$
$$p = 1.35 \, bar = 135 \, kPa$$

$$T = 20°C = 293 \text{ K}$$

Applying equation of state,

$$pV = mRT$$

$$pV = m \times \frac{\bar{R}}{M}T$$

$$135 \times 0.15 = m \times \frac{8.314}{27.62} \times 293$$

$$m = 0.2296 \text{ kg}$$

## Solution : 28

The Fahrenheit and Celsius temperature scale are related by

$$T_{°C} = \frac{5}{9}(T_{°F} - 32)$$

For

$$T_{°C} = T_{°F} = T$$

$\therefore$

$$T = \frac{5}{9}(T - 32)$$

or

$$T = -40$$

$\therefore$

$$T_{°C} = -40°C \text{ and } T_F = -40°F$$

On absolute scales,

$$T_K = T_{°C} + 273.15$$
$$= -40 + 273.15 = 233.15 \text{ K}$$
$$T_R = T_{°F} + 459.67$$
$$= -40 + 459.67 = 419.67 \text{ R}$$

## Solution : 29

Given: $T_1 = 25°C = 298 \text{ K}$; $P_1 = 210 \text{ kPa} + 100 \text{ kPa} = 310 \text{ kPa}$; $V = 0.65 \text{ m}^3$
$P_{atm} = 100 \text{ kPa}$; $R = 0.287 \text{ kJ/kg K}$

When

$$T_2 = 50°C, \text{ let } P_2 \text{ be the pressure inside tire}$$

mass of air

$$m_1 = \frac{P_1 V}{RT_1} = \frac{310 \times 10^3 \times 0.65}{287 \times 298} = 2.356 \text{ kg}$$

As volume of tire is constant,

$$P_2 = \frac{2.356 \times 0.287 \times 10^3 \times (273 + 50)}{0.65}$$

$$P_2 = 336 \text{ kPa}$$

$$\text{Pressure rise in tire} = P_2 - P_2 = 26 \text{ kPa}$$

To restore pressure to its original value ($P_1 = 310 \text{ kPa}$) at $T_2 = 50°C$, let $\Delta m$ be the mass of air that must be bled off.

$$\Delta m = m_1 - m_2$$

where

$$m_2 = \frac{310 \times 10^3 \times 0.65}{287 \times (273 + 50)} = 2.1736 \text{ kg}$$
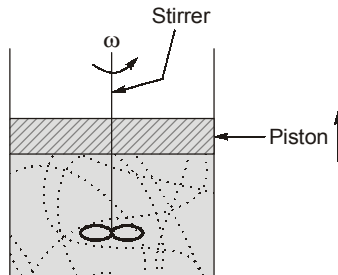
and

$$m_1 = 2.356 \text{ kg}$$
$$\Delta m = 2.356 - 2.1736 = 0.1823 \text{ kg}$$

**Solution : 30**

Consider the system of fluid contained in the leakproof piston along with the stirrer. Work is being done by the piston on the fluid by stirring, through electric motor.

As a result of this fluid moves out against atmospheric pressure and does work.

Work done by fluid against atmospheric pressure,



$$W_{atm} = \int PdV = P_{atm} \times (V_2 - V_1)$$

$$= (1.01325 \times 10^5 \, \text{Pa}) \times \frac{\pi D^2}{4} (\Delta x) \, \text{m}^3$$

$$= 1.01325 \times 10^5 \times \frac{\pi \times 0.4^2}{4} \times 0.485$$

$$W_{atm} = 6.175 \, \text{kJ}$$

Net work done by the fluid $= W_{net} = W_{stirrer} + W_{atm} = 2 \, \text{kJ}$

∴ $\qquad W_{stirrer} = 2 - 6.175 = -4.175 \, \text{kJ}$

Sign is negative, since this work is done on the system.

Speed of rotation of motor, $\quad \omega = \dfrac{2\pi \times 840}{60}$ rad/s = 87.965 rad/s

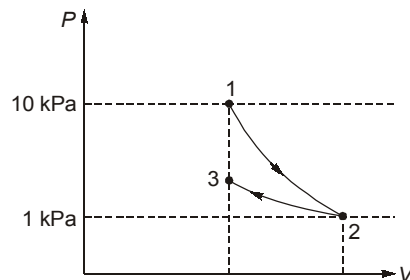Power output of shaft $= \dfrac{W_{stirrer}}{t} = \dfrac{-4.175 \, \text{kJ}}{10 \, \text{min}} = 6.96 \, \text{watt}$

Assume torque of motor as $T$ Nm.

∴ $\qquad$ Power of motor $= T\omega = 6.96 \, \text{watt}$

⇒ $\qquad T = \dfrac{6.96}{87.965} = 7.91 \times 10^{-2} \, \text{Nm}$

**Solution : 31**

Consider the isothermal and adiabatic process as represented on $P\text{-}V$ diagram in figure.



The gas ratio of specific heat capacities

$$= \frac{c_P}{c_V} = \gamma = \frac{1.005}{0.718} \simeq 1.4$$

For process 1-2, Adiabatic expansion (reversible)

$$P^{1-\gamma} T^{\gamma} = \text{Constant}$$

$$\therefore \qquad T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 403 \times \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 208.73 \text{ K}$$
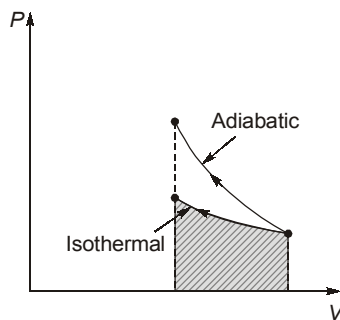
Also $\qquad PV^{\gamma} = \text{Constant}$

$$\Rightarrow \qquad V_2 = V_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma} = 0.5 \times \left(\frac{10}{1}\right)^{1/1.4} = 2.59 \text{ m}^3$$

For process 2-3, Isothermal compression

$$\therefore \qquad T_3 = T_2 = 208.73 \text{ K}$$
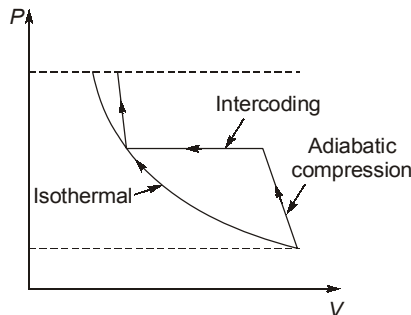
$$PV = \text{Constant}$$

$$\Rightarrow \qquad P_3 = \frac{P_2 V_2}{V_3} = \frac{P_2 V_2}{V_1} = \frac{1 \times 2.59}{0.5} = 5.18 \text{ kPa}$$



As can be seen in figure, slope of an isothermal process is less than that of an isentropic/adiabatic process.

Hence area under the curve, which is equal to work done is minimum in case of an isothermal process.

$\therefore$ Isothermal process should be used in compression



In Practise, for compression involving high compression ratios, adiabatic process with intercooling is employed. This method closely approaches an isothermal process.

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# 2 First law of Thermodynamics (Non-Flow Processes)

## LEVEL 1 Objective Questions

1. (a)
2. (d)
3. (c)
4. (b)
5. (c)
6. (b)
7. (b)
8. (a)
9. (d)
10. (c)
11. (c)
12. (c)

## LEVEL 2 Objective Questions

13. (c)
14. (a)
15. (c)
16. (15)
17. (−55.45)
18. (d)
19. (1.69)
20. (b)
21. (c)
22. (a)
23. (a)
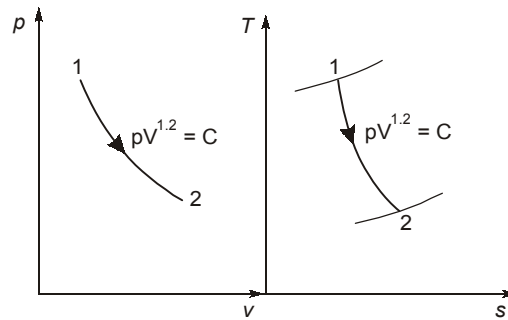
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**LEVEL 3** Conventional Questions

**Solution : 24**

Given  data: $p_1$ = 8 bar = 800 kPa, $V_1$ = 0.02 m³; $p_2$ = 2 bar = 200 kPa; $pV^{1.2}$ = C; m = 0.25 kg; du = – 55 kJ/kg;

$$dU = mdu$$
$$= -0.25 \times 55 = -13.75 \text{ kJ}$$
$$p_1 V_1^{1.2} = p_2 V_2^{1.2}$$
$$8 \times (0.02)^{1.2} = 2 \times V_2^{1.2}$$



or
$$V_2^{1.2} = 0.03658$$
or
$$V_2 = 0.0635 \text{ m}^3$$

Work done: $W = \dfrac{p_1 V_1 - p_2 V_2}{n-1} = \dfrac{800 \times 0.02 - 200 \times 0.0635}{1.2 - 1} = 16.5 \text{ kJ}$

Applying first law of thermodynamics for process 1–2,

$$Q = dU + W$$
$$= -13.75 + 16.5 = 2.75 \text{ kJ}$$

**Solution : 25**

Given data: $V_1$ = 0.1 m³; $p_1$ = 3 bar = 300 kPa; $V_2$ = 0.3 m³; $p_2$ = 1 bar = 100 kPa
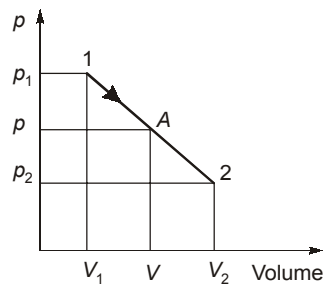


Let $A$ is any point on process 1–2. $p$ and $V$ are the pressure and volume at point $A$. Based on the given data, the p-V relation can be expressed by the linear equation

$$\frac{p_2 - p_1}{V_2 - V_1} = \frac{p - p_1}{V - V_1}$$

$$\frac{100-300}{0.3-0.1} = \frac{p-300}{V-0.1}$$

$$\frac{-200}{0.2} = \frac{p-300}{V-0.1}$$

$$-1000 = \frac{p-300}{V-0.1}$$

or $\qquad -1000V + 100 = p - 300$

or $\qquad\qquad\qquad p = 400 - 1000\,V$

where $p$ is in kPa and $V$ is in m³.

The work done is calculated by

$$W_{1-2} = \int_{V_1}^{V_2} pdV = \int_{0.1}^{0.3}(400-1000V)dV = \left[400V - \frac{1000V^2}{2}\right]_{0.1}^{0.3}$$

$$= 400 \times 0.3 - 500 \times (0.3)^2 - 400 \times 0.1 + 500 \times (0.1)^2$$

$$= 120 - 45 - 40 + 5 = 40 \text{ kJ}$$

**Alternate Method :**

$$W = \frac{1}{2}(V_2 - V_1) \times (P_1 + P_2) = \frac{1}{2} \times 0.2 \times 400 = 40 \text{ kJ}$$

**Solution : 26**

Change in internal energy, $\qquad dU = mc_v(T_2 - T_1)$

$$dU = \frac{mR}{\gamma - 1}(T_2 - T_1) \qquad\qquad \dots \text{(i)}$$

$$\left(\because\ c_v = \frac{R}{\gamma - 1}\right).$$

Work done, $\qquad W = \int_{1}^{2} pdV$

where $\qquad\qquad\qquad pV^n = C$

$$p = \frac{C}{V^n} = CV^{-n}$$

$\therefore \qquad\qquad\qquad W = \int_{1}^{2} V^{-n}dV = C\frac{[V^{-n+1}]_{1}^{2}}{-n+1}$

$$= \frac{p_2\,V_2^n\,V_2^{-n+1} - p_1\,V_1^n\,V_1^{-n+1}}{1-n} = \frac{p_2\,V_2 - p_1\,V_1}{1-n}$$

$$W = \frac{mR(T_2 - T_1)}{1 - n} \qquad \text{...(ii)}$$

Heat transfer,
$$Q = dU + W = \frac{mR(T_2 - T_1)}{\gamma - 1} + \frac{mR(T_2 - T_1)}{1 - n}$$

$$= \left(\frac{1-n}{\gamma - 1} + 1\right)\frac{mR(T_2 - T_1)}{1 - n} = \left(\frac{1 - n + \gamma - 1}{\gamma - 1}\right)\frac{mR(T_2 - T_1)}{1 - n}$$

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right)\frac{mR(T_2 - T_1)}{1 - n} \qquad \text{...(iii)}$$

$$Q \;:\; dU \;:\; W$$

$$\left(\frac{\gamma - n}{\gamma - 1}\right)\frac{mR(T_2 - T_1)}{1 - n} \;:\; \frac{mR(T_2 - T_1)}{\gamma - 1} \;:\; \frac{mR(T_2 - T_1)}{1 - n}$$

$$\left(\frac{\gamma - n}{\gamma - 1}\right)\times\frac{1}{1 - n} \;:\; \frac{1}{\gamma - 1} \;:\; \frac{1}{1 - n}$$

$$(\gamma - n) \;:\; (1 - n) \;:\; (\gamma - 1)$$

Heat transfer,
$$Q = mc_n(T_2 - T_1) \qquad \text{...(iv)}$$

Comparing Eq. (iv) with Eq. (iii), we get

$$c_n = \left(\frac{\gamma - n}{\gamma - 1}\right)\frac{R}{1 - n} = \left(\frac{\gamma - n}{1 - n}\right)c_v$$
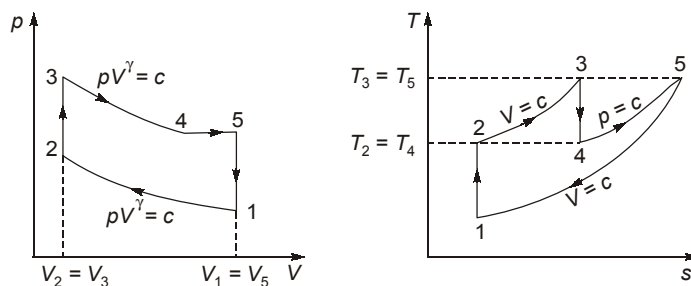
where
$$c_v = \frac{R}{\gamma - 1}$$

**Solution : 27**

Given data: $T_2 = 555\,\text{K}$; $T_3 = 835\,\text{K}$; $T_4 = T_2 = 555\,\text{K}$; $T_5 = T_3 = 835\,\text{K}$

From p-V diagram,

$$\frac{V_2}{V_1} = \frac{V_3}{V_5}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_5} \times \frac{V_4}{V_4}$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \times \frac{V_4}{V_5}$$

$$\left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{T_4}{T_3}\right)^{\frac{1}{\gamma-1}} \times \frac{T_4}{T_5}$$

or

$$\frac{T_1}{T_2} = \frac{T_4}{T_3} \times \left(\frac{T_4}{T_5}\right)^{\gamma-1}$$

$$\frac{T_1}{555} = \frac{555}{835} \times \left(\frac{555}{835}\right)^{1.4-1}$$

or

$$T_1 = 313.28 \text{ K}$$

Net work done: $w_{net} = w_{1-2} + w_{2-3} + w_{3-4} + w_{4-5} + w_{5-1}$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} + 0 + \frac{R(T_3 - T_4)}{\gamma - 1} + R(T_5 - T_4) + 0$$

$$= \frac{0.287(313.28 - 555)}{1.4 - 1} + \frac{0.287(835 - 555)}{1.4 - 1} + 0.287(835 - 555)$$

$$= -173.43 + 200.9 + 80.36 = 107.83 \text{ kJ/kg}$$

## Solution : 28

Given: $\dot{Q}$ = Net heat loss rate = (100 – 75) = 25 kJ/hr, $P_1$ = 1 bar, $T_1$ = 25°C, $V$ = 10 m$^3$

As the process is isochoric i.e., $\Delta V = 0$

∴ Total heat required, $Q = m c_V \Delta T = m \times 0.718 \times 25$

$$m = \frac{P_1 V_1}{R T_1} = \frac{100 \times 10}{0.287 \times 298} = 11.692 \text{ kg}$$

$$Q = 209.877 \text{ kJ}$$

$$\text{Time} = \frac{Q}{\dot{Q}} = \frac{209.877}{25} = 8 \text{ hour } 23 \text{ min } 42.38 \text{ sec}$$
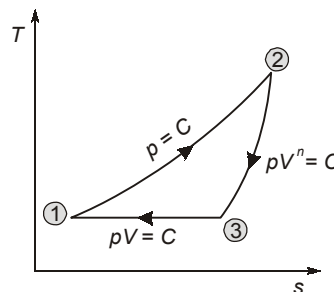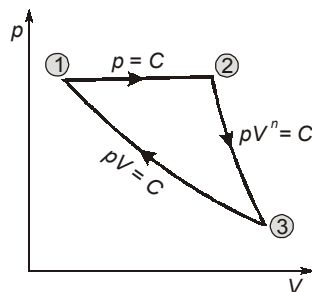
## Solution : 29

Given data:

$$T_1 = 260°C = (260 + 273) \text{ K} = 533 \text{ K}$$
$$p_1 = 700 \text{ kPa}$$
$$V_1 = 0.028 \text{ m}^3$$
$$V_2 = 0.084 \text{ m}^2$$

Applying equation of state at state 1,

$$p_1 V_1 = mRT_1$$
$$700 \times 0.028 = m \times 0.287 \times 533$$
or
$$m = 0.1281 \text{ kg}$$

For isobaric process 1-2,

$$\frac{T_2}{V_2} = \frac{T_1}{V_1} \qquad \text{(Charles law)}$$

$$\frac{T_2}{0.084} = \frac{533}{0.028}$$

or
$$T_2 = 1599 \text{ K}$$

For polytropic process 2-3,

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_3}\right)^{n-1} \qquad (\because T_3 - T_1)$$

$$\frac{533}{1599} = \left(\frac{0.084}{V_3}\right)^{1.5-1} = \left(\frac{0.084}{V_3}\right)^{0.5}$$

or
$$\left(\frac{533}{1599}\right)^{\frac{1}{0.5}} = \frac{0.084}{V_3}$$

$$\left(\frac{533}{1599}\right)^2 = \frac{0.084}{V_3}$$

or
$$V_3 = 0.756 \text{ m}^3$$

For process 1-2,

Work done:
$$W_{1\text{-}2} = p_1(V_2 - V_1) = 700(0.084 - 0.028) = 39.2 \text{ kJ}$$

Heat transfer:
$$Q_{1\text{-}2} = m c_p (T_2 - T_1) = 0.1281 \times 1.005(1599 - 533) = 137.23 \text{ kJ}$$

For process 2-3,

Work done:
$$W_{2\text{-}3} = \frac{mR(T_3 - T_2)}{1-n} = \frac{0.1281 \times 0.287(533 - 1599)}{1 - 1.5} = 78.38 \text{ kJ}$$

Heat transfer:
$$Q_{2\text{-}3} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W_{2\text{-}3} = \left(\frac{1.4 - 1.5}{1.4 - 1}\right) \times 78.38 = -19.59 \text{ kJ}$$

For process 3-1, Work done:
$$W_{3\text{-}1} = Q_{3\text{-}1} = mRT_1 \log_e \frac{V_1}{V_3}$$

$$= 0.1281 \times 0.287 \times 533 \log_e \frac{0.028}{0.756} = -64.584 \text{ kJ}$$

Net work done:
$$W_{net} = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}1} = 39.2 + 78.38 - 64.584 = 52.996 \text{ kJ}$$

Heat supplied:
$$Q_s = Q_{1\text{-}2} = 137.23 \text{ kJ}$$

The efficiency of the cycle,
$$\eta = \frac{\text{Net work done}}{\text{Heat supplied}} = \frac{W_{net}}{Q_s} = \frac{52.996}{137.23} = 0.3862 = 38.62\%$$

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# 3 First law of Thermodynamics (Flow Processes)

## LEVEL 1 Objective Questions

1. (a)
2. (b)
3. (b)
4. (c)
5. (a)
6. (c)
7. (c)
8. (b)
9. (d)
10. (c)
11. (b)
12. (b)

## LEVEL 2 Objective Questions

13. (a)
14. (70)
15. (523.02)
16. (b)
17. (c)
18. (d)
19. (2717)
20. (c)
21. (a)
22. (b)

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**LEVEL 3** Conventional Questions
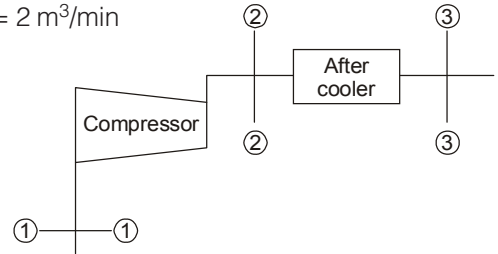
**Solution : 23**

Given data;

$V$ = Volume flow rate = 2 m³/min
$p_1$ = 0.11 MPa
$T_1$ = 20°C = 293 K
$p_2$ = 1.5 MPa
$T_2$ = 111°C = 384 K
$T_3$ = 25°C = 298 K
$W$ = 4.15 kW

Assumption:

(i) Neglecting ΔK.E. and ΔP.E.     (ii) Assume flow is steady.

(iii) There is no work transfer in after cooler.

Applying ideal gas eq. at (1)

$$p_1 V = m R T_1$$

$$m = \frac{P_1 V}{R T_1} = \frac{0.11 \times 10^3 \times 2}{0.287 \times 293 \times 60} = 0.0436 \text{ kg/s}$$

Now applying S.F.E.E. for compressor between (1) and (2)

$$m(h_1) + Q_1 = m(h_2) + W$$

∴

$$Q_1 = m(h_2 - h_1) + W = m c_{pa}(T_2 - T_1) + W$$

$$= 0.0436 \times 1.005 \times (111 - 20) - 4.15 = \mathbf{-\, 0.163 \text{ kW}}$$

Now applying S.F.E.E. for after cooler between (2) and (3)

$$m(h_2) + Q_2 = m(h_3)$$

or

$$Q_2 = m(h_3 - h_2) = m c_{pa}(T_3 - T_2)$$

$$= 0.0436 \times 1.005(25 - 111) = \mathbf{-\, 3.76 \text{ kW}}$$

**Solution : 24**

Given data: $W = -10$ hp $= -10 \times 746$ W $= -7460$ W $= -7.46$ kW;

$d_1 = 6$ cm $= 0.06$ m;

$d_2 = 10$ cm $= 0.1$ m; $V_1 = 10$ m/s;

$$m = \rho A_1 V_1$$

$$= 1000 \times \frac{\pi}{4} d_1^2 \times V_1$$

$$= 1000 \times \frac{3.14}{4} \times (0.06)^2 \times 10$$

$$= 28.26 \text{ kg/s}$$

also
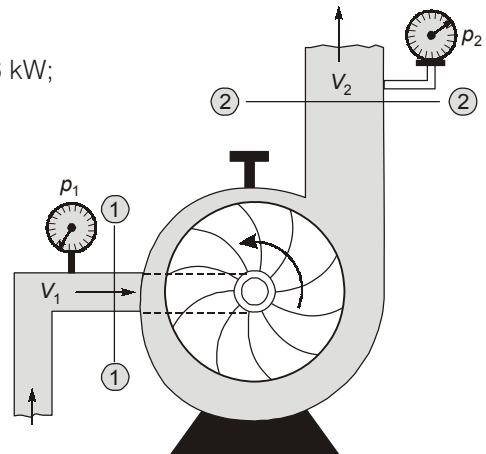
$$m = \rho A_1 V_1 = \rho A_2 V_2$$

$$\rho A_1 V_1 = \rho A_2 V_2$$
$$A_1 V_1 = A_2 V_2 \qquad (\because \rho = \text{constant})$$

$$\frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

$$d_1^2 V_1 = d_2^2 V_2$$
$$(0.06)^2 \times 10 = (0.1)^2 \times V_2$$

or $\qquad V_2 = 3.6 \text{ m/s}$

Applying steady flow energy equation between sections 1 and 2, we get

$$m\left[h_1 + \frac{V_1^2}{2} + gz_1\right] + Q = m\left[h_2 + \frac{V_2^2}{2} + gz_2\right] + W$$

where $\qquad h_1 = u_1 + p_1 v_1 = u_1 + \dfrac{p_1}{\rho_1}$

$$h_2 = u_2 + p_2 v_2 = u_2 + \frac{p_2}{\rho_2}$$

$$z_1 = z_2$$

$\therefore \qquad m\left[u_1 + \dfrac{p_1}{\rho} + \dfrac{V_1^2}{2}\right] = m\left[u_2 + \dfrac{p_2}{\rho} + \dfrac{V_2^2}{2}\right] + W$

where $\qquad u_1 = u_2 \qquad \because T = C$

$\therefore \qquad m\left[\dfrac{p_1}{\rho} + \dfrac{V_1^2}{2}\right] = m\left[\dfrac{p_2}{\rho} + \dfrac{V_2^2}{2}\right] + W$

or $\qquad m\left[\dfrac{p_2 - p_1}{\rho}\right] = \dfrac{m(V_1^2 - V_2^2)}{2} - W$

Unit balancing

$$m\left[\frac{p_2 - p_1}{\rho}\right] = \frac{m(V_1^2 - V_2^2)}{2 \times 1000} - W$$

where $m$ is in kg/s, $p_1$ and $p_2$ are in kPa, $V_1$ and $V_2$ are in m/s, $W$ is in kW, $\rho$ is in kg/m$^3$

$\therefore \qquad 28.26\left[\dfrac{p_2 - p_1}{1000}\right] = 28.26\left[\dfrac{(10)^2 - (3.6)^2}{2000}\right] - (-7.46)$

$$0.02826(p_2 - p_1) = 1.22987 + 7.46$$

or $\qquad p_2 - p_1 = \mathbf{307.49 \text{ kPa}}$

**Solution : 25**

(i)  Applying SFEE,

$$\frac{V_1^2}{2} + C_p T_1 = \frac{V_2^2}{2} + c_p T_2$$

$$T_2 = \frac{V_1^2 - V_2^2}{2c_p} + T_1$$

$$= \frac{400^2 - 700^2}{2 \times 100} + 420 = 255°C$$

(ii)

$$\rho = \frac{p_1}{RT_1} = \frac{7000}{287 \times 693} = 0.0352 \text{ kg/m}^3$$
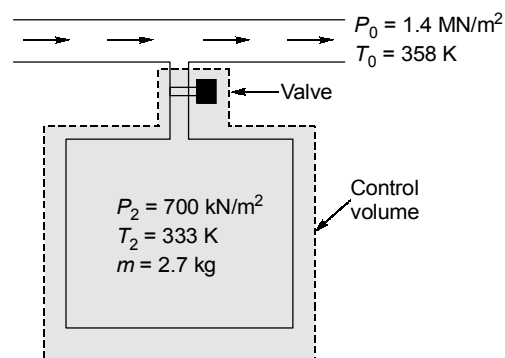
$$m = \rho_1 A_1 V_1 = 0.0352 \times \pi \times 0.1^2 \times 400 = 0.442 \text{ kg/s}$$

(iii)

$$\rho_2 = \rho_1 \left(\frac{T_2}{T_1}\right)^{\frac{1}{r-1}} = 0.0352 \left(\frac{528}{693}\right)^{\frac{1}{1.4-1}} = 0.01784 \text{ kg/m}^3$$

$$d_2 = \sqrt{\frac{\rho_1 d_1^2 V_1}{\rho_2 V_2}} = \sqrt{\frac{0.0352 \times (0.2)^2 \times 400}{0.01784 \times 700}} = 0.212 \text{ m} = 212 \text{ mm}$$

**Solution : 26**

Consider the pressure vessel and main connected by a valve as shown in figure.



Gas flows in side the vessel initially evacuated. Consider control volume as shown in figure.
Applying conservation of energy for variable flow process of this control volume,

$$\text{Rate of change} = \frac{dE_v}{d\tau}$$

$$= \text{Rate of inflow of energy} - \text{Rate of outflow of energy}$$

$$\Rightarrow \qquad \frac{dE_v}{d\tau} = h_p \cdot \frac{dm}{d\tau} + \frac{dQ}{d\tau}$$

$\Rightarrow$ $\qquad\qquad\qquad\qquad\qquad \dfrac{du}{d\tau} = h_p \dfrac{dm}{d\tau} + \dfrac{dQ}{d\tau}$ (since velocity of pipe is negligible)

Integrating, we get, $\qquad m_2\, u_2 - m_1\, u_1 = h_p\,(m_2 - m_1) + Q$

Initial mass of gas in control volume, $\quad m_1 = 0$

Final mass of gas in control volume, $\quad m_2 = 2.7$ kg

$\therefore$ Heat transfer to or from pressure vessel

$$Q = m_2\, u_2 - m_2\, h_p = m_2\,[c_V\, T_2 - c_P\, T_p]$$
$$= \qquad\qquad\qquad\qquad 2.7\,[0.67 \times 333 - 0.88 \times 358]$$
$$Q = -248.211 \text{ kJ}$$

$\therefore$ 248.21 kJ of heat is lost from the pressure vessel. Assume initial volume of gas in pipe before transfer is $V_p$. Since the gas can be assumed to follow ideal gas behaviour –

$$\dfrac{P_P\, V_P}{T_P} = \dfrac{P_2\, V_2}{T_2} = mR = m\,(c_P - c_V)$$

$\Rightarrow$ $\qquad\qquad\qquad\qquad V_P = \dfrac{358 \times 2.7 \times (0.88 - 0.67)}{1.4 \times 10^3} = 0.145 \text{ m}^3$

Hence, volume of gas before filling is 0.145 m$^3$.

■■■■

# 4 Second law of Thermodynamics

**LEVEL 1** Objective Questions

1. (d)
2. (4)
3. (d)
4. (c)
5. (b)
6. (d)
7. (a)
8. (4)
9. (b)
10. (b)
11. (d)
12. (d)

**LEVEL 2** Objective Questions

13. (17.32)
14. (c)
15. (b)
16. (b)
17. (b)
18. (d)
19. (50)
20. (c)

■■■■

**LEVEL 3** Conventional Questions

**Solution : 21**

Consider two heat engines *A* and *B* acting in series as shown in figure.

Efficiency of engine *A*, $\eta_A = \left(1 - \dfrac{T}{700}\right)$

Where, *T* - Temperature of intermediate between *A* and *B*.

Heat received by *A*, $Q_1 = 300$ kJ

Work output of engine *A*, $W_A = Q_1\left(1 - \dfrac{T}{700}\right)$

$W_A = 300\left(1 - \dfrac{T}{700}\right)$

Efficiency of engine *B*, $\eta_B = \left(1 - \dfrac{280}{T}\right)$

Heat rejected by engine *A*, $Q_2 = Q_1 - W_A$

$\Rightarrow \qquad Q_2 = 300 - 300\left(1 - \dfrac{T}{700}\right) = \dfrac{300 \cdot T}{700} = \dfrac{3T}{7}$ kJ

Work output of engine, $B = \left(1 - \dfrac{280}{T}\right)\cdot\dfrac{3T}{7}$

According to question, $\qquad W_A = 2 \cdot W_B$

$\Rightarrow \qquad 300\left(1 - \dfrac{T}{700}\right) = 2\cdot\dfrac{3T}{7}\left(1 - \dfrac{280}{T}\right)$
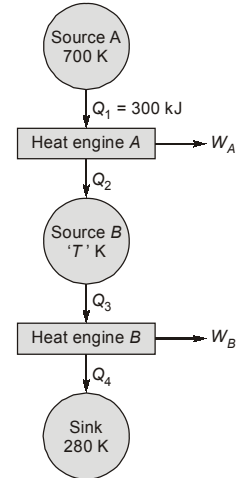
$\Rightarrow \qquad\qquad T = 420$ K

Efficiency of each engine, $\qquad \eta_A = 1 - \dfrac{420}{700} = 0.40 = 40\%$

$\eta_B = 1 - \dfrac{280}{420} = 0.33 = 33.33\%$

Heat rejected by engine *A* and received by engine *B*

$= Q_3 = Q_2 = \dfrac{3T}{7}$ kJ $= 180$ kJ

Heat rejected to the sink $= Q_4 = Q_3 \times \dfrac{T_4}{T_3} = \dfrac{180 \times 280}{420}$

$= 120$ kJ

**Solution : 22**

Consider heat engine and refrigerator operating between source at 1073 K and sink at 303 K.

Efficiency of Carnot heat engine $= 1 - \dfrac{303}{1073} = 0.7176$
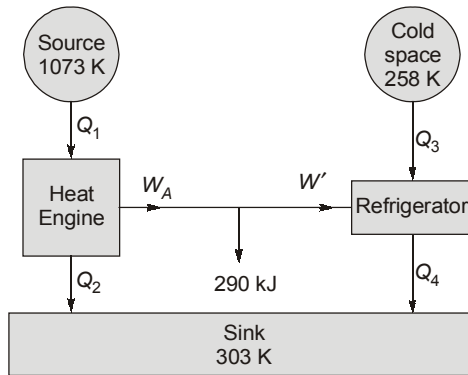
Heat extracted by heat engine, $Q_1$ = 1900 kJ

∴      Work done by heat engine, $W_A$ = $\eta . Q_1 = 0.7176 \times 1900$

     = 1363.47 kJ

Work transferred to refrigerator = $W_A - W_{net}$

     = 1363.47 − 290

⇒      $W'$ = 1073.47 kJ

$$\text{COP}_{refrigerator} = \frac{Q_3}{W'} = \frac{258}{303 - 258} = 5.73$$

⇒      $Q_3$ = 6154.56 kJ

∴      Heat transferred to 303 K reservoir by refrigerator = $Q_3 + W'$

     $Q_4$ = 7228.03 kJ
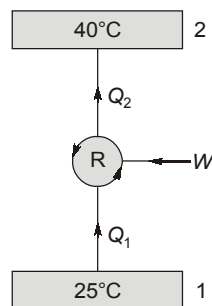
Heat transferred to 303 K reservoir by heat engine, $Q_2$

     $Q_2$ = $Q_1 - W_A$ = 1900 − 1363.47 = 536.53 kJ

Total heat transferred to 30°C thermal reservoir = $Q_2 + Q_4$ = 536.53 + 7228.03 = 7764.56 kJ

## Solution : 23

In summer, ambient at 40°C



**Reversible refrigerator**

For a reversible refrigerator

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

Now $\qquad Q_1 = $ (energy loss per degree $\times \Delta T$) kW

$\qquad\qquad\qquad\qquad = 5 \times 15 = 75$ kW

(As temperature difference $= 40 - 25 = 15°C$)

$\therefore \qquad\qquad\qquad \dfrac{75}{Q_2} = \dfrac{298}{313}$

or $\qquad\qquad\qquad Q_2 = 78.78$ kW

Power required: $\qquad P = (Q_2 - Q_1) = 78.78 - 75 = 3.78$ kW
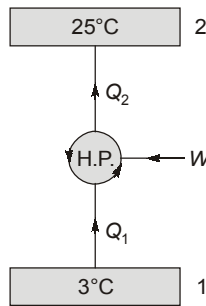


**Reversible heat pump**

In winter, ambient is at 3°C

Now $\qquad\qquad\qquad Q_2 = 5 \times 22 = 110$ kW

and $\qquad\qquad\qquad \dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$

$\therefore \qquad\qquad\qquad Q_1 = \dfrac{276}{298} \times 110 = 101.88$ kW

Work done $= Q_2 - Q_1 = 110 - 108.88 = $ **8.12 kW**

## Solution : 24

Given data: $Q_1 = 900$ kJ, $T_1 = 500$ K, $W = 300$ kJ, $T_2 = 400$ K, $T_3 = 300$ K
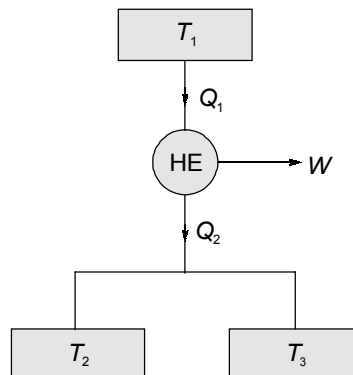


Thermal efficiency,

$$\eta = \dfrac{W}{Q_1} = \dfrac{300}{900} = 0.3333 = 33.33\%$$

Heat rejected :

$\qquad\qquad Q_2 = Q_1 - W$

$\qquad\qquad\quad\; = 900 - 300 = 600$ kJ

Let $Q$ be the heat rejected to sink at 400 K. Then heat rejected to sink at 300 K is (600 – Q).
As the engine is reversible, the clausius theorem gives:

$$\oint \frac{\delta Q}{T} = 0$$

$$\frac{Q_1}{T_1} - \frac{Q}{T_2} - \frac{600 - Q}{T_3} = 0$$

$$\frac{900}{500} - \frac{Q}{400} - \frac{600 - Q}{300} = 0$$

Multiplying by 100, we get

$$\frac{900 \times 100}{500} - \frac{100Q}{400} - \frac{600 \times 100}{300} + \frac{100Q}{300} = 0$$

$$180 - 0.25\,Q - 200 + 0.3333Q = 0$$

or $\qquad\qquad\qquad\qquad Q = 240\ \text{kJ}$

Thus, heat rejected to sink at $T_2$ = (400 K) = 240J
and heat rejected to sink at $T_3$ = 300 K

$$= 600 - 240 = 360\ \text{kJ}$$
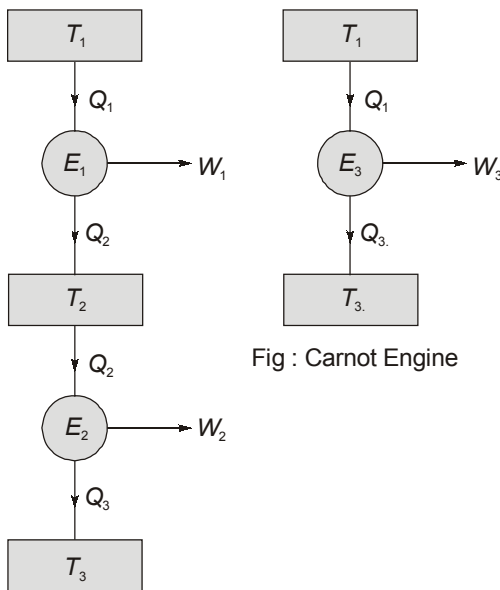
**Solution : 25**



Fig : Carnot Engine

Fig : Carrnot Engine
in series

$$\eta_1 = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

or $\qquad\qquad 1 - \eta_1 = \frac{T_2}{T_1}$

Similarly, $\qquad\quad 1 - \eta_2 = \frac{T_3}{T_2}$

$$\therefore \qquad (1-\eta_1)(1-\eta_2) = \frac{T_3}{T_1} \qquad \qquad \text{. . . (i)}$$

For the Carnot engine operating directly between the temperature limits $T_1$ and $T_3$

$$\eta_3 = \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$$

or

$$1 - \eta_3 = \frac{T_3}{T_1} \qquad \qquad \text{. . . (ii)}$$

Equating equation (i) and (ii), we get

$$1 - \eta_3 = (1-\eta_1)(1-\eta_2)$$
$$= 1 - \eta_1 - \eta_2 + \eta_1\eta_2$$

or

$$\eta_3 = \eta_1 + \eta_2 - \eta_1\eta_2$$

**Solution : 26**
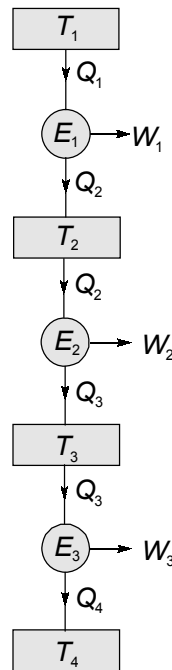


Given data : $Q_1 = 4000$ kJ, $W_1 = 1800$ kJ, $W_2 = 1200$ kJ, $W_3 = 500$ kJ, $T_4 = 300$ K

$$\text{Total work output} = W_1 + W_2 + W_3$$
$$= 1800 + 1200 + 500$$
$$= 3500 \text{ kJ}$$

From the principle of energy conseravtion, the heat rejected to the sink at temperature $T_4$ is

$$Q_4 = Q_1 - W_{net}$$
$$= 4000 - 3500 = 500 \text{ kJ}$$

For the overall Carnot engine, $\qquad \dfrac{T_1}{T_4} = \dfrac{Q_1}{Q_4}$

$$\frac{2000}{T_4} = \frac{4000}{500}$$

or $\qquad T_4 = \mathbf{250\ K}$

For the composite system with engines $E_1$ and $E_2$ in series

$$\begin{aligned} Q_3 &= Q_1 - W_1 - W_2 \\ &= 4000 - 1800 - 1200 \\ &= 1000\ kJ \end{aligned}$$

$$\frac{T_1}{T_3} = \frac{Q_1}{Q_3}$$

$$\frac{2000}{T_3} = \frac{4000}{1000}$$

or $\qquad T_3 = 500\ K$

**Alternate Method:**

For engine I $\qquad \eta = 1 - \dfrac{T_2}{T_1} \Rightarrow \dfrac{1800}{4000} = 1 - \dfrac{T_2}{2000} \quad \Rightarrow \quad T_2 = 1100\ K$

For engine II $\qquad \eta = 1 - \dfrac{T_3}{T_2} \Rightarrow T_3 = \dfrac{1000}{2200} \times T_2$

$$T_3 = \frac{1000}{2200} \times 1100 = 500\ K$$

For engine III $\qquad \eta = 1 - \dfrac{T_4}{T_3} = \dfrac{500}{1000} \Rightarrow 0.5 = \dfrac{T_4}{T_3}$

$$T_4 = 500 \times 0.5 = 250\ K$$

■■■■

# 5 Entropy, Availability and Irreversibility

## LEVEL 1 Objective Questions

1. (a)
2. (a)
3. (a)
4. (400)
5. (c)
6. (600)
7. (b)
8. (b)
9. (a)
10. (b)
11. (b)
12. (c)

## LEVEL 2 Objective Questions

13. (d)
14. (c)
15. (b)
16. (a)
17. (b)
18. (c)
19. (c)

■■■■

**LEVEL 3** Conventional Questions

**Solution : 20**

Given data: For compartment $A$, $V_A = 0.1$ m$^3$ ; $p = 2$ bar; $T = 13°C = (13 + 273)$K $= 286$ K

Molecular weight:
$$M_A = 32 \text{ kg/kmol}$$

For compartment $B$,
$$V_B = 0.2 \text{ m}^3$$
$$p = 2 \text{ bar}$$
$$T = 13°C = 286 \text{ K}$$
$$M_B = 28 \text{ kg/kmol}$$

For compartment $C$,
$$V_C = 0.05 \text{ m}^3$$
$$p = 2 \text{ bar}$$
$$T = 286 \text{ K}$$
$$M_C = 44 \text{ kg/kmol}$$

| Ⓐ | Ⓑ | Ⓒ |
|---|---|---|
| 0.1 m$^3$ | 0.2 m$^3$ | 0.05 m$^3$ |
| 2 bar | 2 bar | 2 bar |
| 13°C | 13°C | 13°C |
| O$_2$ | N$_2$ | CO$_2$ |

The partitions are removed and $O_2$, $N_2$ and $CO_2$ mix,

Final pressure: $p = 2$ bar

Now, partial pressure of $O_2$,

$$p_A = \frac{V_A}{V_A + V_B + V_C} \times p = \frac{0.1}{0.1 + 0.2 + 0.05} \times 2 = 0.5714 \text{ bar}$$

Partial pressure of $N_2$,

$$p_B = \frac{V_B}{V_A + V_B + V_C} \times p = \frac{0.2}{0.1 + 0.2 + 0.05} \times 2 = 1.1428 \text{ bar}$$

Partial pressure of $CO_2$,
$$p_C = \frac{V_C}{V_A + V_B + V_C} \times p = \frac{0.05}{0.1 + 0.2 + 0.05} \times 2 = 0.2857 \text{ bar}$$

For compartment $A$,
$$R_A = \frac{\bar{R}}{M_A} = \frac{8.314}{32} = 0.2598 \text{ kJ/kgK}$$

$$pV_A = m_A R_A T$$
$$200 \times 0.1 = m_A \times 0.2598 \times 286$$

or
$$m_A = 0.2691 \text{ kg}$$

For compartment $B$,
$$R_B = \frac{\bar{R}}{M_B} = \frac{8.314}{28} = 0.2969 \text{ kJ/kgK}$$

$$pV_B = m_B R_B T$$
$$200 \times 0.2 = m_B \times 0.2969 \times 286$$
or
$$m_B = 0.4710 \text{ kg}$$

For compartment $C$,
$$R_C = \frac{\bar{R}}{M_C} = \frac{8.314}{44} = 0.1889 \text{ kJ/kgK}$$

$$pV_C = m_C R_c T$$
$$200 \times 0.05 = m_C \times 0.1889 \times 286$$
or
$$m_C = 0.18509 \text{ kg}$$

Change in entropy of $O_2$,
$$(\Delta S)_A = -m_A R_A \log_e \frac{p_A}{p}$$
$$= -0.2691 \times 0.2598 \log_e \frac{0.5714}{2} = 0.0875 \text{ kJ/K}$$

Change in entropy of $N_2$,
$$(\Delta S)_B = -m_A R_B \log_e \frac{p_B}{p}$$
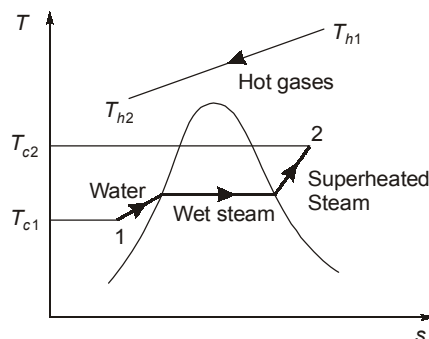$$= -0.4710 \times 0.2969 \log_e \frac{1.1428}{2} = 0.07826 \text{ kJ/K}$$

Change in entropy of $CO_2$,
$$(\Delta S)_C = -m_C R_C \log_e \frac{p_C}{p}$$
$$= -0.18509 \times 0.1889 \log_e \frac{0.2857}{2} = 0.0680 \text{ kJ/K}$$

**Solution : 21**

Given data: $T_{h1} = 1100°C$; $T_{h2} = 550°C$; $p_h = 0.1$ MPa; $p_c = 0.8$ MPa; $T_{c1} = 150°C$; $T_{c2} = 250°C$
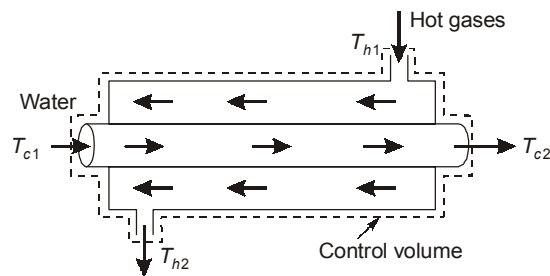


$$p_0 = 100 \text{ kPa}, \ T_0 = 25°C = 298 \text{ K}$$

$$c_{ph} = 1.09 \text{ kJ/kgK}$$
$$h_{c1} = 632.2 \text{ kJ/kg}; \ s_{c1} = 1.8418 \text{ kJ/kgK}$$
$$h_{c2} = 2950 \text{ kJ/kg}; \ s_{c2} = 7.0389 \text{ kJ/kgK}$$

(i) Change in availability of water on unit mass of water basis,

$$\psi_2 - \psi_1 = (h_{c2} - h_{c1}) - T_0(s_{c2} - s_{c1}) \text{ [neglecting change in K.E. and P.E.]}$$
$$= (2950 - 632.2) - 298(7.0389 - 1.8418)$$
$$= 2317.8 - 1548.73 = \textbf{769.07 kJ/kg}$$

(ii) Heat exchanged between combustion products (hot gases) and water in boiler,

Heat lost by combustion products = Heat gained by water

$$m_h c_{ph}(T_{h1} - T_{h2}) = m_c(h_{c2} - h_{c1})$$
$$m_h \times 1.09(1100 - 550) = m_c \times (2950 - 632.2)$$
$$m_h = 3.866 \, m_c$$

For 1 kg of water i.e., $m_c = 1$ kg

∴ $$m_h = 3.866 \times 1 = 3.866 \text{ kg}$$

Change in availability of product of combustion per kg of water,

$$\psi_1 - \psi_2 = m_h(h_{h1} - h_{h2}) - T_0 m_h(s_{h1} - s_{h2})$$
$$= m_h c_{ph}(T_{h1} - T_{h2}) - T_0 m_h c_p \log_e \frac{T_{h1}}{T_{h2}}$$

where
$$m_h = 3.866 \text{ kg}$$
$$c_{ph} = 1.09 \text{ kJ/kgK}$$
$$T_{h2} = 550°C = (550 + 273)K = 823 K$$
$$T_{h1} = 1100°C = (1100 + 273)K = 1373 K$$

∴ $$\psi_2 - \psi_1 = 3.866 \times 1.09(1373 - 823) - 298 \times 3.866 \times 1.09 \log_e \frac{1373}{823}$$
$$= 2317.66 - 642.69 = \textbf{1674.97 kJ}$$

(iii) Process irreversibility per unit mass of water flow,

$$I = T_0(\Delta S)_{uni}$$
$$= T_0[(\Delta S)_{water} + (\Delta S)_{hot \, gas}]$$
$$= 298\left[m_w(s_2 - s_1) + m_h c_{ph} \log_e \frac{T_{h2}}{T_{h1}}\right]$$
$$= 298\left[1 \times (7.0389 - 1.8418) + 3.866 \times 1.09 \log_e \frac{823}{1373}\right]$$
$$= 298[5.1971 - 2.1566] = \textbf{906.06 kJ}$$
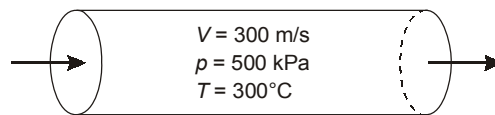
Exergy recovered = 1674.97 − 906.06 = 768.91 kJ

(iv) Second law efficiency, $$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = \frac{768.91}{1674.97} = 0.4590 = \textbf{45.90\%}$$

(v) Entropy generated per kg of water,

$$(\Delta S)_{uni} = (\Delta S)_{water} + (\Delta S)_{hot\ gas}$$

$$= m_w(s_2 - s_1) + m_h c_{ph} \log_e \frac{T_{h2}}{T_{h1}}$$

$$= 1 \times (7.0389 - 1.8418) + 3.866 \times 1.09 \log_e \frac{823}{1373}$$

$$= 5.1971 - 2.1566 = \textbf{3.04 kJ/K per kg of water}$$

## Solution : 22

Given data: $V = 300$ m/s; $p = 500$ kPa; $T = 300°C$; $= (300 + 273)K = 573$ K



$$p_o = 100 \text{ kPa}$$

$$T_o = 20°C = (20 + 273)K = 293 K$$

Gas constant: $R = \dfrac{\bar{R}}{M} = \dfrac{8.314}{28} = 0.2969$ kJ/kgK

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.2969}{1.4 - 1} = 1.039 \text{ kJ/kgK}$$

Availability: $a = (h - h_0) - T_o(s - s_0) + \dfrac{V^2}{2000}$

$$= 1.039(573 - 293) - 293\left(1.039 \log_e \frac{573}{293} - 0.2969 \log_e \frac{500}{100}\right) + \frac{(300)^2}{2000}$$

$$= 290.92 - 293(0.6968 - 0.4778) + 45 = 271.75 \text{ kJ/kg}$$

## Solution : 23

Availability of a system is the maximum useful work potential of a system at the given state. Availability is equal to the maximum work obtainable from the system when it moves from the current state to a state of complete equilibrium with the surroundings (also known as dead state) - temperature, pressure and chemical equilibrium.

Consider a Carnot cycle as shown in figure which extracts heat $Q$ from source at temperature $T$ and rejects it to sink at temperature $T_0$.
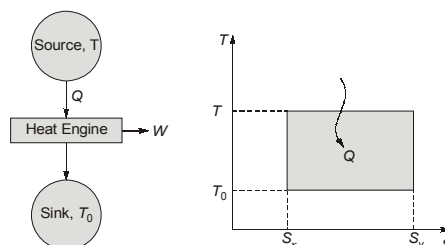
Work '$W$' is produced by the Carnot engine during the process. This is the maximum work which can be obtained by any cycle (reversible work) and hence, as per the definition of availability.

$$\text{Availability} = A = W_{max} = W_{Carnot}$$

$$\text{Efficiency of Carnot engine} = 1 - \frac{T_0}{T}$$

∴ $$\text{Work obtained, } W = \eta_C \times Q$$

⇒ $$W = \left(1 - \frac{T_0}{T}\right)Q$$

∴ $$\text{Availability} = W = \left(1 - \frac{T_0}{T}\right)Q$$

**Solution : 24**

Given data:
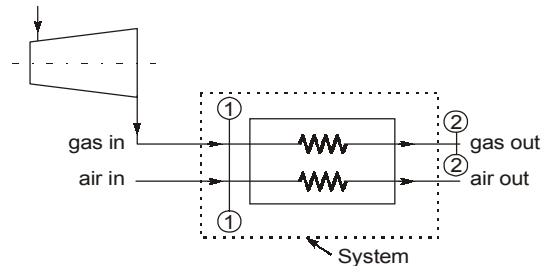


$$p_{1g} = \text{Pressure of gas inlet} = 1.12 \text{ bar}$$
$$T_{1g} = \text{Temperature of gas inlet} = 800 \text{ K}$$
$$T_{2g} = \text{Temperature of gas outlet} = 700 \text{ K}$$
$$T_{1a} = \text{Temperature of air inlet} = 470 \text{ K}$$
$$c_{pg} = \text{Exhaust gas} = 1.08 \text{ kJ/kgK}$$
$$c_{pa} = 1.05 \text{ kJ/kgK} = \text{air}$$
$$m_a = \text{Mass flow rate of air}$$
$$m_g = \text{Mass flow rate of gas}$$

⇒ $$m_a = 2 \, m_g$$
$$T_0 = \text{Surrounding temperature} = 20°C = 293 \text{K}$$
$$P_0 = \text{Surrounding pressure} = 1.03 \text{ bar}$$

Applying heat balance for the system

$$\text{heat lost by gas} = \text{heat gained by air}$$

⇒ $$m_g c_{pg}(T_{1g} - T_{2g}) = m_a c_{pa}(T_{2a} - T_{1a})$$

∴ $$T_{2a} = \text{air outlet temp.} = 521.43 \text{K}$$

(i) Decrease in availability of exhaust gas

$$= m_g \left[ c_{pg}\left(T_{1g} - T_{2g}\right) + T_0 c_{pg} \ln \frac{T_{2g}}{T_{1g}} \right]$$

$$= 1.08\left[(800-700)+293\ln\frac{700}{800}\right] = 65.75 \text{ kJ/kg}$$

(ii) Total entropy production rate $= d(\Delta S) + \Sigma m_e s_e - \Sigma m_i s_i - \displaystyle\int_1^2 \frac{dQ}{T}$

where $d(\Delta S) = 0$; $\Sigma m_e = \Sigma m_i = \Sigma m$      for steady flow

$$\int_1^2 \frac{dQ}{T} = 0 \qquad\qquad\qquad\qquad \text{for heat transfer}$$

∴      Total entropy production rate $= \Sigma m(S_e - S_i)$

$$= m_g\left[S_{2g} - S_{1g}\right] + m_a\left[S_{2a} - S_{1a}\right]$$

$$= m_g\left[c_{pg}\ln\frac{T_{2g}}{T_{1g}} + 2c_{pa}\ln\frac{T_{2a}}{T_{1a}}\right]$$

∴ Total entropy production per kg gas

$$= 1.08\times\ln\frac{700}{800} + 2\times1.05\times\ln\frac{521.43}{470} = 0.0738 \text{ kJ/kgK}$$

(iii) Heat supplied to air in heat exchanger

$$= m_g c_{pg}(T_{1g} - T_{2g}) = 1.08 \times (800 - 700) = 108 \text{ kJ/kg of gas}$$

Now for reversible heat transfer

$$\Delta S_{universe} = 0$$

⇒      $(\Delta S)_{gas} + (\Delta S)_{air} = 0$

⇒      $-(\Delta S)_{gas} = +(\Delta S)_{air}$

⇒      $-m_g c_{pg}\ln\dfrac{T_{2g}}{T_{sg}} = +m_a c_{pa}\ln\dfrac{T_{2a}}{T_{1a}}$

⇒      $m_g c_{pg}\ln\dfrac{T_{1g}}{T_{2g}} = 2m_g c_{pa}\ln\dfrac{T_{2a}}{T_{1a}}$

⇒      $1.08\ln\dfrac{800}{700} = 2\times1.05\ln\dfrac{T_{2a}}{470}$

∴      $T_{2a} = 503.41 \text{ K}$

∴ Now heat supplied to air in heat exchanger

$$= m_a c_{pa}(T_{2a} - T_{1a}) = 2\,m_g \times 1.05(503.41 - 470)$$

$$= 2 \times 1 \times 1.05\,(503.41 - 470)$$

$$= 70.16 \text{ kJ/kg of exhaust gas}$$

Extra output $= 108 - 70.16 = 37.84$ kJ/kg of exhaust gas

**Solution : 25**

Consider the various processes as shown in P-V diagram.

### Process 1-2:

We know

$$Tds = dh - Vdp$$

$$\Rightarrow \qquad dS = \frac{C_P dT}{T} - \frac{R}{P}dP$$
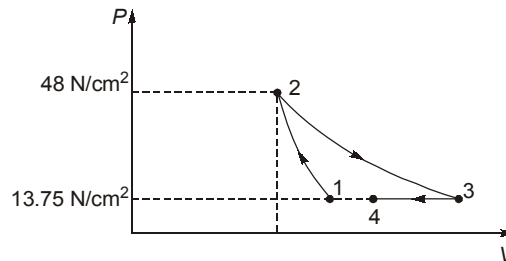


$$\Rightarrow \qquad \Delta S = C_p \ln\frac{T_f}{T_i} - R\ln\frac{P_f}{P_i}$$

$$\Delta S_{1\text{-}2} = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1.004 \times \ln\frac{556}{278} - 0.291\ln\frac{48}{13.75}$$

$$= 0.33212 \text{ kJ/kgK}$$

### Process 2-3: Throttling process ($\Delta h = 0$)

for an ideal gas, in throttling process,

$$\Delta h = C_P dT = 0$$

$$\Rightarrow \qquad dT = 0$$

$$\therefore \qquad \Delta S_{2\text{-}3} = -R\ln\frac{P_3}{P_2} = -0.291 \times \ln\left(\frac{13.75}{48}\right) = 0.36379 \text{ kJ/kgK}$$

### Process 3-4: Constant pressure compression

$$\frac{P_3 V_3}{T_3} = \frac{P_4 V_4}{T_4}$$

$$\Rightarrow \qquad T_4 = T_3 \cdot \frac{V_4}{V_3} = \frac{T_3}{2} = \frac{T_2}{2} = 278 \text{ k}$$

(since $P_3 = P_4$ and $T_3 = T_2$)

$$\Delta S_{3\text{-}4} = C_p \ln\frac{T_4}{T_3} - R\ln\frac{P_4}{P_3} = 1.004 \ln\frac{278}{556} = -0.69591 \text{ kJ/kgK}$$

$\therefore$ Net change in entropy of gas $= m\Delta S_{net}$

$$= m(\Delta S_{1\text{-}2} + \Delta S_{2\text{-}3} + \Delta S_{3\text{-}4})$$

$$= 2 \times (0.33212 + 0.36379 - 0.69591)$$

$$= 0 \text{ kJ/K}$$
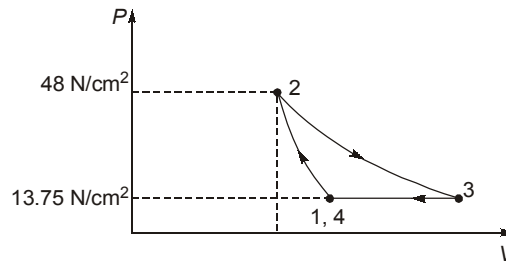
Therefore we can conclude that point 1 and 4 are same.

Hence the graph can be redrawn as shown:

**Solution : 26**

Given: $m = 6$ kg, $P = 5$ bar, $T = 600$ k, $P_0 = 1$ bar, $T_0 = 300$ K

### Assumption:

Air behave, as an ideal gas.

Changes in KE and PE are negligible.

Volume of air at given state, $V = \dfrac{mRT}{P} = \dfrac{6 \times 0.287 \times 600}{5 \times 10^2} = 2.0664$ m$^3$

Volume of air at surrounding conditions,

$$V_0 = \dfrac{mRT_0}{P_0} = \dfrac{6 \times 0.287 \times 300}{1 \times 10^2} = 5.166 \text{ m}^3$$

Entropy change when gas moves from current state to surrounding state

$$\Delta S = S - S_0$$

$$= m\left[C_p \ln \dfrac{T}{T_0} - R \ln \dfrac{P}{P_0}\right] = 6\left[1.005 \ln \dfrac{600}{300} - 0.287 \ln \dfrac{5}{1}\right]$$

$$= 1.408 \text{ kJ/K}$$

Availability of the system at the given state $= \phi - \phi_o$

$$= (U + P_0 V - T_0 S) - (U_0 + P_0 V_0 - T_0 S_0)$$

$$= (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

$$= mC_V(T - T_0) + P_0(V - V_0) - T_0 \cdot \Delta S$$

$$= 6 \times 0.718 \times (600 - 300) + 100 \times (2.0664 - 5.166) - 300 \times 1.408$$

$$= 560.04 \text{ kJ}$$

Now air is cooled from this state to surrounding conditions at constant pressure.

Heat transferred at constant pressure from air for a small $dT$ change in temperature, $dq = mC_p dT$

Maximum work (reversible) done possible with this heat transfer

$$= \eta_{\text{Carnot}} \cdot dq = \left(1 - \dfrac{T_0}{T}\right) mC_p dT$$

∴ Availability of air in this process = Maximum reversible work possible

$$= \int_{T_0}^{T}\left(1 - \dfrac{T_0}{T}\right) mC_p dT = \int_{300}^{600}\left(1 - \dfrac{T_0}{T}\right) \cdot (6 \times 1.005) dT = 555.1 \text{ kJ}$$

**Alternate method:**

Availability $\phi = mC_v(T - T_0) + P_0(V - V_0) - T_0\Delta S$

$$= 6 \times 0.718 \times 300 + 500\left(2.0664 - \frac{2.0664}{2}\right) - \left[300 \times mC_p \ln\frac{600}{300}\right]$$

$$= 1292.4 + 516.6 - 1253.90$$

$$\phi = 555.1 \text{ kJ}$$

**Solution : 27**

Heat lost by steel balls + Heat supplied by motor = Heat gained by oil

$$mc(T_1 - T_2) + \frac{W \times t \times 60}{1000} = m_0 c_0 (T_2 - T)$$



$W = 200$ W for 20 min

$T_1 = 200°C$
$m = 50$ kg
$c = 0.45$

$T = 30°C$
$T_2 = 40°C$
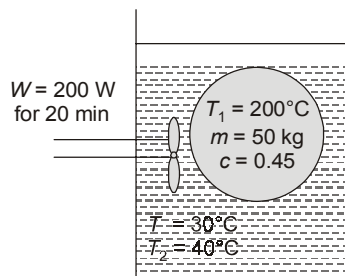
$$50 \times 0.45 (200 - 40) + \frac{200 \times 20 \times 60}{1000} = m_0 \times 2.8 \times (40 - 30)$$

or $m_0 = 137.14 \text{ kg}$

Entropy change of steel ball: $(\Delta S)_{ball} = mc\log_e\frac{T_2}{T_1} = 50 \times 0.45 \times \log_e\frac{313}{473} = -9.29 \text{ kJ/K}$

Entropy change of oil: $(\Delta S)_{oil} = m_0 c_0 \ln\left[\frac{T_2}{T}\right]$

$$(\Delta S)_{oil} = 137.14 \times 2.8 \times \log_e\frac{313}{303} = 12.47 \text{ kJ/K}$$

$$(\Delta S) = (\Delta S)_{soil} + (\Delta S)_{ball}$$

Entropy generated $= 12.47 - 9.29 = 3.18 \text{ kJ/K}$

■■■■

# 6 Properties of Pure Substance

**LEVEL 1** Objective Questions

1. (a)
2. (c)
3. (c)
4. (c)
5. (b)
6. (b)
7. (b)
8. (b)
9. (c)
10. (a)
11. (d)

**LEVEL 2** Objective Questions

12. (b)
13. (d)
14. (b)
15. (a)
16. (a)
17. (c)
18. (b)
19. (3138.56)
20. (4158.97)
21. (a)

■■■■

**LEVEL 3** Conventional Questions

**Solution : 22**

Given data:
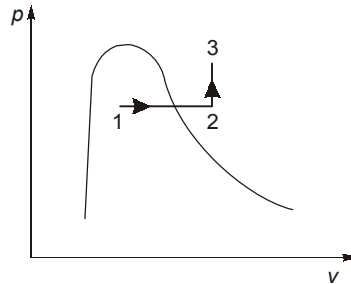


$$x_1 = 0.25$$

Mass of piston: $m_p = 40 \text{ kg}$

$$d = 10 \text{ cm} = 0.10 \text{ m}$$

$$\therefore \quad A = \frac{\pi}{4}d^2 = \frac{3.14}{4} \times (0.10)^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$p_{atm} = 1 \text{ bar} = 100 \text{ kPa}$$

$$y_1 = 1 \text{ cm} = 0.01 \text{ m}$$

$$y_2 = 5 \text{ cm} = 0.05 \text{ m}$$

$$V_1 = Ay_1 = 7.85 \times 10^{-3} \times 0.01 = 7.85 \times 10^{-5} \text{ m}^3$$

$$V_2 = Ay_2 = 7.85 \times 10^{-3} \times 0.05 = 39.25 \times 10^{-5} \text{ m}^3$$

To fix the initial equilibrium state, we have

$$p_1 A = m_p g + p_{atm} A$$

$$p_1 \times 7.85 \times 10^{-3} = \frac{40 \times 9.81}{1000} + 100 \times 7.85 \times 10^{-3}$$

$$p_1 = \frac{40 \times 9.81}{1000 \times 7.85 \times 10^{-3}} + 100$$

$$= 149.98 \text{ kPa} = 1.499 \text{ bar}$$

From saturated steam table,

At

$$p_1 = 1.5 \text{ bar}$$

$$v_f = 0.001053 \text{ m}^3/\text{kg},$$

$$v_g = 1.1593 \text{ m}^3/\text{kg}$$

$$h_f = 467.11 \text{ kJ/kg}$$

$$h_g = 2693.6 \text{ kJ/kg}$$

$$v_1 = v_f + x_1(v_g - v_f) = 0.001053 + 0.25(1.1593 - 0.001053)$$

$$= 0.29061 \text{ m}^3/\text{kg}$$

Thus

$$m = \frac{V_1}{v_1} = \frac{7.85 \times 10^{-5}}{0.29061} = 2.70 \times 10^{-4} \text{ kg}$$

also

$$m = \frac{V_2}{v_2}$$

$\therefore \qquad\qquad\qquad\qquad 2.70 \times 10^{-4} = \dfrac{39.25 \times 10^{-5}}{v_2}$

or $\qquad\qquad\qquad\qquad\qquad v_2 = 1.4537 \text{ m}^3/\text{kg}$

It may be noted that $v_g$ at 1.5 bar $= 1.1593 \text{ m}^3/\text{kg}$

Thus the vapour is in a super heated state when the piston touches the stops. The process is shown on the p-v diagram.

The work done during the constant pressure process,

$$W_{1-2} = p_1(V_2 - V_1) = 149.98 \times (39.25 \times 10^{-5} - 7.85 \times 10^{-5})$$
$$= 0.04709 \text{ kJ}$$

Net heat supplied, $\qquad Q =$ Heat supplied at constant pressure + heat supplied at constant volume

$$= (U_2 - U_1) + W_{1-2} + (U_3 - U_2) + W_{2-3}$$
$$= U_2 - U_1 + W_{1-2} + U_3 - U_2 + 0$$
$$= W_{1-2} + U_3 - U_1 = W_{1-2} + m(u_3 - u_1)$$

where $\qquad u_1 = u_f + x_1(u_g - u_f)$
$$= h_f - p_1 v_f + x_1(h_g - p_1 v_g - h_f + p_1 v_f)$$
$$= 467.11 - 149.98 \times 0.001053 + 0.25(2693.6 - 149.98 \times 1.1593$$
$$- 467.11 + 149.98 \times 0.001053) = 980.14 \text{ kJ/kg}$$

From superheated steam table,

At $\qquad\qquad\qquad p_3 = 3$ bar and $v_3 = v_2 = 1.4537 \text{m}^3/\text{kg}$

By interpolation, $\qquad h_3 = 3703.2 + \dfrac{(3927.1 - 3703.2)(1.4537 - 1.3414)}{1.4957 - 1.3414}$

$$= 3866.15 \text{ kJ/kg}$$

$\therefore \qquad\qquad\qquad u_3 = h_3 - p_3 v_3$
$$= 3866.15 - 300 \times 1.4537 = 3430.04 \text{ kJ/kg}$$

Thus $\qquad\qquad\qquad Q = W_{1-2} + m(u_3 - u_1)$
$$= 0.04709 + 2.70 \times 10^{-4}(3430.04 - 980.14)$$
$$\mathbf{= 0.7085 \text{ kJ}}$$

## Alternate:

$\qquad\qquad\qquad\qquad h_1 = 467.11 + 0.25[2693.6 - 467.11] = 1023.73 \text{ kJ/kg}$

$\qquad\qquad\qquad\qquad h_3 = 3866.15 \text{ kJ/kg}$

Total heat transfer $= Q_{1-2} + Q_{2-3}$
$$= [(h_2 - h_1) - v\Delta p]_{1\text{-}2} + [(h_3 - h_2) - v\Delta p]_{2\text{-}3}$$
$(\Delta p)_{1\text{-}2} = 0$
$$= (h_3 - h_1) - [v\Delta p]_{2\text{-}3}$$
$$= (3866.15 - 1023.73) - 1.4537(300 - 149.98)$$
$$= 2624.336 \text{ kJ/kg}$$

$$Q_{1\text{-}3} = m \times q_{1\text{-}3}$$
$$Q_{1\text{-}3} = 2624.336 \times 2.7 \times 10^{-4}$$
$$Q_{1\text{-}3} = 0.7085 \text{ kJ}$$

**Solution : 23**

Given data: At initial condition: $V_1 = 0.024 \text{ m}^3$; $p_1 = 700$ kPa; $T_1 = 95°\text{C} = (95 + 273)\text{K} = 368\text{K}$

At state 2,

$$p_2 = 70 \text{ kPa} = p_3$$
$$T_3 = T_1 = 368 \text{ K}$$
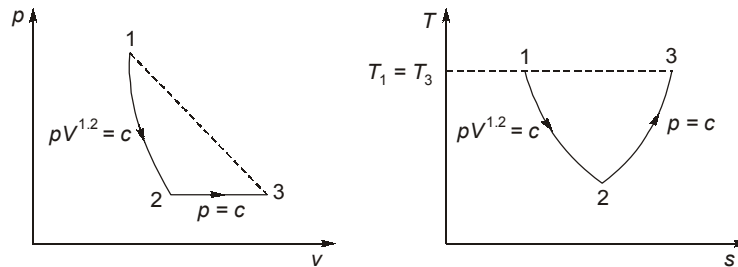


For polytropic process 1–2

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\frac{T_2}{368} = \left(\frac{70}{700}\right)^{\frac{1.2-1}{1.2}} = (0.1)^{0.166} = 0.68129$$

or 
$$T_2 = 0.68129 \times 368 = 250.72\text{K}$$
$$p_1 V_1^{\,n} = p_2 V_2^{\,n}$$

or 
$$\left(\frac{V_2}{V_1}\right)^n = \frac{p_1}{p_2}$$

or 
$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{1/n}$$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/n} = 0.024\left(\frac{700}{70}\right)^{1/1.2} = 0.1635 \text{ m}^3$$

$$R = c_p - c_v = 1.047 - 0.775 = 0.272 \text{ kJ/kgK}$$

For constant pressure process 2–3

$$\frac{V_3}{T_3} = \frac{V_2}{T_2}$$

$$\frac{V_3}{368} = \frac{0.1635}{250.72}$$

or 
$$V_3 = 0.2399 \text{ m}^3$$

Applying equation of state at state -1

$$p_1 V_1 = mRT_1$$
$$700 \times 0.024 = m \times 0.272 \times 368$$

or
$$m = 0.1678\, \text{kg}$$

Work done for process 1–2 :
$$W_{1-2} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{0.1678 \times 0.272(368 - 250.72)}{1.2 - 1} = 26.76\, \text{kJ}$$

Applying first law for process 1–2 :
$$Q_{1-2} = dU + W_{1-2}$$
$$= mc_v(T_2 - T_1) + W_{1-2}$$
$$= 0.1678 \times 0.775(250.72 - 368) + 26.76 = 11.51\, \text{kJ}$$

$$S_2 - S_1 = mc_p \log_e \frac{T_2}{T_1} - mR \log_e \frac{p_2}{p_1}$$

$$= 0.1678 \times 1.047 \log_e \frac{250.72}{368} - 0.1678 \times 0.272 \log_e \frac{70}{700}$$

$$= -0.0674 - (-0.10509) = 0.03769\, \text{kJ/K}$$

Work done for process 2–3,
$$W_{2-3} = p_2(V_3 - V_2)$$
$$= 70(0.2399 - 0.1635) = 5.348\, \text{kJ}$$

$$Q_{2-3} = mc_p(T_3 - T_2)$$
$$= 0.1678 \times 1.047(368 - 250.72) = 20.604\, \text{kJ}$$

$$S_3 - S_2 = mc_p \log_e \frac{T_3}{T_2} = 0.1678 \times 1.047 \times \log_e \frac{368}{250.75}$$

$$= 0.06742\, \text{kJ/K}$$

Work done for whole process,
$$W_{1-2-3} = W_{1-2} + W_{2-3} = 26.76 + 5.35 = 32.11\, \text{kJ}$$

Heat transfer for whole process,
$$Q_{1-2-3} = Q_{1-2} + Q_{2-3} = 11.51 + 20.604 = 32.114\, \text{kJ}$$

Change in entropy of whole process,
$$\Delta S = (S_2 - S_1) + (S_3 - S_2)$$
$$= 0.03769 + 0.06742 = 0.10511\, \text{kJ/K}$$

## Solution : 24

From the superheated steam table
At $p$ = 1 bar & $T_1$ = 150°C, $h_1$ = 2776.4 kJ/kg,
At $p$ = 1 bar and $T_2$ = 200°C, $h_2$ = 2875.3 kJ/kg,
$h_2 = h_1 + c_{ps}(T_2 - T_1)$
2875.3 = 2776.4 + $c_{ps}$(200 – 150)
or $c_{ps}$ = 1.978 kJ/kgK

## Solution : 25

Total volume of the shell $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(0.3)^2 = 0.11309\, \text{m}^3$

$\therefore$ Volume of vapour and that of water,

$$V_g = V_f = \frac{0.11309}{2} = 0.056545 \text{ m}^3$$

From steam table,
At $t = 300°C$

$$v_f = 0.001404 \text{ m}^3/\text{kg}$$
$$v_g = 0.0216 \text{ m}^3/\text{kg}$$

$\therefore$ Mass of vapour ; $\quad m_g = \dfrac{v_g}{v_g} = \dfrac{0.056545}{0.02167} = 2.6094 \text{ kg}$

Mass of water; $\quad m_f = \dfrac{v_f}{v_f} = \dfrac{0.056545}{0.001404} = 40.274 \text{ kg}$

## Solution : 26

From the steam table,

At $p = 5$ bar

$v_f = 0.001093 \text{ m}^3/\text{kg}$, $v_g = 0.375 \text{ m}^3/\text{kg}$, $h_f = 640.2 \text{ kJ/kg}$, $h_g = 2748.7 \text{ kJ/kg}$

$V = 0.25 \text{ m}^3$, $x = 0.85$, $v = v_f + x(v_g - v_f) = 0.001093 + 0.85(0.375 - 0.001093) = 0.31891 \text{ m}^3/\text{kg}$

$$\text{Mass of steam, } m = \frac{V}{v} = \frac{0.25}{0.31891} = 0.78392 \text{ kg}$$

$$\text{Mass of 1 m}^3 \text{ of steam} = \frac{1}{v} = \frac{1}{0.31891} = 3.135 \text{ kg}$$

Enthalpy of 1 kg of steam,

$$h = h_f + x(h_g - h_f)$$
$$= 640.2 + 0.85(2748.7 - 640.2) = 2432.42 \text{ kJ/kg}$$

$\therefore$ Total heat content of steam, $\quad H = 3.135 \times 2432.42$
$$= 7625.63 \text{ kJ}$$

## Solution : 27

Specific enthalpy at inlet of condenser,

$$h_1 = h_{f1} + x h_{fg}$$
$$= 159.2 + 0.95 \times 2411.5$$
$$= 2450.125 \text{ kJ/kg}$$

Specific enthalpy at exit of condenser,

$$h_2 \simeq h_f \text{ at } 30°C$$
$$h_2 = 125.8 \text{ kJ/kg}$$

Applying energy balance equation,

$$\text{Heat lost by steam} = \text{Heat gained by water}$$
$$m_s(h_1 - h_2) = m_w c_{pw} \times (\Delta T)_w$$
$$2500(2450.125 - 125.8) = m_w \times 4.186 \times 10$$

or $$m_W = \frac{2500(2450.125 - 125.8)}{4.186 \times 10}$$

Supply rate of cooling water, $m_w$ = 138815.39 kg/hr

**Solution : 28**

**Given data:** $m = 0.1$ kg; $P_i = 3$ bar; $P_f = 10$ bar; $T_i = -10°C = (-10 + 273)$ K = 263 K

$T_f = 30°C = (30 + 273)$ K = 303 K; $v_i = 0.001002$ m³/kg; $v_f = 0.1321$ m³/kg

Initial total volume: $V_i = mv_i = 1.002 \times 10^{-4}$ m³

Final total volume: $V_f = mv_f = 0.1321 \times 0.1 = 1.321 \times 10^{-2}$ m³

$U_i = mu_i = 0.1 \times 134 \times 10^3 = 1.34 \times 10^4$ J

$U_f = 1347 \times 10^3 \times 0.1 = 1.347 \times 10^5$ J

also $P = a + bV$

$3 \times 10^5 = a + b \times 1.002 \times 10^{-4}$

$10 \times 10^5 = a + b \times 1.32 \times 10^{-2}$

$$b = \frac{7 \times 10^5}{(132.1 - 1.002) \times 10^{-4}} = 5.34 \times 10^7$$

$a = 2.95 \times 10^5$

Applying Ist law of thermodynamics,

$$\delta Q = dU + \delta W$$

or $$\int \delta Q = \int du + \int P dv \quad \text{(Assuming reversible process)}$$

$$Q = (u_f - u_i) + \int_{V_i}^{V_f}(a + bv)dv$$

$$= (1.347 \times 10^5 - 1.34 \times 10^4) + a(v_f - v_i) + \frac{b(v_f^2 - v_i^2)}{2}$$

$$= 121.3 \times 10^3 + 3.8674 \times 10^3 + 4.659 \times 10^3 = 129.82 \text{ kJ}$$

Energy accumulated = 50 W

Total time elapsed $= \frac{Q}{50} = \frac{129.82 \times 10^3}{50} = 43.27$ minutes

Total heat energy infilerated = 129.82 kJ

Total time elapsed = 43.27 minutes

∎∎∎∎

# 7 Thermodynamic Relations and Clapeyron Equation

## LEVEL 1 Objective Questions

1. (a)
2. (b)
3. (c)
4. (c)
5. (b)
6. (a)
7. (c)
8. (b)
9. (c)
10. (c)

## LEVEL 2 Objective Questions

11. (b)
12. (c)
13. (d)
14. (c)
15. (d)
16. (2443.248)
17. (1000)
18. (d)
19. (c)

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## LEVEL 3 Conventional Questions

**Solution : 20**

For incompressible substance like liquids or solids, the specific volume (or density) is constant and the specific internal energy assumed to vary only with temperature.

Since the specific internal energy of an incompressible substance depends only on temperature, the specific heat $c_v$ is also a function of temperature alone.

$$c_v(T) = \frac{du}{dT} \qquad\qquad ..(i)$$

Although the specific volume is constant and internal energy depends on temperature only, enthalpy varies with both pressure and temperature according to

$$h(T, p) = u(T) + pv \qquad\qquad ...(ii)$$

Differentiating equation (ii) with respect to temperature at constant pressure,

$$\left(\frac{dh}{dT}\right)_p = \left(\frac{du}{dT}\right) + 0$$

$$c_p = c_v$$

where $$c_p = \left(\frac{\partial h}{\partial T}\right)_p \text{ and } c_v = \frac{\partial u}{\partial T}$$

Thus, for an incompressible substance there is no difference between $c_p$ and $c_v$, and both can be represent by the same symbol, $c$.

**Solution : 21**

By definition of Joule-Thomson coefficient,

$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h \qquad\qquad ...(i)$$

Consider $$s = f(T, p)$$

Then $$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \qquad\qquad ...(ii)$$

From Tds relations,

$$dh = Tds + vdp$$

Substituting the value of $ds$ from Eq. (ii) in above equation.

$$dh = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp + vdp$$

$$= T\left(\frac{\partial s}{\partial T}\right)_p dT + \left[v + T\left(\frac{\partial s}{\partial p}\right)_T\right]dp$$

where $$T\left(\frac{\partial s}{\partial T}\right)_p = c_p$$

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{from Maxwell's relation}$$

$$\therefore \qquad dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

For throttling process, $\qquad dh = 0$

$$\therefore \qquad 0 = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

$$-c_p dT = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

or $\qquad \left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{c_p}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right] \qquad \qquad \dots(iii)$

and $\qquad \frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = v \times \left(\frac{-1}{T^2}\right) + \frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_p$

$$\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p = \frac{1}{T^2}\left[-v + T\left(\frac{\partial v}{\partial T}\right)_p\right]$$

or $\qquad -v + T\left(\frac{\partial v}{\partial T}\right)_p = T^2\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right] \qquad \qquad \dots(iv)$

From Eqs. (iii) and (iv), we get

$$\left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$$

or $\qquad \mu_J = \left(\frac{\partial T}{\partial p}\right)_h = \frac{T^2}{c_p}\left[\frac{\partial}{\partial T}\left(\frac{v}{T}\right)_p\right]$

## Solution : 22

The diffedrence in specific heats for any substance is given by

$$c_p - c_v = \frac{\beta^2}{\alpha}Tv$$

$\therefore$ For liquid water,

$$c_p - c_v = \frac{(2 \times 10^{-4})^2 \times 300 \times 0.001003}{4.85 \times 10^{-4} \times 10^{-6}} = 24.816 \text{ J/kgK}$$

$$= 0.0248 \text{ kJ/kgK}$$

**Solution : 23**

For an ideal gas, pv = RT

Coefficient of volume expansion,

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{p}{RT}\times\frac{\partial}{\partial T}\left(\frac{RT}{p}\right) = \frac{p}{RT}\times\frac{R}{p} = \frac{1}{T}$$

and isothermal compressibility,

$$\alpha = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{p}{RT}\right)\times\frac{\partial}{\partial p}\left(\frac{RT}{p}\right)_T$$

$$= -\left(\frac{p}{RT}\right)\times\frac{\partial}{\partial p}\left(\frac{RT}{p}\right)_T = -\left(\frac{p}{RT}\right)\times\left(\frac{-RT}{p^2}\right) = \frac{1}{p}$$

$$\therefore \qquad c_p - c_v = \frac{\beta^2}{\alpha}Tv = \frac{1}{T^2}\times\frac{p}{1}\times Tv = \frac{pv}{T} = R$$

**Alternate method:**

$$dQ = dU + PdV$$
$$dH - VdP = dU + PdV$$
$$mC_p dT - mC_v dT = PdV + VdP$$
$$m(C_P - C_V)dT = d(PV)$$
$$m(C_P - C_V)dT = d(mRT)$$
$$C_P - C_V = R$$

**Solution : 24**

We know that third Maxwell relation,

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \qquad\qquad \text{. . .(i)}$$

When a phase change occurs, the saturated pressure $p_s$ depends on saturated temperature $T_s$ only.
That is, $\qquad\qquad p_s = f(T_s)$

The partial derivative, $\left(\frac{\partial p}{\partial T}\right)_v$ can be written as a total

derivative $\left(\frac{dp}{dT}\right)_{sat}$. The total derivative $\left(\frac{dp}{dT}\right)_{sat}$ is the

slope on saturated curve in p-T diagram at a saturated state, as shown in figure, and it is independent of specific

volume. Thus, the slope $\left(\frac{dp}{dT}\right)_{sat}$ is considered as a

constant during the integration of Eq. (i) between two saturated liquid state f and saturated vapour state g.
Equation (i) is written as
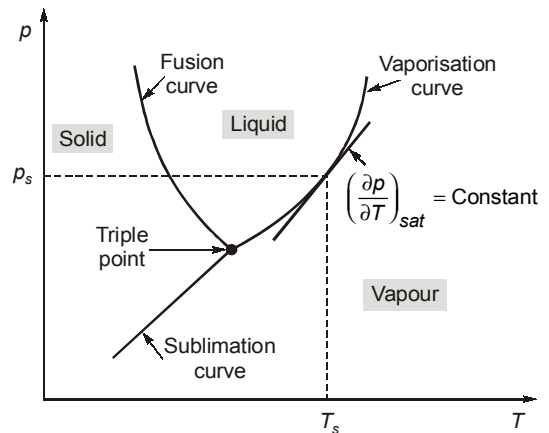


**Phase diagram on p-T planes for water**

$$\left(\frac{dp}{dT}\right)_{sat} dv = ds$$

Integration between saturated liquid state $f$ and saturated vapour state $g$, we get

$$\left(\frac{dp}{dT}\right)_{sat} \int_{v_f}^{v_g} dv = \int_{s_f}^{s_g} ds$$

$$\left(\frac{dp}{dT}\right)_{sat} \left(v_g - v_f\right) = (s_g - s_f)$$

$$\left(\frac{dp}{dT}\right)_{sat} v_{fg} = s_{fg}$$

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{s_{fg}}{v_{fg}}$$

...(ii)

We know that
$$dh = Tds + vdp$$

During the phase change, both temperature and pressure are constant.

*i.e.,* $\quad T = C \quad$ and $\quad p = C$

$\therefore \qquad dh = Tds$

Integration between two saturated states, we get

$$\int_{h_f}^{h_g} dh = T\int_{s_f}^{s_g} ds$$

$$h_g - h_f = T(s_g - s_f)$$

$$h_{fg} = Ts_{fg}$$

where $\quad h_{fg} = h_g - h_f$, specific enthalpy of vaporization

$\quad s_{fg} = s_g - s_f$, change in specific entropy during a phase change process.

$\therefore \qquad\qquad s_{fg} = \frac{h_{fg}}{T}$

Substituting the value of $s_{fg} = \frac{h_{fg}}{T}$ in Eq. (ii), we get

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}}$$

...(iii)

Equation (iii) is called the **Clapeyron equation**. The Clapeyron equation is used to determine the change in enthalpy when phase change takes place during a process *i.e.*, enthalpy of vaporization $h_{fg}$. This equation is valid only for any phase change at constant $T$ and $p$.

For liquid-vapour and solid-vapour phase change process, the Clapeyron equation can be simplified by using some approximations.

At low pressure, $v_g \gg v_f$ for liquid and solid

$\therefore \qquad\qquad v_{fg} = v_g$

$\therefore v_f$ is neglected

If vapour is considered as an ideal gas,

$\therefore \qquad\qquad pv_g = RT$

from equation of state

or
$$v_g = \frac{RT}{p}$$

$\therefore$
$$v_{fg} = \frac{RT}{p}$$

$\because v_g = v_{fg}$

Substituting the values of $v_{fg} = \dfrac{RT}{p}$ in Eq. (iii), we get

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{ph_{fg}}{RT^2}$$

$$\frac{dp}{p} = \left[\frac{h_{fg}}{RT^2}\right]dT \qquad \ldots \text{(iv)}$$

Equation (iv) is called the **Clapeyron-Clausius equation**. This equation is used to calculate the variation of saturated pressure with temperature.

### Solution : 25

Consider variation of saturation pressure with saturation temperature.

During a phase change process, saturation pressure which varies only with saturation temperature.
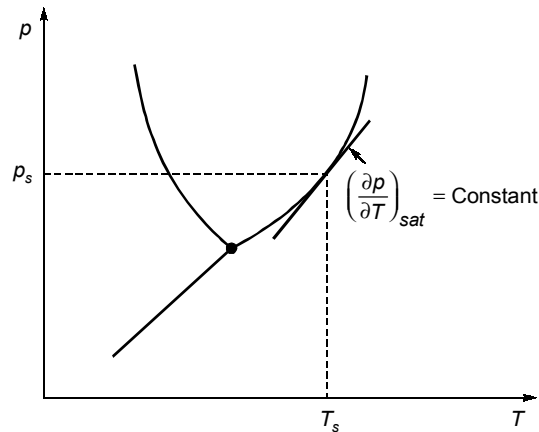


$\therefore$
$$P_{sat} = \int(T_{sat})$$

$\Rightarrow$
$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{dS}{dV}\right)_{sat}$$

From Maxwell's equation, we have:

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

$\therefore$
$$\left(\frac{dP}{dT}\right)_{sat} = \frac{S_g - S_f}{V_g - V_f} \qquad \ldots \text{(i)}$$

Also, $TdS = dh - vdp$

During phase change process, $dp = 0$ (pressure remains constant at $P_{sat}$),

$\therefore$
$$TdS = dh \Rightarrow S_g - S_f = \frac{h_{fg}}{T} \qquad \ldots \text{(ii)}$$

∴ From (i) and (ii), we get: $\left(\dfrac{\partial P}{\partial T}\right)_{sat} = \dfrac{h_{fg}}{T V_{fg}}$

This equation is known as Clapeyron equation. Calculating $h_{fg}$ using Claperon equation

$$\left(\frac{\partial P}{\partial T}\right)_{sat @ -10°C} \cong \left(\frac{\Delta P}{\Delta T}\right)_{sat @ -10°C} = \frac{P_{sat @ 0°C} - P_{sat @ 20°C}}{20}$$

$$= \frac{497.41 - 244.72}{20} = 12.6345 \text{ kPa/K}$$

$$h_{fg} = \left(\frac{\partial P}{\partial T}\right)_{sat} \times T \times V_{fg} = 12.6345 \times 263 \times (0.0654 - 0.0007587)$$

$$= 214.8 \text{ kJ/kg}$$

Tabulated value of the $h_{fg @ -10°C} = 213.136$ kJ/kg,

$$\text{Error \%} = \frac{(214.8 - 213.136)}{213.136} \times 100$$

$$\text{Error \%} = 0.78\%$$

Hence, value of $h_{fg}$ obtained is very close to the tabulated value.

**Solution : 26**

For an ideal gas, internal energy and enthalpy are functions of temperature alone.

∴ $dh = C_p dT$ and $du = C_v dT$

Also, ideal gas equation holds, $PV = RT$

From 1st law of thermodynamics,

$$Q = dU + W = dU + PdV$$

also, $Q = TdS = dU + PdV$ ... (i)

$$H = U + PV$$

⇒ $dH = dU + PdV + VdP$

∴ $TdS = dH - VdP$ ... (ii)

From (i) $TdS = C_V dT + PdV$ ... (iii)

From (ii) $TdS = C_p dT - VdP$ ... (iv)

Combining (iii) and (iv),

$$dT = \frac{TdS - PdV}{C_V} = \frac{TdS + VdP}{C_P}$$

⇒ $T C_p dS - P C_p dV = T C_v dS + V C_v dP$

⇒ $T dS (C_p - C_v) = P C_p dV + V C_v dP$

We know, $C_p - C_v = R$

⇒ $RTdS = PC_p dV + V C_v dP$

⇒ $PVdS = PC_p dV + V C_v dP$

⇒ $dS = C_p \dfrac{dV}{V} + C_v \dfrac{dP}{P}$  Hence, proved

For isentropic process of an ideal gas,

$$dS = 0$$

$$\therefore \qquad dS = C_p \frac{dV}{V} + C_v \frac{dP}{P} = 0$$

$$\Rightarrow \qquad \frac{dP}{dV} = -\frac{C_p}{C_v} \cdot \frac{P}{V} = -\gamma \cdot \frac{P}{V}$$

where γ-ratio of specific heats of ideal gas.

$$\therefore \qquad \frac{dP}{P} = -\gamma \frac{dV}{V}$$

Integrating both sides, we get

$$\ln P = -\gamma \ln V + \ln C, \quad \text{where } C \text{ is a constant} = \ln (V^{-\gamma} \cdot C)$$

$$\Rightarrow \qquad P = V^{-\gamma} \cdot C$$

$$\Rightarrow \qquad PV^{\gamma} = C$$

where $C$ is a constant.

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