2020

RANK Improvement WORKBOOK

Detailed Explanations of
Objective & Conventional *Questions*

Mechanical Engineering

Theory of Machine

MADE EASY Publications

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Publications

1 Mechanisms and Machines

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(a)
4.	(a)
5.	(c)
6.	(a)
7.	(d)
8.	(d)
9.	(d)
10.	(a)
11.	(c)
12.	(d)
13.	(d)

LEVEL 2 Objective Questions

14.	(6)
15.	(1)
16.	(c)
17.	(1)
18.	(1)
19.	(c)
20.	(c)
21.	(b)
22.	(a)
23.	(b)
24.	(b)
25.	(1)
26.	(a)
27.	(180)

LEVEL 3 Conventional Questions

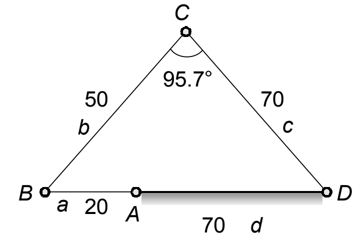
Solution : 28

In this mechanism

Length of the longest linke = 70 mm

Length of the shortest link = 20 mm

Length of other links = 70 and 50 mm



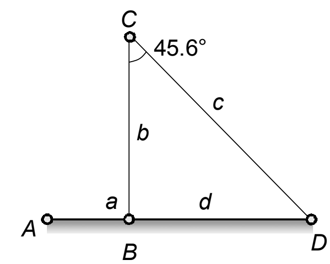
(a)

Since 70 + 20 < 70 + 50, it belongs to the class-I, mechanism.

In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Figure a]

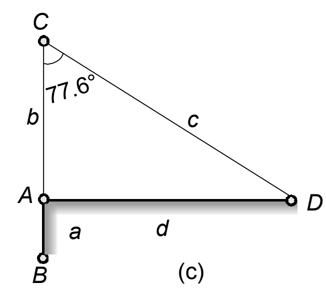
Thus,
$$(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$$
$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70\cos\mu$$
$$8100 = 2500 + 4900 - 7000\cos\mu$$
$$\cos\mu = -0.1$$
$$\mu = 95.7°$$



(b)

Minimum transmission angle is when θ is 0°

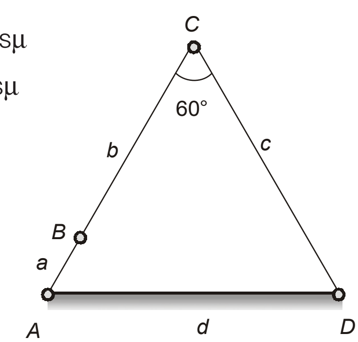
Thus
$$(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70\cos\mu$$
$$2500 = 2500 + 4900 - 7000\cos\mu$$
$$\cos\mu = 0.7$$
$$\mu = 45.6°$$



(c)

The two toggle positions are shown in figure (c) and (d).

Transmission angle for first position,
$$d^2 = (b - a)^2 + c^2 - 2(b - a)c\cos\mu$$
$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70\cos\mu$$
$$4900 = 900 + 4900 - 4200\cos\mu$$
$$\cos\mu = 0.5$$
$$\mu = 60°$$



(d)

(or as all the sides of the triangle of figure are of equal length,

it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60°$

- The above results can also be obtained graphically be drawing the figures to scale and measuring the angles.

Solution : 29

The mechanism has three sub-chains:

(i) *ABC*, a slider-crank chain

(ii) *ABDE*, a four-bar chain

(iii) *AEFG*, a four-bar chain

(DEF is a locked chain as it has only three links.)

- As the length BC is more than the length AB plus the offset of 2 units, AB acts as a crank and can revolve about A.
- In the chain ABDE,

  Length of the longest link = 8

  Length of the shortest link = 4

  Length of the other links = 8 and 6

Since 8 + 4 < 8 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus EF and AG can revolve fully.

- In the chain $AEFG$,

  Length of the longest link = 8

  Length of the shortest link = 4

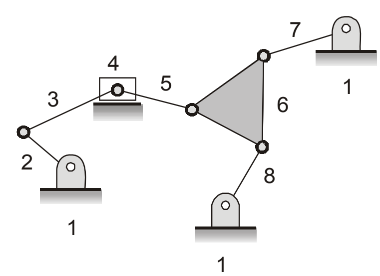
  Length of the other links = 6 and 6

Since 8 + 4 = 6 + 6, it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism and thus EF and AG can revolve fully.

AS DEF is a locked chain with three links, the link EF revolves with the revolving of ED. With the revolving of ED, AG also revolves.

Solution : 30

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Gruebler's criterion. Total number of links = 8



(At the slider, one sliding pair and two turning pairs)

$$F = 3(l-1) - 2j - h$$
$$= 3(8-1) - 2 \times 10 - 0 = 1$$

Thus, it is a mechanism with a single degree of freedom.

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.

$\therefore$ Eeffective degree of freedom $= 3(l-1) - 2j - h - F_r$

$$= 3\,(4-1) - 2 \times 4 - 0 - 1 = 0$$

As the effective degree of freedom is zero, it is a locked system.

■■■■

2 Velocity & Acceleration Analysis

LEVEL 1 Objective Questions

1.	(b)
2.	(0.33)
3.	(d)
4.	(c)
5.	(b)
6.	(d)
7.	(0)
8.	(b)
9.	(1)
10.	(b)
11.	(c)

LEVEL 2 Objective Questions

12.	(67)
13.	(a)
14.	(301.59)
15.	(2.74)
16.	(a)
17.	(3)
18.	(1000)
19.	(45)
20.	(2.69)
21.	(8.0379)
22.	(a)

■■■■

LEVEL 3 Conventional Questions

Solution : 23

$$\omega_{ao} = \frac{2\pi \times 150}{60} = 15.7 \text{ rad/s}$$

$$v_{ao} = w_{ao} \times OA = 15.7 \times 0.1 = 1.57 \text{ m/s}$$

The vector equation for the mechanism OABQ,

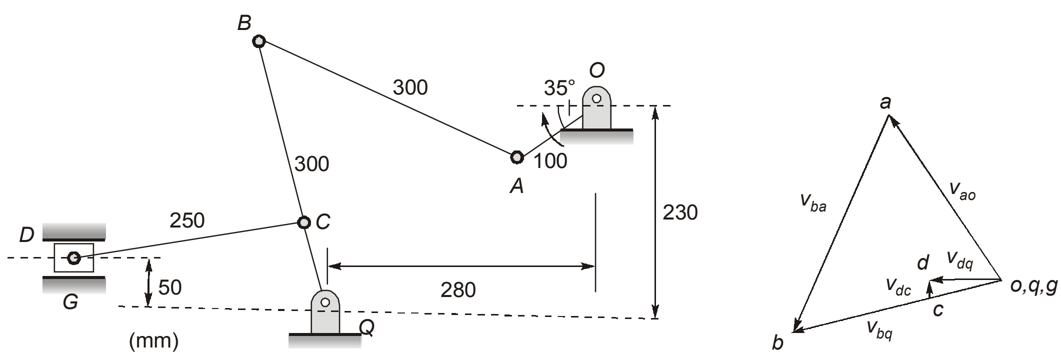
$$V_{bo} = v_{ba} + v_{ao}$$

or $$V_{bq} = v_{ao} + v_{ba}$$

or $$qb = oa + ab$$

Take the vector v_{a0} to a convenient scale in the proper direction and sense

v_{ba} is $\perp AB$, through a;



(a) (b)

v_{bq} is $\perp$ QB, draw a line $\perp$QB through q;

The intersection of the two lines locates the pint b.

Locate the point c on qb such that

$$\frac{qc}{qb} = \frac{100}{300} = 0.3$$

The vector equation for the mechanism QCD,

$$v_{dq} = v_{dc} + v_{cq}$$

or $$v_{dg} = v_{cq} + v_{dc}$$

or $$\text{gd} = \text{qc} + \text{cd}$$

v_{dc} is $\perp$ DC, draw a line $\perp$ DC through c;

For v_{dg}, draw a line through g, parallel to the line of stroke of the slider in the guide G.

The intersection of the two lines locates the point d.

(i) The velocity of slider at D,

$$v_d = gd = 0.56 \text{ m/s}$$

(ii) $$\omega_{bq} = \frac{v_{bq}}{QB} = \frac{1.69}{0.3} = 5.63 \text{ rad/s}$$ counter-clockwise

(iii) $$\omega_{ba} = \frac{v_{ba}}{AB} = \frac{1.89}{0.3} = 6.3 \text{ rad/s}$$ counter-clockwise

As both the links connected at B have counter-clockwise angular velocities,

Velocity of rubbing at the crank pin

$$B = (\omega_{ba} - \omega_{bq})r_b = (6.3 - 5.63) \times 0.04 = 0.0268 \text{ m/s}$$

Solution : 24

$$\text{VP} = 2.5 \times 0.24 = 0.6 \text{ m/s}$$

Locate a point Q on AB beneath point P on the slider.

Solve any of the following velocity vector equations,

$$v_{pa} = v_{pq} + v_{qa}$$

or $$v_{qo} = v_{qp} - v_{po}$$

Produce qa to r such that $\frac{ar}{qa} = \frac{AR}{QA}$

Now, $$v_{sa} = v_{sr} + v_{ra}$$

Complete the velocity diagram as indicated by this equation

(i) v_s = gs = 0.276 m/s

(ii) v_{pq} = qp = 0.177 m/s

(iii) $\omega_{rs} = \frac{v_{rs} \text{ or } v_{sr}}{RS} = \frac{0.12}{0.43} = 0.279 \text{ rad/s}$ clockwise

Solution : 25

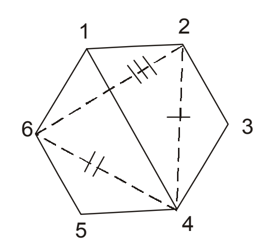
$$\omega_2 = \frac{2\pi \times 150}{60} = 5\pi \text{ rad/s}$$

The velocity of a point A on the link 2 is known.

It is required to find the velocity of a point on the link 6.

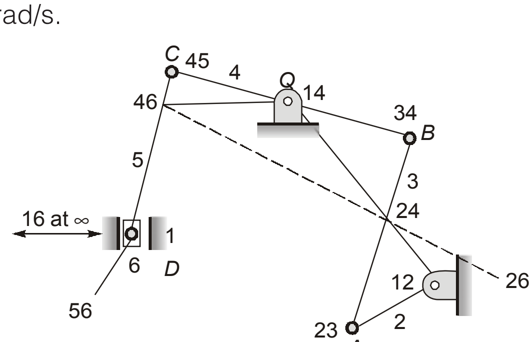
Thus, locate the I-centre 26 as follows:

- Locate I-centres 12, 23, 34, 45, 56, 16 and 14 by inspection.
- Locate 24 which lies on the intersection of 21 – 14 and 23 – 34
- Locate 46 which lies on the intersection of 45 – 56 and 14 – 16 (16 is at ∞)
- Locate 26 which is the intersection 24 – 46 and 21 – 16.



(a)

First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about O with an angular velocity of 5 π rad/s.



(b)

Then, $$v_{26} = \omega_2 \times (12 - 26) = 5\pi \times 0.145 = 2.28 \text{ m/s.}$$

The velocity of the point 26 is in the vertically downward direction if *OA* rotates clockwise.

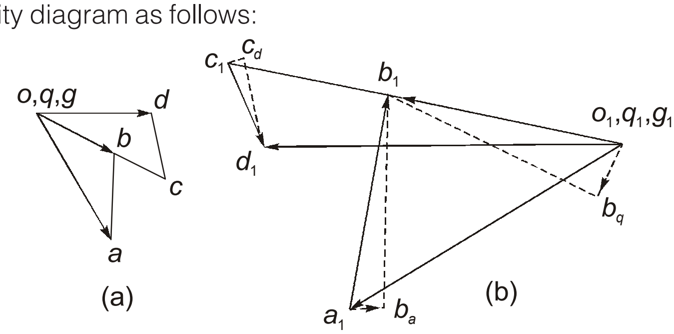
Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the vertical direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

Thus, velocity of the slider, $v_d = v_{26} = 2.28$ m/s

Solution : 26

$$v_a = \frac{2\pi \times 210}{60} \times 0.1 = 2.2 \text{ m/s}$$

Complete the velocity diagram as follows:



- For the four-link mechanism *OABQ,* complete the velocity diagram as usual.
- Locate point *c* on vector *ob* extended so that

$$\frac{cq}{bq} = \frac{CQ}{BQ} = \frac{300}{180} = 1.667$$

- Draw a horizontal line through *g* for the vector v_{dg} and a line $\perp CD$ for the vector v_{dc}, the intersection of the two locates the point *d*.

Thus the velocity diagram is completed.

Set the vector table 4

S.N.	Vector	Magnitude(m/s^2)	Direction	Sense
1.	$f^c_{a\,0}$ or o_1a_1	$\frac{(oa)^2}{OA} = \frac{(2.2)^2}{0.1} = 48.4$	$\parallel OA$	$\rightarrow O$
2.	f^c_{ba} or a_1b_1	$\frac{(ab)^2}{AB} = \frac{(-1.29)^2}{0.3} = 5.55$	$\parallel AB$	$\rightarrow O$
3.	f^t_{ba} or b_1a_1	–	$\perp AB$	–
4.	f^c_{ba} or q_1b_q	$\frac{(bq)^2}{BQ} = \frac{(1.29)^2}{0.18} = 9.25$	$\perp BQ$	–
5	f^t_{bq} or b_qb_1	–	$\parallel CD$	$\rightarrow O$
6.	f^c_{dc} or c_1c_d	$\frac{(cd)^2}{CD} = \frac{(0.01)^2}{0.4} = 2.55$		
7.	f^t_{dc} or c_dd_1	–	$\perp CD$	–
8.	f_{dg} or g_1d_1	–	$\parallel$ to slider motion	–

The acceleration diagram is drawn as follows:

(i) From the pole point o_1 take the first vector o_1a_1.

(ii) Add the second vector by placing its tail at a_1.

(iii) For the third vector f_{ba}^{t}, draw a line $\perp AB$ through b_a

(iv) Add the fourth vector by placing its tail at q_1 and to add the fifth vector f_{ba}^{t}, draw a line $\perp BQ$ through b_q. Intersection of the two lines locates point b_1.

(v) Locate point c_1 on the vector q_1b_1 by extending it so that

$$\frac{c_1q_1}{b_1q_1} = \frac{CQ}{BQ} = \frac{300}{180} = 1.667$$

(vi) Add the vector for centripetal acceleration f_{dc}^{c} of link CD by placing its tail at c_1 and for its tangential component, draw a perpendicular line to it.

(vii) For the vector 8, draw a horizontal line through g, the intersection of this line with the live drawn in (viii) locates point d_1.

This completes the acceleration diagram. Acceleration of slider $D = g_1d_1 = 54.4$ m/s

Angular acceleration on link CD,

$$\alpha_{cd} = \frac{f_{cd}^{t} \text{ or } c_d d_1}{CD} = \frac{13.3}{0.4} = 33.25 \text{ rad/s}^2$$

Solution : 27

As per given data:

Horizontal reciprocating engine mechanism.

Crank, $r = 8$ cm

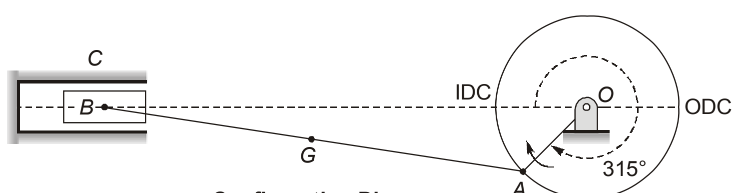
Connecting rod, $l = 36$ cm

Engine speed, $N = 2000$ rpm (CW)

Velocity and acceleration of piston when the crank is 315° from inner dead centre.

Configuration diagram by assuming scale{1 cm = 4 cm}

$OA = 8$ cm ; $AB = 36$ cm



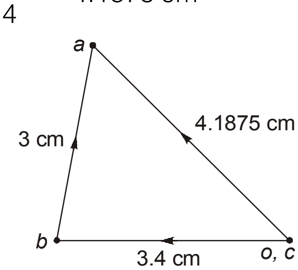
Configuration Diagram

Velocity diagram:

Assuming scale = 1 cm = 4 m/s

Velocity of crank, $OA = r \times \omega = \frac{0.08 \times 2\pi \times N}{60} = \frac{0.08 \times 2\pi \times 2000}{60} = 16.75$ m/s

In diagram, $oa = \frac{16.75}{4} = 4.1875$ cm



Velocity Diagram

From diagram,

$$\text{Velocity piston, } ob = 3.4 \text{ cm} = 3.4 \times 4 \Rightarrow 13.6 \text{ m/s}$$

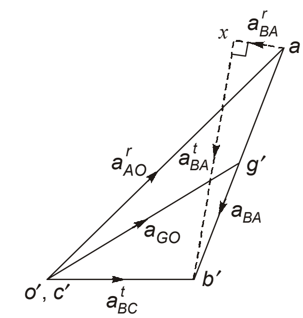
$$\text{Velocity of piston w.r.t. crank} = ba = 3 \text{ cm} \Rightarrow 3 \times 4 = 12 \text{ cm/s}$$

Acceleration diagram:

$$\alpha_{\text{crank}} = 0$$

Assume scale 1 cm = 1000 m/s^2

Point	w.r.t.	Procedure
A	O	$a^r_{AO} = \frac{V^2_{AO}}{AO} = 3.507 \times 10^3$ m/s^2 along $A \to O$ a^t_{AO} $AO \times \alpha_{AO} = 0$ $\perp^{ar}$ to AO
B	A	$a^r_{BA} = \frac{V^2_{BA}}{AB} = \frac{13.6^2}{0.36} = 0.513 \times 10^3$ m/s^2 along $B \to A$ $a^t_{BA} = BA \times \alpha_{BA} =$ unknown $\perp^{ar}$ to BA
B	C	$a^r_{BC} = \frac{V^2_{BC}}{BC} = 0$ along $B \to C$ $a^t_{BC} = BC \times \alpha_{BC} =$ unknown $\perp^{ar}$ to BC



Acceleration Diagram

From acceleration diagram

$$o'a' = 3.507 \text{ cm}$$
$$a'x = 0.513 \text{ cm}$$
$$xb' = 2.6 \text{ cm}$$
$$o'b' = 1.55 \text{ cm}$$
$$a'b' = 2.65 \text{ cm}$$
$$o'g' = 2.35 \text{ cm}$$

The tangential component of acceleration of connecting rod, $xb' = 2.6 \times 10^3$ m/s^2 $= \alpha_{AB} \times AB$

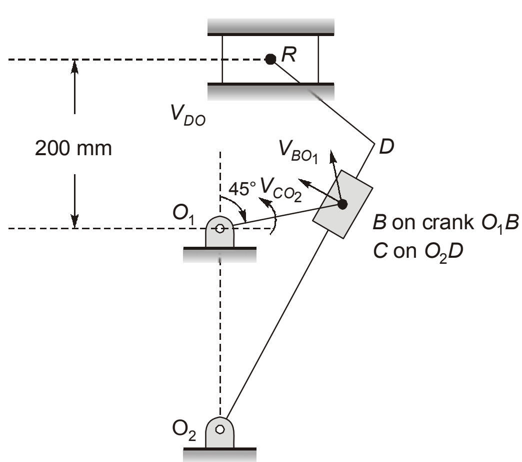
$$\alpha_{AB} = \frac{2.6 \times 10^3}{0.36} = 7.222 \times 10^3 \text{ rad/s}^2$$

The acceleration of piston, $o'b' = 1.55 \times 10^3$ m/s^2.

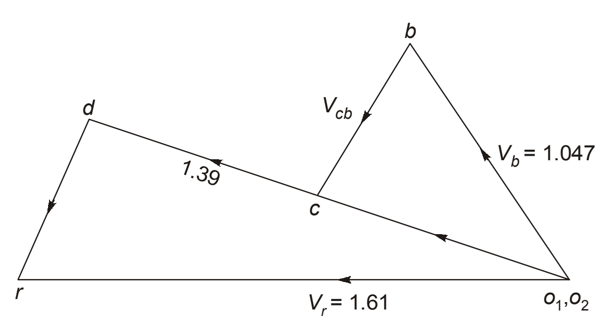
Total acceleration of connecting rod at mid-point,

$$o'g' = 2.35 \times 10^3 \text{ m/s}^2$$

Solution : 28



Space diagram



Velocity diagram

1. **Velocity of Ram R:**
   First of all draw the space diagram or configuration diagram. By assuming suitable scale.
   (i) Since o_1 and o_2 are fixed points therefore these points are marked as one point in velocity diagram, draw vector o_1b perpendicular to O_1B such that

$$o_1b = V_B = \left(\frac{2\pi N}{60}\right)\times O_1B = \left(\frac{2\pi\times 40}{60}\right)\times 0.25 = 1.047 \text{ m/s}$$

   (ii) From point o_2 draw vector $\overrightarrow{o_2c}$ perpendicular to O_2C to represent the velocity of the coincident point c with respect to o_2 or simple velocity of C, and from b draw vector $\overrightarrow{bc}$ parallel to the path of motion of the sliding block (which is along line O_2D) to represent the velocity of c with respect to b (i.e. V_{cb}). The vectors $\overrightarrow{o_2c}$ and bc intersect at c.
   (iii) Since the point d lies on o_2c produced, along line and the ratio $\dfrac{cd}{o_2d}=\dfrac{CD}{O_2D}$ will remain same.
   (iv) Now from point d, draw vector dr perpendicular to dr to represent the velocity of r with respect to d(i.e. V_{rd}) and from point o_1 draw vector $o_1 r$ parallel to the path of motion of R (which is horizontal) to represent velocity of r.
   The vectors dr and o_1r intersect at r.
   By measurement V_r = vector o_1r = 1.61 m/s
2. **Angular velocity of link O_2D**
   From velocity diagram

$$V_{do_2} = V_d = \overrightarrow{o_2d} = 1.39 \text{ m/s}$$

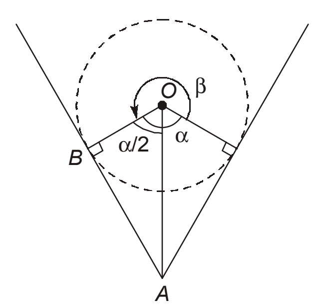
length of link $\quad o_2d = 1250 \text{ mm} = 1.25 \text{ m}$

$$\omega_{do_2} = \frac{o_2d}{O_2D}=\frac{1.39}{1.25} = 1.112 \text{ rad/s anticlockwise about } o_2$$

Solution : 29

$$N_{crank} = 120 \text{ rpm (clockwise)}$$

$$\omega_{crank} = \frac{2\pi\times 120}{60} = 4\pi \text{ rad/s}$$



$$\text{Quick return ratio} > \frac{\alpha}{\beta}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{\beta}{\alpha} = \frac{2}{1}$$

$$360 - \alpha = 2\alpha$$

$$3\alpha = 360°$$

$$\alpha = 120°$$

$$OA = 50 \text{ cm}$$

$$\cos\frac{\alpha}{2} = \frac{OB}{OA}=\frac{OB}{50}$$

$$\cos 60^\circ = \frac{OB}{50}$$

$$\frac{1}{2} = \frac{OB}{50}$$

$$\Rightarrow \quad OB = 25 \text{ cm}$$

Length of crank = 25 cm

Velocity of slider (V_B) = $r_{crank} \times \omega_{crank} = 0.25 \times 4\pi = \pi$ m/s

Maximum velocity of slotted bar in cutting stroke (Mid position)

For maximum velocity position

$$r = \frac{50+25}{100} = 75 \text{ cm} = 0.75 \text{ m}$$

$$V = \omega r \Rightarrow \therefore \quad V = \frac{\pi}{0.75} = 4.1887 \text{ m/s]}$$

■■■■

3 Cams

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(c)
4.	(a)
5.	(a)
6.	(c)
7.	(c)
8.	(c)
9.	(c)
10.	(c)
11.	(b)

LEVEL 2 Objective Questions

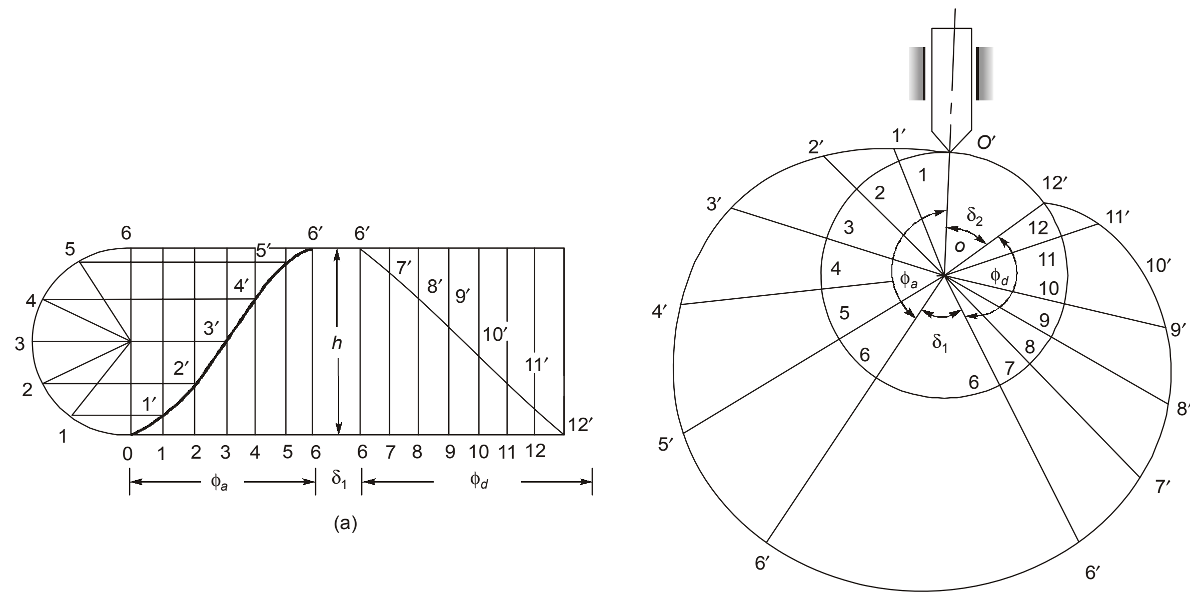
12.	(2.37)
13.	(720)
14.	(0.628)
15.	(1.95)
16.	(d)
17.	(b)
18.	(d)
19.	(d)
20.	(c)
21.	(a)
22.	(c)
23.	(b)
24.	(a)
25.	(b)
26.	(b)
27.	(a)
28.	(d)

■■■■

LEVEL 3 Conventional Questions

Solution : 29

Given data: h = 30 mm, φ_a = 150°, N = 120 rpm, δ_1 = 60°, r_c = 20 mm, φ_d = 100°, δ_2 = (360° – 150° – 100° – 60°) = 50°



(a)

(b)

Draw the displacement diagram of the follower as discussed earlier taking a convenient scale. Construct the cam profile as follows [refer the figure]

(i) Draw the circle with radius r_c.

(ii) If the cam rotates clockwise and the follower remains in vertical direction, the cam profile can be drawn by assuming that the cam is stationary and the follower rotates about the cam in the counter-clockwise direction. From the vertical position, mark angles φ_a, δ_1, φ_d, and δ_2 in the counter-clockwise direction, representing angles of ascent, rest or dwell, descent and rest respectively.

(iii) Divide the angles φ_a and φ_d into same number of parts as is down in the displacement diagram. In this case, each has been divided into 6 equal parts.

(iv) Draw radial lines, O – 1, O – 2, O – 3 etc, O – 1 represents that after an interval of $\varphi_d/6$ of the cam rotation in the clockwise direction it will take the vertical position of O – O'.

(v) On the radial linkes produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e. 1 –1′, 2 – 2′, 3 – 3′, etc.

(vi) Draw a smooth curve passing through O',1′, 2′ . . ., 10′, 11′ and 12′. Draw an arc of radius O-6′ for the dwell period δ_1.

During ascent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

$$V_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a}$$

or
$$v_{max} = \frac{30}{2}\times\frac{\pi\times 12.57}{150\times\frac{\pi}{180}} = 226.3 \text{ mm/s}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{2}\right)^2$$

or
$$f_{max} = \frac{30}{2}\times\left(\frac{\pi\times 12.57}{150\times\frac{\pi}{180}}\right)^2 = 3413 \text{ mm/s}^2 \text{ or } 3.413 \text{ m/s}^2$$

During descent

$$v_{max} = h\frac{\omega}{\varphi_d}$$

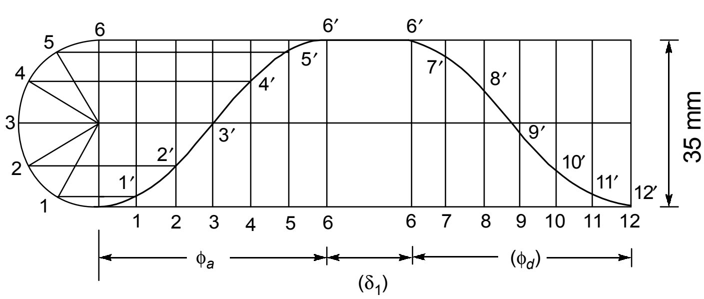
$$v_{max} = 30\times\frac{12.57}{100\times\frac{\pi}{180}} = 216 \text{ mm/s}$$

$$f_{max} = f = 0$$

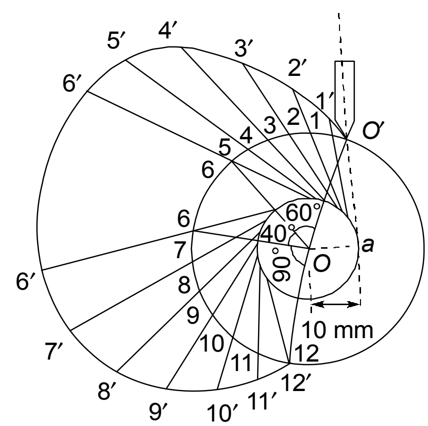
Note that to draw the cam profile, it is not necessary that the interval δ_1 is taken in the displacement diagram. Also, the scales of φ_a and φ_d can be taken different and of any magnitudes.

Solution : 30

Given data : h = 35 mm, φ_a = 60°, N = 150 rpm, δ_1 = 40°, r_c = 25 mm, φ_d = 90°, x = 10 mm



(a)



(b)

Draw the displacement diagram of the follower as discussed earlier. Construct the cam profile as follows :

(i) Draw a circle with radius r_c (= 25 mm).

(ii) Draw another circle concentric with the previous circle with radius x(= 10 mm). If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be a –O′.

(iii) Join O – O'. Divide the circle of radius r_c into four parts as usual with angle j_a, d_1, j_d and d_2 starting from O – O'.

(iv) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3 etc., on the circumference of circle with radius r_c.

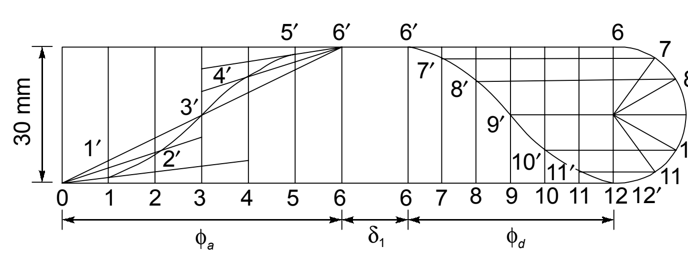
(v) Draw tangents to the circle with radius x from the points 1, 2, 3 etc.
(vi) On the extension of the tangent lines, mark the distances from the displacement diagram.
(vii) Draw a smooth curve through O′, 1′, 2′, etc.
This is the required pitch cure.

During ascent
$$\omega = \frac{2\pi \times 150}{60} = 5\pi \text{ rad/s}$$
$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a}$$
or
$$v_{max} = \frac{35}{2} \times \frac{\pi \times 5\pi}{60 \times \frac{\pi}{180}} = 824.7 \text{ mm/s}$$
$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2$$
or
$$f_{max} = \frac{35}{2} \times \left(\frac{\pi \times 5\pi}{60 \times \frac{\pi}{180}}\right)^2 = 38862 \text{ mm/s}^2 = 38.882 \text{ m/s}^2$$

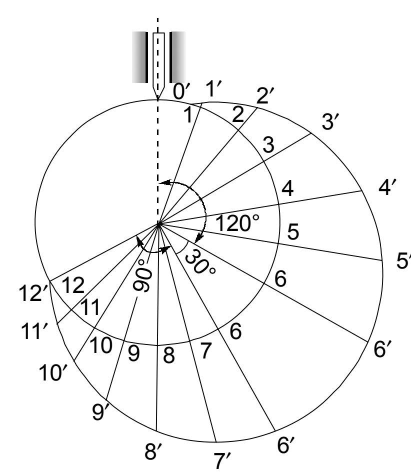
During descent
$$v_{max} = \frac{35}{2} \times \frac{\pi \times 5\pi}{90 \times \frac{\pi}{180}} = 549.8 \text{ mm/s}$$
$$f_{max} = \frac{35}{2} \times \left(\frac{\pi \times 5\pi}{90 \times \frac{\pi}{180}}\right)^2 = 17272 \text{ mm/s}^2 \text{ or } 17.272 \text{ m/s}^2$$

Solution : 31

Given data : h = 30 mm, r_c = 30 mm, φ_a = 120°, δ_1 = 30°, φ_d = 90°, N = 800 rpm, δ_2 = 360° – 120° – 30° – 90° = 120°



(a)



(b)

Draw the displacement diagram of the follower as shown in figure. As the rotation of the cam shaft is counter-clockwise, the cam profile is to be drawn assuming the cam to be stationary and the follwer rotating clockwise about the cam. Construct the cam profile as described below :

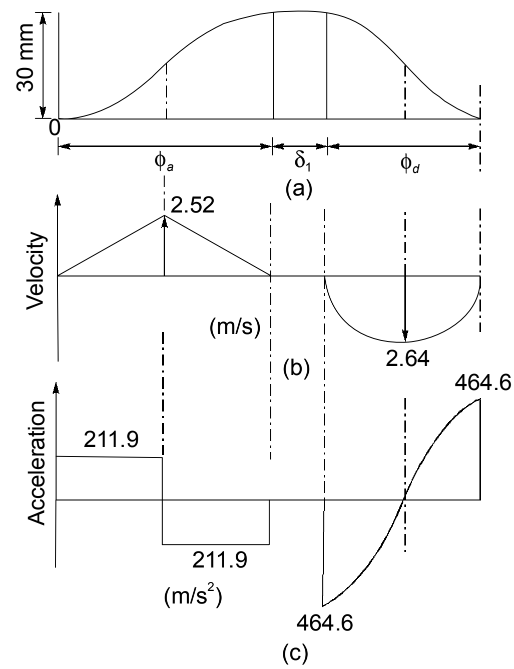
(i) Draw a circle with radius r_c.

(ii) From the vertical position, mark angles φ_a. δ_1, φ_d and δ_2 in the clockwise direction.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, φ_a as well as φ_d have been divided into 6 equal parts.

(vi) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a smooth curve tangential to end points of all the radial lines to obtain the required cam profile.



The displacement diagram is reproduced in figure. The velocity and acceleration diagrams are to be drawn below this figure

$$\omega = \frac{2\pi \times 840}{60} = 88 \text{ rad/s}$$

During ascent

During the ascent period, the acceleration and the deceleration are uniform. Thus, the velocity is linear and is given by

$$v = \frac{4h\omega}{\varphi_a^2}.\theta$$

The maximum velocity is at the end of the acceleration period, i.e., when $\theta = \varphi_{a/2.}$

$$\therefore \qquad v_{max} = 2h\frac{\omega}{\varphi_a} = 2 \times 0.03 \times \frac{88}{\frac{120\pi}{180}} = 2.52 \text{ m/s}$$

The plot of velocity variation during the ascent period is shown in figure

$$f_{uniform} = \frac{4h\omega^2}{\varphi_a^2}$$

or

$$f_{uniform} = \frac{4 \times 0.03 \times 88^2}{\left(\frac{120\pi}{180}\right)^2} = 211.9 \text{ m/s}^2$$

This has been shown in figure.

During descent

During descent, it is simple harmonic motion. The variation of velocity is given by

$$v = \frac{h}{2}\frac{\pi\omega}{\varphi_d}\sin\frac{\pi\theta}{\varphi_d}$$

Maximum value is at

$$\theta = \frac{\varphi_d}{2}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_d} = \frac{0.03}{2}\times\frac{\pi\times 88}{\frac{90\pi}{180}} = 2.64 \text{ mm/s}$$

The plot of velocity variation during the descent period is shown in figure.

The acceleration variation is given by,

$$f = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\frac{\cos\pi\theta}{\varphi}$$

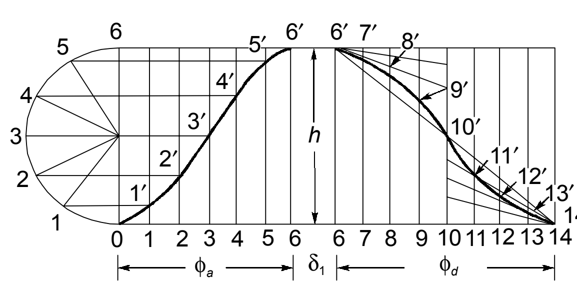
It is maximum at $\theta = 0$ i.e.

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_d}\right)^2 = \frac{0.03}{2}\times\left(\frac{\pi\times 88}{\frac{90\pi}{180}}\right)^2 = 464.6 \text{ m/s}^2$$

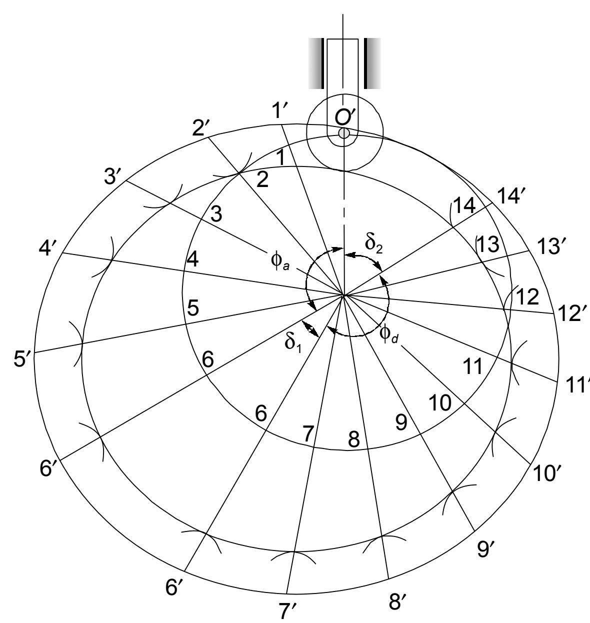
This variation is shown in figure.

Solution : 32

Given data : $h = 30$ mm, $\varphi_a = 120°$, $N = 150$ mm, $\delta_1 = 30°$, $r_c = 25$ mm, $\varphi_d = 150°$, $r_r = 7.5$ mm, $d_2 = 60°$



(a)



(b)

Draw the displacement diagram of the follower as shown in figure. Construct the cam profile as described below.

(i) Draw the circle with radius $(r_c + r_r)$.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the counter-clockwise direction (assuming that the cam is to rotate in the clockwise direction).

(iii) Divide the angle φ_a and φ_d into the same number of parts as is done in the displacement diagram. In this case, φ_a has been divided into 6 equal parts whereas φ_d is divided into 8 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a series of arcs of radii equal to r_r, as shown in the diagram from the points 1′, 2′, 3′ etc.

(vi) Draw a smooth curve tangential to all the arcs which is the required cam profile.

During the descent period, the acceleration and the deceleration are uniform. Therefore, the maximum velocity is at the end of the acceleration period.

$$v_{max} = 2h\frac{\omega}{\varphi_d} = 2\times 30\times\frac{\frac{2\pi\times 150}{60}}{150\times\frac{\pi}{180}} = 360 \text{ m/s}$$

$$f_{max} = f_{uniform} = \frac{4h\omega^2}{\varphi_d^2}$$

$$f_{max} = \frac{4\times 30\times\left(\frac{2\pi\times 150}{60}\right)^2}{\left(150\times\frac{\pi}{180}\right)^2} = 4320 \text{ mm/s}^2 \text{ or } 4.32 \text{ m/s}^2$$

Solution : 33

Given data : e = 40 mm, m = 3 kg, s = 5 N/mm = 5000 N/m, P = 60 N + mg = (60 + 3 × 9.81)N

Consider the rotation of the cam through angle θ,

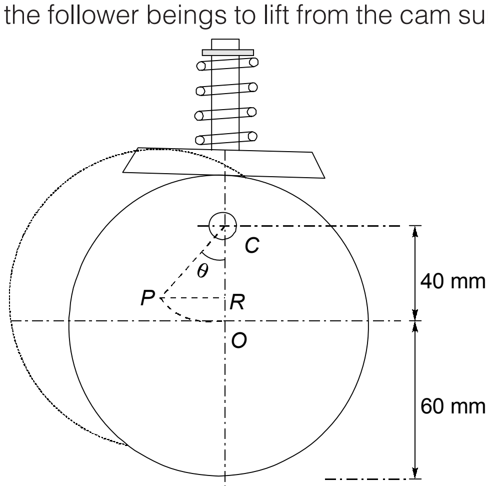
Now $$x = 40 - 40\,e\cos\theta = 40(1-\cos\theta)$$

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = 40\omega\sin\theta$$

$$\ddot{x} = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = 40\omega^2\cos\theta$$

which is the required expression for acceleration of the cam follower system.

To find the speed at which the follower beings to lift from the cam surface or the jump speed,



$$\omega = \sqrt{\frac{2se+P}{me}}$$

$$= \sqrt{\frac{2\times5000\times0.04+60+3\times9.81}{3\times0.04}}$$

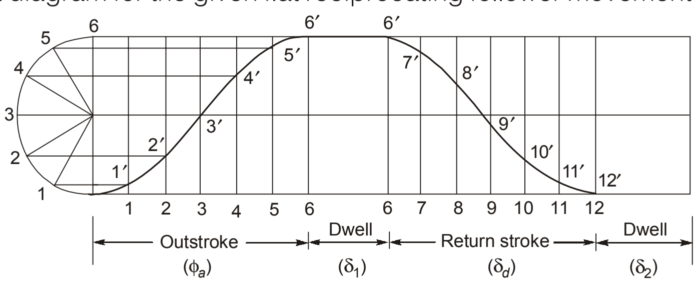
$$= \sqrt{4078.6} = 63.86 \text{ rad/s}$$

or $$\frac{2\pi N}{60} = 63.86$$

or $$N = 609.9 \text{ rpm}$$

Solution : 34

The displacement diagram for the given flat reciprocating follower movement will be as:

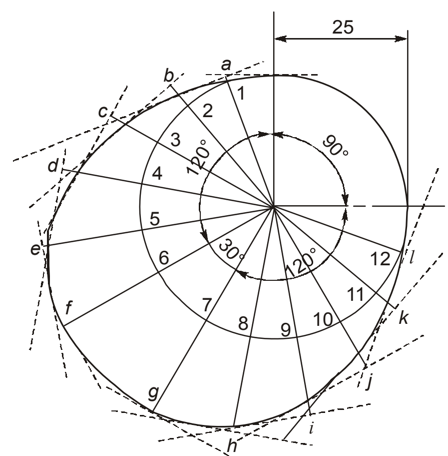


Given: ϕ_a = 120°, h(lift) = 20 mm, δ_1 = 30°, ϕ_d = 120°, δ_2 = 90°

Motion is SHM both during outward and inward stroke, minimum radius of cam (r_c) = 25 mm.

Construction:

1. First draw the displacement diagram now construct the cam profile as follows
2. Draw a circle with radius (r_c = 25 mm)
3. Take angles (ϕ_a, δ_1, ϕ_d and δ_2) in the counter clockwise direction if the cam rotation is assumed clockwise
4. Divide ϕ_a and ϕ_d into same number of parts as in the displacement diagram. (Example take 6 equal parts)
5. Draw radial lines (0-1, 0-2, 0-3,, etc. ....)
6. On the radial lines produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e., 1 - 1′, 2 - 2′, 3 - 3′, etc.
7. Draw the follower in all the positions by drawing perpendiculars to the radial lines at 1′, 2′, 3′, etc. In all the positions, the axis of the follower passes through centre O
8. Draw a curve tangential to the flat faces of the follower representing the cam profile.



Solution : 35

The equation for cycloidal follower motion is:

$$x = h\left(\frac{\pi}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)$$

Angle of ascent during cam rise,

$$\beta = 120° = \frac{120\pi}{180} = \frac{2\pi}{3}\text{ rad}$$

Cam angle (instantaneous) in the ascending stroke,

$$\theta = 100° = \frac{100\times\pi}{180} = \frac{5\pi}{9}\text{rad}$$

Rise, h = 10 mm

Spring preload, $F_s = K x_0 = 15 \times 2.5 = 37.5$ N

$$x = 10\left(\frac{5\pi}{9}\times\frac{3}{2\pi} - \frac{1}{2\pi}\sin\left[2\pi\times\frac{5\pi}{9}\times\frac{3}{2\pi}\right]\right) = 10\left(\frac{5}{6} - \frac{1}{2\pi}\sin\frac{5\pi}{3}\right)$$

$$= 10\left(0.833 + \frac{0.866}{2\times 3.14}\right) = 10(0.833 + 0.1378)$$

$$x = 10 \times 0.97 = 9.7 \text{ mm}$$

$$\ddot{x} = 2h\pi\left(\frac{\omega}{\beta}\right)^2 \sin\left(\frac{2\pi\theta}{\beta}\right)$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times 1000}{60} = 104.72 \text{ rad/s}$$

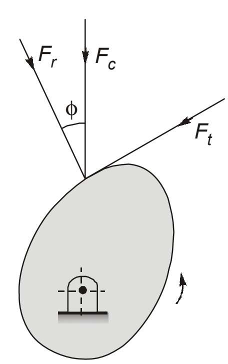
$$\frac{\theta}{\beta} = \frac{5\pi}{9}\times\frac{3}{2\pi} = \frac{5}{6}$$

So, $$\ddot{x} = 2\times 0.01\times 3.14\left(\frac{104.72\times 3}{2\times 3.14}\right)^2 \sin\left(\frac{5\pi}{3}\right) = -136.1 \text{ m/s}^2$$

Velocity of cam follower at 100° cam action angle is given by;

$$\dot{x} = \frac{h\omega}{\beta}\left[1-\cos\left(\frac{2\pi\theta}{\beta}\right)\right] = 0.01\times\frac{104.72}{3}\times 2\pi\left[1-\cos\frac{5\pi}{3}\right] = 1.1 \text{ m/s}$$

Net cam force, $F_c = m\ddot{x} + kx + kx_0$



$$F_c = 1.8 \times (-136.1) + 15(9.7 + 2.5)$$

$$= -244.98 + 183 = -61.98 \text{ N}$$

[Negative sign of F_c is for downward follower motion]

Assuming pressure angle ϕ as 10°

Radial component of cam force is given by;

$$F_r = F_c \cos\phi = 61.98 \times \cos 10° = 61 \text{ N}$$

Power Input = Power output

$$\text{Torque} \times \omega = F_c\, \dot{x}$$

$$\text{Torque} = \frac{61.98\times 1.1}{104.72} = 0.651 \text{ N.m}$$

Solution : 36

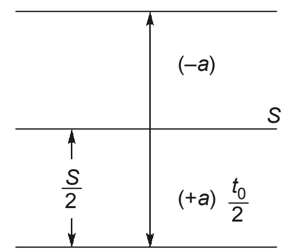
It is uniform acceleration and retardation motion θ_0 (outstrike angle)

$$= 60° = \frac{\pi}{3} \text{ rad}$$

$$N_{cam} = 300 \text{ rpm}$$

$$\omega_{cam} = \frac{2\pi \times 300}{60} = 5\pi \times 2 = 10\pi \text{ rad/s}$$

$$s = 20 \text{ mm} = 0.020 \text{ m}$$



In this motion:

$$(V_0)_{max} = 2(V_0)_{mean}$$

$$= 2\left(\frac{\omega . s}{\theta_0}\right) = \frac{2 \times 10\pi \times 0.020}{\pi / 3}$$

$$= 6 \times 10 \times 0.020 = 6 \times 0.20$$

$$= 1.2 \text{ m/s}$$

In first half of stroke

$$(V_0)_{max} = 0 + a \times \frac{t_0}{2}$$

$$\frac{2\omega \cdot s}{\theta_0} = \frac{a \times \theta_0}{\omega \times 2}$$

$$a = \frac{4\omega^2 \cdot s}{\theta_0^2} = \frac{4 \times 10\pi \times 10\pi \times 0.020}{\frac{\pi^2}{9}} = 8 \times 9 = 72 \text{ m/s}^2$$

■■■■

4 Gear and Gear Train

LEVEL 1 Objective Questions

1.	(c)
2.	(7.48)
3.	(91)
4.	(360)
5.	(a)
6.	(c)
7.	(b)
8.	(d)
9.	(d)
10.	(d)
11.	(c)
12.	(39207.076)

LEVEL 2 Objective Questions

13.	(c)
14.	(c)
15.	(b)
16.	(a)
17.	(b)
18.	(b)
19.	(b)
20.	(a)
21.	(a)
22.	(c)
23.	(0.86)
24.	(10.29)
25.	(9.04)
26.	(c)
27.	(b)
28.	(a)
29.	(c)
30.	(b)
31.	(c)
32.	(b)
33.	(b)
34.	(a)
35.	(d)
36.	(c)
37.	(d)
38.	(70)
39.	(d)
40.	(18)
41.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 42

1 is the gear wheel and 2 is the pinion.

$\varphi = 20°$; $T = 40$; $N_p = 600$ mm; $t = 24$; $m = 4$ mm, Addendum = 1 module = 4 mm

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}; R_a = 80 + 4 = 84 \text{ mm}$$

$$r = \frac{mT}{2} = \frac{4 \times 24}{2} = 48 \text{ mm}; r_a = 48 + 4 = 52 \text{ mm}$$

$$N_g = N_p \times \frac{t}{T} = 600 \times \frac{24}{40} = 360 \text{ rpm}$$

(i) Let pinion (gear 2) be the driver. The tip of the driving wheel is in contact with a tooth of the driven wheel at the end of engagement. Thus, it is required to find the path of recess which is obtained from the dimensions of the driving wheel.

$$\text{Path of recess} = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$= \sqrt{(52)^2 - 48(\cos 20°)^2} - 48\sin 20° = 9.458 \text{ mm}$$

$$\text{Velocity of sliding} = (\omega_p + \omega_g) \times \text{Path of recess}$$

$$= 2\pi(N_P + N_g) \times 9.458 = 2\pi\,(600 + 360) \times 9.458$$

$$= 57049 \text{ mm/min} = 950.8 \text{ mm/s}$$

(ii) In case the gear wheel is the driver, the tip of the pinion will be in contact with the flank of a tooth of the gear wheel at the beginning of contact. Thus, it is required to find the distance of the point of contact from the pitch point, i.e. path of approach. the path of approach is found from the dimensions of the driven wheel which is again pinion.

Thus, $$\text{Path of approach} = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi = 9.458 \text{ mm},$$

as before and velocity of sliding = 950.8 mm/s

Thus, it is immaterial whether the driver is the gear wheel or the pinion, the velocity of sliding is the same when the contact is at the tip of the pinion.

The maximum velocity of sliding will depend upon the larger path considering any of the wheels to be the driver.

Consider pinion to be the driver.

$$\text{Path of recess} = 9.458 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi = \sqrt{(84)^2 - (80\cos 20°)} - 80\sin 20°$$

$$= 10.117 \text{ mm}$$

This is also the path of recess if the wheel becomes the driver

$$\text{Maximum velocity of sliding} = (w_p + w_g) \times \text{Maximum path}$$

$$= 2p(600 + 360) \times 10.117 = 61024 \text{ mm/min}$$

$$= 1017.1 \text{ mm/s}$$

Solution : 43

$$R = \frac{mT}{2} = \frac{8 \times 60}{2} = 240 \text{ mm};$$

$$r = \frac{mT}{2} = \frac{8 \times \left(\frac{60}{1.5}\right)}{2} = 160 \text{ mm}$$

Let the pinion be the driver.

Maximum possible length of path of approach = rsinφ

Actual length of path of approach= 0.4 × r sinφ

Similarly, actual length of path of approach of recess = 0.4 R sinφ

Thus, we have

$$0.4r \sin\varphi = \sqrt{R_a^2 - (R\cos\varphi)^2} - R\sin\varphi$$

$$0.4 \times 160 \sin 20° = \sqrt{R_a^2 - (240\cos 20^2)^2} - 240\sin 20°$$

$$R_a^2 - 50862 = 10809.8$$

$$R_a^2 = 61671.8$$

$$R_a = 248.3 \text{ mm}$$

$$\text{Addendum of the wheel} = 248.3 - 240 = 8.3 \text{ mm}$$

Also,
$$0.4R\sin\varphi = \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$0.4 \times 240 \sin 20° = \sqrt{r_a^2 - (160\cos 20°)^2} - 160\sin 20°$$

or
$$r_a^2 - 22605 = 7666$$

or
$$r_a^2 = 30271$$

or
$$r_a = 174 \text{ mm}$$

$$\text{Addendum of the pinion} = 174 - 160 = 14 \text{ mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi} = 0.4\left(\frac{r\sin\varphi + R\sin\varphi}{\cos\varphi}\right) = 58.2 \text{ mm}$$

Solution : 44

Given: $\phi = 20°$, $T = 42$, $t = 19$, $m = 6$ mm, Addendum = 1 m = 6 mm

$$\text{Radius of gear, } R = \frac{mT}{2} = \frac{6 \times 42}{2} = 126 \text{ mm}$$

$$\text{Radius of pinion, } r = \frac{mt}{2} = \frac{6 \times 19}{2} = 57 \text{ mm}$$

Addendum radius of gear,

$$R_a = R + m = 126 + 6 = 132 \text{ mm}$$

Addendum radius of pinion,

$$r_a = r + m = 57 + 6 = 63 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$

$$= \sqrt{(132)^2 - (126\cos 20°)^2} - 126\sin 20° = 15.26 \text{ mm}$$

$$\text{Path of recess} = \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi = \sqrt{(63)^2 - (57\cos 20°)^2} - 57\sin 20°$$

$$= 13.672 \text{ mm}$$

$$\text{Path of contact} = \text{Path of approach} + \text{Path of recess} = 15.26 + 13.672 = 28.932 \text{ mm}$$

$$\text{Length of arc of contact} = \frac{\text{Path of contact}}{\cos\phi} = \frac{28.932}{\cos 20°} = 30.788 \text{ mm}$$

Number of pairs of teeth in contact or contact ratio is given by

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \frac{30.788}{\pi \times 6} = 1.6334$$

$$\text{Angle of action by the pinion, } \theta_p = \frac{\text{Arc of contact}}{\text{Pitch circle radius of pinion}} = \frac{30.788}{57} = 0.54 \text{ radian}$$

$$\theta_p = 30.95°$$

(a) $$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\text{Pitch line velocity}} = \frac{(\omega_p + \omega_g) \times \text{Path of approach}}{\omega_p \times r}$$

$$= \left(1 + \frac{\omega_g}{\omega_p}\right) \times \frac{\text{Path of approach}}{r} = \left(1 + \frac{19}{42}\right) \times \frac{15.26}{57} = 0.388$$

(b) $$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \left(1 + \frac{\omega_g}{\omega_p}\right) \times \frac{\text{Path of recess}}{r} = \left(1 + \frac{19}{42}\right) \times \frac{13.672}{57} = 0.348$$

(c) $$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g) \times 0}{\text{Pitch line velocity}} = 0$$

Solution : 45

Given: Pressure angle, $\phi = 20°$, Pinion speed, $N = 120$ rpm, Module, $m = 3$ mm, Addendum = 1.1 m = 3.3 mm, Velocity ratio, $VR = 3$

(1) Minimum number of teeth on each gear wheel to avoid interference:

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} = \frac{2 \times 1.1}{\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 1\right)\sin^2 20°} - 1} = 49.44$$

Taking the higher whole number divisible by the velocity ratio, i.e. $T = 51$ and $t = \frac{T}{3} = \frac{51}{3} = 17$

(2) Contact ratio or number of pairs of teeth in contact,

$$n = \frac{\text{Arc of contact}}{\text{Circular pitch}} = \left(\frac{\text{Path of contact}}{\cos\phi}\right) \times \frac{1}{\pi m}$$

$$R = \frac{mT}{2} = \frac{3 \times 51}{2} = 76.5 \text{ mm}$$

$$R_a = R + a = 76.5 + 3.3 = 79.8 \text{ mm}$$

$$r = \frac{mT}{2} = \frac{3 \times 17}{2} = 25.5 \text{ mm}$$

$$r_a = r + a = 25.5 + 1.1 \times 3 = 28.8 \text{ mm}$$

$$\text{Path of approach} = \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi$$

$$= \sqrt{(79.8)^2 - (76.5\cos 20^\circ)} - 76.5\sin 20^\circ = 8.482 \text{ mm}$$

$$\text{Path of recess} = \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

$$= \left(\sqrt{28.8^2 - (25.5\cos 20^\circ)} - (25.5\sin 20^\circ)\right) = 7.255$$

$$\text{Path of contact} = 8.482 + 7.255 = 15.737 \text{ mm}$$

$$n = \frac{15.737}{\cos 20^\circ} \times \frac{1}{\pi \times 3} = 1.78$$

Thus, 1 pair of teeth will always remain in contact whereas for 78% of the time, 2 pairs of teeth will be in contact.

Solution : 46

$$\text{Velocity ratio} = 3 : 1$$

$$\text{Pressure angle} = 20^\circ$$

$$\text{Addendum} = 1 \text{ module}$$

Minimum teeth required on pinion

$$t = \frac{2 \times a}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

Where Gear ratio, $G = \dfrac{T}{t} = 3$

$$\Rightarrow \quad t = \frac{2 \times 1}{3\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20} - 1\right]} = 14.98 \approx 15$$

If a pinion of 12 teeth is to be used, the addendum has to be modified

$$3 \times 12 = \frac{2 \times \frac{a}{m}}{\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20} - 1\right]} = \frac{2 \times \left(\frac{a}{m}\right)}{0.0445}$$

$$\frac{a}{m} = 0.8010$$

$$\Rightarrow \quad \text{Addendum, } a = 0.8010 \text{ m}$$

Solution : 47

As S and P are in external mesh and F meshes internally with P. So module of S, P and F will be same.

$$\Rightarrow \quad m_s = m_p = m_f \qquad [\because m \text{ is same for all}]$$

$$r_s + 2r_p = r_F$$

$$T_s + 2T_p = T_F$$

$$m = \frac{1}{\text{Diametral Pitch}} = \frac{1}{3}\text{cm}$$

Arm	S (T_S)	P (T_P)	F (T_F)
Arm is fixed	x	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_P}\times\frac{T_P}{T_F}$
Arm rotates with y rpm in the direction of S.	$y+x$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_F}$

Given: $N_S = 500$, $N_A = 100$

as S is driver so F (annular gear) is fixed so,

$$N_F = 0$$

$$\Rightarrow \quad N_S = y + x = 500 \quad \text{...(i)}$$

$$N_A = y = 100 \quad \text{...(ii)}$$

From eq. (i) and (ii)

$$x = 400 \text{ rpm}$$

$$N_F = y - x\frac{T_S}{T_F} = 0$$

$$100 = 400\frac{T_S}{T_F}$$

$$T_F = 4T_S$$

⇒ As it is given that D_F is close to 25 cm but by taking $D_F = 25$ cm, values of number of teeth of S and P don't come out a whole number so we are taking $D_F = 24$ cm

$$m_F = \frac{D_F}{T_F}$$

$$T_F = \frac{D_F}{m_F} = \frac{24}{1/3} = 72$$

We know that, $$T_S = \frac{T_F}{4} = \frac{72}{4} = 18$$

and $$T_S + 2T_P = T_F$$

$$18 + 2T_P = 72$$

$$2T_P = 72 - 18 = 54$$

$$T_P = 27$$

Speed of P, $$N_P = y - x\frac{T_S}{T_P} = 100 - 400\times\frac{18}{27} = 100 - \frac{800}{3} = \frac{-500}{3} = -166.67 \text{ rpm}$$

Therefore, speed of planet Gear P is 166.67 rpm in opposite direction to S and A.

Solution : 48

Given: $T_B = 20$; $T_C = 80$; $T_D = 60$; $T_E = 30$; $T_F = 32$; $N_B = 1000$ rpm (counter clockwise)

The table of motions is given below:

Step No.	Conditions of motion	Revolutions of elements				
		Arm A	Gear B (or input shaft)	Compound wheel D-E	Gear C	Gear F (or output shaft)
1.	Arm fixed, gear B rotated through + 1 revolution (i.e. 1 revolution anticlockwise)	0	+ 1	$+\frac{T_B}{T_D}$	$-\frac{T_B}{T_D}\times\frac{T_D}{T_C} = -\frac{T_B}{T_C}$	$-\frac{T_B}{T_D}\times\frac{T_E}{T_F}$
2.	Arm fixed, gear B rotated through $+x$ revolution	0	$+x$	$+x\times\frac{T_B}{T_D}$	$-x\times\frac{T_B}{T_C}$	$-x\times\frac{T_B}{T_D}\times\frac{T_E}{T_F}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y	+ y	+ y
4.	Total motion	+ y	$x+y$	$y+x\times\frac{T_B}{T_D}$	$y-x\times\frac{T_B}{T_C}$	$y-x\times\frac{T_B}{T_D}\times\frac{T_E}{T_F}$

1. **Speed of the output shaft when gear C fixed**

Since the gear C is fixed, therefore from the fourth row of the table

$$y - x\times\frac{T_B}{T_C} = 0$$

or

$$y - x\times\frac{20}{80} = 0$$

$$y - 0.25x = 0 \quad \text{...(i)}$$

We know that the input speed (or the speed of gear B) is 1000 rpm counter clockwise, therefore from the fourth row of the table,

$$x + y = 1000 \quad \text{...(ii)}$$

From equations (i) and (ii),

$$x = 800$$

and

$$y = 200$$

$\therefore$ Speed of output shaft = Speed of gear F = $y - x\times\frac{T_B}{T_D}\times\frac{T_E}{T_F}$

$$= 200 - 800\times\frac{20}{60}\times\frac{30}{32} = 200 - 250 = -50 \text{ rpm}$$

$$= -50 \text{ rpm (clockwise)}$$

2. **Speed of the output shaft when gear C is rotated at 10 rpm counter clockwise**

Since the gear C is rotated at 10 rpm counter clockwise, therefore from the fourth row of the table,

$$y - x\times\frac{T_B}{T_C} = 10$$

or

$$y - x\times\frac{20}{80} = 10$$

$\therefore$ $$y - 0.25x = 10 \quad \text{...(iii)}$$

From equations (ii) and (iii),

$$x = 792$$

and $$y = 208$$

$\therefore$ Speed of output shaft = Speed of gear F

$$= y - x \times \frac{T_B}{T_D} \times \frac{T_E}{T_F}$$

$$= 208 - 792 \times \frac{20}{60} \times \frac{30}{32}$$

$$= 208 - 247.5 = -39.5 \text{ rpm} = 39.5 \text{ rpm} \quad \text{(clockwise)}$$

■■■■

5 Flywheel and Governors

LEVEL 1 Objective Questions

1. (c)
2. (d)
3. (d)
4. (d)
5. (243.17)
6. (0.04)
7. (0.38)
8. (d)
9. (a)
10. (d)
11 (206.04)
12. (b)
13. (a)
14. (a)
15. (c)
16. (d)
17. (a)

LEVEL 2 Objective Questions

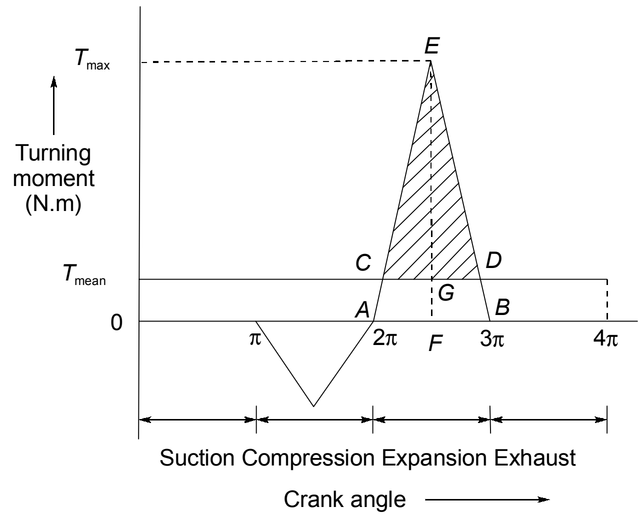
18. (d)
19. (323.6)
20. (991.735)
21. (183.33)
22. (22360)
23. (0.38)
24. (d)
25. (b)
26. (0.51)
27. (31.98)
28. (0.0444)
29. (51.3)
30. (31.4)
31. (b)
32. (c)
33. (c)
34. (c)
35. (a)
36. (d)

■■■■

LEVEL 3 Conventional Questions

Solution : 37

It is a four-stroke engine, thus, a cycle is completed in 4π radians. The turning moment diagram is shown in figure.



The energy is produced only in the expansion stroke whereas in the other three strokes, it is spent only.

Net energy produced in one cycle= [7200 – (440 + 1600 + 660)] × 3 = 13500 N.m

Also $$T_{mean} \times 4\pi = 13500$$

or $$T_{mean} = 1074 \text{ N.m}$$

Energy produced during expansion stroke = $\text{Area} \times \dfrac{\text{Energy}}{\text{mm}^2} = 7200 \times 3 = 21600$ N.m

As the area of the turning-moment diagram during the expansion stroke indicates the energy produced during the expansion stroke,

$\therefore$ $$\frac{T_{max} \times \pi}{2} = 21600$$

or $$T_{max} = 13751 \text{ N.m}$$

In triangle *ABE*, $$\frac{CD}{AB} = \frac{EG}{EF} = \frac{13751 - 1074}{13751} = \frac{12677}{13751} = 0.9219$$

or $$\text{CD} = 0.9219 \times \pi = 2.896 \text{ rad}$$

and maximum fluctuation of energy,

$$e = \text{Area } CDE = \frac{CD \times EG}{2} = \frac{2.896 \times 12677}{2} = 18356 \text{ N.m}$$

Now, $$e = \frac{1}{2} I\left(\omega_1^2 - \omega_2^2\right)$$

$$18356 = \frac{1}{2} mk^2\left(\omega_1^2 - \omega_2^2\right)$$

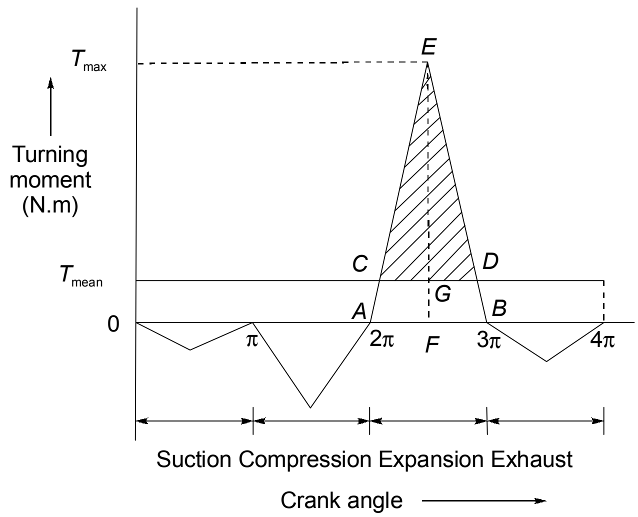
$$= \frac{1}{2} \times m \times 1.25^2 \left[\left(\frac{2\pi}{60}\right)^2 (222^2 - 218^2)\right] = 15.0786 \text{ m} = 1217.4 \text{ kg}$$

Solution : 38

P = 14 kW, N = 280 rpm, K = 1.5%,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 280}{60} = 29.32 \text{ rad/s}$$

It is a four-stroke engine, thus, a cycle is completed in 4π radians. Thus the number of working strokes per minute is half the rpm, i.e., 140. The turning-moment diagram is shown in figure.



$$\text{Net energy produced/s} = 14000 \text{ N.m}$$

$$\text{Net energy produced/minute} = 14000 \times 60 \text{ N.m}$$

$$\text{Net energy produced/cycle} = \frac{14000 \times 60}{140} = 6000 \text{ N.m}$$

Now, during the compression stroke, the energy is absorbed whereas during the expansion stroke, it is produced.

Thus if E is the energy produced during the expansion stroke,

Then $$E - \frac{E}{3} = 6000$$

or $$E = 9000 \text{ N.m}$$

Also $$\frac{T_{max} \times \pi}{2} = 9000$$

or $$T_{max} = 5730 \text{ N.m}$$

and $$T_{mean} \times 4\pi = 6000$$

$\therefore$ $$T_{mean} = 477.5 \text{ N.m}$$

In triangle ABE, $$\frac{CD}{AB} = \frac{EG}{EF} = \frac{5730 - 477.5}{5730} = \frac{5252.5}{5730} = 0.9167$$

or $$CD = 0.9167 \times \pi = 2.88 \text{ rad}$$

and maximum fluctuation of energy,

$$e = \text{Area } CDE = \frac{CD \times EG}{2} = \frac{2.88 \times 5252.5}{2} = 7564 \text{ N.m}$$

$$K = \frac{e}{I\omega^2}$$

or $$0.03 = \frac{7564}{I \times 29.32^2}$$

or $$I = 293.3 \text{ kg.m}^2$$

Solution : 39

Let the energy required for punching be "E" Joules.

In 60 seconds 5 operations are done therefore each operations require 12 seconds hence cycle time is 12 seconds.

$$\text{Power of Motor} = \frac{E}{12} \text{Watt}$$

Energy given by motor in during punching

$$= \frac{E}{12}(\text{Watt}) \times \text{punching time (seconds)} = \frac{8E}{12} = \frac{2E}{3} \text{Joules}$$

$$\text{Energy given by flywheel} = E - \frac{2E}{3} = \frac{3E - 2E}{3} = \frac{E}{3} \text{Joules}$$

Hence, $$\frac{E}{3} = I\omega^2_{mean}\, c_s$$

$$I = Mk^2 = 200 \times \left[\frac{400}{1000}\right]^2 = 32 \text{ kg.m}^2$$

$$\omega_{mean} = \frac{2\pi\, N_{mean}}{60} = \frac{2\pi}{60} \times \left[\frac{400 + 250}{2}\right] = 34.034 \text{ rad/s}$$

$$c_s = \text{coefficient of fluctuation of speed}$$

$$= \frac{N_1 - N_2}{N_{mean}} = \frac{400 - 250}{\left[\frac{400 + 250}{2}\right]} = 0.4615$$

Calculating "E" $$\frac{E}{3} = 32 \times 34.034^2 \times 0.4615$$

$$E = 51317.91 \text{ Joules}$$

Energy required for each punching operation

$$= 51.317 \text{ kJ}$$

$$\text{Power of motor} = \frac{E}{12} = 4276.5 \text{ Watt} = 4.28 \text{ kW}$$

Solution : 40

$\because$ Machine punches 70 holes per hr. and each hole requires 15 kN.m of energy

$\therefore$ $$\text{Motor power} = \frac{70 \times 15}{60 \times 60} \approx 0.3 \text{ kW}$$

$\because$ Each operation requires 2-sec

$\therefore$ Energy supplied by the motor during operation

$$E_2 = 2 \times 0.3 = 0.6 \text{ kJ}$$

Rest of energy by flywheel, $\Delta E = 15 - 0.6 = 14.4$ kW

$$\text{Coeff. of fluctuation of speed} = \frac{2(225-200)}{(225+200)} = 0.11765$$

$$\therefore \qquad \Delta E = 2E.C_s$$

$$E = \frac{\Delta E}{2\,C_s}$$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}\cdot\frac{\Delta E}{C_s} \quad \Rightarrow \quad I\omega^2 = \frac{\Delta E}{C_s}$$

$$MR^2\omega^2 = \frac{\Delta E}{C_s}$$ [Here radius of gyration taken as R]

$$M = \frac{\Delta E}{R^2\bar{\omega}^2 C_s}$$

$$\bar{\omega} = \frac{2\pi N}{60} = \frac{2\pi}{60}\,\frac{(225+200)}{2}$$

$$\bar{\omega} = 22.253 \text{ rad/sec}$$

$$M = \frac{\Delta E}{R^2\bar{\omega}^2 C_s} = \frac{14.4\times 10^3}{0.25\times(22.253)^2\times 0.11765} = \mathbf{988.68\ kg}$$

Solution : 41

Given: $BP = BD = 300$ mm; $DH = 40$ mm; $M = 70$ kg; $m = 10$ kg; $r = BG = 200$ mm

Equilibrium speed when the radius of rotation $r = BG = 200$ mm

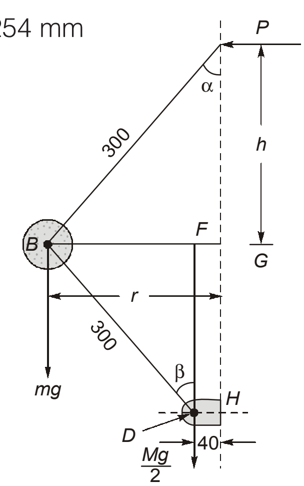
Let N = Equilibrium speed

The equilibrium position of the governor is shown in figure. From the figure, we find that height of the governor,

$$h = PG = \sqrt{(BP)^2-(BG)^2} = \sqrt{(300)^2-(200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

$$\therefore \qquad BF = BG - FG = 200 - 40 = 160 \text{ mm} \qquad ...(\because FG = DH)$$

$$DF = \sqrt{(DB)^2-(BF)^2} = \sqrt{(300)^2-(160)^2} = 254 \text{ mm}$$



$$\therefore \qquad \tan\alpha = \frac{BG}{PG} = \frac{200}{224} = 0.893$$

and
$$\tan\beta = \frac{BF}{DF} = \frac{160}{254} = 0.63$$

$$\therefore \qquad q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.893} = 0.705$$

We know that

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m}\times\frac{895}{h}$$

$$N^2 = \frac{10 + \frac{70}{2}(1+0.705)}{10}\times\frac{895}{0.224} = 27840$$

or $N = 167$ rpm

Range of the speed

$$N_{maximum} = \frac{m + \frac{Mg+F}{2g}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{(70\times 9.81+20)}{2\times 9.81}\times 1.705}{10} \times \frac{895}{0.224} = 168.918 \text{ rpm}$$

$$N_{minimum} = \frac{m + \frac{Mg-F}{2g}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{(70\times 9.81-20)}{2\times 9.81}\times 1.705}{10} \times \frac{895}{0.224} = 164.755 \text{ rpm}$$

Range of speed : $N_2 - N_1$ or $N_{max} - N_{min}$

$$= 168.918 - 164.755 = 4.163 \text{ rpm}$$

Solution : 42

Given: $x = y$; $d = 130$ mm; or $r = 65$ mm $= 0.065$ m; $N = 450$ rpm

or $$\omega = 2\pi \times \frac{450}{60} = 47.13 \text{ rad/s}$$

$$h = 25 \text{ mm} = 0.025 \text{ m}$$

$$M = 4 \text{ kg}$$

$$F = 30 \text{ N}$$

1. Value of each rotating mass

Let m = Value of each rotating mass in kg and

S = Spring force on the sleeve at mid position in newtons.

Since the change of speed at mid position to overcome friction is 1 per cent either way (i.e. ± 1%), therefore

Minimum speed at mid position,

$$\omega_1 = \omega - 0.01\,\omega = 0.99\,\omega = 0.99 \times 47.13 = 46.66 \text{ rad/s}$$

and maximum speed at mid position,

$$\omega_2 = \omega + 0.01\,\omega = 1.01\,\omega = 1.01 \times 47.13 = 47.6 \text{ rad/s}$$

∴ Centrifugal force at the minimum speed,

$$F_{C1} = m(\omega_1)^2 r = m(46.66)^2\, 0.065 = 141.5m \text{ N}$$

and centrifugal force at the maximum speed,

$$F_{C2} = m(\omega_2)^2 r = m\,(\omega_2)^2 r = m(47.6)^2\, 0.065 = 147.3\,m \text{ N}$$

We know that fore minimum speed at mid position

$$S + (Mg - F) = 2F_{C1} \times \frac{x}{y}$$

or $$S + (4 \times 9.81 - 30) = 2 \times 141.5\,m \times 1 \qquad ...(\because x = y)$$

$\therefore$ $$S + 9.24 = 283\,m \qquad ...(i)$$

and for maximum speed at mid-position,

$$S + (M.g + F) = 2F_{C2} \times \frac{x}{y}$$

or $$S + (4 \times 9.81 + 30) = 2 \times 147.3\,m \times 1 \qquad ...(\because x = y)$$

$\therefore$ $$S + 69.24 = 294.6\,m \qquad ...(ii)$$

From equations (i) and (ii),

$$m = 5.2 \text{ kg}$$

2. Spring stiffness in N/mm

Let s = Spring stiffness in N/mm

Since the maximum variation of speed, considering friction is ± 5% of the mid-position speed, therefore,

Minimum speed considering friction,

$$\omega_1 = \omega - 0.05\,\omega = 0.95\,\omega = 0.95 \times 47.13 = 44.8 \text{ rad/s}$$

and maximum speed considering friction,

$$\omega_2 = \omega + 0.05\,\omega = 1.05\,\omega = 1.05 \times 47.13 = 49.5 \text{ rad/s}$$

We know that minimum radius of rotation considering friction,

$$r_1 = r - h_1 \times \frac{x}{y} = 0.065 - \frac{0.025}{2} = 0.0525 \text{ m} \qquad ...\left(\because x = y \text{ and } h_1 = \frac{h}{2}\right)$$

and maximum radius of rotation considering friction,

$$r_2 = r + h_2 \times \frac{x}{y} = 0.065 + \frac{0.025}{2} = 0.077 \text{ m} \qquad ...\left(\because x = y \text{ and } h_2 = \frac{h}{2}\right)$$

$\therefore$ Centrifugal force at the minimum speed considering friction,

$$F_{C}{}'_1 = m(\omega'_1)^2 r_1 = 5.2(44.8)^2\,0.0525 = 548 \text{ N}$$

and centrifugal force at the maximum speed considering friction,

$$F_{C2}' = m(\omega'_2)^2 r_2 = 5.2(49.5)^2\,0.0775 = 987 \text{ N}$$

S_1 = Spring force at minimum speed considering friction, and

S_2 = Spring force at maximum speed considering friction

We know that for minimum speed considering friction,

$$S_1 + (M{\cdot}g - F) = 2F'_{C1} \times \frac{x}{y}$$

$$S_1 + (4 \times 9.81 - 30) = 2 \times 548 \times 1$$

$\therefore$ $$S_1 + 9.24 = 1096 \qquad ...(\because x = y)$$

or $$S_1 = 1096 - 9.24 = 1086.76 \text{ N}$$

and for maximum speed considering friction,

$$S_2 + (M{\cdot}g + F) = 2F'_{C2} \times \frac{x}{y}$$

$$S_2 + (4 \times 9.81 + 30) = 2 \times 987 \times 1 \qquad ...(\because x = y)$$

$$\therefore \quad S_2 + 69.24 = 1974$$

or $$S_2 = 1974 - 69.24 = 1904.76 \text{ N}$$

We know that stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{1904.76 - 1086.76}{25} = 32.72 \text{ N/mm}$$

3. Initial compression of the spring

We know that initial compression of the spring

$$= \frac{S_1}{s} = \frac{1086.76}{32.72} = 33.2 \text{ mm}$$

Solution : 43

Given: Number of holes per minute = 30, Motor Power (P) = 1.5 kW

Coefficient of fluctuation of speed$= \frac{20}{100} = 0.2$

Actual punching of each hole is accomplished during 30° of crank rotation of machine.

Thus, the actual punching takes place in $\frac{30^\circ}{360^\circ} = \left(\frac{1}{12}\right)^{th}$ of the crank rotation of machine

Time required to punch one hole = $\frac{60}{30} = 2$ sec

Energy supplied per stroke or per hole = 1.5 × 2 = 3000 N-m

As actual punching is done in $\left(\frac{1}{12}\right)^{th}$ of cycle, the energy is stored during the remaining $\left(\frac{11}{12}\right)^{th}$ of cycle in the flywheel.

∴ Maximum fluctuation of energy = Energy stored in the flywheel/stroke

$$\Delta E = \left(\frac{11}{12}\right) \times 3000 = 2750 \text{ N-m}$$

Also, $$\Delta E = I\omega^2 C_s$$

As the number of punching strokes is 30 holes per minute,

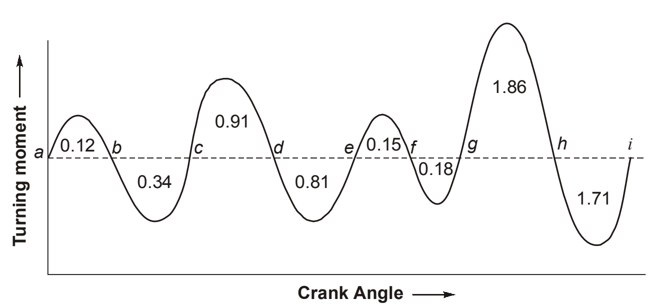
$$\omega = \frac{2\pi(30)}{60} = 3.14 \text{ rad/sec}$$

$$\Delta E = I(3.14)^2(0.2)$$

$$2750 = I(3.14)^2(0.2)$$

$$I = 1394.58 \text{ kg-m}^2$$

Solution : 44



Let the flywheel KE, at $a = E$

at $b = E + 0.12$

at $c = E + 0.12 - 0.34 = E - 0.22$

at $d = E - 0.22 + 0.91 = E + 0.69$

at $e = E + 0.69 - 0.81 = E - 0.12$

at $f = E - 0.12 + 0.15 = E + 0.03$

at $g = E + 0.03 - 0.18 = E - 0.15$

at $h = E - 0.15 + 1.86 = E + 1.71$

at $i = E + 1.71 - 1.71 = E$

Maximum flywheel energy, $E_{max} = E + 1.71$

Minimum flywheel energy, $E_{min} = E - 0.22$

$$\Delta E = [(E + 1.71) - (E - 0.22)] \times \text{Horizontal scale} \times \text{Vertical scale}$$

$$= 1.93 \times \frac{15° \times \pi}{180} \times 15 \text{ ton}\cdot\text{m}$$

$$= 7.58 \text{ ton-m} = 7.58 \times 8896.44 = 67435.04 \text{ N.m}$$

[1 ton = 907.185 kg; 1 tonne = 1000 kg]

Coefficient of fluctuation of speed, $K = \dfrac{\Delta E}{I\omega^2}$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

$$I = 100 \text{ ton-m}^2 = 90718.5 \text{ kg.m}^2$$

$$K = \frac{67435.04}{90718.5 \times (10.47)^2} = 6.78 \times 10^{-3} \text{ or } K = 0.678\%$$

Solution : 45

Given: Mass of each governor ball, $m = 7$ kg, Speed range = 420 to 440 rpm,

Range of ball path radius = 12.4 cm to 13.2 cm,

Controlling force at $r_1 = 12.4$ cm and $N_1 = 420$ rpm,

$$F_1 = m\omega_1^2 r_1$$

$$= m\left(\frac{2\pi N_1}{60}\right)^2 r_1 = 7 \times \left(\frac{2\pi \times 420}{60}\right)^2 \times 0.124 = 1679.1 \text{ N}$$

Controlling force at $r_2 = 13.2$ cm and $N_2 = 440$ rpm,

$$F_2 = 7 \times \left(\frac{2\pi \times 440}{60}\right)^2 \times 0.132 = 1961.7 \text{ N}$$

Let us take linear relationship between ball path radius and controlling force as:

$$F = ar + b$$

$$F_1 = a r_1 + b$$

$$F_2 = a r_2 + b$$

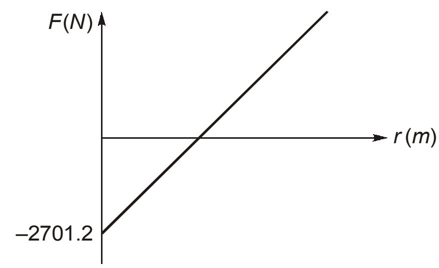
$$a = \frac{F_2 - F_1}{r_2 - r_1} = \frac{1961.7 - 1679.1}{0.132 - 0.124} = 35325 \text{ N/m}$$

$$b = F_1 - a r_1$$

$$b = 1679.1 - 35325 \times 0.124 = -2701.2 \text{ N}$$

So, the relationship is $F = 35325\, r - 2701.2$

At r = 12.8 cm or 0.128 m,



Graph between Controlling Force (F) and Radius of Rotating Ball (r)

$$F = 35325 \times 0.128 - 2701.2 = 1820.4 \text{ N}$$

$$m\,\omega^2\, r = 1820.4$$

$$\omega = \left(\frac{1820.4}{7 \times 0.128}\right)^{\frac{1}{2}} = 45.074 \text{ rad/s}$$

$$N = \frac{60\,\omega}{2\pi} = \frac{60 \times 45.074}{2 \times 3.14} = 430.43 \text{ rpm}$$

Solution : 46

(i) Power of engine $= T_{mean} \times \omega,\ \omega = \frac{2\pi N}{60} = 26.18$ rad/sec

$\therefore$ Power of engine = 10000 × 26.18 = 261.8 kW

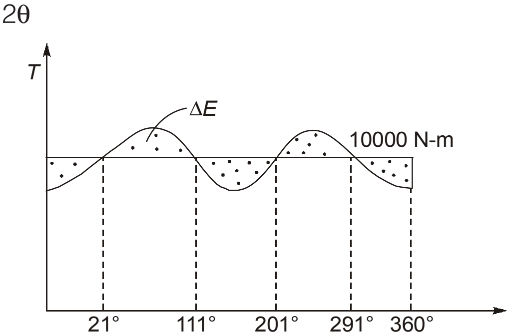
(ii) $T = 10000 + 2000 \sin 2\theta - 1800 \cos 2\theta$

$$(T - T_m) = \Delta T = 2000 \sin 2\theta - 1800 \cos 2\theta$$

$\because$ $\Delta T = 0$

$$\tan 2\theta = \frac{1800}{2000} = 0.9$$

$$\theta = \frac{n\pi}{2} + 21°, \quad n = 0, 1, 2$$



$\therefore$ Max. flucluation of energy

$$\Delta E = \int_{21°}^{111°} \Delta T \cdot d\theta$$

$$= \int_{21°}^{111°} (2000 \sin 2\theta - 1800 \cos 2\theta)\, d\theta$$

$$= 1000[-\cos 2\theta]_{21^\circ}^{111^\circ} - \frac{1800}{2}[\sin 2\theta]_{21^\circ}^{111^\circ} = 1486.3 + 900 \times 1.3383$$

$$= 2690.735 \text{ N-m}$$

$$\therefore \quad \Delta E = 2E \cdot C_s = I\omega^2 \cdot C_s$$

$$2690.735 = I \times 26.18^2 \times \frac{0.5}{100}$$

$$I = 785.166 \text{ kg-m}^2$$

(iii) At $\theta = 45°$

$$\Delta T = 2000 \sin 90° - 1800 \cos 90° = 2000 \text{ N-m}$$

$$\Delta T = I\alpha$$

$$\alpha = \frac{\Delta T}{I} = \frac{2000}{785.166} = 2.547 \text{ rad/sec}^2$$

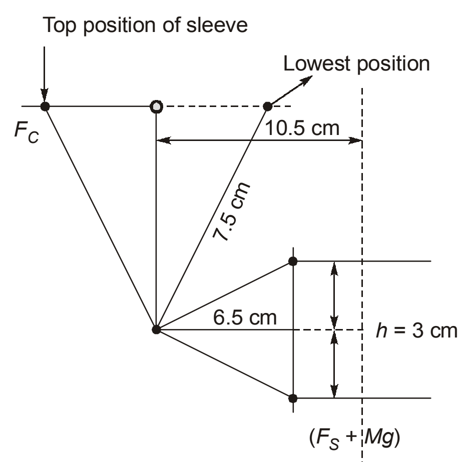
Solution : 47

$$m = 1.5 \text{ kg}$$

In the top position of sleeve

$$\frac{x}{7.5} = \frac{1.5}{6.5}$$

$$x = 1.7308 \text{ cm}$$



Radius of rotation in top position

$$r_2 = 10.5 + 1.7308 = 12.23 \text{ cm}$$

Radius of rotation in lowest position = 10.5 – 1.7308 = 8.77 cm

Equilibrium speed is ω_2 = 415 rpm = 43.46 rad/sec

$$Fc_2 \times 7.5 = \frac{(Fs_2 + Mg)}{2} \times 6.5$$

$$1.5 \times 0.1223 \times (43.46)^2 \times \frac{7.5 \times 2}{6.5} = (Fs_2 + Mg)$$

$$Fs_2 + Mg = 799.604 \quad \text{...(i)}$$

In the lowest position

$$Fc_1 \times 7.5 = \left(\frac{Fs_1 + Mg}{2}\right) \times 6.5$$

$$mr_1\omega_1^2 \times 7.5 = \left(\frac{Fs_1 + Mg}{2}\right) \times 6.5$$

$$1.5 \times 0.0877 \times \left(\frac{43\pi}{3}\right)^2 \times \left(\frac{7.5 \times 2}{6.5}\right) = (Fs_1 + Mg)$$

$$Fs_1 + Mg = 615.56 \quad \text{...(ii)}$$

Assuming $M = 0$

(i) Stiffness & Initial compression

$$(Fs_2 - Fs_1) = kh$$

$$k = \frac{Fs_1 - Fs_1}{h} = \frac{799.604 - 615.56}{0.03} = 6134.8 \text{ N/m}$$

$$\text{Initial compression}, S_1 = \frac{Fs_1}{k} = \frac{615.56}{6286.1} = 0.1003 \text{ m} = 10.03 \text{ cm}$$

(ii) Initial compression which gives 10 rpm more in topmost position

The lowest position rpm = 430 rpm

$\therefore$ Highest position rpm = 440 rpm

$$\therefore \quad \omega_2 = 440 \times \frac{2\pi}{60} = 46.077 \text{ Rad/sec}$$

$$\therefore \quad Fc_2 \times 7.5 = \frac{Fs_2}{2} \times 6.5 \quad \rightarrow \quad (\because$$

$M = 0$ assume)

$$1.5 \times 0.1223 \times (46.077)^2 \times \frac{7.5 \times 2}{6.5} = Fs_2$$

$$Fs_2 = 898.8 \text{ N}$$

$$\therefore \quad Fs_2 = Fs_1 + k \cdot h$$

$$898.8 = Fs_1 + 6134.8 \times 0.03$$

$$Fs_1 = 898.8 - 184.044 = 714.759 \text{ N}$$

$$x_1 = \frac{714.759}{6134.8} \text{ m} = 0.1165 \text{ m} = 11.65 \text{ cm}$$

■■■■

6 Balancing and Gyroscope

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(b)
4.	(a)
5.	(d)
6.	(c)
7.	(a)
8.	(a)
9.	(b)

LEVEL 2 Objective Questions

10.	(d)
11.	(c)
12.	(a)
13.	(a)
14.	(a)
15.	(a)
16.	(a)

■■■■

LEVEL 3 Conventional Questions

Solution : 17

Given data : $I_w = \frac{32}{2} = 16\ \text{kg.m}^2$, $m =$ 3000 kg, $r = 0.45$ m, $h = 1$m, $I_m = 16$ kg.m^2, $w = 1.4$ m,

$R = 250$ m

(i) Reaction due to weight

$$R_w = \frac{mg}{4} = \frac{3000 \times 9.81}{4} = 7357.5 \text{ N (upwards)}$$

(ii) Reaction due to gyroscopic couple

$$C_w = 4I_w.\frac{v^2}{r.R} = 4 \times 16 \times \frac{v^2}{0.45 \times 250} = 0.569v^2$$

$$C_m = 2I_m G\omega_w\omega_p \quad \text{(as there are two motors)}$$

$$= 2 \times 16 \times 3 \times \frac{v^2}{0.45 \times 250} = 0.853\ v^2$$

$$C_G = C_w - C_m \quad \text{(motors rotate in opposite direction)}$$

$$= 0.569\ v^2 - 0.853\ v^2 = 0.284\ v^2$$

Reaction on each outer wheel, $R_{Go} = 0.1014\ v^2$ (upwards)

(iii) Reaction due to centrifugal couple

$$C_c = \frac{mv^2}{R}h = 3000 \times \frac{v^2}{250} \times 1 = 12\ v^2$$

$$R_{co} = \frac{C_c}{2w} = \frac{12v^2}{2 \times 1.4} = 4.286v^2 \quad \text{(upwards)}$$

$$R_{ci} = \frac{C_c}{2w} = 4.286v^2 \quad \text{(downwards)}$$

Total reaction on outer wheel $= 7357.5 - 0.1014v^2 + 4.286\ v^2$

$= 7357.5 + 4.1846v^2$

Total reaction on inner wheel $= 7357.5 + 0.1014v^2 - 4.286\ v^2 = 7357.5 - 4.184\ v^2$

Thus, the reaction on the outer wheel is always positive(upwards). There are chances that the inner wheels leave the rails.

For maximum speed, $7357.5 - 4.1846v^2 = 0$

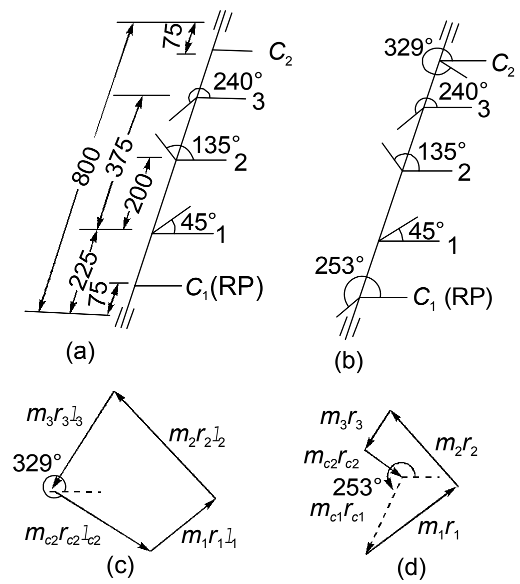
or $v^2 = 1758.2$

$v = 41.93$ m/s

or $v = \frac{41.93 \times 3600}{1000} = 151$ km/h

Solution : 18

Figures shown the planes of unbalanced masses as well as the planes of the countermasses. Plane C_1 is to be taken as the reference plane and the various distances are to be considered from this plane.



Analytical solution

$$I_{c2} = (800 - 75 \times 2) = 650 \text{ mm}$$
$$l_1 = 225 - 75 = 150 \text{ mm}$$
$$l_2 = 150 + 200 = 350 \text{ mm}$$
$$I_3 = 150 + 375 = 525 \text{ mm}$$
$$m_1r_1l_1 = 4 \times 75 \times 150 = 45000$$
$$m_1r_1 = 4 \times 75 = 300$$
$$m_2r_2l_2 = 3 \times 85 \times 350 = 89250$$
$$m_2r_2 = 3 \times 85 = 255$$
$$m_3r_3l_3 = 2.5 \times 50 \times 525 = 65625$$
$$m_3r_3 = 2.5 \times 50 = 125$$
$$\Sigma mrl + m_{c2}r_{c2}l_{c2} = 0$$

or $\quad 4500\cos 45° + 89250\cos 135° + 65625 \cos 240° + m_{c2}r_{c2}r_{c2} \cos \theta_{c2} = 0$

and $\;45000 \sin 45° + 89250 \sin 135° + 65625 \sin 240° + m_{c2}r_{c2}l_{c2} \sin\theta_{c2} = 0$

Squaring, adding and then solving,

$$m_{c2}r_{c2}l_{c2} = \begin{bmatrix} (4500\cos 45° + 89250\cos 135° + 65625\cos 240°)^2 \\ + (4500\sin 45° + 89250\sin 135° + 65625\sin 240°)^2 \end{bmatrix}$$

$$= [(-64102)^2 + (38096)^2]^{1/2}$$

or $\quad m_{c2} \times 40 \times 650 = 74568$

$$m_{c2} = 2.868 \text{ kg}$$

$$\tan\theta_{c2} = \frac{-38096}{-(-64102)} = -0.594$$

$$\theta_{c2} = 329.3° \text{ or } 329°18'$$

Now, $\Sigma mr + m_{c1}r_{c1} + m_{c2}r_{c2} = 0$

or $300\cos45° + 255\cos135° + 125\cos240° + m_{c1}r_{c1}\cos\theta_1 + 2.868 \times 40 \cos329.3 = 0$

and $300\sin45° + 255\sin135° + 125\sin240° + m_{c1}r_{c1}\sin\theta_1 + 2.868 \times 40 \sin329.3 = 0$

Squaring, adding and then solving,

$$m_{c1}r_{c1} = \begin{bmatrix}(300\cos45° + 255\cos135° + 125\cos240° + 2.868\times40\cos329.3°)^2 \\ +(300\sin45° + 255\sin135° + 125\sin240° + 2.868\times40\sin329.3°)^2\end{bmatrix}$$

$$m_{c1} \times 75 = [(67.96)^2 + (225.62)^2]^{1/2} = 235.63$$

$$m_{c1} = 3.14 \text{ kg}$$

$$\tan\theta_{c1} = \frac{-225.62}{-67.96} = 3.32$$

$$\theta_{c1} = 253.2° \text{ or } 253°.12'$$

Graphical solution

The graphical solution has also been shown in figure (c) and (d). From figure (c)

$$m_{c2}r_{c2}l_{c2} = 74000$$

$\therefore$ $$m_{c2} = \frac{74000}{40 \times 650} = 2.846 \text{ kg at } 329°$$

From figure (d), $$m_{c1}r_{c1} = 235$$

$\therefore$ $$m_{c1} = \frac{235}{75} = 3.13 \text{ kg at } 253°$$

Figure (b), represents the position of the balancing masses on the rotating shaft.

Solution by using complex numbers

$$m_1r_1l_1\angle\theta_1 = (4 \times 75 \times 150)\angle45° = 4500\angle45° = 31820 + j\,31820$$

$$m_2r_2l_2\angle\theta_2 = (3 \times 85 \times 350)\angle135° = 89250\angle135° = -63109 + j\,63109$$

$$m_3r_3l_3\angle\theta_3 = (2.5 \times 50 \times 525)\angle240° = -65625\angle240° = -32813 - j\,56833$$

Now, $m_1r_1l_1\angle\theta_1 + m_2r_2l_2\angle\theta_2 + m_3r_3l_3\angle\theta_3 + m_{c2}r_{c2}l_{c2}\angle\theta_{c2} = 0$

$$(31820 + j\,31820) + (-63109 + j\,63109) + (-32813 - j\,56833) + m_{c2}r_{c2}l_{c2}\angle\theta_{c2} = 0$$

$$m_{c2}r_{c2}l_{c2}\angle\theta_{c2} = 64102 - j38096 = 74568Ð329.3°$$

$$m_{c2} \times 40 \times 650 = 74568$$

$$m_{c2} = 2.868 \text{ kg}$$

Similarly, $$m_1r_1\angle\theta_1 = (4 \times 75)\angle45° = 300\angle45° = 212.1 + j212.1$$

$$m_2r_2\angle\theta_2 = (3\times 85)\angle 135° = 225\angle 135° = -180 + j180.3$$

$$m_3r_3\angle\theta_3 = (2.5\times 50)\angle 240° = 125\angle 240° = -62.5 - j108.3$$

$$m_{c2}r_{c2}\angle\angle\theta_{c2} = (2.868\times 40)\angle 329.3° = 114.72$$

$$\angle 329.3° = 98.6 - j58.6$$

Now, $m_1r_1\angle\theta_1 + m_2r_2\angle\theta_2 + m_3r_3\angle\theta_3 + m_{c2}r_{c2}\angle\theta_{c2} + m_{c1}r_{c1}\angle\theta_{c1} = 0$

$$(212.1 + j\,212.1) + (-180.3 + j180.3) + (-62.5 - j108.3) + (98.6 - j58.6) + m_{c2}r_{c2}\angle 329.3° = 0$$

$$m_{c1}r_{c1}\angle\theta_{c1} = -67.9 - j225.5 = 235.5\angle 253.2°$$

or $$m_{c1}\times 75 = 235.63$$

$$m_{c1} = 3.14 \text{ kg}$$

Solution : 19

$$m_br_b = 25\times 100 = 5000$$

$$m_cr_c = 40\times 100 = 4000$$

$$m_dr_d = 35\times 180 = 6300$$

For complete balance, taking $\theta_b = 0°$

$$\Sigma mr\cos\theta = 0 \quad \text{and} \quad \Sigma mr\sin\theta = 0$$

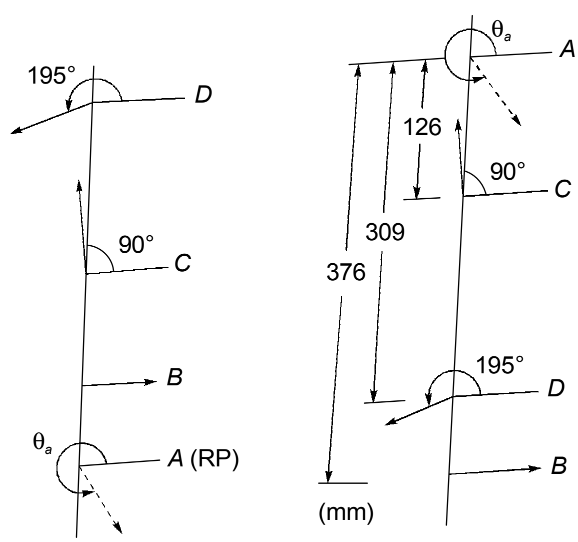
i.e. $m_a\times 150\times\cos\theta_a + 5000\cos 0° + 4000\cos 90° + 6300\cos 195° = 0$

or $m_a\times 150\times\cos\theta_a + 5000 + 0 - 6085 = 0$

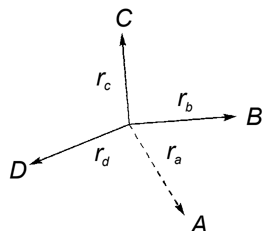
or $$150\,m_a\cos\theta_a = 1085 \quad \text{. . . (i)}$$

and $m_a\times 150\times\sin\theta_a + 5000\sin 0° + 4000\sin 90° + 6300\sin 195° = 0$

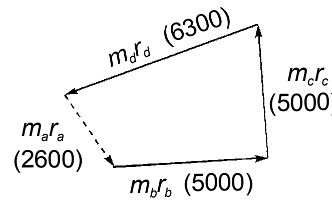
or $$150\,m_a\sin\theta = -2369 \quad \text{. . . (ii)}$$



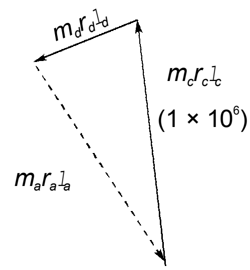
(a) (b)



(c)



(d)



(e)

Squaring and adding (i) and (ii)

$$22500\, m_a^2 = (1085)^2 + (-2369)^2$$

or $$m_a^2 = 30175$$

or $$m_a = 17.37 \text{ kg}$$

Dividing (ii) by (ii) $$\tan \theta_a = \frac{-236.9}{108.5} = -2.184$$

or $$\theta_a = 294.6° \quad \text{or } 294°36'$$

For complete balance, the couple equations are

$$\Sigma mrl \cos \theta = 0 \quad \text{and} \quad \Sigma mrl \sin \theta = 0$$

Taking A as the reference plane,

$$5000\, l_b \cos 0° + 4000\, l_c \cos 90° + 63000\, l_d \sin 195° = 0$$

or $$5000\, l_b = 6085\, l_d$$

or $$l_b = 1.217\, l_d$$

and $500\, l_b \sin 0° + 4000\, l_c \sin 90° + 6300\, l_d \sin 195° = 0$

or $$4000\, l_c = 1631\, l_d$$

or $$l_c = 0.4078\, l_d$$

or $$l_b + 250 = 0.4078\, l_d$$

or $$1.217\, l_d + 250 = 0.4078\, l_d$$

or $$0.8092\, l_d = -250$$

or $$l_d = -309 \text{ mm}$$

$$l_b = 1.217\, l_d = 1.217 \times (-309) = -376 \text{ mm}$$

$$l_c = l_b + 250 = -376 + 250 = -126 \text{ mm}$$

The correct positions of the planes have been shown in figure.

To solve the problem graphically, $m_a r_a$ is is obtained from the vector sum of $m_b r_b$, $m_c r_c$ and $m_d r_d$ (figure). On measuring,

$$m_a r_a = 2600$$

$$\therefore \quad m_a = \frac{2600}{150} = 17.3 \text{ kg} \quad \text{and } \theta_a = 294.5°$$

Now, $m_a r_a l_a = 4000 \times 250 = 1 \times 10^6$, taking B as the reference plane. Take the vector $m_c r_c l_c$ and from its two ends, draw lines parallel to $m_a r_a$ and $m_d r_d$. Thus, forming a triangle. Measuring the two sides,

$$m_a r_a l_a = 985000, \; l_a = \frac{985000}{17.3 \times 150} = 379 \text{ mm}$$

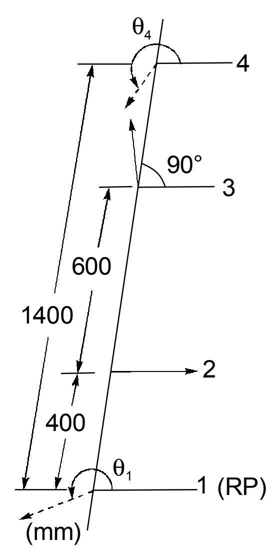
$$m_d r_d l_d = 437000, \; l_d = \frac{437000}{6300} = 69 \text{ mm}$$

l_a and l_d establish the relative positions of the planes.

Solution : 20

$$\text{Total mass to be balanced} = m_p + cm$$

$$= 280 + \frac{2}{3} \times 300 = 480 \text{ kg}$$



(i) Take 1 as the reference plane and angel $\theta_2 = 0°$. Writing the couple equations,

$$m_2 r_2 l_2 \cos\theta_2 + m_3 r_3 l_3 \cos\theta_3 + m_4 r_4 l_4 \cos\theta_4 = 0$$

or $\quad 480 \times 300 \times 400 \cos 0° + 480 \times 300 \times 1000 \cos 90° + m_4 \times 620 \times 1400 \cos\theta_4 = 0$

or $$m_4 \cos\theta_4 = -66.36 \quad \text{. . . (i)}$$

and $\quad m_2 r_2 l_2 \sin\theta_2 + m_3 r_3 l_3 \sin\theta_3 + m_4 r_4 l_4 \sin\theta_4 = 0$

or $\quad 480 \times 300 \times 400 \sin 0° + 480 \times 300 \times 1000 \sin 90° + m_4 \times 620 \times 1400 \sin\theta_4 = 0$

or $$m_4 \sin\theta_4 = -165.9 \quad \text{. . . (ii)}$$

Squaring and adding (i) and (ii),

$$m_4 = 178.7 \text{ kg}$$

Dividing (ii) by (i), $\tan\theta_4 = \dfrac{-165.9}{-66.36} = 2.5$

$$\theta_4 = 248.2°$$

Taking 4 as the reference plane and writing the couple equations,

$$m_2 r_2 l_2 \cos\theta_2 + m_3 r_3 l_3 \cos\theta_3 + m_1 r_1 l_1 \sin\theta_1 = 0$$

$$480 \times 300 \times 1000 \cos 0° + 480 \times 300 \times 400 \cos 90° + m_1 \times 620 \times 1400 \sin\theta_1 = 0$$

or $m_1 \sin\theta_1 = -165.9$ . . . (iii)

Similarly, $m_1 \sin\theta_1 = -66.36$ . . . (iv)

From (iii) and (iv), $m_1 = 178.7 \text{ kg} = m_4$

$$\tan\theta_1 = \frac{-66.36}{-165.9} = 0.4 \quad \text{or } \theta_1 = 201.8°$$

The treatment shows that the magnitude of m_1 could have directly been written equal to m_4.

(ii) $\omega = \dfrac{50 \times 1000 \times 1000}{60 \times 60} \times \dfrac{1}{\frac{1800}{2}} = 15.43 \text{ rad/s}$

$$\text{Swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$$

$$= \pm\frac{1}{\sqrt{2}}\left(1-\frac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2 \times 0.6 = 3030.3 \text{ N.m}$$

(iii) Variation in tractive force $= \pm\sqrt{2}(1-c)mr\omega^2 = \pm\sqrt{2}\left(1-\dfrac{2}{3}\right) \times 300 \times 0.3 \times (15.43)^2 = 10100 \text{ N}$

(iv) Balance mass for reciprocating parts only $= 178.7 \times \dfrac{\frac{2}{3} \times 300}{480} = 74.46 \text{ kg}$

$$\text{Hammer-blow} = mr\omega^2 = 74.46 \times 0.62 \times (15.43)^2 = 10\,991 \text{ N}$$

$$\text{Dead load} = 3.5 \times 1000 \times 9.81 = 34335 \text{ N}$$

$$\text{Maximum pressure on rails} = 34335 + 10991 = 45326 \text{ N}$$

$$\text{Minimum pressure on rails} = 64335 - 10991 = 23344 \text{ N}$$

(v) Maximum speed of the locomotive without lifting the wheels from the rails will be when the dead load becomes equal to the hammer-blow.

i.e., $74.46 \times 0.62 \times \omega^2 = 34335$

or $\omega = 27.27 \text{ rad/s}$

$$\text{Velocity of wheels} = \omega r = \left(27.27 \times \frac{1.80}{2}\right) \text{ m/s}$$

$$= \left(27.27 \times \frac{1.8}{2} \times \frac{60 \times 60}{1000}\right) \text{ km/h} = 88.36 \text{ km/h}$$

Solution : 21

$$\omega = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

$$n = \frac{800}{200} = 4$$

Figure represents the relative position of the cylinders and the cranks.

Taking 2 as the reference plane, primary couples about the RF,

$$m_1 r_1 l_1 = 200 \times 0.2 \times 0.4 = 16$$

$$m_2 r_2 l_2 = 0$$

$$m_3 r_3 l_3 = m_3 \times 0.2 \times (-0.6) = -0.12\, m_3$$

$$m_4 r_4 l_4 = 200 \times 0.2 \times (-1.1) = -44$$

The couple polygon is drawn in figure

$m_3 r_3 l_3$ or the crank 3 from the diagram = 53.7 at 135°

$\therefore$ $$m_3 r_3 l_3 = m_3 \times 0.12 = 53.7 \text{ or } m_3 = 448 \text{ kg}$$

As its direction is to be negative, its direction is (135° + 180°) or 315°.

Primary force (mr) along each of outer cranks = 200 × 0.2 = 40

Primary force (*mr*)along crank 3 = 448 × 0.2 = 89.6

The force polygon dis drawn in figure.

$m_2 r_2$ of crank 2 from the diagram = 87.6 at 161.4°

$\therefore$ $$m_2 r_2 = m_2 \times 0.2 = 87.6 \text{ or } m_2 = 438 \text{ kg}$$

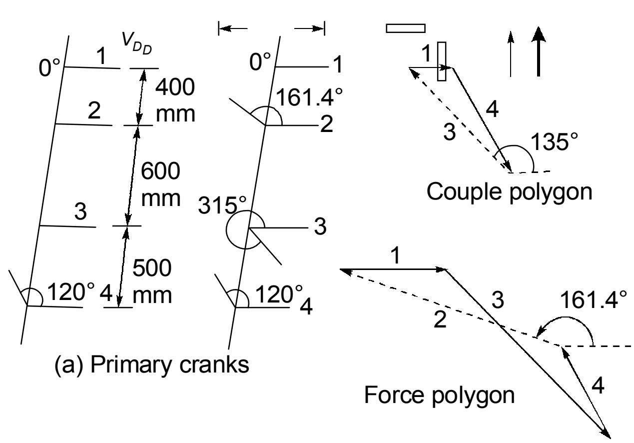
Its angular position is 161.4°

Figure represents the relative position of the cylinders and the cranks.

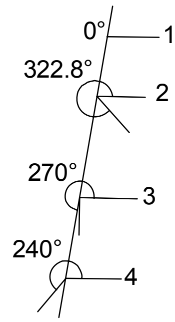
From secondary unbalanced force polygon,

$$mr = 198$$

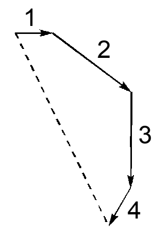
$$\text{Maximum unbalanced} = 198 \times \frac{\omega^2}{n} = 198 \times \frac{22^2}{4} = 23958 \text{ N}$$



(a) Primary cranks



(b) Secondary cranks



Force polygon

Solution : 22

M = 2200 kg, m = 140 kg, w = 1.4 m, k = 0.15 m, b = 2.4 m, I_w = 0.7 kg.m^2

$$r = \frac{0.8}{2} = 0.4 \text{ m}$$

$$R = 100 \text{ m}$$

$$v = \frac{72 \times 1000}{3600} = \text{m/s}$$

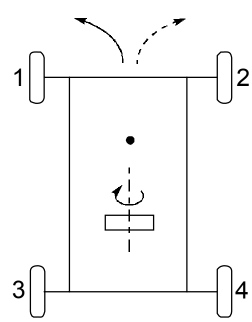
(i) Car turning left

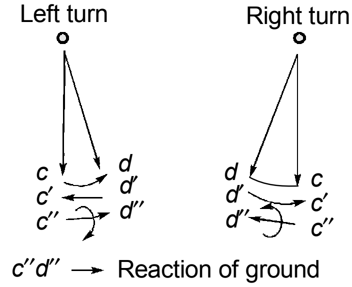
(a) Reaction due to weight

$$\text{Total weight} = 2200 \times 9.81 = 21582 \text{ N}$$

$$R_{w1,2} = \left(21582 \times \frac{1.4}{2.4}\right) \times \frac{1}{2} = 6295 \text{ N(upwards)}$$

$$R_{w3,4} = \left(21582 \times \frac{1}{2.4}\right) \times \frac{1}{2} = 4496 \text{ N(upwards)}$$





(b) Reaction due to gyroscopic couples

$$C_w = 4I_w \frac{v^2}{rR} = 4 \times 0.7 \times \frac{(20)^2}{0.4 \times 100} = 28 \text{ N.m}$$

For outer wheels, $$R'_{G2,4} = \frac{C_w}{2w} = \frac{28}{2 \times 1.4} = 10\text{N (upwards)}$$

For inner wheels, $$R'_{G1,3} = 10 \text{ N (downwards)}$$

$$I_e = mk^2 = 140 \times (0.15)^2 = 3.15 \text{ kg.m}^2$$

$$C_e = I_e G\omega_w \omega_p = 3.15 \times 5 \times \frac{(20)^2}{0.4 \times 100} = 157.5 \text{ N}$$

For front wheels, $R''_{G1,2} = \dfrac{C_e}{2b} = \dfrac{157.5}{2 \times 2.4} = 32.8$ N(upwards)

For real wheels, $R''_{G3,4} = 32.8$ N (downwards)

(c) Reaction due to centrifugal couple :

$$C_c = M\frac{v^2}{R}h = 2200 \times \frac{(20)^2}{100} \times 0.6 = 5280 \text{ N.m}$$

For outer wheels, $R_{c2,4} = \dfrac{C_c}{2w} = \dfrac{5280}{2 \times 1.4} = 1886$ N (upwards)

For rear wheels, $R_{c1,3} = 1886$ N (downwards)

Therefore, reaction on wheels: $R = R_w + R'_G + R''_G + R_c$

$$R_1 = 6295 - 10 + 32.8 - 1886 = 4431.8 \text{ N}$$
$$R_2 = 6295 - 10 + 32.8 + 1886 = 8223.8 \text{ N}$$
$$R_3 = 4496 - 10 - 32.8 - 1886 = 2567.2 \text{ N}$$
$$R_4 = 4496 + 10 - 32.8 + 1886 = 6359.2 \text{ N}$$

(ii) Car turning right :

All the reactions due to gyroscopic couples and centrifugal couple change signs. Therefore,

$$R_1 = 6295 + 10 - 32.8 + 1886 = 8158.2 \text{ N}$$
$$R_2 = 6295 - 10 - 32.8 - 1886 = 4366.2 \text{ N}$$
$$R_3 = 4496 + 10 + 32.8 + 1886 = 6426.8 \text{ N}$$
$$R_4 = 4496 - 10 + 32.8 - 1886 = 2632.8 \text{ N}$$

Solution : 23

$m = 4$ kg, $N = 800$ rpm, $k = 0.06$ m, $N_p = 50$ rpm

$$I = mk^2 = 4 \times (0.06)^2 = 0.0144 \text{ kg.m}^2$$
$$l = 80 \text{ mm} = 0.08 \text{ m}$$
$$\omega = \frac{2\pi \times 800}{60} = 83.78 \text{ rad/s}$$
$$\omega_p = \frac{2\pi \times 50}{60} = 5.24 \text{ rad/s}$$

$\therefore$ $C = I\omega\omega_p = 0.0144 \times 83.78 \times 5.24 = 6.32$ N.m

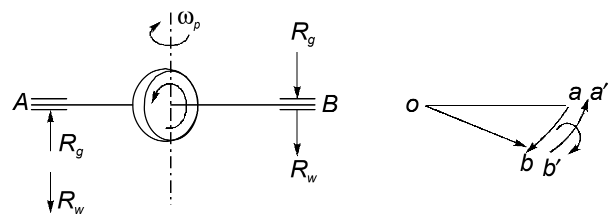
The applied (active) and reaction couples are shown in figure. The reaction couple is clockwise when viewed from front and tends to raise the bearing *A* and lower the bearing *B*. Thus, reaction of each bearing in turn is downwards at *A* and upwards at *B*.

Reaction at bearing *A* due to gyro. couple $= \dfrac{C}{l} = \dfrac{6.32}{0.08} = 79$ N (downwards)

Reaction at bearing B due to gyro. couple = 79 N (upwards)

Force at each bearing due to weight of the disc

$$= \frac{4 \times 9.81}{2} = 19.6 \text{ N}$$

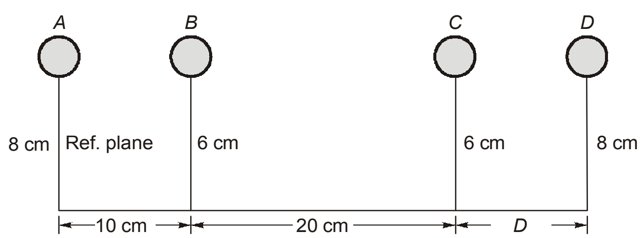


or Reaction at each bearing due to weight = 19.6 N (upwards)

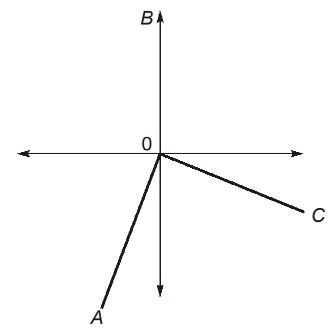
Reaction at bearing, B = 79 + 19.6 + 98.6 N (downwards)

∴ Reaction at bearing, A = 79 – 19.6 = 59.4 N

Solution : 24



Position of Planes



Position of Crank

As the mass at the plane A is unknown, we take the plane A as our reference for fixing the axial locations of the other planes. For locating angular positions, we take the plane B as the reference.

$$\theta_B = 0,\ \theta_C = 100°,\ \theta_A = 190° \ (\theta_D \text{ is not known})$$

With A as the reference plane for couple vectors, we have

$$x_A = 0,\ x_B = 10 \text{ cm},\ x_C = 30 \text{ cm} \ (x_D \text{ is not known})$$

Let us resolve forces and couples in two mutually perpendicular planes as done in table below. Summing up the last four columns of the table and equating each to zero, we get the following equations:

$$m_D x_D \cos\theta_D = -86.17 \quad \text{...(i)}$$

$$m_D x_D \sin\theta_D = -276.5 \quad \text{...(ii)}$$

$$8\, m_D \cos\theta_D - 7.88\, m_A = -94.98 \quad \text{...(iii)}$$

$$8\, m_D \sin\theta_D - 1.39\, m_A = -73.73 \quad \text{...(iv)}$$

Plane	Mass, *m* (kg)	Eccentricity, *e* (cm)	Distance from RP A, x (cm)	Angle with ref. line *B*, θ(deg.)	Couple Vector		Force Vector	
					$mex\cos\theta$	$mex\sin\theta$	$me\cos\theta$	$me\sin\theta$
A	m_A	8	0	190	0	0	$-7.88\, m_A$	$-1.39\, m_A$
B	18	6	10	0	1080	0	108	0
C	12.5	6	30	100	–390.6	2212	–13.02	73.73
D	m_D	8	x_D	θ_D	$8m_D x_D\cos\theta_D$	$8m_D x_D\sin\theta_D$	$8m_D\cos\theta_D$	$8m_D\sin\theta_D$

From equation (i) and (ii), $\tan\theta_D = 3.21$.

Since x_D is known to be positive, both $\sin\theta_D$ and $\cos\theta_D$ are negative. So $\theta_D = 252.7°$.

Here, θ_D is the angle between the masses at *D* and *B*.

$$\cos\theta_D = -0.2975 \text{ and } \sin\theta_D = -0.955$$

Substituting $\cos\theta_D$ and $\sin\theta_D$ in equation (iii) and (iv), we have

$$-2.38\, m_D - 7.88\, m_A = -94.98 \quad \text{...(v)}$$

$$-7.64\, m_D - 1.39\, m_A = -73.73 \quad \text{...(vi)}$$

Solving (v) and (vi), we get

$$m_A = 9.67 \text{ kg},\ m_D = 7.89 \text{ kg}$$

From equation (i), we get $x_D = 36.57$ cm and the distance from *C* to *D* is $(x_D - 30) = 6.57$ cm.

Mass at *A* = 9.67 kg

Mass at *D* = 7.89 kg

Distance between the planes *C* and *D* = 6.57 cm

Angular position of the mass at *D* = 252.7° (with respect to *B*)

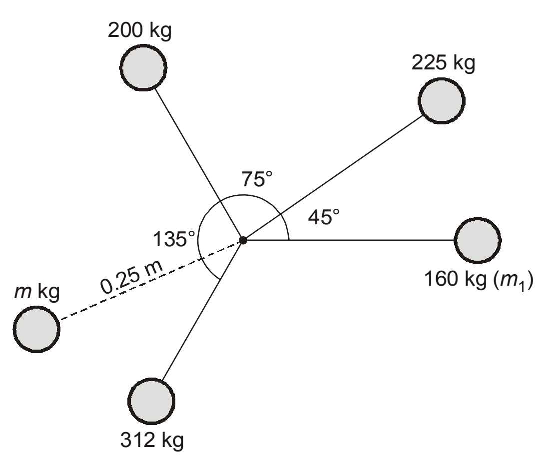
Solution : 25

Let the balancing mass be *m*.

Then the angle which the balancing mass makes with m_1 is θ.

Given: $m_1 = 160$ kg, $r_1 = 0.25$ m, $m_2 = 225$ kg, $r_2 = 0.2$ m, $m_3 = 200$ kg, $r_3 = 0.3$ m, $m_4 = 312$ kg, $r_4 = 0.25$ m

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius, therefore



$$m_1r_1 = 160 \times 0.25 = 40 \text{ kg.m}$$

$$m_2r_2 = 225 \times 0.2 = 45 \text{ kg.m}$$

$$m_3r_3 = 200 \times 0.3 = 60 \text{ kg.m}$$

$$m_4r_4 = 312 \times 0.25 = 78 \text{ kg.m}$$

$$\Sigma H = 0 = m_1r_1\cos\theta_1 + m_2r_2\cos\theta_2 + m_3r_3\cos\theta_3 + m_4r_4\cos\theta_4$$

$$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$$

$$= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg.m}$$

$$\Sigma V = 0 = m_1r_1\sin\theta_1 + m_2r_2\sin\theta_2 + m_3r_3\sin\theta_3 + m_4r_4\sin\theta_4$$

$$= 40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255°$$

$$= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg.m}$$

$$R = \left[(\Sigma H)^2 + (\Sigma V)^2\right]^{\frac{1}{2}} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2 \text{ kg.m} = mr$$

$$m = \frac{23.2}{0.25} = \mathbf{92.8 \text{ kg}}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{8.5}{21.6} = 0.3935 \text{ or } \theta = 21.48°$$

Since θ is the angle of the resultant R from the horizontal mass of 160 kg, therefore the angle of balancing mass from the horizontal mass of 160 kg.

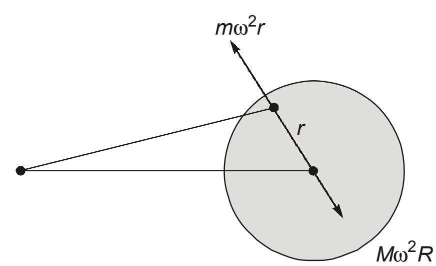
$$\theta = 180 + 21.48 = \mathbf{201.48°}$$

Solution : 26

No the reciprocating engine cannot be completely balanced.

To minimize unbalance we introduce a fraction of unbalanced mass at the crank opposite to unbalance

$$M\omega^2 R = cm\,\omega^2 r$$



If C is the fraction mass unbalanced

$$MR = cmr$$

$$\text{Unbalance along line of stroke} = m\,\omega^2 r\cos\theta - M\,\omega^2 R\cos\theta$$

$$= m\,\omega^2 r\cos\theta - cm\,\omega^2 r\cos\theta$$

$$= m\,\omega^2 r\cos\theta\,(1 - c)$$

$$\text{Unbalance along vertical} = M\,\omega^2 R\sin\theta = cm\,w^2 r\sin\theta$$

$$\text{Resultant} = \sqrt{(m\omega^2 r)^2 c^2 \sin^2\theta + (m\omega^2 r)^2 \cos^2\theta(1-c)}$$

$$= m\omega^2 r\sqrt{c^2\sin^2\theta + \cos^2\theta(1-c)^2}$$

If $C = \frac{1}{2}$ the forces are minimum

$$\text{Resultant} = \frac{m\omega^2 r}{2} \times \left(\frac{2\pi \times 250}{60}\right)^2 \times 0.2 = 8224.6 \text{ N}$$

mass required at a radius of 30 cm

$$M = \frac{1/2 \times 120 \times 20}{30} = 40 \text{ kg}$$

■■■■

7 Vibrations

LEVEL 1 Objective Questions

1.	(d)
2.	(d)
3.	(d)
4.	(a)
5.	(a)
6.	(3)
7.	(b)
8.	(d)
9.	(d)
10.	(d)
11.	(b)
12.	(a)

LEVEL 2 Objective Questions

13.	(b)
14.	(a)
15.	(1.11)
16.	(c)
17.	(b)
18.	(2.28)
19.	(0.56)
20.	(26.74)
21.	(1.9052)
22.	(a)
23.	(0.667)
24.	(1)
25.	(8)
26.	(2)
27.	(0.05)

LEVEL 3 Conventional Questions

Solution : 28

(a) Newton's Method

Let a = area of cross section of the tube

ρ = mass density of water

l = total length of water column

Inertia force + External force = 0

Mass × Acceleration + weight of water column above $h - h$ = 0

$$(al\rho)\times\ddot{x} + (a\times 2x)\rho g = 0$$

or
$$x + \frac{2g}{l}x = 0$$

Energy method,

At any instant, $\frac{d}{dt}(KE + PE) = 0$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(al\rho)\dot{x}^2$$

PE = work to transfer a water column of length x from the right-hand side to the left-hand side.

$$= mgx = (a\ x\ \rho)gx = a\rho gx^2$$

$$= \frac{d}{dt}\left(\frac{1}{2}al\rho\dot{x}^2 + a\rho gx^2\right) = 0$$

$$\frac{1}{2}al\rho\times 2\dot{x}\ddot{x} + a\rho g\times 2x\dot{x} = 0$$

$$\ddot{x} + \frac{2g}{l}x = 0\ \omega_n = \sqrt{\frac{2g}{l}}$$

Solution : 29

m = 15 kg, Δ = 12 mm, F_0 = 100 N, f = 6 Hz

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.012}} = 4.55\text{ Hz}$$

The motion becomes aperiodic when the damped frequency is zero or when it is critically damped ($\zeta = 1$)

and
$$\omega = \omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{0.012}} = 28.59\text{ rad/s}$$

$$c = c_c = 2\ m,\ \omega_n = 2\times 15\times 28.59$$
$$= 857\text{ N/m/s} = 0.857\text{ N/mm/s}$$

Thus, the force needed is 0.857 N at a speed 1 mm/s.

$$A = \frac{F_0}{\sqrt{(s - m\omega^2)^2 + (c\omega)^2}}$$

But
$$\omega = 2\pi\times f = 2\pi\times 6 = 37.7\text{ rad/s}$$

and s can be found from $$f_n = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

or $$4.55 = \frac{1}{2\pi}\sqrt{\frac{s}{15}}$$

or $$s = 12260 \text{ N/m}$$

$\therefore$ $$A = \frac{100}{\sqrt{[12260 - 15\times(37.7)^2]^2 + (857\times 37.7)^2}}$$

$$= 0.00298 \text{ m} = 2.98 \text{ mm}$$

Solution : 30

d = 40 mm = 0.04 m, l = 2.5

$$I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}\times(0.04)^4 = 0.1257 \times 10^{-6} \text{ m}^4$$

We have $$f_n = \frac{0.4985}{\sqrt{\Delta_1 + \Delta_2 + \Delta_3 + \ . \ . \ . \ \frac{\Delta_s}{1.27}}}$$

$$\Delta_1 = \frac{mga^2b^2}{3EIl}$$

Here, m = 90 kg, a = 0.8 m and b = 1.7 m.

$\therefore$ $$\Delta_1 = \frac{90\times 9.81\times(0.8)^2\times(1.7)^2}{3\times 200\times 10^9\times 0.1257\times 10^{-6}\times 2.5} = 0.00866 \text{ m}$$

For Δ_2, m = 140 kg, a = 1.5 m, b = 1 m

$\therefore$ $$\Delta_2 = \frac{140\times 9.81\times(1.5)^2\times(1)^2}{3\times 200\times 10^9\times 0.1257\times 10^{-6}\times 2.5} = 0.1639 \text{ m}$$

For D$_3$, m = 60 kg, a = 2 m, b = 0.5 m

$\therefore$ $$\Delta_3 = \frac{60\times 9.81\times(2)^2\times(0.5)^2}{3\times 200\times 10^9\times 0.1257\times 10^{-6}\times 2.5} = 0.00312$$

$$\Delta_s = \frac{5mgl^4}{384EI} = \frac{5\times 15\times 9.81\times(2.5)^4}{384\times 200\times 10^9\times 0.1257\times 10^{-6}} = 0.00298 \text{ m}$$

$$f_n = \frac{0.4985}{\sqrt{0.00866 + 0.01639 + 0.00312 + \frac{0.00298}{1.27}}} = 2.85 \text{ Hz}$$

Solution : 31

(a) Say initial displacement is θ, it decay 50% so

$$\delta = \log_e\left(\frac{\theta}{\theta/2}\right) = \log_e 2 = 0.693$$

(b) $$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

So $$(1-\varepsilon)^2\delta^2 = 4\pi^2\varepsilon^2$$
or $$\delta^2 = (4\pi^2+\delta^2)\varepsilon^2$$
$$\varepsilon = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = \frac{0.693}{\sqrt{4\pi^2+(0.693)^2}} = 0.109$$

We know that $$\varepsilon = \frac{C}{C_c} \quad \text{or} \quad C = \varepsilon C_c$$

But $$C_c = 2\sqrt{k_t J}$$
$$C = 2\varepsilon\sqrt{k_t J}$$
$$J = .05 \text{ kg-m}^2$$
$$k_t = \frac{GJ}{l}\left[k_t = \frac{T}{\theta} = \frac{GJ}{l}\right] = \frac{4.5\times10^{10}\times\frac{\pi}{32}\times(.1)^4}{0.5}\left(J = \frac{\pi}{32}d^4\right)$$
$$k_t = 8.83125\times10^5 \text{ Nm/rad}$$

So, $$C = \varepsilon\times2\sqrt{8.83125\times10^5\times.05}$$
$$= 0.109\times2\times2.1013\times10^2 = 45.809 \text{ Nm/rad}$$

This is damping torque per unit velocity.

(c) periodic time of oscillation $= \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{\omega_n\sqrt{1-\varepsilon^2}} = \dfrac{2\pi}{\sqrt{\dfrac{k_t}{J}[1-(.109)^2]}}$

$$= \frac{2\pi}{\sqrt{\frac{8.83125\times10^5}{.05}(.9881)}} = 1.503\times10^{-3} \text{ sec}$$

(d) When the disc is removed from viscous fluid, the natural frequency is given

$$f = \frac{\omega_n}{2\pi}$$

But $$\omega_n = \sqrt{\frac{k_t}{J}} = \sqrt{\frac{8.83125\times10^5}{.05}} = 4202.67 \text{ rad/sec}$$

So $$f_n = \frac{4202.67}{2\pi} = 669.2 \text{ Hz}$$

Solution : 32

The equation of motion can be written as

$$ml^2\ddot{\theta} = -k_1l_1^2\theta - Cl_2^2\dot{\theta} - mgl\theta$$
$$ml^2\ddot{\theta} + Cl_2^2\dot{\theta} + (k_1l_1^2 + mgl)\theta = 0$$
$$\ddot{\theta} + \frac{Cl_2^2\dot{\theta}}{ml^2} + \frac{(k_1l_1^2+mgl)}{ml^2} = 0$$

The general equation for such a system is $\ddot{\theta} + 2\varepsilon\omega_n\dot{\theta} + \omega_n^2\theta = 0$

Let us compare this equation with general from, we have

$$2\varepsilon\omega_n = \frac{Cl_2^2}{ml^2} \quad \text{. . . (i)}$$

and
$$\omega_n^2 = \frac{\left(k_1 l_1^2 + mgl\right)}{ml^2} \quad \text{. . . (ii)}$$

So,
$$\omega_n = \frac{Cl_2^2}{ml^2 \times 2\varepsilon} \quad \text{from equation (i) . . . (iii)}$$

or
$$\omega_n^2 = \left(\frac{Cl_2^2}{2\varepsilon ml^2}\right)^2 \quad \text{. . . (iv)}$$

From equation (iv) and (ii), we get

$$\left(\frac{Cl_2^2}{2\varepsilon ml^2}\right)^2 = \left(\frac{k_1 l_1^2 + mgl}{ml^2}\right)$$

$$(1-\varepsilon^2) = 1-\frac{c^2 l_2^4}{4ml^2(k_1 l_1^2 + mgl)} \quad \text{. . . (v)}$$

We know that damped frequency ω_d is given by the expression

$$\omega_d^2 = \left(1-\varepsilon^2\right)\omega_n^2$$

Using equation (v) in the above expression, we get

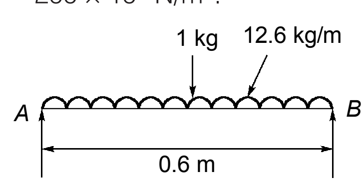
$$\omega_d^2 = \left(1-\frac{C^2 l_2^4}{4ml^2\left(k_1 l_1^2 + mgl\right)}\right)\left(\frac{k_1 l_1^2 + mgl}{ml^2}\right) \quad \text{(Putting w from eq.(ii))}$$

$$= \left(\frac{4ml^2(k_1 l_1^2 + mgl) - C^2 l_2^4}{4ml^2(k_1 l_1^2 + mgl)}\right)\left(\frac{k_1 l_1^2 + mgl}{ml^2}\right)$$

So,
$$\omega_d = \sqrt{\frac{k_1 l_1^2 + mgl}{ml^2} - \left(\frac{Cl_2}{2ml^2}\right)^2}$$

Solution : 33

Given : d = 20 mm = 0.02 m; l = 0.02 m; l = 0.6 m, m_1 = 1 kg; ρ = 40 Mg/m^3 = 40 × 10^6 g/m^3 = 40 × 10^3 kg/m^3; E = 200 GN/m^2 = 200 × 10^9 N/m^2.



The shaft is shown in figure.

We known that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.02)^4 \text{m}^4 = 7.855 \times 10^{-9} \text{ m}^4$$

Since the density of shaft material is 40 × 10^3 kg/m^3, therefore mass of the shaft per metre length,

$$m_s = \text{Area} \times \text{length} \times \text{density}$$

$$= \frac{\pi}{2}(0.02)^2 \times 1 \times 40 \times 10^3 = 12.6 \text{ kg/m}$$

$$\delta = \frac{Wl^3}{48EI} = \frac{1. \times 9.81(0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}}$$

$= 28 \times 10^{-6}$ m and static deflection due to mass of the shaft,

$$\delta = \frac{5wl^4}{384EI} = \frac{5 \times 12.6 \times 9.81(0.6)^4}{384 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 0.133 \times 10^{-3} \text{m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta + \dfrac{\delta_s}{1.27}}} + \frac{0.4985}{\sqrt{28 \times 10^{-6} + \dfrac{0.133 \times 10^{-3}}{1.27}}} = \frac{0.4985}{11.52 \times 10^{-3}} = 43.3 \text{ Hz}$$

Let N_c = whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse vibration in Hz, therefore

$$N_c = 43.4 \text{ r.p.s.} = 43.3 \times 60 \text{ r.p.m.} = 2598 \text{ r.p.m}$$

Solution : 34

Given: Stiffness of each spring = 7.5 N/mm, Total/equivalent stiffness of springs = 4 × 7.5 = 30 N/mm, x_1 = 17.2 mm, x_3 = 3.2 mm

$$\frac{x_1}{x_3} = \left(\frac{x_1}{x_2} \times \frac{x_2}{x_3}\right)$$

We know that,

$$\frac{x_1}{x_2} = \frac{x_2}{x_3}$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/2} = \left(\frac{17.2}{3.2}\right)^{1/2} = 2.32$$

Natural circular frequency of motion is given by

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{30 \times 10^3}{76}} = 19.87 \text{ rad/s}$$

We know that,

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{2\pi a}{\sqrt{\omega_n^2 - a^2}}$$

$$\log_e 2.32 = \frac{2\pi a}{\sqrt{\omega_n^2 - a^2}}$$

$$(\omega_n^2 - a^2)(0.8416)^2 = 4\pi^2 a^2$$

$$(19.872^2 - a^2) \times 0.708 = 39.48\,a^2$$

$$279.53 = (39.48 + 0.708)a^2$$

$$a = \left(\frac{279.53}{40.188}\right)^{1/2} = 2.637$$

$$a = \frac{C}{2\,m} \text{ and } C = 2\,ma = 2 \times 76 \times 2.637 = \mathbf{400.824\ N/m/s}$$

$$\frac{f_d}{f_n} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \sqrt{\frac{(19.87)^2 - (2.637)^2}{19.87}} = 0.99$$

$$\text{Periodic time of damped vibration} = \frac{2\pi}{\omega_d}$$

$$T_d = \frac{2\pi}{\sqrt{\omega_n^2 - a^2}} = \frac{2\pi}{\sqrt{19.7^2 - 2.637^2}} = 0.32\text{ sec}$$

Solution : 35

Given: $I = 600$ kg cm^2, $d = 10$ cm, $l = 40$ cm, $\theta_1 = 9°$, $\theta_2 = 6°$, $\theta_3 = 4°$, $G = 4.4 \times 1010$ N/m^2

Polar moment of inertia of shaft:

$$J = \frac{\pi d^4}{32} = \frac{\pi \times (0.1)^4}{32} = 9.8175 \times 10^{-6}\text{m}^4$$

$$\text{Torsional stiffness of shaft}, q = \frac{GJ}{l} = \frac{4.4 \times 10^{10} \times 9.8175 \times 10^{-6}}{0.4} = 1079922\text{ N.m/rad}$$

$$\omega_n\,(\text{Natural frequency}) = \sqrt{\frac{q}{I}} = \left(\frac{1079922}{600 \times 10^{-4}}\right)^{\frac{1}{2}} = 4242.5\text{ rad/s}$$

(i) Logarithmic decrement, $\delta = \ln\left(\frac{\theta_n}{\theta_{n+1}}\right) = \ln\left(\frac{9}{6}\right) = \ln(1.5) = 0.405$

(ii) Critical damping coefficient,

$$C_c = 2\sqrt{Iq} = \sqrt{600 \times 10^{-4} \times 1079922} = 509.1\text{ N.M.s/rad}$$

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$(0.405)^2(1-\zeta^2) = (2\pi)^2\zeta^2$$

$$0.1640 = 39.642\,\zeta^2$$

$$\zeta = 0.0643$$

Damping coefficient, $C = C_c\zeta = 509.1 \times 0.0643 = 32.745$ N.m.s/rad

(iii)

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4242.5\sqrt{1-(0.0643)^2} = 4233.72\text{ rad/s}$$

$$T_d = \frac{2\pi}{\omega_d} = \frac{2 \times 3.14}{4233.72} = 1.484 \times 10^{-3}\text{ s}$$

Solution : 36

Given: m_1 = 100 kg; m_2 = 2 kg; l = 80 mm = 0.08 m; e = $\frac{1}{25}$; N = 1000 rpm; or

$$\omega = 2p \times \frac{1000}{60} = 104.7 \text{ rad/s}$$

Combined stiffness of springs

Let s = Combined stiffness of springs in N/m and

ω_n = Natural circular frequency of vibration of the machine in rad/s

We know that transmissibility ratio (ε)

$$\frac{1}{25} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(104.7)^2 - (\omega_n)^2}$$

or $$(104.7)^2 - (\omega_n)^2 = 25(\omega_n)^2$$

or $$(\omega_n)^2 = 421.6$$

or $$\omega_n = 20.5 \text{ rad/s}$$

We know that $$\omega_n = \sqrt{\frac{s}{m_1}}$$

$\therefore$ $$s = m_1(\omega_n)^2 = 100 \times 421.6 = 42160 \text{ N/m}$$

1. Force transmitted to the foundation at 1000 rpm

Let F_T = Force transmitted, and

x_1 = Initial amplitude of vibration

Since the damping reduces the amplitude of successive free vibrations by 25%, therefore final amplitude of vibration,

$$x_2 = 0.75x_1$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$\{a = z\omega_n\}$

or $$\log_e\left(\frac{x_1}{0.75x_1}\right) = \frac{a \times 2\pi}{\sqrt{421.6 - a^2}}$$

Squaring both sides,

$$(0.2877)^2 = \frac{a^2 \times 4\pi^2}{421.6 - a^2}$$

or $$0.083 = \frac{39.5a^2}{421.6 - a^2} \qquad \because \left[\log_e\left(\frac{1}{0.75}\right) = \log_e 1.333 = 0.2877\right]$$

$$35 - 0.083\,a^2 = 39.5a^2$$

or $$a^2 = 0.884$$

or $$a = 0.94$$

We know that damping coefficient or damping force per unit velocity,

$$c = a \times 2m_1 = 0.94 \times 2 \times 100 = 188 \text{ N/m/s}$$

and critical damping coefficient,

$$c_c = 2m\,\omega_n = 2 \times 100 \times 20.5 = 4100 \text{ N/m/s}$$

$\therefore$ Actual value of transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c\omega}{c_c \cdot \omega_n}\right)}}{\sqrt{\left(\dfrac{2c\omega}{c_c \cdot \omega_n}\right)^2+\left(1-\dfrac{\omega^2}{(\omega_n)^2}\right)^2}} = \frac{\sqrt{1+\left(\dfrac{2\times188\times104.7}{4100\times20.5}\right)^2}}{\sqrt{\left(\dfrac{2\times188\times104.7}{4100\times20.5}\right)^2+\left(1-\left(\dfrac{104.7}{(20.5)^2}\right)^2\right)^2}}$$

$$= \frac{\sqrt{1+0.22}}{\sqrt{0.22+629}} = \frac{1.104}{25.08} = 0.044$$

We know that the maximum unbalanced force on the machine due to reciprocating parts,

$$F = m_2\omega^2 r = 2(104.7)^2(0.08/2) = 877 \text{ N} \qquad ...(r = l/2)$$

$\therefore$ Force transmitted to the foundation,

$$F_T = \varepsilon F = 0.044 \times 877 = 38.6 \text{ N} \qquad ...(\varepsilon = F_T/F)$$

2. Force transmitted to the foundation at resonance

Since at resonance, $\omega = \omega_n$, therefore transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\dfrac{2c}{c_c}\right)^2}}{\sqrt{\left(\dfrac{2c}{c_c}\right)^2}} = \frac{\sqrt{1+\left(\dfrac{2\times188}{4100}\right)^2}}{\sqrt{\left(\dfrac{2\times188}{4100}\right)^2}} = \frac{\sqrt{1+0.0084}}{0.092} = 10.92$$

and maximum unbalanced force on the machine due to reciprocating parts at resonance speed ω_n,

$$F = m_2(\omega_n)^2 r = 2(20.5)^2(0.08/2) = 33.6 \text{ N} \qquad ...(\because r = l/2)$$

$\therefore$ Force transmitted to the foundation at resonance,

$$F_T = \varepsilon F = 10.92 \times 33.6 = 367 \text{ N}$$

3. Amplitude of the forced vibration of the machine at resonance

$$= \frac{\text{Force transmitted at resonance}}{\text{Combined stiffness}} = \frac{367}{42160} = 8.7 \times 10^{-3} \text{ m} = 8.7 \text{ mm}$$

Solution : 37

$$m = 1 \text{ tonne} = 1000 \text{ kg}$$

Logarithmic decrement for n cycle is given by

Here, $$n = 4 \text{ cycles}$$

$$\delta = \frac{1}{n}\log_n \frac{x_1}{x_{n+1}} = \frac{1}{4}\log_e \frac{5}{0.10} = 0.978$$

We have equation, $$\delta = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

or $$0.978 = \frac{2\pi\varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\frac{\varepsilon}{\sqrt{1-\varepsilon^2}} = 0.155$$

$$\varepsilon^2 = 0.023$$

So $$\varepsilon = 0.15$$

Damped frequency $$\omega_d = \frac{2\pi}{T} = \frac{2\pi}{0.64} = 9.8 \text{ rad/sec}$$

$$\frac{\omega_d}{\omega_n} = \sqrt{1-\varepsilon^2}$$

So $$\omega_n = \frac{\omega_d}{\sqrt{1-\varepsilon^2}} = \frac{9.8}{\sqrt{1-0.15\times0.15}} = 9.9 \text{ rad/sec} = \sqrt{\frac{k}{m}}$$

$$\omega_n^2 = \frac{k}{m}$$

So, $$k = m\omega_n^2 = 1000 \times (9.9) = 98010 \text{ N/m}$$

Critical damping, $$C_c = 2\text{ m}\,.\,\omega_n = 2 \times 1000 \times 9.9 = 19800 \text{ N-m/rad}$$

$$\varepsilon = \frac{C}{C_c}$$

So, $$C = \varepsilon \times C_c = 0.15 \times 19800 = 2970 \text{ N-m/rad}$$

Solution : 38

$$k = \text{force/deflection}$$

$$= \frac{70\times9.81}{0.02} = 34.335\times10^3 \text{ N/m}$$

$$\xi = \frac{C}{C_c} = 0.23$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{34.335\times10^3}{70}} = 22.15 \text{ rad/s}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 22.15\sqrt{1-(0.23)^2} = 21.56 \text{ rad/s}$$

Logarithmic decrement, $$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\times3.14\times0.23}{\sqrt{1-(0.23)^2}} = 1.48$$

Ratio of successive amplitudes,

$$\frac{A_1}{A_2} = e^{\delta} = e^{1.48} = 4.39$$

Expression of amplitude, $$A = \frac{F/k}{\sqrt{\left(1-r_\omega^2\right)^2 + \left(2\xi r_\omega^2\right)^2}}$$

$F = 700$ N, $k = 34335$ N/m, $r_\omega = \dfrac{\omega}{\omega_n} = 0.78$, $\xi = 0.23$

$$A = \frac{\left(\frac{700}{34335}\right)}{\sqrt{\left(1-0.78^2\right)^2 - (2\times 0.23\times 0.78)^2}} = 0.13\text{ m} = 130\text{ mm}$$

$$\tan\phi = \frac{2\xi r_\omega}{1-r_\omega^2} = \frac{2\times 0.23\times 0.78}{1-0.78^2} = 0.916$$

$$\therefore \quad \phi = 42.4897°$$

Solution : 39

We know,

Transmissibility ratio, $$\frac{F_{TR}}{Fo} = \frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

as there is no damper, $\xi = 0$

Now, $$\frac{F_{TR}}{F_o} = \frac{1}{-(1-r^2)}$$

$$\Rightarrow \quad 0.1 = \frac{1}{r^2-1}$$

$$\Rightarrow \quad r^2 - 1 = 10$$

$$r^2 = 11$$

$$r = \sqrt{11}$$

and $$r = \frac{\omega}{\omega_n}$$

$$\therefore \quad \frac{\omega}{\omega_n} = \sqrt{11}$$

$$\omega_n = \frac{\omega}{\sqrt{11}}$$

$$\omega = \frac{2\pi N}{60}\text{ rad/s} = 50.265\text{ rad/s}$$

$$\therefore \quad \omega_n = 15.155\text{ rad/s}$$

$$\therefore \quad \sqrt{\frac{k_{eq}}{m}} = 15.155$$

$$k_{eq}/\text{m} = 229.674$$

$$k_{eq} = 35\times 229.674$$

$$k_{eq} = 8038.59$$

$$\therefore \quad 5\,k = 8038.59$$

$$k = 1607.718\text{ N/m}$$

$$k = 1.607\text{ N/mm}$$

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