2020

RANK improvement WORKBOOK

Detailed Explanations of Objective & Conventional *Questions*

Mechanical Engineering

Mechatronics and Robotics

MADE EASY Publications

1 Introduction to Mechatronics

LEVEL 1 Objective Questions

1.	(a)
2.	(b)
3.	(c)
4.	(d)
5.	(c)
6.	(c)
7.	(c)
8.	(b)
9.	(d)
10.	(d)
11.	(d)
12.	(b)
13.	(b)
14.	(d)
15.	(d)
16.	(a)
17.	(c)
18.	(d)

LEVEL 2 Objective Questions

19.	(d)
20.	(c)
21.	(a)
22.	(c)
23.	(b)
24.	(b)
25.	(a)
26.	(c)

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LEVEL 3 Conventional Questions

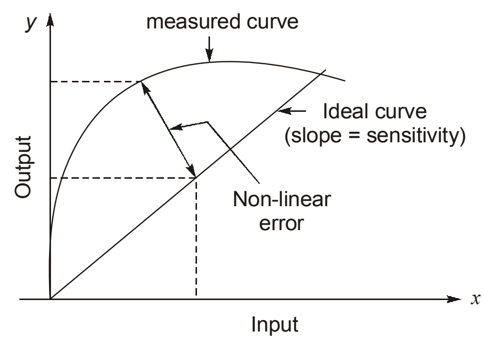
Solution : 27

(i) Sensitivity of sensor is defined as the ratio of change in the output value of a sensor to the change in the input that causes the output.

or

Sensitivity is the ratio between output span of a sensor to Input span of a sensor.
For exp: Thermocouple has a sensitivity of 55 $\mu v/°C$, this means, thermocouple can generate 55 μv for every 1°C rise in input temperature.

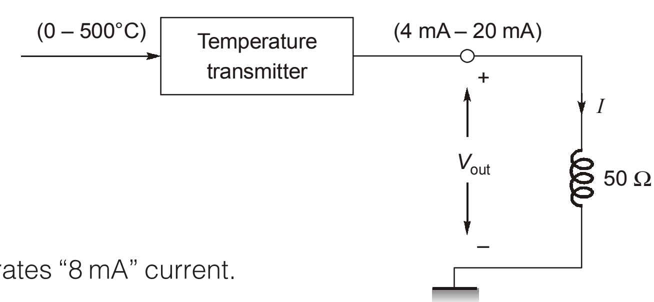
(ii) Linearity: If the data of input and output follows a linear relation ($y = mx$), then we can say the sensor is linear For any sensor consider the following.



Non linear error: The maximum deviation between measured output and actual output at any particular input is called non-linear error.

(iii) Resolution is the Smallest incremental change of input that can be applied to produce the detectable output is called resolution.

Solution : 28



Given that $\quad V_{out} = 50 \times I$

$$\frac{4v}{50\Omega} = I$$

$\Rightarrow \quad 8 \text{ mA} = I$

- This means the temperature transmitter generates "8 mA" current.
- For temperature transmitter

0°C + 125°C ⟶ 4 mA + 4 mA

125°C ⟵ 8 mA

500°C ⟶ 20 mA

$$\text{Sensitivity of transmitter} = \frac{20\text{mA} - 4\text{mA}}{500°\text{C} - 0°\text{C}} = \frac{16\text{mA}}{500°\text{C}}$$

i.e. for a increment of 125°C generated output is 8 mA.

Solution : 29

Given that natural frequency of work piece

$$\omega_n = 2\pi f_n$$
$$= 2 \times 3.14 \times 16 = 100.48 \text{ rad/sec}$$

at steady state (when $\omega << \omega_n$)

The amplitude of acceleration is given by

$$\frac{X_0}{A_i} = \frac{1}{\omega_n^2}$$

$x_0 \rightarrow$ Relative displacement of mass

$A_i \rightarrow$ Input acceleration

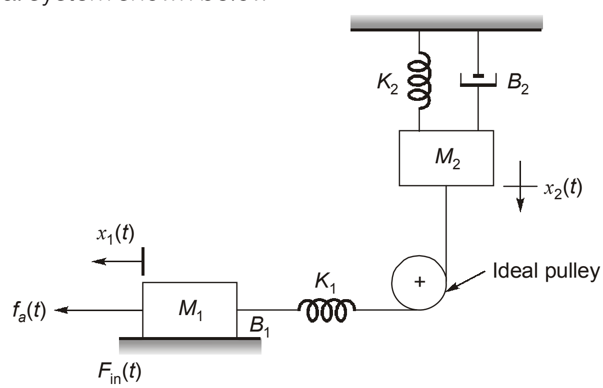
$\omega_n \rightarrow$ Natural frequencies of work piece

$$A_i = \omega_n{}^2 x_0$$

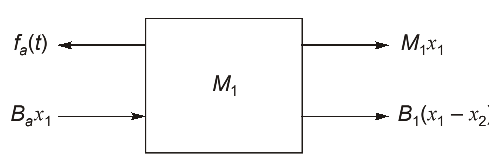
$$= (100.48)^2 \times 5 \times 10^{-3} = 50.481 \text{ m/sec}^2$$

Solution : 30

Consider the mechanical system shown below

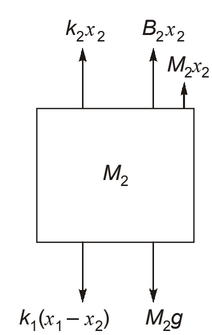


Freebody diagram of Mass 'M',



$$f_a(t) = M_1\ddot{x}_1(t) + B_1\dot{x}_1(t) + k_1\left(x_1(t) - x_2(t)\right)$$

$$\Rightarrow \quad f_a(t) = M_1\frac{d^2}{dt^2}x_1(t) + B_1\frac{d}{dt}x_1(t) + k_1x_1(t) - k_2x_2(t)$$



Free body diagram of Mass 'M_2'

$$M_2g = M_2\,\ddot{x}_2 + B_2\,\dot{x}_2 + (k_1 + k_2) - k_1\,x_1$$

$$\Rightarrow \quad M_2g = M_2\frac{d^2}{dt^2}x_2(t) + B_2\frac{d}{dt}x_2(t) + (k_1 + k_2)x_2(t) - k_1x_1(t)$$

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2 Microprocessors and Microcontrollers

LEVEL 1 Objective Questions

1.	(d)
2.	(b)
3.	(d)
4.	(a)
5.	(b)
6.	(c)
7.	(b)
8.	(d)
9.	(a)
10.	(c)
11.	(d)

LEVEL 2 Objective Questions

12.	(d)
13.	(a)
14.	(d)

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LEVEL 3 Conventional Questions

Solution : 15

ROM: ROM is Read Only Memory. It is used for store elements programs/data. It is not lost when power is removed.

PROM: PROM is Programmable Read Only Memory. It is a type of ROM chip programmed by user for one time only.

EPROM: EPROM is Erasable and Programmable ROM. It is a type of ROM chip programmed by user and contents can be erased for repeated programming.

EEPROM: It is electrically erasable PROM. It is similar to EPROM, but erasing is done electrically by applying high voltage and programmed repeatedly.

RAM: Random Access Memory. It is used to store temporary data/programs during execution. It is lost with power.

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3 Actuators

LEVEL 1 Objective Questions

1.	(b)
2.	(a)
3.	(b)
4.	(b)
5.	(b)
6.	(b)
7.	(b)
8.	(a)
9.	(c)
10.	(a)
11.	(a)
12.	(c)

LEVEL 2 Objective Questions

13.	(c)
14.	(b)
15.	(a)
16.	(c)
17.	(a)
18.	(b)

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4 Sensors and Transducers

LEVEL 1 Objective Questions

1.	(d)
2.	(b)
3.	(c)
4.	(a)
5.	(a)
6.	(c)
7.	(c)
8.	(b)
9.	(b)
10.	(c)
11.	(a)
12.	(a)
13.	(c)
14.	(b)
15.	(b)
16.	(d)

LEVEL 2 Objective Questions

17.	(b)
18.	(b)
19.	(a)
20.	(b)
21.	(a)
22.	(d)
23.	(d)
24.	(a)
25.	(b)
26.	(d)
27.	(a)
28.	(c)
29.	(a)
30.	(c)
31.	(c)
32.	(c)
33.	(d)
34.	(c)
35.	(b)
36.	(c)
37.	(d)
38.	(c)
39.	(d)
40.	(d)

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LEVEL 3 Conventional Questions

Solution : 41

$$K_1 = \text{Transducer sensitivity} = 0.3 \text{ ohm/°C}$$
$$K_2 = \text{Wheatstone bridge sensitivity} = 0.01 \text{ V/ohm}$$
$$K_3 = \text{Amplifier gain} = 80 \text{ V/V}$$
$$K_4 = \text{Pen recorder} = 1.2 \text{ mm/V}$$
$$\text{Overall sensitivity, } K = K_1 \times K_2 \times K_3 \times K_4 = 0.3 \times 0.01 \times 80 \times 1.2$$
$$= 0.288 \text{ mm/°C because } \left[\frac{ohm}{C} \times \frac{V}{ohm} \times \frac{V}{V} \times \frac{mm}{V}\right]$$
$$\text{Static sensitivity, } K = \frac{\text{Change of output signal}}{\text{Change of input signal}}$$
$$0.288 = \frac{30}{\text{Change of input signal}}$$
$$\text{Temperature change} = \frac{30}{0.288} = 104.167\text{°C}$$

Solution : 42

(a) $$\text{Span of potentiometer} = 160° - (-160°) = 320°$$

(b) $$\text{Sensitivity} = \frac{12-0}{320} = 0.0375 \text{ V/deg}$$

(c) $$\text{Average resolution} = \frac{12}{850} = 0.0141 \text{ V}$$

Solution : 43

$$\text{Range of pressure} = \text{Maximum pressure} - \text{Minimum pressure} = 20 - 0 = 20 \text{ bar}$$
$$\text{Percentage of error} = \frac{\text{Calibration pressure}}{\text{Range of pressure}} = \frac{0.125}{20} \times 100 = \pm 0.625\%$$
$$\text{Possible error at 2.5 bar} = \frac{0.125}{2.5} \times 100 = 5\%$$

Solution : 44

$$\text{Maximum voltage, } V = \frac{\text{Power}}{\text{Current}} = \frac{P}{I} \qquad [\because P = VI]$$
$$= \frac{P}{V/R} \qquad [\because V = IR]$$
$$V^2 = PR$$
$$V = \sqrt{PR} = \sqrt{50 \times 10^{-3} \times 8 \times 10^3} = 20 \text{ V}$$
$$\text{Output voltage, } V_0 = V\frac{x_1}{x_r} = 20 \times \frac{2}{5} = 8 \text{ V}$$

Solution : 45

$$\text{Gauge factor, GF} = \frac{\Delta R/R}{\Delta L/L}$$

$$\Delta L = \frac{(\Delta R/R)\times L}{GF} = \frac{(0.015/220)\times 0.15}{2} = 5.114 \times 10^{-6}\ \text{m}$$

$$\text{Stress, } \sigma = \frac{200\times 10^{9}\times 5.114\times 10^{-6}}{0.15\times 10^{6}} = 6.8186\ \text{MPa}$$

$$\text{Force, } F = \sigma \times A$$

$$F = 6.8186 \times 10^{6} \times 5 \times 10^{-4} = 3409.33\ \text{N}$$

Solution : 46

$$\text{Sensitivity of LVDT} = \frac{\text{Output current}}{\text{Displacement}} = \frac{3\times 10^{-3}}{0.75} = 4 \times 10^{-3}\ \text{A/mm}$$

$$\text{Sensitivity of insturment} = \text{Amplification factor} \times \text{sensitivity of LVDT} = 4 \times 10^{-3} \times 20 = 0.8\ \text{A/mm}$$

$$\text{1 scale division} = \frac{10}{100} = 0.1\ \text{A}$$

$$\text{Least current that can be measured} = \frac{1}{10}\times 0.1 = 0.01\ \text{A}$$

$$\text{Resolution} = 0.01\times\frac{1}{0.8} = 0.0125\ \text{mm}$$

Solution : 47

$$\text{Moment of inertia for rectangular section, } I = \frac{bt^3}{12}$$

$$= \frac{0.025\times 0.005^3}{12} = 2.6 \times 10^{-9}\ \text{m}^4$$

(a) $$\text{Deflection, } \delta = \frac{FL^3}{3EI} = \frac{30\times 0.3^3}{3\times 200\times 10^{9}\times 2.6\times 10^{-9}} = 5.19 \times 10^{-4}\ \text{m} = 0.52\ \text{mm}$$

(b) $$\text{Deflection per unit force} = \frac{x}{F} = \frac{5.19\times 10^{-4}}{30} = 0.0173\ \text{mm/N}$$

$$\text{Total sensitivity} = 0.0173 \times 0.6 = 0.0104\ \text{V/N}$$

$$\text{1 scale division} = \frac{20}{100} = 0.2\ \text{V}$$

$$\text{Resolution} = \frac{2}{100}\times 0.2 = 0.04\ \text{V}$$

$$\text{Minimum force} = \frac{0.04}{0.0104} = 3.85\ \text{N}$$

$$\text{Maximum force} = \frac{20}{0.0104} = 1923.08\ \text{N}$$

Solution : 48

$$\text{Capacitance, } C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

Area of plates, $A = 2 \times 30 \times 20 = 1200\ \text{cm}^2$ (number of plates = 2)

$$\text{Displacement sensitivity} = \frac{dC}{\mathrm{d}d}$$

$$\frac{dC}{\mathrm{d}d} = \omega_0 \varepsilon_r A \left(-\frac{1}{d^2}\right) = -\frac{\omega_0 \varepsilon_r A}{d^2}$$

$$= \frac{-8.854 \times 10^{-12} \times 1.0006 \times 0.12}{(1.2 \times 10^{-3})^2}$$

$$= -7.383 \times 10^{-7}\ \text{F/m} = -7383\ \text{nF/m}$$

Note: Minus sign indicates that the capacitance will increase for decreasing value of d.

Solution : 49

$$\text{Force, } F = kx$$

$$= 15 \times 45 \times 10^{-3}$$

$$\text{Weight of the object, } W = \text{Force} = 0.675\ \text{kN}$$

We know that $F = \text{mg}$

$$m = \frac{F}{g} = \frac{0.675 \times 10^3}{9.81} = 68.81\ \text{kg}$$

$$\text{Deflection, } x = \frac{8FD^3 n}{Gd^4}$$

$$45 = \frac{8 \times 0.675 \times 500^3 \times n}{4 \times 10^6 \times 2^4} = 4.267 \simeq 5\ \text{coils}$$

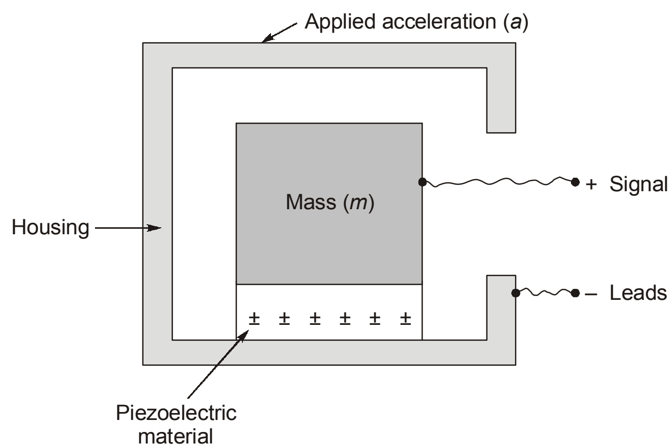
Solution : 50

Piezoelectric Accelerometer: A piezoelectric accelerometer that utilizes the piezoelectric effect of certain materials to measure dynamic changes in mechanical variables (e.g. acceleration, vibration and mechanical stock). As with all transducers, piezoelectric accelerometers convert one form of energy into another and provide an electrical signal in response to a quantity property or condition that is being measured. Using the general sensing method upon which all accelerometers are based, acceleration acts upon a seismic mass that is restrained by a spring or suspended on a cantilever beam and converts a physical force into a electrical signal. Before the acceleration can be converted into an electrical quantity it must first be converted into either a force or displacement. This conversion is done via the mass spring system. Piezoelectric accelerometers are widely accepted as the best choice for measuring absolute vibration.

Accelerometer design is based on

1. Shear system
2. Compression system
3. Bending or flexure system

The reason for using different piezoelectric system is their individual suitability for various measurement tasks and their sensitivity to environmental influences. Shear design is applied in the major part of modern accelerometers due to its better performance.



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5 System Response and Process Controllers

LEVEL 1 Objective Questions

1.	(b)
2.	(a)
3.	(b)
4.	(a)
5.	(a)
6.	(d)
7.	(a)
8.	(c)
9.	(b)
10.	(c)
11.	(a)
12.	(c)
13.	(a)
14.	(a)
15.	(b)

LEVEL 2 Objective Questions

16.	(c)
17.	(a)
18.	(a)
19.	(d)
20.	(a)
21.	(b)
22.	(c)
23.	(c)
24.	(a)

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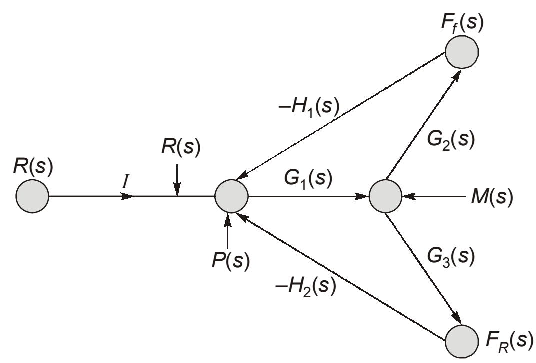
LEVEL 3 Conventional Questions

Solution : 25

Consider the following signal flow graph



Name of unknown nodes

as M(s) and P(s), Then

$$P(s) = R(s) - F_f(s)H_1(s) - F_R(s)\,H_2(s) \rightarrow \quad (1)$$

$$M(s) = G_1(s)[P(s)] \quad \rightarrow \quad (2)$$

$$F_f(s) = G_2(s)\,[M(s)] \quad \Rightarrow \quad G_2(s)\,G_1(s)[P(s)]$$

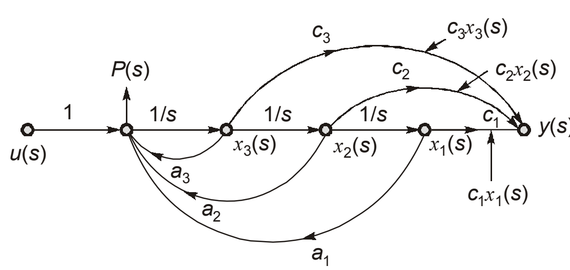
$$F_f(s) = G_1(s)G_2(s)\,[R(s) - F_f(s)\,H_1(s) - F_R(s)H_2(s)] \quad \rightarrow \quad (3)$$

As the problem requires only $F_f(s) | R(s)$, $F_R(s)$ can be taken as zero then from equation '3' we can write as

$$\frac{F_f(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H_1(s)}$$

Solution : 26

Consider the signal flow graph shown



From above

$$Y(s) = C_1x_1(s) + C_2x_2(s) + C_3x_3(s)$$

Apply Inverse Laplace transform on both sides

$$Y(t) = C_1x_1(t) + C_2x_2(t) + C_3x_3(t)$$

The above expression can be written in writer form as

$$Y(t) = [c_1\,c_2\,c_3]\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad \rightarrow \quad \text{(a)}$$

$$x_1(s) = 1/s\, x_2(s); \quad x_2(s) = s x_1(s)$$

Apply inverse Laplace transform as both sides

$$x_2(t) = x_1(t)$$

$$x_1(t) = 0\, x_1(t) + 1 \cdot x_2(t) + 0 \cdot x_3(t) + 0\, u(t) \qquad \text{...(1)}$$

$$\Rightarrow \qquad x_2(s) = \frac{1}{s} x_3(s); \; x_3(s) = s x_2(s)$$

Apply inverse Laplace transform on both sides then

$$x_3(t) = x_2(t)$$

$$\Rightarrow \qquad x_2(t) = 0x_1(t) + 0x_2(t) + 1x_3(t) + 0.u(t) \qquad \text{...(2)}$$

$$\Rightarrow \qquad x_3(s) = \frac{1}{s}[P(s)]; \; P(s) = Sx_3(s)$$

$$\Rightarrow \qquad P(t) = x_3(t)$$

$$P(s) = a_1x_1(s) + a_2x_2(s) + a_3x_3(s) + 1.\, u(s)$$

Apply Inverse Laplace transform on both sides.

$$P(t) = a_1x_1(t) + a_2x_2(t) + a_3x_3(t) + 1.u(t)$$

$$\Rightarrow \qquad x_3(t) = a_1x_1(t) + a_2x_2(t) + a_3x_3(t) + 1.u(t) \qquad \text{...(3)}$$

From equation (1), (2), (3), (a) We can develope "state space representation" as shown below:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$[y(t)] = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$$

From the above state space representation

Control system matrix (or)

System matrix $$[A]_{3\times 3} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$\Rightarrow$ Input matrix $$[B]_{3\times 1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow$ Output matrix $[C]_{1\times 3} = [c_1, c_2, c_3]$

Solution : 27

A closed loop control system, also known as a feedback control system. It is a control system, which uses the concept of an openloop system as it's forward path, but has one or more feedback loops.

In closed loop control system, controller can always monitor the output with the help of feedback (sensor).

Exp: Student is allowed to write any example for closed loop control system.

Solution : 28

$$\text{Controller output}(\%) = \frac{\text{Current output} - \text{Minimum output}}{\text{Maximum output} - \text{Minimum output}} \times 100$$

$$= \frac{15-4}{20-4} \times 100 = 68.75\%$$

Solution : 29

Let us assume the relationship between speed and current as

$$R = mi + R_0$$

Applying given conditions

$$400 = M \times 4 + R_0 \text{ and } 1400 = M \times 20 + R_0$$

Solving for M and R_0, we have $M = 62.5$ rpm/mA and $R_0 = 150$ rpm, At 1000 rpm

$$1000 = 62.5 \times i + 150$$

Therefore $\quad i = 13.6$ mA

The output can be expressed in percent as

$$\text{Controller output}(\%) = \frac{\text{Current output} - \text{Minimum output}}{\text{Maximum output} - \text{Minimum output}} \times 100$$

$$= \frac{13.6-4}{20-4} \times 100 = 60\%$$

Solution : 30

$$\text{Temperature}(\%) = \frac{\text{MV} - \text{Minimum MV}}{\text{Maximum MV} - \text{Minimum MV}} = \frac{60-40}{100-40} \times 100 = 33.33\%$$

Solution : 31

(a) Percent per point

$$\text{SP}(\%) = \frac{\text{SP} - \text{Minimum operating value}}{\text{Operating range}} = \frac{1200-100}{2000-100} \times 100 = 57.9\%$$

(b) Percent measured value

$$\text{MV}(\%) = \frac{\text{Current speed} - \text{Minimum speed}}{\text{Operating range}} \times 100$$

$$= \frac{1100-100}{1900} \times 100\% = 52.63\%$$

(c) Error $\quad \text{Error} = 1200 - 1100 = 100$ rpm

(d) Percent error $\quad \text{Error}(\%) = \dfrac{\text{Error}}{\text{Operating range}} \times 100 = \dfrac{100}{1900} \times 100 = 52.6\%$

Solution : 32

(a) Let us assume that there is a linear relation between level and current

$$h = Ki + h_0$$

where K is a constant and h_0 the initial value of liquid level. We can find K and h_0 by writing two equations:

$$1\text{ m} = \text{K}(4\text{ mA}) + h_0 \quad ...(1)$$

$$2\text{ m} = \text{K}(20\text{ mA}) + h_0 \quad ...(2)$$

Subracting eq. (1) from eq. (2), we have K = 0.0625 m/mA. Putting the value of K in either of the two equations we get

$$h_0 = 0.75\text{ m}$$

The relationship is $h = 0.0625\text{i} + 0.75$

(b) The microswitch closes at 14 mA, which is a maximum level, and opens at 10 mA. The maximum height

$$h_m = 0.0625 \times 14 + 0.75 = 1.625\text{ m}$$

The lowest level $h_1 = 0.0625 \times 10 + 0.75 = 1.375\text{ m}$

Differential gap $h_m - h_1 = 1.625 - 1.375 = 0.25\text{ m}$

Solution : 33

(a) Calculating the period of oscillation of the controlled group

Volume of transmitted signal (V) = Area × Height

$$= \frac{\pi d^2}{4} \times h = \frac{\pi \times 0.6^2}{4} \times 1.2 = 0.339\text{ m}^3 = 3391$$

Volume within 7% differential gap = 0.07 × 339 = 23.731

$$\text{Rate of rise} = 90 - 60 = 30\text{ LPM}$$

$$\text{Time to rise} = \frac{339}{30} = 11.3\text{ m}$$

$$\text{Rate of fall} = 60 - 0 = 60\text{ LPM}$$

$$\text{Time to fall} = \frac{339}{60} = 5.65\text{ min}$$

$$\text{Total time} = 11.3 + 5.65 = 16.95\text{ min}$$

(b) If the load is 50 LPM then

$$\text{Time to rise} = \frac{339}{90-50} = 8.475\text{ min}$$

$$\text{Time to fall} = \frac{339}{50} = 6.78\text{ min}$$

$$\text{Total time} = 15.255\text{ min}$$

Solution : 34

(a)The open-loop transfer function:

$$G(s) = \frac{K_P}{s(s+4)}$$

Therefore the system is type '1' system.

(b) Calculating the steady-state errors

For step input:

$$e_{SS} = \lim_{s\to 0}\left[s\frac{1}{1+G(s)}R(s)\right] \text{ for step input } R(s) = \frac{1}{s}$$

$$= \lim_{x\to\infty}\left[s\frac{1}{1+G(s)}\,\frac{1}{s}\right] = \frac{1}{1+\infty} = 0$$

For ramp input:

$$e_{SS} = \lim_{s\to 0}\left[s\frac{1}{1+\left[K_p/(s+4)\right]}\frac{1}{s^2}\right] = \lim_{s\to 0}\frac{1}{s+\left[K_p/(s+4)\right]} = \frac{4}{K_p}$$

Note that high values of K_p give a smaller steady-state error, but tend to give a greater instability.

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6 Introduction to Robotics

LEVEL 1 Objective Questions

1. (b)
2. (c)
3. (d)
4. (a)
5. (b)
6. (a)
7. (c)
8. (a)
9. (d)
10. (d)
11. (d)
12. (d)
13. (a)

LEVEL 2 Objective Questions

14. (13)
15. (a)
16. (d)
17. (c)
18. (a)
19. (c)
20. (b)
21. (b)
22. (a)
23. (a)
24. (d)
25. (b)
26. (c)
27. (c)
28. (a)
29. (a)
30. (b)
31. (c)

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LEVEL 3 Conventional Questions

Solution : 32

The six characteristics of a hydraulic actuator are —

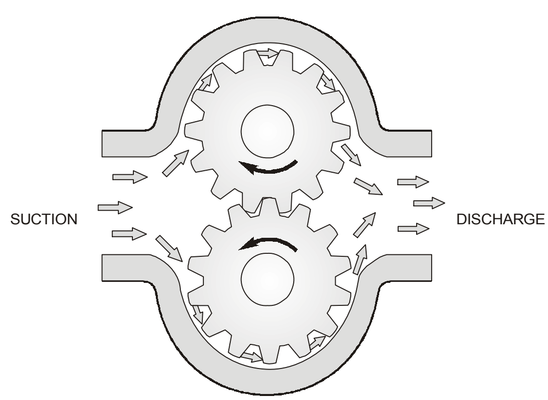
1. It has higher load carring capacity
2. It power to weight ratio is high
3. It can produce large forces to drive loads
4. Its manufacturing cost and maintenance are high
5. It has high accuracy and fast response
6. It poses certain safety concerns as leakage of oils is undesirable and should be avoided.

Solution : 33

Working principle of Gear pump :

A gear pump uses the meshing of gears to pump fluid by displacement. They are one of the most widely used types of pumps for hydraulic fluid power operators. As the gears rotate they separate on the intake side of the pump, creating a void and suction which is filled by fluid. The fluid is carried by the gears to the discharge side of the pump, where the meshing of the gears displaces the fluid. The mechanical clearances are small-in the order of 10 μm. The tight clearances, along with the speed of rotation, effectively prevent the fluid from leaking backwards. The rigid design of the gears and houses allow for very high pressure and the ability to pump highly viscous fluids.

There are two main variations; external gears pump which use two external spur gears and internal gear pump which use an external and an internal spur gears. Gear pumps are positive displacement, meaning they pump a constant amount of fluid for each revolution.



Simple Gear Pump

■■■■

7 Robotics and Transformations

LEVEL 1 Objective Questions

1.	(d)
2.	(b)
3.	(c)
4.	(d)
5.	(a)
6.	(b)
7.	(d)
8.	(a)
9.	(d)
10.	(a)
11.	(b)
12.	(c)
13.	(a)
14.	(d)
15.	(d)
16.	(c)
17.	(b)
18.	(d)
19.	(c)
20.	(a)
21.	(b)

LEVEL 2 Objective Questions

22.	(b)
23.	(d)
24.	(c)
25.	(c)
26.	(a)
27.	(d)
28.	(c)

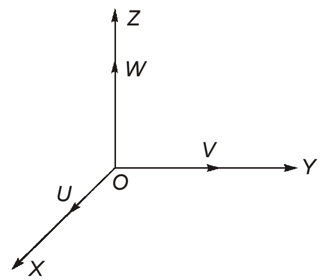
■■■■

LEVEL 3 Conventional Questions

Solution : 29

Given: Coordinates of a point $P(5, 3, 1)^T$ attached to a moving frame F_{uvw}.

Set-A



1. Rotation of (–90°) about z-axis
2. Translation of [5, –3, 4] about the x, y and z axes respectively.
3. Rotation of (–90°) about y-axis.

As Rotation about fixed axes ⇒ Pre multiplication sequence of operations

⇒ R(–90°, y-axis), T(5, –3 ,4), R(–90°, z-axis)

$$\therefore \quad R(-90°, y\text{-axis}) = \begin{bmatrix} \cos-90° & 0 & \sin(-90°) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90°) & 0 & \cos(90°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T(5, -3, 4) = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R(-90°, y\text{-axis}) \times T(5, -3, 4) = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{and} \quad R(-90°, z\text{-axis}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall transformation matrix

$$[T_{overall}] = \begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, new coordinates of point P

$$\begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -8 \\ 8 \\ 1 \end{bmatrix}$$

$$P' = [-5, -8, 8]^T$$

Set - B

1. Rotation of (–90°) about W-axis
2. Translation of [5, –3, 4] about U, V and W axes respectively.
3. Finally a rotation (–90°) about V-axis

A rotation about current frame ⇒ Post multiplication

$\Rightarrow \quad R(-90°, W\text{-axis}) \times T(5, -3, 4) \times R(-90°, V\text{-axis})$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

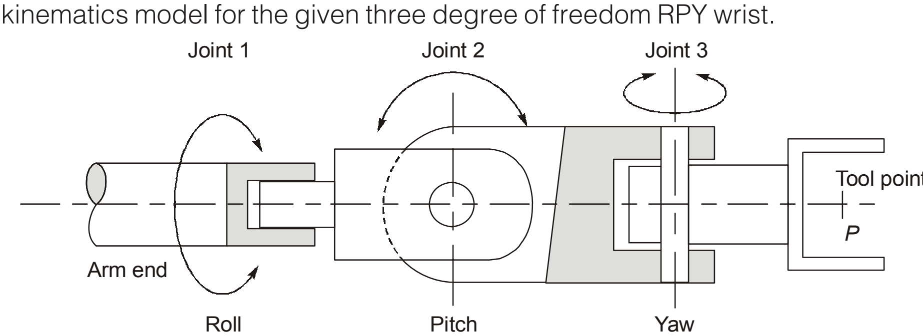
$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -5 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ 9 \\ 1 \end{bmatrix}$$

$$\therefore \quad P'' \rightarrow [0, -4, 9]^T$$

Solution : 30

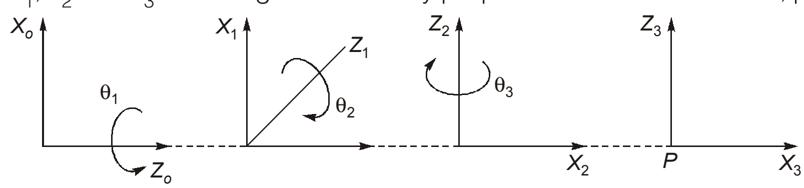
Forward kinematics model for the given three degree of freedom RPY wrist.



A 3-DOF freedom roll, pitch and yaw (RPY) wrist

The 3-DOF RPY wrist has three revolute (RRR) joints which provide any arbitrary orientation to the end effector in 3-D space.

It is assumed that the arm end-point is stationary and can be considered as the stationary base frame [0], for the wrist. The joint are identified and labelled with joint axes as shown in figure. The three joints displacements θ_1, θ_2 and θ_3 are along thee mutually perpendicular directions: roll, pitch and yaw.



Frame assignment for 3 DOF RPY wrist

D-H parameters table

Link	a	α	d	θ
1	0	90°	0	θ_1
2	0	90°	0	$\theta_2 + 90°$
3	0	0	0	θ_3

Individual transformation matrices

$$^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And overall transformation matrix for RPY wrist is

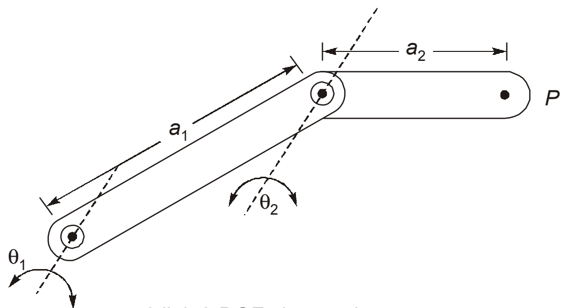
$$^0T_3 = {}^0T_1\,{}^1T_2\,{}^2T_3$$

$$= \begin{bmatrix} -c_1s_2c_3 + s_1s_3 & c_1s_2s_3 + s_1c_3 & c_1c_2 & 0 \\ -s_1s_2c_3 - c_1s_3 & s_1s_2s_3 - c_1c_3 & s_1c_2 & 0 \\ c_2c_3 & -c_2s_3 & s_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

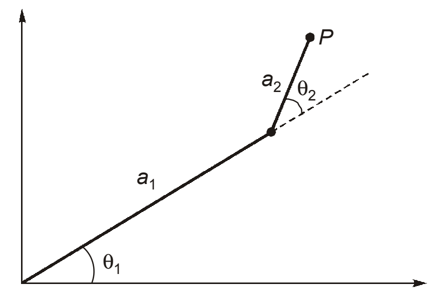
Orientation of the last frame with respect to [0] frame, if $\theta_1 = 0$ and $\theta_2 = \theta_3 = 90°$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution : 31



2 link 2-DOF planar robot arm



D-H Parameters are —

(1) Link length (a_i)

(2) Joint angle (θ_i)

(3) Link twist (α_i)

(4) Joint depth/offset (d_i)

D-H parameters table for 2 DOF planar robot

	a_i	α_i	d_i	θ_i
link 1	a_1	0	0	θ_1
link 2	a_2	0	0	θ_2

Let composite transformation matrix = ${}_0T^2$

As $${}_0T^2 = {}_0T^1 \times {}_1T^2$$

where ${}_0T^1$ and ${}_1T^2$ are individual transformation matrix

$${}_0T^1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $c \rightarrow \cos\theta$ and $s \rightarrow \sin\theta$

$${}_1T^2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

And composite transformation matrix will be—

$${}_0T^2 = {}_0T^1 \times {}_1T^2$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ Forward kinematic equations are

$$P_x = a_1\, c_1 + a_2\, c_{12}$$
$$P_y = a_1\, s_1 + a_2\, s_{12}$$
$$P_z = 0$$

Position of piont, $P = [P_x\ P_y\ P_z]^T$

(b) Given: $a_1 = 15$ units; $a_2 = 10$ units; $\theta_1 = 45°$; $\theta_2 = 45°$ (counter clockwise)

Final position [P] is given as –

$$P = [P_x\ \ P_y\ P_z]^T$$

where $P_x = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2)$

$$= 15\cos 45° + 10\cos(45° + 45°) = \frac{15}{\sqrt{2}}$$

$$P_y = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2) = 15\sin 45° + 10\sin(45° + 45°)$$

$$P_y = \frac{15}{\sqrt{2}} + 10$$

and $P_z = 0$

∴ Position of tool point, $P = \left[\frac{15}{\sqrt{2}},\ \frac{15}{\sqrt{2}} + 10,\ \ 0\right]$

orientation of tool point is given as —

$$\begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(45° + 45°) & -\sin(45° + 45°) & 0 \\ \sin(45° + 45°) & \cos(45° + 45°) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

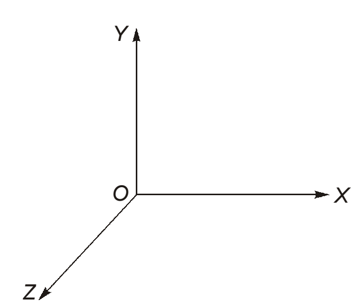
$$= \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution : 32

Given frame representation,

$$F = \begin{bmatrix} ? & 0 & -1 & 5 \\ ? & 0 & 0 & 3 \\ ? & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and *OXYZ* axes system as ⇒



Since position of the frame *F* with respect to *OXYZ* is —

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

and $P_x = 5$; $P_y = 3$; and $P_z = 2$

and orientation of frame F with respect to $OXYZ$ as —

Frame / Origin	X	Y	Z
X	90°	90°	180°
Y	180°	90°	90°
Z	90°	180°	90°

So, orientation of frame F is given by

$$\Rightarrow \begin{bmatrix} \cos 90^\circ & \cos 90^\circ & \cos 180^\circ \\ \cos 180^\circ & \cos 90^\circ & \cos 90^\circ \\ \cos 90^\circ & \cos 180^\circ & \cos 90^\circ \end{bmatrix}$$

$$\Rightarrow \begin{array}{|c|c|c|} \hline 0 & 0 & -1 \\ \hline -1 & 0 & 0 \\ \hline 0 & -1 & 0 \\ \hline \end{array}$$

and the missing element in the frame representation (F) is —

$$\begin{bmatrix} ? & 0 & -1 & 5 \\ ? & 0 & 0 & 3 \\ ? & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & -1 & 5 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution : 33

The 45° rotation of P about the z-axis of frame {1} from equation is

$$T_z(45^\circ) = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For the rotation of vectors, we have:

$$^1Q = R_z(45^\circ)\, ^1P$$

Substituting values of R_z and ^1P.

$$^1Q = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.0 \\ 2.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 3.535 \\ 1 \end{bmatrix}$$

Thus, the coordinates of the new point Q relative to frame {1} are $[0.707 \quad 3.535 \quad 1.0]^T$ or the new position vector is $Q = [0.707 \quad 3.535 \quad 1.0]^T$

Solution : 34

(i) The homogenous transform matrix describing frame {2} with respect to frame {1}, is

$$^1T_2 = \left[\begin{array}{ccc|c} & ^1R_2 & & ^1D_2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Because frame {2} is rotated relative to frame {1} about x-axis by 60°, we have

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60° & -\sin 60° \\ 0 & \sin 60° & \cos 60° \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & -0.866 \\ 0 & 0.866 & 0.500 \end{bmatrix} $$

Substituting ${}^1R_2$ and ${}^1D_2$ in the above equation

$$ {}^1T_2 = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 7.000 \\ 0 & 0.500 & -0.866 & 5.000 \\ 0 & 0.866 & 0.500 & 7.000 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] $$

Given ${}^2P = [2.0 \quad 2.0 \quad 6.0]^T$, point P in frame {1} is given by

$$ {}^1P = {}^1T_2\,{}^2P $$

or $$ {}^1P = [9.000\ 1.804 \quad 13.464 \quad 1]^T $$

The 3 × 1 position vector of point P in frame {1} in physical coordinates is then

$$ {}^1P = [9.000 \quad 1.804 \quad 13.464]^T $$

(ii) The homogenous transformation for describing frame {1} relative to frame {2}, ${}^2T_1$ is given by

$$ {}^2T_1 = \left[\begin{array}{ccc:c} & {}^2R_1 & & {}^2D_1 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] = \left[{}^1T_2\right]^{-1} $$

The inverse of ${}^1T_2$ is given by equation, that is

$$ {}^2T_1 = \left[\begin{array}{ccc:c} & {}^1R_2^T & & -R_2^T\ {}^2D_2 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] $$

From ${}^1T_2$ in part (i), we have

$$ {}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & -0.866 \\ 0 & 0.866 & 0.500 \end{bmatrix} $$

and $$ {}^1D_2 = [7.0 \quad 5.0 \quad 7.0]^T $$

Hence, $$ {}^2R_1 = {}^1R_2^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500 & 0.866 \\ 0 & -0.866 & 0.500 \end{bmatrix} $$

and the position of the origin of frame {1} with respect to frame {2} is given by

$$ {}^2D_1 = -{}^1R_2^T\ {}^1D_2 $$

Substituting values $$ {}^2D_1 = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 5.00 & 0.866 \\ 0 & 0.866 & 0.500 \end{bmatrix} \begin{bmatrix} 7.0 \\ 5.0 \\ 7.0 \end{bmatrix} = \begin{bmatrix} -7.000 \\ -8.562 \\ 0.830 \end{bmatrix} $$

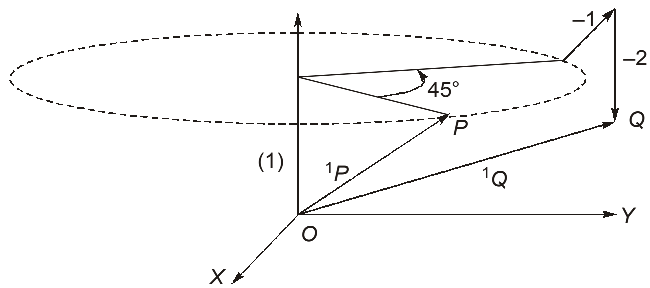
Therefore, $$ {}^2T_1 = \begin{bmatrix} 1 & 0 & 0 & -7.000 \\ 0 & 0.500 & 0.866 & -8.562 \\ 0 & -0.866 & 0.500 & 0.830 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Solution : 35

Figure shows a point P and a vector from origin as ${}^1P$ in frame {1} and its new location after the rotational and transnational transformation as ${}^1Q$. The relation between ${}^1Q$ and ${}^1P$ is described by equation as

$$ {}^1Q = T\,{}^1P $$

where
$$ T = \left[\begin{array}{ccc|c} & R(\theta) & & D \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

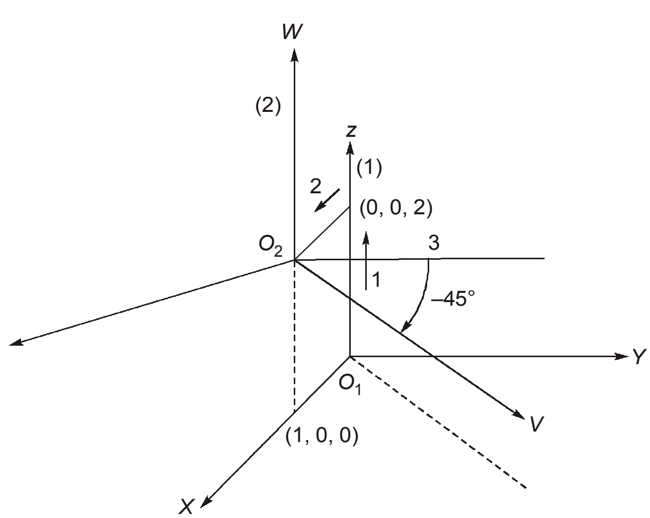


Transformation of point *P* in space

Substituting values gives

$$ {}^1Q = \begin{bmatrix} \cos 45° & -\sin 45° & 0 & -1 \\ \sin 45° & \cos 45° & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^1P = \begin{bmatrix} 0.707 & -0.707 & 0 & -1 \\ 0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^1P $$

The transformation in above equation can be regarded as transformation of two frames {1} and {2}. Assuming frame {1} and frame {2} to be initially coincident, the final position of frame {2} is obtained by translating it by +2 units along z_1-axis (motion 1), and +1 unit along x_1-axis (motion 2) and then rotating it by an angle of –45° about z_1-axis (motion 3). The two frames are shown in figure below.



Transformation of frames corresponding to transformation of vectors

Solution : 36

Let the given rotation matrix which specifies the orientation of frame {2} with respect to frame {1} be

$$^1R_2 = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad \text{...(i)}$$

The equivalent rotation matrix for a set of ZYX-Euler angle rotation ($\theta_1\ \theta_2\ \theta_3$) is given by equation

$$^1R_2 = \begin{bmatrix} C_2C_3 & S_1S_2C_3 - C_1S_3 & S_1S_3 + C_1S_2C_3 \\ C_2S_3 & S_1S_2S_3 + C_1C_3 & -S_1C_3 + C_1S_2S_3 \\ -S_2 & S_1C_2 & C_1C_2 \end{bmatrix} \quad \text{...(ii)}$$

Equating the corresponding elements of these matrices gives nine equations in three independent variables, θ_1, θ_2, θ_3. Apart from the redundancy in equations, additional complication is that these are transcendental in nature.

Equating elements (1, 1) & (2, 1) in equation (i) with corresponding element of (ii), we have

$$C_2C_3 = r_{11} \text{ and } C_2S_3 = r_{21}$$

Squaring and adding gives $\quad C_2 = \cos\theta_2 = \pm\sqrt{r_{11}^2 + r_{21}^2}$

Combining with the element (3, 1), ($-S_2 = r_{31}$), the angle θ_2 is computed as

$$\tan\theta_2 = \frac{S_2}{C_2}$$

which gives $\quad \theta_2 = A\tan 2(-r_{31}, \pm\sqrt{r_{11}^2 + r_{21}^2})$

where $A\tan 2(a, b)$ is a two-argument arc tangent function.

The solution for θ_1 and θ_3 depends on value of θ_2. Here, two cases arise which are worked out as follows:

Case I: $\theta_2 \neq 90°$

From the elements (1, 1) and (2, 1) in equation θ_3 is obtained as

$$\theta_3 = A\tan 2\left[\frac{r_{21}}{C_{21}}, \frac{r_{11}}{C_2}\right]$$

and from elements (3, 2) and (3, 3), θ_1 is

$$\theta_1 = A\tan 2\left[\frac{r_{32}}{C_2}, \frac{r_{33}}{C_2}\right]$$

Note that there is one set of solution corresponding to each value of θ_2.

Case 2 $\quad \theta_2 = \pm 90°$

For $\theta_2 = \pm 90°$, the solution obtained in Case 1 egenerates. However, it is possible to find only the sum or difference of θ_3 and θ_1. Comparing elements (1, 2) and (2, 2)

$$r_{12} = S_1S_2C_3 - C_1S_3$$

and $$r_{22} = S_1S_2C_3 - C_1S_3 \qquad \text{... (i)}$$

If $\theta_2 = +90°$, these equations reduce to

$$r_{12} = \sin(\theta_1 - \theta_3)$$
$$r_{22} = \cos(\theta_1 - \theta_3)$$

and $$\theta_1 - \theta_3 = A\tan2(r_{12}, r_{22}) \qquad \text{... (ii)}$$

Choosing $\theta_3 = 0°$ gives the particular solution

$$\theta_2 = 90°;\ \theta_3 = 0° \text{ and } \theta_1 = A\tan2(r_{12}, r_{22}) \qquad \text{... (ii)}$$

With $\theta_2 = -90°$, the solution is

$$r_{12} = -\sin(\theta_1 + \theta_2)$$
$$r_{22} = \cos(\theta_1 + \theta_2)$$

and $$\theta_1 + \theta_2 = A\tan2(-r_{12}, r_{22})$$

Choosing $\theta_2 = 0°$ gives the particular solution

$$\theta_2 = -90°,\ \theta_3 = 0° \text{ and } \theta_1 = A\tan2(-r_{12}, r_{22})$$

Solution : 37

Rotations are in order X-Y-X about the fixed axes; hence, it is a case of fixed angle representation. Therefore,

$${}^1R_2 = R_x(60°)\, R_y(30°)\, R_x(45°) \qquad \text{... (i)}$$

or
$${}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos60° & -\sin60° \\ 0 & \sin60° & \cos60° \end{bmatrix} \begin{bmatrix} \cos30° & 0 & \sin30° \\ 0 & 1 & 0 \\ -\sin30° & 0 & \cos30° \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos45° & -\sin45° \\ 0 & \sin45° & \cos45° \end{bmatrix}$$

On multiplication,
$${}^1R_2 = \begin{bmatrix} 0.866 & 0.354 & 0.354 \\ 0.433 & -0.177 & -0.884 \\ -0.25 & 0.919 & 0.306 \end{bmatrix} \qquad \text{... (ii)}$$

The reader must verify that the same orientation could have been obtained by performing the same rotations about the moved xyx-axes of the moving frame but in the opposite order. This convention is also referred as XYX- Euler angle representation.

Solution : 38

(a) In screw transformation the moving frame is translated and rotated about same axis. The overall transformation matrix for the given situation is

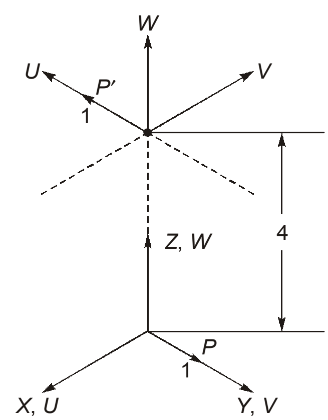
$$T = T(z, \pi)\,T(0, 0, 4)$$

or
$$T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{... (i)}$$

Assume a unit vector along y-axis. This is given by $\hat{p} = [0\,1\,0]^T$. This vector moves with the moving frame and undergoes the two transformations specified. Its position after given translation and rotation will be

$$P' = TP = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \\ 1 \end{bmatrix} \qquad \text{... (ii)}$$

(b) The initial and final positions of two frames and point P are is shown in figure.



(c) If the order of transformation is reversed, that is, rotation followed by translation, the overall transformation matrix will not change. This can be easily verified by the reader and is the property of screw transformation.

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8 Robot Specifications & SCARA Robot

LEVEL 1 Objective Questions

1.	(a)
2.	(a)
3.	(a)
4.	(b)
5.	(b)
6.	(a)
7.	(a)
8.	(c)
9.	(b)
10.	(a)
11.	(a)
12.	(b)
13.	(a)
14.	(c)
15.	(d)
16.	(d)
17.	(c)

LEVEL 2 Objective Questions

18.	(c)
19.	(c)
20.	(b)
21.	(b)
22.	(b)
23.	(d)
24.	(d)
25.	(a)
26.	(d)
27.	(d)

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LEVEL 3 Conventional Questions

Solution : 28

Proportional control (P): In the proportional control algorithm, the controller output is proportional to the error signal, which is the difference between the set point and the process variable.

Derivative control (D): In this algorithm, output is directly proportional to the rate of change of error.

Integral control (I): In this control method, the control system acts in a way that the control effort is proportional to the integral of the error.

PID control: In this control method, the controller output is proportional to present error and integral of error and derivative of error.

Solution : 29

Let three joint variables be (θ_1, θ_2 and θ_3) for a given end effector orientation matrix T_E for RPY wrist.

$$T_E = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Comparing it with the overall transformation matrix ${}^0T_3$ derived before

$$a_x = c_1\, c_2$$

and $$a_y = s_1\, c_2$$

$$\frac{a_y}{a_x} = \frac{s_1}{c_1} = \tan\theta_1$$

$\therefore$ $$\tan^{-1}\left(\frac{a_y}{a_x}\right) = \theta_1$$

Similarly, $$n_z = c_2\, c_3$$

$$o_z = -c_2\, s_3$$

$\therefore$ $$\frac{o_z}{n_z} = -\tan\theta_3$$

$$\theta_3 = \tan^{-1}\left[-\frac{o_z}{n_z}\right]$$

To find θ_2 (pitch) both sides are pre-multiplied by $({}^0T_1)^{-1}$.

$$({}^0T_1)^{-1} \cdot T_E = ({}^0T_1)^{-1} \cdot ({}^0T_1) \cdot ({}^1T_2) \cdot ({}^2T_3)$$

$$({}^0T_1)^{-1} = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $$({}^0T_1)^{-1} \cdot T_E = {}^1T_2\; {}^2T_3$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -s_2 & 0 & c_2 & 0 \\ c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_2 = c_1 a_x + s_1 a_y$$

and

$$s_2 = a_{z7}$$

$$\tan\theta_2 = \frac{a_z}{c_1 a_x + s_1 a_y} \quad \Rightarrow \quad \theta_2 = \tan^{-1}\left[\frac{a_z}{c_1 a_x + s_1 a_y}\right]$$

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