2020

RANK Improvement WORKBOOK

Detailed Explanations of Objective & Conventional *Questions*

Mechanical Engineering

Industrial Engineering

MADE EASY Publications

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Publications

1 Forecasting

LEVEL 1 Objective Questions

1.	(b)
2.	(101.5)
3.	(a)
4.	(b)
5.	(b)
6.	(50)
7.	(d)
8.	(d)
9.	(c)
10.	(c)
11.	(b)
12.	(a)
13.	(c)
14.	(a)

LEVEL 2 Objective Questions

15.	(b)
16.	(2)
17.	(10.4)
18.	(d)
19.	(a)
20.	(b)
21.	(596)
22.	(666.67)
23.	(a)
24.	(a)
25.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 26

Assume that equation of trend line is $y = mx + c$

$$\Sigma y = \Sigma mx + \Sigma c$$
$$\Sigma xy = \Sigma mx^2 + c\Sigma x$$

Year	x	Demand (y_1)	xy	x^2	y^2
2006	1	20,000	20,000	1	4×10^8
2007	2	25,000	50,000	4	625×10^6
2008	3	26,000	78,000	9	676×10^6
2009	4	25,000	1,00,000	16	625×10^6
2010	5	30,000	1,50,000	25	900×10^6
	$\Sigma x = 15$	$\Sigma y = 1{,}26{,}000$	$\Sigma xy = 3{,}98{,}000$	$\Sigma x^2 = 55$	$\Sigma y^2 = 3226 \times 10^6$

$$m = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{5 \times 3{,}98{,}000 - 15 \times 1{,}26{,}000}{5 \times 55 - 15^2} = 2000$$

$$C = \frac{1}{n}[\Sigma y - m\Sigma x] = \frac{1}{5}[1{,}26{,}000 - 2000(15)] = 19{,}200$$

$\Rightarrow$ The trend line is $y = 2000x + 19200$

forecast of 2011, $y_{x=6} = 2000\,(6) + 19200 = 12000 + 19{,}200 = 31{,}200$

forecast of 2012, $y_{x=7} = 2000\,(7) + 19200 = 33200$

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$r = \frac{5 \times 3{,}98000 - (15)(1{,}26{,}000)}{\sqrt{5(55) - 15^2} \times \sqrt{5(3226 \times 10^6) - (1{,}26{,}000)^2}} = 0.88735$$

Solution : 27

$$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) \quad \text{[forecast for the time } t\text{]}$$
$$\alpha = 0.2 \leftarrow \text{smoothing constant.}$$

Month	Actual demand (D_t)	Forecast (F_t)	$D_t - F_t$
March	150	100	150 – 100 = 50
April	200	110	200 – 110 = 90
May	100	128	100 – 128 = –28
June	50	122.4	50 – 122.4 = –72.4
July	150	107.92	150 – 107.92 = 42.08
August		116.33	

$$F_{April} = 100 + 0.2(150 - 100) = 110$$
$$F_{May} = 110 + 0.2(200 - 110) = 128$$
$$F_{June} = 128 + 0.2(100 - 128) = 122.4$$
$$F_{July} = 122.4 + 0.2(50 - 122.4) = 107.92$$
$$F_{August} = 107.92 + 0.2(150 - 107.92) = 116.33$$

$$\text{BIAS} = \frac{\sum_{t=1}^{n}(D_t - F_t)}{n} = \frac{50+90-28-72.4+42.08}{5} = 16.336$$

As the BIAS is positive, so the forecast is under estimated.

Solution : 28

(i) Daily, using a simple five-week moving average.

Days	This week
MON	$\frac{2900+2300+2500+2300+2500}{5} = 2500$
TUE	$\frac{3000+2100+2200+2100+2200}{5} = 2320$
WED	$\frac{3800+2400+2500+2200+2700}{5} = 2720$
THU	$\frac{5000+2200+2100+1900+2000}{5} = 2640$
FRI	$\frac{3700+2700+1900+2000+2100}{5} = 2480$
SAT	—
SUN	$\frac{2800+2900+2900+3200+2900}{5} = 2940$

(ii) Daily, using a weighted average of 0.3, 0.25, 0.2, 0.15, 0.10 for the past five weeks.

MON	$0.3 \times 2500 + 0.25 \times 2300 + 0.2 \times 2500 + 0.15 \times 2300 + 0.1 \times 2900 = 2460$
TUE	$0.3 \times 2200 + 0.25 \times 2100 + 0.2 \times 2200 + 0.15 \times 2100 + 0.1 \times 3000 = 2240$
WED	$0.3 \times 2700 + 0.25 \times 2200 + 0.2 \times 2500 + 0.15 \times 2400 + 0.1 \times 3800 = 2600$
THU	$0.3 \times 2000 + 0.25 \times 1900 + 0.2 \times 2100 + 0.15 \times 2200 + 0.1 \times 5000 = 2325$
FRI	$0.3 \times 2100 + 0.25 \times 2000 + 0.2 \times 1900 + 0.15 \times 2700 + 0.1 \times 3700 = 2285$
SAT	–
SUN	$0.3 \times 2900 + 0.25 \times 3200 + 0.2 \times 2900 + 0.15 \times 2900 + 0.1 \times 2800 = 2965$

Solution : 29

Advantages of exponential smoothing method of forecasting are:–

1. It gives more significance to recent observations.
2. It can produce accurate forecasts.
3. It can produce forecasts quickly.
4. It requires a significantly smaller amount of data to be stored compared to the methods of moving averages.
5. It is easy to learn and apply.
6. It has a relatively low computational cost.

Exponentially smoothed forecast for the periods using.

when $\alpha = 0.1$

Month 1	$D_1 = 10, \quad F_1 = 10$ (Assumed)
Month 2	$F_2 = F_1 + 0.1(D_1 - F_1)$
	$F_2 = 10$
Month 3	$F_3 = F_2 + 0.1(D_2 - F_2) = 10 + 0.1(18 - 10) = 10.8$
Month 4	$F_4 = F_3 + 0.1(D_3 - F_3) = 10.8 + 0.1(29 - 10.8) = 12.62$

Month 5	$F_5 = F_4 + 0.1(D_4 - F_4) = 12.62 + 0.1(15 - 12.62) = 12.858$
Month 6	$F_6 = F_5 + 0.1(D_5 - F_5) = 12.858 + 0.1(30 - 12.858) = 14.5722$
Month 7	$F_7 = F_6 + 0.1(D_6 - F_6) = 14.5722 + 0.1(12 - 14.5722) = 14.314$
Month 8	$F_8 = F_7 + 0.1(D_7 - F_7) = 14.314 + 0.1(16 - 14.314) = 14.482$
Month 9	$F_9 = F_8 + 0.1(D_8 - F_8) = 14.482 + 0.1(8 - 14.482) = 13.835$
when $\alpha = 0.3$	
Month 1	$D_1 = 10; \quad F_1 = 10$
Month 2	$F_2 = F_1 = 10$
Month 3	$F_3 = F_2 + 0.3\,(D_2 - F_2) = 10 + 0.3(18 - 10) = 12.4$
Months 4	$F_4 = F_3 + 0.3(D_3 - F_3) = 12.4 + 0.3\,(29 - 12.4) = 17.38$
Month 5	$F_5 = F_4 + 0.3(D_4 - F_4) = 17.38 + 0.3\,(15 - 17.38) = 16.66$
Month 6	$F_6 = F_5 + 0.3(D_5 - F_5) = 16.66 + 0.3\,(30 - 16.66) = 20.66$
Month 7	$F_7 = F_6 + 0.3(D_6 - F_6) = 20.66 + 0.3\,(12 - 20.66) = 18.06$
Month 8	$F_8 = F_7 + 0.3(D_7 - F_7) = 18.06 + 0.3\,(16 - 18.06) = 17.44$
Month 9	$F_9 = F_8 + 0.3(D_8 - F_8) = 17.44 + 0.3\,(8 - 17.44) = 14.6$

Solution : 30

Months	Demand (D_t)	Forecast (F_t)	Error ($D_t - F_t$)	MAD	BIAS	Tracking Signal
March	350	400	–50	50	–50	–1
April	440	390	50	50	0	0
May	450	400	50	50	16.67	1
June	460	410	50	50	25	2
July	495	420	75	55	35	3.18
August	510	435	75	58.33	41.67	4.29

As
$$F_{t+1} = F_t + \alpha(D_t - F_t)$$
$$F_{April} = 400 + 0.2 \times (350 - 400) = 390$$
$$F_{May} = 390 + 0.2 \times (440 - 390) = 400$$
$$F_{June} = 400 + 0.2 \times (450 - 400) = 410$$
$$F_{July} = 410 + 0.2 \times (460 - 410) = 420$$
$$F_{August} = 420 + 0.2 \times (495 - 420) = 435$$

Mean absolute deviation, $\text{MAD} = \dfrac{\sum_{t=1}^{n} |D_t - F_t|}{n}$

$$\text{BIAS} = \frac{\sum_{t=1}^{n} (D_t - F_t)}{n}$$

Tracking signal, $\text{TS} = \dfrac{(BIAS) \times n}{(MAD)}$

Also, Mean square error, $\text{MSE} = \dfrac{\sum_{t=1}^{n} (D_t - F_t)^2}{N}$

If the value of TS goes beyond $3\sqrt{MSE}$ then it indicates that model needs to be revised.

As we want our forecast to be as close as possible to the actual demand. Here, in this case $\alpha = 0.2$ (small value of α) is not justified as forecast value is close to the last forecasted value and is not close to the actual demand.

Solution : 31

Regression Analysis (Analysis)

Tyres used (y)	Thousands of km driven (x)	x^2	xy
100	1500	2250000	150000
150	2000	4000000	300000
120	1700	2890000	204000
80	1100	1210000	88000
90	1200	1440000	108000
180	2700	7290000	486000
$\Sigma y = 720$	$\Sigma x = 10200$	$\Sigma x^2 = 19080000$	$\Sigma xy = 1336000$

(i) As $\quad y = a + bx \quad$ ... (i)

Taking summation both side

$$\Sigma y = na + b\Sigma x \quad \text{...(ii)}$$

Multiply equation (i) with x and taking summation both side

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{... (iii)}$$

From (ii) and (iii); $\quad 720 = 6a + 10200b$

$$1336000 = 10200a + 19080000b$$

$$a = 10.588,\ b = 0.06436$$

Hence, Linear relationship is given by,

$$y = 10.588 + 0.06436x$$

For Correlation

Tyres used	Thousands of x km driven	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$
100	1500	–200	–20	4000
150	2000	300	30	9000
120	1700	0	0	0
80	1100	–600	–40	24000
90	1200	–500	–30	15000
180	2700	1000	60	60000
$\Sigma y = 720$ $\bar{y} = 120$	$x_i = 10200$ $\bar{x} = 1700$			$\Sigma(x_i - \bar{x})(y_i - \bar{y})$ = 112000

$$\Sigma(x_i - \bar{x})^2 = 1740000$$

$$\Sigma(y_i - \bar{y})^2 = 7400$$

$$\Sigma(x_i - \bar{x})(y - \bar{y}) = 112000$$

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i - \bar{x})^2 \Sigma(y_i - \bar{y})^2}} = \frac{112000}{\sqrt{(1740000) \times 7400}} = 0.987$$

Since correlation coefficient value is very near to 1 hence there is strong linear association between two variables.

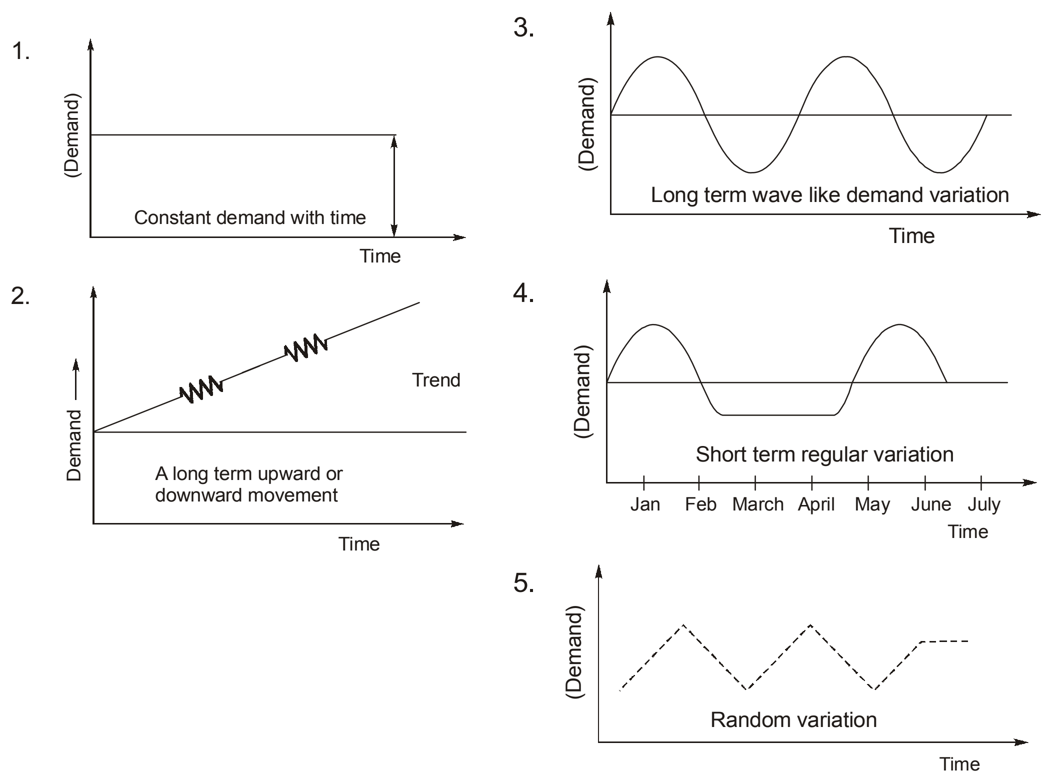
(ii) For expected drive of 3200 thousands km.

Demand for tyres, $y = 10.588 + 0.06436 \times 3200 = 217$ tyres

Solution : 32

(i) Time series forecasting model:

1. Permanent (Example pen)
2. Trend
3. Cyclic (Example sugarcane)
4. Seasonal (Example woollen cloth)
5. Random variation



Simple Exponential Smoothing Method:

These are used to overcome the limitation of simple moving average method i.e. when the value of n is large one has to manage large size of data and the moving average method use only limited number of values to forecast not all of data

$$F_{t+1} = \alpha D_t + (1 - \alpha) F_t$$

$$F_{t+1} = F_t + \alpha(D_t - F_t) \quad \text{...(i)}$$

D_t = Demand of last month

F_t = Forecast of last month

α = Exponential smoothening exponent $(0 \le \alpha \le 1)$

$\left(\alpha = \dfrac{2}{N+1}\right)$ when value of α is not given.

N = Number of value taken

Limitation:

As it can be seen from equation (i) that as the data get older, its weights in the forecasted value decreases smaller the value of α, more will be the smoothening effect i.e. almost similar value to last year forecast value if value of α is large than it seem to be more responsive and value of forecast will be equal to the actual demand of last year

when $\alpha = 0 \longrightarrow F_{t+1} = F_t$ (more stable)

$\alpha = 1 \longrightarrow F_{t+1} = D_t$ (responsive)

$\alpha = 1 \longrightarrow$ Require for new product

$\alpha = 0 \longrightarrow$ Require for old and stable product

(ii)

Months	F_t	D_t	$(D_t - F_t)$	MAD	Running Sum	T.S = $\dfrac{\Sigma(D_t - F_t)}{\text{MAD}}$
Jan	100	97	–3	$\dfrac{\lvert -3 \rvert}{1} = 3$	–3	–1
Feb	100	93	–7	$\dfrac{10}{2} = 5$	–10	–2
March	100	110	+10	$\dfrac{20}{3} = 6.67$	0	0
April	100	98	–2	$\dfrac{22}{4} = 5.5$	–2	–0.36
May	102	130	28	$\dfrac{50}{5} = 10$	26	2.6
June	104	133	29	$\dfrac{79}{6} = 13.16$	55	4.18
July	106	129	23	$\dfrac{102}{7} = 14.57$	78	5.35
Aug	108	138	30	$\dfrac{132}{8} = 16.5$	108	6.55
Sep	110	136	26	$\dfrac{158}{9} = 17.55$	134	7.63
Oct	112	124	12	$\dfrac{170}{10} = 17$	146	8.588
Nov	114	139	25	$\dfrac{195}{11} = 17.72$	171	9.65
Dec	116	125	9	$\dfrac{204}{12} = 17$	180	10.5882

$$\text{Tracking signal} = \frac{\sum_{t=1}^{n}(D_t - F_t)}{(MAD)}$$

MAD = Mean absolute deviation = $\sum_{t=1}^{n} \frac{|D_t - F_t|}{N}$

Comment: Tracking signal – monitors the performance of the forecasting model and automatically indicates whether the model needs to be revised. Upper limit of tracking signal is taken as $3\sqrt{MSE}$. If value of Tracking signal goes beyond this limit than model need to be revised.

Solution : 33

Year	Demand index(x)	Sales (y)	x^2	$x.y$
2011	10	11	100	110
2012	11	13	121	143
2013	14	15	196	210
2014	15	16	225	240
2015	20	18	400	360
	$\Sigma x = 70$	$\Sigma x = 73$	$\Sigma x = 1042$	$\Sigma x.y = 1063$

Let trend line equation is

$$y = a + bx$$

$$\Sigma y = an + b\Sigma x$$

$$73 = 5a + 70b \quad \text{... (i)}$$

$$\Sigma xy = a.\ \Sigma x + b.\Sigma x^2$$

$$1063 = 70a + 1042b \quad \text{...(ii)}$$

Putting the values in equation (i) and (ii) and solving. We get

$$a = 5.342 \text{ and } b = 0.66129$$

Trend line $\quad y = 5.342 + 0.66129x$

For $\quad x = 21,\ y = 19.24$

So for 2016 for demand index sale's must be 1924 units.

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2 Inventory Cantrol and Break Even analysis

LEVEL 1 Objective Questions

1.	(b)
2.	(b)
3.	(c)
4.	(10)
5.	(30)
6.	(141.42)
7.	(b)
8.	(4669)
9.	(b)
10.	(d)
11.	(600)
12.	(d)
13.	(c)
14.	(b)
15.	(80.55)
16.	(258)
17.	(d)
18.	(d)
19.	(a)
20.	(b)

LEVEL 2 Objective Questions

21.	(3)
22.	(75000)
23.	(9.54)
24.	(a)
25.	(b)
26.	(4.133)
27.	(25.2)
28.	(894.78)
29.	(73.25)
30.	(b)
31.	(1)
32.	(1000)
33.	(c)
34.	(17.91)
35.	(100)
36.	(10000)
37.	(a)
38.	(a)

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LEVEL 3 Conventional Questions

Solution : 39

(i) Inventory may be defined as the stock of goods, commodities and other economic resources that are stored or reserved in order to ensure smooth and efficient running of business affairs. Inventory is “usable but idle resource”.

Deterministic Inventory Models: In this model, we deal with situations in which demand is assumed to be fixed and completely known. Models for such situations are called Economic Lot Size Models or Economic Order Quantity Models.

Probabilistic Inventory Models: In this model, one or more of the inputs is not known with certainty and will be represented by a random variable with a probability distribution, for example, demand might be variable and will follow some probability distribution with known parameters.

(ii) Ordering cost, $C_0 = ₹4500$

Demand, $D = 10000$ units/ year

Holding cost, $C_C = 2500 \times 0.1 = ₹250$

$$EOQ_1 = \sqrt{\frac{2C_O D}{C_c}} = \sqrt{\frac{2 \times 4500 \times 10000}{250}} = 600 \text{ units}$$

As per changed data, $C_O = 4500 \times 1.1 = ₹4950,\ C_C = 250 \times 0.9 = 225$

$$EOQ_2 = \sqrt{\frac{2 \times 4950 \times 10000}{225}} = 663.325 \text{ units}$$

$$\%\text{ increase in EOQ} = \frac{663.25 - 600}{600} \times 100 = 10.55\%$$

$$\text{Min. cost } (EOQ_1) = \sqrt{2C_O D C_c}$$

$$= \sqrt{2 \times 4500 \times 10000 \times 250} = ₹150000$$

$$\text{Min. cost } (EOQ_2) = \sqrt{2 \times 4950 \times 10000 \times 225} = ₹149248$$

As minimum inventory cost decreases with 10% decrease in carrying cost and with 10% increase in ordering cost, it is beneficial to accept new *EOQ* for the present sensitivity analysis.

Solution : 40

D = 4000 units per year(Demand)

Carrying cost, C_c = 25% of unit cost = $0.25 \times 6 = ₹\ 1.50$

Ordering cost, C_0 = ₹ 150

$$\text{EOQ} = \sqrt{\frac{2DC_0}{C_c}} = \sqrt{\frac{2 \times 4000 \times 150}{1.5}} = 894.43 \approx 895 \text{ units Ans. (i)}$$

Time between two consecutive order:

$$= \frac{EOQ}{D} = \frac{895}{4000} \text{ years}$$

$= 0.22375$ years or 2.7 months or 81 days Ans. (ii)

$$\text{Number of order per year} = \frac{D}{Q^*} = \frac{4000}{895} = 4.47 \qquad \text{Ans. (iii)}$$

$$\text{Optimal cost} = \text{(Annual demand) (Price of unit item)} + \sqrt{2DC_0C_c}$$

$$= 6 \times 4000 + \sqrt{2 \times 4000 \times 150 \times 6 \times 0.25}$$

$$= ₹\,25342 \qquad \text{Ans. (iv)}$$

Solution : 41

D = 2400 units, C = ₹ 100, C_o = ₹ 150, C_c = 20% of C = ₹ 20

$$\therefore \quad EOQ = \sqrt{\frac{2 \times C_o \times D}{C_c}} = \sqrt{\frac{2 \times 150 \times 2400}{20}}$$

$$Q^* = 190$$

$$\text{Inventory Cost} = \frac{D}{Q^*}.C_0 + \frac{Q^*}{2}C_c = \frac{2400}{190} \times 150 + \frac{190}{2} \times 20$$

$$= ₹\,3794.7$$

Now the cost of inventory in the lot of 300 unit

$$= \frac{2400}{300} \times 150 + \frac{300}{2} \times 20 = 1200 + 3000 = ₹\,4{,}200$$

Hence the inventory cost with 300 lot is high so it is not economical.

Solution : 42

$$\text{Demand, } D = 8000 \text{ units}$$

$$\text{Order cost, } C_0 = \text{` } 1800$$

Starting from the lowest unit cost, C = ` 185

$$\therefore \quad \text{Holding cost, } C_c = 0.1 \times 185 = \text{` } 18.5$$

In this case, EOQ, $$Q^* = \sqrt{\frac{2DC_0}{C_c}} = \sqrt{\frac{2 \times 8000 \times 1800}{18.5}} = 1247.7 \text{ units/order}$$

But as $Q^* < 2000$, hence this unit cost is not possible

If C = 190 per unit then, $$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{190 \times 0.1}} = 1231.17$$

Which again is not possible

If C = 100 per unit then, $$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{200 \times 0.1}} = 1200$$

Which is a feasible case as $1000 < Q^* < 1499$

Now we compute total cost at feasible EOQ and the next higher order size price break points of 1500 & 2000.

$$\therefore \quad (TC)_{1200} = 8000 \times 200 + \frac{8000}{1200} \times 1800 + \frac{1200}{2} \times 0.1 \times 200$$

$$= ₹\, 1624000$$

$$(TC)_{1500} = 8000 \times 190 + \frac{8000}{1500} \times 1800 + \frac{1500}{2} \times 0.1 \times 190$$

$$= ₹\, 1543850$$

$$(TC)_{2000} = 8000 \times 185 + \frac{8000}{2000} \times 1800 + \frac{2000}{2} \times 0.1 \times 185$$

$$= ₹\, 1505700$$

As $(TC)_{2000}$ is minimum hence 2000 is the most economic order size

Solution : 43

Given: $D = 30{,}000$ switches/year

$C_h = 10$/unit

$C_0 = ₹\, 3{,}500$ per run

Optimum run size, $q = \sqrt{\dfrac{2DC_0}{C_h}}$

$$q = \sqrt{\frac{2 \times 30000 \times 3500}{10}}$$

(i) $q = 4582.57$ or 4583 switches

(ii)Optimum level of inventory at the beginning of any period

$$= \frac{Q^*}{2} = \frac{4583}{2} = 2292 \text{ switches}$$

(iii) Re-order cycle time $= t^* = \dfrac{Q^*}{D} = \dfrac{4583}{30000}$

$t^* = 55$ days

(iv) Number of runs $= \dfrac{30{,}000}{4583} \approx 7$

Let cost of each switch be $= ₹\, 10$

Minimum Annual cost $= (30000 \times 10) + \sqrt{2DC_0C_h}$

$= 3{,}00{,}000 + 45825.75 = ₹\, 3{,}45{,}825.75$

Solution : 44

(i) : ABC Analysis – An *ABC* analysis consists of separating the inventory items into three groups *A*, *B*, *C* according to their annual cost volume consumption, i.e. unit cost × annual consumption.

A – items – High value : These are relatively few items whose value accounts for 60-70% of the total value of the inventory. These will usually be 10-15% of the total items.

B – items – Medium value : A large number of items in the middle of the list, usually about 20-25% of the items whose total value accounts for about 20-25% of the total value.

C – items – Low value : The bulk of items usually about 60-70% whose total inventory values are almost negligible. They account for 10-15% of the total value.

Other similar approaches are:

1. HML: (High, Medium, Low) – Materials are classified according to their unit price in 3 categories.
2. VED : (Vital, Essential, Desirable) – Materials are classified according to their operational characteristics.
3. SDE : (Scarce, Difficult, Easy) – Materials are classified on the basis of availability of inventory items for the production system.

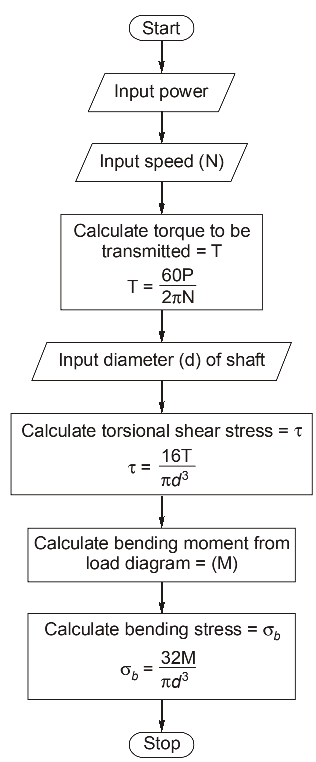
Advantages of such approaches are:–

1. Better control of High-priority inventory.
2. It reduces the cost of inventory items in storage.
3. It ensures control over the costly items in which a large amount of capital is invested.
4. It helps in regular monitoring of critical items and prevents stock out to avoid loss in production.

(ii) : Flow chart is a diagrammatic representation of an algorithm. Flow charts are very helpful in writing program and explaining program to others. A programmer draws a flow chart before writing a computer program. This helps to understand a process and find any flaws and bottlenecks in it. It enables communication between programmers and clients. Once a flow chart is drawn, it becomes comparatively easy to write the program in any high level language. Thus it is good for documentation of a complex program.

Flow Chart:

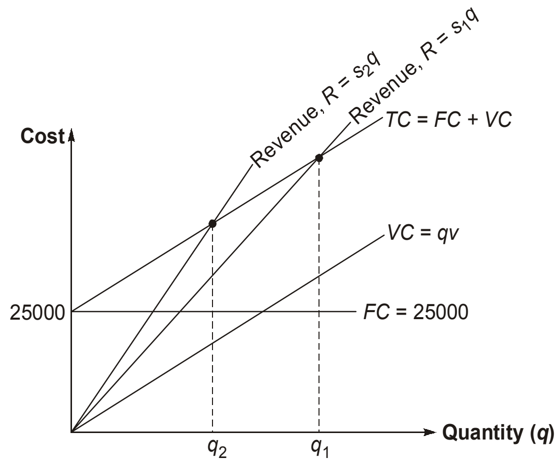
To calculate the torsional and bending stresses in a shaft transmitting power.



Solution : 45

Break-even analysis establishes the relationship among the factors affecting profit. It indicates at what level, cost and revenue are in equilibrium. It also helps in understanding the effect of changes in volume on profit, to understand the effect of alternative decisions that convert costs from variable to fixed, the costs which increase sales volume and revenue. It is powerful tool in evaluating alternative course of action.

Given: Fixed cost, F =25000



Variable cost, v = 10 per unit, Selling price, s = 20

At break-even points,

$$\text{Total cost} = \text{Total revenue}$$

$$F + v.q_1 = s.q_1$$

$$\Rightarrow \quad q_1 = \frac{F}{(s-v)} = \frac{25000}{(20-10)} = 2500$$

Now price becomes, $s = 25$

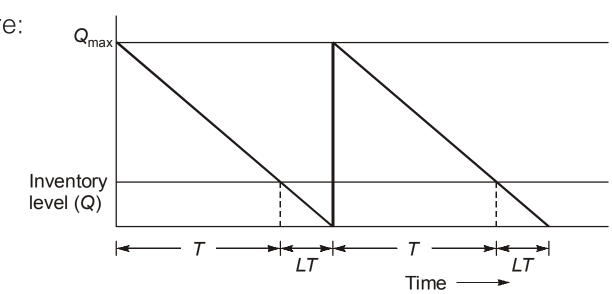
$$\therefore \quad q_2 = \frac{F}{s-v} = \frac{25000}{25-10} = 1667$$

Solution : 46

For a deterministic model. The assumptions made are:

1. Demand is fixed and known.
2. Lead time is known and constant
3. No shortage or stockout is allowed
4. No safety stock is considered
5. Instantaneous replemishment

Model graphical representation:



Let

C = Unit cost

D = Annual demand

C_o = Ordering cost/year

C_h = Holding cost/unit/year

Q = Order quantity

Total inventory cost = Ordering cost + Holding cost

$$\text{TIC} = \frac{D}{Q} \times C_o + \frac{Q}{2} \times C_h$$

for optimum order size (Q^*)

$$\frac{d(\text{TIC})}{dQ} = 0$$

$$\frac{-D}{Q^2} \times C_o + \frac{C_h}{2} = 0$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

Number of orders, $n = \frac{D}{Q^*}$

Optimum time for ordering, $T^* = \frac{Q^*}{D}$

Total inventory cost, $\text{TIC} = \frac{D}{Q^*} C_o + \frac{Q^*}{2} \times C_h = \frac{D}{\sqrt{\frac{2DC_o}{C_h}}} \times C_o + \frac{1}{2}\sqrt{\frac{2DC_o}{C_h}} \times C_h$

$$= \frac{\sqrt{DC_oC_h}}{\sqrt{2}} + \frac{\sqrt{DC_oC_h}}{\sqrt{2}} \quad \text{TIC}(Q^*) = \sqrt{2DC_oC_h}$$

Solution : 47

The analysis which provides a relationship between revenues and costs with respect to volume (quantity) of sales and represents the level of sales at which costs and revenues are in equilibrium, is called break-even analysis. It is useful to the manager in the following ways :

What volume of sales will be necessary to cover.

(i) A reasonable return on capital employed.

(ii) Preference and ordinary dividends.

(iii) Reserves.

- Computing costs and revenues for all possible volumes of output to fix budgeted sales.
- To find the price of an article to give the desired profit.
- To determine variable cost per unit
- To compare a number of business enterprises by arranging their earnings in order of magnitude.

Given, selling price/unit = Rs 100 = S.P.

variable cost/unit = Rs 60 = V.C.

fixed costs = Rs 10,00,000 = F.C.

Break even quantity, $Q_B = \frac{F.C.}{S.P. - V.C.} = \frac{10,00,000}{100 - 60} = 25000$

Given that variable cost have increased by 10%

$\therefore \quad V.C.' = 60 + \frac{10}{100} \times 60 = \text{Rs. } 66$

Fixed cost have increased by 5%

$\therefore \quad F.C. = 1000000 \times \frac{5}{100} + 1000000 = 1050000$

But break even quantity is to remain constant

$\therefore \quad Q_B = 25000 = \frac{1050000}{S.P.' - 66} = \frac{F.C.}{S.P.' - V.C.'} = S.P.' = 108$ (new sales price)

$\therefore$ percentage increase in sales price $= \frac{108 - 100}{100} = 0.08 = 8\%.$

■■■■

3 PERT and CPM

LEVEL 1 Objective Questions

1. (d)
2. (4)
3. (c)
4. (b)
5. (d)
6. (b)
7.. (d)
8. (b)
9. (c)
10. (d)
11. (c)
12. (b)
13. (c)
14. (c)
15. (a)
16. (d)
17. (d)

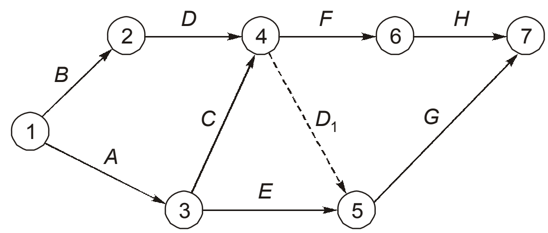
LEVEL 2 Objective Questions

18. (c)
19. (c)
20. (15.87)
21. (c)
22. (d)
23. (b)
24. (1)
25. (d)
26. (b)
27. (13.25)
28. (b)
29. (a)
30. (18)
31. (b)
32. (c)
33. (d)
34. (c)
35. (50)

■■■■

LEVEL 3 Conventional Questions

Solution : 36



Project Network

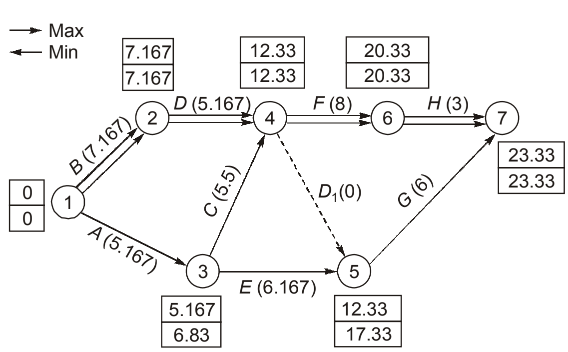
D_1 is dummy activity which connects event no (4) and (5)

We know that, $$t_E = \frac{a+4m+b}{6}$$

Variance,

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2$$

Activity	a	m	b	t_E (Week)	σ^2
A	3	5	8	$\frac{3+20+8}{6} = 5.167$	$\left(\frac{8-3}{6}\right)^2 = 0.694$
B	6	7	9	$\frac{6+28+9}{6} = 7.167$	$\left(\frac{9-6}{6}\right)^2 = 0.25$
C	4	5	9	$\frac{4+20+9}{6} = 5.5$	$\left(\frac{9-4}{6}\right)^2 = 0.694$
D	3	5	8	$\frac{3+20+8}{6} = 5.167$	$\left(\frac{8-3}{6}\right)^2 = 0.694$
E	4	6	9	$\frac{4+24+9}{6} = 6.167$	$\left(\frac{9-4}{6}\right)^2 = 0.694$
F	5	8	11	$\frac{5+32+11}{6} = 8$	$\left(\frac{11-5}{6}\right)^2 = 1$
G	3	6	9	$\frac{3+24+9}{6} = 6$	$\left(\frac{9-3}{6}\right)^2 = 1$
H	1	2	9	$\frac{1+8+9}{6} = 3$	$\left(\frac{9-1}{6}\right)^2 = 1.78$



Path 1-2-4-6-7; duration = 7.167 + 5.167 + 8 + 3 = 23.33 weeks
Path 1-3-4-6-7; duration = 5.167 + 5.5 + 8 + 3 = 21.667 weeks
Path 1-2-4-5-7; duration = 7.167 + 5.167 + 0 + 6 = 18.334 weeks
Path 1-3-4-5-7; duration = 5.167 + 5.5 + 0 + 6 = 16.667 weeks
Path 1-3-5-7; duration = 5.167 + 6.167 + 6 = 17.334 weeks
The critical path is BDFH (1-2-4-6-7)
Expected project completion,

$$T_E = 23.33 \text{ weeks}$$

$$\text{Variance along BDFH} = 0.25 + 0.694 + 1 + 1.78 = 3.724$$

$$\sigma = \sqrt{3.724} = 1.92976$$

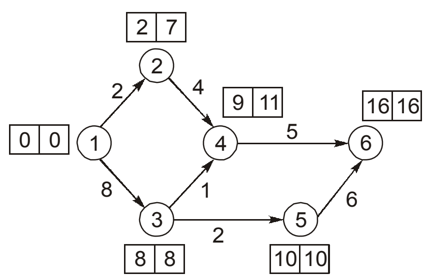
Standard normal variate (SNV)

$$z = \frac{T_S - T_E}{\sigma} = \frac{30 - 23.33}{1.92976} = 3.4$$

$$\therefore \quad P(z) = 0.9997 \text{ or } 99.97\%$$

Solution : 37

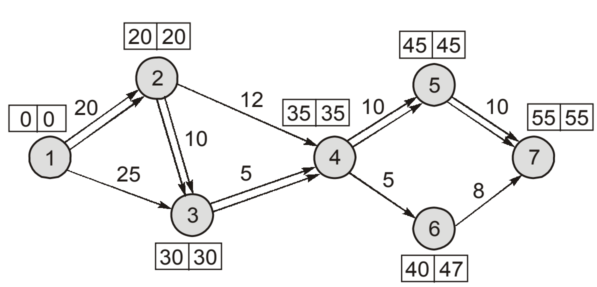
CPM network diagram is



Activity	Duration	EST	EFT	LST	LFT	TF	FF	SF	IF
1-2	2	0	2	5	7	5	0	5	0
1-3	8	0	8	0	8	0	0	0	0
2-4	4	2	6	7	11	5	3	0	-2
3-4	1	8	9	10	11	2	0	2	0
3-5	2	8	10	8	10	0	0	0	0
4-6	5	9	14	11	16	2	2	0	0
5-6	6	10	16	10	16	0	0	0	0

From the above table total float is zero for activities 1-3, 3-5 and 5-6. So critical path is 1-3-5-6. So total project duration is = 8 + 2 + 6 = 16

Solution : 38



Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Cost Slope	EST	EFT	LST	LFT	TF	FF	SF	IF
1-2	20	17	600	720	40	0	20	0	20	0	0	0	0
1-3	25	25	200	200	0	0	25	5	30	5	5	5	5
2-3	10	8	300	440	70	20	30	20	30	0	0	0	0
2-4	12	6	400	700	50	20	32	23	35	3	3	3	3
3-4	5	2	300	420	40	30	35	30	35	0	0	0	0
4-5	10	5	300	600	60	35	45	35	45	0	0	0	0
4-6	5	3	600	900	150	35	40	42	47	7	0	7	0
5-7	10	5	500	800	60	45	55	45	55	0	0	0	0
6-7	8	3	400	700	60	40	48	47	55	7	7	0	0
			3600										

For Activity 1-2: Cost slope $= \dfrac{720-600}{20-17} = 40$

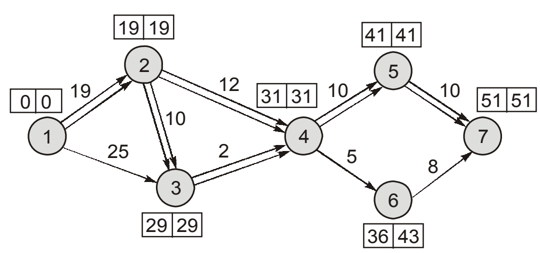
$$\text{EST} = 0,$$
$$\text{LFT} = 20$$
$$\text{EFT} = \text{EST} + t_{1-2} = 0 + 20 = 20$$
$$\text{LST} = \text{LFT} - t_{1-2} = 20 - 20 = 0$$
$$\text{TF} = \text{LST} - \text{EST} = \text{LFT} - \text{EFT} = 0$$

Therefore 1 - 2 is critical activity, similarly 2-3, 3-4, 4-5, 5-7 have 0 total float.

Total float of 1 - 3 is 5 days and total float of 2 - 4 is 3 days. So we can crash 3 days on activity 3 - 4 and 1 day on activity 1 - 2.

Therefore project cost = total normal cost + 4(40) = 3760

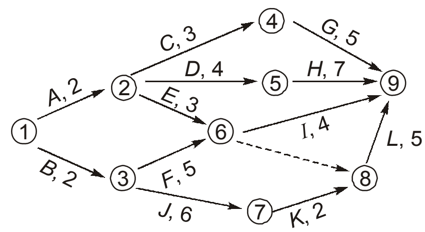
Thus the precedence diagram becomes



Solution : 39

Activity	t_0	t_m	t_p	$t_E = \dfrac{t_0 + 4t_m + t_p}{6}$
A	1	2	3	2
B	1	2	3	2
C	1	3	5	3
D	3	4	5	4
E	2	3	4	3
F	3	5	7	5
G	4	5	6	5
H	6	7	8	7
I	2	4	6	4
J	4	6	8	6
K	1	2	3	2
L	3	5	7	5

(i) Network diagram:



The possible paths are:

	Time (Days)
1 → 2 → 4 → 9	10
1 → 2 → 5 → 9	13
1 → 2 → 6 → 9	9
1 → 2 → 6 → 8 → 9	10
1 → 3 → 6 → 9	11
1 → 3 → 6 → 8 → 9	12
1 → 3 → 7 → 8 → 9	15 ← (Critical path)

(ii) Thus the critical path is 1 → 3 → 7 → 8 → 9 $(B - J - K - L)$

$(T_E = 15 \text{ days})$

(iii)
$$Z = \frac{T_S - T_E}{\sigma}$$

$$\sigma_{C.P} = \sqrt{\sigma_B^2 + \sigma_J^2 + \sigma_K^2 + \sigma_L^2} = \sqrt{\left(\frac{2}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + \left(\frac{2}{6}\right)^2 + \left(\frac{4}{6}\right)^2}$$

$$\sigma_{C.P} = 1.054$$

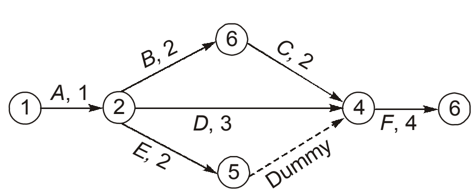
$$Z = \frac{18 - 15}{1.054} = 2.846$$

From the probability table, Probability = 99.77%

Solution : 40

Activity	$t_e = \frac{t_0 + t_p + 4t_m}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
A	1	1/36
B	2	1/9
C	2	0
D	3	1/25
E	2	25/144
F	4	1/4

1. Arrow diagram:



2. Among the six activities, activity '*F*' is most uncertain.
3. Critical path = (A → B → C → F)

$$\text{Time} = 9 \text{ days}$$

$$\sigma_{cp}^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_F^2 = \frac{1}{36} + \frac{1}{9} + 0 + \frac{1}{4}$$

$$\sigma_{cp}^2 = \frac{14}{36} = 0.388$$

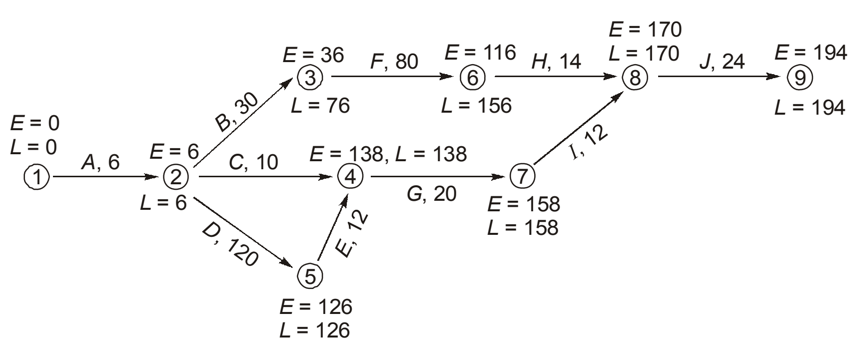
$$\sigma_{cp} = 0.623$$

So, $$Z = \frac{T - T_{cp}}{\sigma_{cp}} = \frac{10 - 9}{0.623} = 1.605$$

From SND chart, Probability = 0.9474 or 94.74%

Solution : 41

Activity	Duration (Days)	Preceding activities
A	6	NONE
B	30	*A*
C	10	*A*
D	120	*A*
E	12	*D*
F	80	*B*
G	20	*C*, *E*
H	14	*F*
I	12	*G*
J	24	*H*, *I*



Activity	t_E	EST	EFT	LST	LFT	Total Float
A, 1–2	6	0	6	0	6	0
B, 2–3	30	6	36	46	76	40
C, 2–4	10	6	16	128	138	122
D, 2–5	120	6	126	6	126	0
E, 5–4	12	126	138	126	138	0
F, 3–6	80	36	116	76	156	40
G, 4–7	20	138	158	138	158	0
H, 6–8	14	116	130	156	170	40
I, 7–8	12	158	170	158	170	0
J, 8–9	24	170	194	170	194	0

Based on the above calculated table, for critical activities, total float = 0.
Therefore critical path is : 1 — 2 — 5 — 4 — 7 — 8 — 9
Duration = 194 days.

Solution : 42

(i) Statistical assumptions made in PERT are:

(a) The activity durations are independent random variables.

(b) The critical paths are independent

(c) The project duration is assumed to follow a normal distribution, independent of the distribution of the activity duration.

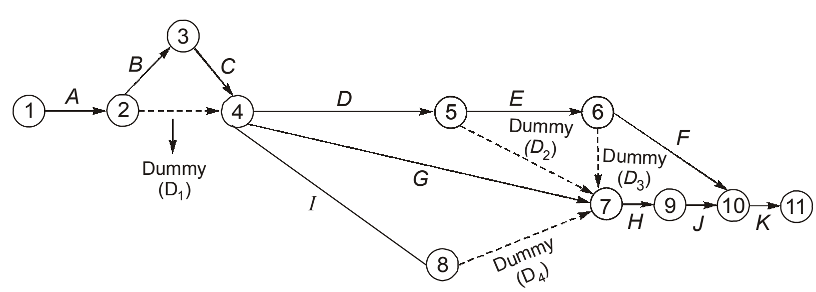
(d) The activity time follows: β – distribution

(e) The estimated time of completion is based on three time estimates (t_0, t_m and t_p).

If a particular activity has high variance then it will increase overall variance of project completion time if it falls on critical path.

Activity Name	$\left(t_e = \dfrac{t_0 + t_p + 4t_m}{6}\right)$	$\sigma^2 = \left(\dfrac{t_p - t_0}{6}\right)^2$
A	5	0.11
B	4.5	0.694
C	2	0.11
D	5.83	0.694
E	8	0.11
F	5.16	0.25
G	4	0.25
H	13.16	0.25
I	2.16	0.25
J	1	0
K	5.33	1

Network diagram :



Possible Paths

1. $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow K$ (35.82)

2. $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow D_3 \rightarrow H \rightarrow J \rightarrow K$ (44.82)

3. $A \rightarrow B \rightarrow C \rightarrow G \rightarrow H \rightarrow J \rightarrow K$ (34.99)

4. $A \rightarrow B \rightarrow C \rightarrow I \rightarrow D_4 \rightarrow H \rightarrow J \rightarrow K$ (33.15)

Critical path $\rightarrow$ ($A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow D_3 \rightarrow H \rightarrow J \rightarrow K$) ($T_E$ = 44.82)

$$\sigma_{cp}^2 = 0.11 + 0.694 + 0.11 + 0.694 + 0.11 + 0 + 0.25 + 0 + 1 = 2.968$$

$$\sigma_{cp} = 1.722$$

$$z = \frac{T - T_\varepsilon}{\sigma_{cp}}$$

$$Z_{.45} = \frac{45 - 44.82}{1.722} = 0.1045$$

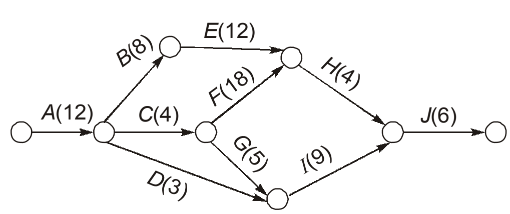
Probability of completion of project in 45 days is 0.0398 + 0.5 = 0.5398

$$Z_{50} = \frac{50 - 44.82}{1.722} = 3$$

Probability of completion in 50 days is 0.4987 + 0.5 = 0.9987

Solution : 43

CPM	PERT
(i) CPM uses activity network	(i) PERT uses event oriented network
(ii) It is used extensitely in construction projects	(ii) Mostly used in *R&D* projects
(iii) Deterministic concept is used	(iii) Probabilistic model is used
(iv) CPM can control both time and cost when planning	(iv) PERT is basically a tool for planning
(v) Duration of activity may be estimated with a fair degree of accuracy	(v) Estimate of time for activities are not so accurate and definite



PATH	DURATION (days)
$A \to B \to E \to H \to J$	12 + 8 + 12 + 4 + 6 = 42
$A \to C \to F \to H \to J$	12 + 4 + 18 + 4 + 6 = 44
$A \to C \to G \to I \to J$	12 + 4 + 5 + 9 + 6 = 36
$A \to D \to I \to J$	12 + 3 + 9 + 6 = 30

Critical path : $A \to C \to F \to H \to J$

Project Duration : 44 days Ans.

Solution : 44

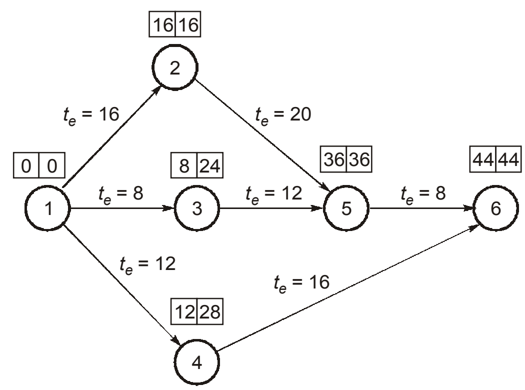
(i)
- Earliest start time (EST): It is the earliest time at which an activity can start. EST = E_j.
- Earliest finish time (EFT): It is the earliest possible time at which an activity can finish. EFT = EST + t_{ij}
- Latest start time (LST): It is the latest time by which the activity must be completed so that the scheduled date for the completion of the project may not be delayed.

LFT = L_j.

- Total float (TF) (or) Total slack (TS): Total float is the extra time which is available with any parallel activity without affecting the project completion time. The calculation of slacks or floats is used to determine the critical activities in a project network.
- Safety float: It is extra time available for starting any activity (t_{ij}) beyond LST of i^{th} node without affecting the LST of j^{th} node. SF = $L_j - L_i - t_{ij}$
- Independent float (IF): It is surplus time available with any activity without affecting any of earliest and latest time of any successor activity. Independent floats take a pessimistic view of the situation of an activity.

  Note: TF + IF = FF + SF
- Critical activities and critical path: The activities for which total float (TF) is zero are "Critical activities", any delay on the critical activities will affect the project completion. Combination of critical activities is called critical path and it tells us that 'in how much time project can be completed". It is the longest path.

  The critical path is represented by double thick arrow line to distinguish it clearly.



(ii) Paths are:

1 - 2 - 5 - 6 = 44 days (maximum time)

1 - 3 - 5 - 6 = 28 days

1 - 4 - 6 = 28

∴ Critical path 1 - 2 - 5 - 6

■■■■

4 Queuing Theory

LEVEL 1 Objective Questions

1.	(0.593)
2.	(a)
3.	(3.2)
4.	(b)
5.	(0.297)
6.	(b)
7.	(b)
8.	(b)
9.	(c)
10.	(d)
11.	(b)
12.	(0.25)
13.	(c)
14.	(c)

LEVEL 2 Objective Questions

15.	(b)
16.	(c)
17.	(0.343)
18.	(15)
19.	(62.5)
20.	(0.5)
21.	(d)
22.	(0.6)
23.	(b)

LEVEL 3 Conventional Questions

Solution : 24

$$\lambda = \frac{1}{9} \text{ per minute}, \quad \mu = \frac{1}{3} \text{ per minute}$$

(i) Probability that a person will have to wait:

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{9}}{\frac{1}{3}} = 0.33$$

(ii) Average queue length

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\left(\frac{1}{9}\right)^2}{\left(\frac{1}{3}\right)\left(\frac{1}{3}-\frac{1}{9}\right)} = \frac{1}{6} \text{ person}$$

(iii) If arrival is greater than the waiting time, the length of queue will go on increasing and that justifies the provision of a second milk booth.

$$E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)}$$

Here $E(W_q) = 4$ minutes, $\mu = \frac{1}{3}$ per min.

We have to find new λ'

$$4 = \frac{\lambda'}{\frac{1}{3}\left(\frac{1}{3}-\lambda'\right)}$$

or, $$\frac{\lambda'}{\frac{1}{3}-\lambda'} = \frac{4}{3}$$

or $$\lambda' = \frac{4}{21} \text{ arrival/minute } = 0.19/\text{minute}$$

$\therefore$ Increase in the flow of arrivals

$$= \frac{4}{21} - \frac{1}{9} = \frac{5}{63} \text{ per minute}$$

(iv) Probability (waiting time $\geq$ 10 min)

$$= \rho\, e^{-(\mu-\lambda)t} = \frac{1}{3} e^{-\left(\frac{1}{3}-\frac{1}{9}\right)10} = \frac{0.1083}{3} = 0.0361 = 3.61\%$$

(v) Probability [time in the system $\geq$ 10]

$$= e^{-(\mu-\lambda)t} = e^{-\left(\frac{1}{3}-\frac{1}{9}\right)10} = 0.1083 = 10.83\%$$

Solutin : 25

It is (M / M / 1) : (∞ / FCFS) model

Mean arrival rate, $\lambda = \frac{30}{60 \times 24} = \frac{1}{48}$ trains / minute

Mean service rate, $\mu = \frac{1}{36}$ trains / minute

Mean queue size, $l_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1-\rho} \left(\because \rho = \frac{\lambda}{\mu}\right)$

We have $\rho = \frac{\lambda}{\mu} = \frac{36}{48} = \frac{3}{4} = 0.75$

$\therefore$ $l_q = \frac{0.75 \times 0.75}{0.25} = 2.25$ trains

Probability (n > 10)

$= \rho^{10+1} = (0.75)^{11} = 0.042$

Solution : 26

Given: Service rate, $\mu = \frac{10}{5} = 2$ customers/minute

Assuming that nine customers (data missing in question) arrive on an average every 5 minutes,

Arrival rate (λ) $= \frac{9}{5} = 1.8$ customers/minute

(i) Average number of customers in the system:–

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{1.8}{2 - 1.8} = 9$$

(ii) Average number of customers in the queue:–

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(1.8)^2}{2(2 - 1.8)} = 8.1$$

(iii) Average time a customer spends in the system:–

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{2 - 1.8} = 5 \text{ minutes}$$

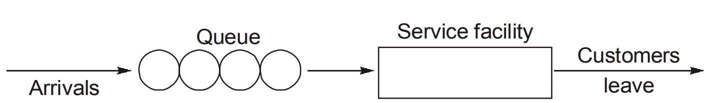
(iv) Average time a customer waits in the queue:–

$$W_q = \frac{\lambda}{\mu}\left(\frac{1}{\mu - \lambda}\right) = \frac{1.8}{2} \times \left(\frac{1}{2 - 1.8}\right) = 4.5 \text{ minutes}$$

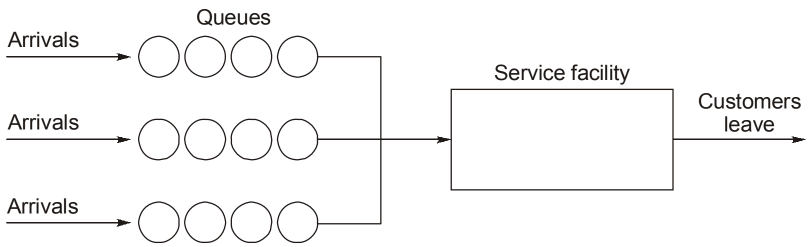
Solution : 27

(i) Different waiting line structures are:

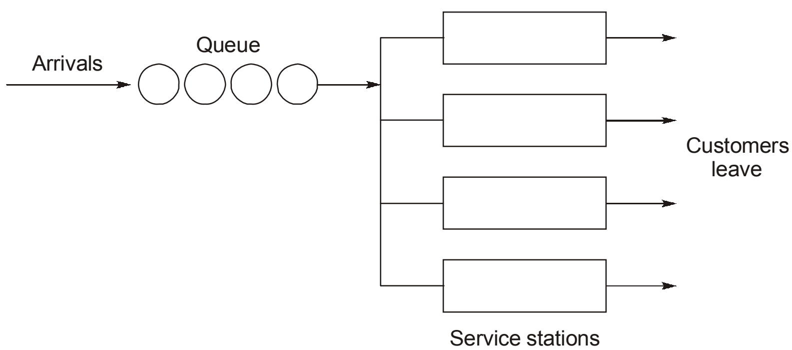
(a) Single server – Single Queue



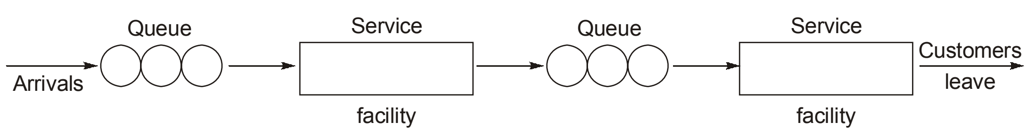
(b) Single Server – Several Queues



(c) Several (Parallel) Servers – Single Queue



(d) Service facilities in series:



(ii) Types of distributions assumed for arrivals and services are:

1. Random arrivals generally follow a Poisson distribution.
2. Service pattern are randomly distributed according to exponential probability distribution.

(iii) Given ($\lambda = \mu$)

$$\text{Utilization factor} = \frac{\lambda}{\mu}$$

$$\rho = 1 \quad (\text{when } \lambda = \mu)$$

and mean waiting time for a customer before being served (W_S) = $\frac{L_S}{\lambda}$

and $$L_S = \left(\frac{\lambda}{\mu - \lambda}\right)$$

$$W_S = \frac{1}{\mu - \lambda} = \underset{(\text{infinite})}{\infty} \quad [\text{when } \lambda = \mu]$$

Solution : 28

Arrival rate, $\lambda =$ 15 per hour

Service rate, μ =3 min. per person = 20 per hour

(i) Utility of teller, $P = \frac{\lambda}{\mu} = \frac{15}{20} = 0.75$

(ii) Average number in waiting line, $L_q = \frac{\lambda}{\mu - \lambda} - \frac{\lambda}{\mu} = \frac{15}{20 - 15} - \frac{15}{20} = 3 - 0.75 = 2.25$ person

(iii) Average number in the system, $L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{15}{20-15} = 3$ person

(iv) Average waiting time in line, $W_q = \dfrac{L_q}{\lambda} = \dfrac{2.25}{15} = 0.15$ hour = 9 min.

(v) Average waiting time in system, $W_s = \dfrac{L_s}{\lambda} = \dfrac{3}{15} = 0.20$ hour = 12 min.

Present level of service for 3 or fewer persons:

$n = 0 \quad P_0 = (1-\rho)\,\rho^0 = 0.25$

$n = 1 \quad P_1 = (1-\rho)\,\rho = 0.188$

$n = 2 \quad P_2 = (1-\rho)\,\rho^2 = 0.141$

$n = 3 \quad P_3 = (1-\rho)\,\rho^3 = 0.105$

$P_0 + P_1 + P_2 + P_3 = 0.684 = 68.4\%$

To attain 95% level of service

$P_0 + P_1 + P_2 + P_3 = 0.95$

$\Rightarrow (1-\rho) + \rho\,(1-\rho) + \rho^2\,(1-\rho) + \rho^3\,(1-\rho) = 0.95$

Onsolving polynomial

$\Rightarrow \qquad \rho = 0.47$

Utilization $\rho = \dfrac{\lambda}{\mu} = 0.47$

Solution : 29

Given, $\lambda = 6$/hour, i.e., machines break-down rate

Ideal time cost of the machines = Rs 15/hour

Repairman *A* : (Whose wage is Rs 8 per hour)

Service rate, $\mu_A = \dfrac{1}{6}$/min = 10/hour

Average number of units in the system, $L_s = \dfrac{\lambda}{\mu_A - \lambda} = \dfrac{6}{10-6} = 1.5$

Machine hours lost in 8 hour day = $1.5 \times 8 = 12$ hours

Total cost/day = cos of idle machines + repairman charges

Rs $(15 \times 12 + 8 \times 8)$ = Rs. 244

Repairman *B* : (whose wage is Rs 10 per hour)

Service rate, μ_B = 1/5 min = 12 /hour

Average number of units in the system, $L_s = \dfrac{\lambda}{\mu_B - \lambda} = \dfrac{6}{12-6} = 1$

machine hours lost in 8 hour day = $1 \times 8 = 8$ hours

total cost/day = cost of idle machines + repairman charges

= Rs $(15 \times 8 + 10 \times 8)$

= Rs 200

$\therefore$ The service of repairman *B* should be used, so that Rs 44

(i.e., Rs 244 – Rs 200) can be saved by the shop per day.

■■■■

5 Scheduling, Line Balacning

LEVEL 1 Objective Questions

1.	(a)
2.	(c)
3.	(c)
4.	(d)
5.	(b)
6.	(c)
7.	(d)
8.	(d)
9.	(c)
10.	(c)
11.	(d)
12.	(c)
13.	(c)
14.	(3)
15.	(35.71)
16.	(c)
17.	(a)

LEVEL 2 Objective Questions

18.	(b)
19.	(a)
20.	(d)
21.	(37)
22.	(22.22)
23.	(23.33)
24.	(14.4)
25.	(87.14)
26.	(6)
27.	14%
28.	(a)
29.	(80)
30.	(a)
31.	(43)
32.	(71.88)
33.	(6)
34.	(b)
35.	(14)
36.	(20.83)

LEVEL 3 Conventional Questions

Solution : 37

$$\text{Max}\,(M_i) \le \text{Min}\,(M_j)$$
$$\text{Max}\,(B) = 5 = \text{Min}\,(C)$$

	X = (A + B)	Y = (B + C)
1	7	11
2	13	14
3	9	6
4	7	8
5	7	11

4-1-5-2-3 (or) 4-5-1-2-3

	A	A	B	B	C	C
	In	Out	In	Out	In	Out
4	0	5	5	7	7	13
1	5	8	8	12	13	20
5	8	12	12	15	20	28
2	12	20	20	25	28	37
3	20	28	28	29	37	42

$$\text{Total cycle time} = 42 \text{ hours}$$

Gantt Charts:

Machine A:

0	5	8	12	20	28	42
4	1	5	2	3	Idle	

$$\text{Idleness of machine A} = \frac{14}{42} \times 100 = 33.33\%$$

$$\text{Utilization of machine A} = \frac{28}{42} \times 100 = 66.67\%$$

Machine B:

0	5	7	8	12	15	20	25	28	29	42
Idle	4	Idle	1	5	Idle	2	Idle	3	Idle	

$$\text{Idleness of machine B} = \frac{27}{42} \times 100 = 64.29\%$$

$$\text{Utilization of machine B} = \frac{15}{42} \times 100 = 35.71\%$$

Machine C:

0	7	13	20	28	37	42
Idle	4	1	5	2	3	

$$\text{Idleness of machine C} = \frac{7}{42} \times 100 = 16.67\%$$

$$\text{Utilization of machine C} = \frac{35}{42} \times 100 = 83.33\%$$

Solution : 38

Lowest processing time is 1 hour for job 6 on machine *B*. Thus we schedule job 6 last on machine *A* as shown below.

						6

Now, the reduced set of processing times becomes

Job :	1	2	3	4	5	7
Machine *A*	3	12	15	6	10	9
Machine *B*	8	10	10	6	12	3

There are two equal minimal values : Processing time of 3 hours for job 1 on m/c *A* and processing time of 3 hours for job 7 on machine *B*. According to rules, job 1 is scheduled first and job 7 next to job 6 as shown below :

1					7	6

The reduced set of processing times becomes

Job :	2	3	4	5
Machine *A*	12	15	6	10
Machine *B*	10	10	6	12

Again there are two equal minimal values : Processing time of 6 hours for job 4 on machine *A* as well as on machine *B*. We may choose arbitrarily to process job 4 next to job 1 or next to job 7 as shown below :

1	4				7	6

or

1				4	7	6

The reduced set of processing times becomes

Job :	2	3	5
Machine *A* :	12	15	10
Machine *B* :	10	10	12

There are three equal minimal values : Processing time of 10 hours for job 5 on machine *A* and for jobs 2 and 3 on machine *B*. According to rules, job 5 is scheduled next to job 4 in the first schedule or next to job 1 in the second schedule. Job 2 is then schedule next to job 7 in the first schedule or next to job 4 in the second schedule. The optimal sequences are shown below.

1	4	5	3	2	7	6

First Schedule

or

1	5	3	2	4	7	6

Second Sechedule

Jobs	Machine *A* Time IN	Machine *A* Time OUT	Machine *B* Time IN	Machine *B* Time OUT	Idle time for machine *B*
1	0	3	3	11	3
4	3	9	11	17	0
5	9	19	19	31	2
3	19	34	34	44	3
2	34	46	46	56	2
7	46	55	56	59	0
6	55	66	66	67	7

∴ minimum elapsed time T = 67 hours, Idle time for m/c A = 1 hour

Idle time for m/c B = 17 hours

Solution : 39

Step I : Select the minimum time of the matrix and if it appears for the first machine, process it first and if it appears for the second machine, process it last.

Step II : If there is a tie, see the processing time of other machine and whichever product has got minimum processing time then take the decision on that job first.

	1	7	4					8	5
Jobs	A	B	C	D	E	F	G	H	I
Machine-I	2	5	4	9	6	8	7	5	4
Machine-II	6	8	7	4	3	9	3	8	11
				6	2		3		

The order comes to be

A - C - I - B - H - F - D - G - E

(or)

A - C - I - H - B - F - D - G - E

	M/C I IN	M/C I OUT	M/C II IN	M/C II OUT
A	0	2	2	8
C	2	6	8	15
I	6	10	15	26
B	10	15	26	34
H	15	20	34	42
F	20	28	42	51
D	28	37	51	55
G	37	44	55	58
E	44	50	58	61 (Cycle time)

Gantt Charts:

Charts used to represent the idle time of a machine

M/c - I

0	2	6	10	15	20	28	37	44	50	61
A	C	I	B	H	F	D	G	E	Idle Time	

M/c - II

0	2	8	15	26	34	42	51	55	58	61
Idle Time	A	C	I	B	H	F	D	G	E	

$$\text{Idleness of M/c I} = \frac{11}{61} \times 100 = 18\%$$

$$\text{Utilization of M/c II} = \frac{50}{61} \times 100 = 81.96\%$$

$$\text{Idleness of M/c II} = \frac{2}{61} \times 100 = 3.3\%$$

$$\text{Utilization of M/c II} = \frac{59}{61} \times 100 = 96.72\%$$

Solution : 40

Using largest candidate rule for line balancing:
Given : line efficiency is 90% and number of stations = 3

Activity	Duration (seconds)	Activity which must precede
7	109	6
3	102	1
5	85	2
4	70	2
2	68	1
6	67	3, 4, 5
9	55	7, 8
1	46	–
8	44	6
10	20	9
	TWC = 666	

Let T_C be the cycle time.

Work Station	Activity	T_{si} (Station time)
I	1 3 2	216
II	5 4 6	222
III	7 8 9 10	228

$$\eta_L = 0.90$$

$$n = \frac{TWC}{T_C}$$

$$T_{C,\,min} = 228 \text{ seconds}$$

$$\eta_L = \frac{TWC}{n \cdot T_c} \times 100$$

$$T_C = 246.66 \text{ sec.}$$

Working days = 300 day/year

Production time/shift = 7 hours

Total time/year = $300 \times 7 = 2100$ hours

$$\text{Actual production rate} = \frac{0.9 \times 2100 \times 60 \times 60}{666} = 10216 \text{ units}$$

Solution : 41

Given: $T_C = 10$ min; Total work content (TWC) = 50 min

n_{min} (Minimum number of workstation)

$$n_{min} = \frac{TWC}{T_C} = \frac{50}{10} = 5 \text{ stations}$$

Using largest candidate rule:

Element	Time of Completion	Predecessor
12	7	11
5	6	2
8	6	7
1	5	–
6	5	5
3	4	2
10	4	6
11	4	7
2	3	1
4	3	1
7	2	6
9	1	6

Work Station	Element	T_{Si}	Idle time
I	1 2	5 + 3 = 8	2
II	5, 3	6 + 4 = 10	0
III	6, 10, 9	5 + 4 + 1 = 10	0
IV	4, 7, 11	3 + 2 + 4 = 9	1
V	12	7	3
VI	8	6	4

$$\text{Balance delay} = \frac{nT_c - TWC}{nT_C} = \frac{6 \times 10 - 50}{6 \times 10} = 16.67\%$$

$$\text{Line efficiency } n_L = 100 - BD\% = 100 - 16.67\% = 83.33\%$$

$$\text{Smoothness Index } (SI) = \sqrt{(10-8)^2 + (10-9)^2 + (10-7)^2 + (10-6)^2} = 5.477$$

Solution : 42

- Shortest Processing Time (SPT)
- Earliest Due Date (EDD)
- Minimum Slack Time (MST)

Slack time Remaining = Due time – processing time

- Critical Ratio Rule

$$\text{Critical ratio} = \frac{\text{Due time}}{\text{Processing time}}$$

This is a problem of n-jobs and two machines. Using Johnson's rule:
Optimum sequence, E - D - C - A - B - F

Job	Preparation time		Point shop time	
	In	Out	In	Out
E	0	2	2	8
D	2	5	8	16
C	5	10	16	23
A	10	20	23	28
B	20	27	28	32
F	27	31	32	35

∴ Make span time = 35 Hours.

Solution : 43

(i) This n-job and 3-machines case, to apply Johnson rule, it must be converted to 2-machines case by creating two artificial machines *G* and *H*.

For equivalent 2-machines conversion, any one of the following cases must be satisfied.

Minimum process time on $X \geq$ Maximum process time on M

Minimum process time on $W \geq$ Maximum process time on M

Minimum process time $X = 4$

Maximum process time M = 4

	$G = X + M$	$H = M + W$
N	11	8
A	8	10
O	10	10
L	9	12
E	10	8

Minimum process time $W = 4$

Both conditions are satisfied hence Jhonson rule can be applied.

∴ Optimal sequence, $A \rightarrow L \rightarrow O \rightarrow N \rightarrow E$

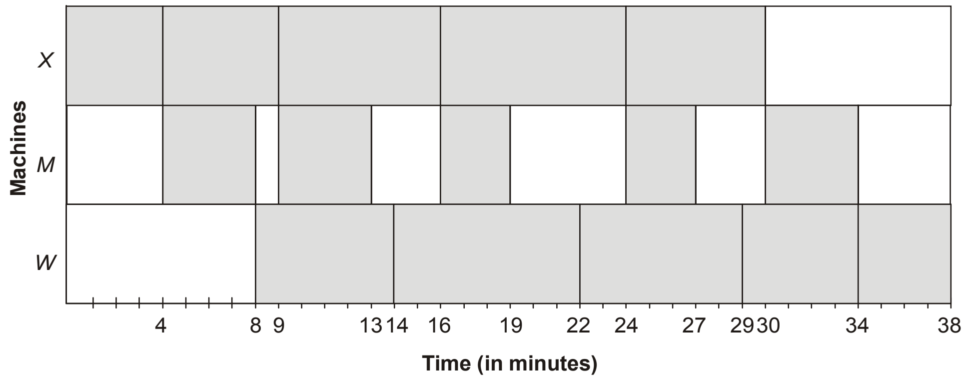
(ii) There is no alternative sequences possible.

(iii)

	X		M		W	
Job	In	Out	In	Out	In	Out
A	0	4	4	8	8	14
L	4	9	9	13	14	22
O	9	16	16	19	22	29
N	16	24	24	27	29	34
E	24	30	30	34	34	38
Process time (PT):	30		18		30	

∴ Make span time = 35 minutes.

Gantt Chart:



(iv) Make span time = 38 minutes.

Idle time for machine, X = MST – PT

= 38 – 30 = 8 min

Utilization % of machine, X = $\frac{\text{PT}}{\text{MST}} \times 100 = \frac{30}{38} \times 100 = 78.95\%$

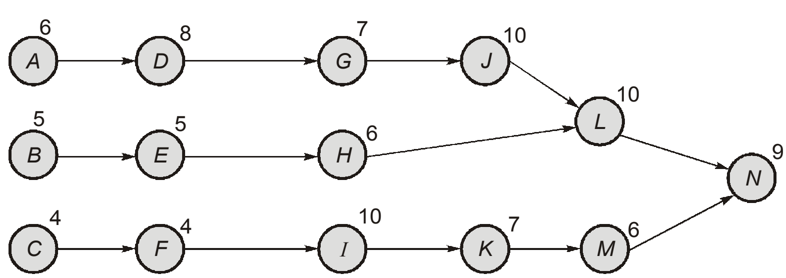
Idle time for machine, M = 38 – 18 = 20 min

Utilization % of machine, M = $\frac{18}{38} \times 100 = 47.37\%$

Idle time for machine, W = 38 – 30 = 8 minutes

Utilization % of machine, M = $\frac{30}{38} \times 100 = 78.95\%$

Solution : 44



Listing all the element is decreasing order of their elemental task time :

Element	I	J	L	N	D	G	K	A	H	M	B	E	C	F
Time (min.)	10	10	10	9	8	7	7	6	6	6	5	5	4	4
Precedence	F	G	J,H	L,M	A	D	I	–	E	K	–	B	–	C

Work Station	Element	Station Time (T_{si})	Idle Time ($T_c - T_{si}$)
I	A-6 B-5 E-5 C-4	20	0
II	D-8 G-7 F-4	19	1
III	I-10 J-10	20	0
IV	K-7 H-6 M-6	19	1
V	L-10 N-9	19	1

Total work duration of all the tasks = 97 min

Cycle time (T_c) = 20 min

$$\text{Number of work stations} = \frac{97}{20} = 4.85$$

∴ The optimal number of work stations = 5

$$\text{Balnce delay} = \frac{\eta T_c - T_{w_c}}{\eta T_c} = \frac{(5 \times 20) - 97}{100} = 3\%$$

$$\text{Line efficiency} = 1 - B.D = 97\%$$

■■■■

6 PPC and MRP

LEVEL 1 Objective Questions

1.	(b)
2.	(d)
3.	(c)
4.	(b)
5.	(a)
6.	(d)
7.	(b)
8.	(d)
9.	(a)
10.	(a)
11.	(c)
12.	(b)
13.	(d)
14.	(c)
15.	(c)
16.	(c)
17.	(a)
18.	(a)

LEVEL 2 Objective Questions

19.	(a)
20.	(d)
21.	(405)
22.	(c)
23.	(d)
24.	(b)
25.	(a)
26.	(d)
27.	(b)
28.	(c)
29.	(b)
30.	(b)
31.	(d)
32.	(c)
33.	(d)
34.	(7)

■■■■

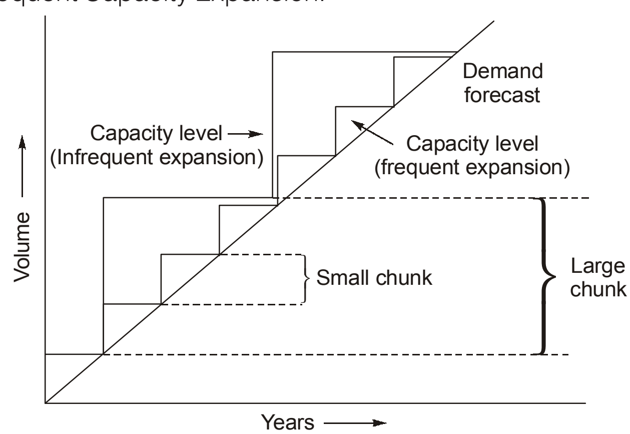
LEVEL 3 Conventional Questions

Solution : 35

Capacity Expansion: There are two types of costs to consider when adding or expanding capacity ®the cost of upgrading too frequently and that of upgrading too infrequently. Upgrading capacity too frequently is expensive. Direct costs including removing and replacing old requirements and training employees on the new equipment. In addition, the new equipment must be purchased, often for considerably more than the selling price of the old. Finally, there is the opportunity cost of idling the plant or service site during the changeover period.

Conversely, upgrading the capacity too infrequently is also expensive. Infrequent expansion means that capacity is purchased into the large chunks. Any excess capacity that is purchased must be carried as overhead until it is utilized.

Frequent versus Infrequent Capacity Expansion.



Solution : 36

Material Requirements Planning (MRP) is a production planning, scheduling, and inventory control system used to manage manufacturing processes. MRP works backward from a production plan for finished goods to develop requirements for components and raw materials.

Objectives:

1. To ensure that material and components are available for production, and final products are ready for dispatch.
2. To ensure minimum-inventory level and right quantity of material is available at the right time to produce right quantity of final products.
3. To ensure planning of all manufacturing processes, this involves scheduling of different job works as to minimize or remove any kind of idle time for machines and workers.

Dependent Demand : It is demand for component parts or subassemblies. Dependent demand depends upon the number of finished goods required or on number of other items.

Lot size : The part quantities issued in the planned order receipt and planned order release sections of an MRP schedule.

Gross Requirements : These are the total requirements for each item and calculated from the planned order release schedule of each final item.

Time phasing : Materials that are planned with the time phased planning technique are provided with the MRP date in the file. It represents the date on which the material is to be planned again and is calculated on the basis of the planning cycle.

Net requirement : The requirements of an item based on its gross requirements, minus stock already on-hand and scheduled receipts is defined as net requirement.

Solution : 37

Given data:

$$\eta_{machine} = 80\%$$
$$\text{scrap} = 25\%$$

Desired output = 1200 pieces per week

In a week = 40 hrs available

In a year = 50 weeks available

∴ useful component coming out of machine = 75%

Number of component required per year = 1200 × 50 = 60000/years

Time available = 40 × 50 = 2000 hrs/years

1 hrs. → 2 components

actual components per machine /hr = 2 × 0.8 × 0.75 = 1.2

∴ $2000 \times 1.2 \times N = 60000$

N = 25 machine

Solution : 38

$$\text{Number of kanbans} = \frac{\text{Hourly demand} \times \text{Order cycle time} \times \text{Safety factor}}{\text{Lot size}}$$

$$\text{(i)Number of } C\text{-kanbans} = \frac{200 \times 0.5 \times 1.1}{25} = 4.4 \text{ Ans.}$$

$$\text{Number of } P\text{-kanbans} = \frac{200 \times 1 \times 1.1}{25} = 8.8 \text{ Ans.}$$

Effective level of protection

Rounding off to higher integer $k_c = 5,\ k_p = 9$

implies we have more inventory in the system then required for a safety factor of 10%

Actual safety

$$\alpha_c = \frac{5 \times 25}{200 \times \frac{1}{2}} - 1 = \frac{1}{4} = 25\%$$

$$\alpha_p = \frac{9 \times 25}{200 \times 1} - 1 = \frac{1}{8} = 12.5\%$$

Rounding off to lower integer

$$k_c = 4,\ k_p = 8$$

$$\alpha_c = \frac{4 \times 25}{200 \times \frac{1}{2}} - 1 = 0$$

$$\alpha_p = \frac{8 \times 25}{200 \times 1} - 1 = 0$$

Reduces safety factor to zero.

■■■■

7 Linear Programming

LEVEL 1 Objective Questions

1. (b)
2. (b)
3. (c)
4. (a)
5. (b)
6. (a)
7. (6)
8. (d)
9. (c)
10. (c)
11. (d)
12. (d)
13. (c)
14. (c)
15. (b)
16. (c)
17. (c)
18. (b)
19. (d)

LEVEL 2 Objective Questions

20. (28000)
21. (a)
22. (b)
23. (3)
24. (d)
25. (–2)
26. (d)
27. (a)
28. (c)
29. (c)
30. (b)

■■■■

LEVEL 3 Conventional Questions

Solution : 31

The definition of the problem indicates that the decisions is to be made are the number of batches of the respective products to be produced per week so as to maximize their total profit.

Let x_1 = number of batches of product 1 produced per week

x_2 = number of batches of product 2 produced per week

z = Total profit (in thousand of rupees) per week from producing these two products

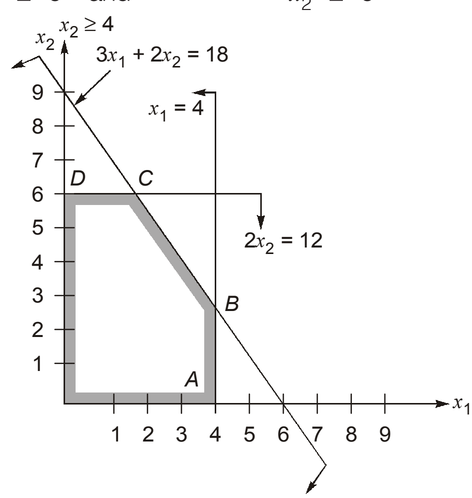
Thus x_1 and x_2 are the decision variables for the said LPP.

The objective is to choose the values of x_1 and x_2 so as to maximize

$$Z = 3x_1 + 5x_2$$

Subject to restriction
$$x_1 \le 4$$
$$2x_2 \le 12$$
$$3x_1 + 2x_2 \le 18$$

and $x_1 \ge 0$ and $x_2 \ge 0$



The values of (x_1, x_2) at all corner points of feasible region are :

$0 \equiv (0, 0)$ $A \equiv (4, 0)$

$B \equiv (4, 3)$ $C \equiv (2, 6)$

$D \equiv (0, 6)$

Z at $(0, 0) = 0$

Z at $(4, 0) = 3 \times 4 + 5 \times 0 = 12$

Z at $(4, 3) = 3 \times 4 + 5 \times 3 = 27$

Z at $(2, 6) = 3 \times 2 + 5 \times 6 = 36 \leftarrow$ Maximum

Z at $(0, 6) = 3 \times 0 + 5 \times 6 = 30$

Maximum profit, Z_{max} = ₹ 36000 per week

Solution : 32

(i) **Degeneracy :** An LP is degenerate if in a basic feasible solution, one of the basic variables takes on a zero value. Degeneracy is caused by a redundant constraints and could cost simplex method extra iterations.

(ii) **Alternative Optima :** An alternative optimal solution also called as an alternate optima, which is when a linear/integer programming problem has more than one optimal solution. Typically, an optimal solution is a solution to a problem which satisfies the set of constraints of the problem and the objective function which is to maximize or minimize. Alternate optima are said to exist when in the final objective function row, there exists $Z_j - C_j$ values (reduced objective function coefficients) which are zero for non basic variables. These non basic variables may then enter the basis (without altering the value of the objective function) and an alternative optimal solution is obtained.

(iii) **Unbounded solution :** In some linear programming models, the value of the variables may be increased indefinitely without violating any of the constraints, meaning that the solution space is unbounded in atleast one direction. As a result, the objective function may increase (maximization case) or decrease (minimization case) indefinitely. In this case both the solution space and the optimum objective value are unbounded. The rule for recognizing unboundedness is that if at any iteration all the constraint coefficients of any non basic variables are zero or negative, then the solution space is unbounded in that direction. If, in addition, the objective coefficient of that variable is negative in the case of maximization or positive in the case of minimization, then the objective function is unbounded as well.

Solution : 33

Let, Product 'A' $= x$

Product 'B' $= y$

(i) Profit, $z = 120x + 100y$

Constraints,

$2x + 2.5y \le 1000$ ... (i)

$3x + 1.5y \le 1200$ ... (ii)

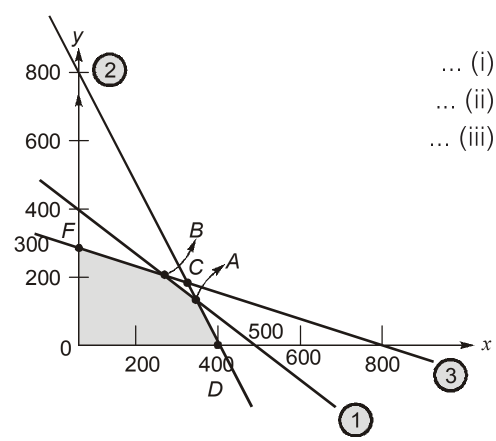
$1.5x + 4.0y \le 1200$ ... (iii)

$x, y \ge 0$

(ii) $\dfrac{x}{500} + \dfrac{y}{400} = 1$ ...(1)

$\dfrac{x}{400} + \dfrac{y}{800} = 1$ ...(2)

$\dfrac{x}{800} + \dfrac{y}{300} = 1$ ...(3)



From (1) and (2)

$\Rightarrow$ $x = \dfrac{1000}{3}, \quad y = \dfrac{400}{3}$ (Point A)

From (1) and (3) $x = \dfrac{4000}{17}, \quad y = \dfrac{3600}{17}$ (Point B)

or $A = \left(\dfrac{1000}{3}, \dfrac{400}{3}\right)$, In integer form (333, 133)

$B = (235, 211)$

$Z_A = (120 \times 333 + 100 \times 133) = 53260$ - Optimum profit.

$Z_B = (120 \times 235 + 100 \times 211) = 49300$

$Z_D = (120 \times 400 + 100 \times 0) = 48000$

$Z_F = (120 \times 0 + 300 \times 100) = 30000$

Solution : 34

Maximize, $z = 3x_1 + 2x_2 + 5x_3$

subject to $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + x_2 \leq 420$

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$

Standard Form:

$$Z = 3x_1 + 2x_2 + 5x_3 + 0.S_1 + 0.S_2 + 0.S_3$$

Subject to,

$$x_1 + 2x_2 + x_3 + S_1 = 430$$
$$3x_1 + 2x_3 + S_2 = 460$$
$$x_1 + x_2 + S_3 = 420$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

Initial solution: $S_1 = 430$, $S_2 = 460$, $S_3 = 420$

Incoming variable

e_i	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i	$\theta_i = b_i/a_{ij}$
0	S_1	1	2	1	1	0	0	430	430
0	S_2	3	0	$\boxed{2}$	0	1	0	460	230 ←
0	S_3	1	1	0	0	0	1	420	∞
	C_j	3	2	5					
	Z_j	0	0	0					
	Δ_j	3	2	5					
				↑					

Leaving variable

Ist iteration:

e_i	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i	θ_{ij}
0	S_1	–0.5	$\boxed{2}$	0	1	–0.5	0	200	100 ←
5	x_3	1.5	0	1	0	0.5	0	230	∞
0	S_3	1	1	0	0	0	1	420	420
	C_j	3	2	5					
	Z_j	7.5	0	5					
	Δ_j	–4.5	2	0					
			↑						

$S_1 = 200, \quad x_3 = 230, \quad S_3 = 420$

IInd iteration:

e_i	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b_i	θ_{ij}
2	x_2	–0.25	1	0	0.5	–0.25	0	100	
5	x_3	1.5	0	1	0	0.5	0	230	
0	S_3	1.25	0	0	–0.5	0.25	1	320	
	C_j	3	2	5					
	Z_j	7	2	5					
	Δ_j	–4	0	0					

This is an optimal solution as all values in Δ_j are negative or zero.

$x_1 = 0; \quad x_2 = 100; \quad x_3 = 230; \quad S_3 = 320$

$Z = 3x_1 + 2x_2 + 5x_3 = 2 \times 100 + 5 \times 230 = 1350$

Solution : 35

Let, x = Number of toy-1 produced.

y = Number of toy-2 produced.

Using simplex method maximum, $z = 10x + 10y + 0.s_1 + 0.s_2$

Subject to,
$$4x + 5y + s_1 + 0.s_2 = 100$$
$$5x + 2y + 0.s_1 + s_2 = 80$$
$$x, y \geq 0$$

Initial solution, $x = 0,\ y = 0,\ s_1 = 100,\ s_2 = 80$

e_i	Basic variable	x	y	s_1	s_2	b_i	$\theta_i = b_i/a_{ij}$
0	s_1	4	5	1	0	100	25
0	s_2	5	2	0	1	80	16 ←

	x	y	s_1	s_2
C_j	10	10	0	0
Z_j	0	0	0	0
Δ_j	10	10	0	0
	↑ key column			

e_i	Basic variable	x	y	s_1	s_2	b_i	θ_i
0	s_1	0	3.4	1	–0.8	36	10.58 ←
10	x	1	2/5	0	1/5	16	40

	x	y	s_1	s_2
C_j	10	10	0	0
Z_j	10	4	0	2
Δ_j	0	6	0	–2
		↑		

e_i	Basic variable	x	y	s_1	s_2	b_i	θ_i
10	y	0	1	1/3.4	–0.8/3.4	36/3.4	–
10	x	1	0	–0.117	0.2941	11.764	–

	x	y	s_1	s_2
C	10	10	0	0
Z_j	10	10	1.77	0.588
Δ_j	0	0	–1.77	–0.588

As all the values in (Δ_j) are either zero or negative, so this is our optimum solution.

$$x = 11.764$$

$$\text{and } y = \frac{36}{3.4} = 10.59$$

Taking whole numbers, $\begin{pmatrix} x = 11 \\ y = 11 \end{pmatrix}$

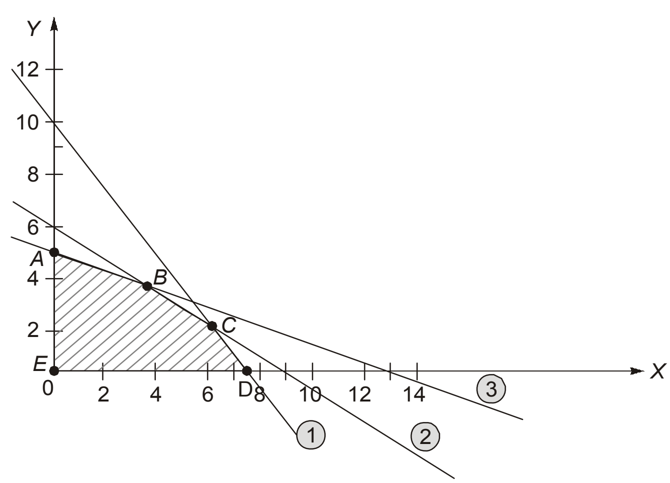
$$\text{Profit } (z) = 10 \times 11 + 10 \times 11 = 220$$

Solution : 36

Products	Types of machines Time taken (hours)		
	P	*Q*	*R*
A	10	6	5
B	7.5	9	13
Total time available (No. of hours/week)	75	54	65

The producer contemplates profit of Rs. 60 and Rs. 70 for the products *A* and *B* each respectively.
Let x_1 be the number of products produced of type *A* and
x_2 be the number of products produced of type *B*
$\therefore$ Maximize, $z = 60\,x_1 + 70\,x_2$
Constraints from the above table can be formed as,

$$10x_1 + 7.5x_2 \le 75 \quad \text{...(i)}$$
$$6x_1 + 9x_2 \le 54 \quad \text{...(ii)}$$
and
$$5x_1 + 13x_2 \le 65 \quad \text{...(iii)}$$



(0, 10) and (7.5, 0) points join the line (1)

(0, 6) and (9, 0) points join the line (2)

(0, 5) and (13, 0) points join the line (3)

Point of intersection of lines (1) and (2) is (6, 2)

Point of intersection of lines (2) and (3) is (3.54, 3.63)

Point of intersection of lines (3) and (1) is (5.27, 2.973)

Common region of all the three line is *EABCD*.

The basic feasible solutions are *E*(0,0)

A(0, 5)

B(3.54, 3.63)

C(6, 2)

$D(7.5, 0)$

Substituting the point *E*, *A*, *B*, *C* and *D* in the maximization function we get,

For point $E(0, 0), Z = 60 \times 0 + 70 \times 0 = 0$

$A(0, 5), Z = 60 \times 0 + 70 \times 5 = 350$

$B(3.54, 3.63), Z = 60 \times 3.54 + 70 \times 3.63 = 466.5$

$C(6, 2), Z = 60 \times 6 + 70 \times 2 = 500$

$D(7.5, 0), Z = 60 \times 7.5 + 70 \times 0 = 450$

Since the maximum profit is Rs. 500 at point $C(6, 2)$, therefore the optimum point is *C*.

Number of products of type *A* to be produced are 6 and

Number of products of type *B* to be produced are 2.

Solution : 37

Assuming that magazine has a reach to 2000 customers and magazine II has a reach to 2500 customers.

Let *X* and *Y* be the number of pages in magazine I and Magazine II respectively which ABC company should buy or opt for.

Maximize $Z = 2000X + 2500Y$ (objective function as in this are the reach of two magazines has to be maximised.)

First Constraint: (Constraint of monthly budget)

$$400X + 600Y \leq 6000 \quad \text{...(i)}$$

Second Constraint: (Constraint of minimum potential customer which should be reached)

$$400X + 200Y \leq 4000 \quad \text{...(ii)}$$

Other real constraints are $X \geq 0$ and $Y \geq 0$

from (i) we have

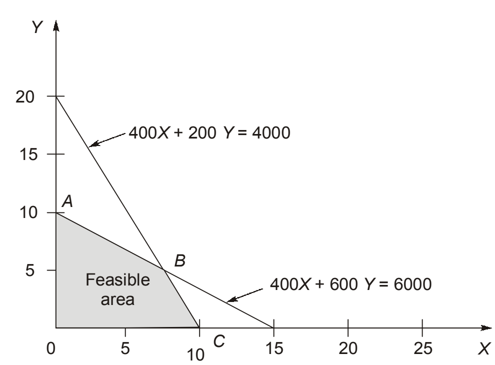
$X = 0, \quad Y = 10 \quad (0, 10)$

$Y = 0, \quad X = 15 \quad (15, 0)$

From (ii), we have $X = 0, \quad Y = 20 \quad (0, 20)$

$Y = 0, \quad X = 10 \quad (10, 0)$

Plotting these equations as straight lines on the graph to find the feasible solution area.



OABC is the feasible solution area.

Point	Coordinates	Value of objective function
O	$X = 0,\ Y = 0$	$Z = 2000 \times 0 + 2500 \times 0 = 0$
A	$X = 0,\ Y = 10$	$Z = 2000 \times 0 + 2500 \times 10 = 25000$
B	$X = 7.5,\ Y = 5$	$Z = 2000 \times 7.5 + 2500 \times 5 = 27500$
C	$X = 10,\ Y = 0$	$Z = 2000 \times 10 + 2500 \times 0 = 20000$

Z_{max} is at point B and equal to 27500.

The company should produce 7.5 pages in magazine I and 5 pages in magazine II to maximize its reach i.e. to 27500 people.

Note : The question is incomplete. Let us make assemption that magazine I has a reach of 2000 potential customer. The assumption can also be made vice-versa. If we take magazine I has reach of 2500 customers and Magazine II has reach of 2000 customers, then $Z_{max} = 2500\,X + 2000\,Y$ would be function to maximized.

$Z_{max}(7.5, 5) = 2500 \times 7.5 + 2000 \times 5 = 28750.$

■■■■

8 Transportation & Assignment

LEVEL 1 Objective Questions

1. (a)
2. (b)
3. (c)
4. (c)
5. (a)
6. (a)
7. (b)
8. (d)
9. (b)

LEVEL 2 Objective Questions

10. (1156)
11. (13)
12. (b)
13. (4550)
14. (128)
15. (20)
16. (c)
17. (b)
18. (c)
19. (a)

■■■■

LEVEL 3 Conventional Questions

Solution : 20

Since the given problem is maximization problem, we have to convert the profit matrix into opportunity loss matrix by substracting all the other element from the highest element 42 of the given matrix. Result is table 1.

9	3	1	13	1
1	17	13	20	5
0	14	8	11	4
19	3	0	5	5
12	8	1	6	2

TABLE - 1

8	0	0	7	0
0	14	12	14	4
0	12	8	6	4
19	1	0	0	5
11	5	0	0	1

TABLE - 2

Now reduce the matrix of table 1 so that there is at least one zero in each row and each column. Result is table 2.

Number or crossed lines in table 2 = 4 < 5

So, table 2 is not an optimal solution.

Now, let us see table 2. We have to look for the smallest element which is not crossed (out of the elements that do not have line crossed through them) is 4. Subtract 4 from all the element that do not have a line through them and add it to every element which lies at the intersection of two lines. Result is table 3.

	A	*B*	*C*	*D*	*E*
1	12	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$
2	$\boxed{0}$	10	8	10	$\cancel{0}$
3	$\cancel{0}$	8	4	2	$\boxed{0}$
4	23	1	$\boxed{0}$	$\cancel{0}$	5
5	15	5	$\cancel{0}$	$\boxed{0}$	1

Optimal assignment is

$1 \rightarrow B$

$2 \rightarrow A$

$3 \rightarrow E$

$4 \rightarrow C$

$5 \rightarrow D$

Maximum profit = ₹ 39000 + ₹ 41000 + ₹ 38000 + ₹ 42000 + ₹ 36000 = ₹ 196000

Solution : 21

1. Row minima:

0	13	49	2	10	18
0	35	29	7	20	5
13	0	63	9	17	5
47	15	0	22	12	5
25	0	46	11	14	7
0	53	50	28	14	25

2. Column minima:

0	13	49	0	0	13
0	35	29	5	10	0
13	0	63	7	7	0
47	15	0	20	2	0
25	0	46	9	4	2
0	53	50	26	4	20

Now, all the zeroes can be covered by a minimum of 5 lines i.e., it is not equal to the number of rows or column. Hence optimal solution is not reached.
Examining rows successively until a row with exactly one unmarked zero is found and making there an assignment and leaving other zeros in that row or column unassigned.

$$\begin{array}{cccccc|c}
\cancel{0} & 13 & 49 & \boxed{0} & \cancel{0} & 13 & \\
\cancel{0} & 35 & 29 & 5 & 10 & \boxed{0} & \checkmark \\
13 & \cancel{0} & 63 & 7 & 7 & \cancel{0} & \checkmark \\
47 & 15 & \boxed{0} & 20 & 2 & \cancel{0} & \\
25 & \boxed{0} & 46 & 9 & 4 & 2 & \checkmark \\
\boxed{0} & 53 & 50 & 26 & 4 & 20 & \checkmark \\
\checkmark & \checkmark & & & & \checkmark &
\end{array}$$

As no. of allocations is 5 which is less than number of rows, optimal solution is not reached. Performing optimality test, we get

$$\begin{array}{cccccc}
4 & 17 & 49 & \boxed{0} & \cancel{0} & 17 \\
\boxed{0} & 35 & 25 & 1 & 6 & \cancel{0} \\
13 & \cancel{0} & 59 & 3 & 3 & \boxed{0} \\
51 & 19 & \boxed{0} & 20 & 2 & 4 \\
25 & \boxed{0} & 42 & 5 & \cancel{0} & 2 \\
\cancel{0} & 53 & 46 & 22 & \boxed{0} & 20
\end{array}$$

We get the optimal solution:
$B \rightarrow a$; $E \rightarrow b$; $D \rightarrow c$; $A \rightarrow d$; $F \rightarrow e$; $C \rightarrow f$

Solution : 22

As the given matrix is maximization of profit per dress. Therefore, converting the profit matrix to an equivalent loss matrix by subtracting all the profit values from the highest value i.e. 55. The loss matrix obtained will be—

Manufactures	*I*	*II*	*III*	*IV*	Total Quantity
A	30	15	5	35	300
B	25	20	0	40	900
C	35	10	10	30	500
	200	400	900	300	1800 / 1700

It is unbalanced transportation problem.
Using Dummy manufacturer *D* to supply the remaining quantity i.e. 100 units. The balanced transportation problem will be —
Again due to Dummy manufacturer, our equivalent loss matrix will change accordingly. Element in loss matrix for Dummy will be (55 – 0 = 55).

	I	*II*	*III*	*IV*	
A	200 (30)	(15)	(5)	100 (35)	300
B	(25)	(20)	900 (0)	(40)	900
C	(35)	400 (10)	(10)	100 (30)	500
D	(55)	(55)	(55)	100 (55)	100
	200	400	900	300	1800 / 1800

Using Vogel's approximation method, initial basic feasible solution is–

$$\text{Total profit} = 200 \times 25 + 100 \times 20 + 900 \times 55 + 400 \times 45 + 100 \times 25 + 100 \times 0$$
$$= 77000$$

Checking for optimality using MODI method.

Here $m + n - 1 = 7$, but no. of allocations are 6 making the solution degenerate. Now, allocating infinitely small but positive value $\in$ (at vacant min cost cell), such that all allocated cells remains at independent position.

		$V_1 = 30$	$V_2 = 15$	$V_3 = 15$	$V_4 = 35$
		I	II	III	IV
$u_1 = 0$	A	200 [30]	[15]	θ [5]	100 $-\theta$ [35]
$u_2 = -15$	B	[25]	[20]	900 [0]	[40]
$u_3 = -5$	C	[35]	400 [10]	$\in -\theta$ [10]	100 $+\theta$ [30]
$u_4 = 20$	D	[55]	[55]	[55]	100 [55]

For unallocated cells, $\Delta_{ij} = C_{ij} - (u_i + v_j)$

$$\Delta_{12} = 15 - (0 + 15) = 0$$
$$\Delta_{13} = 5 - (0 + 15) = -10$$
$$\Delta_{14} = 35 - (0 + 35) = 0$$
$$\Delta_{21} = 25 - (-15 + 30) = 10$$
$$\Delta_{22} = 20 - (-15 + 15) = 20$$
$$\Delta_{24} = 40 - (-15 + 35) = 20$$
$$\Delta_{31} = 35 - (-5 + 30) = 10$$
$$\Delta_{41} = 55 - (20 + 30) = 5$$
$$\Delta_{42} = 55 - (20 + 15) = 20$$
$$\Delta_{43} = 55 - (20 + 15) = 20$$

New matrix will be (and $\theta = \in$)

		$V_1 = 30$	$V_2 = 15$	$V_3 = 5$	$V_4 = 35$
		I	II	III	IV
$u_1 = 0$	A	200 [30]	[15]	$\in$ [5]	100 [35]
$u_2 = -5$	B	[25]	[20]	900 [0]	[40]
$u_3 = -5$	C	[35]	400 [10]	[10]	100 [30]
$u_4 = 20$	D	[55]	[55]	[55]	100 [55]

$$\Delta_{12} = 15 - (0 + 15) = 0$$
$$\Delta_{21} = 25 - (30 - 5) = 0$$
$$\Delta_{22} = 20 - (15 - 5) = 10$$
$$\Delta_{24} = 40 - (-5 + 35) = 10$$
$$\Delta_{31} = 10$$
$$\Delta_{33} = 10 - (-5 + 5) = 10$$
$$\Delta_{41} = 5$$
$$\Delta_{42} = 20$$
$$\Delta_{43} = 25$$

As all Δ_{ij} values for unoccupied cells are positive, we reach the optimal solution.

Maximum profit $= (200 \times 25) + (50 \times \in) + (100 \times 20) + (900 \times 55)$

+ (400 × 45) + (100 × 25) + (100 × 0)

= 77000 + 50∈

= 77000 (As ∈ is very small)

North west corner rule is used to get initial basic feasible solution in transportation problem. This method starts in the cell (route) corresponding to the north west corner or the upper left of the table with maximum amount allocated to the selected cell and adjust the associated supply and demand quantities by subtracted the allocated quantity. This method is repeated until all the allocations are made.

Solution : 23

(i) The North-West Corner Rule is a method adopted to compute the initial feasible solution of the transportation problem. The name North-West Corner is given to this method because the basic variables are selected from extreme left corner.

(ii) If the basic feasible solution of a transportation problem with '*m*' origins and '*n*' destinations has fewer than (*m* + *n* – 1) positive x_{ij} (occupied cells), the problem is said to be a degenerate transportation problem. Degeneracy can occur at two stages.

- At the initial solution
- During the testing of the optimal solution.

To resolve degeneracy, we make use of an artificial quantity (∈). The quantity '∈' is assigned to that unoccupied independent cell (i.e. it should not make close loop), which has the minima transportation cost.

The quantity '∈' is so small that it does not affect the supply and demand constraints.

(iii) Vogel's approximation method (VAM)

	A	B	C	D	Supply	Row Penalty			
P	10	2 (15)	20	11	~~15~~		8	$\boxed{9}$ max.	
Q	12	7	9 (15)	20 (10)	~~25~~	10	2	2	$\boxed{11}$ max.
R	4 (5)	14	16	18 (5)	~~10~~	5	$\boxed{10}$ max.	2	2
Demand	~~5~~	~~15~~	~~15~~	~~15~~					
Column Penalty	6	5	7	7					
		5	7	7					
			7	2					

Final allocation

	A	B	C	D
P		15		
Q			15	10
R	5			5

∴ Transportation cost = (15 × 2) + (15 × 9) + (10 × 20) + (5 × 4) + (5 × 18) = 475

(iv) Optimality check is done with,

- Stepping stone method
- MODI or UV method.

Solution : 24

Using Hungarian Method:

Step 1: Subtract the minimum element of each row from other elements of that row.

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	0	4	12	2
C_2	6	0	8	2
C_3	0	6	12	6
C_4	14	2	10	0

Step 2: Column minima:

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	$\boxed{0}$	4	4	2
C_2	6	$\boxed{0}$	$\cancel{0}$	2
C_3	$\cancel{0}$	6	4	6
C_4	14	2	2	$\boxed{0}$

Step 3: Using method for optimal solution draw minimum number of lines to cover all zeroes.

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	0	4	4	2
C_2	6	0	0	2
C_3	0	6	4	6
C_4	14	2	2	0

Select the smallest element from uncovered elements (i.e. 2) and add it to the element crossed by two lines and subtract from element that is uncovered.

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	$\cancel{0}$	2	2	$\boxed{0}$
C_2	8	$\boxed{0}$	$\cancel{0}$	2
C_3	$\boxed{0}$	4	2	4
C_4	16	2	2	$\cancel{0}$

Again it is not optimal. Again performing the same sequence of steps to get optimal one.

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$
C_2	10	$\boxed{0}$	$\cancel{0}$	4
C_3	$\cancel{0}$	2	$\boxed{0}$	4
C_4	16	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

As there are more than one zero in each row and column, multiple assignments are possible.

$$\begin{bmatrix} C_1 \to M_1 \\ C_2 \to M_2 \\ C_3 \to M_3 \\ C_4 \to M_4 \end{bmatrix}$$

Solution (cost) $= 10 + 10 + 20 + 6 = 46$

or other solution,

Casting	Machines			
	M_1	M_2	M_3	M_4
C_1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
C_2	10	$\boxed{0}$	$\cancel{0}$	4
C_3	$\boxed{0}$	2	$\cancel{0}$	4
C_4	16	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

$$\begin{bmatrix} C_1 \to M_3 \\ C_2 \to M_2 \\ C_3 \to M_1 \\ C_4 \to M_4 \end{bmatrix}$$

Cost $= 8 + 10 + 22 + 6 = 46$

Solution : 25

Total supply = 550
Total demand = 490
Surplus supply = 60

		D	*E*	*F*	*G*	Supplies
	A	44	50	40	39	180
Factories	*B*	42	51	54	53	170
	C	41	40	42	45	200
		90	100	120	180	

Here we have a surplus of 60 units, therefore we create a fictitious (dummy) destination (side). The associated cost coefficients are taken as zero.

Finding the initial feasible solution :

	D	E	F	G	d	Supply	
A	44	50	40	39	0	180	[39]
B	42	51	54	53	0 (60)	170/110	[42] ←
C	41	40	42	45	0	200	[40]
Demand	90	100	120	180	60		
	[1]	[10]	[2]	[6]	0		

	D	E	F	G	Supply	
A	44	50	40	39	180	[1]
B	42	51	54	53	110	[9]
C	41	40 (100)	42	45	200/100	[1]
Demand	90	100	120	180		
	[1]	[10] ↑	[2]	[6]		

	D	F	G	Supply	
A	44	40	39	180	[1]
B	42 (90)	54	53	110/20	[11] ←
C	41	42	45	100	[1]
Demand	90	120	180		
	[1]	[2]	[6]		

	F	G	Supply	
A	40	39 (180)	180	[1]
B	54	53	20	[1]
C	42	45	100	[3]
Demand	120	180		
	[2]	[6] ↑		

	F	Supply
B	54 (20)	20
C	42 (100)	100
Demand	120	

The initial feasible solution is

	D	E	F	G	d	Supply
A	44	50	40	39 (180)	0	180
B	42 (90)	51	54 (20)	53	0 (60)	170
C	41	40 (100)	42 (100)	45	0	200
Demand	90	100	120	180	60	

Performing optimality test :

From the above solution, we can find that

(a) Number of allocations $= m + n - 1$, here $m = 3$, $n = 5$

$= m + n - 1 = 3 + 5 - 1 = 7$

But the number of allocations $= 6$, i.e., $m + n - 1 < 6$

therefore optimality test cannot be performed. Such a solution is called degenerate solution.

(b) The allocations are in independent positions.

	D	E	F	G	d
A				180	
B	90		20		60
C		100	100		

Now to apply optimality test, we have to allocate infinitesimally small but positive value $\in$ to the least cost cell so that the allocation should not form a closed loop.

i.e.,

	D	E	F	G	d
A				180	$\in$
B	90		20		60
C		100	100		

Sub Step-1 : Set up a cost matrix containing the costs associated with the cells for which allocation have been made.

	D	E	F	G	d	
A				39	0	u_1
B	42		54		0	u_2
C		40	42			u_3
	v_1	v_2	v_3	v_4	v_5	

$$
\begin{aligned}
u_1 + v_4 &= 39 \qquad & \text{take, } u_1 &= 0\\
u_1 + v_5 &= 0 & v_4 &= 39\\
u_2 + v_1 &= 42 & v_5 &= 0,\ u_2 = 0\\
u_2 + v_3 &= 54 & v_3 &= 54\\
u_2 + v_5 &= 0 & u_3 &= -12\\
u_3 + v_2 &= 40 & v_2 &= 52\\
u_3 + v_3 &= 42 & v_1 &= 42
\end{aligned}
$$

	D	E	F	G	d	
A				39	0	$u_1 = 0$
B	42		54		0	$u_2 = 0$
C		40	42			$u_3 = -12$
	v_1	v_2	v_3	v_4	v_5	
	42	52	54	39	0	

Filling the vacant cells with the sums of u_i and v_i.

42	52	54		
	52		39	
30			27	–12

Substract cell values of the above matrix from the original cost matrix

44–42	50–52	40–54		
	51–52		53–39	
41–30			45–27	0–(–12)

=

2	– 2	– 14		
	– 1		14	
11			18	12

∴ –ve values are there, the initial solution is not optimal.

In this matrix the most negative value is – 14. Randomly choose the cell (1, 3).

Write down the initial feasible solution,

			180	∈
90		20		60
	100	100		

Put mark (✓) for the most negative cell i.e., for (1, 3) cell

Now trace the path in this matrix consisting of a series of alternately horizontal and vertical lines. Mark the identified cell as positive and each occupied cell at the corners of the path alternatively –ve, +ve, –ve. Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned a –ve sign. Add and substract this new allocation from the cells at the corners of the path.

		✓ ⊕	180	∈ ⊖
90		20 ⊖		60 ⊕
	100	100		

This is the II feasible solution

		∈	180	
90		20		60
	100	100		

Checking for optimality :

$$m + n - 1 = 3 + 5 - 1 = 7$$

and number of allocation $s = 7$.

These 7 allocations are in independent positions.

∴ optimality test can be performed

By following the above procedure we get the following matrices,

v_1	v_2	v_3	v_4	v_5	
		40	39		u_1
42		54		0	u_2
	40	42			u_3

$$\begin{aligned} u_1 + v_3 &= 40 \\ u_1 + v_4 &= 39 \\ u_2 + v_1 &= 42 \\ u_2 + v_3 &= 54 \\ u_2 + v_5 &= 0 \\ u_3 + v_2 &= 40 \\ u_3 + v_3 &= 42 \end{aligned}$$

$$\begin{aligned} \text{take, } u_1 &= 0 \\ v_3 &= 40 \\ v_4 &= 39 \\ u_3 &= 2 \\ v_2 &= 38,\ u_2 = 14 \\ v_5 &= -14 \\ v_1 &= 28 \end{aligned}$$

	28	38	40	39	–14
= 0	28	38			–14
14		52		53	
2	30			41	–12

=

44–28	50–38			14
	51–52		53–53	
41–30			45–41	0 – (–12)

=

16	12			14
	– 1		0	
11			4	12

=

		∈	180	
90	✓ ⊕	20 ⊖		60
	100 ⊖	100 ⊕		

			$\in$	180	
=	90	20			60
		80	120		

This is the III feaside solution.

$$m + n - 1 = 3 + 5 - 1 = 7$$

number of allocation = 7,

All these 7 allocations are in independent positions.

Checking for optimality.

	v_1	v_2	v_3	v_4	v_5	
			40	39		u_1
	42	51			0	u_2
		40	42			u_3

$u_1 + v_3 = 40$ take, $u_2 = 0$

$u_1 + v_4 = 39$ $u_3 = -11$

$u_2 + v_1 = 42$ $v_2 = 51$

$u_2 + v_2 = 51$ $v_1 = 42$

$u_3 + v_2 = 40$ $v_3 = 53$

$u_3 + v_3 = 42$ $v_5 = 0$

$u_2 + v_5 = 0$ $u_1 = -13$

$v_4 = 52$

42	51	53	52	0	
29	38			–13	–13
		53	52		0
31			41	–11	–11

$\Rightarrow$

44–29	50–38			0–(–13)
		54–53	53–52	
41–31			45–41	0–(–11)

15	12			13
		1	1	
10			4	11

Since all values are positive hence solution is optimum.

Feasible solution (optimum) :

44	50	40 ($\in$)	39 (180)	0
42 (90)	51 (20)	54	53	0 (60)
41	40 (80)	42 (120)	45	0

Transportation cost is $40 \times \in + 39\times180 + 42 \times 90 + 51 \times 20 + 0\times60 + 40\times80 + 42\times120$ = Rs 20060.

Solution : 26

Add dummy row to make matrix square

18	24	28	32
8	13	17	18
10	15	19	22
0	0	0	0

$\boxed{0}$	6	10	14
$\cancel{0}$	5	9	10
$\cancel{0}$	5	9	12
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

$\boxed{0}$	6	10	14	✓
$\cancel{0}$	5	9	10	✓
$\cancel{0}$	5	9	12	✓
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	
✓				

$\boxed{0}$	1	5	9	✓
$\cancel{0}$	$\boxed{0}$	4	5	✓
$\cancel{0}$	0	4	7	✓
5	$\boxed{0}$	$\boxed{0}$	$\cancel{0}$	
✓	✓			

$\boxed{0}$	1	1	5
$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	3
9	4	$\cancel{0}$	$\boxed{0}$

Jobs		Machines
A	→	W
B	→	X
C	→	Y

Solution : 27

	1	2	3	4
A	20	36	31	27
B	24	34	45	22
C	22	45	38	18
D	37	40	35	28

Sub-Step : 1

Substract the minimum element of the each row from all the elements of the row. i.e.,

	1	2	3	4
A	0	16	11	7
B	2	12	23	0
C	4	27	20	0
D	9	12	7	0

Sub-Step : 2

Since columns 2 and 3 contain no zero entry, we have to substract minimum element of each column from all the elements of the column. i.e.,

	1	2	3	4
A	0	4	4	7
B	2	0	16	0
C	4	15	13	0
D	9	0	0	0

This is initial basic feasible solution.

Checking if optimal assignment can be made.

Examine rows successively until a row with exactly one unmarked zero is found. Mark (□) this zero, indicating that an assignment will be made there. Mark (*X*) all other zeroes in the same column showing that they cannot be used for making other assignments. i.e.,

	1	2	3	4
A	$\boxed{0}$	4	4	7
B	2	0	16	0
C	4	15	13	0
D	9	0	0	0

Next examine columns for single unmarked zeroes, making them (□) and also marking (*X*) any other zeros in this rows. Keep repeating the same procedure for rows and columns. i.e.,

Since there is one assignment in each row and in each column, the optimal assignment can be made in the current solution.

Optimum assignment is

$A \rightarrow 1$, $B \rightarrow 2$, $C \rightarrow 4$, $D \rightarrow 3$

Total work time = 20 + 34 + 18 + 35 = 107 hours.

■■■■

9 Element of Computation and Maintenance

LEVEL 1 Objective Questions

1.	(a)
2.	(c)
3.	(a)
4.	(b)
5.	(b)
6.	(d)
7.	(d)
8.	(d)
9.	(a)
10.	(c)
11.	(d)
12.	(d)

LEVEL 2 Objective Questions

13.	(a)
14.	(c)
15.	(c)
16.	(a)
17.	(0.025)
18.	(2)
19.	(9.48)
20.	(d)
21.	(c)

■■■■