2020

RANK improvement WORKBOOK

Detailed Explanations of Objective & Conventional *Questions*

Mechanical Engineering

Strength of Materials

MADE EASY Publications

1 Mechanical Properties of Materials & Elastic Constants

LEVEL 1 Objective Questions

1. (c)
2. (c)
3. (b)
4. (c)
5. (b)
6. (b)
7. (c)
8. (b)
9. (b)
10. (a)
11. (d)

LEVEL 2 Objective Questions

12. (c)
13. (b)
14. (c)
15. (d)
16. (d)
17. (b)
18. (a)
19. (c)
20. (0.4)
21. (c)
22. (b)

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LEVEL 3 Conventional Questions

Solution : 23

Given: D = 50 mm, L = 600 mm, P = 150 kN, δ = 0.23 mm, Torque (T) = 1.4 kN-m, θ = 1°

$$\because \quad \delta = \frac{PL}{AE}$$

$$0.23 = \frac{150 \times 10^3 \times 600}{\frac{\pi}{4}(50)^2 \times E}$$

$$\text{Elastic modulus, } E = 199.289 \approx 199.3 \text{ GPa}$$

And
$$\theta = \frac{TL}{GJ}$$

$$\left(1° \times \frac{\pi}{180°}\right) = \frac{1.4 \times 10^3 \times 600 \times 10^3}{G \times \frac{\pi}{32}(50)^4}$$

$$\text{Modulus of Rigidity, } G = 78.44 \text{ GPa}$$

Also,
$$G = \frac{E}{2(1+\nu)}$$

$$1 + \nu = \frac{E}{2G} = \frac{199.3}{2 \times 78.44} = 1.27$$

i.e.
$$\nu = 1.27 - 1 = 0.27 \text{ (Poisson's ratio)}$$

we know,
$$K = \frac{E}{3(1-2\nu)} = \frac{199.3}{3(1-2 \times 0.27)}$$

$$\text{Bulk modulus, } K = = 144.42 \text{ GPa}$$

Solution : 24

When σ_2 is applied along the two lateral directions,

$$e_2' = e_3' = \frac{\sigma_2}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

However, if σ_2 is not applied,
$$e_2 = -\frac{\mu\sigma_1}{E}$$

$$\therefore \quad \frac{\sigma_2}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_1}{E} = -\frac{1}{3}\frac{\mu\sigma_1}{E}, \quad \frac{\sigma_2}{E}(1-\mu) = \frac{\mu\sigma_1}{E}\left(1 - \frac{1}{3}\right)$$

Hence
$$\sigma_2 = \frac{2}{3}\frac{\mu}{1-\mu}\sigma_1$$

Net strain in the longitudinal direction :

$$e_1' = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_2}{E} = \frac{\sigma_1}{E}\left[1 - 2\mu \times \frac{2}{3}\frac{\mu}{1-\mu}\right]$$

or
$$e_1' = \frac{\sigma_1}{E}\left[\frac{3 - 3\mu - 4\mu^2}{3(1-\mu)}\right] \text{ compressive.}$$

Substituting $\mu = \frac{1}{m}$, the above expression reduces to

$$e_1' = \frac{\sigma_1}{E}\left[\frac{3m^2 - 3m - 4}{3m(m-1)}\right]$$

Solution : 25

We have $E = 2G(1+\mu)$

From which $\mu = \frac{E}{2G} - 1 = \frac{1.9\times 10^5}{2\times 0.75\times 10^5} - 1 = 0.267$

Also, $K = \frac{E}{3(1-2\mu)} = \frac{1.9\times 10^5}{3(1-2\times 0.267)} = 1.359\times 10^5 \text{ N/mm}^2$

For maximum percentage error in the derived value of μ, the error in the values of E and G should be of different sign. Let % error in E be +1 and that in G be –1.

Now $\mu = \frac{E}{2G} - 1$

Hence $\mu' = \frac{E'}{2G'} - 1$

where, E' = incorrect value of $E = (1.9\times 10^5)\times 1.01$

G' = incorrect value of $G = (0.75\times 10^5)\times 0.99$

and μ' = computed incorrect value of μ

$\therefore$ $\mu' = \frac{1.9\times 10^5\times 1.01}{2(0.75\times 10^5\times 0.99)} - 1 = 0.2923$

$\therefore$ % error in $\mu = \frac{\mu' - \mu}{\mu}\times 100 = \frac{0.2923 - 0.267}{0.267}\times 100 = 9.475\%$

Solution : 26

Given: $K_A = K_B = K;\ E_B = 1.01\,E_A$

From equation, we have $E = \frac{9\,KG}{G + 3K}$

or $EG + 3\,EK = 9\,KG$

or $3\,K(3G - E) = EG$, from which $K = \frac{EG}{3(3G-E)}$

Hence $\frac{E_A G_A}{3(3G_A - E_A)} = K = \frac{E_B G_B}{3(3G_B - E_B)}$

$\therefore$ $E_A G_A(3\,G_B - E_B) = E_B G_B(3G_A - E_A)$

or $3E_A G_A G_B - E_A G_A E_B = 3E_B G_B G_A - E_A E_B G_B$

or $G_B(3E_A\,G_A - 3E_B G_A + E_A E_B) = E_A G_A E_B$

From which
$$G_B = \frac{E_A G_A E_B}{3E_A G_A - 3E_B G_A + E_A E_B}$$
$$= \frac{1.01 E_A G_A E_A}{3E_A G_A - 3 \times 1.01 E_A G_A + 1.01 E_A \cdot E_A}$$
or
$$G_B = \frac{1.01 E_A G_A}{1.01 E_A - 3(1.01-1)G_A} = \frac{101 E_A G_A}{101 E_A - 3G_A}$$

Solution : 27

$$A = \frac{\pi}{4}(12)^2 = 113.1\,\text{mm}^2$$
$$\sigma = \frac{14 \times 10^3}{113.1} = 123.79\,\text{N/mm}^2$$
Now,
$$\text{lateral strain} = \frac{\delta d}{d} = \frac{3.6 \times 10^{-3}}{12} = 3 \times 10^{-4}$$
$$\text{But lateral strain} = \frac{\sigma}{mE}\ (\mu \times \text{longitudinal strain} = \frac{1}{m} \times \frac{\Delta L}{L})$$
$\therefore$
$$\frac{\sigma}{mE} = 3 \times 10^{-4}$$
or
$$mE = \frac{\sigma}{3 \times 10^{-4}} = \frac{123.79}{3 \times 10^{-4}} = 41.26 \times 10^4 \quad \text{...(i)}$$
Now
$$E = 2G\left(1 + \frac{1}{m}\right)$$
or
$$mE = 2\,G\,(m+1)$$
or
$$41.26 \times 10^4 = 2 \times 0.5 \times 10^5\,(m+1)$$
From which
$$m = 3.126 \text{ and } \frac{1}{m} \approx 0.32$$
$\therefore$
$$E = \frac{41.26 \times 10^4}{m} = \frac{41.26 \times 10^4}{3.126} = 1.32 \times 10^5\ \text{N/mm}^2$$
Also,
$$K = \frac{E}{3\left(1 - \frac{2}{m}\right)} = \frac{1.32 \times 10^5}{3\left(1 - \frac{2}{3.126}\right)} = 1.22 \times 10^5\ \text{N/mm}^2$$
Alternatively from equation
$$K = \frac{2G\left(1 + \frac{1}{m}\right)}{3\left(1 - \frac{2}{m}\right)} = \frac{2 \times 0.5 \times 10^5 (1 + 0.32)}{3(1 - 2 \times 0.32)} = 1.22 \times 10^5\ \text{N/mm}^2$$

Solution : 28

(a) Volume of bar,
$$V = b^2 \cdot L$$
$\therefore$
$$\frac{\delta V}{V} = 2\frac{\delta b}{b} + \frac{\delta L}{L} = 2e_b + e_L$$
or
$$\frac{\delta V}{V} = -\frac{2\sigma}{mE} + \frac{\sigma}{E} = \frac{\sigma}{E}\left(1 - \frac{2}{m}\right)$$

Now, $E = 2G\left(1+\frac{1}{m}\right)$

$\therefore$ $\frac{1}{m} = \frac{E}{2G} - 1 = \frac{2 \times 10^5}{2 \times 0.81 \times 10^5} - 1 = 0.2346$

or $m = 4.263$

Hence $\frac{\delta V}{V} = \frac{\sigma}{E}(1 - 2 \times 0.2346) = \frac{15000}{1600(2 \times 10^5)} \times 0.5309$

$= 2.488 \times 10^{-5}$

$\therefore$ % reduction in volume $= \frac{\delta V}{V} \times 100 = 2.488 \times 10^{-5} \times 100 = 0.00249$

(b) On the cube, $p = \sigma = wh$

Here, $w = 10080$ N/m³ (for sea water) and $h = 4$ km $= 4000$ m.

$\therefore$ $\sigma = 10080 \times 4000 = 40.32 \times 10^6$ N/m² $= 40.32$ N/mm²

Now, $K = \frac{mE}{3(m-2)} = \frac{4.263 \times 2 \times 10^5}{3(4.263 - 2)} = 1.256 \times 10^5 \text{ N/mm}^2$

Now $e_v = \frac{\delta V}{V} = \frac{\sigma}{K}$ (by definition)

$\therefore$ $\delta V = \frac{\sigma}{K} \times V = \frac{40.32}{1.256 \times 10^5} \times (100)^3 = 321 \text{ mm}^3$

Solution : 29

Volume of bar, $V = \frac{\pi}{4}d^2L = 0.785 \times (30)^2 \times 1500 = 1059750 \text{ mm}^3$

Given $\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1}{4} = 0.25 = \nu$

$E = 2G(1 + \nu)$

$\Rightarrow$ $G = \frac{50}{1.25} = 40 \text{ GPa}$ Ans.

$E = 3K(1 - 2\nu)$

$\Rightarrow$ $K = \frac{100}{3(1 - 2 \times 0.25)} = 66.7 \text{ GPa}$ Ans.

$K = \frac{100}{\left(\frac{dV}{V}\right)} \Rightarrow \frac{dV}{V} = \frac{100}{66.7 \times 10^3} = 1.499 \times 10^{-3}$

$\therefore$ Change in volume $= dV = 1059750 \times 1.499 \times 10^{-3}$

$\therefore$ $dV = 1588.83 \text{ mm}^3$

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2 Stress and Strain

LEVEL 1 Objective Questions

1.	(b)
2.	(a)
3.	(d)
4.	(c)
5.	(b)
6.	(c)
7.	(c)
8.	(a)
9.	(a)
10.	(c)
11.	(b)

LEVEL 2 Objective Questions

12.	(d)
13.	(d)
14.	(c)
15.	(a)
16.	(b)
17.	(b)
18.	(b)
19.	(d)

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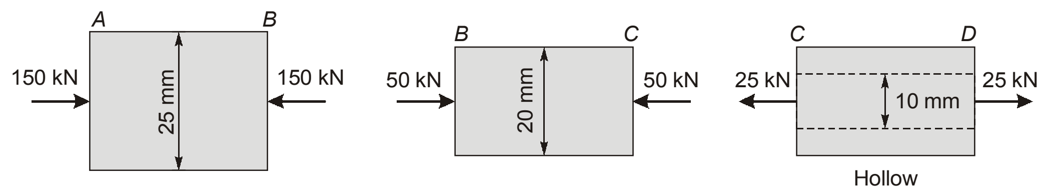
LEVEL 3 Conventional Questions

Solution : 20



Considering compressive stress as (–ve),

$$\sigma_{AB} = -\frac{150{,}000}{625} = -240 \text{ MPa (square section)}$$

$$\sigma_{BC} = -\frac{50{,}000}{400} = -125 \text{ MPa}$$

$$\sigma_{CD} = +\frac{25000}{400-100} = +83.33 \text{ MPa}$$

$$\sigma_{max} = -240 \text{ MPa} \quad \text{(in portion } AB\text{)}$$

$$\text{Change in length} = -\frac{240}{200{,}000}\times 2000 - \frac{125\times 2000}{200{,}000} + \frac{83.33\times 2000}{200{,}000}$$

$$= -2.4 - 1.25 + 0.833 = -2.817 \text{ mm (contraction)}$$

Solution : 21

Given: d_s = 1.8 cm = 18 mm, $(d_c)_i$ = 2 cm = 20 mm, $(d_c)_0$ = 3 cm = 30 mm, l = 50 cm = 500 mm, $E_s = 2 \times 10^5$ N/mm²

$$E_c = \frac{1}{2}\times E_s = 1\times 10^5 \text{ N/mm}^2$$

Let the compressive force developed in copper be P_c. Then, the force developed in steel is P_s.
From equilibrium condition;

$$P_s = P_c$$

$$\sigma_s \times A_s = \sigma_c \times A_c$$

$$\sigma_s \times \frac{\pi}{4}\times (18)^2 = \sigma_c \times \frac{\pi}{4}\times \left[(30)^2 - (20)^2\right]$$

$$\sigma_s = 1.543\,\sigma_c$$

As, nut is turned through 45° against the washer and pitch is 0.24 cm

$$\therefore \quad \text{Displacement} = \frac{45°}{360}\times 0.24 = 0.03 \text{ cm} = 0.3 \text{ mm}$$

∴ Due to tightening of nut,

$$0.3 \text{ mm} = \text{Extension of bolt + Contraction in tube}$$

$$= \frac{\sigma_s}{E_s}\times L + \frac{\sigma_c}{E_c}\times L = \left[\frac{(1.543\sigma_c)}{2\times 10^5} + \left(\frac{\sigma_c}{1\times 10^5}\right)\right]\times 500$$

$$0.3 = (885.75 \times 10^{-5}) \times \sigma_c$$

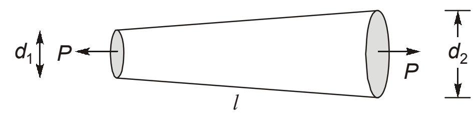
$$\sigma_c = -33.86 \text{ N/mm}^2$$

$$\sigma_s = 1.543 \times \sigma_c = 52.26 \text{ N/mm}^2$$

where, σ_c and σ_s are the compressive stress in copper tube and tensile stress in steel bolt respectively.

Solution : 22

(i)



$$\text{Elongation, } \Delta = \frac{4Pl}{\pi d_1 d_2 E}$$

$$\therefore \quad \frac{4Pl}{\pi d_1 d_2 E} = \frac{4Pl}{\pi d^2 E}$$

$$\therefore \quad d = \sqrt{d_1 d_2}$$

Maximum stress for tapered bar

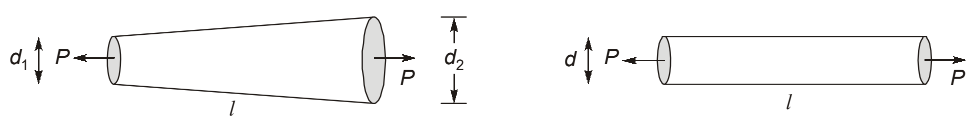
$$\sigma_{bar} = \frac{P}{A_{min}} = \frac{P}{\frac{\pi}{4}d_1^2} = \frac{4P}{\pi d_1^2}$$

Maximum stress in uniform C.S. bar

$$\sigma_{avg} = \frac{P}{A} = \frac{4P}{\pi d^2}$$

$$\sigma_{max} = 2\,\sigma_{avg} = \frac{8P}{\pi d^2}$$

(ii)



Solution : 23

Force on central bar = 200 kN (tensile)

Let the diameter of central portion be d_2.

$$\therefore \quad A_2 = \frac{\pi}{4}d_2^2$$

$$\therefore \quad \text{Stress} = \frac{200 \times 10^3}{\frac{\pi}{4}d_2^2} = \frac{254648}{d_2^2}\ \text{N/mm}^2$$

But this should not exceed 150 N/mm^2.

$$\therefore \quad 150 = \frac{254648}{d_2^2}$$

From which d_2 = 41.2 mm. Hence $A_2 = \frac{\pi}{4}(41.2)^2 = 1333.3\ \text{mm}^2$

Also, $A_1 = \frac{\pi}{4}(50)^2 = 1963.5\ \text{mm}^2$

Let the length of middle portion be L_2 mm

Hence total length of the two end portions = $2L_1 = (500 - L_2)$ mm

$$\therefore \qquad \Delta = \frac{P}{E}\left[\frac{2L_1}{A_1}+\frac{L_2}{A_2}\right] = \frac{200\times10^3}{200\times10^3}\left[\frac{500-L_2}{1963.5}+\frac{L_2}{1333.3}\right]$$

But Δ is limited to 0.3 mm.

$$\therefore \qquad 0.3 = \frac{500-L_2}{1963.5}+\frac{L_2}{1333.3}$$

$$= 0.2546 - 5.093\times10^{-4}\,L_2 + 7.5\times10^{-4}\,L_2$$

or $\qquad 2.407\,L_2\times10^{-4} = 0.0454$

From which $\qquad L_2 = 188.62$ mm

$$\therefore \qquad L_1 = \frac{1}{2}(500-188.62) = 155.69 \text{ mm}$$

Solution : 24

$$\text{Area of cross-section of wire, } A = \frac{\pi}{4}(10)^2 = 78.54 \text{ mm}^2$$

$$\text{Extension of wire due to the load, } \Delta_1 = \frac{PL}{AE} = \frac{2.5\times10^3\times150\times10^3}{78.54\times2.04\times10^5} = 23.41 \text{ mm}$$

$$\text{Extension of wire due to self weight, } \Delta_2 = \frac{wL^2}{2E}$$

Here, $\qquad w = \rho\, g = 7950\times9.81 = 77.99$ kN/m^3

$\qquad = 77.99\times10^3/10^9$ N/mm^3 $= 7.799\times10^{-5}$ N/mm^3

$$\therefore \qquad \Delta_2 = \frac{7.799\times10^{-5}(150\times1000)^2}{2\times2.04\times10^5} = 4.30 \text{ mm}$$

$\therefore \qquad$ Total elongation $= \Delta_1 + \Delta_2 = 23.41 + 4.30 = 27.71$ mm

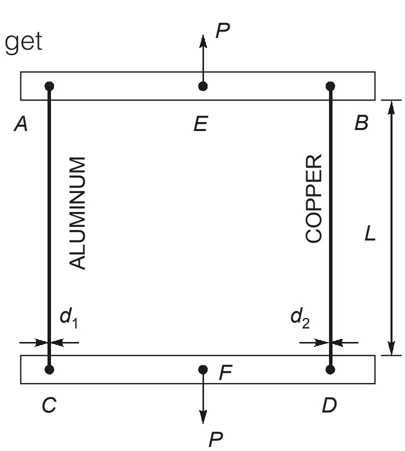
Solution : 25

Let us use suffix 1 for aluminium wire and suffix 2 for copper wire.

The extensions Δ_1 and Δ_2 of the two wires will be different. Hence the displacement Δ between points E and F is given by

$$\Delta = \frac{1}{2}(\Delta_1+\Delta_2)$$

Also, from statical equilibrium, by taking moments about plane AC, we get



$$P_2\times AB = P\times AE$$

$$\therefore \qquad P_2 = P\frac{AE}{AB} = \frac{1}{2}P$$

$$\text{Hence} \qquad P_1 = \frac{1}{2}P$$

$$\text{Now} \qquad \Delta_1 = \frac{P_1L}{A_1E_1} = \frac{\left(\frac{P}{2}\right)L}{\frac{\pi}{4}d_1^2E_1} = \frac{2PL}{\pi d_1^2E_1}$$

$$\text{Similarly} \qquad \Delta_2 = \frac{2PL}{\pi d_2^2E_2}$$

$$\therefore \qquad \Delta = \frac{1}{2}(\Delta_1 + \Delta_2) = \frac{1}{2}\left[\frac{2PL}{\pi d_1^2 E_1} + \frac{2PL}{\pi d_2^2 E_2}\right]$$

or
$$\Delta = \frac{PL}{\pi}\left[\frac{1}{d_1^2 E_1} + \frac{1}{d_2^2 E_2}\right]$$, which is the required expression.

If, however, both the wires are of the same material, $E_1 = E_2 = E$

$$\therefore \qquad \Delta = \frac{PL}{\pi E}\left[\frac{1}{d_1^2} + \frac{1}{d_2^2}\right]$$

Solution : 26

As the material is FG 300, σ_{ult} = 300 MPa

P = 25000 N, σ_{ult} = 300 N/mm², FOS = 3

$$\sigma_t = \frac{\sigma_{ult}}{FOS} = \frac{300}{3} = 100\,\text{N/mm}^2$$

$$\sigma_t = \frac{P}{A} + \frac{My}{I} = \frac{25000}{2t^2} + \frac{25000(10+t)t}{\frac{1}{12}t(2t)^3} = 100$$

$$\Rightarrow \qquad \frac{12500}{t^2} + \frac{37500(10+t)}{t^3} = 100$$

or
$$t^3 - 500t - 3750 = 0$$

Solving the above equation by trial and error method, t = 25.45 mm, say 26 mm.

Solution : 27

Given:

Length, l = 16 m = 16 × 10³ mm

Cross sectional area, A = 4 mm²

Weight of the wire ABC, W = 20 N

Modulus of elasticity, E = 200 GPa = 200 × 10³ N/mm².

Deflection at C

We know that deflection of wire at C due to self weight of the wire AC,

$$dl_c = \frac{Wl}{2AE} = \frac{20 \times (16 \times 10^3)}{2 \times 4 \times (200 \times 10^3)} = 0.2 \text{ mm}$$

Deflection at B

We know that the deflection at B consists of deflection of wire AB due to self weight plus deflection due to weight of the wire BC. We also know that deflection of the wire at B due to self weight of wire AB

$$\delta l_1 = \frac{\left(\frac{W}{2}\right) \times \left(\frac{l}{2}\right)}{2AE} = \frac{10 \times (8 \times 10^3)}{2 \times 4 \times (200 \times 10^3)} = 0.05 \text{ mm} \qquad \text{...(i)}$$

and deflection of the wire at B due to weight of the wire BC.

$$\delta l_2 = \frac{\left(\frac{W}{2}\right)\times\left(\frac{l}{2}\right)}{AE} = \frac{10\times\left(8\times10^3\right)}{4\times\left(200\times10^3\right)} = 0.1 \text{ mm} \quad \text{...(ii)}$$

∴ Total deflection of the wire at *B*.

$$\delta l_B = \delta l_1 + \delta l_2 = 0.05 + 0.1 = 0.15 \text{ mm}$$

Solution : 28

Longitudinal pull, $P = 125$ kN

Considering equilibrium of the bar, we have

$$P_{cu} + P_{zn} + P_{AL} = P = 125 \text{ kN} \quad \text{...(1)}$$

$$\frac{P_{cu}L}{A_{cu}E_{cu}} = \frac{P_{zn}L}{A_{zn}E_{zn}} = \frac{P_{Al}L}{A_{Al}E_{Al}}$$

$$P_{zn} = P_{cu}\frac{E_{zn}}{E_{cu}}\frac{A_{zn}}{A_{cu}} = \frac{100}{130}\times\frac{375}{250}P_{cu} = \frac{15}{13}P_{cu}$$

$$P_{Al} = \frac{A_{Al}}{A_{cu}}\times\frac{E_{Al}}{E_{cu}}P_{cu} = \frac{500}{250}\times\frac{80}{130}P_{cu} = \frac{16}{13}P_{cu}$$

Now, substituting the values of P_{zn} and P_{Al} in equation (i), we get

$$P_{cu} + \frac{15}{13}P_{cu} + \frac{16}{13}P_{cu} = 125$$

or
$$\frac{44P_{cu}}{13} = 125$$

$$P_{cu} = \frac{13\times125}{44} = 36.93 \text{ kN}$$

$$P_{zn} = \frac{15}{13}P_{cu} = \frac{15}{13}\times36.93 = 42.61 \text{ kN}$$

$$P_{Al} = \frac{16}{13}P_{cu} = \frac{16}{13}\times36.93 = 45.45 \text{ kN}$$

$$\sigma_{cu} = \frac{P_{cu}}{A_{cu}} = \frac{36.93\times10^3}{250\times10^{-6}} = 147.72 \text{ MPa}$$

$$\sigma_{zn} = \frac{P_{zn}}{A_{zn}} = \frac{42.61\times10^3}{375\times10^{-6}} = 113.63 \text{ MPa}$$

$$\sigma_{Al} = \frac{P_{Al}}{A_{Al}} = \frac{45.45\times10^3}{500\times10^{-6}} = 90.9 \text{ MPa}$$

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3 Shear force & Bending Moment

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(c)
4.	(a)
5.	(a)
6.	(d)
7.	(c)
8.	(d)
9.	(d)
10.	(c)
11.	(d)
12.	(c)
13.	(a)

LEVEL 2 Objective Questions

14.	(a)
15.	(a)
16.	(c)
17.	(c)
18.	(b)
19.	(c)
20.	(a)
21.	(d)
22.	(c)
23.	(b)
24.	(b)
25.	(4.47)
26.	(50)
27.	(d)
28.	(d)
29.	(150)
30.	(a)
31.	(a)
32.	(b)
33.	(c)

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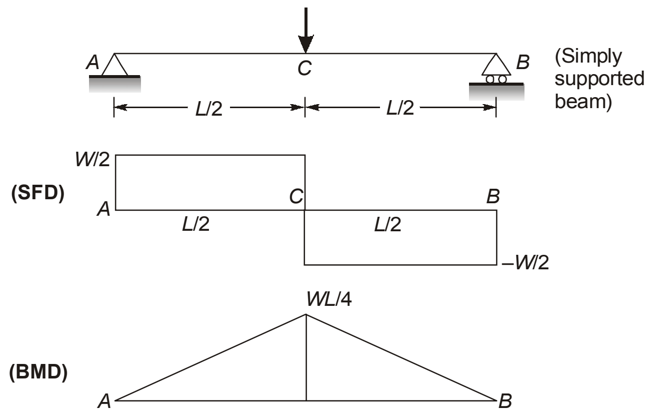
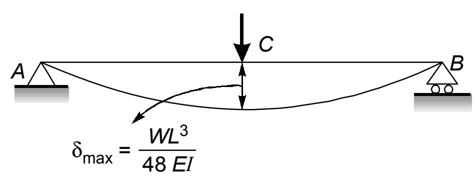
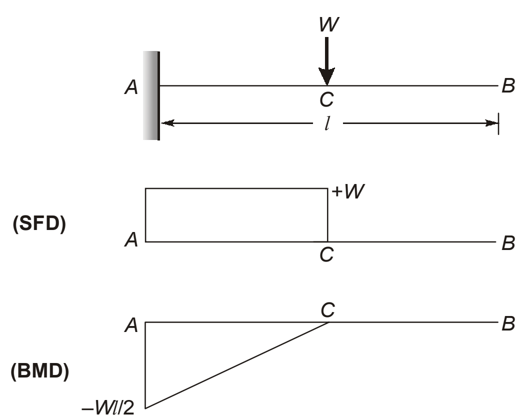
LEVEL 3 Conventional Questions

Solution : 34

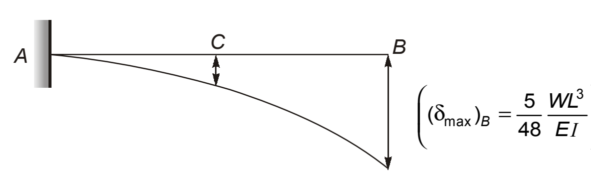


Deflection diagram:





Deflection diagram:



Structural members subjected to compression and which are relatively long compared to their lateral dimensions are called columns or struts. Generally, the term column is used to denote vertical members and the term strut denotes inclined members.

Columns are generally fixed at the both ends while strut can have any end fixation conditions like both end fixed, both ends hinged, one end fixed other end free, etc.

According to Rankine's formulae,

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

where P_e = buckling load

P_c = crushing load

For short column $P_e >>>> Pc$

or $\frac{1}{P_e} <<< \frac{1}{P_c} \Rightarrow \frac{1}{P_e}$ can be neglected ,

$$\frac{1}{P_R} = \frac{1}{P_c}$$

$$P_R \approx P_c \approx A\sigma_c$$

For long columns $P_c >>> P_e$

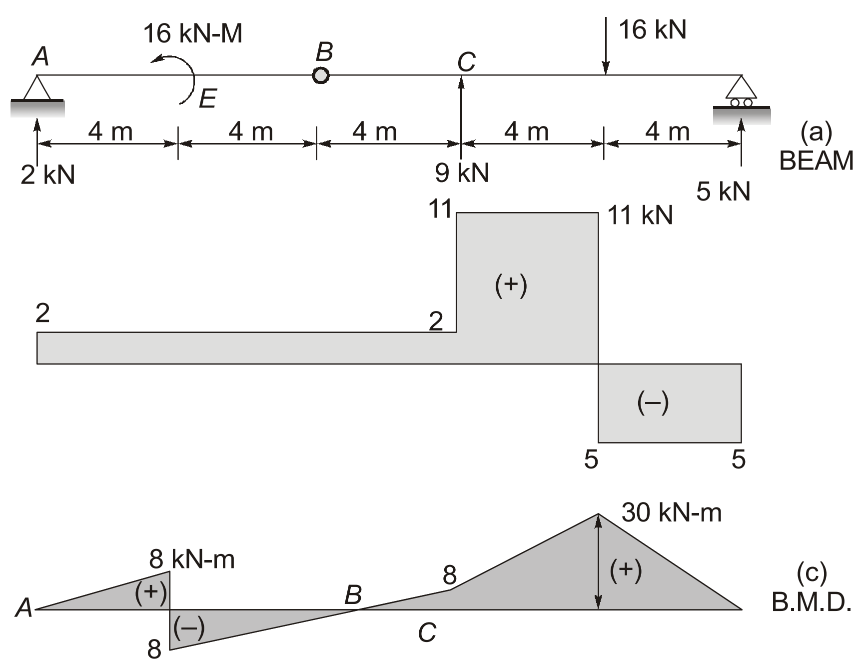
$\frac{1}{P_c} <<< \frac{1}{P_e} \Rightarrow \frac{1}{P_c}$ can be neglected

$$P_R \approx P_e = \frac{\pi^2 EI_{min}}{L_e^2}$$

$$\therefore \quad P_R = \frac{\sigma_c A}{1 + c(Se)^2}$$

where, c = Rankine's constant and Se = slenderness ratio

Solution : 35



Considering L.H.S $M_B = 0 = R_A \times 8 - 16$

$$\therefore \quad R_A = \frac{16}{8} = 2\ \text{kN}(\uparrow)$$

Considering R.H.S., $M_B = 0 = R_D.12 + R_C.4 - 16 \times 6$

or $R_C + 3R_D = 24$ . . . (i)

For the whole beam $R_A + R_C - R_D = 16$

$\therefore$ $R_C + R_D = 16 - 2 = 14$ . . .(ii)

From (i) and (ii), we get

$R_D = 5$ kN(↑) and $R_C = 9$ kN(↑)

S.F. Diagram : For aC : $F_x = 2$kN, which is constant from A to C.

For CF : $F_x = 2 + 9 = 11$ kN, which is a constant from C to F.

For FD : $F_x = 2 + 9 - 16 = -5$, which is constant from F to D.

The S.F. diagram is shown in figure.

B.M. diagram : For AE : $M_x = 2x$, which is a linear variation

At $x = 0$, $M_A = 0$. At $x = 4$ m, M_E(left) $= 2 \times 4 = 8$ kN-m

For EC : $M_x = 2x - 16$, which is a linear variation

At $x = 4$ m, $M_x = 2 \times 4 - 16 = -8$ kN. At $x = 8$, $M_B = 2 \times 8 - 16 = 0$, as expected.

At $x = 12$, $M_x = 2 \times 12 - 16 = 8$ kN-m

For CF : $M_x = 2x - 16 + 9(x - 12)$, which is a linear variation.

At $x = 14$ m, $M_{max} = 2 \times 14 - 16 + 9(14 - 12) = +30$ kN-m

For FD : $M_x = 2x - 16 + 9(x - 12) - 16(x - 14)$, which is a linear variation.

At $x = 20$m, M $= 2 \times 20 - 16 + 9(20 - 12) - 16(20 - 14) = 0$

as expected.

The B.M.D. is shown in figure.

Question : 36

Deflection at C due to UDL,

$$Y_C = \frac{wL^4}{8EI} + \frac{wL^3}{6EI} \cdot BC$$

$$= \frac{10 \times 4^4}{8EI} + \frac{10 \times 4^3 \times 2}{6EI}$$

$$Y_C = \frac{1600}{3EI}$$

Deflection due to vertical force at C.

$$Y_C = \frac{R_C \cdot l^3}{3EI} = \frac{R_C \cdot 216}{3EI}$$

$\therefore$ Equate defl^n as net deflection zero.

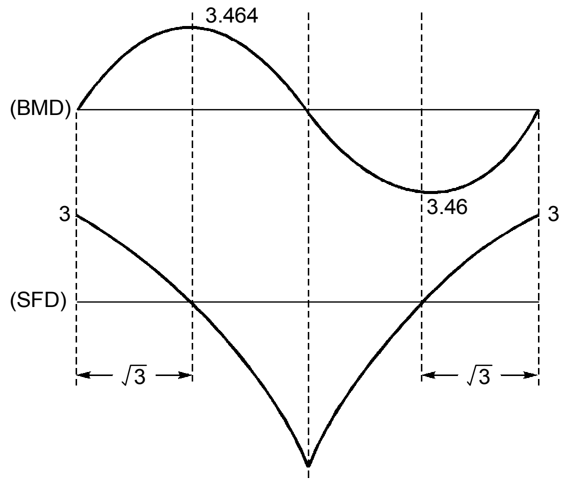
$$R_C = \frac{1600}{216} = \frac{200}{27} \text{ kN}$$

Solution : 37

$R_A = R_B = 3$ KN but opposite in direction

$$S_x = R_A - \frac{1}{2} \cdot 2x \cdot x = 3 - x^2$$

$$S_x = 0 \text{ at } x = \sqrt{3}$$



$$M_x = R_A \cdot x - \frac{1}{2} \cdot x \cdot 2x \cdot \frac{1}{3} x = 3x - \frac{x^3}{3}$$

Solution : 38

R_B and R_E be the vertical reactions at supports B and E respectively

$\therefore \quad R_B + R_E = 20 \times 0.5 + 50 + 40 = 100$ kN

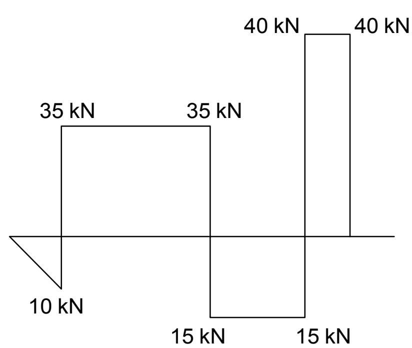
Taking moments about *B*

From $\Sigma M_B = 0$, we have

$\therefore \quad (20 \times 0.5) \times 0.25 = -100 + 50 \times (2+1) - R_E \times 4.5 + 5 \times 40$

Solving we get, $\quad R_E = 55$ KN

$\therefore \quad R_B = 100 - 55 = 45$ KN



Bending moment

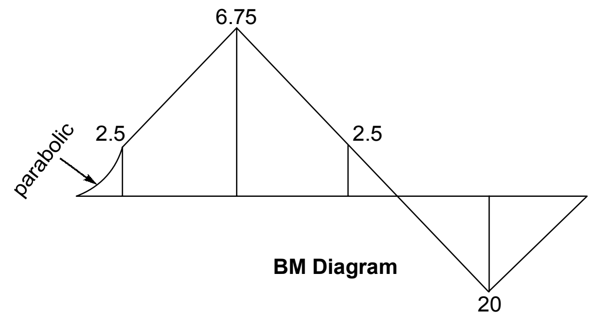
At *B*, $\quad BM = (20 \times 0.5) \times \frac{0.5}{2} = -2.5$ kNm

At *C*, $\quad BM = -(20 \times 0.5) \times 1.25 + 45 \times 1 - 100 = -67.5$ kNm

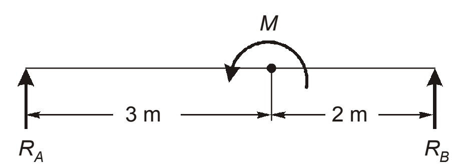
At *D*, $\quad BM = 40 \times (1.5 + 0.5) - 55 \times 1.5 = -2.5$ kNm

At *E*, $\quad BM = 40 \times 0.5 = 20$ kNm

At *G*, $\quad BM = BM = 0$



Solution : 39



Above is the loading diagram as observed from the bending moment diagram.

$$\therefore \quad R_A \times 3 - M = -8 \quad \text{...(i)}$$

$$R_A + R_B = 0 \quad \text{...(ii)}$$

$$R_A \times 5 - M = 0 \quad \text{...(iii)}$$

From equation (iii),

$$R_A = \frac{M}{5}$$

From equation (i),

$$3R_A - M = -8$$

$$\frac{3M}{5} - M = -8$$

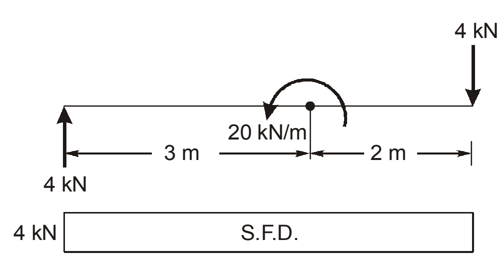
$$\frac{3M - 5M}{5} = -8$$

$$\frac{-2M}{5} = -8$$

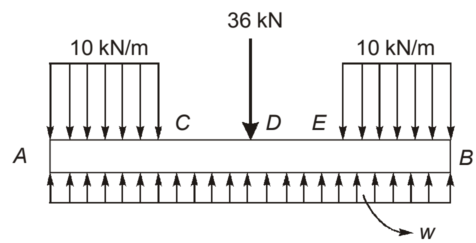
$$M = 20 \text{ kNm}$$

$$R_B = -R_A = -\frac{M}{5} = -\frac{20}{5} = -4 \text{ kN}$$

$$R_A = \frac{M}{5} = \frac{20}{5} = 4 \text{ kN}$$



Solution : 40



Let w be UDL generated due to uniform ground level

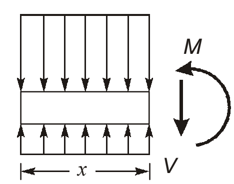
$$\Sigma F_y = 0 \text{ (for whole beam)}$$

$$AC = CD = DE = ED = 0.9 \text{ m}$$

$$3.6\,w - 0.9 \times 10 - 36 - 0.9 \times 10 = 0$$

or $\quad w = 15$ kN/m

for A to C ($0 < z < 0.9$ m):



$$15x - 10x - V = 0$$

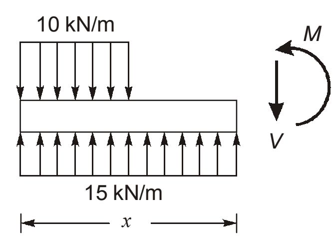
$$V = 5x$$

$$M - \frac{1}{2}(15x)x + \frac{1}{2}(10x)x = 0$$

At c, $\quad V = 4.5$ kN

and $\quad M = 2.025$ kNm

from C to D ($0.9 \text{ m} < x < 1.8$ m)



$$\Sigma F_y = 0$$

$\Rightarrow$
$$15x - 9 - V = 0$$

$$V = 15x - 9$$

$$\Sigma M = 0$$

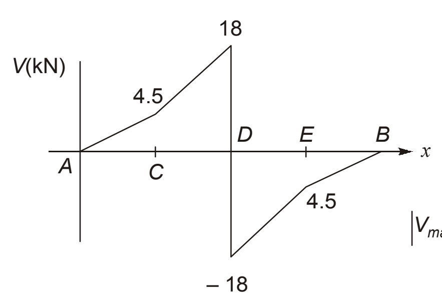
$\Rightarrow$
$$(-15x)\frac{x}{2} + 9(x - 0.45) + M = 0$$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

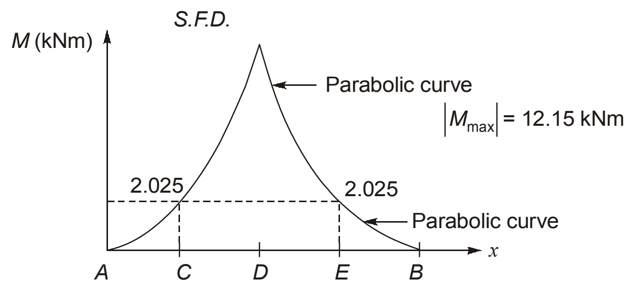
At D, $\quad V = 15 \times 1.8 - 9 = 18$ kN

$$M = 12.15 \text{ kNm}$$

form D to $B \rightarrow$ we can use symmetricity of the given loading.



$|V_{max}| = 18$ kN


S.F.D.
M (kNm)
Parabolic curve
|M_max| = 12.15 kNm
2.025
2.025
Parabolic curve
x
A
C
D
E
B

■■■■

4 Shear Stress and Bending Stress

LEVEL 1 Objective Questions

1.	(a)
2.	(c)
3.	(b)
4.	(c)
5.	(c)
6.	(b)
7.	(d)
8.	(a)
9.	(a)
10.	(c)
11.	(b)
12.	(c)
13.	(b)
14.	(a)
15.	(c)
16.	(b)
17.	(d)
18.	(a)
19.	(b)

LEVEL 2 Objective Questions

20.	(c)
21.	(d)
22.	(b)
23.	(b)
24.	(b)
25.	(1.36)
26.	(89.05)
27.	(b)
28.	(56.25)
29.	(d)
30.	(6.25)
31.	(266.67)
32.	(b)
33.	(166.67)
34.	(a)
35.	(c)
36.	(120)

■■■■

LEVEL 3 Conventional Questions

Solution : 37

Having same material and equal weight per unit length means same area of cross-section for all three beams.

I-section

Area of cross-sections $= 2 \times 150 \times 20 + 260 \times 12 = 6000 + 3120 = 9120 \text{ mm}^2$

$$I_{xx} = \frac{150 \times 300^3}{12} - \frac{138 \times 260^3}{12}$$

$$= 337.5 \times 10^6 - 202.124 \times 10^6 = 135.376 \times 10^6 \text{ mm}^4$$

$$Z_1 = \frac{135.376 \times 10^6}{150} = 0.9025 \times 10^6 \text{ mm}^3$$

Rectangular section

$$\text{Area} = 2B^2 = 9120 \text{ (same area of cross-section)}$$

$$B = 67.53 \text{ mm}$$

$$D = 135.055 \text{ m}$$

$$Z_2 = \frac{BD^2}{6} = \frac{67.53 \times 135.055^2}{6} = 0.20529 \times 10^6 \text{ mm}^3$$

Circular section, $\dfrac{\pi d^2}{4} = 9120$

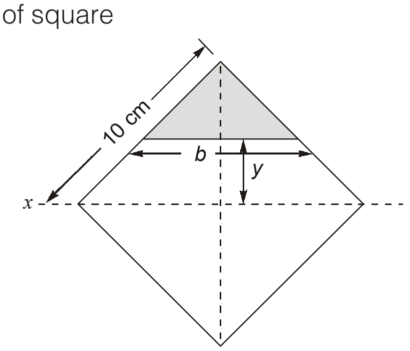
Diameter, $d = 107.758$ mm

$$Z_3 = \frac{\pi d^3}{32} = \frac{\pi \times (107.758^3)}{32} = 0.122845 \times 10^6 \text{ mm}^4$$

$$Z_1 : Z_2 : Z_3 = 0.9025 : 0.20529 : 0.122845 = 7.346 : 1.671 : 1$$

Solution : 38

Given: a = Side of square = 10 cm, F = Shear force, d = diagonal of square



As, $\tau = \dfrac{V\theta}{Ib}$

Here, $I_{xx} = \dfrac{a^4}{12}$, $V = F$

Consider the section at distance 'y' from neutral axis.

θ = First moment = Area × (Distance of CG from neutral axis)

$$\text{Area} = \frac{1}{2} \times \left(\frac{d}{2} - y\right) \times b$$

Distance of centroid from neutral axis,

$$\Rightarrow y + \frac{1}{3} \times \left(\frac{d}{2} - y\right) \Rightarrow y + \frac{1}{6} - \frac{y}{3} \Rightarrow \frac{2y}{3} + \frac{d}{6}$$

So,
$$\tau = \frac{V\theta}{Ib} = \frac{F \times \frac{1}{2} \times \left(\frac{d}{2} - y\right) \times b \times \left(\frac{2y}{3} + \frac{d}{6}\right)}{\left(\frac{d^4}{12}\right) \times b}$$

As,
$$a = \frac{d}{\sqrt{2}}$$

$$\tau = \frac{F \times \frac{1}{2} \times \left(\frac{d}{2} - y\right) \times b \times \left(\frac{2y}{3} + \frac{d}{6}\right)}{\frac{d^4}{45}}$$

Here, τ = Function (y) [As value of force F, diagonal d are constant]

For maximum shear stress,
$$\frac{d\tau}{dy} = 0$$

$$\therefore \quad \left(\frac{d}{2} - y\right)\left(\frac{2}{3}\right) - \left(\frac{2y}{3} + \frac{d}{6}\right) = 0$$

$$\frac{d}{3} - \frac{2y}{3} - \frac{2y}{3} - \frac{d}{6} = 0$$

$$\frac{d}{6} - \frac{4y}{3} = 0$$

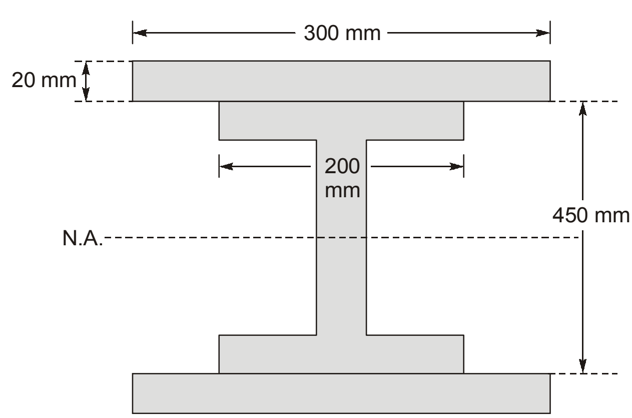
$$\frac{d}{6} = \frac{4y}{3}$$

$$y = \frac{d}{8}$$

$$d = a \times \sqrt{2} = \sqrt{2} \times 10 = 10\sqrt{2} \text{ cm}$$

Position of the layer at which shear stress is maximum is,
$$y = \frac{10\sqrt{2}}{8} = 1.76 \text{ cm from neutral axis.}$$

Solution : 39



Second moment of area of whole structure

$$I_{NA} = 35{,}060 + 2\left[\frac{1}{12}\times 30\times 2^3 + 30\times 2\times 23.5^2\right]$$
$$= 101370 \text{ cm}^4$$

Now as maximum stress will be near supporting plates

$$\Rightarrow \quad \sigma_{max} = \frac{My}{I}$$

$$\text{Permissible stress} = 125 \text{ MPa}$$

$$125\times 10^6 = \frac{M\times 24.5\times 10^{-2}}{101370\times 10^{-8}}$$

$$\Rightarrow \quad M = 517.19 \text{ kNm}$$

(i) Let the central load be W

If joist is simply supported $\quad M = \frac{WL}{4}$ at mid point

$$10^3\times 517.19 = \frac{W\times 5}{4}$$
$$W = 413.76 \text{ kN}$$

(ii) Let the plates be riveted at a distance x meters from each support.

Bending moment at this reaction

$$M = \frac{W}{2}\times x = \frac{413.76}{2}\times 10^3\times x = 206.88\, x \text{ kNm}$$

Now as $\quad I = 35{,}060 \text{ cm}^4$

$$Z = \frac{I}{y} = \frac{35060}{22.5} = 1558 \text{ cm}^3 = 1558\times 10^{-6} \text{ m}^3$$

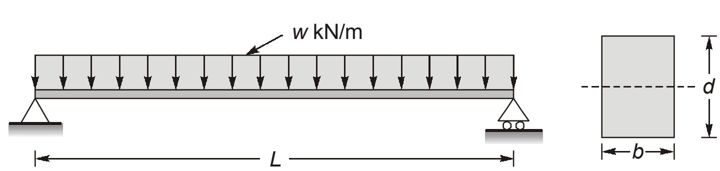
$$\frac{M}{Z} = \sigma_{max}$$

$$\Rightarrow \quad 206.88\, x\times 10^3 = 125\times 10^6\times 1558\times 10^{-6}$$

$$x = \frac{125\times 1558}{206.88\times 10^3} = 0.9415 \text{ m} = 0.942 \text{ m}$$

$$\text{Minimum length of plate} = 5 - (0.942\times 2) = 3.116 \text{ m}$$

Solution : 40



Given, $L = 8$ m, $P = 60$ kN, $\delta_{max} = 10$ mm, $E = 210$ GPa, $\sigma_{max} = 50$ MPa, $\delta_{max} = \frac{5wL^4}{384EI}$ $\quad [\because P = wl]$

So, $$\delta_{max} = \frac{5PL^3}{384EI}$$

$\therefore$ $$I = \frac{5PL^3}{384E\delta_{max}} = \frac{5 \times 60 \times 10^3 \times 8^3}{384 \times 210 \times 10^9 \times 10 \times 10^{-3}} = 1.9 \times 10^{-4}\ \text{m}^4$$

Maximum bending moment,

$$M_{max} = \frac{wL^2}{8} = \frac{PL}{8} = \frac{60 \times 10^3 \times 8}{8} = 60 \times 10^3\ \text{Nm}$$

$$\sigma_{max} = \frac{M_{max}}{Z}$$

So, $$Z = \frac{M_{max}}{\sigma_{max}} = \frac{60 \times 10^3}{50 \times 10^6} = 1.2 \times 10^{-3}\ \text{m}^3$$

$$I = \frac{bd^3}{12},\ Z = \frac{bd^2}{6}$$

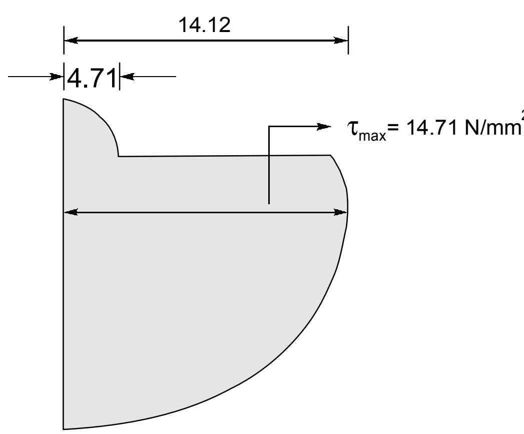
$\therefore$ $$\frac{I}{Z} = \frac{d}{2}$$

$\therefore$ $$d = \frac{2I}{Z} = \frac{2 \times 1.9 \times 10^{-4}}{1.2 \times 10^{-3}} = 0.3167\ \text{m}$$

$$b = \frac{12I}{d^3} = \frac{12 \times 1.9 \times 10^{-4}}{(0.3167)^3} = 0.0718\ \text{m}$$

Breadth of beam, $b = 71.8$ mm
Depth of beam, $d = 316.7$ mm } Ans.

Solution : 41



Shear Stress Distribution

Maximum shear stress occurs at the neutral axis.

Distance of neutral axis from the top edge : $\frac{(150 \times 50) \times 25 + (150 \times 50) \times [75 + 50]}{(150 \times 50) + (150 \times 50)}$

$$\bar{y} = \frac{25 + 125}{2} = 75\ \text{mm}$$

Centroidal position is given also.

Thus, $$\tau_{max} = \frac{F}{Ib} \times A\bar{y}$$

$$= \frac{100 \times 1000}{5312.5 \times 10^4 \times 50}[(150 \times 50) \times 50 + (50 \times 25) \times 12.5]$$
$$= 14.71 \text{ N/mm}^2$$

Shear stress in the flange at the junction of flange and web,

$$= \frac{100 \times 1000}{5312.5 \times 10^4 \times 150} \times (150 \times 50 \times 50)$$
$$= 14.12 \text{ N/mm}^2$$

The shear stress distribution is shown in figure.

Solution : 42

$$R_A + R_B = 60 \text{ kN}$$

or $\Sigma M_A = 0$

$\Rightarrow$ $25 \times 150 + 35 \times 750 = R_B \times 950$

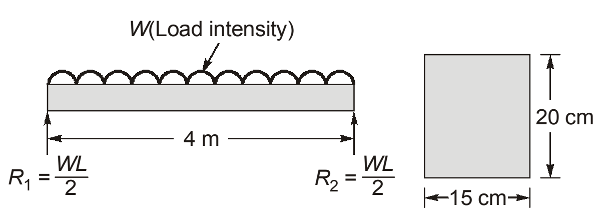
$\Rightarrow$ $R_B = \frac{30000}{950} = 31.58 \text{ kN}$

$$R_A = 60 - R_B = 28.42 \text{ kN}$$
$$M_C = R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N.mm}$$
$$M_D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6 \text{ N.mm}$$
$$M_{max} = M_D = 6.316 \times 10^6 \text{ N.mm}$$
$$\sigma_{max} = \frac{M_{max}}{Z} = \frac{32 M_{max}}{\pi d^3}$$

$\therefore$ $d^3 = \frac{32 \times 6.316 \times 10^6}{100 \times 3.14} = 643668.79 \text{ mm}^3$

$\therefore$ $d = 86.3$ mm say 90 mm

Solution : 43



Maximum bending moment is at the centre $= \left(\frac{WL}{2}\right) \times \frac{L}{2} - \frac{WL}{2}\frac{L}{4} = \frac{WL^2}{8}$

Maximum bending stress $= \sigma_b = \frac{M}{I/y} = \frac{M_{max} \times d/2}{\frac{bd^3}{12}} = \frac{6 M_{max}}{bd^2}$

$\Rightarrow$ $30 \times 10^6 = \frac{6WL^2}{8bd^2} = \frac{3}{4}\frac{WL^2}{bd^2} = \frac{3 \times W \times 16}{4 \times 0.15 \times (0.2)^2}$

$\Rightarrow$ $W = 15$ kN/m

Transverse stress $\tau_{max} = 1.5\frac{F}{150 \times 200} = 3$

Shear force, $F = 60{,}000\text{ N} = 60\text{ kN} = \dfrac{WL}{2}$

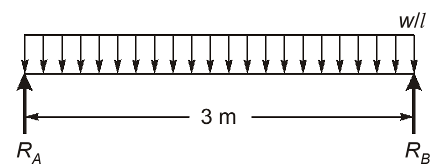
$$60 = W \times 2$$

$$W = 30\text{ kN/m}$$

∴ Allowable load, W = 15 kN/m (taking lower of two)

Solution : 44

Let the diameter of rod be "D" m.



Weight of rod = ρ × volume × g

$$= 7860 \times \frac{\pi}{4} \times D^2 \times 3 \times 9.81 = 181{,}678.15\ D^2\text{N}$$

Weight per unit length of rod $= \dfrac{181{,}678.15 D^2}{3}\text{N/m} = 60{,}559.38\ D^2\text{N/m}$

Maximum bending moment for simply supported and uniformly loaded beam

$$= \frac{wl^2}{8} = \frac{60559.38 D^2 \times 9}{8} = 68129.303\ D^2(\text{N.m.})$$

$$\sigma_{max} = \frac{M_{max}}{Z}$$

$$28 \times 10^6 = \frac{68129.303 D^2}{Z}$$

$$Z = \frac{I}{y} = \frac{\pi}{64}\frac{D^4}{\frac{D}{2}} = \frac{\pi}{32}D^3$$

$$28 \times 10^6 = \frac{68129.303 D^2}{\frac{\pi}{32} \times D^3}$$

$$D = 0.0248\text{ m} = 24.78\text{ mm}$$

Solution : 45

Let
D = inside diameter of the drum (1.25 m)
d = diameter of rod (6 mm)
l = radius of curvature of centreline of rods when bent

As the rods are coiled inside the drum,

$$\rho = \frac{D-d}{2} = \frac{1250-6}{2} = 622\text{ mm}$$

$$I = \frac{\pi}{64}d^4 = \frac{3.14}{64} \times (6)^4 = 63.617 \text{ mm}^4$$

As per theory of simple bending,

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{\rho}$$

so, $$\sigma_{max} = \frac{E}{\rho}y_{max} = \frac{200 \times 10^3}{622} \times 3 = 964.63 \text{ MPa}$$ Ans. (i)

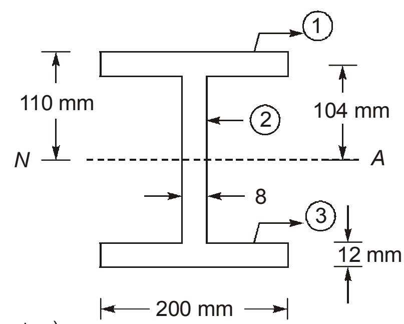
$$M = \frac{EI}{\rho} = \frac{200 \times 10^3 \times 63.617}{622} = 20455.627 \text{ Nmm}$$

Bending moment corresponding to maximum stress,

$$M_{max} = 20.455 \text{ Nm}$$

Solution : 46

MOI about neutral axis:



$$I_1 = \frac{1}{12} \times 200 \times 12^3 + 200 \times 12 \times (104)^2$$

$$I_1 = 25.9872 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} \times 8 \times (196)^3 = 5.0197 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 25.9872 \times 10^6 \text{ mm}^4 \quad \text{(from symmetry)}$$

$$I = I_1 + I_2 + I_3$$

$$I = (25.9872 + 5.0197\ 5 + 25.9872) \times 10^6 \text{ mm}^3 = 56.944 \times 10^6 \text{ mm}^3$$

$$\sigma_{max} = \frac{M}{z}$$

Largest bending moment, $M = \sigma_{max} \times z$

$$M = 155 \times 0.5177 \times 10^6 \text{ Nmm}$$

$$M = 80.24 \times 10^6 \text{ Nmm}$$

or $$M = 80.24 \times 10^3 \text{ Nm}$$

Solution : 47

Calculating reaction forces

$$\Sigma F_V = 0$$

$$R_A + R_B = (2 \times 3) + 4 = 10 \text{ kN}$$

$$R_A + R_B = 10000 \text{ N}$$

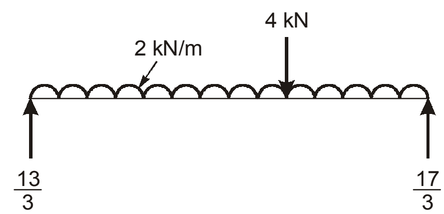
$$\Sigma M_B = 0$$

$$R_A \times 3 - \left(6 \times \frac{3}{2}\right) - (4 \times 1) = 0$$

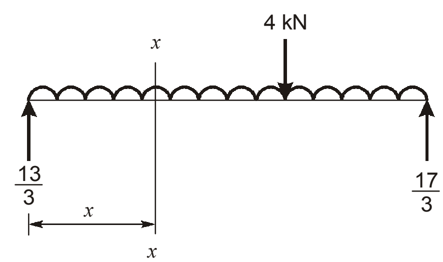
$$3R_A = 9 + 4$$

$$R_A = \frac{13}{3}\text{kN}$$

$$R_B = 10 - \frac{13}{3} = \frac{30-13}{3} = \frac{17}{3}\text{kN}$$



Calculating bending stress at "C"



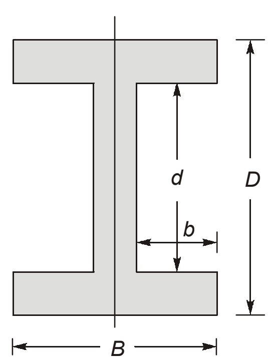
$$M = \frac{13}{3} \times x - 2x\frac{x}{2} \forall x \in (0, 2)$$

$$M = \frac{13}{3}x - 2x\frac{x}{2} - 4(x-2) \forall x \in (2, 3)$$

Bending moment at $x = 2$ m

$$M_2 = \frac{13}{3} \times 2 - 2 \times 2 \times \frac{\cancel{2}}{\cancel{2}} = 8.67 - 4 = 4.67 \text{ kNm}$$

$$Z = \frac{I}{y}$$



B = 75 mm, D = 200 mm

$$b = \frac{75}{2} - \frac{15}{2}$$

$$d = 200 - 20 = 180 \text{ mm}$$

$$= \frac{60}{2} = 30 \text{ mm}$$

$$I = \frac{BD^3}{12} - \frac{2 \times bd^3}{12} = \frac{75 \times 200^3}{12} - \frac{2 \times 30 \times 180^3}{12}$$

$$= 20840,000 \text{ mm}^3$$

$$y = \frac{I}{y} = \frac{20840000}{100} = 203400 \text{ mm3}$$

Bending stress, $$\sigma = \frac{M}{Z}$$

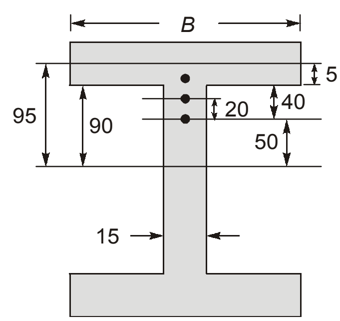
$$\sigma_b = \frac{4.67 \times 10^3 \times 10^3}{208400} = 22.4088 \text{ N/mm}^2$$

It is compressive in nature

Calculating shear stress at location "*D*".

$$\tau = \frac{FA\bar{y}}{ZI}$$

$$F = R_A + (2 \times 2) + 4 = \frac{13}{3} + 4 + 4 = 12.33 \text{ kN}$$



All dimensions are(in mm)

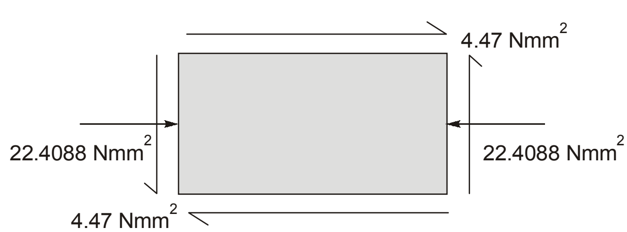
$$A\bar{y} = A_1y_1 + A_2y_2$$

$$= (75 \times 10 \times 95) + \{(90 - 50) \times 15 \{20 + 50\}$$

$$= 71250 + 42000 = 113250 \text{ m}^3$$

$$\tau = \frac{F(A\bar{y})}{Z \times I} = \frac{12.33 \times 10^3 \times 113250}{20840,000 \times 15} = 4.47 \text{ N/mm}^2$$

Principal stresses



$$= \frac{\sigma_b}{2} \pm \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$$

$$\sigma_{1,2} = \frac{-22.4088}{2} \pm \frac{1}{2}\sqrt{(22-4088)^2 + 4 \times 4.47^2}$$

$$= -11.2044 \pm 12.063$$

$$\sigma_1 = -23.3674 \text{ N/mm}^2$$

$$\sigma_2 = 0.8586 \text{ N/mm}^2$$

■■■■

5 Torsion of Shafts

LEVEL 1 Objective Questions

1.	(b)
2.	(d)
3.	(a)
4.	(d)
5.	(d)
6.	(a)
7.	(b)
8.	(b)
9.	(d)
10.	(b)
11.	(b)
12.	(d)
13.	(d)
14.	(c)
15	(b)
16.	(d)
17.	(b)

LEVEL 2 Objective Questions

18.	(b)
19.	(20.00)
20.	(400)
21.	(b)
22.	(b)
23.	(0.12)
24.	(b)
25.	(d)
26.	(34.17)

■■■■

LEVEL 3 Conventional Questions

Solution : 27

Torque T is applied at section B and is shared by shaft AB and BC, as shown

$$T_1 + T_2 = T \quad \text{...(i)}$$

$$\theta_{AB} = \theta_{BC} \quad (\text{since } \theta_{AC} = 0)$$

$$\frac{T_1 \times 0.4}{GJ_1} = \frac{T_2 \times 0.4}{GJ_2}$$

$J_1 = 16\, J_2$ as $d_1 = 40$ mm, $d_2 = 20$ mm$(d_1 = 2d_2)$

$$\frac{T_1}{J_1} = \frac{T_2}{J_2}$$

$$T_1 = 16T_2$$

So $$T_1 = 0.94117\, T$$

$$T_2 = 0.05883\, T$$

Assume that maximum shear stress occurs in AB,

$$T_1 = \frac{\pi}{16}\tau \times 40^3 = \frac{\pi}{16} \times 100 \times 40^3 = 1256637 \text{ Nmm} = 0.94117\, T$$

$$\text{Torque, } T = 1335186 \text{ Nmm} = 1.335 \text{ kNm}$$

Then, $$T_2 = 78540.35 \text{ Nmm} = \frac{\pi}{16} \times \tau' \times 20^3$$

$$\tau' = 50 \text{ N/mm}^2 \text{ in portion } BC. \quad \text{(Less than permissible hence, OK)}$$

Hence, $$T = 1.335 \text{ kNm}$$

Solution : 28

Given: d = Diameter of solid shaft, d_0= Diameter (outside) of hollow shaft, d_i = Inside (diameter) of hollow shaft, $d_i/d_o = 2/3$

As both shafts transmit given torque with same maximum shear stress, then,

$$(Z_P)_s = (Z_P)_h,$$

$$Z_P = \text{Polar section modulus}$$

$$\frac{\pi}{16}d^3 = \frac{\pi}{16}d_0^3 \times \left(1 - \left(\frac{d_i}{d_0}\right)^4\right);$$

$$d^3 = d_0^3 \times \left[1 - \left(\frac{2}{3}\right)^4\right]$$

$$d = 0.929\, d_0$$

$\therefore$ $$d_0 = 1.076\, d$$

$$d_i = \frac{2}{3}d_0 = \frac{2}{3} \times 1.076d = 0.7174d$$

$$A_s = \text{Cross section area of solid shaft} = \frac{\pi}{4}d^2$$

A_h = Cross section area of hollow shaft

$$= \frac{\pi}{4}\left(d_0^2 - d_i^2\right) = \frac{\pi}{4}d^2[(1.076)^2 - (0.7174)^2] = (0.6431)\times\frac{\pi}{4}d^2$$

Comparison of weights of equal lengths and same material, $\frac{A_h}{A_s} = 0.6431$

∴ Weight of hollow shaft is 0.6431 times the weight of solid shaft.

Solution : 29

Mean torque to be transmitted

$$T = \frac{P}{\omega} = \frac{600000}{\frac{2\pi\times 110}{60}} = 52087.07 \text{ N-m}$$

Max. torque transmitted

$$T_m = 1.2\times 52087.07$$
$$= 62504.5 \text{ N-m}$$

$$\because \quad \frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{l} \quad \text{...(i)}$$

Considering shear stress

$$\therefore \quad \frac{\tau}{R} = \frac{G\theta}{l}$$

$$\frac{63}{R} = \frac{G\theta}{l} = \frac{84000\times 1.4\times\frac{\pi}{180}}{3000} = 0.68417$$

$$R = \text{outer radius} = \frac{63}{0.68417} = 92.08 \text{ mm}$$

$$r = \text{Inner Radius} = \frac{3}{8}\times 92.08 = 34.53 \text{ mm}$$

Now consider maximum torque

$$\frac{T}{J} = \frac{G\theta}{l} = 0.6814$$

$$\frac{62504500}{J} = 0.6814$$

$$J = 91729527.44$$

$$\frac{\pi}{32}\cdot(d_2^2 - d_1^2) = 91729527.44$$

$$\because \quad d_1 = \frac{3}{8}d_2$$

$$\frac{\pi}{32}\times d_2^4\left[1-\left(\frac{3}{8}\right)^4\right] = 91729527.44$$

$$d_2 = 175.7 \text{ mm}$$

$$r_2 = \text{outer radius} = \frac{d_2}{2} = 87.85 \text{ mm}$$

$$r_1 = \frac{3}{8} r_2 = 32.94 \text{ mm}$$

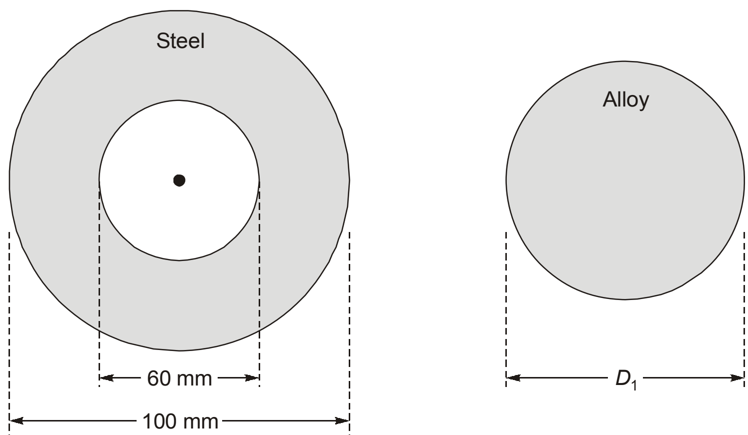
Consider maximum combination out of two so the answer is

$$r_2 = 92.08 \text{ mm}, \; r_1 = 34.53 \text{ mm}$$

or $$d_2 = 184.16 \text{ mm}, \; d_1 = 69.06 \text{ mm}$$

Solution : 30

Given, $d = 60$ mm, $D = 100$ mm, $G_s = 2\,G_A$



$$(Z_P)_{\text{Hollow shaft}} = \frac{J_{\text{Hollow}}}{D/2} = \frac{\pi}{16}\frac{\left(D^4 - d^4\right)}{D}$$

$$(Z_P)_{\text{Solid shaft}} = \frac{J_{\text{solid}}}{D_1/2} = \frac{\pi}{16} D_1^3$$

From the given condition, $$\frac{D^4 - d^4}{D} = D_1^3$$

$$\therefore \quad D_1^3 = \frac{100^4 - 60^4}{100}$$

or $$D_1 = 95.5 \text{ mm} \quad \textbf{Ans.}$$

Ratio of torsional rigidity $$= \frac{(GJ)_{\text{Steel}}}{(GJ)_{\text{Alloy}}} = 2\frac{J_{\text{Steel}}}{J_{\text{Alloy}}}$$

or $$\text{Ratio} = \frac{2\left(D^4 - d^4\right)}{D_1^4} = 2\left(\frac{100^4 - 60^4}{95.5^4}\right) = 2.093 \quad \textbf{Ans.}$$

Solution : 31

We know,

$$\text{Power} = \frac{2\pi NT}{60 \times 10^3} \text{ kW}$$

$$350 = \frac{2\pi \times 150 \times T}{60 \times 10^3}$$

or $$T = 22.282 \text{ kN-m}$$

For the hollow shaft $J_h = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi d_o^4}{32}[(1-(0.7)^4] = 0.0746\ d_o^4\ m^4$

For the solid shaft, $J_s = \frac{\pi}{32} d^4$

Now we know that $\tau = \frac{T}{J} \times \frac{d_o}{2}$

but T and τ are equal for solid and hollow shafts, hence

$$\frac{J_h}{d_o} = \frac{J_s}{d}$$

or $$\frac{0.746\, d_o^4}{d_o} = \frac{\pi d^4}{32 \times d}$$

or $$0.7599\, d_o^3 = d^3 \quad \text{... (i)}$$

Also, for solid shaft

$$\tau_s = \frac{T}{J_s} \times \frac{d}{2}$$

or $$90 \times 10^6 = \frac{22.282 \times 10^3 \times d \times 32}{\pi\, d^4 \times 2}$$

or $$d = 0.108\ \text{m}$$

now, from equation (i), we have

$$0.7599 \times d_o^3 = (0.108)^3$$

$$d_o = 0.1183\ \text{m}$$

Hence $$d_i = 0.7 \times 0.1183 = 0.0828\ \text{m}$$

Now, area of solid shaft

$$A_s = \frac{\pi}{4} \times (0.108)^2 = 9.16 \times 10^{-3}\ \text{m}^2$$

Area of hollow shaft $$A_h = \frac{\pi}{4}(0.1183^2 - 0.0828^2) = 5.607 \times 10^{-3}\ \text{m}^2$$

Hence, percentage savings in weight $$= \frac{A_s - A_h}{A_s} \times 100 = \frac{9.16 - 5.607}{9.16} \times 100 = 38.78\%$$

Solution : 32

Given

$$\frac{d}{D} = 0.7,\ P = 500\ \text{kW},$$

$$N = 300\ \text{r.p.m.},\ \tau_{allowable} = 65\ \text{N/mm}^2$$

$$\theta = 1^\circ = \frac{\pi}{180}\ \text{radian}$$

$$G = 8.2 \times 10^4\ \text{N/mm}^2$$

From, $$P = T \cdot \omega$$

Torque $$T = \frac{P}{\omega} = \frac{P \times 60}{2\pi N} = \frac{500 \times 10^3 \times 60}{2 \times \pi \times 300} = 15923.6\ \text{N-m}$$

$$\frac{T}{J} = \frac{G\theta}{l} = \frac{\tau}{R}$$

where polar moment of inertia, $J = \frac{\pi}{32}(D^4 - d^4)$

$$R = \frac{D}{2}$$

For shear stress, $\tau = \frac{T}{J}R$

$$65 = \frac{15923.6 \times 10^3}{\frac{\pi}{32}(D^4 - d^4)} \cdot \frac{\Delta}{2}$$

$$D = 117.98 \text{ mm}$$

For angle of twist, $\theta = \frac{TL}{GJ}$

$$\frac{\pi}{180} = \frac{15923.6 \times 10^3 \times 2.4 \times 10^3}{8.2 \times 10^4 \times \frac{\pi}{32}(D^4 - d^4)}$$

$$D = 137.57 \text{ mm}$$

Hence minimum external diameter of shaft = 137.57 mm (taking bigger one value).

Solution : 33

Let
Diameter of solid beam = D
Diameters of hollow beam = D_0 and D_i
L = Length of each beam
W = Weight of each beam
ρ = Density of the material of each shaft
Weight of solid beam = Weight of hollow beam

$$\Rightarrow \quad \rho \times g \times \frac{\pi}{4}D^2 \times L = \rho \times g \times \frac{\pi}{4}\left(D_0^2 - D_i^2\right) \times L$$

$$\Rightarrow \quad D^2 = D_0^2 - D_i^2 \quad \text{...(i)}$$

$$Z_{\text{solid}} = \frac{\pi}{16}D^3$$

$$Z_{\text{hollow}} = \frac{\pi}{16 D_0}\left[D_0^4 - D_i^4\right]$$

$$\therefore \quad \frac{Z_s}{Z_h} = \frac{D_0 D^3}{D_0^4 - D_i^4} = \frac{D \times D_0 \times D^2}{\left(D_0^2 + D_i^2\right)\left(D_0^2 - D_i^2\right)}$$

or

$$\frac{Z_s}{Z_h} = \frac{D \times D_0 \times \left(D_0^2 - D_i^2\right)}{\left(D_0^2 + D_i^2\right)\left(D_0^2 - D_i^2\right)} \quad \text{[from equation (i)]}$$

or

$$\frac{Z_s}{Z_h} = \frac{D D_0}{D_0^2 + D_i^2} = \frac{D \times D_0}{\left(2D_0^2 - D^2\right)}$$

Solution : 34

Torque on the segment AB = 1020 + 1020 + 1360 = 3400 Nm

Torque on the segment BC = 1020 + 1020 = 2040 Nm

Torque on the segment CD = 1020 Nm

Maximum shear stress in the segment,

$$AB = \frac{16 \times 3400 \times 10^3}{\pi \times 75^3} = 41.05\,\text{N/mm}^2$$

Maximum shear stress in the segment,

$$BC = \frac{16 \times 2040 \times 10^3}{\pi \times 62.5^3} = 42.56\,\text{N/mm}^2$$

Maximum shear stress in the segment,

$$CD = \frac{16 \times 1020 \times 10^3}{\pi \times 50^3} = 41.56\,\text{N/mm}^2$$

∴ Maximum shear stress in the shaft = 42.56 N/mm^2

$$\text{Twist of the segment } AB = \frac{600 \times 3400 \times 10^3 \times 32}{8 \times 10^4 \times \pi \times 75^4} = 0.008210\,\text{radian}$$

$$\text{Twist of the segment } BC = \frac{600 \times 2040 \times 10^3 \times 32}{8 \times 10^4 \times \pi \times 62.5^4} = 0.010213\,\text{radian}$$

$$\text{Twist of the segment } CD = \frac{600 \times 1020 \times 10^3 \times 32}{8 \times 10^4 \times \pi \times 50^4} = 0.012468\,\text{radian}$$

∴ Twist of the end D with respect to the end A = 0.008210 + 0.010213 + 0.012468 = 0.03089 radian

$$= \frac{0.03089 \times 180}{\pi}\text{degrees} = 1.77 \text{ degrees}$$

Solution : 35

$$\phi = \int_0^L d\phi = \int_0^L \frac{\{T_A + Tx\}dx}{GJ}$$

Given $t(x) = Ax + B$ (varies linearly)

at $x = 0$ $t(x) = 0$

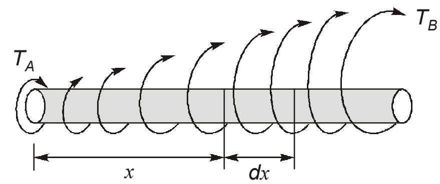
at $x = L$ $t(L) = t_0$

$0 = A \cdot o + B$

∴ $B = O$

$t_0 = A \cdot L + O$

$$A = \frac{t_o}{L}$$



$$t(x) = \frac{t_o x}{L}$$

Let T_A and T_3 be the reaction torques.

Considering "dx" element length at a distance x from end "A".

$$\therefore \quad T_x = \frac{1}{2} \times t(x) \times x$$

$$T_x = \frac{1}{2} \times x \times \frac{t_o x}{L} = \frac{t_o x^2}{2L}$$

$$\phi = \int_0^L d\phi = \int_0^L \left(T_A + \frac{t_o x^2}{2L}\right)\frac{dx}{GJ}$$

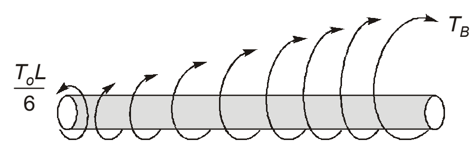
$$= \frac{1}{GJ}\left[T_A(L) + \frac{t_o}{2L}\left[\frac{L^3}{3}\right]\right] = \frac{1}{GJ}\left[T_A L + \frac{t_o L^2}{6}\right]$$

As shaft is fixed $\quad \phi = 0$

$$\therefore \quad 0 = \frac{1}{GJ}\left[T_A \cdot L + \frac{t_o L^2}{6}\right]$$

$$0 = T_A \cdot L + \frac{t_o L^2}{6}$$

$$T_A = -\frac{t_o L}{6}$$

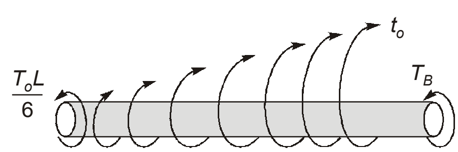


$$T_A = \frac{1}{2} \times L \times t_o + T_B$$

$$\frac{t_o L}{6} = \frac{t_o L}{2} - T_B$$

$$T_B = \frac{t_o L}{6} - \frac{t_o L}{2} = \frac{t_o L - 3t_o L}{6}$$

$$T_B = -\frac{t_o L}{3}$$



Expression for $\phi(x)$

$$\phi(x) = \int_o^x \frac{(T_A + T_x) \cdot dx}{GJ}$$

$$= \frac{1}{GJ}\left[\int_o^x T_A \cdot dx \int_o^x T_x \cdot dx\right] = \frac{1}{GJ}\left[\int_o^x -\frac{t_o L}{6} dx + \int_o^x \frac{t_o x^2}{2L} \cdot dx\right]$$

$$= \frac{1}{GJ}\left[-\frac{t_o L}{6} \cdot x + \frac{t_o}{2L} \times \frac{x^3}{3}\right]$$

To calculate : ϕ_{max}

$$\frac{d\phi}{dx} = \frac{1}{GJ}\left[\frac{d}{dx}\left[-\frac{t_o L x}{6}\right] + \frac{d}{dx}\left[\frac{t_o x^3}{6L}\right]\right]$$

$$o = \frac{1}{GJ}\left[-\frac{t_o L}{6} + \frac{t_o}{6L} \times 3x^2\right]$$

$$\frac{t_o L}{6} = \frac{t_o x^2}{2L}$$

$$\frac{L^2}{3} = x^2$$

$$x = \frac{L^2}{\sqrt{3}}$$

$$\phi \text{ is } \phi_{max} \text{ at } x = \frac{L}{\sqrt{3}} \qquad \text{[Location of } \phi_{max} \text{ along the bar]}$$

$$\phi_{max} = \frac{1}{GJ}\left[-\frac{t_o L}{6} \times x + \frac{t_o}{2L} \times \frac{x^3}{3}\right] = \frac{1}{GJ}\left[-\frac{t_o L}{6} \times \frac{L}{\sqrt{3}} + \frac{t_o}{2L} \times \frac{1}{3} \times \left(\frac{L}{\sqrt{3}}\right)^3\right]$$

$$= \frac{1}{GJ}\left[-\frac{t_o L^2}{6\sqrt{3}} + \frac{t_o}{6L} \times \frac{L^3}{3\sqrt{3}}\right] = \frac{1}{GJ}\left[-\frac{t_o L^2}{6\sqrt{3}}\right]\left[-1 + \frac{1}{3}\right] = \frac{1}{GJ}\left[\frac{t_o L^2}{6\sqrt{3}}\right]\left[\frac{-3+1}{3}\right]$$

$$= \frac{1}{GJ}\left[-\frac{t_o L^2}{9\sqrt{3}}\right] = \frac{t_o L^2}{9\sqrt{3}GJ} = \frac{-\sqrt{3} t_o L^2}{27GJ}$$

Solution : 36

The given shafts are connected in series. The ratio of the torque to the angle of twist (T/θ) is defined as spring constant.

$$\theta_{AD} = \theta_1 + \theta_2 + \theta_3$$

$$\theta_{AD} = \frac{T}{k_1} + \frac{T}{k_2} + \frac{T}{k_3}$$

$$\frac{6 \times \pi}{180} = T\left[\frac{1}{K_1} + \frac{1}{K} + \frac{1}{K_3}\right]$$

$$k = \frac{T}{\theta} = \frac{GJ}{L}$$

$$k_1 = \frac{59 \times 10^3 \times \pi \times (45)^4}{64 \times 800} = 3.02 \times 10^7 \text{ Nmm/rad}$$

$$k_2 = \frac{71 \times 10^3 \times \pi \times (45)^4}{64 \times 800} = 3.57 \times 10^7 \text{ Nmm/rad}$$

$$k_3 = \frac{82 \times 10^3 \times \pi \times (35)^4}{64 \times 400} = 3.02 \times 107 \text{ Nmm/rad}$$

Torque = 1.1 kNm

Solution : 37

Angle of twist will be same for this case.

$$\frac{T_s}{T_b} = \frac{G_s J_s}{G_b J_b} = \frac{80}{44} \times \frac{0.04^4}{0.06^4 - 0.04^4} = 0.4475 \quad \text{...(i)}$$

$$T_s = \tau_s \times \frac{\pi}{16} \times 0.04^3 \quad \text{...(ii)}$$

$$T_b = \tau_b Z_b = \tau_b \times \frac{\pi}{16} \times \frac{0.06^4 - 0.04^4}{0.06} \quad \text{...(iii)}$$

$$\frac{T_s}{T_b} = \frac{\tau_s}{\tau_b} \times \frac{24}{65} = 0.4475 \qquad \text{from equation ....(i)}$$

$$\frac{\tau_s}{\tau_b} = 1.212$$

It is therefore evident that the bronze will reach its limiting stress of 38 mN/m^2 before the steel reaches its limiting stress of 60 MPa. The maximum stress in the bronze is therefore 38 MPa and the maximum stress in the steel is 38 × 1.212 = 46.1 MPa

Therefore,
$$T_s = 46.1 \times 10^6 \times \frac{\pi}{16} \times (0.04)^3$$

$$T_s = 580 \text{ Nm} \qquad \text{from equation ....(ii)}$$

and
$$T_b = 38 \times 10^6 \times \frac{\pi}{16} \times \frac{0.06^4 - 0.04^4}{0.06}$$

$$= 1293 \text{ Nm} \qquad \text{from equation ....(iii)}$$

$$T = 1293 + 580 = 1873 \text{ N.m}$$

$$\text{Power} = \text{Torque} \times \omega$$

$$= 1873 \times \frac{2\pi \times 500}{60} = 98 \text{ kW}$$

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6 Principle Stress & Strain, Mohr's Circle

LEVEL 1 Objective Questions

1.	(d)
2.	(c)
3.	(a)
4.	(c)
5.	(d)
6.	(c)
7.	(a)
8.	(b)
9.	(c)
10.	(b)
11.	(d)
12.	(a)
13.	(c)
14.	(c)
15.	(c)
16.	(c)
17.	(d)
18.	(c)

LEVEL 2 Objective Questions

19.	(a)
20.	(70)
21.	(d)
22.	(75)
23.	(b)
24.	(a)
25.	(a)
26.	(c)
27.	(b)
28.	(a)
29.	(b)
30.	(6.98)
31.	(0)
32.	(86.60)

■■■■

LEVEL 3 Conventional Questions

Solution : 33

For $\theta = 0°$: $\varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

For $\theta = 40°$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_B = 1496 \times 10^{-6}$

and $\varepsilon_x = 1100 \times 10^{-6}$;

Then simplify and rearrange: $0.4132\,\varepsilon_y + 0.49240\phi_{xy} = 850.52 \times 10^{-6}$ .....(i)

For $\theta = 140°$

$$\varepsilon_{x_2} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_2} = \varepsilon_C = -39.44 \times 10^{-6}$

and $\varepsilon_x = 1100 \times 10^{-6}$;

Then simplify and rearrange:

$0.4132\,\varepsilon_y - 0.49240\phi_{xy} = -684.92 \times 10^{-6}$ .....(ii)

Solve equation (i) and (ii)

$\varepsilon_y = 200.4 \times 10^{-6}$ and $\phi_{xy} = 1559.2 \times 10^{-6}$

Hooke's Law, $\sigma_x = \dfrac{E}{1-\mu^2}\left(\varepsilon_x + \mu\varepsilon_y\right) = 91.6$ MPa

Solution : 34

Given: σ_1, σ_2 are the principal stresses.

(i) According to plane stress transformation equations

$$\sigma_x' = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{... (i)}$$

$$\sigma_y' = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{... (ii)}$$

Adding equation (i) and (ii),

$$\sigma_x' + \sigma_y' = \sigma_x + \sigma_y$$

Similarly,
$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (i)}$$

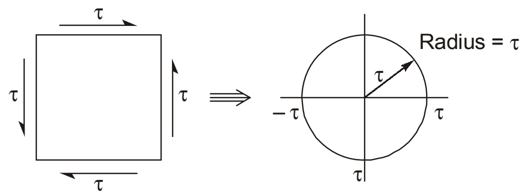
$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{... (ii)}$$

Adding, (i) and (ii),

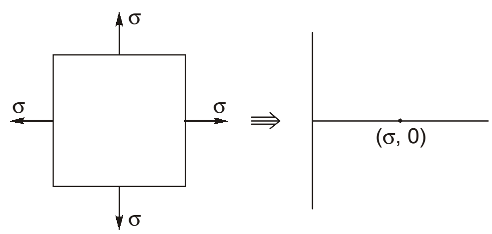
$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$\therefore \quad \sigma_x + \sigma_y = \sigma_x' + \sigma_y' = \sigma_x + \sigma_y$$

(ii) Mohr's circle for pure shear stress state.



Mohr's circle for hydrostatic state.



∴ Mohr's circle in this case will be reduced to a point.

Solution : 35

Given data: $D = 100$ mm; $R = 50$; $t = 5$ mm; $T = 1000$ N-m

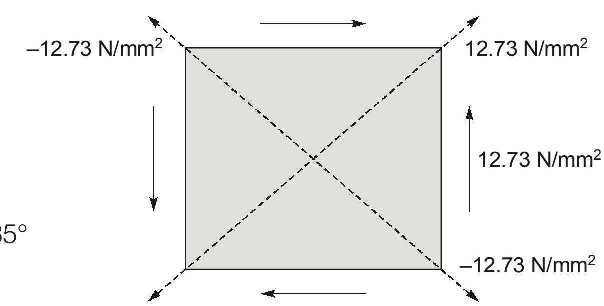
Moment of area

$$J = 2\pi R^3 t = 3.925 \times 10^6 \text{ mm}$$

Shear stress,
$$\tau = \frac{T \times 50}{3.925 \times 10^6} = \frac{1000 \times 10^3 \times 50}{3.925 \times 10^6}$$

$$\tau_{XY} = 12.73 \text{ N/mm}^2$$

Principal stress



$$\sigma = \tau_{XY} \sin 2\theta$$

$$\sigma_{1,2} = 12.73 \sin 2\theta$$

$$\sigma_1 = 12.73 \text{ N/mm}^2, \theta = 45°$$

$$\sigma_2 = -12.73 \text{ N/mm}^2, \theta = 135°$$

θ's are with direction of torsion moment.

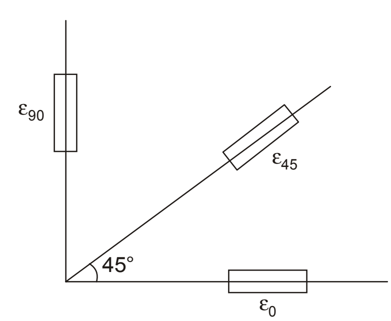
Solution : 36

Rectangular strain gauge rosette

$\varepsilon_{0°} = -220 \times 10^{-6}$; $\varepsilon_{45°} = 120 \times 10^{-6}$

$\varepsilon_{90°} = 220 \times 10^{-6}$; $E = 2 \times 10^5$ N/mm²; $\mu = 0.3$

Now, principal strains are given by



$$\epsilon_{45} = \frac{\epsilon_0 + \epsilon_{90}}{2} + \frac{\epsilon_0 - \epsilon_{90}}{2}\cos 2\theta + \frac{\phi}{2}\sin 2\theta$$

$$120 \times 10^{-6} = \phi/2$$

∴ Principal strain

$$\epsilon_1, \epsilon_2 = \frac{\epsilon_0 + \epsilon_{90}}{2} \pm \sqrt{\left(\frac{\epsilon_0 - \epsilon_{90}}{2}\right)^2 + \left(\frac{\phi}{2}\right)^2}$$

$$= \pm\sqrt{\left(220 \times 10^{-6}\right)^2 + \left(120 \times 10^{-6}\right)^2} = \pm 250.6 \times 10^{-6}$$

Principal stress

$$\sigma_1 = \frac{E}{1-\mu^2}(\epsilon_1 + \mu\,\epsilon_2) = \frac{2\times10^5}{1-0.09}\left(250.6\times10^{-6} - 75.18\times10^{-6}\right)$$

$$= \frac{2\times10^5}{0.91}\times175.42\times10^{-6} = 38.5 \text{ N/mm}^2$$

$$\sigma_2 = \frac{E}{1-\mu^2}(\epsilon_2 + \mu\,\epsilon_1) = -38.55 \text{ N/mm}^2$$

Direction of principal stress

$$\tan 2\theta_p = \frac{\phi}{\epsilon_0 - \epsilon_{90}} = \frac{240}{-440}$$

$$2\theta_p = -28.61, 151.4$$

$$\theta_p = -14.3°, 75.7°$$

Solution : 37

The maximum principal stress is given by

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$150 = \frac{1}{2}(100+60) + \frac{1}{2}\sqrt{(100-60)^2 + 4\tau_{xy}^2}$$

$$150 = 80 + \frac{1}{2}\sqrt{1600 + 4\tau_{xy}^2}$$

$$140 = \sqrt{1600 + 4\tau_{xy}^2}$$

$$19600 = 1600 + 4\tau_{xy}^2$$

$$\therefore \quad \tau_{xy} = 67.082 \text{ MPa}$$

The principal planes are given by

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times67.082}{100-60} = 3.3541$$

$$\theta_1^\circ = 36.6992, \quad \theta_2 = 126.6992°$$

The minimum principal stress is given by

$$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$$

$$100 + 60 = 150 + \sigma_2$$

$$\sigma_2 = 10 \text{ MPa}$$

OR

$$\sigma_2 = \frac{1}{2}(100+60) - \frac{1}{2}\sqrt{(100-60)^2 + 4(67.082)^2}$$

$$\sigma_2 = 80 - 70 = 10 \text{ MPa}$$

Maximum shear stress, $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}(150-10) = 70 \text{ MPa}$

Solution : 38

If σ_{11}, σ_{22}, σ_{33} be the three principle stresses, then the principle strains t_{11}, t_{22}, t_{33} obtained by Generalized Hooke's law as:

$$\epsilon_{11} = \frac{\sigma_{11}}{E} - \frac{\sigma_{22}+\sigma_{33}}{mE} \quad ...(i)$$

$$\epsilon_{22} = \frac{\sigma_{22}}{E} - \frac{\sigma_{33}+\sigma_{11}}{mE} \quad ...(ii)$$

$$\epsilon_{33} = \frac{\sigma_{33}}{E} - \frac{\sigma_{11}+\sigma_{22}}{mE} \quad ...(iii)$$

where, E = Modulus of elasticity

and $\frac{1}{m}$ = Poisson's Ratio

From equation (i), we have $$E\epsilon_{11} = \sigma_{11} - \frac{\sigma_{22}}{m} - \frac{\sigma_{33}}{m} \quad ...(iv)$$

From equation (ii), we have $$E\epsilon_{22} = \sigma_{22} - \frac{\sigma_{33}}{m} - \frac{\sigma_{11}}{m} \quad ...(v)$$

From equation, (iii) we have $$E\epsilon_{33} = \sigma_{33} - \frac{\sigma_{11}}{m} - \frac{\sigma_{22}}{m} \quad ...(vi)$$

Subtracting equation (v) from equation (iv), we get

$$E(\epsilon_{11}-\epsilon_{22}) = (\sigma_{11}-\sigma_{22})\left(1+\frac{a}{m^2}\right) \quad ...(vii)$$

From equations (i) and (iii), (vi), we get

$$E(\epsilon_{11}+\epsilon_{33}) = \sigma_{11}\left(1-\frac{1}{m^2}\right) - \sigma_{22}\left(1+\frac{1}{m^2}\right)\frac{1}{m} \quad ...(viii)$$

Multiplying equation (vii) by $\left(\frac{1}{m}\right)$ and subtracting from equation (viii), we get

$$E\left[\left(1-\frac{1}{m}\right)\epsilon_{11} + \frac{1}{m}\right](\epsilon_{11}+\epsilon_{33}) = \sigma_{11}\left(1+\frac{1}{m}-\frac{2}{m^2}\right) = \sigma_{11}\left(1+\frac{1}{m}\right)\left(1-\frac{2}{m}\right)$$

$$\sigma_{11} = \frac{E\left(1-\frac{1}{m}\right)\epsilon_{11} + \frac{1}{m}(\epsilon_{22}+\epsilon_{33})}{\left(1+\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Similarly, $$\sigma_{22} = \frac{E\left(1-\frac{1}{m}\right)\epsilon_{22} + \frac{1}{m}(\epsilon_{33}+\epsilon_{22})}{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Similarly, $$\sigma_{33} = \frac{E\left(1-\frac{1}{m}\right)\epsilon_{33} + \frac{1}{m}(\epsilon_{11}+\epsilon_{22})}{\left(1-\frac{1}{m}\right)\left(1-\frac{2}{m}\right)}$$

Solution : 39

Given that, $\sigma_x = -8$ kN/cm^2, $\sigma_y = 14$ kN/cm^2, $\tau_{xy} = -5$ kN/cm^2
the principal stresses can be found as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-8+14}{2} \pm \sqrt{\left(\frac{-8-14}{2}\right)^2 + (-5)^2} = 3 \pm \sqrt{121+25}$$

$$= 3 \pm 12.08$$

$$= 15.08, -9.08 \text{ kN/cm}^2$$

Strain in the direction of σ_1, is given by

$$\in_1 = \frac{1}{E}(\sigma_1 - \nu\,\sigma_2) = \frac{1}{2.1\times 10^6}[15.08 + (0.28 \times 9.08)] \times 10^3$$

$$= 8.39 \times 10^{-3}$$

Strain in the direction of σ_2

$$\in_2 = \frac{1}{E}(\sigma_2 - \nu\,\sigma_1) = \frac{1}{2.1\times 10^6}[-9.08 - (0.28 \times 15.08)] \times 10^3$$

$$= -6.33 \times 10^{-3}$$

Change in length of the diameter along σ_1

$$= \in_1 \times 100 = 8.39 \times 10^{-3} \times 100 = 0.839 \text{ mm}$$

Change in length of the diameter along σ_2

$$= \in_2 \times 100 = -6.33 \times 10^{-3} \times 100 = -0.633 \text{ mm}$$

Hence major axis of the ellipse

$$= 100 + 0.839 = 100.839 \text{ mm}$$

Minor axis of the ellipse

$$= 100 - 0.633 = 99.367 \text{ mm}$$

Principal planes's direction is given by

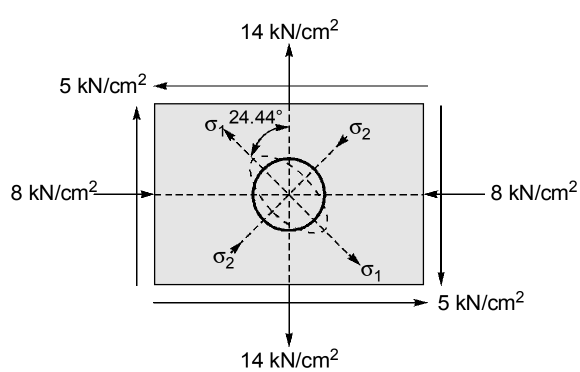
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times(-5)}{-8-14} = 0.455$$

$$2\,\theta = 24.44$$

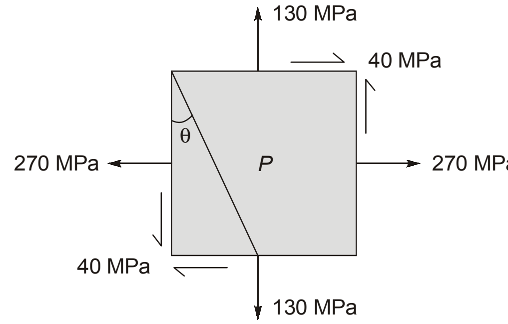
hence $\theta_1 = 12.22°$

$$\theta_2 = 12.22 + 90 = 102.22°$$

Principal planes are shown in the figure.



Solution : 40



(i)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{270+130}{2} \pm \frac{1}{2}\sqrt{(270-130)^2 + 4\times 40^2}$$

$$= 200 \pm \frac{1}{2}\sqrt{140^2 + 4\times 1600}$$

$$= 200 \pm 80.62$$

$$\sigma_{max}(\sigma_1) = 280.62 \text{ MPa}$$

$$\sigma_{min}(\sigma_2) = 119.38 \text{ MPa}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 40}{140} \qquad \therefore 2\theta = 29.74°$$

$$\therefore \quad \theta_1 = 14.87°$$

and $\theta_2 = 104.87°$

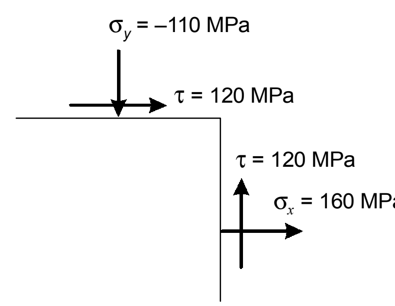
(ii) Maximum shearing stress $= \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(280.62 - 119.38)$

$= \pm 80.62$ MPa

Direction of shearing stress $= \theta_1 + 45°$ and $\theta_2 + 45°$

i.e., 59.87° and 149.87°

Solution : 41



Given:

$$\sigma_x = 160 \text{ MPa}$$

$$\sigma_y = -110 \text{ MPa}$$

$$\tau = 120 \text{ MPa}$$

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}\left[(160-110) \pm \sqrt{(270)^2 + 4\times 120^2}\right]$$

$$= \frac{1}{2}[50 \pm 361.25]$$

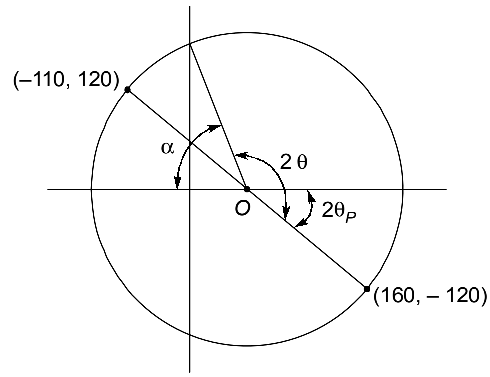
$\sigma_1 = 205.625$ MPa, $\sigma_2 = -155.625$ MPa Ans. (i)

$$\tan 2\theta_P = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\times 120}{270} = 0.889$$

$$2\theta_P = 41.63° \text{ and } 221.63°,$$

$\theta_P = 20.815°$ and $110.815°$ Ans. (i)

$$\text{Centre of Mohr's circle} = \frac{\sigma_x + \sigma_y}{2} = \frac{160-110}{2} = 25$$



$$\text{Radius of Mohr's circle} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{160 + 110}{2}\right)^2 + 120^2} = 180.6239$$

$$\cos\alpha = \frac{25}{180.6239}$$

$$\alpha = 82.04$$

$$2\theta = 2\theta_P + 180 - \alpha = 139.58$$

∴ Position of the plane on which normal stress is zero.

$$= \theta = 69.8°$$

■■■■

7 Strain Energy and Thermal Stress

LEVEL 1 Objective Questions

1. (b)
2. (a)
3. (a)
4. (d)
5. (a)
6. (a)
7. (d)
8. (a)
9. (80)
10. (d)

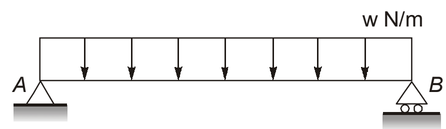
LEVEL 2 Objective Questions

11. (c)
12. (1)
13. (187.5)
14. (d)
15. (c)
16. (a)
17. (b)
18. (d)
19. (b)

■■■■

Solution : 20

Given:

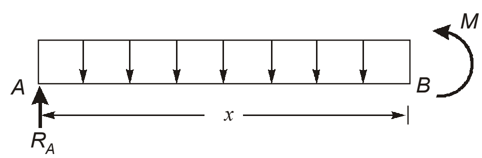


Simple supported beam loaded by uniformly distributed load.

Let U_e be the elastic strain energy due to bending of beam.

$$U_e = \int_0^L \frac{M^2 dx}{2EI}$$

$$R_A = \frac{wL}{2}$$



$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$U_e = \frac{1}{2EI}\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx$$

$$U_e = \frac{1}{2EI} \times \frac{w^2}{4}\int_0^L (L^2x^2 + x^4 - 2Lx^3)dx$$

$$U_e = \frac{w^2}{8EI}\left[\frac{L^5}{3} + \frac{L^5}{5} - \frac{2L^5}{4}\right] = \frac{w^2}{8EI} \times \left[\frac{20+12-30}{60}\right]L^5$$

$$U_e = \left(\frac{w^2L^5}{240\ EI}\right) = \frac{w^2L^5}{240 \times E \times \frac{1}{12}bh^3} = \frac{w^2L^5}{20\,E\,bh^3} \quad \text{...(i)}$$

Now,

$$\sigma_{max} = \frac{My}{I} = \frac{6M}{bd^2} = \frac{6 \times wL^2}{8 \times bh^2} = \frac{3wL^2}{4bh^2}$$

Volume of beam $= A \times L = (b \times h)L$

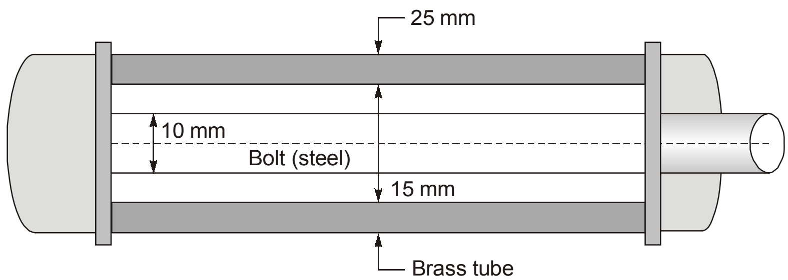
$$= \left(\frac{\sigma_{max}^2}{2E}\right) \times \left(\frac{8}{45} \times \text{volume of beam}\right)$$

$$= \frac{9w^2L^4}{32b^2h^4E} \times \frac{8}{45} \times (b \times h) \times L = \frac{w^2L^5}{240\ Ebh^3} = U_e$$

This expression is same as obtained by equation (i).

Solution : 21

The bolt is tightened so as to reduce the length of tube by 1.5 mm.



Here length of the tube is not given. Assume length = 1000 mm

Case-1: Before heating

Let, σ_{s1} = stress induced in steel bolt

σ_{b1} = stress induced in brass tube

Therefore,

$$\sigma_{s1} \times A_s = \sigma_{b1} \times A_b$$

where, A_s = Area of steel bolt = $\frac{\pi}{4} \times (10)^2 = 78.5 \text{ mm}^2$

A_b = Area of brass tube = $\frac{\pi}{4} \times (25^2 - 15^2) = 314 \text{ mm}^2$

$$\therefore \quad \sigma_{s1} \times 78.5 = \sigma_{b1} \times 314$$

$$\sigma_{s1} = 4\sigma_{b1}$$

Since change in length of tube = 0.15 mm (given)

Assume, length of tube = 1000 mm

$$\therefore \quad \text{Strain in Brass tube} = \left(\frac{\Delta\ell}{\ell}\right) = \left(\frac{0.15}{1000}\right)$$

Stress induced in tube

$$\sigma_{b1} = (E_{tube}) \times (\delta)_{tube}$$

where E_b = Young's Modulus of elasticity of brass tube = 1×10^5 M/mm^2

$$\therefore \quad \sigma_{b1} = 1 \times 10^5 \times \left(\frac{0.15}{1000}\right) = 15 \text{ N/mm}^2$$

$$\sigma_{s1} = 4\,\sigma_{b1} = 60 \text{ N/mm}^2$$

Case-2: After heating

Let, σ_{s2} = stress induced in Steel Bolt

σ_{b2} = stress induced in Brass Tube

$$\frac{\sigma_{b_2}}{E_b} + \frac{\sigma_{s_2}}{E_s} = (\alpha_b - \alpha_s)t$$

$$\sigma_{b2} A_b = \sigma_{s2} A_s$$

α_b = coefficient of thermal expansions = 1.9×10^{-5}/°C

α_s = coefficient of thermal expansions = 1.2×10^{-5}/°C

$$\sigma_{b2} \times 314 = \sigma_{s2} \times 78.5$$

$$\sigma_{s2} = 4\,\sigma_{b2} \quad \text{...(i)}$$

$$\frac{\sigma_{b_2}}{1\times10^5}+\frac{\sigma_{s_2}}{2\times10^5} = (1.9-1.2)\times10^{-5}\times40$$

$$2\,\sigma_{b2}+\sigma_{s2} = 2\times10^5\times0.7\times10^{-5}\times40$$

$$2\,\sigma_{b2}+\sigma_{s2} = 56 \qquad \text{...(ii)}$$

from equ. (i) and (ii), we find the value of σ_{b2} and $\sigma_{s2}\rightarrow$

Put, $\sigma_{s2} = 4\,\sigma_{b2}$ in equation (ii), we get

$$2\,\sigma_{b2}+4\,\sigma_{s2} = 56$$

$$6\sigma_{b2} = 56$$

$$\sigma_{b2} = 9.33\text{ N/mm}^2,\ \sigma_{s2} = 37.33\text{ N/mm}^2$$

∴ Total stress induced in Brass Tube:

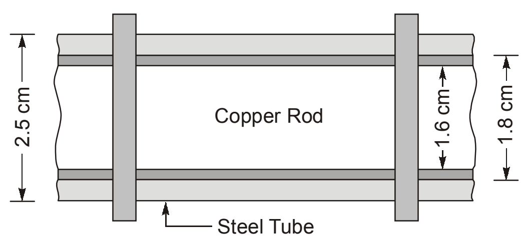
$$\sigma_b = 9.33+15 = 24.33\text{ N/mm}^2 \quad \text{(compressive)}$$

and total stress induced in steel bolt

$$\sigma_s = 37.33+60 = 97.33\text{ N/mm}^2 = 97.33\text{ N/mm}^2$$

$$\sigma_b = 24.33\text{ N/mm}^2 \quad \text{(tensile)}$$

Solution : 22



$$A_c = 0.785\times(1.6\times10^{-2})^2 = 2\times10^{-4}\text{ m}^2$$

$$A_s = 0.785\times(2.5^2-1.8^2)\times10^{-4} = 2.3628\times10^{-4}\text{ m}^2$$

Let L is the length of the copper bar and steel tube.

Due to increase in temp copper will be in compression and steel will be in tension.

Let σ_c = Compressive stress in copper rod

σ_s = Tensile stress in steel tube

Now $\Delta_{st} = \Delta_c$

$$L\alpha_sT+\frac{\sigma_s}{E_s}L = L\alpha_cT-\frac{\sigma_c}{E_c}L$$

$$\therefore \quad (\alpha_c-\alpha_s)\,T = \frac{\sigma_s}{E_s}+\frac{\sigma_c}{E_c} \qquad \text{...(i)}$$

and compressive force in copper = Tensile force in steel

$$\sigma_cA_c = A_s\sigma_c$$

$$2\sigma_c = 2.363\,\sigma_s$$

$$\therefore \quad \sigma_c = 1.18\,\sigma_s \qquad \text{...(ii)}$$

From eq. (i), $$(20-12)\times10^{-6}\times190 = \frac{\sigma_s}{210\times10^3}+\frac{1.18\sigma_s}{100\times10^3}$$

$$8\times10^{-6}\times190\times10^3 = \sigma_s\left[\frac{1}{210}+\frac{1.18}{100}\right]$$

$$12520\times10^{-3} = 0.0165\,\sigma_s$$

$$\therefore \quad \sigma_s = 91.77\text{ MPa} \quad \text{(tensile)}$$

$$\sigma_c = 108.3\text{ MPa} \quad \text{(compressive)}$$

Solution : 23

Free elongation of copper section $= \alpha_c t l_c = 16 \times 10^{-6} \times 60 \times 25 = 24 \times 10^{-3}$ cm

Free extension of aluminium section $= \alpha_a t l_a = 20 \times 10^{-6} \times 60 \times 50 = 60 \times 10^{-3}$ cm

Free extension of steel section $= \alpha_s t l_s = 12 \times 10^{-6} \times 60 \times 25 = 18 \times 10^{-3}$ cm

Total free extension of the composite bar $= (24 + 60 + 18)10^{-3} = 0.102$ cm $= 1.02$ mm

Since this extension is prevented by the rigid supports, therefore, compressive stresses are induced in the bar.

Let P be the compressive force in the bar in N.

∴ Stresses are:

$$\sigma_c = \frac{P}{A_c} = \frac{4P}{\pi \times 2500}\text{N/mm}^2$$

$$\sigma_a = \frac{P}{A_a} = \frac{4P}{\pi \times 10^4}\text{N/mm}^2$$

$$\sigma_s = \frac{P}{A_s} = \frac{4P}{\pi \times 2500}\text{N/mm}^2$$

Strains are:

$$\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{4P}{\pi \times 2500 \times 100 \times 10^3} = \frac{4P}{\pi \times 25 \times 10^7}$$

$$\varepsilon_a = \frac{\sigma_a}{E_a} = \frac{4P}{\pi \times 10^4 \times 90 \times 10^3} = \frac{4P}{\pi \times 90 \times 10^7}$$

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{4P}{\pi \times 2500 \times 200 \times 10^3} = \frac{4P}{\pi \times 50 \times 10^7}$$

Extensions are:

$$\Delta l_c = \varepsilon_c \times l_c = \frac{4P \times 250}{\pi \times 25 \times 10^7} = \frac{4P}{\pi \times 10^6}\text{mm}$$

$$\Delta l_a = \frac{4P \times 500}{\pi \times 90 \times 10^7} = \frac{20P}{\pi \times 9 \times 10^6}\text{mm}$$

$$\Delta l_s = \frac{4P \times 250}{\pi \times 50 \times 10^7} = \frac{2P}{\pi \times 10^6}\text{mm}$$

(i) Now $\Delta l_c + \Delta l_a + \Delta l_s = 1.02$

$$\therefore \quad \frac{P}{\pi \times 10^6}\left[4 + \frac{20}{9} + 2\right] = 1.02$$

$$\frac{P}{\pi \times 10^6}[4 + 2.222 + 2] = 1.02$$

$$\frac{P \times 8.222}{\pi \times 10^6} = 1.02$$

$$P = \frac{\pi \times 10^6 \times 1.02}{8.222} = 389.727\,\text{kN}$$

$$\sigma_c = \frac{4 \times 389727}{\pi \times 2500} = 198.486\,\text{MPa}$$

$$\sigma_a = \frac{4 \times 389727}{\pi \times 10^4} = 49.62\,\text{MPa}$$

$$\sigma_s = \frac{4 \times 389727}{\pi \times 2500} = 198.486\,\text{MPa}$$

(ii) When the supports yield by 0.05 cm, then $\Delta l_c + \Delta l_a + \Delta l_s = 1.02 - 0.50 = 0.52$ mm

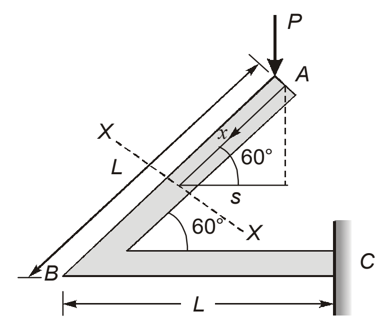
$$\therefore \quad P = \frac{\pi \times 10^6 \times 0.52}{8.822} = 185176\,\text{N}$$

$$\sigma_s = \sigma_c = \frac{4 \times 185176}{\pi \times 2500} = 94.309\,\text{MPa}$$

$$\sigma_a = \frac{4 \times 185176}{\pi \times 10^4} = 23.577\text{ MPa}$$

Solution : 24

Take lexural rigidity of the rod be EI.



$$\cos 60° = \frac{s}{x}$$

$$s = x\cos 60° = 0.5x = \frac{x}{2}$$

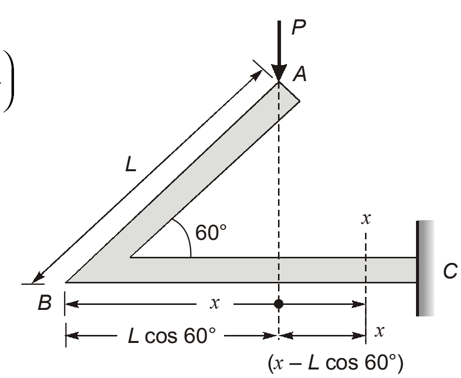
$$M_{AB} = P \times s = P \times \frac{x}{2}$$

$$\frac{\partial M_{AB}}{\partial P} = \frac{x}{2}$$

$$U_{AB} = \int_o^L \frac{M_{AB}^2 dx}{2EI}$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_o^L M_{AB} \times \frac{\partial M_{AB}}{2P} \times dx = \frac{1}{EI}\int_o^L \left[P \times \frac{x}{2}\right] \times \frac{x}{2} \times dx$$

$$= \frac{1}{4EI}\int_o^L P \times x^2 \times dx = \frac{P}{4EI}\int_o^L x^2 \times dx = \frac{PL^3}{12EI}$$



$$M_{BC} = -P \times (x - L\cos 60) = -P \times \left(x - \frac{L}{2}\right)$$

$$\frac{\partial M_{BC}}{\partial P} = -\left(x - \frac{L}{2}\right)$$

$$U_{BC} = \int_o^L \frac{M_{BC}^2\, dx}{2EI}$$

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_o^L M_{BC} \times \frac{\partial M_{BC}}{\partial P} \times dx$$

$$= \frac{1}{EI}\int_o^L -P\left(x-\frac{L}{2}\right)\times\left(-\left(x-\frac{L}{2}\right)\right)dx = \frac{P}{EI}\int_o^L \left(x-\frac{L}{2}\right)^2 dx$$

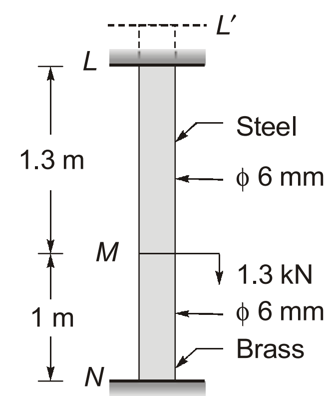
$$= \frac{P}{EI}\left[\frac{L^3}{3}+\frac{L^2}{4}\times L - 2\times\frac{L}{2}\times\frac{L^2}{2}\right] = \frac{P}{EI}\left[\frac{L^3}{3}+\frac{L^3}{4}-\frac{L^3}{2}\right]$$

$$= \frac{PL^3}{EI}\left[\frac{4+3-6}{12}\right] = \frac{PL^3}{12EI}$$

$$\text{Vertical deflection} = \frac{\partial U_{AB}}{\partial P}+\frac{\partial U_{BC}}{\partial P} = \frac{PL^3}{12EI}+\frac{PL^3}{12EI} = \frac{PL^3}{6EI}$$

Solution : 25

The statement of the problem can be shown as



$$A_s = A_b = \frac{\pi}{4}\times(6\times10^{-3})^2 = 28.27\times10^{-6}\text{ m}^2$$

Initial stress/residual stress which can be carried by composite bar LMN is

$$\sigma_{s1} = \sigma_{b1} = \frac{3.5\times10^3}{28.27\times10^{-6}} = 123.8\text{ MPa}$$

As the load is tensile, so σ_{s1} and σ_{b1} are also tensile stress.

Now load of 1.3 kN acts at junction M:

$$P_s + P_b = 1.3\text{ kN}$$

$$\delta l_s = \delta l_b \quad \text{(both bar will undergo equal elongation)}$$

$$\frac{P_sL_s}{A_sE_s} = \frac{P_bL_b}{A_bE_b}$$

$$P_s = \frac{A_s}{A_b}\frac{E_s}{E_b}\frac{L_b}{L_s}P_b$$

$$P_s = 1\times\frac{200}{85}\times\frac{1}{1.3}\times P_b \text{ - } 1.81\,P_b$$

So, $1.81\,P_b + P_b = 1.3$

or $P_b = 462.6\text{ N}$

and $P_s = 837.4\text{ N}$

Stresses due to application of 1.3 kN are

$$\sigma_{s2} = \frac{P_s}{A_s} = \frac{837.4}{28.27\times10^{-6}} = 29.62\text{ MPa (tensile)}$$

$$\sigma_{b2} = \frac{P_b}{A_b} = \frac{462.4}{28.27\times10^{-6}} = 16.36\text{ MPa (compressive)}$$

In the absence of fixtures, the rod *LN* would have expanded freely due to temperature rise to *NL'*. But the pressure of fixtures effectively compress the rod *LN* by an amount *LL'* which is equal to $(\alpha_s l_s + \alpha_b l_b)t$. Thus if σ is the compressive stress in steel and brass (it will be same in both as the cross sectional areas are the same) due to temperature rise, then we have

$$\frac{\sigma L_s}{E_s} + \frac{\sigma L_b}{E_b} = \alpha_s L_s t + \alpha_b L_b t$$

$$\sigma\left[\frac{1.3}{200\times10^9} + \frac{1}{85\times10^9}\right] = 30(12\times10^{-6}\times1.3 + 19\times10^{-6}\times1)$$

or $$\frac{\sigma}{10^9}\left[\frac{1.3}{200} + \frac{1}{85}\right] = 30\times10^{-6}(12\times1.3 + 19)$$

or $$\sigma = \frac{30\times10^3\times34.6}{6.5\times10^{-3}+11.76\times10^{-3}} = 56.84 \text{ MPa (compressive)}$$

Final stress before temperature rise,

$$\sigma_{si} = 123.8 + 29.62 = 153.42 \text{ MPa (tensile)}$$

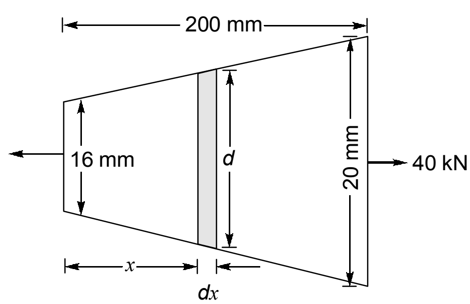
$$\sigma_{bi} = 123.8 - 16.36 = 107.44 \text{ MPa (tensile)}$$

Final stress after temperature rise,

$$\sigma_{sf} = 153.42 - 56.84 = 96.58 \text{ MPa (tensile)}$$

$$\sigma_{bf} = 107.44 - 56.84 = 50.6 \text{ MPa (tensile)}$$

Solution : 26



Referring to above figure:

$$d = d_1 + \frac{d_2 - d_1}{L}x$$

$$= d_1 + kx \qquad k = \frac{d_2 - d_1}{L}$$

Total extension of the bar, $\Delta_T = \dfrac{4PL}{\pi d_1 d_2 E} = \dfrac{31.85\times10^6}{E}$

As given, $$\Delta_T = \frac{31.85\times10^6}{E} = 0.0004 \text{ m}$$

$$E = 79.6 \text{ GPa}$$

Strain energy stored in the bar,

$$U_T = \int_0^L \frac{P^2dx}{2A_xE} = \int_0^L \frac{2P^2dx}{\pi d^2E} = \int_0^L \frac{2P^2dx}{\pi E(d_1+kx)^2} = \frac{2P^2}{\pi E}\left[\frac{-1}{k(d_1+kx)}\right]_0^L = \frac{2P^2L}{\pi d_1 d_2 E}$$

$$= 8 \text{ N-m}$$

For a bar of uniform diameter 0.018 m, extension under a load of 40 kN is given by

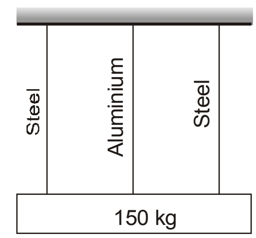
$$\Delta_U = \frac{Pl}{AE} = \frac{40\times10^3\times0.2}{\frac{\pi}{4}\times0.018^2\times79.6\times10^9} = 0.000395 \text{ m}$$

∴ ratio of strain energy = ratio of extensions for the same load

Note: Strain energy of prismatic bar due to axial load is given by

$$U = \frac{1}{2}P\Delta = 7.9 \text{ N-m}$$

Solution : 27



Initial stress in steel = Initial stress in aluminium

$$= \frac{50 \times 9.81}{8 \times 10^{-6}} = 61.3 \text{ MN/m}^2 \text{ (tensile)}$$

$$\text{Load sheared by each wire} = \frac{150}{3}\text{kg} = 50 \text{ kg}$$

Let σ_a and σ_s be the stress in the aluminium and steel respectively due to the temperature rise only. Then,

tensile force in steel wire = compressive force in aluminium wire

$$2\,\sigma_s\, a_s = \sigma_a\, a_a$$

i.e. $$2\,\sigma_s = \sigma_a \quad \text{...(i)}$$

Also $$\frac{\sigma_s l}{210 \times 10^9} + \frac{\sigma_a l}{70 \times 10^9} = l\,(24 - 12)\,5\;10^{-6} \times 50$$

$$\sigma_s + 3\,\sigma_a = 126 \quad \text{...(ii)}$$

fjrom (i) and (ii), σ_s = 18 MPa (tensile)

σ_a = 36 MPa (compressive)

Resultant stress in aluminium

= 61.3 – 36 = 25.3 MPa (tensile)

Resultant stress in steel = 61.3 + 18 = 79.3 MPa (tensile)

When aluminium wire becomes slack,

σ_a = 61.3 MPa (compressive)

Therefore, since σ_a is propo0rtional to the change in temperature,

$$\Delta T = \frac{61.3}{36} \times 50 = 85.138°\text{C}$$

Solution : 28

Let suffices 1 and 2 refer to the square and circular portions respectively.

$$\sigma_1 = \frac{20 \times 10^3}{(0.01)^2} = 200 \text{ MPa}$$

$$\sigma_2 = \frac{20 \times 10^3}{\frac{\pi}{4}d^2} = \frac{0.08}{\pi d^2} \text{ MN/m}^2$$

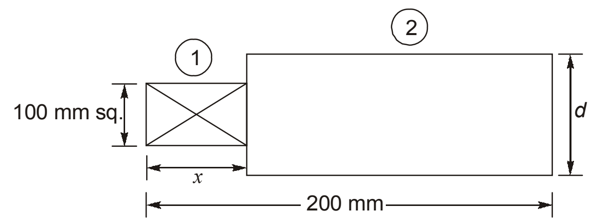
$$\text{Strain energy, } U = \frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2$$

$$= \frac{1}{2E}\left\{(200\times10^6)^2\times x\times0.01^2+\left(\frac{0.08\times10^6}{\pi d^2}\right)^2\times24\times10^{-6}\right\}$$

As given, volume of circular part

$$= 24\times10^{-6}\ \text{m}^3$$

$$\frac{\pi}{4}d^2(0.2-x) = 24\times10^{-6}$$



$$\pi d^2 = \frac{96\times10^{-6}}{0.2-x}$$

Now, $$U = \frac{1}{2E}\left\{4x\times10^{12}+\left[\frac{0.08}{96}(0.2-x)\times10^{12}\right]^2\times24\times10^{-6}\right\}$$

$$U = \frac{10^{12}}{12E}\left\{100x^2-16x+4\right\}$$

For the strain energy to be minimum,

$$\frac{dU}{dx} = 0$$

$$200x-16 = 0$$

$$x = 0.08\text{ m or }80\text{ mm}$$

$$\text{length of circular portion} = 0.12\text{ m or }120\text{ mm}$$

$$\pi d^2 = \frac{96\times10^{-6}}{(0.2-0.12)}$$

$$d = 0.0255\text{ m or }25.5\text{ mm}$$

$$\text{strain energy} = \frac{10^{12}}{12\times200\times10^9}\left\{100\times0.08^2-16\times0.08+4\right\} = 1.4\text{ J}$$

■■■■

8 Deflection of Beams

1.	(b)
2.	(d)
3.	(d)
4.	(c)
5.	(a)
6.	(b)
7.	(c)
8.	(a)
9.	(d)
10.	(a)
11.	(d)
12.	(d)
13.	(d)
14.	(b)
15.	(d)
16.	(d)
17.	(0.67)
18.	(0.78)
19.	(0.21)
20.	(2.2)
21.	(b)
22.	(a)
23.	(a)
24.	(b)
25.	(d)
26.	(b)
27.	(34.91)
28.	(c)
29.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 30

Given: Simply supported beam with uniformly distributed load 'w' per unit length.

Let the reaction forces be R_A and R_B.

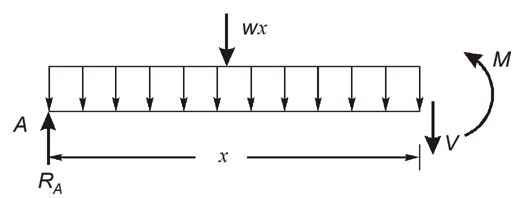
$$R_A + R_B = wL$$

and
$$R_B \times L = wL \times \frac{L}{2}$$

$$R_B = \frac{wL}{2}; \qquad R_A = \frac{wL}{2}$$

Consider a section at a distance x from A,

$$M + \frac{wx^2}{2} = R_A x$$



$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

Also, we know,
$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1x + C_2$$

Boundary conditions at, $x = 0, \quad y = 0$

and $x = L, \quad y = 0$

$\therefore$ $C_2 = 0$

and
$$0 = \frac{wL^4}{12} - \frac{wL^4}{24} + C_1L = \frac{wL^4}{24} + C_1L$$

$$C_1 = -\frac{wL^3}{24}$$

Equation of elastic curve:
$$EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3}{24}x$$

Due to symmetry, maximum deflection of beam occurs at $\left(x = \frac{L}{2}\right)$

So,

$$EIy_{max} = \frac{wL}{12}\left(\frac{L}{2}\right)^3 - \frac{w}{24}\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}\left(\frac{L}{2}\right) = \frac{wL^4}{96} - \frac{wL^4}{384} - \frac{wL^4}{48}$$

$$EIy_{max} = \frac{-5wL^4}{384}$$

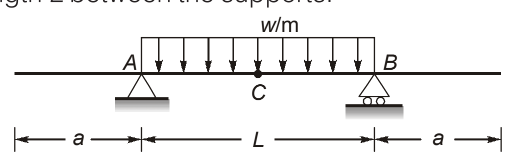
$$y_{max} = -\frac{5wL^4}{384EI}$$

Solution : 31

Case-I

When uniformly distributed load w/unit length covering the length L between the supports.

Let R_A and R_B be the reaction forces



$$R_A + R_B = wL$$

and

$$R_B \cdot L = \frac{wL^2}{2}$$

$$R_B = \frac{wL}{2} \quad \text{and} \quad R_A = \frac{wL}{2}$$

and

$$M_x = R_A(x-a) - \frac{w(x-a)^2}{2}$$

$$= \frac{wL}{2}(x-a) - w\frac{(x-a)^2}{2} = \frac{w}{2}(x-a)(L-x+a)$$

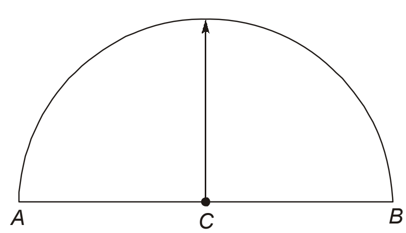
$$M_x = \frac{w}{2}(x-a)(L+a-x)$$

$$M_c = \frac{w}{2}\left(a+\frac{L}{2}-a\right)\left(L+a-a-\frac{L}{2}\right) = \frac{w}{2}\left(\frac{L}{2}\times\frac{L}{2}\right) = \frac{wL^2}{8}$$

and $M_A = 0$, $M_B = 0$

Therefore the curve of area moment diagram between A and B is parabolic in nature.

Using area moment diagram between C and B



M/EI diagram

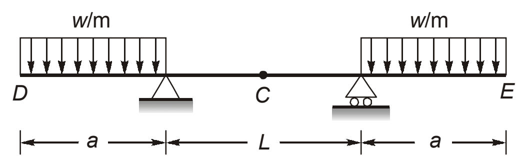
$$y_B - y_C = \frac{-1}{EI}\left[\frac{2}{3}\times\left(\frac{wL^2}{8}\right)\times\left(\frac{L}{2}\right)\times\frac{5L}{16}\right]$$

$$0 - y_C = \frac{-5wL^4}{384EI}$$

$$y_C = \frac{5wL^4}{384EI} \text{ (downwards)}$$

Case-II

When load is covering only two overhanging lengths



$$R_A = R_B = wa$$

Between section D to A and B to E

$$M_x = -\frac{wx^2}{2}$$

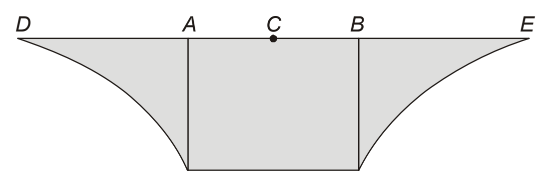
and between section A and B

$$M_x + wa\left(x - \frac{a}{2}\right) - wa(x - a) = 0$$

$$M_x + wax - \frac{wa^2}{2} - wax + wa^2 = 0$$

$$M_x = -\frac{wa^2}{2}$$

Using area moment diagram



M/EI diagram

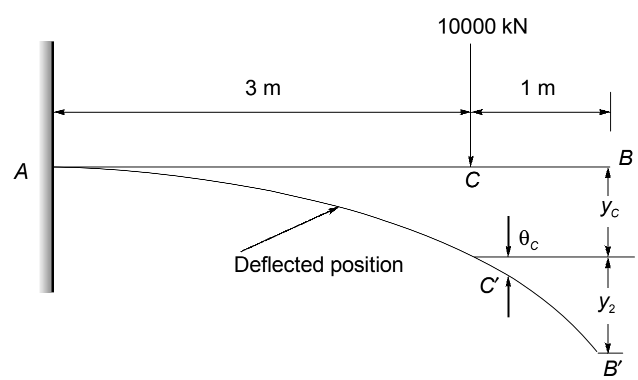
$$y_B - y_C = -\frac{1}{EI}\left[\frac{-wa^2}{2} \times \frac{L}{2} \times \frac{L}{4}\right]$$

$$0 - y_C = \frac{wa^2L^2}{16EI}$$

$$y_C = -\frac{wa^2L^2}{16\,EI} \text{(upward direction)}$$

Solution : 32

For portion AC, taking distance x from the fixed end.



$$EI\frac{d^2y}{dx^2} = 10000(300 - x)$$

integrating and putting the same end conditions before,

$$\theta_C = \frac{Pl^2}{2EI} = \frac{10000 \times (300)^2}{2 \times 40000 \times 20 \times 10^5} = \frac{9}{1600} \text{rad}$$

and $$y_C = \frac{Pl^3}{3EI} = \frac{10000 \times (300)^3}{3 \times 40000 \times 20 \times 10^5} = 1.125 \text{ cm}$$

For portion CB, $$EI\frac{d^2y}{dx^2} = 0$$

$EI\dfrac{dy}{dx}$ = constant, which shows that slope in the portin BC is constant as the bent portin $B'C'$ is straight while AC' is curved.

So $$\theta_B = \theta_C = \frac{9}{1600} \text{radians}$$

$$Y_B = y_C + y_2 = 10125 + \theta_C \times \text{BC} = 1.125 + \frac{9}{1600} \times 100 = 1.6875 \text{ cm.}$$

Solution : 33

$$I = \frac{1}{12} \times 100(200)^3 = 66.67 \times 10^6 \text{ mm}^4$$

Deflection due to UDL, $$Y_{B1} = \frac{wL^4}{8EI} - \left[\frac{wL_1^4}{8EI} + \frac{wL_1^3}{6EI}(L - L_1)\right]$$

$$= \frac{2000}{EI}\left[\frac{(2)^4}{8} - \frac{(1)^4}{8} - \frac{(1)^3(2-1)}{6}\right](1000)^3$$

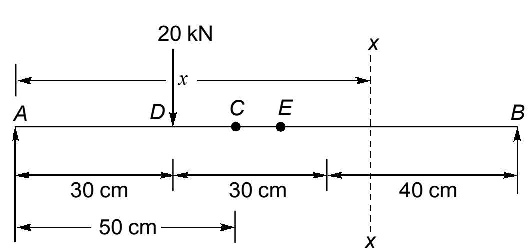
$$= \frac{3.417 \times 10^{12}}{EI} \quad \text{where } E \text{ and } I \text{ are in mm units.}$$

$$= \frac{3.417 \times 10^{12}}{0.105 \times 10^5 \times 66.67 \times 10^6} = 4.88 \text{ mm}$$

Due to point load. $$y_{B2} = \frac{wL^3}{3EI} = \frac{1000(2)^3(1000)^3}{3 \times 0.105 \times 10^5 \times 66.67 \times 10^6} = 3.81 \text{ mm}$$

∴ Total deflection of B = 4.88 + 3.81 = 8.69 mm

Solution : 34



Taking moment about A,

$$20 \text{ kN} \times 30 = R_B \times 100$$

$$R_B = 6 \text{ kN}$$

$$R_A = 20 - 6 = 14 \text{ kN}$$

At a distance x from A in the DB portion,

$$EI\frac{d^2y}{dx^2} = R_A \cdot x - W(x-30) \quad ...(1)$$

Integrating, (For BD section)

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{W}{2}(x-30)^2 + C_1 \quad ...(2)$$

Again integrating, (For BD section)

$$EIy = R_A\frac{x^3}{6} - \frac{W}{2\times 3}(x-30)^3 + C_1x + C_2 \quad ...(3)$$

Boundary conditions,

At $x = 0$, $y = 0$, So, $C_2 = 0$

At $x = L$, $y = 0$ $\quad 0 = R_A\frac{L^3}{6} - \frac{W}{6}\times 70^3 + C_1L + 0$

$$\left[\frac{20}{6}\times 70^3 - \frac{14\times 100^3}{6}\right]\frac{10^3}{100}\times 10^{-4} = C_1 \quad [C_1 = -1190]$$

Putting value of C_1 in equation (2) and (3),

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - \frac{W}{2}(x-30)^2 - 1190 \quad ...(4)$$

$$EIy = R_A\frac{x^3}{6} - \frac{W}{6}(x-30)^3 - 1190x \quad ...(5)$$

For slopes at different points,

Slope at point, A, $\theta_A = -\frac{1190}{EI}$ radian

Slope at point B, $\theta_B = \frac{1}{EI}\left[14\times 10^3 \times \frac{100^2}{2}\times 10^{-4} - \frac{20\times 10^3}{2}\times (70)^2 \times 10^{-4} - 1190\right]$

$$= \frac{1}{EI}[7000 - 4900 - 1190] = \frac{910}{EI} \text{ radian}$$

Slope at point C, $\theta_C = \frac{1}{EI}\left[14\times 10^3 \times \frac{50^2}{2}\times 10^{-4} - \frac{20\times 10^3}{2}\times 20^2 \times 10^{-4} - 1190\right]$

$$= \frac{1}{EI}[1750 - 400 - 1190]$$

$$\theta_C = +\frac{160}{EI} \text{ radian}$$

Slope at point D, $\theta_D = \frac{1}{EI}\left[\frac{14}{2}\times 10^3 \times 30^2 \times 10^{-4} - 1190\right] = -\frac{560}{EI}$ radian

Slope at point E, $\theta_E = \frac{1}{EI}\left[\frac{14\times 10^3}{2}\times 0.6^2 - \frac{20}{2}\times 10^3 \times 0.3^2 - 1190\right]$

$$= \frac{430}{EI} \text{ radian}$$

Deflection at different points,

Deflection of point *A*, $y_A = 0$

Deflection of point *B*, $y_B = 0$

Deflection of point C, $$y_C = \frac{1}{EI}\left[14\times10^3\times\frac{0.5^3}{6}-\frac{20\times10^3}{2}\times0.2^3-1190\times0.5\right]$$

$$= \frac{-1150}{3EI}\text{ units}$$

Deflection of point *D*, $$y_D = \frac{1}{EI}\left[14\times10^3\times\frac{0.3^3}{6}-1190\times0.3\right] = \frac{-294}{EI}\text{ units}$$

Deflections of point *E*, $$y_E = \frac{1}{EI}\left[14\times10^3\times\frac{0.6^3}{6}-\frac{20000}{6}\times0.3^3-1190\times0.6\right]$$

$$= \frac{-300}{EI}\text{ units}$$

Solution : 35

Slope due to load at free end of cantilever beam:

$$\theta = \frac{WL^2}{2EI}$$

Deflection due to load at free end of cantilever beam:

$$\delta = \frac{WL^3}{3EI}$$

Deflection at any other point at a distance L from free end.

$$= \frac{WL^3}{3EI}+\theta\times L$$

Load point	Load	Length *L* from *A*, *m*	Slope, θ	Length *L* from *D*, *m*	Deflection
B	W	2	$2W/EI$	4	$\frac{8W}{3EI}+\frac{8W}{EI}=\frac{32W}{3EI}$
C	2W	4	$\frac{16W}{EI}$	2	$\frac{128W}{3EI}+\frac{32W}{EI}=\frac{224W}{3EI}$
D	3W	6	$\frac{54W}{EI}$	0	$\frac{216W}{EI}$

∴ $$\theta_D = (2+16+54)\frac{W}{EI} = \frac{72W}{EI}$$

$$\delta_D = \frac{W}{EI}\left(\frac{32}{3}+\frac{224}{3}+216\right) = \frac{301.33W}{EI}$$

∴ $$\theta_D = \frac{72\times1\times10^3}{200\times10^9\times10^{-4}} = 3.6\times10^{-3}\text{ rad}$$ Ans.

$$\delta_D = \frac{301.33\times10^3}{200\times10^9\times10^{-4}} = 0.01506\text{ m} = 15.06\text{ mm}$$ Ans.

Alternate

Given,

$$E = 200 \times 10^9 \text{ Pa}$$
$$I = 1 \times 10^{-4} \text{ m}^4$$
$$W_1 = 3w = 3 \text{ kN}$$
$$W_2 = 2w = 2 \text{ kN}$$
$$W_3 = w = 1 \text{ kN}$$

Flexural rigidity $= EI$

$$= 200 \times 10^9 \times 1 \times 10^{-4} = 2 \times 10^7 \text{ Nm}^2$$

(i) Slope of the free end

$$\theta_D = \theta_1 + \theta_2 + \theta_3$$

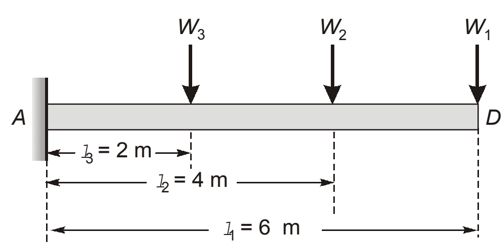
$$= \frac{W_1 l_1^2}{2EI} + \frac{W_2 l_2^2}{2EI} + \frac{W_3 l_3^2}{2EI} = \frac{1}{2EI}\left[W_1 l_1^2 + W_2 l_2^2 + W_3 l_3^2\right]$$

$$= \frac{1}{2 \times 2 \times 10^7} \times 10^3 \left[3 \times 6^2 + 2 \times 4^2 + 1 \times 2^2\right]$$

$$= 3.6 \times 10^{-3} \text{ rad}$$

(ii) Deflection at the free end

$$\delta_D = \delta_1 + \delta_2 + \theta_2(l_1 - l_2) + \delta_3 + \theta_3 \times (l_1 - l_3)$$



$$= \frac{W_1 l_1^3}{3EI} + \frac{W_2 l_2^3}{3EI} + \frac{W_2 l_2^2}{2EI}(l_1 - l_2) + \frac{W_3 l_3^3}{3EI} + \frac{W_3 l_3^2}{2EI}(l_1 - l_3)$$

$$= \frac{1}{EI}\left[\frac{W_1 l_1^3}{3} + \frac{W_2 l_2^3}{3} + \frac{W_2 l_2^2}{2}(l_1 - l_2) + \frac{W_3 l_3^3}{3} + \frac{W_3 l_3^2}{2}(l_1 - l_3)\right]$$

$$= \frac{1}{2 \times 10^7} \times 10^3 \times \left[\frac{3 \times 6^3}{3} + \frac{2 \times 4^3}{3} + \frac{2 \times 4^2}{2}(6 - 4) + \frac{1 \times 2^3}{3} + \frac{1 \times 2^2}{2}(6 - 2)\right] = 0.01506 \text{ m} = 15.06 \text{ mm}$$

■■■■

9 Theories of Failure & Springs

LEVEL 1 Objective Questions

1.	(c)
2.	(4.5)
3.	(a)
4.	(b)
5.	(d)
6.	(d)
7.	(c)
8.	(b)
9.	(a)
10.	(c)
11.	(d)
12.	(b)
13.	(d)
14.	(d)
15.	(b)
16.	(c)

LEVEL 2 Objective Questions

17.	(d)
18.	(112.51)
19.	(b)
20.	(d)
21.	(d)
22.	(b)
23.	(d)
24.	(c)
25.	(b)
26.	(a)
27.	(d)
28.	(d)
29.	(d)

■■■■

LEVEL 3 Conventional Questions

Solution : 30

Given: Axial Pull, $P = 10$ kN, Shear force (V) = 5 kN

Let d be the diameter of rod.

$$\sigma_x = \frac{10}{A}\text{ kN/mm}^2$$

$$\tau_{xy} = \frac{5}{A}\text{ kN/mm}^2$$

σ_1, σ_2 be the principal stresses,
$$\sigma_{1,2} = \frac{5}{A} \pm \sqrt{\left(\frac{5}{A}\right)^2 + \left(\frac{5}{A}\right)^2} = \frac{5}{A} \pm \frac{7.071}{A}$$

$$\sigma_1 = \frac{12.071}{A} \text{ and } \sigma_2 = \frac{-2.071}{A}$$

(i) Strain energy theory

$$\sigma_1^2 + \sigma_2^2 - 2\upsilon\sigma_1\sigma_2 = \left(\frac{\sigma_{yt}}{FOS}\right)^2$$

$$\left\{\left(\frac{12.071}{A}\right)^2 + \left(\frac{-2.071}{A}\right)^2 - 2\times 0.3 \times\left(\frac{12.071}{A}\right)\left(\frac{-2.071}{A}\right)\right\}\times 10^6 = \left(\frac{270}{3}\right)^2$$

$$\left[\frac{145.7}{A^2} + \frac{4.289}{A^2} + \frac{15}{A^2}\right]\times 10^6 = (90)^2 = 8100$$

$$\frac{164.989\times 10^6}{A^2} = 8100$$

$$A = 142.72\text{ mm}^2$$

$$\frac{\pi}{4}d^2 = 142.72\text{ mm}^2$$

$$d = 13.48 \text{ or } 14\text{ mm}$$

(ii) Shear strain energy theory

$$(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2) = \left(\frac{\sigma_{yt}}{FOS}\right)^2$$

$$\left[\left(\frac{12.071}{A}\right) + \left(\frac{-2.071}{A}\right)^2 - \left(\frac{12.071}{A}\right)\left(\frac{-2.071}{A}\right)\right]\times 10^6 = \left(\frac{270}{3}\right)^2$$

$$\left[\frac{145.7}{A^2} + \frac{4.289}{A^2} + \frac{25}{A^2}\right]\times 10^6 = 8100$$

$$\frac{174.989\times 10^6}{A^2} = 8100$$

$$A = 146.981 \text{ mm}^2$$

$$\frac{\pi}{4}d^2 = 146.981 \text{ mm}^2$$

$$d = 13.68 \text{ mm}$$

Solution : 31

(i) According to Von-misses criterion.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \le 2(\sigma_y)^2 \quad \text{[Considering bi-axial stress]}$$

For stress state,

$$\sigma_3 = 0$$

$$\sigma_1 = \tau$$

$$\sigma_2 = -\tau$$

$$[\tau - (-\tau)]^2 + \tau^2 + \tau^2 \le \sigma_y{}^2$$

$$6\tau_{ys}{}^2 \le 2\sigma_{yt}{}^2$$

$$\frac{\sigma_{yt}}{\sqrt{3}} = \tau_{ys} \quad \text{[Relation between tensile and shear yield stress]}$$

$$\tau = \text{Shear yield stress}$$

$$\sigma_y = \text{Tensile stress}$$

(ii) Accroding to Tresca criteria:

$$\frac{\sigma_1 - \sigma_2}{2} \le \frac{\sigma_y}{2}$$

For Stress State,

$$\sigma_1 = \tau \quad \text{[Considering bi-axial stress]}$$

$$\sigma_2 = -\tau, \sigma_3 = 0$$

$$\frac{\tau - (-\tau)}{2} \le \frac{\sigma_y}{2}$$

$$\tau_{ys} = \frac{\sigma_y}{2} \quad \text{[Relation between tensile and shear yield stress.]}$$

Solution : 32

Given: $C = \frac{D}{d} = 6$, δ = Deflection(s) is 30 mm under a load of W = 500 N, τ_{max} = 350 MPa = 350 N/mm^2

G = 80 GPa = 80 × 10^9 N/m^2

Let the spring stiffness be K.

$$W = K\delta$$

$$K = \frac{W}{\delta} \Rightarrow \frac{500}{30} = 16.67 \text{ N/mm}$$

Let the Wahl's correction factor be k_ω,

We know,

$$k_\omega = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times(6)-1}{4\times(6)-4} + \frac{0.615}{6} = 1.2525$$

Also,

$$\tau = k_\omega\left(\frac{8\,WC}{\pi d^2}\right)$$

$$350 = (1.2525)\left(\frac{8\times 500\times 6}{\pi d^2}\right)$$

$$d = 5.229 \text{ or } 6 \text{ mm (say)}$$

$$\text{Coil diameter} = 6 \times 6 = 36 \text{ mm}$$

As,
$$K = \frac{Gd^4}{8D^3N_a}$$
$$N_a = \text{No. of active coils}$$
$$16.67 = \frac{80\times10^3\times6}{8\times(6)^3\times N_a}$$
$$N_a = 16.67 \text{ or } 17 \text{ coils}$$

Total length of wire = Length of one coil × no. of active coils
$$= \pi D\times n = 3.14\times36\times17 = 1922.65 \text{ mm or } 1.92 \text{ m}$$

Solution : 33

Direct stress,
$$s_d = \frac{25}{\pi d^2/4} = \frac{100}{\pi d^2}\ \text{kN/m}^2$$

Shear stress,
$$\tau = \frac{15}{\pi d^2/4} = \frac{60}{\pi d^2}\ \text{kN/m}^2$$

The principal stresses are given by
$$\sigma_{1,3} = \frac{\sigma_d}{2} \pm \sqrt{\left(\frac{\sigma_d}{2}\right)^2 + \tau^2}$$
$$= \frac{50}{\pi d^2} \pm \sqrt{\left(\frac{50}{\pi d^2}\right)^2 + \left(\frac{60}{\pi d^2}\right)^2} = \frac{50}{\pi d^2} \pm \frac{78.103}{\pi d^2}$$

Hence
$$\sigma_1 = \frac{40.78}{d^2}\ \text{kN/m}^2$$
$$\sigma_2 = 0$$
$$\sigma_3 = -\frac{8.95}{d^2}\ \text{kN/m}^2$$

Given that,
$$\sigma = \frac{300}{3} = 100\ \text{MN/m}^2.$$

(i) According to maximum principal stress theory
$$\sigma_1 = \sigma$$
$$\frac{40.78\times10^3}{d^2} = 100\times10^6$$
$$d = 2.029\times10^{-2}\ \text{m} = 20.29\ \text{m}$$

(ii) According to shear strain energy theory
$$2\sigma^2 = (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2$$

But
$$\sigma_2 = 0$$

Hence, $2\sigma^2 = 2(\sigma_1{}^2 + \sigma_3{}^2 - \sigma_1\sigma_3)$
$$\sigma^2 = \sigma_1{}^2 + \sigma_3{}^2 - \sigma_1\sigma_3$$
$$(100)^2\times10^{12} = \left[\left(\frac{40.78}{d^2}\right)^2 + \left(\frac{-8.95}{d^2}\right)^2 - \left(\frac{40.78}{d^2}\right)\left(\frac{-8.95}{d^2}\right)\right]\times10^6$$
$$10^6 = \frac{1}{d^4}[2108.092]\times10^6$$
$$d = 0.02143\ m = 21.43\ \text{mm}$$

Solution : 34

Given, $W = 120$ N, $d = 80$ mm, $T = 500$ N.mm, $\theta = 90°$, $D = 25$ mm

$$\delta\text{(Deflection of close coiled spring)} = \frac{8WD^3n}{Gd^4}$$

or $$1 = \frac{8WD^3n}{Gd^4} \text{ (for unit deflection)}$$

$$\theta = \frac{64TDn}{Ed^4}$$

for unit angular rotation, $$1 = \frac{64TDn}{Ed^4}$$

$$\therefore \quad \frac{8WD^3n}{Gd^4} = \frac{64TDn}{Ed^4}$$

$$\frac{T}{W} = \frac{D^2E}{8G} = \frac{D^2 \times 2G(1+\nu)}{8G} = \frac{D^2(1+\nu)}{4}$$

Torque/unit angular rotation, $$T = \frac{500}{90 \times \frac{\pi}{180}} = 318.3 \text{ N-mm/rad}$$

Axial load applied/unit deflection,

$$W = \frac{120}{80} = 1.5 \text{ N/mm}$$

$$\therefore \quad \frac{T}{W} = \frac{D^2(1+\nu)}{4} \text{ or, } 1+\nu = \frac{4T}{WD^2}$$

or $$\nu + 1 = \frac{4 \times 318.3}{1.5 \times 625} = 1.358$$

or $$\nu = 0.358$$

Solution : 35

Let M = Bending moment and T = Twisting moment

(i) Equivalent bending moment

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}\left(3T + \sqrt{9T^2 + T^2}\right)$$

$$= \frac{T}{2}\left(3 + \sqrt{10}\right) = 3.08114T$$

Safe stress in bending, $$\sigma = \frac{370}{4} = 92.5 \text{ N/mm}^2$$

$$M_e = \sigma \frac{\pi d^3}{32}$$

$$\therefore \quad 3.08114\,T = 92.5\pi \times \frac{100^3}{32} \times \frac{1}{10^6} \text{ kNm}$$

$$T = 2.947 \text{ kNm}$$

(ii) Equivalent torque, $T_e = \sqrt{M^2 + T^2} = \sqrt{9T^2 + T^2} = T\sqrt{10}$

Safe stress, $\tau_s = \frac{\sigma}{2} = \frac{92.5}{2} = 46.25\,\text{N/mm}^2$

$$T_e = \tau_s \cdot \frac{\pi d^3}{16}$$

$$T\sqrt{10} = 46.25\pi \times \frac{100^3}{16} \times \frac{1}{10^6}\,\text{kNm}$$

$$T = 2.872\,\text{kNm}$$

Solution : 36

$$W = 500\text{ N},\ D = 10\text{ d},$$

$$\tau_{allow} = 75\text{ MPa}$$

$$\tau = \frac{8WD}{\pi d^3} = \frac{8 \times 500 \times 10d}{\pi d^3} = \frac{12732.4}{d^2}$$

$$d^2 = \frac{12732.4}{75} = 169.765$$

or $d = 13\text{ mm}$ (wire diameter)

D(mean diameter) $= 10\,d = 130$ mm

Solution : 37

$$d = 120\text{ mm}$$

$$T = 20\text{ kNm}$$

$$M = 12\text{ kNm}$$

$$\sigma_t = 220\text{ MPa}$$

$$\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 12 \times 10^6}{3.14 \times (120)^3} = 70.74\text{ MPa}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 20 \times 10^6}{3.14 \times (120)^3} = 58.95\text{ MPa}$$

Principal stresses are given by

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \frac{70.74}{2} \pm \sqrt{\left(\frac{70.74}{2}\right)^2 + (58.95)^2}$$

$$= 104.12\text{ MPa or } -33.38\text{ MPa}$$

$N(\sigma_1 - \sigma_2) = \sigma_t$ (According to maximum shear stress theory)

$$N = \frac{\sigma_t}{\sigma_1 - \sigma_2} = \frac{220}{104.12 + 33.38} = 1.6$$

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10 Euler's theory of column

LEVEL 1 Objective Questions

1.	(b)
2.	(b)
3.	(c)
4.	(c)
5.	(a)
6.	(a)
7.	(c)
8.	(d)
9.	(a)
10	(c)

LEVEL 2 Objective Questions

11.	(b)
12.	(d)
13.	(c)
14.	(c)
15.	(a)
16.	(b)
17.	(d)

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LEVEL 3 Conventional Questions

Solution : 18

Structural members subjected to compression and which are relatively long compared to their lateral dimensions are called columns or struts. Generally, the term column is used to denote vertical members and the term strut denotes inclined members.

Columns are generally fixed at the both ends while strut can have any end fixation conditions like both end fixed, both ends hinged, one end fixed other end free, etc.

According to Rankine's formulae,

$$\frac{1}{P_R} = \frac{1}{P_e} + \frac{1}{P_c}$$

where P_e = buckling load

P_c = crushing load

For short column $P_e >>>> Pc$

or $\frac{1}{P_e} <<< \frac{1}{P_c} \Rightarrow \frac{1}{P_e}$ can be neglected ,

$$\frac{1}{P_R} = \frac{1}{P_c}$$

$$P_R \approx P_c \approx A\sigma_c$$

For long columns $P_c >>> P_e$

$\frac{1}{P_c} <<< \frac{1}{P_e} \Rightarrow \frac{1}{P_c}$ can be neglected

$$P_R \approx P_e = \frac{\pi^2 E I_{min}}{L_e^2}$$

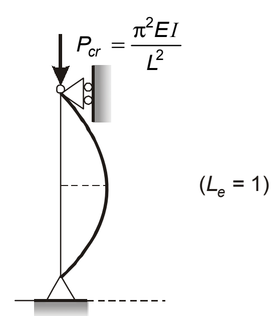
$$\therefore \quad P_R = \frac{\sigma_c A}{1 + c(Se)^2}$$

where, c = Rankine's constant and Se = slenderness ratio

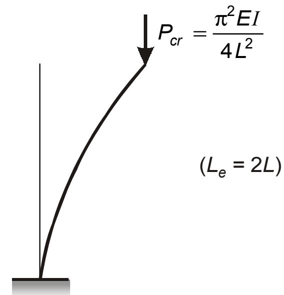
Solution : 19

(i) Buckling of columns under axial load with four different end conditions

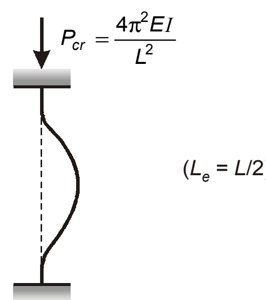
(a) Pinned Ends:



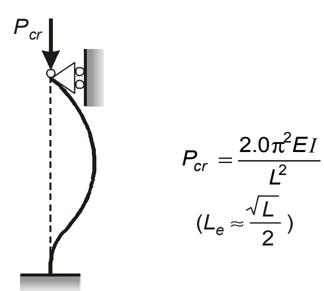
(b) One end fixed, other end free



(c) Both ends fixed



(d) One end fixed, other end pinned



$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

$$L_e \approx \frac{L}{\sqrt{2}}$$

Solution : 20

For both ends hinged $P_{cr} = \frac{\pi^2 EI}{L^2}$

For both ends fixed $P_{cr} = \frac{4\pi^2 EI}{L^2}$

$$\frac{d_0}{d_i} = 1.25 \text{ and } \frac{d_i}{d_0} = 0.8$$

Now if P is load for both ends hinged and $P + 300$ is load for both ends fixed

$$P = \frac{\pi^2 EI}{L^2} \text{ and } P + 300 = \frac{4\pi^2 EI}{I^2}$$

we get,
$$\frac{\pi^2 EI}{L^2} + (300 \times 10^3) = \frac{4\pi^2 EI}{L^2}$$

$$\frac{3\pi^2 EI}{L^2} = 300 \times 10^3$$

$$I = \frac{300 \times 10^3 \times 9}{3 \times \pi^2 \times 100 \times 10^9} = 9.1189 \times 10^{-7}$$

$$\left(\therefore \text{Where } k = \frac{d_i}{d_0} = 0.8\right)$$

$$I = 9.12 \times 10^{-7} \text{ m}^4$$

Now
$$I = \frac{\pi}{64} d_0^4 (1 - k^4) \text{ for hollow column.}$$

$$\frac{\pi}{64} d_0^4 (1 - k^4) = 9.12 \times 10^{-7}$$

$$d_0 = 74.9 \text{ mm}$$

Solution : 21

Here, we can calculate I_{min} as follows :

$$I_{min} = \frac{50 \times 30^3}{12} = 112.5 \times 10^3 \text{ mm}^4$$

$$L = 2000 \text{ mm}$$

Using Euler's equation, we get

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{(\pi^2 \times 2 \times 10^5 \times 112.5 \times 10^3)}{(2000 \times 2000)}$$

$$P_{cr} = 55.51 \times 10^3 \text{ N}$$

and
$$\sigma_{cr} = \frac{P_{cr}}{A} = 37.01 \text{ N/mm}^2$$

$$r_{min} = \sqrt{\left(\frac{I_{min}}{A}\right)} = 8.66 \text{ mm}$$

and
$$\frac{L}{r_{min}} = 230.94$$

or, using Equation

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} = 37.01 \text{ N/mm}^2$$

Since, $\sigma_{cr} < \sigma_p$,
$$\frac{L}{r} > \pi\sqrt{\left(\frac{E}{\sigma_p}\right)} = 88.86$$

As $\sigma_{cr} < \sigma_p$ $\left(\text{i.e.} \dfrac{L}{r_{min}} > 88.86\right)$ for $L = 2$ m

Euler's expression is valid for limiting value of minimum length

$$\frac{L_{min}}{r_{min}} = 88.86$$

$$L = 769.5 \text{ mm} \approx 0.77 \text{ m which is minimum length.}$$

Solution : 22

Shown in the figure is the composite section of the column. As the column section is symmetrical about the axes x-x and y-y, the centroid 'C' is given by the intersection of x-x and y-y axes. It will be seen that the dimension parallel to the y-y axis is more. hence M.I. will be minimum about the y-y axis.

$$I_{yy} = 2\left[\frac{10\times 240^3}{12}\right] + 2\left[\frac{12\times 200^3}{12}\right] + \frac{276\times 10^3}{12}$$

$$= 3.91 \times 10^7 \text{ mm}^4$$

But, it is always better to calcualte I_{xx} also.

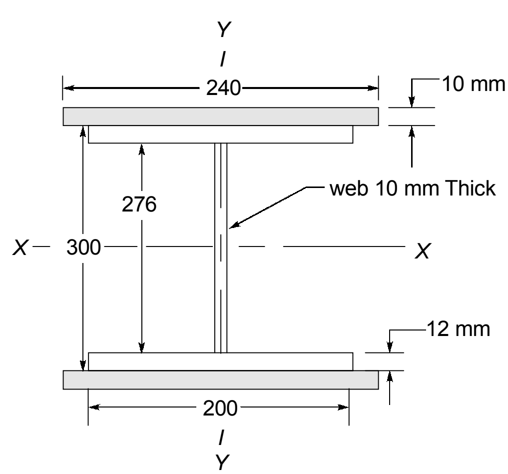
$$I_{xx} = I_{xx} \text{ of flange plates} + I_{xx} \text{ of flanges} + I_{xx} \text{ of web}$$

$$I_{xx} = 2\left[\frac{240\times 10^3}{12} + 240\times 10\times(150+5)^2\right]$$

$$+2\left[\frac{200\times 12^3}{12} + 200\times 12\times(150-6)^2\right] + \frac{10\times 276^3}{12}$$

$$I_{xx} = 21.495 \times 10^7 \text{ mm}^4$$

$$I_{xx} > I_{yy}, \text{ Hence } I_{yy} = I_{min} = 3.91 \times 10^7 \text{ mm}^4$$



For column with both ends fixed,

$$l_e = \frac{l}{2}$$

or $$l_e = \frac{6000}{2} = 3000 \text{ mm}$$

Using Euler's formula,

$$\text{Crippling Load} = P = \frac{\pi^2 E I_{min}}{l_e^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 3.91 \times 10^7}{(3000)^2}$$

$$= 85.76 \times 10^5 \text{ N}$$
$$= 85.76 \times 10^2 \text{ kN}$$

For the given column, Euler's buckling load = 8576 kN.

Solution : 23

$$\text{Length, } l = 4 \text{ m}$$
$$\text{UDL, } \omega = 30 \text{ kN/m}$$
$$\text{Deflection at centre, } \delta = 15 \text{ mm} = 0.015 \text{ m}$$

For a simply supported beam carrying UDL over the whole span, the deflection at the centre is given by

$$\delta = \frac{5}{384}\frac{\omega l^4}{EI}$$

$\therefore$ $$EI = \frac{5}{384}\frac{\omega l^4}{\delta} = \frac{5}{384} \times \frac{30 \times (4)^4}{0.015} = 6666.67 \text{ kN.m}^2$$

(i) $$P_{cr} = \frac{2\pi^2 EI}{l^2} \quad \left(\text{for one end fixed and other hinged, } l_e = \frac{l}{\sqrt{2}}\right)$$

$$= \frac{2 \times 3.14^2 \times 6666.67}{(4)^2} = 8216.33 \text{ kN}$$ Ans.

(ii) $$P_{cr} = \frac{\pi^2 EI}{l^2} \quad (\text{for both end hinged, } l_e = l)$$

$$= \frac{(3.14)^2 \times 6666.67}{(4)^2} = 4108.167 \text{ kN}$$ Ans.

Solution : 24

$$I_{xx} = \frac{1}{2}(240 \times 880^3 - 220 \times 800^3) = 4242.8 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{1}{2}(800 \times 20^3 + 2 \times 40 \times 240^3) = 92.7 \times 10^6 \text{ mm}^4$$

$$\delta = \frac{5wl^4}{384EI}$$

$$l^4 = \frac{384EI}{5w} \times \delta = \frac{384 \times 205 \times 10^3 \times 4242.8 \times 10^6 \times 12}{5 \times 50}$$

$$L = 11252 \text{ mm}$$

$$L_e = \frac{l}{2} = \frac{11252}{2} = 5626 \text{ mm}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{(3.14)^2 \times 205 \times 10^3 \times 92.7 \times 10^6}{(5626)^2} = 5925.6 \times 10^3 \text{ N}$$

$$\text{Safe load} = \frac{P_{cr}}{N} = \frac{5925.6 \times 10^3}{5} = 1185.12 \text{ kN}$$

■■■■

11 Pressure Vessel

LEVEL 1 Objective Questions

1.	(c)
2.	(c)
3.	(a)
4.	(b)
5.	(c)
6.	(b)
7.	(b)
8.	(b)
9.	(b)
10.	(b)
11.	(b)

LEVEL 2 Objective Questions

12.	(c)
13.	(d)
14.	(a)
15.	(c)
16.	(c)
17.	(6.68)
18.	(34.17)
19.	(b)
20.	(d)

■■■■

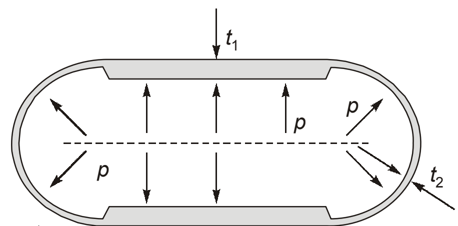
LEVEL 3 Conventional Questions

Solution : 21

In a pressure vessel, D is internal diameter and t is wall thickness, if $\frac{D}{t} > 20$, then it is a thin shell. When the shell is subjected to internal pressure, the hoop stress developed in the shell does not vary much across the thickness. If $\frac{D}{t} < 20$. Then it is thick shell; i.e., thickness of shell is considerable in comparison to diameter, then there is variation of hoop and radial stresses across the thickness of thick shell.

Figure shows a thin cylindrical shell with hemispherical ends, subjected to internal pressure p.

In cylindrical portion:



$$\text{Hoop stress, } \sigma_{h_c} = \frac{pD}{2t_1}$$

$$\text{Axial stress, } \sigma_{a_c} = \frac{pD}{4t_1}$$

$$\text{Hoop strain, } \varepsilon_{h_c} = \frac{pD}{2t_1E} - \frac{\nu pD}{4t_1E} = \frac{pD}{4t_1E}(2-\nu)$$

In hemispherical portion at junction

$$\text{Hoop stress, } \sigma_{h_s} = \frac{pD}{4t_2}$$

$$\text{Hoop strain, } \varepsilon_{h_s} = \frac{pD}{4t_2E}(1-\nu)$$

For no distortion,

$$\varepsilon_{h_c} = \varepsilon_{h_s}$$

$$\frac{pD}{4t_1E}(2-\nu) = \frac{pD}{4t_2}(1-\nu)\frac{1}{E}$$

or

$$\frac{t_2}{t_1} = \frac{1-\nu}{2-\nu} = \frac{1-0.3}{2-.07} = \frac{0.7}{1.7} = \frac{7}{17}$$

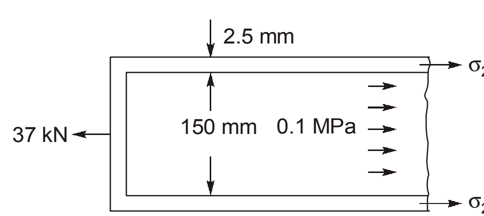
$$\text{Maximum stress in cylindrical portion} = \frac{pD}{2t_1} = \sigma_{h_c}$$

$$\text{Maximum stress in hemispherical portion} = \frac{pD}{4t_2} = \sigma_{h_s}$$

$$\sigma_{h_c} = \sigma_{h_s}$$

$$\frac{pD}{2t_1} = \frac{pD}{4t_2}; \quad \frac{t_2}{t_1} = 0.5$$

Solution : 22



$$\Delta p = 0.1 \text{ N/mm}^2$$

$$\text{Reduction in volumetric strain of cylinder} = \frac{\Delta p \times D}{4tE}(5-4\nu)$$

$$\text{Reduction in volumetric strain of water} = \frac{\Delta p}{K} = \frac{0.1}{2200}$$

$$\text{Axial force} = 37000 \text{ N}$$

$$\text{Axial stress} = \frac{37000}{\pi Dt} = \frac{37000}{\pi \times 150 \times 2.5} = 31.406 \text{ N/mm}^2$$

$$\text{Increase in volumetric strain due to } \sigma = \frac{\sigma}{E}(1-2\nu) = \frac{31.406}{E} \times (1-2\nu)$$

Substituting the values, we get

$$\frac{0.1 \times 150}{4 \times 2.5E}(5-4\nu) + \frac{0.1}{2200} = \frac{31.406}{E}(1-2\nu)$$

$$\Rightarrow \quad 1.5(5-4\nu) + \frac{0.1 \times 140000}{2200} = 31.406\,(1-2\nu)$$

$$\Rightarrow \quad 7.5 - 6\nu + 6.3636 = 31.406 - 62.812\nu$$

$$\Rightarrow \quad 56.812\nu = 31.406 - 6.3636 - 7.5 = 17.5424$$

$$\text{Poisson's ratio, } \nu = \frac{17.5424}{56.812} = 0.308$$

Solution : 23

For steel ring. $R_1 = 5$ cm

$R_0 = 5$ cm

$R_0 = 7.5$ cm

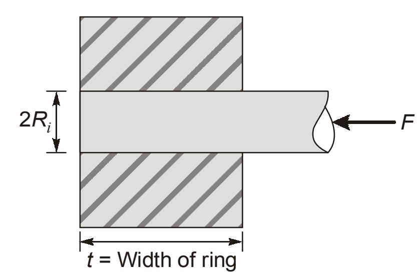
Let, P = Pressure developed at interface

Circumferential strain at outer surface

$$\epsilon_{no} = 1.55 \times 10^{-4}$$

∴ Circumferential stress at outer surface

$$\sigma_{no} = \epsilon_{no} \cdot E = 31 \text{ MPa}$$



According to Lame's equation

Circumferential stress at outer surface

$$\sigma_{no} = \frac{2PR_i^2}{R_o^2 - R_i^2}$$

$$31 = \frac{2P \times 5^2}{7.5^2 - 5^2}$$

∴ $P = 19.375$ MPa

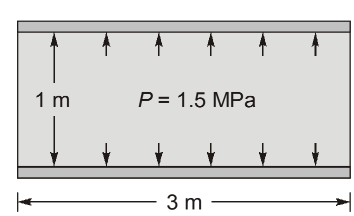
Force required to push the rod

$$F = 2\pi R_i t \,.\, p.\mu,$$

μ = Coefficient of friction

$$F = 2\pi \times 0.05 \times 0.06 \times 19.375 \times 10^6 \times 0.25 = 91256.2 = 91.25 \text{ kN}$$

Solution : 24



Given data: $d = 1$ m; $t = 10$ mm; $P = 1.5$ MPa; $\mu = 0.3$;
$E = 200$ GPa; $L = 3$ m

$$\text{Change in length} = \frac{Pd}{4tE}[1-2\mu]L$$

$$= \frac{1.5 \times 10^6}{4 \times 10 \times 10^{-3} \times 200 \times 10^9}[1 - 2 \times 0.3] \times 3$$

$$= 2.25 \times 10^{-4}\ \text{m}$$

$$\text{Change in diameter} = \frac{Pd^2}{4tE}[2-\mu] = \frac{1.5\times10^6\times1^2}{4\times10\times10^{-3}\times200\times10^9}[2-0.3] = 3.188\times10^{-4}\ \text{m}$$

$$\text{Now hoop stress} = \frac{Pd}{2t} = \frac{1.5\times10^6\times1}{2\times10\times10^{-3}} = 75\ \text{MPa} = \sigma_1$$

$$\text{longitudinal stress} = \frac{Pd}{4t} = \frac{1.5\times10^6}{4\times10\times10^{-3}} = 37.5\ \text{MPa} = \sigma_2$$

$$\text{Maximum shear stress} = \frac{\sigma_1+p}{2} = \frac{75+1.5}{2} = 38.25\ \text{MPa}$$ (Considering the effect of internal pressure)

$$\text{Change in volume,}\quad \frac{\delta V}{V} = \frac{pd}{4tE}[5-4\mu] = \frac{1.5\times10^6\times1}{4\times10\times10^{-3}\times200\times10^9}[5-4\times0.3] = 7.125\times10^{-4}$$

$$\delta V = 7.125\times10^{-4}\times\left(\frac{\pi}{4}D^2L\right) = 7.125\times10^{-4}\times\left(\frac{\pi}{4}\times1\times3\right)$$

$$\delta V = 1.67\times10^{-3}\ \text{m}^3$$

Solution : 25

Given that, $d = 0.5$ m, thickness, $t = 0.005$ m, $P = 3.5$ MPa

Longitudinal stress due to pressure P,

$$\sigma_l = \frac{Pd}{4t} = \frac{3.5\times0.5}{4\times0.005} = 87.5\ \text{MPa}$$

When the pipe is closely wound with a wire of diameter (d_w) 2.5 mm, carrying a tension $T = 75$ MPa, then compressive hoop stress induced in the cylinder

$$(\sigma_c)_w = -\frac{\pi}{4}\times\frac{d_w}{t}\times T$$

$$= -\frac{\pi}{4}\times\frac{2.5\times10^{-3}}{5\times10^{-3}}\times75$$

$$= -29.45\ \text{MPa} \qquad \text{(compressive)}$$

Let $(\sigma_c)_P$ and $(\sigma_w)_P$ be the final stresses in the cylinder and the winding respectively due to pressure P, then

$$d\times p = (\sigma_c)_P\times2t + (\sigma_w)_P\times\frac{\pi}{2}d_w$$

$$50\times10^{-2}\times3.5 = (\sigma_c)_P\times2\times5\times10^{-3} + (\sigma_w)_P\times\frac{\pi}{2}\times2.5\times10^{-3}$$

$$175 = (\sigma_c)_P + 0.3927(\sigma_w)_P \qquad \text{... (i)}$$

Change of hoop strains in the cylinder and the winding at the junction are equal. Hence

$$\frac{(\sigma_w)_P - T}{E_w} = \frac{1}{E_c}[(\sigma_c)_P - (\sigma_c)_w - \nu_c\sigma_l]$$

Where $E_w = E_c$ = Modulus of elasticity of winding and cylinder

Also ν_c = Poisson's ratio of cylinder

$\nu_s = 0.25$

Hence,

$$(\sigma_w)_P - 75 = (\sigma_c)_P + 29.45 - 0.25 \times 87.5$$

$$(\sigma_w)_P = (\sigma_c)_P + 82.575 \quad \text{... (ii)}$$

From equation (i) and (ii)

$$175 = (\sigma_w)_P - 82.575 + 0.3927(\sigma_w)_P$$

$$(\sigma_w)_P = 184.95 \text{ MPa}$$

and $$(\sigma_c)_P = 184.95 - 82.575 = 102.37 \text{ MPa}$$

Solution : 26

Let the radial pressure and the hoop stress at any radius be given by

$$\text{Radial stress } (p_x) = \frac{2b}{x^3} - a \qquad \therefore \qquad \text{hoop stress } (f_x) = \frac{b}{x^3} + a$$

At $$x = 75 \text{ mm}, \quad p_x = 20 \text{ N/mm}^2$$

$$\therefore \quad \frac{2b}{421875} - a = 20 \quad \text{...(i)}$$

At $$x = 75 \text{ mm}, \quad f_x = 100 \text{ N/mm}^2$$

$$\therefore \quad \frac{b}{421875} + a = 100 \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get,

$$a = 60$$

and $$b = 16875000$$

Let the external radius be r_1

At $$x = r_1, \quad p_x = 0$$

$$\therefore \quad \frac{2 \times 16875000}{r_1^3} - 60 = 0$$

$$\therefore \quad r_1 = 82.55 \text{ mm}$$

$$\therefore \quad \text{Thickness of the shell} = 82.55 - 75 = 7.55 \text{ mm}$$

Solution : 27

Additional quantity of fluid pumped in = Increase in volume of pressure vessel + Compression of fluid

Let K be the bulk modulus of fluid.

$$\frac{\delta V}{V} = \frac{3pR}{2tE}(1-\nu) + \frac{p}{K}$$

$$\frac{900 \times 10^{-6}}{\frac{4}{3}\pi(0.4)^3} = \frac{3 \times 5.5 \times 10^6 \times 0.4(1-0.286)}{2 \times 10 \times 10^{-3} \times 204 \times 10^9} + \frac{5.5 \times 10^6}{K}$$

$$3.3572 \times 10^{-3} = 1.155 \times 10^{-3} + \frac{5.5 \times 10^6}{K}$$

$$K = \frac{5.5 \times 10^6}{2.1791 \times 10^{-3}} = 2.497 \text{ GPa}$$

Solution : 28

Diametral strain = Circumferential strain

$$\epsilon_D = \epsilon_C = \frac{PD}{4tE}(1-\nu) = 0.0005$$

$$P = \frac{4tE \times 0.0005}{D(1-\nu)} = \frac{4\times 3\times 200\times 10^3 \times 0.0005}{600\times 0.7}$$

$$P = 2.86 \text{ N/mm}^2$$

Solution : 29

Dia. of cylindrical shell, d = 800 mm

Thickness of cylindrical shell,

t = 10 mm

length of shell, L = 2 m or 2000 mm

Internal pressure, P = 1.5 MPa

Elastic modulus, E = 205 GPa

Poisson's ratio, ν = 0.3

$$\text{Hoop stress, } \sigma_c = \frac{Pd}{2t} = \frac{1.5\times 800}{2\times 10} = 60 \text{ MPa (tensile)}$$

$$\text{Longitudinal stress, } \sigma_L = \frac{Pd}{4t} = \frac{1.5\times 800}{4\times 10} = 30 \text{ MPa (tensile)}$$

Maximum shear stress at any point in the thickness of metal

$$\tau_{max} = \frac{\sigma_c + P}{2} = \frac{60+1.5}{2} = 30.75 \text{ MPa} \quad \text{Ans. (i)}$$

$$\tau_{max} = \frac{\sigma_t}{2} = 30 \text{ MPa (it } P \text{ is neglected)} \quad \text{Ans. (i)}$$

$$\text{Change in diameter, } \delta d = \epsilon_c d = \frac{\sigma_c - \nu\sigma_L}{E}\cdot d$$

$$\delta d = \frac{60-0.3\times 30}{205\times 10^3}\times 800 = 0.199 \text{ mm (+ve means increases)}$$

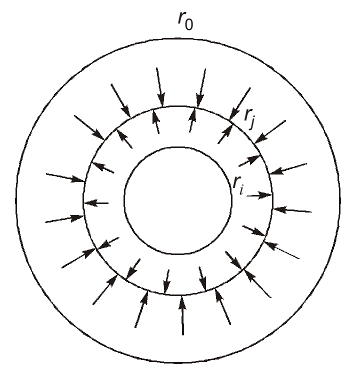
$$\delta L = \epsilon_l L = \frac{\sigma_L - \nu\sigma_c}{E}L = \frac{30-0.3\times 60}{205\times 10^3}\times 2000 = 0.117 \text{ mm(increase) Ans. (ii)}$$

$$\frac{\delta V}{V} = \frac{Pd}{2tE}(2.5-2\nu) = \frac{\sigma_c}{E}(2.5-2\nu)$$

$$\delta V = \frac{60}{205\times 10^3}(2.5-2\times 0.3)\times \frac{\pi}{4}\times 800^2 \times 2000$$

$$\delta V = 559050 \text{ mm}^3 \text{ (increase)} \quad \text{Ans. (iii)}$$

Solution : 30



$$r_0 = 150 \text{ mm}$$
$$r_J = 125 \text{ mm}$$
$$r_i = 75 \text{ mm}$$
$$P_J = 28 \text{ N/mm}^2 \quad \text{(radial pressure at junction)}$$

Radial interference is given by

$$I = \frac{2P_j r_j^3 \left(r_0^2 - r_i^2\right)}{E\left(r_0^2 - r_i^2\right)\left(r_0^2 - r_i^2\right)}$$

$$I = \frac{2 \times 28 \times (125)^3 \left(150^2 - 75^2\right)}{210 \times 10^3 \left(150^2 - 125^2\right)} = 0.12784 \text{ mm}$$

Solution : 31

$$\text{Circumferential stress, } \sigma_c = \frac{Pd}{2t\eta_1} = \frac{1.25 \times 2}{2 \times 0.015 \times 0.8} = 104 \text{ MN/m}^2$$

$$\text{Longitudinal stress, } \sigma_l = \frac{Pd}{4t\eta_c} = \frac{1.25 \times 2}{4 \times 0.015 \times 0.6} = 69.5 \text{ MN/m}^2$$

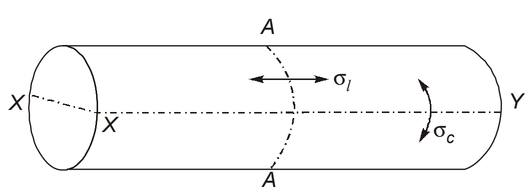
$$\text{Factor of safety} = \frac{450}{104} = 4.32$$

Note:

- X–Y : Longitudinal joint resisting hoop stress
- A – A : Circumferential joint resisting longitudinal stress

(i) If the cylinder is made up from riveted plates and the efficiency of the longitudinal joints is η_l then the average stress in the joint is given by

$$\sigma_c = \frac{Pd}{2t\eta_l}$$



(ii) If the cylinder is made up from riveted plates and the efficiency of the circumferential joints is η_c then the average stress in the joint is given by

$$\sigma_l = \frac{Pd}{4t\eta_c}$$

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