2020

RANK improvement WORKBOOK

Detailed Explanations of
Objective & Conventional *Questions*

Mechanical Engineering

Machine Design

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1 Design for Static Loading

LEVEL 1 Objective Questions

1. (a)

2. (a)

3. (a)

4. (d)

5. (b)

6. (b)

7. (b)

8. (d)

9. (b)

10. (c)

11. (a)

LEVEL 2 Objective Questions

12. (a)

13. (a)

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LEVEL 3 Conventional Questions

Solution : 14

Pitch of the threads (p) = 2.4 mm

Angle through which nut is tightened = 45°

$$\therefore \quad \text{Axial movement of nut} = \frac{2.4 \times 45°}{360°} = 0.3 \text{ mm}$$

$$A_{steel} = 0.785 \times 18^2 = 254.47 \text{ mm}^2$$

$$A_{copper} = 0.785(30^2 - 20^2) = 392.70 \text{ mm}^2$$

The stress developed in bolt = $+\sigma_b$ (tensile)

stress developed in tube = $-\sigma_t$ (compressive)

Due to tightening of the nut, the bolt is extended and tube is contracted

$$\sigma_b A_b - \sigma_t A_t = 0$$

$$\sigma_b \times 254.47 = \sigma_t \times 392.70$$

or $$\sigma_b = 1.543\, \sigma_t$$

$$\text{Extension in bolt} = \frac{\sigma_b}{E_s} \times L = \frac{\sigma_b \times 500}{2 \times 10^6} = \frac{\sigma_b}{400}$$

$$\text{Contraction in tube} = \frac{\sigma_t}{E_c} \times L = \frac{\sigma_1 \times 500}{1 \times 10^6} = \frac{\sigma_t}{200}$$

Axial movement of nut = extension nut + contraction in tube

or $$0.3 = \frac{\sigma_b}{400} + \frac{\sigma_t}{200}$$

or $$\sigma_b + 2\sigma_t = 120$$

or $$(1.543 + 2)\sigma_t = 120$$

$$\sigma_t = \frac{120}{3.543} = 33.87 \text{ N/mm}^2 \text{ (compressive)}$$

$$\sigma_b = 1.543 \times 33.87 = 52.26 \text{ N/mm}^2 \text{ (tensile)}$$

Solution : 15

Given: $P = 2.5$ kN, $S_{ut} = 300$ N/mm², $(fs) = 3$

Step I: Calculation of permissible stress

$$\sigma_{max} = \frac{S_{ut}}{(fs)} = \frac{300}{3} = 100 \text{ N/mm}^2$$

Step II: Bending stress at fillet section

Due to symmetry, the reaction at each bearing is 1250 N. The stresses are critical at two sections: (i) at the centre of span and (ii) at the fillet. At the fillet section,

$$\sigma_o = \frac{32 M_b}{\pi d^3} = \frac{32(1250 \times 350)}{\pi d^3} \text{ N/mm}^2$$

$$K_t = 1.61$$

$$\therefore \quad \sigma_{max} = K_t \sigma_o = 1.61\left[\frac{32(1250 \times 350)}{\pi d^3}\right] = \left(\frac{7174704.8}{d^3}\right) \text{N/mm}^2$$

Step III: Bending stress at centre of the span

$$\sigma_o = \frac{32M_b}{\pi d^3} = \frac{32(1250\times 500)}{\pi(1.1d)^3} = \left(\frac{4783018.6}{d^3}\right)\text{N/mm}^2$$

Step IV: Diameter of beam

From (i) and (ii), it is seen that the stress is maximum at the fillet section. Equating it with permissible stress,

$$\left(\frac{7174704.8}{d^3}\right) = 100;\ d = 41.55 \text{ mm}$$

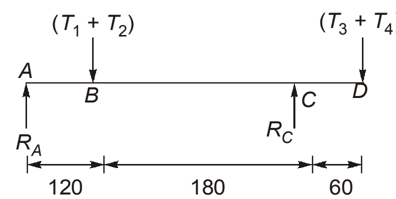
Solution : 16

Given: Shaft Power = 2 kW at N = 750 rpm, K_b = 1.5, K_t = 1.0, $\tau_{(allow)}$ = 65 MPa

Let the tensions in belt of pulley B be T_1, T_2.

Then tensions in belt of pulley D are T_3, T_4.

M_t = torque transmitted



$$P = \frac{2\pi N M_t}{60}$$

$$M_t = \frac{60\times 2\times 10^3}{2\pi(750)} = 25.47 \text{ N-m}$$

Also,
$$M_t = (T_1 - T_2)R_B = (T_3 - T_4)R_D$$
$$25.47\times 10^3 = (T_1 - T_2)\times 150$$

And
$$\frac{T_1}{T_2} = \frac{T_3}{T_4} = 3 \text{ (Given)} \quad \text{(i)}$$
$$25.47\times 10^3 = (2T_2)\times 150$$
$$T_2 = 84.92 \text{ N}$$

from (i) we get
$$T_1 = 254.77 \text{ N}$$
$$T = (T_3 - T_4)\times R_D$$
$$25.47\times 10^3 = (2T_4)\times 75$$
$$T_4 = 169.8 \text{ N}$$

from (i) we get
$$T_3 = 509.4 \text{ N}$$

R_A and R_C be the reaction forces,
$$R_A + R_C = (T_1 + T_2) + (T_3 + T_4) = 1018.89 \text{ N} \quad \text{...(ii)}$$

Taking moment about point A
$$(T_1 + T_2)\times 120 + (T_3 + T_4)\times 360 = R_C\times 300$$
$$40762.8 + 244512 = R_C\times 300$$
$$R_C = 950.916 \text{ N}$$

from (ii) we get
$$R_A = 67.974 \text{ N}$$

Bending moment at B
$$M_B = R_A\times 120 = 8156.88 \text{ N-mm}$$
$$M_C = (T_3 + T_4)\times 60 = 40752 \text{ N-mm}$$

Maximum Bending moment will be at 'C'.

Let
$$T_e = \text{Equivalent torque} = \sqrt{(K_bM_b)^2 + (K_tM_t)^2}$$
$$= \sqrt{(1.5\times 40752)^2 + (1\times 25.47\times 10^3)^2} = 66{,}222 \text{ N-mm}$$

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Also, $\tau = \dfrac{16T_e}{\pi d^3}$

$$65 = \frac{16\times 66,222}{\pi d^3} \Rightarrow d = 17.315 \text{ mm} \;\; \text{or}$$

Diameter of shaft = 20 mm

Solution : 17

b = 120 mm, t = ?, P_{max} = 250 kN, P_{min} = 100 kN,

σ_e = 225 MPa, σ_y = 300 MPa, FOS(N) = 1.5

Let t = thickness of the plate in mm

Area, $A = bt = 120\ t\ \text{mm}^2$

$$P_{mean} = \frac{P_{max}+P_{min}}{2} = \frac{250+100}{2} = 175 \text{ kN}$$

$$P_{variable} = \frac{P_{max}-P_{min}}{2} = \frac{250-100}{2} = 75 \text{ kN}$$

$$\sigma_{mean} = \frac{P_m}{A} = \frac{175\times 10^3}{120t} = \frac{1458.33}{t} \text{ N/mm}^2$$

$$\sigma_{variable} = \frac{P_v}{A} = \frac{75\times 10^3}{120t} = \frac{625}{t} \text{ N/mm}^2$$

According to soderberg's formula:

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{1458.33}{300t} + \frac{625}{225t}$$

or $$t = 1.5\left(\frac{1458.33}{300} + \frac{625}{225}\right) = 11.458 \text{ mm}$$

or thickness, t = 11.5 mm

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2 Design Against Fluctuating Loading

LEVEL 1 Objective Questions

1.	(a)
2.	(a)
3.	(c)
4.	(d)
5.	(b)
6.	(d)
7.	(b)
8.	(a)
9.	(b)
10.	(c)

LEVEL 2 Objective Questions

11	(a)
12	(b)
13.	(b)
14.	(a)
15.	(b)
16.	(c)
17.	(d)
18.	(a)
19.	(2.7)
20.	(b)
21.	(5)
22.	(c)

LEVEL 3 Conventional Questions

Solution : 23

Given M_t = –100 N-m to + 400 N-m, S_{ut} = 500 N/mm², S_{yt} = 300 N/mm², R = 90%, (fs) = 2

Step I: Endurance limit stress for shaft

$$S'_e = 0.5\ S_{ut} = 0.5\ (500) = 250\ \text{N/mm}^2$$

Cold drawn steel and S_{ut} = 500 N/mm²,

$$K_a = 0.79$$

Assuming 7.5 < d < 50 mm $\quad K_b = 0.85$

For 90% reliability, $\quad K_c = 0.897$

$$S_e = K_a K_b K_c S'_e = 0.79(0.85)(0.897)(250) = 150.58\ \text{N/mm}^2$$

Step II: Construction of modified Goodman diagram

Using the distortion-energy theory, $S_{se} = 0.577\ S_e = 0.577\ (150.58) = 86.88\ \text{N/mm}^2$

(a) $\quad S_{sy} = 0.577\ S_{yt} = 0.577(300) = 173.1\ \text{N/mm}^2$

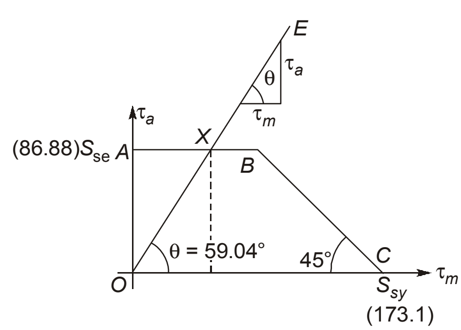
(b)
$$(M_t)_m = \frac{1}{2}[(M_t)_{max} + (M_t)_{min}] = \frac{1}{2}[400 - 100] = 150\ \text{N-m}$$

$$(M_t)_a = \frac{1}{2}[(M_t)_{max} + (M_t)_{min}] = \frac{1}{2}[400 + 100] = 250\ \text{N-m}$$

$$\tan\theta = \frac{(M_t)_a}{(M_t)_m} = \frac{250}{150} = 1.67$$

$$\theta = 59.04°$$

The modified Goodman diagram for this example is shown in figure.



Step III: Permissible shear stress amplitude

Refer to figure. The ordinate of point X is S_{se} or 86.88 N/mm².

$\therefore \quad S_{sa} = 86.88\ \text{N/mm}^2.$

Step IV: Diameter of shaft

Since
$$\tau_a = \frac{S_{sa}}{(fs)};\quad \therefore \frac{16(M_t)_a}{\pi d^3} = \frac{S_{sa}}{(fs)}$$

$$\frac{16(250 \times 10^3)}{\pi d^3} = \frac{86.88}{2}$$

$$d = 30.83\ \text{mm}$$

Solution : 24

Given: $m = 50$ kg, $h = 25$ mm, $l = 250$ mm, $S_{yt} = 400$ N/mm², $(fs) = 2$, $E = 207000$ N/mm²

Step I: Impact stress (σ_i)

$$\sigma_i = \sigma_s\left[1+\sqrt{1+\frac{2h}{\delta_s}}\right] \quad \text{...(i)}$$

Step II: Static stress (σ_s)

For simply supported beam, $W = m\,g = 50(9.81) = 490.5$ N

$$M_b = \frac{Wl}{4} = \frac{490.5(250)}{4} = 30656.25 \text{ N-mm}$$

$$I = \frac{bd^3}{12} = \frac{a(a)^3}{12} = \frac{a^4}{12}\text{mm}^4, \qquad y = \frac{a}{2}$$

where a is the side of square cross-section. Therefore,

$$\sigma_s = \sigma_b = \frac{M_b y}{I} = \frac{(30656.25)\left(\frac{a}{2}\right)}{\left(\frac{a^4}{12}\right)} = \frac{183973.5}{a^3} \text{ N/mm}^3$$

Step III: Static deflection $\delta_s = \frac{Wl^3}{48\,EI} = \frac{(490.5)(250)^3(12)}{48(207000)a^4} = \frac{9256.11}{a^4}$ mm

Step IV: Cross-section of beam

Equating impact stress to permissible stress,

$$\sigma_i = \frac{S_{yt}}{(fs)} = \frac{400}{2} = 200 \text{ N/mm}^2$$

Substituting the values of σ_s, σ_i and δ_s in equation (i)

$$200 = \frac{183973.5}{a^3}\left[1+\sqrt{1+\frac{2(25)a^4}{9256.11}}\right]$$

or

$$\frac{a^3}{919.87} = \left[1+\sqrt{1+\frac{2(25)a^4}{9256.11}}\right]$$

$$\left(\frac{a^3}{919.87}-1\right)^2 = 1+\frac{a^4}{185.12}$$

On solving $a = 67.808$ (or) 70 mm

The cross-section of the beam is 70 × 70 mm

Check for Impact Stress: $\sigma_s = \frac{183973.5}{3} = 0.5373$ MPa

$$\delta_s = \frac{9256.11}{a^4} = 3.855\times10^{-4}\text{mm}$$

from equation (i): $\sigma_i = 193.68 \text{ N/mm}^2$

$\sigma_i < 200 \text{ MPa}$

Hence, design is safe.

Solution : 25

Given: Bending moment (m) = 40 N-m, Twisting moment (T) = 50 N-m, $\frac{D_0}{D_i} = 2$, $\sigma_{yt} = 280$ MPa, FOS = 2, $k_b = 1.5$, $k_t = 1.0$

Using ASME Code, permissible shear stress,

$$\tau_{max} = 0.30\,\sigma_{yt} = 0.30 \times 280 = 84 \text{ Mpa}$$

$$(Z_{permissible}) = \frac{Z_{max}}{FOS} = \frac{84}{2} = 42 \text{ MPa}$$

Also, Equivalent Torque, Te $= \sqrt{(k_b m)^2 + (k_t T)^2} = \sqrt{(1.5 \times 40)^2 + (1 \times 50)^2}$

$$\text{Te} = 78.102 \text{ N-m}$$

On the basis of maximum shear stress theory,

$$\tau = \frac{T}{Z_P}$$

$$Z_P = \text{Polar section modulus}$$

$$= \frac{\pi}{16} D_0^3 \left[1 - \left(\frac{D_i}{D_0}\right)^4\right] = \frac{\pi}{16} D_0^3 \times \left[1 - (0.5)^4\right] = 0.1839 D_0^3$$

$\because$ $$\tau = \frac{T_e}{Z_P}$$

$$42 = \frac{78.102 \times 10^3}{Z_P} = Z_P = 1859.57 \text{ mm}^3$$

$$Z_P = 0.1839\, D_0^3$$

$$D_0 = 22 \text{ mm}$$

and $$D_i = \frac{D_0}{2} = 11 \text{ mm}$$

Solution : 26

Stress concentration is the localized stress considerably higher than average, even in uniformly loaded cross-section of uniform thickness due to abrupt change in the geometry or localised loading. Stress concentration factor is not considered harmful for ductile materials in static loading because of the phenomenon of local yielding in ductile materials when relieves the stress concentration. When the stress in the vicinity of the discontinuity reaches the yield point there is a plastic deformation, resulting in re-distribution of stresses. This plastic deformation prevents the harmful effects of stress concentration in ductile materials.

While in Brittle materials, stress concentration factor is important in both static and dynamic loading. Brittle materials fail due to fracture. So there is little deformation to relax the concentrated stresses and thus has damaging effects.

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3 Theory of Failure & Spring

LEVEL 1 Objective Questions

1.	(d)
2.	(c)
3.	(d)
4.	(b)
5.	(d)
6.	(b)
7.	(c)
8.	(b)
9.	(c)
10.	(b)

LEVEL 2 Objective Questions

11.	(c)
12.	(a)
13.	(b)
14.	(d)
15.	(b)
16.	(c)
17.	(2118)
18.	(c)
19.	(c)
20.	(a)
21.	(a)

LEVEL 3 Conventional Questions

Solution : 22

Given data: $n_A = n_B$, $l_A = l_B$, $G_A = G_B$, $D_A = 90$ mm, $D_B = 60$ mm, $W = W_A + W_B = 210$ N, $d_A = 12$ mm, $d_B = 7$ mm

$\because$ Compression in both the springs are same

$$\therefore \quad \delta_A = \delta_B$$

$$\left(\frac{8\,WD^3 n}{Gd^4}\right)_A = \left(\frac{8\,WD^3 n}{Gd^4}\right)_B$$

$$\therefore \quad \frac{W_A \cdot D_A^3}{d_A^4} = \frac{W_B D_B^3}{d_B^4}$$

$$\frac{W_A}{W_B} = \frac{d_A^4}{d_B^4} \times \frac{D_B^3}{D_A^3} = \left(\frac{12}{7}\right)^4 \times \left(\frac{60}{90}\right)^3 = 2.56$$

$$W_A + W_B = 210$$

$$(2.56 + 1)W_B = 210$$

$$W_B = \frac{210}{3.56} = 59 \text{ N}; \quad W_A = 151 \text{ N}$$

Shear stress in spring A

$$\tau_A = \frac{8\,W_A D_A}{\pi d_A^3} = \frac{8 \times 151 \times 90}{\pi \times 12^3} = 20 \text{ N/mm}^2$$

Shear stress in spring B

$$\tau_B = \frac{8\,W_B D_B}{\pi d_B^3} = \frac{8 \times 59 \times 60}{\pi \times 7^3} = 26.3 \text{ N/mm}^2$$

Solution : 23

(i) If axial force P is applied on the spring work done by axial force P is converted into strain energy and stored in the spring.

strain energy U = work done by P

$$= (\text{Average torque}) \times (\text{angular displacement}) = \frac{M_t}{2} \times \theta$$

But $\quad \theta = \dfrac{M_t \times l}{J\,G}$ (from torsion equation)

Where $\quad M_t$ = torque acting on spring wire = $\left(\dfrac{PD}{2}\right)$

l = length of the spring wire = $(\pi\, D\, n)$

J = polar moment of inertia of the wire = $\left(\dfrac{\pi d^4}{32}\right)$

G = Shearing modulus

Substituting these values in equation we get

Strain energy $\quad U = \dfrac{4P^2 D^3 N}{Gd^4}$

Now according to Castigliano's theorem, the displacement corresponding to force P is obtained by partially differentiating strain energy w.r.t. that force.

$$\delta = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\left(\frac{4P^2D^3N}{Gd^4}\right) = \frac{8PD^3N}{Gd^4}$$

Where δ = axial deflection of the spring (mm)

The stiffness of the spring (k) is defined as the force required to produce unit deflection. Therefore,

$$K = \frac{P}{\delta} = \frac{Gd^4}{8D^3n} = \frac{Gd^4}{8D^3n}$$

(ii) $$\frac{D}{d} = 6 \qquad \text{(Given)}$$

Also deflection δ = 3 cm

Axial load P = 500 N

Shearing modulus G = 80 GPa = 80×10^9 N/m^2

Now, $$\tau = \frac{8PD}{\pi d^3} = \frac{8P}{\pi d^2}\left(\frac{D}{d}\right)$$

$\Rightarrow$ $$300 \times 10^6 = \frac{8 \times 500}{\pi d^2} \times 6$$

$$d^2 = \frac{8 \times 500 \times 6}{\pi \times 300 \times 10^6}$$

$$d = 5.05 \text{ mm}$$

Also, $$\delta = \frac{8PD^3}{Gd^4}n$$

$$3 \times 10^{-2} = \frac{8 \times 500 \times \left(\frac{D}{d}\right)^3 \times n}{80 \times 10^9 \times d}, \text{ where } \frac{D}{d} = 6, \text{ spring index}$$

$$n = \frac{3 \times 10^{-2} \times 80 \times 10^9 \times d}{8 \times 500 \times \left(\frac{D}{d}\right)^3} = \frac{240 \times 10^7 \times 5.05 \times 10^{-3}}{8 \times 500 \times (6)^3} \simeq 14$$

Hence number of active coils = 14

As $$\frac{D}{d} = 6$$

$$D = 6 \times d = 30.3 \text{ mm}$$

$\therefore$ Length of spring wire = $\pi n D = \pi \times 14 \times 30.3$

$= 1.33$ m

Solution : 24

Given data: $N_A = 9$; $N_B = 8$; $\delta_A = 8$ mm; $\delta_B = 3$ mm; $D_A = 80$ mm; $D_B = ?$

$$\delta_A = \frac{8PD_A^3N_A}{Gd^4} \quad \text{and} \quad \delta_B = \frac{8PD_B^3N_B}{Gd^4}$$

$$\frac{\delta_B}{\delta_A} = \left(\frac{D_B}{D_A}\right)^3 \frac{N_B}{N_A}$$

or
$$\frac{3}{8}\times\frac{9}{8} = \left(\frac{D_B}{D_A}\right)^3$$

or
$$\left(\frac{D_B}{D_A}\right)^3 = \frac{3^3}{4^3}$$

or
$$\frac{D_B}{D_A} = \frac{3}{4}$$

∴
$$D_B = 0.75 \times 80 = 60 \text{ mm}$$

Solution : 25

Given: $P = 2000$ N, $\delta = 5$ mm, $C = 5$, $A = 1753$, $m = 0.182$, $G = 81370$ N/mm^2, $\tau = 0.5\,S_{ut}$

Permissible shear stress

$$S_{ut} = \frac{A}{d^m} = \frac{1753}{d^{0.182}}$$

$$\tau = 0.5 \times S_{ut} = \frac{0.5(1753)}{d^{0.182}} = \frac{876.5}{d^{0.182}} \text{ N/mm}^2$$

Wire diameter

$$K = \frac{4C-1}{4C-4}+\frac{0.615}{C} = \frac{4(5)-1}{4(5)-4}+\frac{0.615}{5} = 1.3105$$

$$\tau = K\left(\frac{8PC}{\pi d^2}\right) \text{or } \frac{876.5}{d^{0.182}} = (1.3105)\left\{\frac{8(2000)(5)}{\pi d^2}\right\}$$

$$\frac{d^2}{d^{0.182}} = \frac{1.3105(8)(2000)(5)}{\pi(876.5)} = 38.07$$

$$d^{1.818} = 38.07 \text{ or } d = (38.07)^{(1/1.818)}$$

∴
$$d = 7.40 \text{ or } 8 \text{ mm} \quad \text{(i)}$$

Mean coil diameter

$$D = C\,d = 5(8) = 40 \text{ mm} \quad \text{(ii)}$$

Number of active coils

From equation
$$\delta = \frac{8PD^3N}{Gd^4} \text{ or } 5 = \frac{8(2000-1000)(40)^3N}{(81370)(8)^4}$$

∴
$$N = 3.25 \text{ or } 4 \text{ coils} \quad \text{(iii)}$$

Total number of coils

For square and ground ends, the number of inactive coils is 2. Therefore,

$$N_t = N + 2 = 4 + 2 = 6 \quad \text{(iv)}$$

Solid length of spring

$$\text{solid length of spring} = N_t d = 6(8) = 8 \text{ mm} \qquad \text{(v)}$$

Free length of spring

The actual deflection of the spring under the maximum force of 2000 N is given by,

$$\delta = \frac{8PD^3N}{Gd^4} = \frac{8(2000)(40)^3(4)}{(81370)(8)^4} = 12.29 \text{ mm}$$

It is assumed that there will be a gap of 0.5 mm between the consecutive coils when the spring is subjected to the maximum force of 2000 N. The total number of coils is 6. Therefore, total axial gap will be (6 – 1) × 0.5 = 2.5 mm

$$\text{Free length} = \text{solid length} + \text{total axial gap} + \delta$$

$$= 48 + 2.5 + 12.29 = 62.79 \text{ or } 63 \text{ mm} \qquad \text{...(vi)}$$

Required spring rate

$$k = \frac{P_1 - P_2}{\delta} = \frac{2000 - 1000}{5} = 200 \text{ N/mm} \qquad \text{...(vii)}$$

Actual spring rate

$$k = \frac{Gd^4}{8D^3N} = \frac{(81370)(8)^4}{8(40)^3(4)} = 162.74 \text{ N/mm} \qquad \text{...(viii)}$$

Solution : 26

Initial compression of spring, δ_1 = 30 mm

Further compression of spring, δ = 50 mm

Total compression of spring, δ_2 = 30 + 50 = 80 mm

Energy absorbed by the spring, E = 250 J = 250 N-m = 250×10^3 N-mm

Spring index, C = 6

S_{ut} = 1500 N/mm^2

G = 81370 N/mm^2

Permissible shear stress, τ_{per} = 0.3 S_{ut}

S_{ut} = 0.3 × 1500 = 450 N/mm^2

To find:

(i) Wire diameter (d)

(ii) Number of active turns (n)

(iii) Free length (L_f)

(iv) Pitch of the turns (p)

Step 1: Calculate the wire diameter of spring

Let, W_1 be the load required for initial compression of spring δ_1 and W_2 be the total load required for total compression of spring δ_2 we know that,

$$K = \frac{W_1}{\delta_1} \qquad \therefore \qquad W_1 = K\delta_1 = 30K, N$$

Similarly,

$$K = \frac{W_2}{\delta_2} \qquad \therefore \qquad W_2 = K\delta_2 = 80K, N \qquad \text{...(ii)}$$

Now, average load during compression $= \dfrac{30K + 80K}{2} = 55K, N$

Energy absorbed during shock is, E = Average force × Further compression of spring

$\therefore \quad 250 \times 10^3 = 55\,K \times 50$

$\therefore \quad K = 90.909$ N/mm

$\therefore \quad W_2 = 80\,K = 80 \times 90.909 = 7272.72$ N

Wahl's factor is, $K_W = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C} = \dfrac{4\times 6-1}{4\times 6-4} + \dfrac{0.615}{6} = 1.2525$

Maximum shear stress is, $\tau_{per} = \dfrac{8W_2C}{\pi d^2} \times K_W$

$\therefore \quad 450 = \dfrac{8\times 7272.72\times 6}{\pi d^2} \times 1.2525$

$\therefore \quad d = 17.5863 \text{ mm} \simeq 18 \text{ mm}$

$\therefore \quad D = Cd = 6 \times 18 = 108$ mm

Step 2: Calculate the number of active turns

Spring rate is, $K = \dfrac{Gd}{8C^3 n}$

$\therefore \quad 90.909 = \dfrac{81370 \times 18}{8 \times 6^3 \times n}$

$\therefore \quad n = 9.3236 \simeq 10$ turns

Step 3: Calculate the free length of the spring

The maximum deflection is, $\delta_{max} = \dfrac{8W_2C^3 n}{Gd} = \dfrac{8\times 7272.72 \times 6^3 \times 10}{81370 \times 18}$

$\therefore \quad \delta_{max} = 85.8032$ mm

Assuming squared and ground ends,

$\therefore \quad n' = n + 2 = 10 + 2 = 12$

Free length is, $L_F = n'd + \delta_{max} + 0.15\,\delta_{max}$

$\therefore \quad L_F = 12 \times 18 + 85.8032 + 0.15 \times 85.8032$

$\therefore \quad L_F = 314.6737 \text{ mm} \simeq 315 \text{ mm}$

Step 4: Calculate the pitch of the turns

Pitch of the turns is, $p = \dfrac{\text{Free length}}{n'-1} = \dfrac{314.6737}{12-1}$

$\therefore \quad p = 28.6067$ mm

Solution: 27

Let M = Bending moment and T = Twisting moment

(i) Equivalent bending moment

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}\left(3T + \sqrt{9T^2 + T^2}\right)$$

$$= \frac{T}{2}\left(3 + \sqrt{10}\right) = 3.08114\,T$$

Safe stress in bending, $\sigma = \dfrac{370}{4} = 92.5 \text{ N/mm}^2$

$$M_e = \sigma \frac{\pi d^3}{32}$$

$$\therefore \quad 3.08114\,T = 92.5\pi \times \frac{100^3}{32} \times \frac{1}{10^6}\,\text{kNm}$$

$$T = 2.947\ \text{kNm}$$

(ii) Equivalent torque,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{9T^2 + T^2} = T\sqrt{10}$$

Safe stress,

$$\tau_s = \frac{\sigma}{2} = \frac{92.5}{2} = 46.25\ \text{N/mm}^2$$

$$T_e = \tau_s \cdot \frac{\pi d^3}{16}$$

$$T\sqrt{10} = 46.25\pi \times \frac{100^3}{16} \times \frac{1}{10^6}\,\text{kNm}$$

$$T = 2.872\ \text{kNm}$$

Solution : 28

Given data: $d = 50$ mm; $M = 1.5$ kNm; $\sigma_y = 210$ MPa

(i) According to maximum principal stress theory

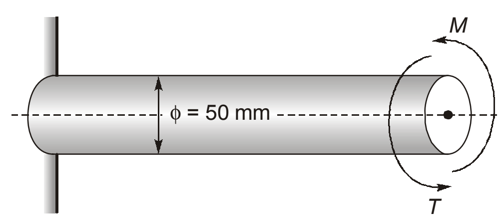
$$\sigma_1 < \sigma_{per}$$

In this case combine bending and twisting moment

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}\left[1.5 + \sqrt{(1.5)^2 + T^2}\right]$$

Bending stress,

$$\sigma_b = \frac{32 M_e}{\pi d^3}$$



$$\therefore \quad 210 \times \frac{\pi}{32}(50)^3 = \frac{1}{2}\left[1.5 + \sqrt{(1.5)^2 + T^2}\right] \times 10^6$$

$$T = 3.3321\ \text{kNm}$$

(ii) According to maximum shear stress theory

$$\tau_{max} = \frac{\sigma_y}{2} = \frac{210}{2} = 105\ \text{MPa}$$

In this case combine twisting and bending moment is

$$T_e = \sqrt{T^2 + M^2} = \sqrt{T^2 + (1.5)^2} \times 10^6$$

Shear stress,

$$\tau = \frac{16 T_e}{\pi d^3}$$

$$105 \times \frac{\pi}{16}(50)^3 = \sqrt{T^2 + (1.5)^2} \times 10^6$$

$$T = 2.0955\ \text{kN.m}$$

Solution : 29

Given: $\sigma_1 = 60\ \text{N/mm}^2$ and $\sigma_2 = -36\ \text{N/mm}^2$

$$\sigma_{yt} = 100\ \text{N/mm}^2$$

(i) Factor of safety according to maximum principal stress criteria

$$\sigma_1 \le \frac{\sigma_{yt}}{\text{fos}}$$

$$60 \le \frac{100}{\text{fos}}$$

$$\text{fos} = 1.67$$

(ii) Factor of safety according to maximum shear stress criteria

$$\tau_{max} \le \frac{\sigma_{yt}}{2\,\text{fos}}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{60+36}{2} = 48\ \text{N/mm}^2$$

$$48 \le \frac{100}{2\,\text{fos}}$$

$$\text{fos} = 1.04$$

(iii) Factor of safety according to maximum strain energy criteria

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \le \left(\frac{\sigma_{yt}}{\text{fos}}\right)^2$$

$$(60)^2 + (-36)^2 + 2 \times 0.3 \times 60\ (36) \le \left(\frac{100}{\text{fos}}\right)^2$$

$$3600 + 1296 + 1296 \le \left(\frac{100}{\text{fos}}\right)^2$$

$$\text{fos} = \frac{100}{79.69} = 1.27$$

(iv) Factor of safety according to maximum distortion energy criteria or maximum shear strain energy criteria

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \le \left(\frac{\sigma_{yt}}{\text{fos}}\right)^2$$

$$(60)^2 + (-36)^2 + 60 \times 36 \le \left(\frac{100}{\text{fos}}\right)^2$$

$$3600 + 1296 + 2160 \le \left(\frac{100}{\text{fos}}\right)^2$$

$$84 \le \frac{100}{\text{fos}}$$

$$\text{fos} = \frac{100}{84} = 1.19$$

■■■■

4 Welded joint, Riveted joint and Bolted Joints

LEVEL 1 Objective Questions

1.	(c)
2.	(b)
3.	(c)
4.	(d)
5.	(a)
6.	(d)
7.	(b)
8.	(b)
9.	(a)
10.	(a)
11.	(a)
12.	(c)
13.	(d)
14.	(c)
15.	(b)
16.	(d)

LEVEL 2 Objective Questions

17.	(d)
18.	(b)
19.	(b)
20.	(d)
21.	(d)
22.	(a)
23.	(a)
24.	(b)
25.	(42.43)
26.	(b)
27	(c)
28.	(c)
29.	(3.22)
30.	(0.75)

■■■■

LEVEL 3 Conventional Questions

Solution : 31

Diameter of rivet hole, $d = 6\sqrt{t} = 6\sqrt{12.5} = 21.2$ mm

As per table, for the rivet hole of 21.5 mm size, diameter of rivet is 20 mm. This size is standard size.

$\therefore$ Diameter of rivet, $d = 20$ mm

Let, n = number of rivets

P_t = Maximum pull on the joint i.e. tearing resistance of plate at the outer row which has only one rivet.

$$P_t = (b-d)\,t \times \sigma_t = (200-21.5) \times 12.5 \times 80 = 178{,}500 \text{ N}$$

$$\text{Shearing resistance of one rivet, } P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau = 1.75 \times \frac{3.14}{4} \times (21.5)^2 \times 65$$

$$\therefore \quad P_s = 41{,}300 \text{ N}$$

$$\text{Crushing resistance of one rivet, } P_c = 21.5 \times 12.5 \times 160 = 43000 \text{ N}$$

$$\text{Number of rivets, } n = \frac{P_t}{\text{Least of } P_s \text{ or } P_c} = \frac{178{,}500}{41{,}300} = 4.32$$

$$\therefore \quad n = 5$$

Solution : 32

Given diameter, d = 5 cm = 50 mm

size of the fillet, s = 1 cm = 10 mm

$$\text{permissible shear stress}, \tau_s = 8 \text{ kN/cm}^2 = \frac{8000}{100} \text{N/mm}^2 = 80 \text{ N/mm}^2$$

we know that maximum shear stress, (τ_{max})

$$\tau_{max} = \frac{2.83T}{\pi s d^2} \Rightarrow \frac{2.83T}{\pi \times 10 \times 50^2} = 80 \text{ N/mm}^2$$

$$\Rightarrow \quad T = \frac{80 \times \pi \times 10 \times 50^2}{2.83} = 2.22 \times 10^6 \text{ N-mm} = 2.22 \text{ kN-m}$$

T = torque applied on the shaft

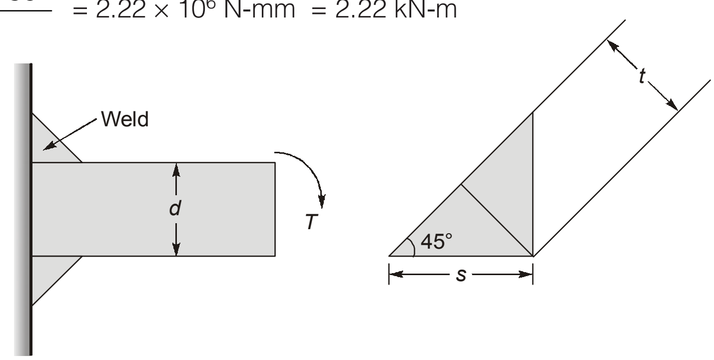
d = diameter of the shaft

S = leg or (side) of the weld

t = throat thickness

J = polar moment of inertia

$$= \frac{\pi t d^3}{4}$$



$$\text{We know that, } \frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{J} \times r = \frac{T}{\frac{\pi t d^3}{4}} \times \frac{d}{2} = \frac{2T}{\pi t d^2}$$

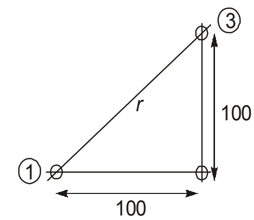
But maximum shear stress occurs at the throat of the weld which is inclined at 45° to the horizontal plane

$$\therefore \quad \tau_{max} = \frac{2T}{\pi s(\sin 45°) d^2} = \frac{2.83T}{\pi s d^2} \qquad (\because\ t = s \sin 45°)$$

Solution : 33

Given: Eccentric load = 12.5 kN, Z_{max} = 40 MPa = 40 N/mm²

Primary shear force on each rivet



$$P' = \frac{p}{5} = \frac{12.5}{5} = 2500 \text{ N}$$

$$r = \sqrt{100^2 + 100^2} \Rightarrow 141.42 \text{ mm}$$

$$e = 100 + 50 + 400 = 550 \text{ mm}$$

Secondary shear force on rivet (3)

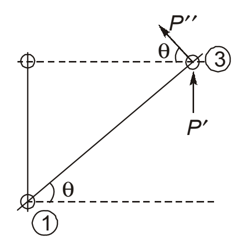
$$P'' = \frac{Per_1}{r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2} \qquad [\because r_s = 0]$$

$$P'' = \frac{(12.5 \times 10^3) \times 550 \times 141.42}{4 \times (141.42)^2}$$

$$P'' = 12153.28 \text{ N}$$

$$\tan\theta = \frac{100}{100} = 1$$

$$\theta = (45°)$$



So, resultant force on rivet (3),

$$P_{resultant} = \sqrt{(P' + P''\sin\theta)^2 + (P''\cos\theta)^2}$$

$$= \sqrt{(2500 + 12153.28\sin 45°)^2 + (12153.28\cos 45°)^2}$$

$$= \sqrt{(123069440.9) + (73851122.57)}$$

$$P_{resultant} = 14032.83 \text{ N}$$

Let, A = Cross sectional area of rivet

$$\tau_{max} = \frac{P_{resultant}}{A}$$

$$A = \frac{14032.83}{40} \text{mm}^2 = 350.82 \text{ mm}^2$$

As, $$A = \frac{\pi}{4} d^2 = 350.82$$

$$d = 21.140 \text{ mm}$$

or from preferred series (given) d = diameter of rivet = 22 mm

Solution : 34

Given, t = 20 mm, d = 25 mm, p = 100 mm, σ_t = 120 MPa, σ_c = 150 MPa, τ = 100 MPa

Tearing resistance of the plate

$$F_t = (p - d)\, t\sigma_t = (100 - 25) \times 20 \times 120 = 180 \text{ kN}$$

Shearing resistance of the rivets

$$F_s = n \times 2 \times \frac{\pi}{4} d^2 \times t = 2 \times 2 \times \frac{\pi}{4} (25)^2 \times 100 = 196.375 \text{ kN}$$

Crushing resistance of the rivets

$$F_c = n \times d \times t \times \sigma_t = 2 \times 25 \times 20 \times 150 = 150 \text{ kN}$$

$\therefore$ Strength of the joint $= \min\{f_t, f_s, f_c\} = 150$ kN

Strength of the unriveted or solid plate, $F = p \times t \times \sigma_t = 100 \times 20 \times 120 = 240$ kN

$\therefore$ Efficiency of the joint $= \dfrac{\text{least of } F_t, F_s \text{ and } F_c}{F} = \dfrac{150000}{240000} = 0.625$ or 62.5 %

Solution : 35

(i)

$$\sigma_{b(max)} = \frac{5.66M}{\pi s d^2}$$

$$M = PL = 5 \times 10^3 \times 200 = 1 \times 10^6 \text{ Nmm}$$

$$d = 50 \text{ mm}$$

$$\sigma_{b(max)} = 100 \text{ N/mm}^2$$

$$s = \frac{5.66 \times 1 \times 10^6}{3.14 \times 100 \times 50 \times 50} = 7.21 \text{ mm or } 8 \text{ mm}$$

(ii)

$$\tau_{max} = \frac{2.83T}{\pi s d^2}$$

Given,

$$T = 3 \text{ kNm} = 3 \times 10^6 \text{ Nmm}$$

$$d = 65 \text{ mm}$$

$$\tau_{max} = 70 \text{ MPa} = 70 \text{ N/mm}^2$$

$$s = \text{size of weld}$$

$$s = \frac{2.83T}{\pi\, \tau_{max} d^2}$$

$$s = \frac{2.83 \times 3 \times 10^6}{3.14 \times 70 \times 65 \times 65} = 9.14 \text{ mm or } 10 \text{ mm}$$

Solution : 36

Given:

$$t = 15 \text{ mm}$$

$$d = 25 \text{ mm};$$

$$p = 75 \text{ mm}$$

$$\sigma_{tu} = 400 \text{ MPa} = 400 \text{ N/mm}^2; \ \tau_u = 320 \text{ MPa}$$

$$= 320 \text{ N/mm}^2$$

$$\sigma_{cu} = 640 \text{ MPa} = 640 \text{ N/mm}^2$$

Minimum force per pitch which will rupture the joint

Since the ultimate stresses are given, therefore we shall find the ultimate values of the resistances of the joint. We know that ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)\, t \times \sigma_{tu} = (75 - 25)\, 15 \times 400 = 300000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u$$

$$= 2 \times \frac{\pi}{4}(25)^2\, 320 = 314200 \text{ N} \qquad (\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300000 N or 300 kN

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300000/4 = 75000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75000 = (p-d)t \times \sigma_{ta} = (75-25)\,15 \times \sigma_{ta} = 750\,\sigma_{ta}$$

$$\therefore \quad \sigma_{ta} = \frac{75000}{750} = 100 \text{ N/mm}^2 = 100 \text{ MPa}$$

Actual shearing resistance of the rivets (P_{sa}),

$$7500 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4}(25)^2\,\tau_a = 982\,\tau_a$$

$$\therefore \quad \tau_a = \frac{75000}{982} = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\,\sigma_{ca}$$

$$\therefore \quad \sigma_{ca} = \frac{75000}{750} = 100 \text{ N/mm}^2 = 100 \text{ MPa}$$

Solution : 37

$$\text{Width} = 100 \text{ mm}$$
$$\text{Thickness} = 12.5 \text{ mm}$$
$$\text{Load, } P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$
$$\tau_{max} = 56 \text{ MPa} = 56 \text{ N/mm}^2$$

Length of weld for static loading

Let l = length of weld

s = weld size = plate thickness = 12.5 mm

Maximum load carried by plate is dependent upon type of parallel fillet weld whether it is single or double, so the maximum load will be carried by double parallel fillet welds.

$$60 \times 10^3 = 1.4145 \times l \times \tau$$
$$60 \times 10^3 = 1.414 \times 12.5 \times l \times 56$$

or l = 60.62 mm

Adding 12.5 mm for starting and stopping of weld run,

$$\text{Design length of weld} = 60.62 + 12.5 = 73.12 \text{ mm}$$

Length of weld for fatigue loading

$$\text{Permissible stress, } \tau = \frac{\tau_{static}}{\text{Stress concentration factor}} = \frac{56}{2.7} = 20.74 \text{ N/mm}^2$$

$$\text{Maximum load} = 2 \times 0.7075 \times l \times \tau$$

$$l = \frac{60 \times 10^3}{1.414 \times 12.5 \times 20.74} = 163.675 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run,

$$\text{Design length of weld} = 163.675 + 12.5 = 176.175 \text{ mm}$$

(Selecting the higher length for the design purpose)

Solution : 38

Steam pressure on cylinder head,

$$P = 0.7 \text{ N/mm}^2$$

Number of bolts, $n = 12$

Effective cylinder diameter, $D = 300$ mm

Tensile stress, $\sigma_t = 100 \text{ MPa} = 100 \text{ N/mm}^2$

$$\text{Total external load} = \frac{\pi}{4}D^2 \times P = \frac{3.14}{4} \times (300)^2 \times 0.7 = 49490 \text{ N}$$

External load on the cylinder head for one bolt,

$$P_2 = \frac{49490}{12} = 4124 \text{ N}$$

Let d = nominal diameter of the bolt, and

d_c = core diameter of the bolt ($d_c = 0.84\,d$)

Initial tension due to tightening of bolt,

$$P_1 = 2840 \, d \text{ N}$$

$$\text{Resultant axial load, } P = P_1 + kP_2$$

$$= P_1 + \frac{a}{1+a}P_2$$

$$a = 1, \; k = \frac{1}{2} = 0.5$$

$$P = \frac{\pi}{4}d_c^2$$

$$\sigma_t = \frac{\pi}{4} \times (0.84\,d)^2 \times 100 = 55.4 \, d^2$$

Now, $P = 2840\,d - 2062 = 55.4 \, d^2$

or $55.4 \, d^2 - 2840\,d - 2062 = 0$

$$d^2 - 51.3\,d - 37.2 = 0$$

$$\therefore \quad d = \frac{51.3 \pm \sqrt{(51.3)^2 + 4 \times 37.2}}{2} = \frac{51.3 \pm 52.7}{2} = 52 \text{ mm}$$

Size of bolt = M52

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5 Clutches and Brakes

LEVEL 1 Objective Questions

1.	(b)
2	(a)
3.	(c)
4.	(b)
5.	(d)
6.	(a)
7.	(a)
8.	(c)
9.	(d)
10.	(a)
11.	(0.127)
12.	(c)

LEVEL 2 Objective Questions

13.	(d)
14.	(d)
15.	(1738.13)
16.	(c)
17.	(a)
18.	(39.5)
19.	(1173.54)
20.	(a)
21.	(d)
22.	(d)
23.	(b)

■■■■

LEVEL 3 Conventional Questions

Solution : 24

Given data:

Mass of vehicle, $m = 110$ kg

Moment of inertia of each wheel,

$I_w = 0.22$ kg-m^2

Radius of each wheel, $R = 0.35$ m

Moment of inertia of engine,

$I_E = 0.7$ kg-m^2

Engine speed, $\omega_E = 3 \times$ speed of wheel $= 3 \times \omega_1$

Initial velocity of vehicle, $V_1 = 60$ km/hr $= 16.667$ m/sec

Distance travelled by vehicle after braking,

$S = 12$ m

To find Energy absorbed

Calculate the energy absorbed by each brake, (E)

(i) Kinetic energy of vehicle is

$$E_v = \frac{1}{2}m\left(V_1^2 - V_2^2\right) = \frac{1}{2} \times 110 \times \left[(16.667)^2 - 0\right] \quad ...(\because V_2 = 0)$$

$\therefore$ $E_v = 15.2783 \times 10^3$ J

Initial angular velocity of wheel is,

$V_1 = R\omega_1 \quad \Rightarrow \quad \therefore 16.667 = 0.35 \times \omega_1$

$\therefore$ $\omega_1 = 47.62$ rad/sec

(ii) Kinetic energy of two wheels is,

$$E_w = 2 \times \frac{1}{2} I_w \left(\omega_1^2 - \omega_2^2\right) = 2 \times \frac{1}{2} \times 0.22 \times \left[(47.62)^2 - 0\right]$$

$...(\because \omega_2 = 0)$

$\therefore$ $E_w = 498.8861$ J

Initial angular velocity of engine is,

$\omega_{E1} = 3 \times \omega_1 = 3 \times 47.62 = 142.86$ rad/sec

(iii) Kinetic energy of engine is,

$$E_E = \frac{1}{2} I_E \left(\omega_{E1}^2 - \omega_{E2}^2\right) = \frac{1}{2} \times 0.7\left[(142.86)^2 - 0\right] \quad ...(\because \omega_{E2} = 0)$$

$\therefore$ $E_E = 7.1431 \times 10^3$ J

(iv) Now, total energy absorbed by brakes is,

$E_T = E_v + E_w + E_E = 15.2783 \times 10^3 + 498.8861 + 7.1431 \times 10^3$

$\therefore$ $E_T = 22.9202 \times 10^3$ J

Energy absorbed by each brake is,

$$E = \frac{\text{Total energy}}{\text{Number of brakes}} = \frac{E_T}{2} = \frac{22.9202 \times 10^3}{2}$$

$\therefore$ $E = 11.4601 \times 10^3$ J

Solution : 25

Maximum torque produced by the engine,

$$T_{max} = 200 \text{ N-m}$$

Rotational speed, $N = 2000$ r.p.m.

Clutch outer diameter, $d_1 = 230$ mm

$= r_1 = 115$ mm

Coefficient of friction, $\mu = 0.3$

Permissible pressure, $p_{max} = 0.25 \text{ N/mm}^2$

Using uniform wear theory:

We know that $w = 2\pi C(r_1 - r_2)$, where $p_{max} \times r_2 = C$

and $T = n\mu\omega R,$

where, $R = \dfrac{r_1 + r_2}{2}$

Taking maximum torque, i.e.

$$T_{max} = 200 \text{ N-m}$$

$$200 \times 10^3 \text{ N-m} = 2 \times 0.3 \times 2\pi \times 0.25 \times r_2(115 - r_2) \times \frac{(115 + r_2)}{2}$$

($\because$ For a single disc or plate clutch, both sides of the discs are effective, i.e., $n = 2$ and $r_2 \to$ inner radius)

$$424413.18 = (115^2 - r_2^2)r_2$$

By hit and trial method, we get

$$r_2 \simeq 93 \text{ mm}$$

Hence clutch inner radius $= 93 \text{ mm} = r_2$

Outer radius $= 115 \text{ mm (given)} = r_1$

Hence width of the clutch plate,

$$w = r_1 - r_2 = 22 \text{ mm}$$

$$\text{Speed of the car} = 60 \text{ km/hr} = V = \frac{60 \times 1000}{3600} \text{ m/s}$$

$\therefore$ Final angular speed $\omega_f = \dfrac{V}{r} \times GR$ ($r \to$ radius of the wheel, $= \dfrac{710}{2}$mm , $= 0.355$ m)

($GR \to$ Gear reduction $= 4.12$)

$$\omega_f = \frac{\dfrac{60 \times 1000}{3600}}{\left(0.\dfrac{710}{2}\right)} \times 4.12 = 193.42 \text{ rad/s}$$

Initial angular velocity (at the time of engagement),

$$\omega_i = \frac{2 \times \pi \times 2000}{60} = 209.43 \text{ rad/s}$$

Initial angular acceleration at the time of engagement,

$$\alpha_i = \frac{T_1 - T_2}{I} = \frac{100 - 200}{1.5} = -66.67 \text{ rad/s}^2$$

where, I - mass moment of inertia of engine side = 1.5 kg-m^2

T_1 - torque at the time of engagement

Torque available at the rear wheels, $T = 105 \text{ N-m} = 105 \times 10^3$ N-mm (Given)

Accelerating force on automobile $F = \dfrac{T}{\text{Radius of the wheel}}$

$$\frac{105\times10^3\text{ N-mm}}{355} = 295.77\text{ N}$$

Acceleration, $a_t = \dfrac{F}{\text{mass of car}} = \dfrac{295.77\text{N}}{1500\text{kg}} = 0.1971\text{ m/s}^2$

Final angular acceleration,

$$\alpha_f = \frac{0.1971}{0.355}\times GR = 2.288\text{ rad/s}^2$$

$GR \rightarrow$ Gear reduction $= 4.12$ (Given)

$$\Delta T = \frac{\omega_i - \omega_f}{\alpha_f - \alpha_i} = \frac{209.43-193.42}{2.288-(-66.67)} = 0.2321\text{ sec.}$$

Solution : 26

Given, Power, $P = 15$ kW

$$N = 720\text{ r.p.m.} = \omega = \frac{2\pi N}{60} = \frac{2\pi\times720}{60} = 75.39\text{ rad/s}$$

Number of shoes, $n = 4$

Inside radius of the drum, $R = \dfrac{325}{2} = 162.5\text{ mm} = 0.1625\text{ m}$

Radial distance of C.G. of each shoe from the axis shaft r = 120 mm

Coefficient of friction, $\mu = 0.25$

Since the speed at which engagement begins (ω_1) is 80% of full speed (ω) therefore

$$\omega_1 = 0.8\times\ \omega = 0.8\times75.39 = 60.312\text{ rad/s}$$

Power transmitted $P = T\omega$

$$15\times10^3 = T\times75.39$$

$$T = 198.96\text{ N-m}$$

(i) Mass of each shoe:

Let the mass of each shoe be m kg

Centrifugal force acting on each shoe, $P_c = m\omega^2 r$

$$= m\times(75.39)^2\times0.12 = (682.03\text{ m})\text{N}$$

The inward force on each shoe exerted by the spring i.e., centrifugal force at the engagement speed ω_1,

$$P_s = m\omega_1^2 r = m\times(60.312)^2\times0.12$$

$$= (436.5\,m)\text{N}$$

Frictional force acting tangentially on each shoe,

$$F = \mu(P_c - P_s) = 0.25\,(682.03\,m - 436.5\text{ m})$$

$$= (61.38\,m)\text{N}$$

Torque transmitted, $T = nFR$

$$198.96 = 4\times61.38\text{ m}\times0.1625 = m = 4.986\text{ kg}$$

(ii) Size of the shoe:

Let, l = Contact length of shoe in mm,

b = Width of the shoe in mm,

θ = Angle subtended by the shoes at the centre of the spider in radians,

$$= 60° = \frac{\pi}{3} \text{ radian}$$

P = Pressure exerted on the shoe in N/mm^2

= 0.1 MPa = 0.1 N/mm^2

We know that $$l = R\theta = 162.5 \times \frac{\pi}{3} = 170.16 \text{ mm}$$

and $$lbp = P_c - P_s = 682.03\text{ m} - 436.5\text{ m} = 170.16 \times b \times 0.1$$
$$= 245.53\text{ m}$$
$$= 245.53 \times 4.986$$

$\Rightarrow$ $$b = 71.94 \text{ mm} \simeq 72 \text{ mm}$$

$\therefore$ Contact length of the shoe, l = 170.16 mm

and width of the shoe, b = 72 mm

Spring design:

Centrifugal force on each spring, $$P_s = \frac{436.5 \times 4.986}{4} = 544.69 \text{ N}$$

Given, C = 6,

τ(shear stress) = 600 MPa = 600 × 10^6 N/m^2

We know that $$\tau = \frac{8P_sC}{\pi d^2}\left\{\frac{4C-1}{4C-4} + \frac{0.615}{C}\right\}$$

$$600 \times 10^6 = \frac{8 \times 544.09}{\pi d^2} \times 6\left\{\frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{4}\right\}$$

$$d = 4.25 \times 10^{-3}\text{ m} = 4.25 \text{ mm}$$

and $$D = md = 6 \times 4.25 = 25.5 \text{ mm}$$

And $$\delta = \frac{8P_sD^3n}{Gd^4}, \ \delta \rightarrow \text{radial clearance} = 0.01 \text{ m} = 10 \text{ mm}$$

$$n = \frac{\delta G d^4}{8P_sD^3} = \frac{0.01 \times (80 \times 10^3 \times 10^6) \times (0.00425)^4}{8 \times 544.09 \times (0.0255)^3}$$

$$n = 3.616 \simeq 4 \text{ (take)}$$

$\therefore$ Number of active turns of the coil, n = 4.

Solution : 27

Given: m = 2500 kg, v_1 = 1.5 m/s, v_2 = 0,

For drum, m = 50 kg, k = 0.7 m, h = 0.5 m, R = 0.75

KE of the mass:

$$\text{KE} = \frac{1}{2}m(v_1^2 - v_2^2) = \frac{1}{2}(2500)[(1.5)^2 - 0]$$
$$= 2812.5 \text{ J} \quad \text{(a)}$$

KE of the drum: $$\omega_1 = \frac{v_1}{R} = \frac{1.5}{0.75} = 2 \text{ rad/s and } \omega_2 = 0$$

$$KE = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2) = \frac{1}{2}(50)(0.7)^2[(2)^2 - 0] = 49 \text{ J} \quad \text{(b)}$$

PE of the mass $PE = mgh = (2500)(9.81)(0.5) = 12262.5 \text{ J}$ (c)

The total energy absorbed by the brake is given by,

$$E = 2812.5 + 49 + 12262.5 = 15124 \text{ J} \quad \text{(i)}$$

Step II: Torque capacity of brake

During the braking action, the mass moves through a distance of 0.5 m. If θ is the angle through which the drum rotates during the braking period.

$$\theta \times (\text{drum radius}) = 0.5;\ \theta \times (0.75) = 0.5$$

or
$$\theta = \frac{0.5}{0.75} = 0.667 \text{ rad};$$

$\therefore$
$$M_t = \frac{E}{\theta} = \frac{15124}{0.667} = 22686 \text{ N-m} \quad \text{(ii)}$$

Solution : 28

Given: $D = 1$ m, $t = 0.2$ m, $n_1 = 350$ rpm, $t = 1.5$ s

Step I: Energy absorbed by brake

The brake absorbs the kinetic energy of the rotating flywheel. The mass density of cast iron is taken as 7200 kg/m^3. The mass of flywheel is given by,

$$m = \frac{\pi}{4}D^2 t\rho = \frac{\pi}{4}(1)^2(0.2)(7200) = 1130.97 \text{ kg}$$

The radius of gyration (k) of solid disk about its axis of rotation is $(d/\sqrt{8})$.

$$k^2 = \frac{D^2}{8} = \frac{1}{8}\text{m}^2$$

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi(350)}{60} = 36.65 \text{ rad/s}$$

and
$$\omega_2 = 0$$

$$E = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2) = \frac{1}{2}(1130.97)\left(\frac{1}{8}\right)(36.65)^2$$

$$= 94946.52 \text{ J} \quad \text{...(i)}$$

Step II: Torque capacity of brake

The average velocity during the braking period is $(\omega_1 + \omega_2)/2$ or $(\omega_1/2)$. Therefore

$$\theta = \left(\frac{\omega_1}{2}\right)t = \left(\frac{36.65}{2}\right)(1.5) = 27.49 \text{ rad}$$

$$M_t = \frac{E}{\theta} = \frac{94946.52}{27.49} = 3453.86 \text{ N-m} \quad \text{...(ii)}$$

Solution : 29

Given: $M_t = 1500$ N-m, $R_o = 150$ mm, $R_i = 100$ mm, $p_a = 2$ MPa, $\mu = 0.35$, number of pads = 2

Step I: Actuating force

Since, there are two pads, the torque capacity of one pad is (1500/2) or 750 N-m

$$R_f = \frac{2}{3}\frac{(R_o^3 - R_i^3)}{(R_o^2 - R_i^2)} = \frac{2}{3}\frac{(150^3 - 100^3)}{3(150^2 - 100^2)} = 126.67 \text{ mm}$$

$$M_t = \mu P R_f$$

$$750(10^3) = 0.35\,P(126.67) \text{ or } P = 16916.85 \text{ N}$$

Step II: Angular dimension of pad

$$P = \text{average pressure} \times \text{area of pad}$$

$$16916.85 = 2\,A \text{ or } A = 8458.42 \text{ mm}^2$$

$$A = \frac{1}{2}\theta(R_o^2 - R_i^2)$$

or
$$8458.42 = \frac{1}{2}\theta(150^2 - 100^2)$$

$$\theta = 1.3533 \text{ radians or } \theta = 1.3533\left(\frac{180}{\pi}\right) = 77.54°$$

The angular dimension of of pad can be taken as 80°.

Solution : 30

$$\text{Power, } P = 15 \text{ kW}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 720}{60} = 75.39 \text{ rad/s}$$

$$\text{Number at shoes, } n = 4$$

$$\text{Inside radius of the drum, } R = \frac{325}{2} = 162.5 \text{ mm} = 0.1625 \text{ m}$$

$$\mu = 0.2$$

$$\text{Engagement speed, } \omega_1 = 80\% \text{ of full speed}$$

$$\omega_1 = 0.8 \times 75.39 = 60.312 \text{ rad/s}$$

$$\text{Torque, } T = \frac{P}{\omega} = \frac{15 \times 10^3}{75.39} = 198.96 \text{ Nm}$$

(i) Mass of each shoe

Let mass of each shoe be m kg.

Centrifugal force acting on each shoe, $P_c = \text{m}\omega^2\text{r}$

$$P_c = \text{m}(75.39)^2 \times 0.12$$

$$P_c = (682\,\text{m})N$$

$$\text{Tangential frictional force, } F = \mu(P_c - P_s)$$

$$\text{Torque, } T = nFR$$

$$T = n\mu(P_c - P_s)\text{R}$$

or
$$198.96 = 4 \times 0.2\,(682 - 436.5)\text{m} \times 0.1625$$

or
$$\text{m} = 6.234 \text{ kg}$$

(ii) Size of the shoe

$$\theta = \text{angle subtended by the shoe at the centre}$$

$$\theta = 60° \text{ or } \frac{\pi}{3} \text{ radian}$$

$$\text{Contact length of shoe, } l = R\theta$$

$$l = 162.5 \times \frac{\pi}{3} = 170.16 \text{ mm}$$

Pressure exerted on the shoe,
$$P = 0.15 \text{ MPa} = 0.15 \text{ N/mm}^2$$

$$l\,bp = P_c - P_s$$

$$l\,bp = (682 - 436.5) \times 6.234$$

or
$$(170.16)\,b \times 0.15 = 245.5 \times 6.234$$

or b = 59.96 mm or 60 mm

Contact length of the shoe, l = 170.16 mm

Contact width of the shoe, b = 60 mm

Solution : 31

Given: $P = 45$ kW $= 45 \times 10^3$ W, $N = 1000$ rpm, $\alpha = 12.5°$, $D = 500$ mm, or $R = 250$ mm, $\mu = 0.2$, $p_n = 0.4$ N/mm^2

(i) Face width

Let b = Face width of the clutch in mm

We know that torque developed by the clutch,

$$T = \frac{P \times 60}{2\pi N} = \frac{45 \times 10^3 \times 60}{2\pi \times 1000} = 430 \text{ Nm} = 430 \times 10^3 \text{ Nmm}$$

We also know that torque developed by the clutch (T),

$$430 \times 10^3 = 2\pi \times \mu \times p_n \times R^2 \times b = 2\pi \times 0.2 \times 0.1(250)^2 b = 7855\, b$$

$$\therefore \quad b = 430 \times 10^3/7855 = 54.7 \text{ say } 55 \text{ mm}$$

(ii) Axial spring force necessary to engage the clutch

We know that the normal force acting on the contact surfaces,

$$W_n = p_n \times 2\pi R\, b = 0.1 \times 2\pi \times 250 \times 55 = 8640 \text{ N}$$

$\therefore$ Axial spring force necessary to engage the clutch,

$$W_e = W_n(\sin\alpha + 0.25\mu\cos\alpha) \quad \text{[Experimentally found]}$$

$$= 8640(\sin 12.5° + 0.25 \times 0.2 \cos 12.5°) = 2292 \text{ N}$$

Solution : 32

Given: $n = 1440$ rpm; $\mu = 0.2$; $r_m = 2b$

$p_a = 0.1$ N/mm^2; $\alpha = 12.5°$

For machine, $m = 150$ kg; $k = 250$ mm; $t = 40$s

Step I: Inner and outer diameters

$$\omega_1 = 0$$

$$\omega_2 = \frac{2\pi n}{60} = \frac{2\pi(1440)}{60} = 150.80 \text{ rad/s}$$

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{150.80 - 0}{40} = 3.77 \text{ rad.s}^2$$

$$M_t = I\alpha = \text{mk}^2; \alpha = 150(0.25)^2(3.77)$$

$$= 35.34292 \text{ Nm} = 35342.92 \text{ Nmm}$$

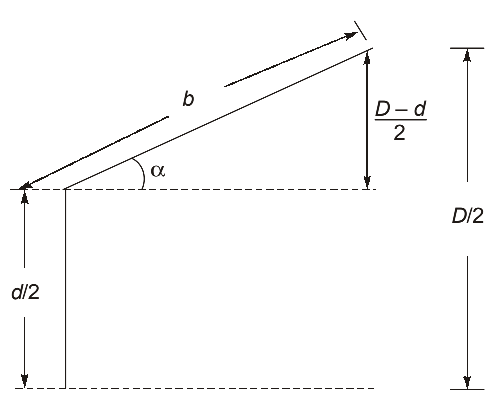


fig. (a)

For cone clutch $$M_t = \frac{\pi\mu p_a d}{8\sin\alpha}\left(D^2 - d^2\right)$$

$$\therefore \quad 35342.92 = \frac{\pi(0.2)(0.1)d}{8\sin(12.5°)}\left(D^2 - d^2\right)$$

Rearranging the terms, we have

$$d(D^2 - d^2) = 973978.34 \quad \text{...(a)}$$

From figure (a) $$D - d = 2b\sin\alpha \quad \text{...(b)}$$

Since the mean radius of the clutch is twice the face width,

$$\frac{D+d}{4} = 2b \qquad \text{or } D + d = 8b \quad \text{...(c)}$$

Dividing eq. (c) by (b), $$\frac{D+d}{D-d} = \frac{8b}{2b\sin\alpha} = \frac{4}{\sin\alpha}$$

Therefore, $$\frac{D}{d} = \frac{4+\sin\alpha}{4-\sin\alpha} = \frac{4+\sin(12.5°)}{4-\sin(12.5°)} = 1.1144$$

$$\frac{D^2}{d^2} = 1.2419 \quad \text{or } D^2 = 1.2419\, d^2$$

Substituting this value in eq. (a) we get

$$d(1.2419\,d^2 - d^2) = 973978.34$$

$$0.2419 d^3 = 973978.34$$

$$d^3 = \frac{973978.34}{0.2419}$$

$$\therefore \quad d = 159.09 \text{ mm}$$

$$D = 1.1144\,d = 1.1144(159.09) = 177.29 \text{ mm} \quad \text{...(i)}$$

Step II: Face width of friction lining

From eq. (c) $$b = \frac{D+d}{8} = \frac{177.29+159.09}{8} = 42.05 \text{ mm} \quad \text{...(ii)}$$

Step III: Force required to engage clutch

From eq. $$P = \frac{4M_t\sin\alpha}{\mu(D+d)} = \frac{4(35342.92)\sin(12.5°)}{0.2(177.29+159.09)} = 454.82 \text{ N} \quad \text{...(iii)}$$

Step IV: Heat generated during each engagement

$$\omega_{ave} = \frac{\omega_1+\omega_2}{2} = \frac{0+150.8}{2} = 75.4 \text{ rad/s}$$

$$\theta = \omega_{ave}(\text{time}) = 75.4\,(40) = 3016 \text{ radians}$$

Heat generated during engagement = work done by frictional torque

$$H_g = M_t\theta = 35.34\,(3016)$$

$$= 106585.44 \text{ Nm} \quad \text{or } J = 106.59 \text{ kJ} \quad \text{...(iv)}$$

■■■■

6 Gears

LEVEL 1 Objective Questions

1. (b)
2. (c)
3. (c)
4. (c)
5. (c)
6. (a)
7. (a)
8. (b)
9. (a)

LEVEL 2 Objective Questions

10. (c)
11. (b)
12. (4)
13. (a)
14. (b)
15. (144.22)
16. (c)

■■■■

LEVEL 3 Conventional Questions

Solution : 17

$$z_p = 25,\ z_g = 60$$

Power at pinion = 25 kW at 1440 rpm

C_S-service factor = 1.5

$$\sigma_p = 140\ \text{N/mm}^2$$

$$\sigma_g = 70\ \text{N/mm}^2$$

Factor of safety = 1.5

Form factor for 25 teeth, Y = 0.34

Form factor for 60 teeth, Y = 0.421

$$\therefore \quad (\sigma Y)_p = 140 \times 0.34 = 47.6\ \text{N/mm}^2$$

$$(\sigma Y)_g = 70 \times 0.421 = 29.47\ \text{N/mm}^2$$

Hence gear is weak so it will be designed

$$b = 10\ \text{module}$$

Beam strength of gear -Lewis equation

$$S = bmY\sigma$$

$$\text{FoS} \times \frac{P_t.C_s}{C_v} = (10\,m)\,m\,(29.47)$$

$$\frac{1.5 \times P_t \times 1.5}{C_v} = 294.7\ m^2 \qquad \ldots \text{(i)}$$

The torque to be transmitted

$$M_t = \frac{\text{Power}}{\omega} = \frac{25000}{\frac{2\pi \times 1440}{60}}$$

$$M_t = 165.786 \times 10^3\ \text{N-mm}$$

$$P_t = \frac{2\,M_t}{d_p} = \frac{2\,M_t}{m.z_p}$$

$$P_t = \frac{2 \times 165.786 \times 10^3}{25\,m}$$

$$P_t = \frac{13262.88}{m}$$

Assuming velocity < 10 m/sec

$$C_v = \left(\frac{3}{3+v}\right)$$

$$C_v = \frac{3}{3+\frac{\pi d_p N_p}{60}} = \frac{3}{3+\frac{\pi \times m \times z_p N_p}{60}} = \frac{3}{3+\frac{\pi \times 25 \times 1440\,m}{60 \times 1000}}$$

$$C_v = \left(\frac{3}{3+1.885\,m}\right)$$

Put P_t and C_v in equation (i)

$$\frac{\frac{1.5 \times 1.5}{3}}{(3 + 1.885\,\text{m})} \times \frac{13262.88}{\text{m}} = 294.7\ \text{m}^2$$

$$\frac{2.25(3 + 1.885\,\text{m}) \times 13262.88}{3\,\text{m}} = 294.7\ \text{m}^2$$

$$33.75\,(3 + 1.885\ \text{m}) = m^3$$

$$m^3 - 63.62\ \text{m} - 101.26 = 0$$

which gives $m = 8.68$ mm

∴ The module is 10 mm

face width b = 100 mm

dia. of pinion, $d_p = 25 \times 10 = 250$ mm

which gives the velocity

$$v = \frac{\pi d_p N_P}{60} = \frac{\pi \times 0.25 \times 1440}{60} = 18.85\ \text{m/sec}$$

Hence the assumption was wrong in velocity.

∴ use the velocity factor

$$C_v = \frac{6}{6 + v} \quad \text{for } 10 < V < 20\ \text{m/sec}$$

∴ $$C_v = \left(\frac{6}{6 + 1.885\,\text{m}}\right)$$

Now put it in equation (i)

$$\frac{1.5^2 \times 13262.88}{\left(\frac{6}{6 + 1.885\,\text{m}}\right)\text{m}} = 294.7\ \text{m}^2$$

$$16.876\,(6 + 1.885\ \text{m}) = m^3$$

$$m^3 - 31.8\ \text{m} - 101.26 = 0$$

This equation gives m = 6.83 mm

∴ Now available module = 8 mm

∴ dia. of pinion = 8 × 25 = 200 mm

dia. of gear = 8 × 60 = 480 mm

Face width, b = 10 m = 80 mm

Solution : 18

Given, $\phi = 20°$

Power = 20 kW = 20 × 10^3 W

N_P = 300 rpm

Velocity ratio $= \frac{T_G}{T_P} = 3$,

σ_{OG} = 100 MPa = 100 × 10^6 N/m^2 = 100 N/mm^2

σ_{OP} = 120 MPa = 120 × 10^6 N/m^2 = 120 N/mm^2

T_P = 15,

and $T_G = 3T_P = 3 \times 15 = 45$

$$b = 14 \text{ m}$$
$$\sigma_{es} = 600 \text{ MPa} = 600 \text{ N/mm}^2$$
$$E_P = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2 = 200 \times 10^3 \text{ N/mm}^2$$
$$E_G = 100 \text{ GPa} = 100 \times 10^9 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$$

Module:

Let, m = module in mm and

D_P = pitch circle diameter of the pinion in mm

Pitch line velocity,
$$V = \frac{\pi D_P N_P}{60} = \frac{\pi m T_P N_P}{60} \qquad \left(\because m = \frac{D}{T}\right)$$
$$= \frac{\pi \times m \times 15 \times 300}{60} = (235.61 \text{ m}) \text{ mm/s} = (0.235 \text{ m}) \text{ m/s}$$

Assuming steady load conditions and 8 – 10 hours of service per day, the service factor, C_s is taken as 1. i.e., $C_s = 1$

We know that the design tangential tooth load,
$$W_T = \frac{P}{V} \times Cs = \frac{20 \times 10^3}{0.235m} \times 1 = \frac{85.10}{m} \times 10^3 \text{ N}$$

and velocity factor,
$$C_V = \frac{3}{3+V} = \frac{3}{3+0.235m}$$

Tooth form factor for pinion,
$$y_p = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{15} = 0.0932$$

And tooth form factor for gear,
$$y_G = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{45} = 0.133$$

$\therefore$
$$\sigma_{OP} \times y_P = 120 \times 0.0932 = 11.184$$
$$\sigma_{OG} \times y_G = 100 \times 0.133 = 13.3$$

Since $(\sigma_{OP} \times y_P)$ is less than $(\sigma_{OG} \times y_G)$, therefore the pinion is weaker. Now using the Lewis equation to the pinion, we have,
$$W_T = (\sigma_{WP}\, b\pi m\, y_P) = (\sigma_{OP} \times C_V)\, b\pi\, m y_P \qquad (\because\ \sigma_{WP} = \sigma_{OP} \times C_V)$$
$$\frac{85.1 \times 10^3}{m} = 120 \times \left(\frac{3}{3+0.235m}\right) \times (14m) \times \pi \times m \times 0.0932$$
$$\frac{85.1 \times 10^3}{m} = \frac{1475.69m^2}{(3+0.235m)}$$
$$\frac{57.66}{m} = \frac{m^2}{3+0.235m} = 172.98 + 13.55 \text{ m} = m^3$$
$$\Rightarrow \quad m^3 - 13.55 \text{ m} - 172.98 = 0$$

By trial and error method, we get $m = 6.37$ mm

We can consider the standard module, (of second choice), $m = 7$

Face width:

Given that face width $b = 14 \text{ m} = 14 \times 7 = 98$ mm

Pitch diameters of gears:

We know that pitch diameter of the pinion,
$$D_P = mT_P = 7 \times 15 = 105 \text{ mm}$$

Pitch diameter of the gear,

$$D_G = mT_G = 7 \times 45 = 315 \text{ mm}$$

Checking the gears for wear:

We know that the ratio factor, $Q = \dfrac{2 \times VR}{VR + 1} = \dfrac{2 \times 3}{3+1} = 1.5$

Load stress factor, $K = \dfrac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\dfrac{1}{E_P} + \dfrac{1}{E_G}\right)$

$$= \frac{600^2 \sin 20°}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{100 \times 10^3}\right) = 1.3192 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_W = D_P\, b\, QK = 105 \times 98 \times 1.5 \times 1.319 = 20358.765 \text{ N}$$
$$= 20.358 \text{ kN}$$

Tangential load on tooth (or beam strength of the tooth)

$$W_T = \frac{85.1 \times 10^3}{m} = \frac{85.1 \times 10^3}{7} = 12157.14 \text{ N} = 12.157 \text{ kN}$$

Since the maximum wear load (20.358 kN) is more than the tangential load (12.157 kN) on the tooth, the design is satisfactory from the stand point of wear.

Solution : 19

Given,
$$n = 1450 \text{ rpm},\quad z_p = 20,\; z_g = 41$$
$$m = 3 \text{ mm},\quad b = 40 \text{ mm},\; C_s = 1.75 \quad (f_s) = 1.5$$
$$\text{BHN} = 400,\quad S_{ut} = 600 \text{ N/mm}^2$$

Step 1: Beam Strength

Since the same material is used for the pinion and the gear, the pinion is weaker than the gear. The Lewis form factor is 0.32 for 20 teeth.

$$\sigma_b = \left(\frac{1}{3}\right)S_{ut} = \left(\frac{1}{3}\right)(600) = 200 \text{ N/mm}^2$$

$$S_b = mb\sigma_b,\; Y = 3(40)(200)(0.32) = 7680 \text{ N}$$

Step 2: Wear Strength $Q = \dfrac{2z_g}{z_g + z_p} = \dfrac{2(41)}{41+20} = 1.344$

$$K = 0.16\left(\frac{BHN}{100}\right)^2 = 0.16\left(\frac{400}{100}\right)^2 = 2.56$$

$$d'_p = mz_p = 3(20) = 60 \text{ mm}$$
$$S_w = bQd'_pK = 40(1.344)(60)(2.56)$$
$$= 8257.54 \text{ N}$$

Step 3: $C_v = \dfrac{3}{3+v} = \dfrac{3}{3+4.5553} = 0.397$

$$P_{\text{eff}} = \frac{C_s}{C_v}P_t = \frac{1.75}{0.397}P_t = (4.41P_1) \text{ N}$$

Step 4: Static Load

In this example, the beam strength is lower than the wear strength. Therefore, beam strength is the extension of design.

$$S_b = P_{eff}\,(f_s)\;P_1\;7680 = (4.41\;P_1)(1.5)$$
$$P_r = 1161\text{ N}$$

Step 5: Rated Powder

$$M = \frac{P_1 d_p'}{2} = \frac{1161(60)}{2} = 34.830\text{ N-mm}$$

$$\text{kW} = \frac{2\pi n_p M_t}{60\times10^6} = \frac{2\pi(1450)(34.830)}{60\times10^6} = 5.29$$

Solution : 20

Given: $T_P = 18$ teeths; $T_G = 45$ teeths; $m = 10$ mm; $d_P = 180$ mm; $d_G = 450$ mm; $\phi = 20°$

Centre distance, $C = R + r = \frac{1}{2}(180 + 450) = 315$ mm

But actual centre distance, $C' = C + 8\text{ mm} = 315 + 8 = 323$ mm

Assuming the teeth profile as involute, the centre distance variation will not alter the velocity ratio.

$$VR = \frac{d_G}{d_P} = \frac{d_G'}{d_P'} = \frac{450}{180} = 2.5$$
$$d_G' = 2.5\,d_P'$$

and $$C' = \frac{1}{2}(d_P' + d_G')$$

$$323 = \frac{1}{2}(d_P' + 2.5 d_P')$$

New pitch circle diameter, $d_P' = 184.5$ mm

and $d_G' = 2.5 \times 184.5 = 461.42$ mm

Also, the base circle radius remains unaltered. Let ϕ' be the new increased pressure cycle.

$$R'\cos\phi' = R\cos\phi$$
$$230.71\cos\phi' = 225\cos 20°$$
$$\phi' = \cos^{-1}(0.9164) = 23.6°$$

Power transmitted, $P = 20$ kW

$$N_P = 950\text{ rpm}$$

Torque, $$T = \frac{60\times P}{2\pi N_p} = \frac{60\times20\times10^3}{2\times\pi\times950} = 201.14\text{ N-m}$$

$$F_t = \text{tangential force}$$
$$T = F_t \times r_P'$$
$$F_t = \frac{201.14\times10^3}{92.25} = 2180.4\text{ N}$$
$$F_r = \text{radial load}$$
$$F_t\tan\phi' = 2180.4\tan 23.6°$$
$$F_r = 952.6\text{ N}$$

and normal tooth load

$$F_N = \frac{F_t}{\cos\phi'} = \frac{2180.4}{\cos 23.6} = 2379.4\text{ N}$$

Solution : 21

Given data: $\phi = 20°$; $T_P = 24$ tooth; $N_P = 950$ rpm; $T_G = 60$ tooth; $m = 6$ mm; $b = 60$ mm;

$\sigma_y = 330$ N/mm^2; $\sigma_u = 680$ N/mm^2; $\sigma_e' = 0.55\,\sigma_u = 0.55 \times 680 = 374$ N/mm^2

$$C_o = \text{overload factor} = 1.8$$

$$\text{Surface endurance limit} = 1500 \text{ MPa}$$

Dynamic factor, $C_v = 2.5$

(i) Beam strength $= \sigma_b\, b\, m\, \pi\, y$

when pinion and shaft are of same material, pinion is made weaker

$$d_p = mT_p = 6 \times 24 = 144 \text{ mm}$$

$$V = \frac{\pi d_p N_P}{60} = \frac{\pi \times (0.144) \times 950}{60} = 7.1592 \text{ m/sec}$$

for 20° full depth,

$$y = 0.154 - \frac{0.912}{T_P} = 0.154 - \frac{0.912}{24} = 0.116$$

Beam strength

$$P_b = \frac{\sigma_b \cdot bm\pi y}{C_o C_v} = \frac{\left(\frac{680}{3}\right) \times 60 \times \pi \times 6 \times 0.116}{1.8 \times 2.5}$$

$$= 6604.88 \text{ N}$$

(ii) Wear strength

$$P_w = d_p\, K\, Q\, b$$

$$Q = \frac{2VR}{VR+1} = \frac{2T_G}{T_G + T_P} = \frac{2 \times 60}{60 + 24} = 1.428$$

$$K = \frac{(\sigma_e)^2 \sin\phi}{1.4}\left[\frac{1}{E_p} + \frac{1}{E_G}\right] = \frac{(1500)^2 \times \sin 20°}{1.4}\left[\frac{1}{2.1 \times 10^5} \times 2\right]$$

$$= 5.235 \text{ MPa}$$

$$P_w = (24 \times 6) \times 5.235 \times 1.428 \times 60$$

$$= 64589$$

(iii)

$$\text{Rated power} = P_{\text{eff}} \times V = \frac{P_b}{\text{FOS}} = \frac{6604.88}{2.5} = 2641.952 \text{ N}$$

$$\text{Power} = 2641.952 \times 7.1592 = 18.92 \text{ kW}$$

Solution : 22

Let

m = required module

T_P = number of teeth on the pinion

T_G = number of teeth on the gear

D_P = PCD of the pinion

D_G = PCD of the gear

Minimum number of the teeth on the pinion to avoid interference:

$$T_P = \frac{2A_w}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]}$$

$$T_P = \frac{2 \times 1}{10\left[\sqrt{1 + 0.1 \times 2.1 \times \sin^2 22.5°} - 1\right]}$$

$$T_P = 13.3 \qquad \text{say } 14$$

$$T_G = G \times T_P = 10 \times 14 = 140 \text{ mm}$$

$$\text{Centre distance, } C = \frac{D_P + D_G}{2} = \left(\frac{10+1}{2}\right) D_P = 5.5\, D_P$$

or

$$660 = 5.5\, D_P$$

$$D_P = \frac{6600}{55} = 120 \text{ mm}$$

$$D_P = m \times T_P$$

$$m = \frac{120}{14} = 8.6 \text{ mm}$$

Nearest standard value of module,

$$m = 8 \text{ mm}$$ Ans. (i)

Number of teeth on the pinion,

$$T_P = \frac{D_P}{m} = \frac{120}{8} = 15$$ Ans(ii)

Number of teeth on the gear,

$$T_G = 10 \times 15 = 150 \text{ mm}$$ Ans(iii)

■■■■

7 Bearings

LEVEL 1 Objective Questions

1.	(d)
2.	(c)
3.	(b)
4.	(d)
5.	(a)
6.	(b)
7.	(d)
8.	(c)
9.	(c)
10.	(a)
11.	(a)
12.	(d)
13.	(c)
14.	(d)
15.	(b)

LEVEL 2 Objective Questions

16.	(a)
17.	(*)
18.	(10.22)
19.	(a)
20.	(157.46)
21.	(3.55)
22.	(d)
23.	(4.4)
24.	(b)
25.	(21.78)
26.	(64.45)
27.	(c)
28.	(c)
29.	(b)

LEVEL 3 Conventional Questions

Solution : 30

Given journal diameter, $D = 60$ mm, $\Rightarrow r = 30$ mm

Bearing length, $L = 60$ mm, $L/D = 1$

Radial load, $W = 2.8$ kN

Journal speed, $N = 1020$ rpm

$$n_s = \frac{1020}{60} \text{ rps}$$

Radial clearance, $C = 0.05$ mm

Viscosity of oil, $r = 80 \times 10^{-9}$ N-s/mm^2

(i) Sommerfeld number $S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{p}$

Bearing pressure $P = \frac{W}{LD} = \frac{2.8 \times 10^3 N}{60 \times 60} = 0.78$ N/mm^2

$$\therefore \quad S = \left(\frac{30}{0.05}\right)^2 \times \frac{(80 \times 10^{-9}) \times \left(\frac{1020}{60}\right)}{0.78} = 0.627 \simeq 0.630$$

From the given table,

for the value of $S = 0.630$, $\left(\frac{r}{c}\right) f = 12.8$

$$\frac{h_0}{c} = 0.8, \text{ and}$$

$$\frac{Q}{rcn_s l} = 3.59$$

where coefficient of friction variable C.F.V. $= \left(\frac{r}{c}\right) f$ and flow variable F.V. $= \frac{Q}{rcn_s l}$

$$\left(\frac{r}{c}\right) f = 3.22$$

Coefficient of friction $f = 12.8 \times \frac{c}{r} = 12.8 \times \frac{0.05}{30} = 0.0213$

(ii) Power loss in friction, (kW)

$$= \frac{2\pi n_s f w r}{10^6} = \frac{2\pi \times \left(\frac{1020}{60}\right) \times 0.0213 \times 2.8 \times 10^3 \times 30}{10^6}$$

$$= 0.1911 \text{ kW}$$

$$\frac{h_0}{c} = 0.8, \; h_0 = 0.8 \times c = 0.8 \times 0.05 = 0.04 \text{ mm}$$

$$\frac{Q}{rcn_s l} = 3.59 = FV$$

(iii) We know temperature rise,

$$\Delta T = \frac{8.3 \times p \times (CFV)}{(FV)} = \frac{8.3 \times 0.78 \times 12.80}{3.59} = 23.082°\text{C}$$

(iv) From the table for $\frac{h_0}{c} = 0.8$, S = 0.630

we get $\phi = 74.02°$ i.e., angle of eccentricity (or location)

$$\frac{h_0}{c} = 0.8$$

Minimum film thickness

$$h_0 = 0.8 \times c = 0.8\ 0 \times 0.05 = 0.04 \text{ mm} = 40 \text{ microns}$$

Solution : 31

Given: $\frac{l}{D} = 1$, $w = 1.1$ kN, $\mu = 55.2 \times 10^{-3}$ Pa.s, $n_s = 18.33$ rps, $f = 4\left(\frac{C}{r}\right)$, $\frac{h_o}{C} = 0.58$

Hole limit 25.038 mm
Shaft limit 25 mm

Radial clearance,
$$C = \frac{25.038 - 25}{2} = 0.019 \text{ mm}$$

(a)
$$\text{Sommerfield number} = \left(\frac{r}{C}\right)^2 \frac{\mu n_s}{P}$$

Where, μ is in MPa.s
P is in N/mm^2 or MPa
n_s is in rps

$$P = \frac{w}{lD} = \frac{1.1 \times 10^3}{25 \times 25} = 1.76 \text{ N/mm}^2$$

$$\therefore \quad S = \left(\frac{12.5}{0.019}\right)^2 \times \frac{55.2 \times 10^{-9} \times 18.33}{1.76 \times 10^6} = 0.2488$$

(b)
$$\frac{h_o}{C} = 0.58$$

$$\therefore \quad h_o = 0.58 \times 0.019 = 0.0110 \text{ mm}$$

(c)
$$f = 4\left(\frac{C}{r}\right) = 4 \times \frac{0.019}{12.5} = 0.006$$

$$\text{Frictional force} = 0.006 \times 1100 = 6.088 \text{ N}$$

$$\text{Frictional torque} = 6.688 \times \frac{25}{2} = 83.6 \text{ N-mm} \quad \text{Ans.}$$

Solution : 32

A ball bearing operates on the given work cycle as:
Considering the work cycle of one minute duration. Then,

$$N_1 = 0.3 \times 1440 = 432 \text{ revolutions}$$
$$N_2 = 0.35 \times 750 = 262.5 \text{ revolutions}$$
$$N_3 = 0.35 \times 1440 = 504 \text{ revolutions}$$

$\therefore$ Equivalent load,

$$P_e = \sqrt[3]{\frac{P_1^3 N_1 + P_2^3 N_2 + P_3^3 N_3}{N_1 + N_2 + N_3}}$$

$$= \sqrt[3]{\frac{432 \times (3500)^3 + 262.5 \times (6000)^3 + 504 \times (2500)^3}{432 + 262.5 + 504}}$$

$$P_e = 4108.18 \text{ N}$$

Bearing life = 10,000 hours

$$L_{10} = \frac{60 \times (432 + 262.5 + 504) \times 10{,}000}{10^6} = 719.1 \text{ million rev.}$$

Let C be its dynamic load carrying capacity

$$L_{10} = \left(\frac{C}{P_{eq}}\right)^3$$

$$719.1 = \left(\frac{C}{4108.18}\right)^3$$

$$C = 36.8 \text{ kN}$$

Solution : 33

Given: $C = 26$ kN, $L_{10h} = 8000$ hr, $n = 300$ rpm

Step I: Bearing life (L_{10})

$$L_{10} = \frac{60 n L_{10h}}{10^6} = \frac{60(300)(8000)}{10^6} = 144 \text{ million rev.}$$

Step II: Equivalent radial load

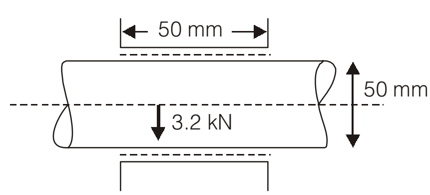
$$C = P(L_{10})^{0.3};$$

$$\therefore \quad P = \frac{C}{(L_{10})} = \frac{26000}{(144)^{0.3}} = 5854.16 \text{ N}$$

Since the bearing is subjected to purely radial load ,

$$F_r = P = 5854.16 \text{ N}$$

Solution : 34



$$\text{Bearing pressure P} = \frac{3200}{d \cdot l} = 1.28 \text{ N/mm}^2 = 1.28 \times 10^6 \text{ N/m}^2$$

(i) Sommerfield No

$$S = \left(\frac{\mu N_s}{P}\right)\left(\frac{r}{C}\right)^2 = \frac{0.025 \times \frac{1500}{60}}{1.28 \times 10^6}\left(\frac{25}{0.05}\right)^2 = 0.122$$

(ii) P_{loss} = Friction torque $\times\, \omega = f \times \text{load} \times r \times \omega$

$$= 0.00322 \times 3200 \times 0.025 \times \frac{2\pi}{60} \times 1500 = 40.46 \text{ watt}$$

(iii)
$$\epsilon = 1 - \frac{h_0}{C}$$
$$\frac{e}{C} = 1 - \frac{h_0}{C}$$
$$\frac{h_0}{C} = 1 - \frac{e}{C} = 1 - \frac{0.03}{0.05} = 1 - 0.6 = 0.4$$
$$h_0 = 0.4 \times 0.05 = 0.02 \text{ mm}$$

(iv)
$$\text{Bearing modulus} = \frac{\mu N_s}{P} = \frac{0.025 \times 25}{1.28 \times 10^6} = 0.4883 \times 10^{-6}$$

Solution : 35

75 mm lies in the diameter steps of 50 to 80 mm, so
$$D = \sqrt{50 \times 80} = 63 \text{ mm}$$

Standard tolerance unit,
$$i = 0.45\sqrt[3]{D} + 0.001D = 0.45(63)^{1/3} + 0.001 \times 63$$
$$= 1.79 + 0.063 = 1.853 \text{ miron}$$
$$\text{Standard tolerance for hole} = 25\,i = 25 \times 1.853 \times 10^{-3} = 0.046 \text{ mm}$$
$$\text{Standard tolerance for shaft} = 16\,i = 16 \times 1.853 \times 10^{-3} = 0.03 \text{ mm}$$

(ii)
$$\text{Length} = \text{Diameter} = 90 \text{ mm}$$
$$\text{Speed, } N = 450 \text{ rpm}$$
$$\text{Viscosity, } Z = 0.06 \text{ kg/ms}$$
$$\text{Diametral clearance, } C = 0.1 \text{ mm}$$
$$\text{Sommerfeld number} = 14.3 \times 10^6$$
$$\frac{ZN}{P}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6$$

or
$$\frac{0.06 \times 450}{P}\left(\frac{90}{0.1}\right)^2 = 14.3 \times 10^6$$
$$P = 1.53 \text{ N/mm}^2$$
$$\text{Safe bearing load, } W = \text{Pressure x Projected Area}$$
$$W = 1.53 \times 90 \times 90 = 12393 \text{ N}$$

Solution : 36

$$\text{Journal diameter } d = 50 \text{ mm} = 0.05 \text{ m}$$
$$\text{length of journal, } l = 100 \text{ mm} = 0.1 \text{ m}$$
$$\text{Bearing pressure, } p = 1.4 \text{ N/mm}^2$$
$$\text{Viscosity at 75°C, Z} = 0.011 \text{ kg/ms}$$
$$\text{Operating temperature, } t_a = 35°\text{C}$$
$$\text{Specific heat, } C_p = 1850 \text{ J/kg/°C}$$
$$\text{Coefficient of friction, } \mu = \frac{33}{10^8}\left(\frac{ZN}{P}\right)\left(\frac{d}{c}\right) + k$$
$$\mu = \frac{33}{10^8}\left(\frac{0.011 \times 900}{1.4}\right) \times 1000 + 0.002$$
$$\mu = 0.00233 + 0.002 = 0.00433$$

$$\text{Load on the bearing, } W = p \times dl = 1.4 \times 50 \times 100$$

$$W = 7000 \text{ N}$$

$$\text{Rubbing velocity, } V = \frac{\pi dN}{60} = \frac{3.14 \times 0.05 \times 900}{60} = 2.36 \text{ m/s}$$

$$\text{Heat generated, } Q_g = \mu WV = 0.00433 \times 7000 \times 2.36$$

$$Q_g = 71.5 \text{ J/s}$$

Temperature of the bearing surface = t_b

$$t_b - b_a = \frac{1}{2}(t_0 - t_a) = \frac{75.35}{2} = 20°\text{C}$$

$$\text{Heat dissipated, } Q_d = hA(t_b - t_a)$$

$$Q_d = 280 \times 0.05 \times 0.1 \times 20(20)$$

$$Q_d = 28 \text{ J/s}$$

∴ Amount of artificial cooling required

$$= Q_g - Q_d = 71.5 - 28 = 43.5 \text{ W}$$

Let, m = mass of the lubricating oil required in kg/s

$$\therefore \quad Q_g = Q_t$$

$$m = \frac{71.5}{18500} = 0.00386 \text{ kg/s} = 0.23 \text{ kg/min}$$

■■■■

8 Shafts

LEVEL 1 Objective Questions

1. (c)
2. (d)
3. (b)
4. (d)
5. (c)
6. (a)
7. (b)
8. (c)
9. (d)
10. (26.89)

LEVEL 2 Objective Questions

11. (d)
12. (d)
13. (a)
14. (a)
15. (d)
16. (1280)
17. (d)
18. (30)
19. (d)

■■■■

LEVEL 3 Conventional Questions

Solution : 20

Given

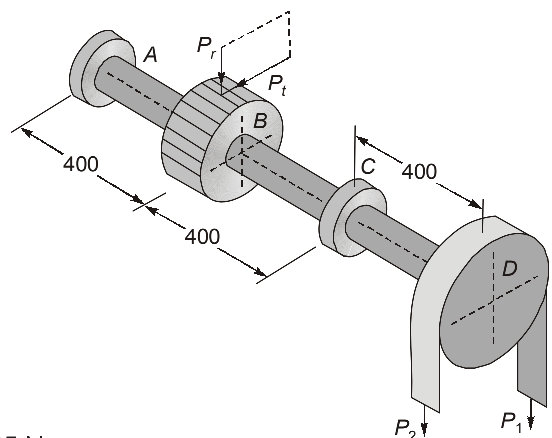
For pulley, $T_1 = 3T_2$

For gear, pressure angle , $\phi = 20°$

$D_2 = 300$ mm

power, $P = 20$ kW

$N = 500$ rpm



Torque transmitted, $$T = \frac{P}{\omega} = \frac{20\times10^3}{\left(\dfrac{2\pi\times500}{60}\right)}$$

$$= \frac{20\times10^3}{52.36} = 381.97 \text{ N-m}$$

$$= 381971 \text{ N-mm}$$

The same torque applies for pulley

i.e. $(T_1 - T_2)225 = 381971 = T_1 - T_2 = 1697.65$ N

$\Rightarrow$ $3T_2 - T_2 = 1697.65$ $\quad (\because\ T_1 = 3T_2)$

$\Rightarrow$ $T_2 = 848.83$N

$T_1 = 2546.48$ N

$= T_1 + T_2 = 3395.31$ N

For the gear, torque $T = F_t \times 150$, $\quad F_t \rightarrow$ Tangential force (horizontal)

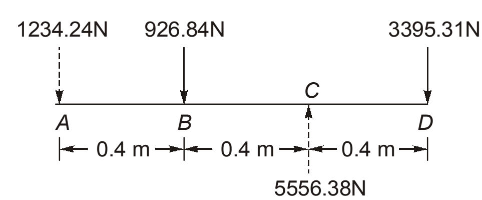
$381971 = F_t \times 150$

$\Rightarrow$ $F_t = 2546.48$ N

$F_r = F_t \times \tan 20°$, $\quad F_t \rightarrow$ Radial force (vertical)

$= 2546.48 \times \tan 20° = 926.84$ N

Invertical plane:



$R_{AV} + R_{CV} = 926.84 + 3395.31$

$= 4322.15$ N

$\Sigma M_A = 0$

$R_{CV} \times 0.8 = 926.84 \times 0.4 + 3395.31 \times 1.2$

$R_{CV} = 5556.38$ N (upwards)

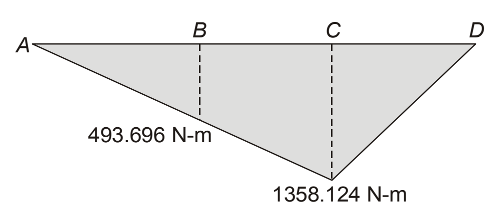
$\Rightarrow$ $R_{AV} = 4322.15 - 5556.38$

$= 1234.24$ N (downwards) i.e. – 1234.24 N

$BM_B = -1234.24 \times 0.4 = -493.696$ N-m

$BM_C = 3395.31 \times 0.4$

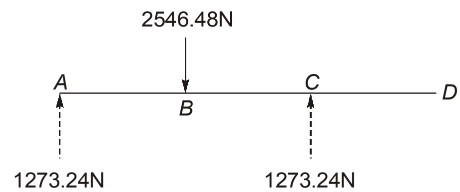
$= 1358.124$ N-m



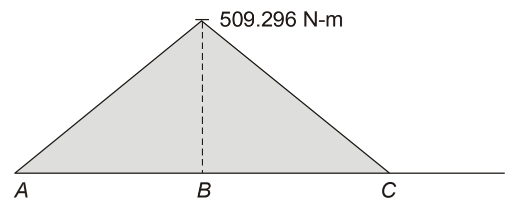
In horizontal plane:

$R_{AH} + R_{CH} = 2546.48$, $\quad R_{DH} = 0$

$$R_{AH} = R_{CH} = \frac{2546.48}{2} = 1273.24 \text{ N}$$



$$BM_B = 1273.24 \times 0.4 = 509.296 \text{ N-m}$$



The maximum $B.M$ is at point C,

i.e. $M_C = 1358.124$ N-m

and maximum torque, $T = 381.971$ N-m

Equivalent torque, $T_e = \sqrt{(k_b M)^2 + (k_c T)^2}$

$$= \sqrt{(1.5 \times 1358.124)^2 + (1.5 \times 381.971)^2}$$

$$T_e = 2116.224 \text{ N-m} \qquad [\because k_b = k_t = 1.5 \text{ (given)}]$$

mostly shafts are designed on the basis of maximum shear stress theory,
given tensile strength of the shaft and key is 700 MPa.

$$= 700 \times 10^6 \text{ N/m}^2$$

Allowance for the key way for stress as 0.75 and factor of safety = 5

$$\therefore \quad \tau = \frac{0.75 \times 700 \times 10^6}{5} = 52.5 \times 10^6$$

$$\therefore \quad 52.5 \times 10^6 = \frac{16 T_e}{\pi d^3} = \frac{16 \times 2116.224}{\pi \times d^3} \qquad (\because \tau = 16\, T/\pi d^3)$$

$$d = 0.05899 \text{ m} = 58.99 \text{ mm} \qquad \text{(diameter of the shaft)}$$

$$\simeq 60 \text{ mm}$$

maximum shear stress for the key

$$\tau = \frac{\sigma}{2 \times F.S} \times 0.75 = \frac{700}{2 \times 5} \times 0.75 = 52.5 \text{ N/mm}^2$$

For the key, take width w = thickness, $t = \dfrac{d}{4}$

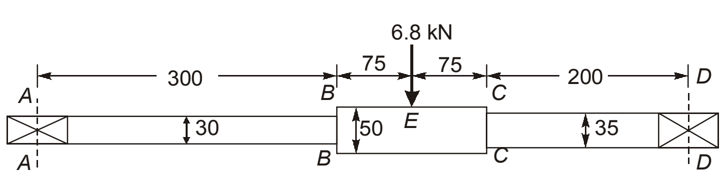
$$w = t = \frac{60}{4} = 15 \text{ mm}$$

$$\therefore \text{ Torque} \quad T = \tau \times \frac{d}{2} l w \qquad \text{(for crushing)}$$

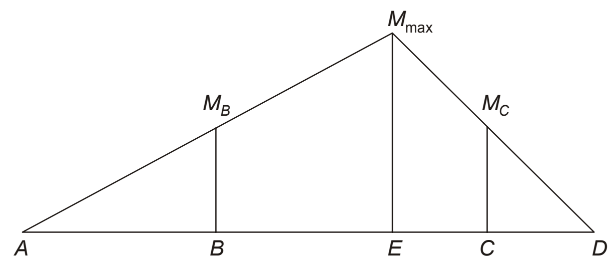
$$= 381971 \text{ N-m} = 52.5 \times \frac{60}{2} \times l \times 15$$

$$\Rightarrow \quad l = 16.16 \text{ mm}$$

Solution : 21



All dimension in mm



Let R_A and R_D be the reaction forces

$$R_A + R_D = 6.8 \text{ kN}$$

and $$R_D \times 650 = 6.8 \times 375$$

$$R_D = 3.923 \text{ kN}$$

and $$R_A = 2.877 \text{ kN}$$

$$M_B = \text{Bending moment at section } BB$$

$$= R_A \times 300 = 863.1 \text{ N-m}$$

$$M_E = R_A \times (375) = 1078.875 \text{ N.m}$$

and $$M_C = R_D \times 200 = 784.6 \text{ N.m}$$

Thus critical section can be B, C and E.

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32 \times 863.1 \times 10^3}{\pi (30)^3} = 325.77 \text{ MPa}$$

$$\sigma_E = \frac{32M_E}{\pi d^3} = \frac{32 \times 1078.875 \times 10^3}{\pi (50)^3} = 87.95 \text{ MPa}$$

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32 \times 784.6 \times 10^3}{\pi (35)^3} = 186.5 \text{ MPa}$$

$$(K_t)_B = 1.5 \quad \text{and} \quad (K_t)_C = 1.4$$

$$(\sigma_{max})_B = 1.5 \times 325.77 = 488.65 \text{ MPa}$$

$$(\sigma_{max})_C = 1.4 \times 186.5 = 261.1 \text{ MPa}$$

Hence, section B-B is the most critical section among B, E and C.

Also, $$q = \frac{K_f - 1}{K_t - 1}$$

$$(K_f)_B = 0.95 \times (1.5 - 1) + 1 = 1.475$$

$$K_d = \frac{1}{K_f} = \frac{1}{1.475} = 0.6779$$

$$S_e = C_r \times K_d \times S_e'$$

and $$S_e' = 0.55 \times \sigma_u = 0.55 \times 650 = 375.5 \text{ N/mm}^2$$

$$S_e = 0.897 \times 0.6779 \times 357.5 = 217.38 \text{ N/mm}^2$$

Therefore, $$\text{FOS} = \frac{(\sigma_{max})_B}{S_e} = \frac{488.65}{217.38} = 2.25$$

Solution : 22

Let d as ammeter of shaft in mm

Section modulus of shaft, $$Z = \frac{\pi}{32} d^3 = 0.0982\, d^3 \text{ mm}^3$$

Tensile stress due to bending moment,

$$\sigma = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982 d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

shear stress due to torsional moment,

$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 30 \times 10^6}{3.14 \times d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma}{2}+\frac{1}{2}\sqrt{\sigma^2+4\tau^2}$$

$$\sigma_1 = \frac{101.8\times10^2}{2d^3}+\frac{1}{2}\left[\sqrt{\left(\frac{101.8\times10^6}{d^3}\right)^2+4\left(\frac{152.3\times10^6}{d^3}\right)^2}\right]$$

$$\sigma_1 = \frac{50.9\times10^6}{d^3}+\frac{10^6}{2d^3}\sqrt{(101.8)^2+4(152.8)^2}$$

$$\sigma_1 = \frac{50.9\times10^6}{d^3}+\frac{161+10^6}{d^3}=\frac{211.9\times10^6}{d^3}\text{N/mm}^2$$

According to maximum strain energy theory,

$$\sigma_1{}^2+\sigma_2{}^2-2\nu\sigma_1\sigma_2=\left(\frac{\sigma_y}{N}\right)^2$$

or $\left(\frac{211.9\times10^6}{d^3}\right)^2+\left(\frac{110.1\times10^6}{d^3}\right)^2+0.5\left(\frac{211.9\times10^6}{d^3}\right)\left(\frac{110.\times10^6}{d^3}\right)=(350)^2$

or $\frac{44902\times10^{12}}{d^6}+\frac{12122\times10^{12}}{d^6}+\frac{11665\times10^{12}}{d^6} = 122500$

or $\frac{68689\times10^{12}}{d^6} = 122500$

or $d^6 = 0.5607\times10^{12}$

or $d = 90.8$ mm

Solution : 23

Torque, $T = 5400$ Nm $= 5400\times10^3$ Nmm

Allowable stress, $\tau_{all} = 150$ MPa

$$\tau = \frac{Tc}{J}$$

$$\Rightarrow \quad 150\times10^6 = \frac{5400\times d/2}{\frac{\pi}{32}\left[d^4-(0.75d)^4\right]}=\frac{40230}{d^3}$$

$$d^3 = \frac{40230}{150\times10^6}=2.682\times10^{-4}$$

$d = 6.45\times10^{-2}$ m or 64.5 mm

The next preferred size is d = 80 mm

Outer diameter = 80 mm

Inner diameter = 60 mm   Ans. (i)

$$J = \frac{\pi}{32}\left(80^4-60^4\right) = 2.749\times10^6\ \text{mm}^4$$

shear stress, $\tau = \frac{5400\times10^3\times60}{2\times2.749\times10^6} = 58.9$ MPa   Ans. (ii)

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