2020

RANK improvement WORKBOOK

Detailed Explanations of Objective & Conventional *Questions*

Mechanical Engineering

Fluid Mechanics & Hydraulic Machines

MADE EASY Publications

MADE EASY

Publications

1 Fluid Properties

LEVEL 1 Objective Questions

1.	(c)
2.	(b)
3.	(c)
4.	(576)
5.	(3.61)
6.	(0.0258)
7.	(b)
8.	(d)
9.	(c)
10.	(a)
11.	(b)

LEVEL 2 Objective Questions

12.	(d)
13.	(c)
14.	(c)
15.	(c)
16.	(c)
17.	(c)
18.	(c)
19.	(b)
20.	(b)
21.	(d)

LEVEL 3 Conventional Questions

Solution : 22

Given data: Ram diameter = 280 mm = 0.280 m, Cylinder diameter = 280.18 mm; $\nu = 0.00042\ m^2/s$

Specific gravity = 0.86; Velocity = 0.22 m/s

$$\tau = \mu \frac{du}{dy}$$

$$\frac{F}{A} = \frac{\mu V}{y}$$

$$F = \frac{\mu AV}{y} = \frac{0.00042 \times 860 \times \pi \times 0.280 \times 2 \times 0.22}{\left(\frac{280.18 - 280}{2}\right) \times 10^{-3}} = \mathbf{1553.33\ N}$$

Solution : 23

$$E_v = \frac{\Delta p}{\delta V / V} = \frac{\Delta p}{(-\Delta V / V)}$$

$$V = 1000\ cm^3;$$

$$\text{Weight} = \text{Volume} \times \text{Density}$$

$$600g = \Delta V \times (13.6 \times 1\ g/cm^3)$$

where ΔV = Change in volume of water

or $$\Delta V = \frac{600}{13.6} = 44.12\ cm^3$$

$$\left(-\frac{\Delta V}{V}\right) = \frac{44.12}{1000} = 0.04412$$

Making substitutions, $2.13 \times 10^4 = \dfrac{\Delta p}{0.04412}$

or $$\Delta p = 939.76\ kg/cm^2 = 939.76 \times 10^4\ kg/m^2$$

From the pressure – depth relationship,

$$h = \frac{\Delta p}{w_{sea}} = \frac{939.76 \times 10^4}{1050} = 8950\ m$$

$\therefore$ Depth of the sea = **8950 m**

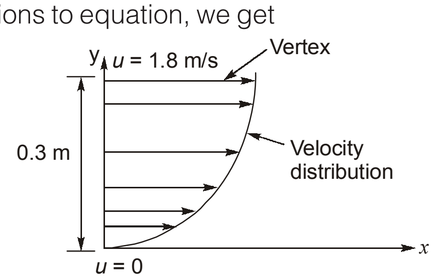
Solution : 24

The equation for velocity profile is given by

$$u = ly^2 + my + n \quad \quad \ldots \text{(i)}$$

where l, m and n are constants.

Applying the boundary conditions to equation, we get



At $y = 0$ m, $u = 0$; therefore $0 = l(0)^2 + m(0) + n \quad \Rightarrow \quad n = 0$

At $y = 0.3$ m, $\frac{du}{dy} = 0$; therefore, $\frac{du}{dy} = 2\,ly + m$

$$0 = 2 \times l \times 0.3 + m \qquad \text{. . . (ii)}$$

at y = 0.3 m, $u = 1.8$ m/s; therefore,

$$1.8 = l \times 0.3^2 + m$$

Solving equations (ii) and (iii), we get

$$l = -20,\ m = 12$$

Therefore, the velocity profile will be

$$u = -20\,y^2 + 12\,y$$

and

$$\frac{du}{dy} = -40\,y + 12$$

To find the velocity gradient,

$$\left.\frac{du}{dy}\right|_{y=0} = 12\ s^{-1}$$

$$\left.\frac{du}{dy}\right|_{y=0.15} = 6\ s^{-1}$$

$$\left.\frac{du}{dy}\right|_{y=0.3} = 0\ s^{-1}$$

To find the shearing stress,

$$\tau|_{y=0} = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau|_{y=0} = 0.9 \times 12 = 10.8\ \text{N/m}^2$$

$$\tau|_{y=0.15} = 0.9 \times 6 = 5.4\ \text{N/m}^2$$

$$\tau|_{y=0.3} = 0.9 \times 0 = 0$$

Solution : 25

Given

$$\frac{v}{v_c} = 1 - \left\{\frac{r}{r_0}\right\}^2$$

or

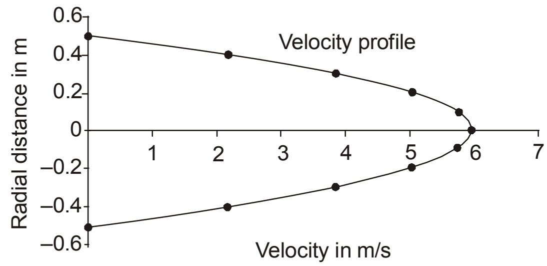
$$v = 6\left[1 - \left\{\frac{r}{0.5}\right\}^2\right]$$

Therefore,

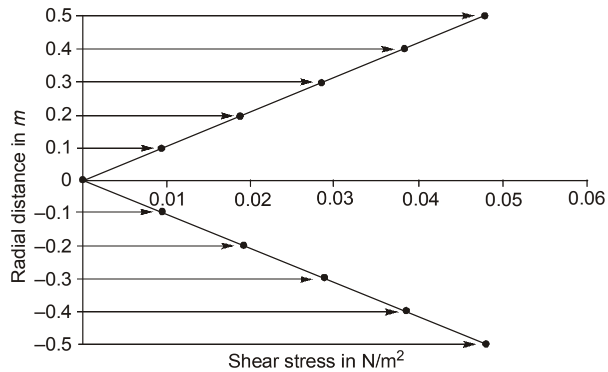
$$v = 6 - 24r^2$$

which gives velocity profile.

$$\frac{dv}{dr} = -24 \times 2r = -48r$$



Velocity profile



Shear stress distribution

$$\tau = -\mu \frac{dv}{dr} \qquad \left[\text{because } \frac{du}{dy} - \frac{dv}{dr}\right]$$

$$= -0.002(-48r) = 0.096\ r$$

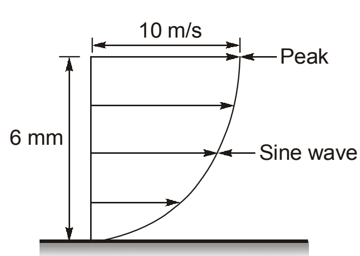
which gives shearing stress profile

Table: gives computed values for velocity profile and shear stress profile.

R	*v*	τ	Remarks
0	6	0	Centerline velocity
0.1	5.76	0.0096	
0.2	5.04	0.0192	
0.3	3.84	0.0283	
0.4	2.16	0.0384	
0.5	0	0.048	Boundary

Solution : 26

Shear stress is given by



Velocity distribution

$$\tau = \mu \frac{dv}{dy}$$

Given, $v = v_{max} \sin\left[\frac{\pi y}{2\delta}\right] = 2618 \cos(261.8y)$

at, $y = 0, \tau = 0.04739 \cos\{261.8(0)\}$

$= 0.0474 \text{ N/m}^2$

at $y = 3 \text{ mm}, \tau = 0.0335 \text{ N/m}^2$

at $y = 6 \text{ mm}, \tau = 0$

Solution : 27

The torque applied = the resisting torque by the fluid

= shear stress × surface area × radius

Hence, at any radial location r from the axis of rotation

$$0.88 = \tau \times (2\pi r \times 0.3)r$$

or $\tau = \dfrac{0.467}{r^2}$

we have $\tau = \mu \dfrac{dv}{dy}$

Therefore, $\dfrac{dv}{dy} = \dfrac{\tau}{\mu} = \dfrac{0.467}{\mu r^2}$

Rearranging the above expression and substituting ($-dr$) in place of dy (the minus sign indicates that r, the radial distance, decreases as v increase), we obtain

$$\int_{v_{outer}}^{v_{inner}} dv = \frac{0.467}{\mu} \int_{0.13}^{0.12} -\frac{dr}{r^2}$$

Hence,

$$(v_{inner} - v_{outer}) = \frac{0.467}{\mu} \left\{\frac{1}{r}\right\}_{0.13}^{0.12}$$

But $v_{inner} = 0.754$ m/s i.e., $(2 \times \pi \times 0.12)$

$v_{outer} = 0$ m/s (fixed)

Therefore, substituting the above values, we get

$$(0.754 - 0) = \frac{0.467}{\mu}\left[\frac{1}{0.12} - \frac{1}{0.13}\right]$$

or $\mu = 0.397$ Pa.s

Solution : 28

Let the radius of the bubble at any time be R, then the change in volume, $dV = 4\,\pi R^2 dR$ and the work done in blowing the soap bubble of radius R_0 is given by

$$W = \int_0^{R_0} (\Delta P)dv = \int_0^{R_0} \frac{4\sigma}{R} 4\pi R^2 dR$$

$$= 8\pi\sigma R_0^2 = 0.006$$

$$\therefore \qquad R_0 = \left[\frac{0.006}{8\pi \times 0.08}\right]^{1/2} = 0.0546 \text{ m}$$

$$= 54.6 \text{ mm}$$

Force required to separate the bubble into two identical halves is given by

$$F = 4\pi R_0\,\sigma = 4\pi \times 54.6 \times 10^{-3} \times 0.08 = 0.05489 \text{ N}$$

and

$$\Delta P = \frac{4\sigma}{R_0} = \frac{4 \times 0.08}{54.6 \times 10^{-3}}$$

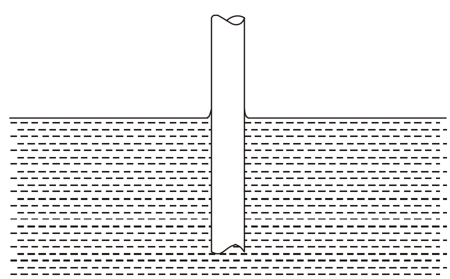
$$= 5.86 \text{ N/m}^2$$

Or, the pressure inside the bubble is more than the outside pressure by 5.86 Pa.

Solution : 29

The difference in pressure between inside and outside the air bubble is given by

$$\Delta P = \frac{2\sigma}{R} = \frac{2 \times 0.0725}{0.001} = 145 \text{ N/m}^2$$



The pressure at the outside surface of the air bubble at a depth of 15 mm below the free surface is given by ρgh,

$$\therefore \qquad p_{outer} = 10^3 \times 9.81 \times 0.015 = 147.15 \text{ N/m}^2$$

$\therefore$ the pressure required to blow the air bubble, or the inside pressure,

$$p_i = p_0 + \Delta p = 147.15 + 145$$

$$= 292.15 \text{ N/m}^2 = 292.15 \text{ Pa}$$

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2 Fluid Statics

LEVEL 1 Objective Questions

1.	(b)
2.	(c)
3.	(b)
4.	(b)
5.	(c)
6.	(b)
7.	(c)
8.	(c)
9.	(d)
10.	(a)
11.	(a)
12.	(d)

LEVEL 2 Objective Questions

13.	(109.9)
14.	(a)
15.	(c)
16.	(d)
17.	(a)
18.	(a)
19.	(0.338)
20.	(33.5)
21.	(a)

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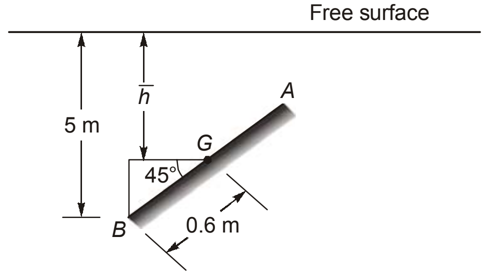
LEVEL 3 Conventional Questions

Solution : 22

Given data: Width of the gate; b = 5 m; Depth of the gate; d = 1.2 m; Inclination of gate; θ = 45°



$\therefore$ Area; $A = b \times d = 5 \times 1.2 = 6\ \text{m}^2$

Depth of CG of the gate from free surface of the water = $\bar{h}$;

$$\bar{h} = 5 - 0.6 \sin 45° = 4.5757\ \text{m}$$

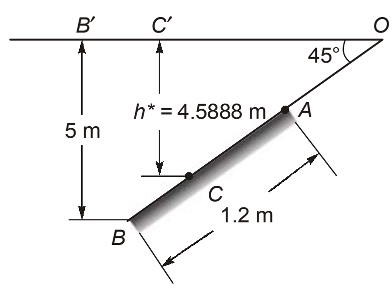
The total pressure force (F) acting on the gate:

$$F = \rho g A\bar{h} = 1000 \times 9.81 \times 6 \times 4.5757 = 269325.702\ \text{N}$$

The position of the centre of pressure from free surface of liquid : h^*

$$h^* = \bar{h} + \frac{I_G \sin^2\theta}{A\bar{h}}$$

where I_G = MOI of rectangular gate about centroid



$$= \frac{bd^3}{12} = \frac{5 \times 1.2^3}{12} = 0.72\ \text{m}^4$$

$$\therefore \quad h^* = 4.5757 + \frac{0.72 \times \sin^2 45°}{6 \times 4.5757} = 4.5888\ \text{m}$$

From $\Delta COC'$; we get

$$\sin 45° = \frac{CC'}{CO}$$

or

$$CO = \frac{CC'}{\sin 45°} = \frac{4.5888}{0.7071}$$

$$CO = 6.4896\ \text{m}$$

and from $\Delta BOB'$, we get

$$\sin 45^\circ = \frac{BB'}{BO}$$

or $$BO = \frac{BB'}{\sin 45^\circ} = \frac{5}{0.7071}$$

$$BO = 7.0711 \text{ m}$$

$\therefore$ $$BC = BO - CO \quad \text{(from figure)}$$

$$= 7.0711 - 6.4896 = 0.5815 \text{ m}$$

and $$CA = BA - BC = 1.2 - 0.5815 = 0.6185 \text{ m}$$

Now taking the moments about the hinge A, we get

$$F_1 \times BA = F \times CA$$

$$F_1 = \frac{F \times CA}{BA} = \frac{269325.702 \times 0.6185}{1.2} = 138814.95 \text{ N}$$

Solution : 23

Let 'V' be the total volume of the iceberg

visible volume of the iceberg above the water level is 600 m^3 and volume of ice in sea water = (V – 600) m^3

density of sea water, $\rho_s = 1025 \text{ kg/m}^3$

density of iceberg, $\rho_i = 915 \text{ kg/m}^3$

we know that

$$\text{weight of iceberg} = \text{weight of water displaced by ice.}$$

$$\rho_i \times V \times g = \rho_s \times (V - 600) \times g$$

$$915\,V = 1025\,(V - 600)$$

$$V = 5590.9 \text{ m}^3 \text{ i.e. total volume}$$

$\therefore$ $$\text{weight of iceberg} = \rho_i \times V \times g$$

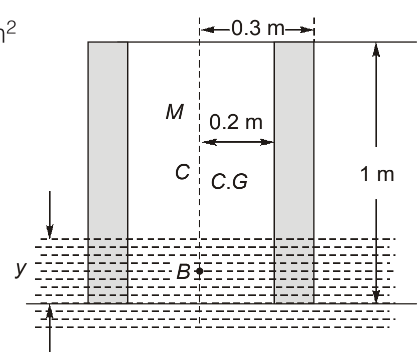
$$= 915 \times 5590.9 \times 9.81$$

$$= 50.184 \times 10^6 \text{ N.}$$

Solution : 24

Cross section of cylinder

$$A = \pi(r_2^2 - r_1^2) = \pi(0.3^2 - 0.2^2) = 0.15708 \text{ m}^2$$



Let the y depth is submerged in water.

$\therefore$ $$700 = 0.15708\,y \times 9.81 \times 1000$$

$$y = 0.4543 \text{ m}$$

For stable equilibrium in vertical direction

$$MC = MB - CB > 0$$

$$MB = \frac{I}{\forall}$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = \frac{\pi}{4}\cdot(r_2^4 - r_1^4) = \frac{\pi}{4}(0.3^4 - 0.2^4) = 5.105 \times 10^{-3} \text{ m}^4$$

and $\forall = A.y$ = volume of displaced liquid

$= 0.15708 \times 0.4543 = 0.07136 \text{ m}^3$

$$\therefore \quad BC = \frac{h}{2} - \frac{y}{2} = 0.5 - \frac{0.4543}{2} = 0.27285$$

$$MC = \frac{I}{\forall} - BC = \frac{5.105 \times 10^{-3}}{0.07136} - 0.27285 = 0.07154 - 0.27285 = -0.20135 \text{ m}$$

The metacentric height is negative. Hence cylinder will not float with vertical axis.

Solution : 25

Writing the manometric equation, we get

$h_A - 0.75 \times 0.8 - 0.25 \times 13.6 = 0$

Therefore, $h_A = 4$ m of water

Solution : 26

Here, the unknown height is H.

Applying the manometric equation from A, we get

$0 + (0.2 \times 13.6) - 0.15 - x = 0$

$x = 2.57$ m

and $H = x + 0.3 = 2.87$ m

Solution : 27

Pressure at A = Pressure at B = pressure at C

So,
$$\frac{p_M}{\gamma} + h = \frac{p_N}{\gamma} + \{h - (120 - 10)\} + (120 - 10)13.6$$

$$= \frac{p_R}{\gamma} + \{h - (160 - 10)\} + (160 - 10) \times 13.6$$

or
$$\frac{p_M}{\gamma} - \frac{p_N}{\gamma} = 110 \times 12.6 = 1386 \text{ cm} = 13.86 \text{ m}$$

or
$$\frac{p_N}{\gamma} - \frac{p_R}{\gamma} = 18.9 - 13.86 = 5.04 \text{ m}$$

Finally,
$$\frac{p_M}{\gamma} - \frac{p_R}{\gamma} = 18.9 \text{ m}$$

Solution : 28

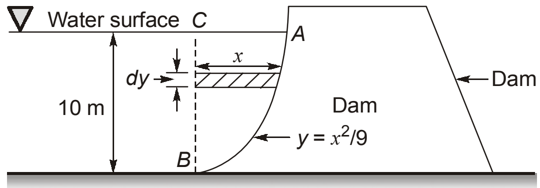
Equation of curve AB is

$$y = \frac{x^2}{9}$$

or
$$x^2 = 9y$$

$$x = \sqrt{9y} = 3\sqrt{y}$$

Data given : $h = 10$ m, $b = 1$ m



$$F_x = \rho g A\bar{h}$$

$$= 1000 \times 9.81 \times (10 \times 1) \times 5 = 490.5 \text{ kN}$$

F_y = weight of water supported by the curve AB

$$F_y = \rho g \left\{ \int_0^{10} x.dy \right\} \times 1.0 = 1000 \times 9.81 \left\{ \int_0^{10} 3\sqrt{y}dy \right\} \times 1.0 = 620.439 \text{ kN}$$

Resultant water pressure in the dam

$$F = \sqrt{F_x^2 + F_y^2} = 790.907 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 51°40'$$

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3 Fluid Kinematics

LEVEL 1 Objective Questions

1.	(b)
2.	(c)
3.	(b)
4.	(c)
5.	(d)
6.	(d)
7.	(c)
8.	(b)
9.	(c)
10.	(d)
11.	(b)
12.	(b)
13.	(c)

LEVEL 2 Objective Questions

14.	(b)
15.	(8)
16.	(8)
17.	(c)
18.	(8)
19.	(d)
20.	(b)
21.	(c)
22.	(a)
23.	(a)
24.	(0)

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LEVEL 3 Conventional Questions

Solution : 25

(i) From the continuity equation in one-dimensional flow

$$Q = AV$$

∴ Velocity at section (1)

$$V_1 = \frac{Q}{\frac{\pi}{4}(d_1)^2} = \frac{4Q}{\pi d_1^2}$$

Similarly,
$$V_2 = \frac{Q}{\frac{\pi}{4}d_2^2} = \frac{4Q}{\pi d_2^2} \text{ and } V_3 = \frac{Q}{\frac{\pi}{4}d_3^2}$$

(ii)
$$V_1 = \frac{0.3}{\frac{\pi}{4}\times(0.4)^2} = 2.39 \text{ m/s}$$

$$V_2 = \frac{0.3}{\frac{\pi}{4}\times(0.2)^2} = 9.55 \text{ m/s and } V_3 = \frac{0.3}{\frac{\pi}{4}\times(0.75)^2} = 0.679 \text{ m/s}$$

(iii) From the principle of conservation of mass, mass rate of flow in a continuous fluid is the same, thus

$$\rho_1 V_1 \frac{\pi}{4} d_1^2 = \rho_2 V_2 \frac{\pi}{4} d_2^2 = \rho_3 V_3 \frac{\pi}{4} d_3^2$$

it is given that at section (1), $Q = 0.3$ m^3/s, hence,

$$\rho_1 Q_1 = \rho_2 V_2 \frac{\pi}{4} d_2^2$$

$$V_2 = \frac{\rho_1 \times 0.3}{0.5\rho_1 \times \frac{\pi}{4}(0.2)^2} = 19.10 \text{ m/s}$$

also
$$\rho_1 Q_1 = \rho_3 \frac{\pi}{4} d_3^2 \cdot V_3$$

∴
$$V_3 = \frac{\rho_1 \times 0.3}{1.3\rho_1 \times \frac{\pi}{4}(0.75)^2} = 0.52 \text{ m/s and } \rho_1 Q_1 = \rho_1 \frac{\pi d^2}{4} V_1$$

$$V_1 = \frac{4Q_1}{\pi d_1^2} = \frac{4\times 0.3}{\pi\times(0.4)^2} = 2.39 \text{ m/s}$$

Solution : 26

(i) Local Acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field. In the equations

$$a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ are known as local accelerations

Convective acceleration is defined as the rate of change of velocity due to the changed position of fluid particles in fluid flow. The expressions other than $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ in the above equations are known as convective accelerations.

(ii) $V = 3xy^2\hat{i} + 2xy\hat{j} + (2zy + 3t)\hat{k}$

Rotational velocity vector is $\vec{\omega} = \omega_x\hat{i} + \omega_j\hat{j} + \omega_z\hat{k}$

where
$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial y}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

and $u = 3xy^2$; $v = 2xy$, $w = 2zy + 3t$

$\Rightarrow$
$$\omega_x = \frac{1}{2}[2z - 0] = 3$$

$$\omega_y = \frac{1}{2}[0 - 0] = 0 \qquad \text{[At (1, 2, 1) and at } t = 3)]$$

$$\omega_z = \frac{1}{2}[2y - 6xy] = y - 3xy = -4$$

$\therefore$
$$\vec{\omega} = 1\hat{i} + 0\hat{k} + (-4)\hat{k} = \hat{i} - 4\hat{k}$$

Solution : 27

$$u = -2xy,\ v = y^2 - x^2,\ w = 0$$

Continuity equation,
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$\Rightarrow$
$$-2y + 2y = 0$$

Satisfies continuity equation hence represent possible flow field.

$$\rho\left[\frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\rho[(-2xy) \times (-2y) + (y^2 - x^2) \times (-2x)] = -\frac{\partial p}{\partial x} + 0$$

$$\rho\left[2x^2y^2+\frac{x^4}{2}-x^2y^2\right]+f(y)=-p$$

Differentiate w.r.t. to y

$$\rho[4x^2y-2x^2y]+f'(y)=-\frac{\partial p}{\partial y}$$

$$\rho\left[\frac{u\partial v}{\partial x}+\frac{v\partial v}{\partial y}\right]=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]$$

$$\rho[(-2xy)\times -2x+(y^2-x^2)\times 2y)]=\rho[4x^2y-2x^2y]+f'(y)+\mu[-2+2]$$

$$\rho[4x^2y+2y^3-2x^2y]=\rho[4x^2y-2x^2y]+f'(y)$$

$$\rho(2y^3)=f'(y)$$

$$f'(y)=\rho(2y^3)$$

$$f(y)=\rho\left(\frac{y^4}{2}\right)+C$$

$$-P=\rho\left[2x^2y^2+\left(\frac{x^4}{2}\right)-x^2y^2\right]+\rho\left(\frac{y^4}{2}\right)+C$$

$$-P=\rho\left[2x^2y^2+\left(\frac{x^4+y^4}{2}\right)-x^2y^2\right]+C$$

At, $x=0,\ y=0,\ P=P_a$

$$-P_a=C$$

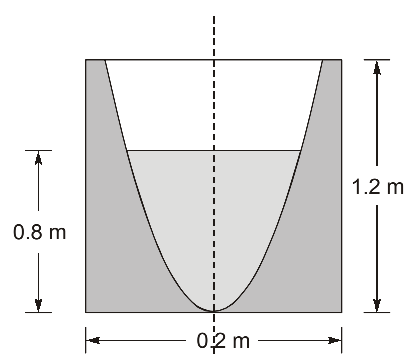
$$-P=\rho\left[2x^2y^2+\frac{x^4}{2}+\frac{y^4}{2}-x^2y^2\right]-P_a$$

$$P_a-P=\rho\left[2x^2y^2+\frac{x^4}{2}+\frac{y^4}{2}-x^2y^2\right]=\rho\left[\frac{x^4}{2}+\frac{y^4}{2}+x^2y^2\right]$$

Solution : 28

(i) Given: $D=0.2$ m, $L=1.2$ m

Let the angular speed, when axial depth of water is zero, be ω



$$Z=\frac{\omega^2r^2}{2g}$$

$$\omega^2r^2=2\times 1.2\times 9.81$$

$$=23.52\text{ m} \quad ...(i)$$

Volume of air before rotation = Volume of air after paraboloid

$\pi R^2(1.2-0.8)$ = Volume of paraboloid

$$\pi\times 0.1^2\times 0.4=\frac{1}{2}(\pi r^2)\times Z=\left(\frac{\pi r^2}{2}\right)\times 1.2$$

$$r^2=6.67\times 10^{-3} \quad ...(ii)$$

From (i) & (ii) $\omega\times(6.67\times 10^{-3})=23.52$ ($\omega=59.4$ rad/sec) and $$\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times 59.4}{2\pi} = 567.22 \text{ rpm}$$

Solution : 29

For one-dimensional flow, the continuity equation can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$$

This is, $$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = -\frac{\partial u}{\partial x} = -[(1-y^2)\sin \omega t]$$

$$\frac{d\rho}{\rho} = -(1 - y^2) \sin \omega t dt$$

Upon integration, we get

$$In(\rho) = (1-y^2)\frac{\cos \omega t}{\omega} + f(y)$$

At $t = \frac{\pi}{2\omega}, \rho = \rho_0$. Substituting these in the above equation, we get

$$In\ (\rho_0) = \frac{(1-y^2)\cos\left(\frac{\omega\pi}{2\omega}\right)}{\omega} + f(y)$$

That is $$In\ p_0 = f(y)$$

Therefore, $$In\ (\rho) = (1-y^2)\frac{\cos \omega t}{\omega} + In\rho_0$$

That is, $$\frac{\rho}{\rho_0} = \exp[(1-y^2)\cos \omega t]/\omega$$

Solution : 30

If u, v and w satisfy the equation of continuity, then it will be possible liquid motion.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2yz.\frac{3x^2 - y^2}{(x^2+y^2)^3} + 2yz.\frac{y^2 - 3x^2}{(x^2+y^2)^3} + 0 = 0$$

This shows that the liquid motion is possible.

For irrotational motion, we have

$$\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} = 0$$

$$\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = 0$$

and $$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$$

So, $$\frac{\partial v}{\partial z}-\frac{\partial w}{\partial y} = \frac{x^2-y^2}{(x^2+y^2)^2}-\frac{x^2-y^2}{(x^2+y^2)^2}=0$$

Also, $$\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z} = \frac{-2xy}{(x^2+y^2)^2}+\frac{2xy}{(x^2+y^2)^2}=0$$

and $$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x} = \frac{2xz(3y^2-x^2)}{(x^2+y^2)^3}-\frac{2xz(3y^2-x^2)}{(x^2+y^2)^3}=0$$

which satisfy the condition. Hence the motion is irrotational.

Solution : 31

Here, $u = 2x^3$ and $v = -6x^2y$

Therefore, $$\frac{\partial u}{\partial x} = 6x^2 \text{ and } \frac{\partial v}{\partial y}=-6x^2$$

Therefore, the continuity equation becomes,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 6x^2 - 6x^2 = 0$$

Hence, the flow is physically possible.

To find rotational or irrotational:

$$\frac{\partial u}{\partial y} = 0 \quad \text{and} \quad \frac{\partial v}{\partial x}=-12xy$$

Because $\frac{\partial u}{\partial y}\neq\frac{\partial v}{\partial x}$, the flow is rotational.

To find angular velocity;

$$\omega_z = \frac{1}{2}\left\{\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right\}=-6xy$$

Note at the origin (0, 0), because $\omega_z = 0$, the flow is rotational except at the origin.

To find vorticity :

vorticity, $\xi = 2\omega = 2(-6xy) = -12xy$

To find shear strain:

$$\gamma_{xy} = \frac{1}{2}\left\{\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right\}=-6xy$$

Note: The negative sign for angular velocity and shear strain indicates clockwise motion.

To find dilatancy (linear strain):

$$\epsilon_x = \frac{\partial u}{\partial x}=6x^2$$

$$\epsilon_y = \frac{\partial v}{\partial y}=-6x^2$$

To find the circulation:

Circulation $\Gamma = \xi \times \text{area of circle} = -12xy(\pi a^2)\ \text{m}^2/\text{s}$

where a is the radius of circle $x^2 + y^2 - 2ay = 0$

■■■■

4 Fluid Dynamics & Flow Measurement

LEVEL 1 Objective Questions

1.	(c)
2.	(d)
3.	(12.11)
4.	(d)
5.	(c)
6.	(d)
7.	(c)
8.	(b)
9.	(34.90)
10.	(a)
11.	(d)
12.	(c)

LEVEL 2 Objective Questions

13.	(2.47)
14.	(80.79)
15.	(137500)
16.	(d)
17.	(a)
18.	(d)
19.	(d)
20.	(25.56)
21.	(55.45)
22.	(0.9848)
23.	(135)
24.	(b)
25 .	(b)
26 .	(8.76)

■■■■

LEVEL 3 Conventional Questions

Solution : 27

Given data: $P_1 = 1.02 \times 10^5$ N/m^2; $T_1 = 28°C = 30$ K; $R = 287$ kJ/kgK; $V_1 = 50$ m/s; $V_2 = 420$ m/s

$$\rho_{air} = \frac{P}{RT} = \frac{1.02\times10^5}{287\times(273+28)} = 1.1807 \text{ kg/ m}^3$$

$$\Delta P = (\rho_m - \rho_{air})gh$$

$$V = \sqrt{\frac{2\Delta P}{\rho_{air}}}$$

$\therefore$ $$\Delta P = \frac{\rho V^2}{2} = (\rho_m - \rho_a)gh$$

$\therefore$ $$h = \frac{V^2}{2g}\frac{1}{\left(\frac{\rho_m}{\rho_{air}} - 1\right)}$$

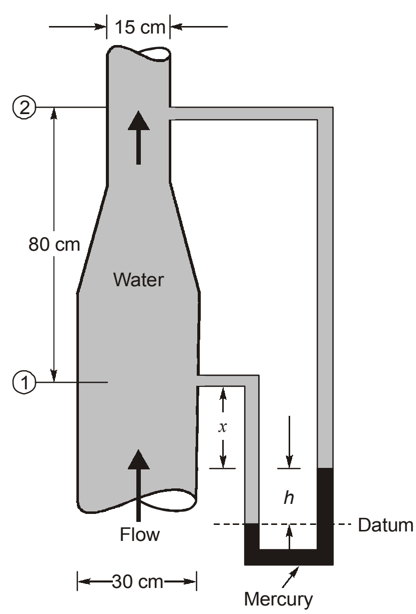
We have $$\rho_m = 13.6 \times 1000 = 13600 \text{ kg/m}^3$$
$$\rho_{air} = 1.1807 \text{ kg/m}^3$$

(i) $$h = \frac{2500}{2\times9.81}\frac{1}{\left(\frac{13600}{1.1807} - 1\right)} \text{ m} = 127.42\times\frac{1}{11517.6} = 0.011 \text{ m}$$

$\therefore$ $$h = 11 \text{ mm}$$

(ii) $$h = \frac{(420)^2}{2\times9.81}\frac{1}{\left(\frac{13600}{1.1807} - 1\right)} \text{m} = \frac{176400}{2\times9.81\times11517.6} = 0.78 \text{ m or } 780 \text{ mm}$$

Solution : 28



Let $\quad S =$ Relative density of mercury.

For the manometer :

$$\frac{p_1}{\rho g} + x + h = \frac{p_2}{\gamma} + 0.8 + x + Sh$$

$$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 0.8 = (S - 1)\, h = (13.6 - 1)h = 12.6\, h \quad \text{. . . (i)}$$

By continuity equation, $\quad Q = \frac{\pi}{4} \times (0.30)^2 \times V_1 = \frac{\pi}{4} \times (0.15)^2 \times V_2 = 0.120 \ \ \text{m}^3/\text{s}$

$$V_1 = 1.6977 \text{ m/s}, \ V_2 = 6.79 \text{ m/s}$$

Applying Bernoulli's equation at points 1 and 2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 0.8 = \frac{V_2^2 - V_1^2}{2g} = \frac{(6.79)^2 - (1.6977)^2}{2 \times 9.81} = 2.203$$

$$\left(\frac{p_1 - p_2}{\rho g}\right) - 0.8 = 12.6 \text{ h} = 2.203$$

$\therefore$ $$h = \frac{2.203}{12.6} = 0.175 \text{ m} = \mathbf{17.5 \text{ cm}}$$

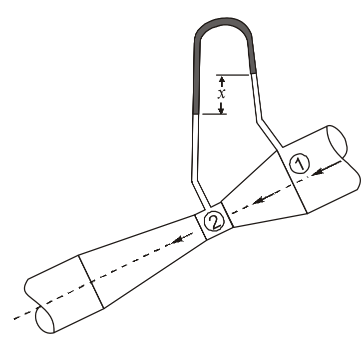
Deflection : As the value of *'h'* comes positive, hence our assumed direction is correct i.e. the manometer limb connected to section 1 will be having a smaller column of mercury than the other limb.

Solution : 29

Given data: Diameter at inlet; $D = 200$ mm $= 0.2$ m;

$\therefore$ Cross-sectional area at inlet,

$$A = \frac{\pi}{4} D^2 = \frac{3.14}{4} \times (0.2)^2 = 0.0314 \text{ m}^2$$



$\therefore$ Cross-sectional area at throat :

$$a = \frac{\pi}{4} d^2 = \frac{3.14}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$$

Specific gravity of liquid used in manometer : $S = 0.75$

$\therefore$ Density of manometer liquid :

$$\rho = S \times 1000 = 0.75 \times 1000 = 750 \text{ kg/m}^3$$

Manometer reading :

$$x = 300 \text{ mm of manometer liquid} = 0.3 \text{ m of manometer liquid}$$

Difference of pressure head:

$$h = x\left[1-\frac{\rho_{mano}}{\rho_{pipe}}\right]$$

for $\rho_{pipe} > \rho_{mano}$

where ρ_{mano} = density of liquid used in manometer

$\therefore$ ρ_{pipe} = density of liquid flow through pipe

$$h = 0.3\left[1-\frac{750}{1000}\right] = 0.3 \times 0.25 = 0.075 \text{ m of water}$$

also
$$h = \left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)$$

Loss of head:
$$h_L = 0.3 \times \text{kinetic head of the pipe}$$
$$= 0.3\times\frac{V_1^2}{2g}$$

Now applying the modified Bernoulli's equation for real fluid at sections (1) and (2), we get

$$\frac{p}{\rho g}+\frac{V_1^2}{2g}+z_1 = \frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_L$$

or
$$\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)+\frac{V_1^2}{2g} = \frac{V_2^2}{2g}+0.3\frac{V_1^2}{2g}$$

$$0.075+\frac{V_1^2}{2g}-0.3\frac{V_1^2}{2g}-\frac{V_2^2}{2g} = 0$$

$$0.075+0.7\frac{V_1^2}{2g}-\frac{V_2^2}{2g} = 0 \qquad \ldots(i)$$

Applying the continuity equation at sections (1) and (2), we get

$$A_1V_1 = aV_2$$
$$0.0314\,V_1 = 0.00785\,V_2$$

or $4V_1 = V_2$

or $V_2 = 4V_1$

Putting the value of V_2 in equation (i), we get

$$0.075+0.7\frac{V_1^2}{2g}-\frac{(4V_1)^2}{2g} = 0$$

$$0.075+0.7\frac{V_1^2}{2g}-\frac{16V_1^2}{2g} = 0$$

$$0.075-15.3\frac{V_1^2}{2g} = 0$$

or
$$0.075 = 15.3\frac{V_1^2}{2g}$$

or
$$\frac{0.075\times 9.81}{15.3} = V_1^2$$

or
$$V_1^2 = 0.096176$$

$$V_1 = 0.310 \text{ m/s}$$

$\therefore$ Discharge: $Q = AV_1$

$$= 0.0314 \times 0.310 = 0.009734 \text{ m}^3\text{/s} = \mathbf{9.734 \text{ litre/s}}$$

Solution : 30

$$V = \frac{Q}{A} = \frac{0.05}{\frac{\pi}{4} \times 0.15^2} = 2.83 \text{ m/s}$$

$$p = \gamma h = 13.6 \times 9.81 \times (-0.15) = -20.01 \text{ kN/m}^2$$

$$H = z + \frac{V^2}{2g} + \frac{p}{\gamma} = -1.2 + \frac{2.83^2}{2 \times 9.81} - \frac{20.01}{0.9 \times 9.81} = -3.058 \text{ m}$$

Solution : 31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + h_L$$

and $$0 + 0 + 5 = 0 + \frac{V_3^2}{2g} + 0 + 3.9$$

Therefore, $$V_3 = 4.645 \text{ m/s}$$

$$Q = A_3 V_3 = \frac{\pi \times 0.03^2}{4} \times 4.645$$

$$= 3.28 \times 10^{-3} \text{ cumecs}$$

Applying Bernoulli's equation between point (1) and (2), we get

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

and $$0 + 0 + 5 = \frac{p_2}{\gamma} + \frac{4.645^2}{2 \times 9.81} + 7 + 1.5$$

$$\frac{p_2}{\gamma} = -4.6 \text{ m}$$

and $$p_2 = 0.9 \times 9.81(-4.6)$$

$$= -40.61 \text{ kN/m}^2$$

Solution : 32

Applying Bernoulli's equation between point A and top of the jet T, we get

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{p_T}{\gamma} + \frac{V_T^2}{2g} + z_T + h_L$$

here $$\frac{p_A}{\gamma} = \frac{55}{9.81} = 5.6 \text{ m}, \frac{p_T}{\gamma} = 0, \frac{V_T^2}{2g} = 0,$$

$$z_T = 1.1 + h, \text{ and } h_L = 0 \text{ (given)}$$

Therefore, $$5.6 + \frac{V_A^2}{2g} + 0 = 0 + 0 + (1.1 + h) + 0$$

$$h = 4.5 + \frac{V_A^2}{2g}$$

Applying Bernoulli's equation between point A and N, we get

$$5.6 + \frac{V_A^2}{2g} + 0 = 0 + \frac{V_N^2}{2g} + 1.1 + 0$$

From continuity equation, we have

$$A_A V_V = A_N V_N$$

here, $A_A = 0.03142 \text{ m}^2$

and $A_N = 0.007854 \text{ m}^2$

Therefore, $V_N = 4.0\, V_A$

Substituting this in the above equation we get

$$5.6 + \frac{V_A^2}{2\times 9.81} = \frac{(4.0V_A)^2}{2\times 9.81} + 1.1$$

$$V_A = 2.426 \text{ m/s}$$

Hence, $h = 4.5 + \dfrac{2.426^2}{2\times 9.81} = 4.7999 \text{ m}$

Solution : 33

Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2U_0^2}\sec^2\theta = U_0\sin\theta \times \frac{x}{U_0\cos\theta} - \frac{1}{2}g\left(\frac{x}{U_0\cos\theta}\right)^2$$

$$= 5\times\sin 60^\circ + \frac{x}{5\times\cos 60^\circ} - \frac{1}{2}\times 9.81\left(\frac{x}{5\times\cos 60^\circ}\right)^2$$

$$= 1.73x - 0.7848x^2$$

Maximum elevation of the jet

$$s = \frac{U_0^2\sin^2\theta}{2g} = \frac{25\times(\sin 60)^2}{2\times 9.81} = 0.955 \text{ m}$$

Maximum horizontal distance

$$0 = 1.73x - 0.7848x^2$$

or $x(1.73 - 0.7848x) = 0$

That is, $x = 0$ or $x = 2.2$ m

So maximum horizontal distance = 2.2 m.

Solution : 34

The static pressure at a depth of 15 m below the sea surface given by $p = \rho gh$

$$= 1026 \times 9.81 \times 15 = 150.976 \text{ kPa gauge}$$

$$\text{The dynamic pressure} = \frac{\rho V^2}{2} = 1026 \times \frac{\left(\dfrac{16\times 10^3}{3600}\right)^2}{2} = 10.133 \text{ kPa}$$

$\therefore$ stagnation pressure = 150.976 + 10.133

= 161.109 kPa gauge

Solution : 35

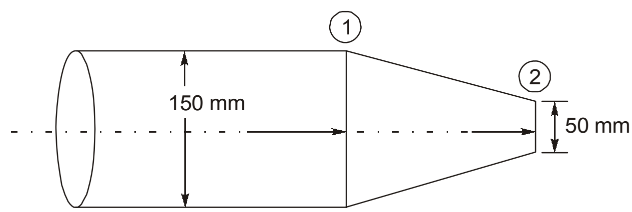
The density of air at 1 km is

$$\rho = \frac{p}{RT} = \frac{0.9\times10^5}{287\times278} = 1.128 \text{ kg/m}^3$$

$$V_1 = \sqrt{\frac{2(\Delta p)}{\rho}} = \sqrt{\frac{2\times0.02\times10^5}{1.128}} = 59.55 \text{ m/s} = 214.37 \text{ kmph}$$

Solution : 36

A nozzle is a converging passage as shown in figure. From the continuity equation



$$V_1 = \frac{Q}{\text{area}} = \frac{40\times10^{-3}}{\frac{\pi}{4}(0.15)^2} = 2.26 \text{ m/s}$$

$$V_2 = \frac{40\times10^{-3}}{\frac{\pi}{4}(0.05)^2} = 20.37 \text{ m/s}$$

From the linear momentum equation,

$$\Sigma F_x = \dot{m}(V_2 - V_1)$$

or $$p_1A_1 - p_2A_2 + F = \dot{m}(V_2 - V_1) = \rho Q(V_2 - V_1)$$

$$200\times10^3 \times \frac{\pi}{4}(0.15)^2 - 0 + F = 10^3 \times 40 \times 10^{-3} \,(20.37 - 2.26)$$

$\therefore$ $F = -2810$ N

[In this problem, $p_2 = 0$ has been assumed as if $p_1 = 200$ kPa = above p_2]

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5 Dimensional Analysis

LEVEL 1 Objective Questions

1.	(d)
2.	(a)
3.	(c)
4.	(c)
5.	(b)
6.	(c)
7.	(a)
8.	(c)
9.	(c)
10.	(c)
11.	(c)
12.	(a)

LEVEL 2 Objective Questions

13.	(c)
14.	(d)
15.	(a)
16.	(b)
17.	(d)
18.	(a)
19.	(d)

LEVEL 3 Conventional Questions

Solution : 20

Buckingham's π-theorem states that "If there are n-variable (Independent and dependent) in a physical phenomenon and if these variable contain m fundamental dimensions then the variables are arranged into $(n - m)$ dimensionless terms which are called π-terms".

Let $X_1, X_2, X_3 \ldots\ldots X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3, X_4 \ldots X_n$ are the independent variables on which X_1 depends. Mathematically it can be written as

$$X_1 = f(X_2, X_3, X_4, \ldots. X_n)$$

and, $\quad f(X_1, X_2, X_3, X_4 \ldots X_n) = 0$

Hence, $\quad f(\pi_1, \pi_2, \pi_3 \ldots. \pi_{n-m}) = 0$

$$\text{Drag force, } F = f(V, L, K, \rho, g)$$

Here,
$$n = 6$$
$$m = 3$$
$$\text{Number of } \pi\text{-terms} = n - m = 3$$

Let these be π_1, π_2 and π_3 .

The selected repeating variables is (V, L, ρ)

$$\pi_1 = V^{a_1} L^{b_1} \rho^{c_1} F$$

$$M^\circ L^\circ T^\circ = \left(LT^{-1}\right)^{a_1} (L)^{b_1} \left(ML^{-3}\right)^{c_1} MLT^{-2}$$

$$M^\circ L^\circ T^\circ = M^{c_1+1} L^{a_1+b_1-3c_1+1} T^{-a_1-2}$$

$$-a_1 - 2 = 0$$
$$\Rightarrow \quad a_1 = -2$$
$$c_1 + 1 = 0$$
$$\Rightarrow \quad c_1 = -1$$
$$a_1 + b_1 - 3c_1 + 1 = 0$$
$$\Rightarrow \quad b_1 = -2$$

$$\pi_1 = V^{-2} L^{-2} \rho^{-1} F = \frac{F}{\rho L^2 V^2}$$

$$\pi_2 = V^{a_2} L^{b_2} \rho^{c_2} K$$

$$M^\circ L^\circ T^\circ = \left(LT^{-1}\right)^{a_2} (L)^{b_2} \left(ML^{-3}\right)^{c_2} L$$

$$= M^{c_2} L^{a_2+b_2-3c_2+1} T^{-a_2}$$

$$c_2 = 0$$
$$a_2 = 0$$
and $\quad a_2 + b_2 - 3c_2 + 1 = 0$
$$\Rightarrow \quad b_2 = -1$$

$$\pi_2 = L^{-1} K = \frac{K}{L}$$

Similarly, $\quad \pi_3 = V^{a_3} L^{b_3} \rho^{c_3} g$

$$M^\circ L^\circ T^\circ = \left(LT^{-1}\right)^{a_3} L^{b_3} \left(ML^{-3}\right)^{c_3} LT^{-2}$$

$$= M^{c_3} L^{a_3+b_3-3c_3+1} T^{-a_3-2}$$

$$c_3 = 0$$

$$a_3 = -2$$

and, $a_3 + b_3 - 3c_3 + 1 = 0$

$\Rightarrow$ $b_3 = 1$

$$\pi_3 = V^{-2}Lg = \frac{gL}{V^2}$$

Since, $f_1(\pi_1, \pi_2, \pi_3) = 0$

$$f_1\left(\frac{F}{\rho V^2 L^2}, \frac{K}{L}, \frac{gL}{V^2}\right) = 0$$

$\therefore$ $F\text{(Drag force)} = \rho L^2 V^2 \phi\left(\frac{K}{L}, \frac{gL}{V^2}\right)$

Solution:21

Given: Total output = 200 MW

Number of turbines = 4

$$\text{Output of each turbine} = \frac{200}{4} = 50 \text{ MW}$$

$$\eta_p = 90\%$$

$$N_p = 100 \text{ rpm}$$

$$H_p = 60 \text{ m}$$

Data for the proposed model

$$Q_m = 400 \text{ lit/sec} = 0.4 \text{ m}^3\text{/sec}$$

$$H_m = 5 \text{ m}$$

$$\eta_m = 0.87$$

$$\text{For prototype, } \eta_p = \left(\frac{P}{\rho gQH}\right)_P$$

$$0.9 = \frac{200 \times 10^6}{10^3 \times 9.81 \times Q \times 60}$$

Total discharge, $Q_p = 377.54 \text{ m}^3\text{/sec}$

Discharge through each turbine

$$(Q_p)' = \frac{Q_p}{4}$$

$$Q_p' = \frac{377.54}{4} = 94.385 \text{ m}^3\text{/sec}$$

For Model

$$\eta_m = \left(\frac{P}{\rho gQH}\right)_P$$

Power developed by each model

$$(P_m) = 0.87 \times 10^3 \times 9.81 \times 0.4 \times 5$$
$$P_m = 17.06 \text{ kW}$$

As,
$$\frac{Q_m}{N_m D_m^3} = \frac{Q_p}{N_p D_p^3}$$
$$\frac{0.4}{N_m D_m^3} = \frac{94.385}{N_p D_p^3}$$

or,
$$\left(\frac{N_p}{N_m}\right) \times L_r^3 = 235.9625 \quad \text{...(i)}$$

Also,
$$\frac{N_m D_m}{\sqrt{H_m}} = \frac{N_p D_p}{\sqrt{H_p}}$$
$$\left(\frac{N_p}{N_m}\right) \times L_r = \sqrt{\frac{H_p}{H_m}} = \sqrt{\frac{60}{5}} = 3.46$$

From equation (i) and (ii),
$$L_r^2 = \frac{235.9625}{3.46} = 68.1165$$
$$L_r = 8.25 = \text{scale ratio}$$

and
$$\frac{N_p}{N_m} = \frac{3.46}{8.25} = 0.4193$$
$$N_m = 238.5 \text{ rpm}$$

Specific speed,
$$(N_s)_m = \frac{N_m\sqrt{P_m}}{H_m^{5/4}} = \frac{238.5\sqrt{17.06}}{(5)^{5/4}} = 131.754 \text{ rpm}$$
$$(N_s)_p = \frac{N_p\sqrt{P_p}}{(H_p)^{5/4}} = \frac{100 \times \sqrt{50 \times 10^3}}{(60)^{5/4}} = 133.90 \text{ rpm}$$

Type of turbine runner is Francis.

Solution : 22

Importance of Dimensional Analysis: Dimensional analysis is an important tool in designing fluid machines, structures and various aspects of fluid flow on them.

(i) In dimensional analysis, model of actual structure is analysed under various flow conditions in order to see various effects of flow on actual structure.

(ii) Dimensional analysis predicts the performance of actual structure with the help of model.

(iii) The model of actual structure made with the help of dimensionless analysis is the replica of actual structure. So it eliminates the chances of error, redesign and reduces the cost.

Rayleigh's Method: This method is used only for simple problems involving 3 to 4 variables only and there is no calculation for dimension less groups. The dimensionless groups are formed directly.

The power to maintain the rotation of cylinder is directly proportional to density ρ, kinematic viscosity v, and diameter D of cylinder.

$\therefore$ $P \propto \rho^a \nu^b D^c V^d N^e$ where a, b, c, d & e are exponents of corresponding variables

$P = k \rho^a \nu^b D^c V^d N^e$, k-dimension less constant

Write down, the dimensions of various variables

$$[ML^2T^{-3}] = k[ML^{-3}]^a[L^2T^{-1}]^b[L]^c[LT^{-1}]^d[T^{-1}]^e$$
$$[ML^2T^{-3}] = k[M]^a[L]^{-3a+2b+c+d}[T]^{-b-d-e}$$

Comparing the power

$$a = 1$$
$$-3a + 2b + c + d = 2$$
$$2b + c + d = 5 \qquad (\because a = 1)$$
$$b + d + e = 3$$
$$b + d = 3 - e \qquad \text{...(i)}$$

we can write $2b + c + d$ as

$$\therefore \quad b + c + (b + d) = 5$$
$$b + c + 3 - e = 5 \qquad (\because b + d = 3 - e)$$
$$b + c = 2 + e \qquad \text{...(ii)}$$
$$c = 2 + e - b = 2 + e - (3 - e - d) = -1 + 2e + d$$
$$b = 3 - e - d$$
$$a = 1$$
$$\therefore \quad P = k\rho\nu^{3-e-d} \,.\, D^{-1+2e+d} \,.\, V^d \,.\, N^e$$
$$= k\rho\nu^3 D^{-1} \,.\, (\nu^{-1} \,.\, D^2 N)^e, (\nu^{-1} \,.\, D \,.\, V)^d$$
$$P = k\frac{\rho\nu^3}{D}\cdot\left(\frac{DV}{\nu}\right)^d\cdot\left(\frac{D^2N}{\nu}\right)^e = \frac{\rho\nu^3}{D}\cdot f\left(\frac{VD}{\nu}, \frac{ND^2}{\nu}\right)$$

Solution : 23

Given data: $\dfrac{L_m}{L_p} = \dfrac{1}{13}$; $V_p = 380$ m/s; $T_p = 23°\text{C} = 296$ K; $P_p = 95$ kPa; $T_m = -20°\text{C} = 253$ K

$P_m = 89$ kPa; $F_p = 400$ N

Assuming dynamic viscosity to be some for prototype and model

$$\mu_m = \mu_p$$

Now,
$$\rho_{\text{model}} = \frac{P_m}{RT_m} = \frac{89\times10^3}{287\times253} = 1.226\ \text{kg/m}^3$$

$$\rho_{\text{proto}} = \frac{P_p}{RT_p} = \frac{95000}{287\times296} = 1.12\ \text{kg/m}^3$$

Applying Reynold's model

$$\frac{\rho_m V_m L_m}{\mu} = \frac{\rho_p V_p L_p}{\mu}$$

$$V_m = \frac{L_p}{L_m}\cdot\frac{\rho_p}{\rho_m}V_p = 4512.89\ \text{m/s}$$

Drag force
$$F \propto \rho V^2 L^2$$

$$\frac{F_p}{F_m} = \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 = \left(\frac{1.12}{1.226}\right)\left(\frac{380}{4512.89}\right)^2\left(\frac{13}{1}\right)^2 = 1.095$$

Drag on prototype
$$F_p = 1.095 \times 400 = 437.86\ \text{N}$$

Solution : 24

The specific speed of a turbine is defined as the speed of operation of a geometrically similar model of the turbine which produces 1 kW power when operating under 1 m head. The expression for specific speed is derived as follows:

We have $$N = f(\rho, gH, P, D)$$

Wherein 'gH' called shaft work is considered to be one variable only.

Thus in all there are 5 variables involved in the phenomenon which can be completely described by 3 fundamental dimensions. As such the above functional relationship may be replaced by another one involving only 2 dimensionless π-terms.

Choosing ρ, 'gH' and P as repeating variables, we have

$$\pi_1 = \rho^{a_1}(gH)^{b_1}P^{c_1}N$$

and $$\pi_2 = \rho^{a_2}(gH)^{b_2}P^{c_2}D$$

Now, $$\pi_1 = \rho^{a_1}(gH)^{b_1}P^{c_1}N$$

Inserting fundamental dimensions

$$[M^oL^oT^o] = \left[\frac{M}{L^3}\right]^{a_1}\left[\frac{L^2}{T^2}\right]^{b_1}\left[\frac{ML^2}{T^3}\right]^{c_1}\left[\frac{1}{T}\right]$$

Equating the powers of M, L and T, we get

for M: $$0 = a_1 + c_1$$

for L: $$0 = -3a_1 + 2b_1 + 2c_1$$

for T: $$0 = -2b_1 - 3c_1 - 1$$

from which $$a = -\frac{1}{2},\ b_1 = \frac{5}{4},\ c_1 = \frac{1}{2}$$

$\therefore$ $$\pi_1 = \left[\frac{NP^{1/2}}{(gH)^{5/4}\rho^{1/2}}\right]$$

By adopting the same procedure, we get

$$\pi_2 = \left[\frac{D(gH)^{3/4}\rho^{1/2}}{P^{1/2}}\right]$$

Thus, we have $$\left[\frac{NP^{1/2}}{(gH)^{5/4}\rho^{1/2}}\right] = \phi\left[\frac{D(gH)^{3/4}\rho^{1/2}}{P^{1/2}}\right]$$

Now if a geometrically similar model of the turbine is considered, then complete similarity between the two will exist when the value of function ϕ is same for both. According to which the value of the parameter on the LHS must be same for the model and its prototype. That is

$$\left[\frac{NP^{1/2}}{(gH)^{5/4}\rho^{1/2}}\right]_{\text{model}} = \left[\frac{NP^{1/2}}{(gH)^{5/4}\rho^{1/2}}\right]_{\text{prototype}}$$

The specific speed N_2 is defined as the speed of a geometrically similar turbine which when working under a unit head develops unit power.

Thus substituting $P = 1$, $H = 1$ and $N = N_s$ for the model and assuming g and ρ to be same for the model and the prototype, we get

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

Solution : 25

Given: η is a function of ρ, μ, ω, D, Q

$$\eta = f(\rho, \mu, \omega, D, Q)$$

Total number of variables $n = 6$

and Fundamental dimensions $m = 3$

$$\text{No. of } \pi\text{-terms} = 6 - 3 = 3$$

$$\therefore \quad f(\pi_1, \pi_2, \pi_{3)} = 0$$

Choosing D, ω and ρ as repeating variables,

$$\pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}\cdot\eta$$

$$\pi_2 = D^{a_2}\omega^{b_2}\rho^{c_2}\cdot\mu$$

$$\pi_3 = D^{a_3}\omega^{b_3}\rho^{c3}\cdot Q$$

First π-term

$$\pi_1 = D^{a_1}\omega^{b_1}\rho^{c_1}\cdot\eta$$

$$M^0L^0T^0 = L^{a_1}T^{-b_1}(ML^{-3})^{c_1}\cdot M^0L^0T^0$$

$$0 = c_1 + 0 \Rightarrow c_1 = 0$$

$$0 = a_1 + 0 \Rightarrow a_1 = 0$$

$$0 = -b_1 + 0 \Rightarrow b_1 = 0$$

$$\pi_1 = D^0\,\omega^0\,\rho^0\,\eta$$

$$\pi_1 = \eta$$

Second π-term

$$\pi_2 = D^{a_2}\cdot\omega^{b_2}\rho^{c_2}\mu$$

$$M^0L^0T^0 = L^{a_2}T^{-b_2}(ML^{-3})^{c_2}ML^{-1}T^{-1}$$

$$0 = c_2 + 1 \Rightarrow c_2 = -1 = a_2 - 3c_2 - 1 \Rightarrow a_2 = -3 + 1 = -2$$

$$0 = -b_2 - 1 \Rightarrow b_2 = -1$$

$$\therefore \quad \pi_2 = D^{-2}\,\omega^{-1}\,\rho^{-1}\,\mu$$

$$\pi_2 = \frac{\mu}{D^2\omega\rho}$$

Third π-term

$$\pi_3 = D^{a_3}\omega^{b_3}\rho^{c_3}Q$$

$$M^0L^0T^0 = L^{a_3}T^{-b_3}(ML^{-3})^{c_3}L^{-3}T^{-1}$$

$0 = c_3 \Rightarrow c_3 = 0$

$0 = a_3 - 3c_3 + 3 \Rightarrow a_3 = -3$

$0 = -b_3 - 1 \Rightarrow b_3 = -1$

$$\therefore \quad \pi_3 = D^{-3}\omega^{-1}\rho^0 Q = \frac{Q}{D^3\omega}$$

$$\therefore \quad f(\pi_1, \pi_2, \pi_3) = 0$$

$$f\left(\eta, \frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right) = 0$$

or

$$\eta = f\left[\frac{\mu}{D^2\omega\rho}, \frac{Q}{D^3\omega}\right]$$

Solution : 26

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. For a distorted model different scale ratios for the linear dimensions are adopted. The models of river and harbours are made as distorted models as two different scale ratios, one for the horizontal dimensions and other for vertical dimensions are taken. If for the river and harbours, the horizontal and vertical scale ratios are taken to be same so that the model is undistorted, then the depth of water in the model of the river will be very-very small which may not be measured accurately. Therefore these are made as distorted models.

Advantages:

1. The vertical dimensions of the model can be measured accurately.
2. The cost of the model can be reduced.
3. Turbulent flow in the model can be maintained.

Disadvantages:

1. Due to different scales in the different directions, the velocity and pressure distribution in the model is not same as that in the prototype.
2. Waves are not simulated in distorted models.
3. The results of the distorted model cannot be directly transferred to its prototype.

■■■■

6 Flow Through Pipes

LEVEL 1 Objective Questions

1. (d)
2. (b)
3. (b)
4. (c)
5. (6.035)
6. (0.151)
7. (c)
8. (a)
9. (d)
10. (c)
11. (a)
12. (c)
13. (c)

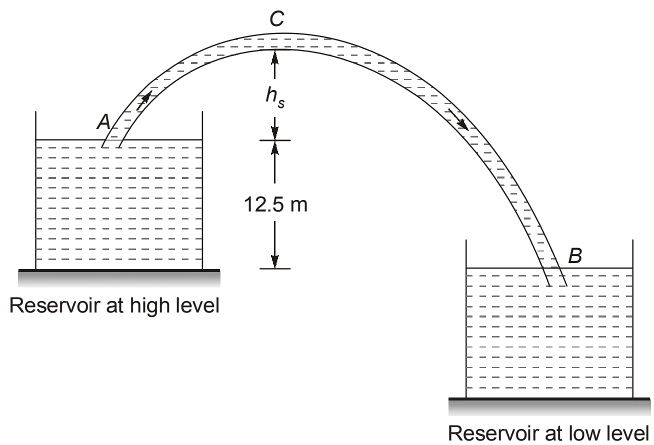
LEVEL 2 Objective Questions

14. (c)
15. (4)
16. (4.8)
17. (12.788)
18. (0.32768)
19. (d)
20. (b)
21. (c)
22. (d)
23. (1.82)

LEVEL 3 Conventional Questions

Solution : 24

Given data: Length of pipe from upper reservoir to the summit:



$$l_1 = 450 \text{ m}$$

Length of siphon, l = 1000 m

Diameter of siphon, d = 300 mm = 0.3 m

Difference between level of two reservoirs, $z_A - z_B$ = 12.5 m

Coefficient of friction, f = 0.01

Absolute pressure head at summit = 2.44 m of water

∴ Vacuum pressure head at summit:

$$\frac{p_C}{\rho g} = 10.3 - 2.44 = 7.86 \text{ m of water}$$

i.e., Pressure head at summit,

$$\frac{p_C}{\rho g} = -7.86 \text{ m of water}$$

Now applying modified Bernoulli's equation at points A and B, we get

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \text{losses of head}$$

$$0 + 0 + z_A = 0 + 0 + z_B + \text{losses of head}$$

or $$z_A - z_B = \text{losses of head}$$

where $$\text{losses of head} = h_i + h_f + h_o$$

$$= 0.5\frac{V^2}{2g} + \frac{4flV^2}{2gd} + \frac{V^2}{2g}$$

∴ $$z_A - z_B = \frac{0.5V^2}{2g} + \frac{4flV^2}{2gd} + \frac{V^2}{2g}$$

$$z_A - z_B = \left[0.5 + \frac{4fl}{d} + 1\right]\frac{V^2}{2g}$$

$$12.5 = \left[0.5 + \frac{4 \times 0.01 \times 1000}{0.3} + 1\right]\frac{V^2}{2g}$$

$$12.5 = 134.83 \times \frac{V^2}{2 \times 9.81}$$

or $$V^2 = 1.8189$$

or $$V = 1.348 \text{ m/s}$$

Now applying modified Bernoulli's equation at points A and C, we get

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\rho g} + \frac{V_C^2}{2g} + z_C + \text{losses of head}$$

$$0 + 0 + z_A = -7.86 + \frac{V_C^2}{2g} + z_C + \text{losses of head}$$

or $$7.86 - \frac{V_C^2}{2g} - \text{losses of head} = z_C - z_A$$

where $$\text{losses of head} = h_i + h_{f1} + h_o = 0.5\frac{V^2}{2g} + \frac{4fl_1V^2}{2gd} + \frac{V^2}{2g}$$

and $$V_c = V$$

$$7.86 - \frac{V^2}{2g} - \left(0.5\frac{V^2}{2g} + \frac{4fl_1V^2}{2gd} + \frac{V^2}{2g}\right) = h_s$$

$$7.86 - \frac{V^2}{2g} - 0.5\frac{V^2}{2g} - \frac{4fl_1V^2}{2gd} - \frac{V^2}{2g} = h_s$$

$$7.86 - \left(1 + 0.5 + \frac{4fl_1}{d} + 1\right)\frac{V^2}{2g} = h_s$$

$$7.86 - \left(1 + 0.5 + \frac{4 \times 0.01 \times 450}{0.3} + 1\right)\frac{(1.348)^2}{2g} = h_s$$

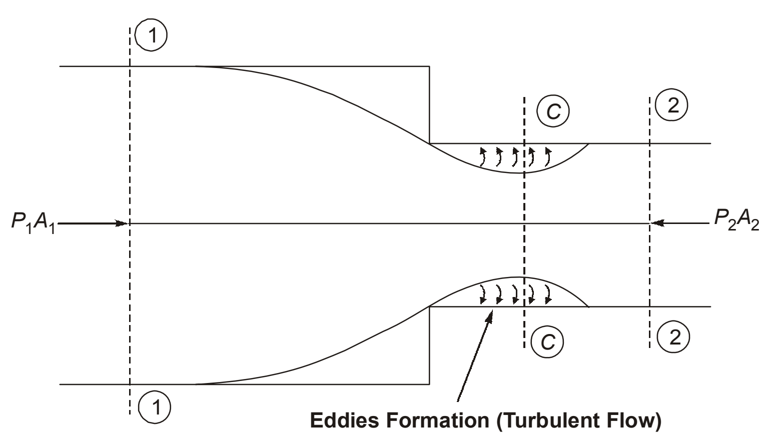
$$7.86 - (1 + 0.5 + 60 + 1) \times 0.0926 = h_s$$

$$7.86 - 5.787 = h_s$$

or $$h_s = \mathbf{2.073 \text{ m}}$$

Solution: 25

Consider a fluid (air) flowing in a pipe which has a sudden contraction in area



As the fluid (air) flows from a large pipe to smaller pipe, the area of flows goes on decreasing and becomes minimum at a section C–C as shown. This section C–C is called vena contracta. After section C–C a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena contracta to smaller pipe.

Let the dynamic loss coefficient be K and coefficient of contraction be C_C.

As, $$K = \left[\frac{1}{C_C} - 1\right]^2$$

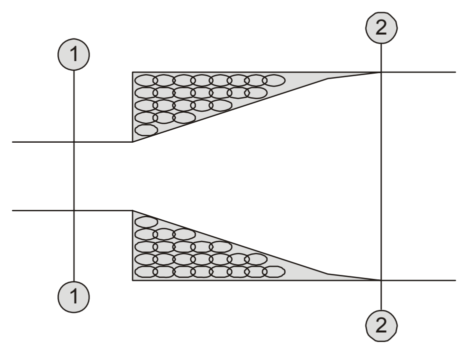
when, $$C_C = 0.62$$

$$K = \left[\frac{1}{0.62} - 1\right]^2 = 0.3756 = 0.3756$$

Solution : 26

If there is a sudden expansion in the pipe we know that head loss $h_e = \dfrac{(V_1 - V_2)^2}{2g}$

Applying Bernoulli's equation at points (i) and (ii) we get,



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$A_1 V_1 = A_2 V_2$$

$$\frac{P_2 - P_1}{\rho g} = \frac{V_1^2 - V_2^2}{2g} - h_e$$

$$\frac{P_2 - P_1}{\rho g} = \frac{(V_1^2 - V_2^2)}{2g} - \frac{(V_1 - V_2)^2}{2g}$$

$$\frac{\Delta P}{\rho g} = \frac{V_1^2}{2g} - \frac{A_1^2 V_1^2}{A_2^2 (2g)} - \frac{\left(V_1 - \frac{A_1 V_1}{A_2}\right)^2}{2g} \qquad (\because A_1 V_1 = A_2 V_2)$$

$$\frac{\Delta P}{\rho g} = \frac{V_1^2}{2g}\left[\left(1 - \frac{D_1^4}{D_2^4}\right) - \left(1 - \frac{D_1^2}{D_2^2}\right)^2\right]$$

$$\Delta P = \frac{\rho V_1^2}{2}\left[2\frac{D_1^2}{D_2^2} - 2\left(\frac{D_1}{D_2}\right)^4\right]$$

Differentiating the above equation with D_1/D_2 to get the maximum differential pressure (ΔP) and equating to zero

$$\frac{d(\Delta P)}{d\left(\frac{D_1}{D_2}\right)} = \frac{\rho V_1^2}{2}\left[4\frac{D_1}{D_2} - 8\left(\frac{D_1}{D_2}\right)^3\right] = 0 \Rightarrow \frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$$

$\therefore$ for maximum differential pressure, the ratio of the diameters is $\dfrac{D_1}{D_2} = \dfrac{1}{\sqrt{2}}$

corresponding loss of head, $$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

$$= \frac{V_1^2}{2g}\left[1-\left(\frac{D_1}{D_2}\right)^2\right]^2 \qquad \left[\text{since } A_1V_1 = A_2V_2; \frac{V_2}{V_1} = \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2}\right]$$

$$= \frac{V_1^2}{2g}\left[1-\left(\frac{1}{\sqrt{2}}\right)^2\right]^2 = \frac{V_1^2}{2g}\times\frac{1}{4} = 0.25\frac{V_1^2}{2g}$$

and corresponding differential pressure head:

as
$$\Delta P = \frac{\rho V_1^2}{2}\left(2\left(\frac{D_1}{D_2}\right)^2 - 2\left(\frac{D_1}{D_2}\right)^4\right) = \frac{\rho V_1^2}{2}\left(2\times\left(\frac{1}{\sqrt{2}}\right)^2 - 2\times\left(\frac{1}{\sqrt{2}}\right)^4\right)$$

$$\Delta P = \frac{\rho V_1^2}{2}\left(1-\frac{1}{2}\right) = 0.5\frac{\rho V_1^2}{2}$$

$$\frac{\Delta P}{\rho g} = 0.5\frac{V_1^2}{2g}$$

Solution : 27

The mean velocity of flow,

$$V = \frac{Q}{\text{area}} = \frac{20\times10^{-3}}{\frac{\pi}{4}(0.1)^2} = 2.546 \text{ m/s}$$

The Reynolds number for this flow,

$$Re = \frac{\rho VD}{\mu}$$

or
$$Re = \frac{1.26\times10^3\times2.546\times10^{-1}}{0.9} = 356 < 2000$$

and therefore, the flow is laminar.

Coefficient of friction,
$$f = \frac{16}{\text{Re}} = \frac{16}{356} = 0.045$$

and the head lost due to friction,

$$h_f = \frac{4fLV^2}{2gd} = \frac{4\times0.45\times45\times(2.546)^2}{2\times9.81\times0.1} = 26.72 \text{ m}$$

By applying Bernoulli equation between the two ends,

$$\frac{p_1}{\rho} + gz_1 = \frac{p_2}{\rho} + gz_2 + h_f g \qquad (V_1 = V_2)$$

the pressure at the outlet end, p_2, is given by

$$p_2 = p_1 - \rho g(z_2 - z_1 + h_f)$$

$$= 590 - 1.26\times10^3\times\frac{9.81(45\tan15° + 26.72)}{10^3} = 110.68 \text{ kPa}$$

Since the head lost is due to the viscous forces, we make a balance of forces :

$$\tau \times 2\pi r_0 L = \Delta P(\pi r_0^2) = \rho g h_f(\pi r_0^2)$$

$$\tau = \frac{\rho g h_f r_0}{2L}$$

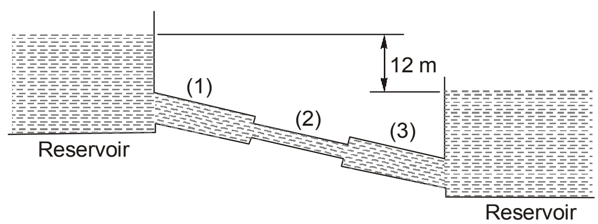
$$= \frac{1.26\times10^3\times26.72\times9.81\times0.05}{2\times45} = 183.486 \text{ N/m}^2$$

Solution : 28

The system is shown in figure. Let the velocity in sections 1 and 3 be given by V_1 and that in section 2 by V_2. From the continuity equation,

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = 2.25V_1$$



Losses in Section 1 :

Loss at entry = the head loss at entrance to the pipe line from a reservoir, usually taken as 0.5 $V^2/2g$

$$\text{Frictional loss} = \frac{4fLV^2}{2gd} = \frac{4\times0.0054\times10}{0.06}\times\frac{V_1^2}{2g} = \frac{3.6V_1^2}{2g}$$

Losses in Section 2:

Loss at entry = The head loss due to sudden contraction.

$$= \frac{0.5V_2^2}{2g} = \frac{0.5\times(2.25)^2V_1^2}{2g} = \frac{2.53125V_1^2}{2g}$$

$$\text{Frictional loss} = \frac{4\times0.0054\times10}{0.04}\frac{V_2^2}{2g}$$

$$= \frac{5.4(2.25)^2V_1^2}{2g} = \frac{27.3375V_1^2}{2g}$$

Losses in Section 3 :

Loss at entry = the head loss due to sudden expansion

$$= \frac{(V_2-V_1)^2}{2g} = \frac{(1.25V_1)^2}{2g} = 1.5625\frac{V_1^2}{2g}$$

$$\text{Frictional loss} = \frac{4\times0.0054\times10}{0.06}\frac{V_1^2}{2g} = \frac{3.6V_1^2}{2g}$$

$$\text{Loss at exit} = \frac{V_1^2}{2g}$$

Thus, the total loss in the system

$$= \frac{[0.5+3.6+2.53125+27.3375+1.5625+3.6+1]V_1^2}{2g} = 12 \text{ m}$$

$$\therefore \quad \frac{V_1^2}{2g} = \frac{12}{40.13125} = 0.299$$

and $\quad V_1 = 2.422$ m/s

and the flow rate,

$$Q = \frac{\pi}{4}D_1^2 \times V_1 = \frac{\pi}{4}(0.06)^2 \times 2.422 = 6.848 \text{ lit/s}$$

Solution : 29

Let the quantity of water flowing through the pipe system be Q. The power produced by the turbine will, then, be $P = \rho Qgh$, where h is the head utilized by the turbine.

$$\therefore \quad h = \frac{P}{\rho Qg} = \frac{100\times10^3}{10^3\times Q\times9.81} = \frac{10.194}{Q} \text{ m}$$

Neglecting the losses in the pipe system due to fittings, we can write $50 = h + h_f + \frac{V^2}{2g}$, where V is velocity in the pipe (equal to outlet velocity)

$$= \frac{10.194}{\frac{\pi}{4}(0.3)^2 V} + \frac{4f\times100\times V^2}{2\times9.81\times0.3} + \frac{V^2}{2g}$$

Let $\quad f = 0.0035$

$$\therefore \quad 50 = \frac{144.19}{V} + 0.238V^2 + 0.051V^2$$

or $\quad 0.2890V^3 - 50V + 144.19 = 0$

By trial and error, $\quad V = 3.0475$ m/s

The Reynolds number

$$Re = \frac{10^3\times3.0475\times0.3}{10^{-3}} = 9.1425\times10^5, \text{ a turbulent flow.}$$

From the Moody's chart, we get f almost equal to 0.0035 and therefore, we can accept the value of V as 3.0475 m/s.

$$\therefore \quad Q = \frac{\pi}{4}d^2V = 0.215 \text{ m}^3/\text{s}$$

Solution : 30

Applying Bernoulli's equation between section 1 and 2, we get

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2+(h_L)_{1-2}$$

$$\frac{12.0}{9.81}+\frac{v_1^2}{2g}+10 = \frac{11.5}{9.81}+\frac{v_2^2}{2g}+10+(h_L)_{1-2}$$

But $$\frac{v_1^2}{2g} = \frac{v_2^2}{2g}$$

Therefore, $$(h_L)_{1-2} = 0.051\text{ m};$$

$$v_1 = \frac{Q}{a_1} = 2.54\text{ m/s; and } v_3 = \frac{Q}{a_3} = 7.07\text{ m/s}$$

Similarly, applying Bernoulli's equation between sections 1 and 3, we get

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1 = \frac{p_3}{\gamma}+\frac{v_3^3}{2g}+z_3+(h_L)_{1-3}$$

$$\frac{12.0}{9.81}+\frac{2.54^2}{2\times 9.81}+10 = \frac{10.3}{9.81}+\frac{7.07^2}{2\times 9.81}+0+(h_L)_{1-3}$$

Therefore, $$(h_L)_{1-3} = 7.95\text{ m}$$

Solution : 31

Applying Bernoulli's equation between section 1 and 2, we get

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2+h_L$$

But $$v_2 = \frac{Q}{a_2} = \frac{\frac{50}{3600}}{\frac{\pi}{4}\times 0.05^2} = 7.074\text{ m/s}$$

Using the relations $$h_L = h_f = \frac{fLv^2}{2gd}$$

$$Re = \frac{\rho vd}{\mu} = \frac{1000\times 7.074\times 0.05}{1.02\times 10^{-3}} = 3.46\times 10^5$$

From Moody's chart for smooth pipe, we get $f = 0.0140$

Therefore, $$h_f = \frac{0.014\times(60+80+30)7.04^2}{2\times 9.81\times 0.05} = 121.4\text{ m}$$

Thus, $$\frac{p_1}{9.81}+0+10 = 0+\frac{7.074^2}{2\times 9.81}+80+121.4$$

$$p_1 = 1902.65\text{ kPa}$$

Solution : 32

Given : diameter of pipe 1 = d; diameter of pipe 2 = $2d$; diameter of pipe 3 = $3d$; discharge through pipe 1 is Q_1 = 1.2 cumecs; length of all pipes = L.

Let h_L = loss of head due to friction. Loss of head remains the same since the pipes have been arranged in parallel. Therefore,

$$h_f = \frac{32fLQ^2}{\pi^2 g d_1^5}$$

$$\frac{32f_1L_1Q_1^2}{\pi^2 g d_1^5} = \frac{32f_2L_2Q_2^2}{\pi^2 g d_2^5} = \frac{32f_3L_3Q_3^2}{\pi^2 g d_3^5}$$

Since the lengths and the coefficient of friction for all the pipes are same, we have

$$\frac{Q_1^2}{d^5} = \frac{Q_2^2}{(2d)^5} = \frac{Q_3^2}{(3d)^5}$$

or
$$Q_1^2 = \frac{Q_2^2}{32} = \frac{Q_3^2}{243}$$

$$(1.2)^2 = \frac{Q_2^2}{32} = \frac{Q_3^2}{243}$$

∴ Q_2 = 6.788 cumecs

and Q_3 = 18.70 cumecs

■■■■

7 Laminar & Turbulent Flow

LEVEL 1 Objective Questions

1.	(c)
2.	(a)
3.	(b)
4.	(d)
5.	(d)
6.	(c)
7.	(b)
8.	(d)
9.	(a)
10.	(c)
11.	(26.4)
12.	(2.66)

LEVEL 2 Objective Questions

13.	(c)
14.	(d)
15.	(a)
16.	(16)
17.	(0.00784)
18.	(1)
19.	(d)
20.	(c)
21	(c)
22.	(a)
23.	(c)

■■■■

LEVEL 3 Conventional Questions

Solution : 24

For 150 mm diameter pipe with absolute roughness,

$$\frac{r_o}{k_s} = \frac{150/2}{0.3} = 250$$

and for 210 mm diameter pipe with absolute roughness,

$$\frac{r_o}{k_s} = \frac{210/2}{0.3} = 350$$

For 150 mm diameter pipe, $\dfrac{1}{\sqrt{f_1}} = 2\log_{10}250 + 1.74$

or $f_1 = 0.0234$

For 210 mm diameter pipe, $\dfrac{1}{\sqrt{f_2}} = 2\log_{10}350 + 1.74$

or $f_2 = 0.021448$

Head loss, $h_f = \dfrac{flv^2}{2gd} = \dfrac{fl}{2gd}\times\left(\dfrac{4Q}{\pi d^2}\right)^2 = \dfrac{8flQ^2}{\pi g d^5}$

$h_f \propto \dfrac{f}{d^5}$ for the same discharge Q and same length

$$\therefore \quad \frac{h_{f1}}{h_{f2}} = \frac{f_1/d_1^5}{f_2/d_2^5} = \frac{f_1 d_2^5}{f_2 d_1^5}$$

Cost of pumping will be proportional to the power required for pumping

Power required $= \rho Q g h_f$

Since ρ and Q are the same for both the pipes, the ratio of

$$\frac{\text{Cost of pumping 150 mm diameter pipe}}{\text{Cost of pumping 210 mm diameter pipe}} = \frac{h_{f1}}{h_{f2}} = \frac{f_1 d_2^5}{f_2 d_1^5} = \frac{0.0234\times(210)^5}{0.021448\times(150)^5} = 5.8677$$

Solution : 25

Reynolds number, $\text{Re} = \dfrac{Vd}{\nu} = \dfrac{4Q}{\pi d\nu}$

Kinematic viscosity, $\nu = \dfrac{\mu}{\rho} = \dfrac{0.883}{1260} = 0.7\times10^{-3}\ \text{m}^2/\text{s}$

$$\therefore \quad \text{Re} = \frac{4\times(180\times10^{-3}/60)}{3.14\times0.01\times0.7\times10^{-3}} = 545.95$$

Since $R < 2000$, flow is laminar.

Pressure loss, $\Delta p = \dfrac{32\mu VL}{d^2} = \dfrac{128\mu QL}{\pi d^4}$

$$= \frac{128 \times 0.883 \times 3 \times 10^{-3} \times 65}{3.14 \times (0.01)^4} = 7019 \times 10^5 \text{ N/m}^2$$

Maximum flow rate for laminar flow will be obtained for Re = 2000,

$$V = \frac{\nu \times \text{Re}}{d} = \frac{0.7 \times 10^{-3} \times 2000}{0.01} = 140 \text{ m/s}$$

$$Q_{max} = \frac{\pi d^2}{4} \times V = \frac{3.14(0.01)^2}{4} \times 140 = 0.01099 \text{ m}^3\text{/s} = 10.99 \ l\text{/s}$$

$$= 659.4 \text{ litres/minute}$$

Solution : 26

Given data: Viscosity of oil, μ = 7.5 poise = $\frac{7.5}{10}$ Ns/m^2 = 0.75 Ns/m^2; Specific gravity, S = 0.85

$\therefore$ Density of oil, $\rho = S \times 1000$ kg/m^3 = 0.85 × 1000 kg/m^3 = 850 kg/m^3; Diameter of pipe, D = 50 mm = 0.05 m

$\therefore$ Radius of pipe, $R = \frac{D}{2} = \frac{0.05}{2}$ = 0.025 m; Pressure drop, $p_1 - p_2$ = 18 kN/m^2 per m length of pipe

i.e.,
$$\frac{p_1 - p_2}{l} = 18 \text{ kN/m}^3 = 18 \times 10^3 \text{ N/m}^3$$

Pressure drop for laminar flow through pipe is,

$$p_1 - p_2 = \frac{32\mu \bar{u} l}{D^2}$$ [From Hagen–Poiseuille equation]

$$\frac{p_1 - p_2}{l} = \frac{32\mu \bar{u}}{D^2}$$

$$18 \times 10^3 = \frac{32 \times 0.75 \times \bar{u}}{(0.05)^2}$$

or
$$\bar{u} = 1.875 \text{ m/s}$$

Reynolds number,
$$\text{Re} = \frac{\rho D \bar{u}}{\mu} = \frac{850 \times 0.05 \times 1.875}{0.75} = 106.25$$

Reynolds number is less than 2000.
Hence, the flow in pipe is laminar.

(i) Flow rate:
$$Q = A\bar{u} = \frac{\pi}{4} D^2 \bar{u} = \frac{3.14}{4} \times (0.05)^2 \times 1.875 = 0.003679 \text{ m}^3\text{/s}$$

The maximum velocity occurs at centre line of the pipe and it equals twice the average flow velocity.

$$U_{max} = 2\bar{u} = 2 \times 1.875 = 3.75 \text{ m/s}$$

(ii) Power required to maintain the flow in 100 m length of pipe: P

$$P = mgh_f = \rho Q g h_f$$

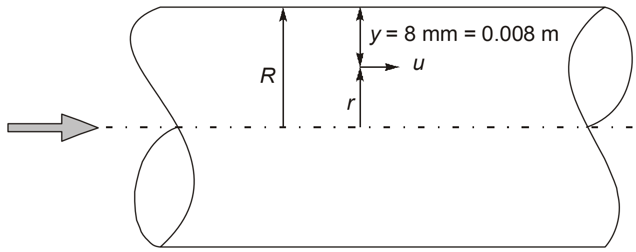
where
$$h_f = \frac{32\mu \bar{u} l}{\rho g D^2}$$

[Hagen-Poiseuille equation]

$$P = \frac{\rho Q g \times 32\mu \bar{u} l}{\rho g D^2} = \frac{32\mu \bar{u} Q l}{D^2}$$ ($\because l$ = 100 m)

$$= \frac{32 \times 0.75 \times 1.875 \times 0.003679 \times 100}{(0.05)^2} = 6622.2 \text{ W}$$

(iii) Velocity at radius r is given by



$$u = -\frac{1}{4\mu}\frac{dp}{dx}(R^2 - r^2)$$

Corresponding to 8 mm from the wall,

$$r = R - y = 0.025 - 0.008 = 0.017 \text{ m}$$

and $\quad \dfrac{p_1 - p_2}{l} = 18 \times 10^3 \text{ N/m}^3$

or $\quad \dfrac{p_2 - p_1}{l} = -18 \times 10^3 \text{ N/m}^3$

or $\quad \dfrac{dp}{dx} = -18 \times 10^3 \text{ N/m}^3$

$\therefore \quad u = -\dfrac{1}{4 \times 0.75} \times \left(-18 \times 10^3\right) \times \left[(0.025)^2 - (0.017)^2\right] = 2.01 \text{ m/s}$

and shear stress; $\quad \tau = -\dfrac{dp}{dx}\dfrac{r}{2}$

$$= -(-18 \times 10^3) \times \frac{0.017}{2} = 153 \text{ N/m}^2$$

Solution : 27

$$\rho = 917 \text{ kg/m}^3 \qquad D = 15 \text{ cm} = 0.15 \text{ m}$$

$$Q = 800 \text{ L/min} = 0.0133 \text{ m}^3/\text{s}$$

$\therefore \quad Q = \dfrac{\pi}{4} D^2 \times V$

$$0.0133 = \frac{\pi}{4} \times (0.15)^2 \times V$$

$\therefore \quad V = 0.75262 \text{ m/s}$

Assuming laminar flow through pipe,

$$P_2 - P_1 = \frac{32 \mu V L}{D^2}$$

$$95 \times 10^3 = \frac{32 \times \mu \times 0.75262 \times 800}{0.15^2}$$

$\therefore \quad \mu = 0.11094 \text{ Ns/m}^2$

Solution : 28

Given data: diameter, $d = 0.1$ m; $Q = 50\ l/s = 50 \times 10^{-3}$ m^3/s; $K = 0.15$ mm

Mean velocity $$V = \frac{Q}{A} = \frac{50\times10^{-3}}{\frac{\pi}{4}(0.1)^2} = 6.37 \text{ m/s}$$

Reynolds number $$Re = \frac{VD}{\nu} = \frac{6.37\times0.1}{1.0\times10^{-6}} = 6.37 \times 10^5$$

Hence the flow is turbulent in the rough pipe.

For rough turbulent flow we know that

$$\frac{1}{\sqrt{f}} = 2.0 \log_{10}\left(\frac{R}{k}\right) + 1.74$$

$$R = 0.05 \text{ m and } k = 0.15 \times 10^{-3} \text{ m}$$

Thus by substitution, we get

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}\left(\frac{0.05}{0.15\times10^{-3}}\right) + 1.74$$

or $$f = 0.0217$$

Shear stress at the pipe surface is given by

$$\tau_0 = \frac{\rho V^2 f}{8}$$

in SI Units $$\tau_0 = \frac{1000\times(6.37)^2\times0.0217}{8} = 110.065 \text{ N/m}^2$$

Shear velocity $$V^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{110.065}{1000}} = 0.332 \text{ m/s}$$

Also $$V^* = \sqrt{\frac{11.227}{102}} = 0.332 \text{ m/s}$$

The equation for the velocity distribution in rough pipe is

$$\frac{\nu}{V^*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5$$

For $$y = R,\ v = v_{max}$$

Hence by substitution, we get

$$\frac{v_{max}}{0.332} = 5.75 \log_{10}\left(\frac{0.05}{0.15\times10^{-3}}\right) + 8.5$$

$\therefore$ $$v_{max} = 7.64 \text{ m/s.}$$

Solution : 29

The velocity distribution in laminar flow in a circular tube is given by

$$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\{R^2 - r^2\} \quad \text{...(i)}$$

$$u_{avg} = \frac{1}{8\mu}\left\{-\frac{\partial p}{\partial x}\right\}R^2 \quad \text{...(ii)}$$

Equating equations (i) and (ii), we get

$$\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\{R^2 - r^2\} = \frac{1}{8\mu}\left\{-\frac{\partial p}{\partial x}\right\}R^2$$

that is $$r^2 = \frac{R^2}{2} \Rightarrow r = 0.707R$$

Distance from the boundary $y = R - r = 0.293R$

Solution : 30

Energy correction factor is given by

$$\alpha = \frac{\int_A u^3 dA}{Au_{avg}^3}$$

But $$u = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\{R^2 - r^2\}$$

and $$u_{avg} = \frac{1}{8\mu}\left\{-\frac{\partial p}{\partial x}\right\}R^2$$

Therefore, substituting u and u_{avg} in the above equation, we get

$$\alpha = \frac{\int_0^R \left(\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\{R^2 - r^2\}\right)^3 2\pi r dr}{\pi R^2\left(\frac{1}{8\mu}\left\{-\frac{\partial p}{\partial x}\right\}R^2\right)^3}$$

Upon simplification, we get

$$\alpha = \frac{2}{R^8}\times 8\left[\frac{R^6.r^2}{2} - \frac{3R^4\cdot r^4}{4} + \frac{3R^2r^6}{6} - \frac{r^8}{8}\right]_0^R$$

$$\alpha = \frac{2\times 8}{R^8}\left[\frac{1}{2} - \frac{3}{4} + \frac{3}{6} - \frac{1}{8}\right]R^8 = 2$$

Similarly, momentum correction factor is given as

$$\beta = \frac{\int_A u^2 dA}{Au_{avg}^2}$$

$$= \frac{\int_0^R \left(\frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\{R^2 - r^2\}\right)^2 2\pi r dr}{\pi R^2\left(\frac{1}{8\mu}\left\{-\frac{\partial p}{\partial x}\right\}R^2\right)^2}$$

Upon simplication, we get $$\beta = \frac{8}{R^6}\left[\frac{R^4r^2}{2} + \frac{r^6}{6} - \frac{2R^2r^4}{4}\right]_0^R = \frac{8}{R^6}\left[\frac{1}{2} + \frac{1}{6} - \frac{1}{2}\right]R^6$$

$$\beta = \frac{4}{3} = 1.33$$

Solution : 31

The liquid flows in the downward direction. The pressure at the upstream station is

$$p_1 + \rho g h_1 = 150 + \frac{1260 \times 9.81 \times 1.5}{1000}$$

$$= 168.54 \text{ kPa.}$$

The pressure at the downstream station,

$$p_2 + \rho g h_2 = p_2 \qquad (h_2 = 0)$$

Since $(p + \rho gh)$ is decreasing in the positive x-direction,

$$\therefore \quad \frac{d}{dx}(p + \rho gh) = \frac{168.54 - 100}{1.5 \times \frac{2}{\sqrt{3}}} = -39.572 \text{ kPa}$$

$$u = \frac{Uy}{b} + \frac{1}{2\mu}\left[y^2 - by\right]\frac{d}{dx}(p + \rho gh)$$

$$= \frac{-0.05y}{0.005} - \frac{1}{2 \times 0.9}[y^2 - 0.005y]39.572 \times 10^3$$

$$= 99y - 21980y^2$$

The shear stress at the upper plate,

$$\tau = \mu \left.\frac{du}{dy}\right)_{y=b}$$

or $\quad \tau = 0.9[99 - 2 \times 21980 \times 0.005] = -108.72 \text{ N/m}^2$

and it must be resisting the plate motion. The discharge through the channel is given by

$$Q = \int_0^b u\,dy = \int_0^b \left[\frac{Uy}{b} + \frac{1}{2\mu}(y^2 - by)\frac{d}{dx}(p + \rho gh)\right]dy$$

$$= \frac{Ub}{2} - \frac{1}{2\mu}\frac{d}{dx}(p + \rho gh)\frac{b^3}{6}$$

for no discharge,

$$\frac{Ub}{2} = \frac{1}{12\mu}\frac{d}{dx}(p + \rho gh)b^3$$

$$\therefore \quad U = \frac{b^2}{6\mu}\frac{d}{dx}(p + \rho gh) = -0.183 \text{ m/s}$$

■■■■

8 Boundary Layer Theory, Drag & Lift

LEVEL 1 Objective Questions

1.	(c)
2.	(b)
3.	(c)
4.	(d)
5.	(c)
6.	(c)
7.	(a)
8.	(d)
9.	(c)
10.	(b)
11.	(b)
12.	(c)

LEVEL 2 Objective Questions

13.	(a)
14.	(b)
15.	(d)
16.	(c)
17.	(b)
18.	(a)
19.	(c)
20.	(a)
21.	(c)
22.	(b)

■■■■

LEVEL 3 Conventional Questions

Solution : 23

Given kinematic viscosity,

$$v = 1.49 \times 10^{-5};\ U = 300 \text{ km/hr} = 83.33 \text{ m/s}$$

Now,
$$Re_x = \frac{Ux}{v}$$

$$2 \times 10^5 = \frac{83.33 \times x}{1.49 \times 10^{-5}}$$

$$x = 0.03576 \text{ m or } 35.76 \text{ mm}$$

To find maximum thickness of the boundary layer

$$\delta_{max} = \frac{5.48x}{\sqrt{R_{ex}}} = \frac{5.48 \times 0.03576}{\sqrt{2 \times 10^5}} = 4.37 \times 10^{-4} \text{ m}$$

Solution : 24

When 0.3 m side is parallel to flow

$$Re_x = \frac{Ux}{v} = \frac{3.5 \times 0.3}{1.5 \times 10^{-5}} = 70{,}000$$

When 1.0 m side is parallel to flow

$$Re_x = \frac{3.5 \times 1.0}{1.5 \times 10^{-5}} = 2{,}33{,}333.33$$

The boundary layer is laminar up to the trailing edge in both cases.

To calculate drag force:

When 0.3 m side is parallel to flow

$$C_D = \frac{1.46}{\sqrt{Re_L}} = \frac{1.46}{\sqrt{70{,}000}} = 5.51 \times 10^{-3}$$

Total drag due to both sides

$$F_D = 2 \times A \times C_D \times \frac{1}{2} \times \rho \times U^2$$

$$= 2 \times 0.3 \times 1 \times 0.00551 \times 0.5 \times 1.2 \times 3.5^2 = 0.024 \text{ N}$$

When 1.0 m side is parallel to flow

$$C_D = \frac{1.46}{\sqrt{Re_L}} = \frac{1.46}{\sqrt{2{,}33{,}333.33}} = 3.022 \times 10^{-3}$$

$$F_D = 2 \times A \times C_D \times \frac{1}{2} \times \rho \times U^2$$

$$= 2 \times 0.3 \times 3.022 \times 10^{-3} \times 0.5 \times 1.2 \times 3.5^2 = 0.0133\text{N}$$

Solution : 25

At 25°C, $\nu = 15.33 \times 10^{-5}$ m^2/s

$$Re_x = \frac{xU_0}{\nu} = \frac{1 \times 3}{15.33 \times 10^{-5}} = 1{,}93{,}175$$

1. Blasius solution

$$\delta = \frac{5x}{\sqrt{\text{Re}_x}} = 0.011376 \text{ m}$$

$$C_D = \frac{0.664}{\sqrt{\text{Re}_x}} = 1.511 \times 10^{-3}$$

2. Approximate solution with assumption of cubic velocity profile

$$\delta = \frac{4.64 \times x}{\sqrt{\text{Re}_x}} = \frac{4.64 \times 1}{\sqrt{1{,}93{,}175}} = 0.010557 \text{ m}$$

$$C_D = \frac{0.646}{\sqrt{\text{Re}_x}} = 1.469 \times 10^{-3}$$

Approximate solution deviate from the exact solution by

$$\frac{1.1376 - 1.0557}{1.1376} \times 100 = 7.2\%$$

Boundary layer thickness

$$\frac{1.511 \times 10^{-3} - 1.469 \times 10^{-3}}{1.511 \times 10^{-3}} \times 100 = 2.78\%$$

Solution : 26

$$C_D = \frac{0.074}{\text{Re}_L^{1/5}}$$

For the front two-third portion of the plate

$$Re_L = \frac{U\left(\frac{2}{3}\right)L}{\nu}$$

Thus,

$$C_D = \frac{0.074}{\left(\frac{U\left(\frac{2}{3}\right)L}{\nu}\right)^{1/5}}$$

Therefore, drag for the front two-third portion of the plate is

$$F_{D1} = C_D \times B \times L \times \frac{\rho U^2}{2} = \frac{0.074}{\left(\frac{U\left(\frac{2}{3}\right)L}{\nu}\right)} \times B \times \frac{2}{3}L \times \frac{\rho U^2}{2}$$

Similarly, drag for the entire plate is

$$F_D = \frac{0.074}{\left(\frac{UL}{\nu}\right)^{1/5}} \times B \times L \times \frac{\rho U^2}{2}$$

Therefore, drag for the one-third of the plate is

$$F_{D2} = F_D - F_{D1}$$

$$= \frac{0.074}{\left(\frac{UL}{\nu}\right)^{1/5}} \times B \times L \times \frac{\rho U^2}{2} - \frac{0.074}{\left(\frac{U\left(\frac{2}{3}\right)L}{\nu}\right)^{1/5}} \times B \times \frac{2}{3} L \times \frac{\rho U^2}{2}$$

Therefore, $\frac{F_{D_1}}{F_{D_2}} = 2.61$

Solution : 27

The local value of Reynolds number is given by

$$Re_x = \frac{U_\infty x}{\nu} = \frac{60 \times 10^3}{3600} \times \frac{0.3}{15 \times 10^{-6}} = 3.33 \times 10^5 < 5 \times 10^5$$

therefore, we can assume that the flow is laminar. The boundary layer thickness,

$$\delta = \frac{x \times 5.0}{\sqrt{\text{Re}_x}} = \frac{0.3 \times 5}{(3.33 \times 10^5)^{1/2}} = 0.26 \text{ cm}$$

The displacement thickness is given by equation. By substituting approximate velocity profile expression as given by equation we get,

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left[1 - \frac{3}{2}\frac{y}{\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy = \frac{3}{8}\delta$$

$\therefore$ $\delta^* = 0.0975$ cm

And the momentum thickness can be computed by substituting

Thus, $$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right]\left[1 - \frac{3y}{2\delta} + \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right] dy$$

$$= 0.1393\delta = 0.1393 \times 0.26 = 0.0362 \text{ cm}$$

Solution : 28

The Reynolds number is

$$Re_L = \frac{\rho U_\infty L}{\mu} = 10^3 \times \frac{10 \times 10^3}{3600} \times \frac{0.25}{1 \times 10^{-3}} = 6.94 \times 10^5$$

The flow is almost laminar over the entire plate and therefore, the drag coefficient

$$\bar{C}_f = C_D = \frac{1.328}{\text{Re}_L^{1/2}} = \frac{1.328}{(6.94 \times 10^5)^{1/2}} = 0.159 \times 10^{-2}$$

The force required to two the plate is

$$D = C_D \rho \frac{U_\infty^2}{2} \times \text{area}$$

$$= 0.159 \times 10^{-2} \times 10^3 \times \left(\frac{10 \times 10^3}{3600}\right) \times \frac{1}{2} \times 2 \times 0.25 \times 0.25$$

$$= 0.766 \text{ N}$$

Here, area has been taken for both sides of the plate because frictional forces act on both sides.

Solution : 29

Using the integral momentum analysis, equation can be written as

$$\frac{\tau_w}{\rho U_\infty^2} = \frac{d}{dx}\int_0^\delta\left(1-\frac{u}{U_\infty}\right)\frac{u}{U_\infty}dy = \frac{d}{dx}\int_0^\delta\left[1-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy$$

$$= \frac{d}{dx}\left[\left(\int_0^\delta\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy\right)-\int_0^\delta\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)dy\right]$$

$$= \frac{d}{dx}\left[-\frac{\cos(\pi/2, y/\delta)}{\frac{\pi}{2\delta}}\right]_0^\delta - \frac{d}{dx}\left[\int_0^\delta\frac{1-\cos 2(\pi/2\cdot y/\delta)}{2}dy\right]$$

$$= \frac{d}{dx}\left[\frac{2\delta}{\pi}\right]-\frac{d}{dx}\left[\frac{\delta}{2}-\frac{\sin^2(\pi/2.y/\delta)}{2\times\frac{2\pi}{2\delta}}\right]_0^\delta = \frac{d}{dx}\left[\frac{2\delta}{\pi}-\frac{\delta}{2}\right] = \frac{(4-\pi)d\delta}{2\pi dx}$$

or $$\tau_w = \rho U_\infty^2\left(\frac{4-\pi}{2\pi}\right)\frac{d\delta}{dx}$$

also, $$\tau_w = \mu\frac{\partial u}{\partial y}\Big)_{y=0} = \mu U_\infty\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)\cdot\frac{\pi}{2\delta}\Big)_{y=0} = \mu\frac{\pi U_\infty}{2\delta}$$

or $$\mu\frac{\pi U_\infty}{2\delta} = \rho U_\infty^2\left(\frac{4-\pi}{2\pi}\right)\frac{d\delta}{dx}$$

∴ $$\delta d\delta = \frac{\mu}{\rho}\frac{\pi^2}{4-\pi}\frac{dx}{U_\infty}$$

and, $$\frac{\delta^2}{2} = \frac{\mu}{\rho U_\infty}\frac{\pi^2}{4-\pi}x + c$$

when $$x = 0,\ \delta = 0,\ \therefore\ c = 0$$

∴ $$\delta = \sqrt{\frac{2\mu x}{\rho U_\infty}\frac{\pi^2}{4-\pi}}$$

and $$\frac{\delta}{x} = \left[\frac{2\mu\pi^2}{(4-\pi)\rho U_\infty x}\right]^{1/2} = \frac{4.795}{Re_x^{1/2}}$$

The displacement thickness,

$$\delta^* = \int_0^\delta\left(1-\frac{u}{U_\infty}\right)dy = \int_0^\delta\left[1-\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)\right]dy$$

or $$\delta^* = \left[\delta+\frac{\cos\left(\frac{\pi}{2}\frac{y}{\delta}\right)}{\frac{\pi}{2\delta}}\right]_0^\delta = \delta\left[1-\frac{2}{\pi}\right] = 0.363\delta$$

$$\theta = \int_0^\delta\frac{u}{U_\infty}\left(1-\frac{u}{U_\infty}\right)dy = \int_0^\delta\sin\left(\frac{\pi y}{2\delta}\right)\left(1-\sin\frac{\pi y}{2\delta}\right)dy$$

$$= \frac{2\delta}{\pi} - \frac{\delta}{2} = 0.1366\delta$$

Shear stress,

$$\tau_w = \frac{\pi}{2}\mu\frac{U_\infty}{\delta} = \frac{\pi}{2}\frac{\mu U_\infty \mathrm{Re}_x^{1/2}}{4.795x} = 0.3276\frac{\mu U_\infty}{x}\mathrm{Re}_x^{1/2}$$

Solution : 30

Let the length of the plate be L and L_1 be the distance from the leading edge, where the drag is equal to 2/3 of the total drag.

$$D_{\text{Total}} = C_D.(bL)\rho\frac{U_\infty^2}{2}$$

and
$$D_{L_1} = C_{D_{L_1}}(bL_1)\rho\frac{U_\infty^2}{2} = \frac{2}{3}D_{\text{Total}}$$

$\therefore$
$$\frac{C_{D_{L_1}}}{C_D} \times \frac{L_1}{L} = \frac{2}{3}$$

Also,
$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}}}$$

therefore,
$$C_D = \frac{1.328}{\left(\frac{U_\infty L}{\nu}\right)^{1/2}}$$

and
$$C_{D_{L_1}} = \frac{1.328}{\left(\frac{U_\infty L_1}{\nu}\right)^{1/2}}$$

$\therefore$
$$\frac{2}{3}.\frac{L}{L_1} = \frac{C_{D_{L_1}}}{C_D} = \left(\frac{L}{L_1}\right)^{1/2}$$

or
$$\left(\frac{L_1}{L}\right)^{1/2} = \frac{2}{3}$$

or
$$\frac{L_1}{L} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

■■■■

9 Hydraulic Machines

LEVEL 1 Objective Questions

1. (d)
2. (d)
3. (d)
4. (1.41)
5. (662.57)
6. (a)
7. (a)
8. (b)
9. (b)
10. (b)
11. (b)
12. (a)
13. (52.80)
14. (b)

LEVEL 2 Objective Questions

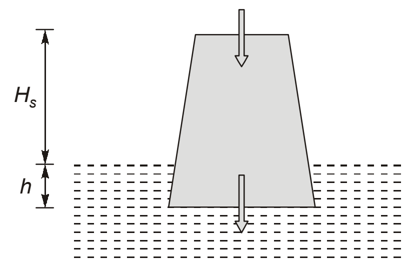
15. (109.52)
16. (d)
17. (c)
18. (b)
19. (a)
20. (d)
21. (c)
22. (c)
23. (d)
24. (b)
25. (d)
26. (d)

■■■■

LEVEL 3 Conventional Questions

Solution : 27

Given data: Head, $H = 100$ m; Power, $P = 20$ MW = 20000 kW; Speed, $N = 200$ rpm; $h = 1$ m; $H_a = 10.3$ m; $H_v = 2.5$ m; Specific speed, $N_s = \frac{N\sqrt{P}}{H^{5/4}}$



Draft tube

Where N is in rpm, P is in kW and H is in m

$$\therefore \quad N_s = \frac{200\sqrt{20000}}{(100)^{5/4}} = 89.44$$

$$\sigma_c = 0.625\left(\frac{N_s}{387.78}\right)^2 = 0.625\left(\frac{89.44}{387.78}\right)^2 = 0.0332$$

also
$$\sigma_c = \frac{H_a - H_v - H_s}{H}$$

$$\therefore \quad 0.0332 = \frac{10.3 - 2.5 - H_s}{100}$$

or
$$0.0332 \times 100 = 10.3 - 2.5 - H_s$$

or
$$H_s = 4.48 \text{ m}$$

Hence, the maximum length of the draft tube = $H_s + h = 4.48 + 1 = 5.48$ m

Solution : 28

Given data: Coefficient of velocity, $C_v = 0.97$; Position of the nozzle from the water level of a lake = 400 m Diameter of jet, $d = 80$ mm = 0.08 m, Diameter of pipe, $D_{pipe} = 0.6$ m; Length of pipe, $l = 4$ km = 4000 m; Friction factor, $f' = 0.032$; Angle of deflection = 165°;

$$\therefore \quad \beta_o = 180° - 165° = 15°$$

Velocity of bucket,
$$u = 0.48\, V_i$$

$$V_{ro} = 0.85\, V_{ri}$$

Mechanical efficiency,
$$\eta_m = 90\% = 0.90$$

Now by continuity equation :

Discharge passing through pipe=discharge issuing from the nozzle

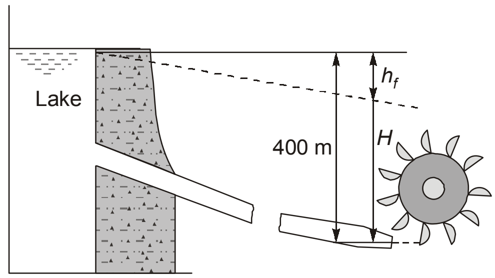
$$AV = aV_i$$

$$\frac{\pi}{4} D_{pipe}^2 \times V = \frac{\pi}{4} d^2 \times V_i$$

$$D_{pipe}^2 \times V = d^2 V_i$$

or
$$V = \left(\frac{d}{D_{pipe}}\right)^2 V_i = \left(\frac{0.08}{0.6}\right)^2 V_i = 0.0177\, V_i$$

Applying Bernoulli's equation to free surface of water in the lake and the base of the nozzle, we get (from figure).



Head of lake = head available at the base of nozzle + head lost due to friction in pipe

$$400 = H + h_f = \frac{V_i^2}{2gC_v^2} + \frac{f'lV^2}{2gD_{pipe}} = \frac{V_i^2}{2gC_v^2} + \frac{f'l(0.0177V_i)^2}{2gD_{pipe}}$$

$$400 = \frac{V_i^2}{2g}\left[\frac{1}{C_v^2} + \frac{f'l \times (0.0177)^2}{D_{pipe}}\right]$$

$$400 = \frac{V_i^2}{2 \times 9.81}\left[\frac{1}{(0.97)^2} + \frac{0.032 \times 4000 \times (0.0177)^2}{0.6}\right]$$

$$400 = \frac{V_i^2}{19.62}[1.0628 + 0.0668]$$

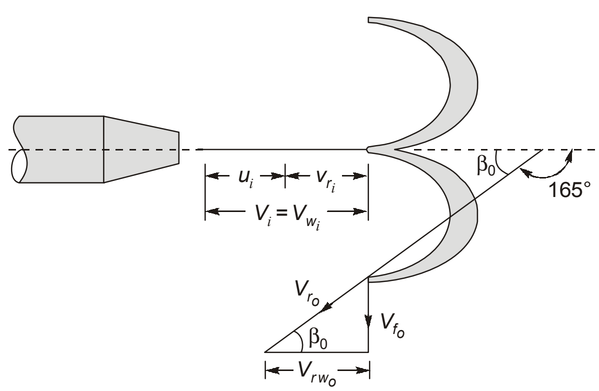
or $V_i^2 = 6947.59$

or $V_i = 83.35$ m/s

Now velocity of bucket, $u = 0.48 \times V_i = 0.48 \times 83.35 = 40$ m/s

At the inlet of Pelton turbine, $V_{ri} = V_i - u_i$ $[\because V_i = V_o = u]$

$= 83.35 - 40 = 43.35$ m/s



and $V_{ro} = 0.85\ V_{ri} = 0.85 \times 43.35 = 36.84$ m/s

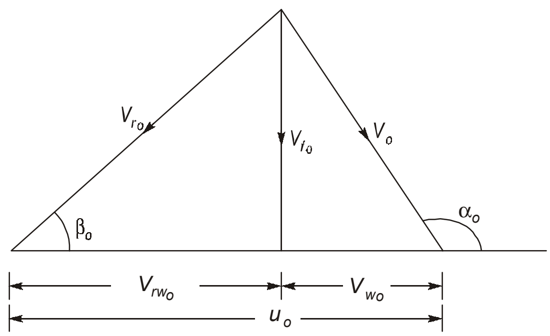
From outlet velocity triangle,

$$\cos\beta_o = \frac{V_{rwo}}{V_{ro}}$$

or $$V_{rwo} = V_{ro}\cos\beta_o = 36.84 \times \cos 15° = 35.58 \text{ m/s}$$

Now suitable data available to draw the complete outlet velocity triangle.

i.e. $u_o > V_{rwo}$, complete outlet velocity triangle is shown in figure.



(a) Flow rate: $$Q = \frac{3.14}{4}\times(0.08)^2\times 83.35 = 0.4157 \text{ m}^3/\text{s}$$

(b) Shaft power: P

Whirl velocity at outlet: $$V_{wo} = u_o - V_{rwo} = 40 - 35.58$$
$$= 4.42 \text{ m/s}$$

Power developed by the runner $$= \rho Q[V_{wi} - V_{wo}]\,u$$
$$= 1000 \times 0.4187[83.35 - 4.42] \times 40$$
$$= 1321919.64 \text{ W} = 1321.919 \text{ kW}$$

Mechanical efficiency: $$\eta_m = \frac{\text{Shaft power:P}}{\text{Power developed by the runner}}$$

or Shaft power: $$P = \eta_m \times \text{Power developed by the runner}$$
$$= 0.90 \times 1321.919 = 1189.71 \text{ kW}$$

Solution : 29

Given data: Shaft power, $P = 22500$ kW $= 22.50 \times 10^6$ W; Head, $H = 20$ m; Speed, $N = 148$ rpm; Hydraulic efficiency, $\eta_H = 95\% = 0.95$; Overall efficiency, $\eta_0 = 89\% = 0.89$; Diameter of the runner, $D = 4.5$ m; Diameter of the hub, $d = 2$ m; Runner vane angle at outlet, $\beta_0 = 34°$

$$V_{fi} = V_{fo} = V_f$$

$\therefore$ Velocity of flow is constant.

Now, peripheral velocity of the runner,

$$u_i = u_0 = u = \frac{\pi DN}{60} = \frac{3.14\times4.5\times148}{60} = 34.85 \text{ m/s}$$

Overall efficiency, $$\eta_o = \frac{P}{\rho QgH}$$

$$0.89 = \frac{22.50\times10^6}{1000\times Q\times9.81\times20}$$

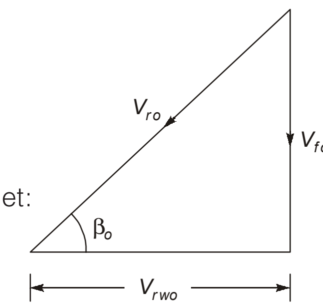
or Discharge, $$Q = 128.85 \text{ m}^3/\text{s}$$

also $$Q = \frac{\pi}{4}[D^2 - d^2]V_{fi}$$

$$128.85 = \frac{3.14}{4}\left[(4.5)^2 - (2)^2\right]\times V_{fi}$$

or $$V_{fi} = 10.10 \text{ m/s} = V_{fo}$$

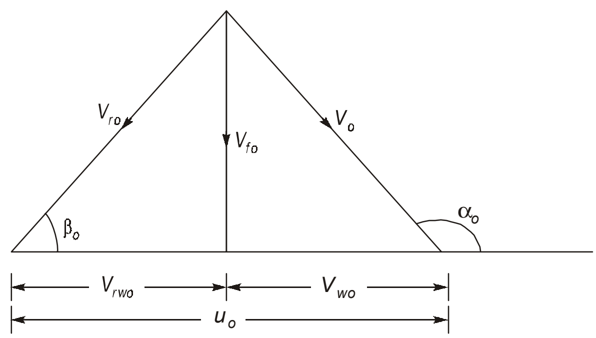
According to given data the partially velocity triangle at outlet as shown in figure.



$$\tan\beta_o = \frac{V_{fo}}{V_{rwo}}$$

or $$V_{rwo} = \frac{V_{fo}}{\tan\beta_o} = \frac{10.10}{\tan 34°} = 14.97 \text{ m/s}$$

Now, we know the suitable data to draw the complete velocity triangle at outlet:



$$u_o > V_{rwo}$$

$$V_{wo} = u_o - 14.97 = 19.88 \text{ m/s}$$

$$V_o = \sqrt{V_{fo}^2 + V_{wo}^2} = \sqrt{(10.10)^2 + (19.88)^2} = 22.30 \text{ m/s}$$

According to energy balance equation,

$$\rho QgH = \rho Q[V_{wi} - V_{wo}]u + \rho Q\frac{V_o^2}{2}$$

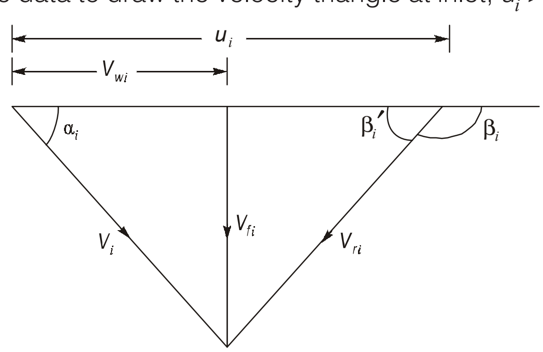
$$gH = [V_{wi} - V_{wo}]u + \frac{V_o^2}{2}$$

$$9.81 \times 20 = [V_{wi} - 19.88] \times 34.85 + \frac{(22.30)^2}{2}$$

$$196.2 = [V_{wi} - 19.88] \times 34.85 + 248.645$$

or $$V_{wi} = 18.375 \text{ m/s}$$

Now, we know the suitable data to draw the velocity triangle at inlet, $u_i > V_{wi}$



(i) Guide vane angle at inlet: α_1

$$\tan\alpha_i = \frac{V_{fi}}{V_{wi}} = \frac{10.10}{18.375} = 0.54966$$

$$\alpha_i = \tan^{-1}(0.54966) = \mathbf{28.796^\circ}$$

(ii) Runner vane angle at inlet: β_i

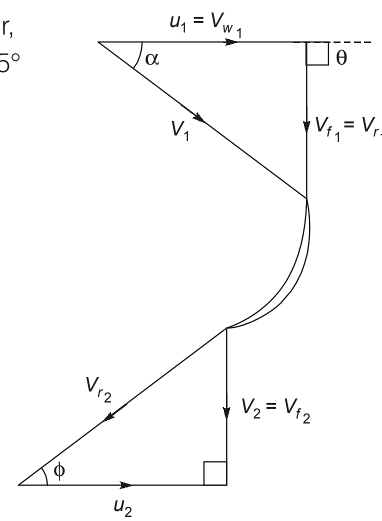
$$\tan\beta_i' = \frac{V_{fi}}{u_i - V_{wi}} = \frac{10.10}{34.85 - 18.375} = 0.613$$

$$\beta_i' = \tan^{-1}(0.613) = 31.5^\circ$$

and $$\beta_i = 180^\circ - 31.5^\circ = \mathbf{148.5^\circ}$$

Solution: 30

Given data: Head, $H = 60$ m; Inlet diameter, $D_1 = 1.5$ m; Outlet diameter, $D_2 = 0.75$ m; Vane angle at entrance, $\theta = 90^\circ$; Guide blade angle, $\alpha = 15^\circ$



From inlet velocity triangle, $\tan\alpha = \dfrac{V_{f_1}}{V_{w_1}}$

$$\Rightarrow \quad \tan 15^\circ = \frac{V_{f_1}}{u_1}$$

$$\Rightarrow \quad V_{f1} = u_1 \tan 15^\circ$$

$$H = \frac{V_2^2}{2g} + \frac{V_{w_1}u_1}{g} = \frac{V_{f2}^2}{2g} + \frac{u_1^2}{g}$$

$$\because \quad V_{w1} = u_1$$

$$\Rightarrow \quad 60 = \frac{V_{f2}^2}{2g} + \frac{u_1^2}{g}$$

$$\Rightarrow \quad 60 = \frac{(u_1 \tan 15^\circ)^2}{2g} + \frac{u_1^2}{g}$$

$$\Rightarrow \quad 60 \times 9.81 = 1.0359\, u_1^2$$

$$\Rightarrow \quad u_1 = 23.837 \text{ m/s}$$

$$\Rightarrow \quad \frac{\pi D_1 N}{60} = 23.837$$

$$\Rightarrow \quad N = \frac{23.837 \times 60}{\pi \times 1.5} = 303.5 \text{ rpm}$$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{u_1 \tan 15^\circ}{u_2}$$

$$\Rightarrow \quad \tan\phi = \frac{D_1}{D_2}\tan 15^\circ = \frac{1.5}{0.75}\tan 15^\circ$$

$$\therefore \quad \phi = 28.19^\circ$$

Solution : 31

Energy per unit weight imparted to the fluid by the pump = $\rho g Q(\Delta H)$

where, ΔH = The head imparted to water by the pump

$$\therefore \quad \Delta H = \frac{11 \times 10^3}{9810 \times \dfrac{150}{1000}} = 7.5 \text{ m of water}$$

Velocity at A, $$V_A = \frac{Q}{\text{Area}} = \frac{150/1000}{\frac{\pi}{4} \times (0.2)^2} = 4.77 \text{ m/s}$$

Velocity at B, $$V_B = \left(\frac{20}{15}\right)^2 \times V_A = 8.48 \text{ m/s} \qquad [A_1V_1 = A_2V_2]$$

Applying the Bernoulli's equation to the free surface in the reservoir and the point A, taking the horizontal line through A as the datum.

$$2 + 0 + 0 = \frac{p_A}{\rho g} + \frac{(4.77)^2}{2 \times 9.81} = \frac{p_A}{\rho g} + 1.159$$

or $$\frac{p_A}{\rho g} = 0.841 \text{ m of water.}$$

$\therefore$ $$p_A = 9810 \times 0.841 = 8250.21 \text{ N/m}^2.$$

Energy per unit weight of water at B is greater than at A by the energy per unit weight supplied by the pump. Applying Bernoulli's equation between A and B,

$$0 + \frac{V_A^2}{2g} + \frac{p_A}{\rho g} + \Delta H = 3.5 + \frac{V_B^2}{2g} + \frac{p_B}{\rho g}$$

$$\frac{(4.77)^2}{2 \times 9.81} + 0.841 + 7.5 = 3.5 + \frac{(8.48)^2}{2 \times 9.81} + \frac{p_B}{\rho g}$$

or $$9.5 = 3.5 + 3.665 + \frac{p_B}{\rho g}$$

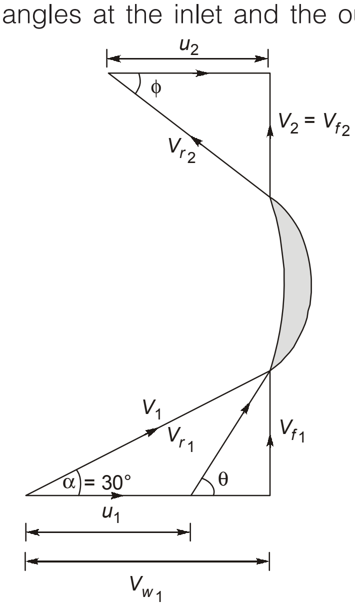
$$\frac{p_B}{\gamma} = 6.0 - 3.665 = 2.335 \text{ m of water}$$

$\therefore$ $$p_B = 9810 \times 2.335 = 22906.35 \text{ N/m}^2 = 22.906 \text{ kN/m}^2 = 22.90 \text{ kPa}$$

Solution : 32

Given: velocity of jet V_1 = 60 m/s, velocity of vanes V_1 = 25 m/s, angles at which the jet enters the vanes α = 30°, and relative velocity of the jet and the vane at the outlet is given by $V_{r_2} = 0.85\ V_{r_1}$.

Refer to figure to find the vane angles at the inlet and the outlet.



(i) **Vane angle at the inlet:**

From the inlet triangle, velocity of whirl at the inlet, is given by

$$V_{w_1} = V_1 \cos 30° = 51.96 \text{ m/s}$$

Velocity of flow at the inlet is given by

$$V_{f_1} = V_1 \sin 30° = 30 \text{ m/s}$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{30}{51.96 - 25} = 1.113$$

Vane angle at the inlet, $\theta = 48.06°$

Relative velocity is given by

$$V_{r_1} = \frac{V_{f_1}}{\sin\theta} = 40.42 \text{ m/s}$$

(ii) **Vane angle at the outlet:**

Relative velocity of the jet and the vane at the outlet is

$$V_{r_2} = 0.85 \times V_{r_1} = 0.85 \times 40.42 = 34.357 \text{ m/s}$$

$$\cos\phi = \frac{u_2}{V_{r_2}} = \frac{25}{34.357} = 0.7276$$

Vane angle at the outlet,

$$\phi = 43°18'$$

Solutin : 33

Data given: effective head H = 150 m, speed N = 300 rpm, overall efficiency = 0.85, and d/D = 1/10

Assuming coefficient of velocity = 0.98.

Velocity (absolute) of jet is given by

$$V_1 = C_V\sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 150} = 53.16 \text{ m/s}$$

To find diameter of wheel (D)

Peripheral velocity of the runner is given by

$$u = 0.46\ V_1 = 24.46 \text{ m/s}$$

Also, $$u = \frac{\pi D N}{60}$$

$$24.46 = \frac{\pi \times D \times 300}{60}$$

$$D = 1.56 \text{ m}$$

To find diameter of the jet (d)

we have $$\frac{d}{D} = \frac{1}{10}$$

$$d = \frac{1.56}{10} = 0.156 \text{ m}$$

To find the width of the bucket

Using the relation, width $= 5 \times d = 5 \times 0.156 = 0.78$ m

Depth of buckets, $1.2 \times d = 1.2 \times 0.156 = 0.187$ m

For number of buckets, $\dfrac{D}{2d} + 15 = \dfrac{1.56}{2 \times 0.156} + 15 = 20$

Solution : 34

Here we use the concept of specific speed.

Given scale model, $= \dfrac{1}{10} = \dfrac{D_m}{D_p}$

Actual turbine (prototype)

$$\text{Power } P_p = 30\,000 \text{ kW}$$

$$\text{Head } H_p = 50 \text{ m}$$

$$N_p = 300 \text{ r.p.m.}$$

Model $H_m = 12$ m

As $u = \pi DN \propto \sqrt{H} = DN \propto \sqrt{H} = D^2 N^2 \propto H$

$$= \frac{H}{D^2N^2} = \text{constant i.e., } \frac{H_p}{D_p^2N_p^2} = \frac{H_m}{D_m^2N_m^2}$$

$$N_m{}^2 = \frac{H_m}{H_p} \times \frac{D_p^2}{D_m^2} \times N_p^2 = \frac{12}{50} \times 10^2 \times 300^2$$

(i) $N_m = 1469.69$ r.p.m.

(ii) $P = \rho g QH$

$$P \propto QH$$

$$\propto A.V.H.$$

$$\propto D^2\sqrt{H}.H = P \propto D^2 H^{3/2} \qquad \left\{\because \sqrt{H} \propto DN\right\}$$

$\Rightarrow$ $P \propto D^2 \times D^3 N^3$ $\qquad H^{3/2} \propto D^3N^3\}$

$$\left(\frac{P}{D^5N^3}\right) = \text{constant} \Rightarrow \frac{P_p}{D_p^5N_p^3} = \frac{P_m}{D_m^5N_m^3}$$

$$P_m = P_p\left(\frac{D_m}{D_p}\right)^5 \times \left(\frac{N_m}{N_p}\right)^3$$

$$P_m = 30000 \times \left(\frac{1}{10}\right)^5 \times \left(\frac{1469.69}{300}\right)^3$$

$$P_m = 35.272 \text{ kW}$$

(iii) Given $\eta_0 = 0.88$

$$\eta_0 = \frac{P_m}{\rho g Q_m H_m}$$

$$\Rightarrow \quad Q_m = \frac{P_m}{\rho g \eta_0 H_m} = \frac{35.2727\times10^3}{1000\times9.81\times0.88\times12} = 0.34\ \text{m}^3/\text{s}$$

As $\quad Q = AV$

$$\propto D^2 \times DN$$

$$\propto D^3 N$$

i.e. $\quad \dfrac{Q}{D^3N} = \text{constant}$

$$\frac{Q_m}{D_m^3 N_m} = \frac{Q_p}{D_p^3 N_p}$$

$$\Rightarrow \quad Q_p = Q_m\left(\frac{D_p}{D_m}\right)^3 \frac{N_p}{N_m} = 0.34\times10^3\times\frac{300}{1469.69} = 69.4\ \text{m}^3/\text{s}.$$

Solution : 35

Discharge, $\quad Q = 4.5\ \text{m}^3/\text{s}$

$N = 750$ rpm

Diameter at inlet, $\quad D_1 = 53$ cm $= 0.53$ m

Diameter at outlet $\quad D_2 = 76$ cm $= 0.76$ m

$V_1 = V_{f1} = 15$ m/s

Vane outlet angle, $\quad \phi = 70°$

width of outlet, $\quad B_2 = 10$ cm $= 0.1$ m

and $\quad u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi\times0.76\times750}{60}$

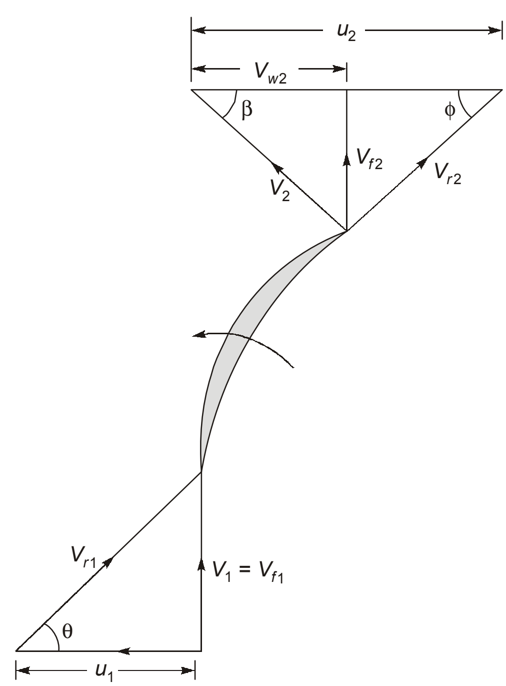
$= 29.8451$ m/s

$Q = \pi D_2 B_2 V_{f2}$

$4.5 = \pi \times 0.76 \times 0.1 \times V_{f2}$

$\Rightarrow \quad V_{f2} = 18.847$ m/s

From outlet velocity triangle,



$$\sin\phi = \frac{V_{f_2}}{V_{r_2}}$$

$$\sin 70° = \frac{18.847}{V_{r_2}}$$

$V_{r2} = 20.05$ m/s

and $\quad V_{w2} = u_2 - V_{r2}\cos\phi$

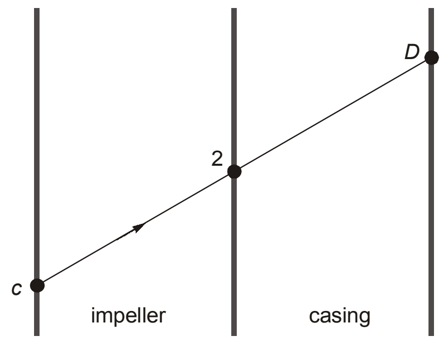
$= 29.8451 - 20.05\cos 70°$

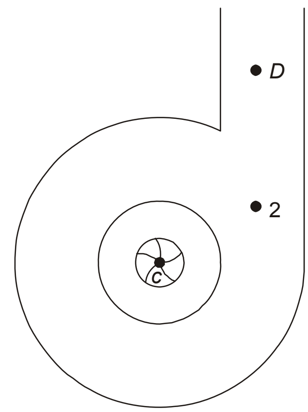
$= 22.985$ m/s

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{18.847}{22.985} = 0.8199$$

$\beta = 39.35°$

$$\cos\beta = \frac{V_{w_2}}{V_2} \Rightarrow V_2 = \frac{22.985}{\cos 39.35} = 29.7237 \text{ m/s}$$





$$H_m = \frac{P_D - P_C}{\rho g} = \frac{P_D - P_2}{\rho g} + \frac{P_2 - P_C}{\rho g}$$

Volute casing gives 30% recovery of outlet velocity head

$$\therefore \qquad \frac{P_D - P_2}{\rho g} = 0.30\frac{V_2^2}{2g}$$

Applying energy equation for impeller

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + \frac{V_{w2}u_2}{g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{P_C}{\rho g} + \frac{15^2}{2\times 9.81} + \frac{22.985\times 29.8451}{9.81} = \frac{P_2}{\rho g} + \frac{29.73^2}{2\times 9.81} + \frac{0.25\times 29.73^2}{2\times 9.81}$$

$$\frac{P_2 - P_C}{\rho g} = 25.084 \text{ m}$$

$$\therefore \qquad H_m = 0.30\times\frac{29.73^2}{2\times 9.81} + 25.084 = 38.60 \text{ m}$$

$$\eta_m = \frac{gH_m}{V_{w2}u_2} = \frac{9.81\times 38.60}{22.985\times 29.8451} = 0.552 = 55.2\%$$

If $\qquad P_1 = P_{atm} = 1$ bar

Discharge pressure $\qquad P_2 = P_1 + \rho g H_m = 4.78$ bar

Solution : 36

Given: diameter of the bore = 0.15 m, area of the bore cylinder $A = 0.01767$ m^2, length of stroke $L = 0.3$ m, actual discharge $Q_a = 250$ l/m, and speed $N = 50$ rpm.

Using the relation for theoretical discharge,

$$Q_t = \frac{ALN}{60} = \frac{0.01767\times 0.3\times 50}{60}$$

$$= 4.4175 \times 10^{-3} \text{ cumecs or } 265.05 \ l/\text{m}$$

Coefficient of discharge,

$$C_d = \frac{250}{265.05} = 0.9432$$

Slip of the pump = 265.05 – 250 = 15.05 l/m

Slip of the pump in percentage = $\frac{265.05-250}{265.05} = 5.68\%$

Alternatively, $(1 - C_d) \times 100 = 5.68\%$

Solution : 37

Given : single acting reciprocating pump, plunger diameter = 0.2 m, cross-sectional area of the plunger = 0.0314 m^2, length of stroke L = 0.3 m, actual discharge Q = 0.6/60 = 0.01 cumecs, speed = 65 rpm, suction head = 6 m and delivery head = 18 m

Note : Plunger pump is similar to that of piston pump, However, in this case, plunger is replacing the piston and its rod. Hence, consider A = area of the plunger.

In the case of single acting reciprocating pump, theoretical discharge is given by

$$Q_t = \frac{ALN}{60} = \frac{0.0314 \times 0.3 \times 65}{60} = 0.0102 \text{ cumecs}$$

Coefficient of discharge,

$$C_D = \frac{Q_a}{Q_t} = \frac{0.01}{0.0102} = 0.98$$

Slip is given by

$$Q_t - Q_a = 0.0102 - 0.01 = 0.0002 \text{ cumecs}$$

$$\text{Percentage slip} = \frac{Q_t - Q_a}{Q_t} \times 100 = \frac{0.0102 - 0.01}{0.0102} \times 100 = 2\%$$

Power required to drive the pump

$$= \gamma Q(h_s + h_d) = 9.81 \times 0.0102 \times (6 + 18) = 2.401 \text{ kW}$$

Solution : 38

Given: diameter of the cylinder = 0.15 m, area of the cylinder A = 0.01767 m^2, stroke length L = 0.3 m, crank radius r = 0.15 m, speed of the pump N = 50 rpm, total height through which water is lifted = 25 m, length of delivery pipe = 22 m, diameter of the delivery pipe = 0.1 m, and actual discharge Q_a = 0.0042 cumecs.

Theoretical discharge is given by the relation

$$Q_t = \frac{ALN}{60} = \frac{0.01767 \times 0.3 \times 50}{60} = 0.0044175 \text{ cumecs}$$

$$\text{The percentage slip} = \frac{Q_t - Q_a}{Q_t} \times 100 = \frac{0.044175 - 0.0042}{0.0044175} \times 100 = 4.92\%$$

Theoretical power, $P_t = \frac{\gamma Q_t H}{1000} = \frac{9810 \times 0.0044175 \times 25}{1000} = 1.083 \text{ kW}$

Acceleration head at the beginning of delivery stroke:

Acceleration head in the delivery pipe is given by

$$h_{ad} = \frac{l_d}{g} \times \frac{A}{A_d} \omega^2 r \cos\theta$$

But $$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 50}{60} = 5.236 \text{ rad/s}$$

At the beginning of the stroke $\theta = 0$ and $\cos\theta = 1$

Therefore,

$$h_{ad} = \frac{22\times 0.01767\times 5.236^2\times 0.15}{9.81\times 0.007854} = 20.75 \text{ m}$$

At the middle of the delivery stroke:

$\theta = 90°$ and $\cos\theta = 0$

Therefore, $h_{ad} = 0$

Solution : 39

Given : single acting reciprocating pump, plunger diameter = 0.06 m, area of the plunger $A = 0.0028$ m^2, stroke length L = 0.12 m. Crank radius $r = 0.06$ m, suction pipe length = 4 m, and diameter of the suction pipe = 0.04 m. Therefore, area of the suction pipe, $A_s = 0.00125$ m^2. Diameter of the delivery pipe = 0.025 m. Therefore, area of the delivery pipe, $A_d = 0.00049$ m^2. Separation occurs at 0.75 bars and specific weight of water = 9.81 kN/m^3. Separation head is given by

$$h_{sep} = \frac{P_{sep}}{\gamma} = \frac{0.75\times 10^5}{9810} = 7.64 \text{ m}$$

Speed of the pump without separation during suction stroke:

Pressure head due to acceleration in the suction pipe is given by

$$h_{as} = \frac{l_s}{g}\times\frac{A}{A_s}\omega^2 r = \frac{4\times 0.0028\times\omega^2\times 0.06}{9.81\times 0.00125} = 0.055\ \omega^2$$

For no separation to take place,

$$(h_{atm} - h_{sep}) = (h_s + h_{as})$$

$$7.64 = 3 + 0.055\ \omega^2$$

$$\omega = 9.18 \text{ rad/s}$$

But $$\omega = \frac{2\pi N}{60}$$

$$9.18 = \frac{2\times\pi\times N}{60}$$

$$N = 87.66 \text{ rpm}$$

Speed of the pump without separation during delivery stroke:

Pressure head due to acceleration in the delivery pipe,

$$h_{ad} = -\frac{l_d}{g}\times\frac{A}{A_d}\omega^2 r = -\frac{15\times 0.0028\times\omega^2\times 0.06}{9.81\times 0.00049} = -0.52\ \omega^2$$

Under limiting condition for no separation to take place,

$$(h_{atm} - h_{sep}) = -(h_d + h_{ad})$$

$$7.64 = -(10 - 0.52\ \omega^2)$$

$$\omega = 5.82 \text{ rad/s}$$

$$\omega = 5.82 = \frac{2\pi N}{60}$$

But, $N = 55.58$ rpm

Therefore, maximum permissible speed = 55.58 rpm (minimum of the two speeds obtained above is considered).

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