2020

RANK Improvement WORKBOOK

Answer key and Hint of
Objective & Conventional *Questions*

Electronics Engineering

Communication Systems

MADE EASY Publications

MADE EASY
Publications

1 Amplitude Modulation

LEVEL 1 Objective Solutions

1.	(4.05)
2.	(d)
3.	(40)
4.	(b)
5.	(a)
6.	(c)
7.	(c)
8.	(a)
9.	(a)
10.	(a)
11.	(c)

LEVEL 2 Objective Solutions

12.	(d)
13.	(a)
14.	(b)
15.	(b)
16.	(c)
17.	(d)
18.	(c)
19.	(c)
20.	(a)

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LEVEL 3 Conventional Solutions

Solution: 1

(a) $$A = \sqrt{200} = 14.14$$
$$B = 8.16$$

(b) $$m = \frac{2B}{A} = 1.1547$$

Solution: 2

$$s(t) = 4\cos(\omega_c t) + 4m(t)\cos(\omega_c t) = 4[1 + m(t)]\cos(\omega_c t)$$

Modulation index, $\mu = |m(t)|_{max} = M$

To get $\mu = 0.10$, M should be equal to 0.10.

Solution: 3

(a) $A_m = 8$ V and $f_m = 5$ kHz

(b) $A = 2A_m = 16$ V

Solution: 4

$$m_a = \sqrt{2}\sqrt{\left(\frac{I_t}{I_c}\right)^2 - 1} = 0.812 \text{ or } 81.2\ \%$$

When modulation index is 0.75,

$$I_t = I_c\sqrt{1 + \frac{m_a^2}{2}} = 8.489 \text{ A}$$

Solution: 5

(a) **Expression for AM wave $v(t)$:**

Let $v_m(t) = V_m \cos \omega_m t$ .... modulating signal

$v_c(t) = V_c \cos \omega_c t$ .... carrier signal

modulated or AM signal is given by,

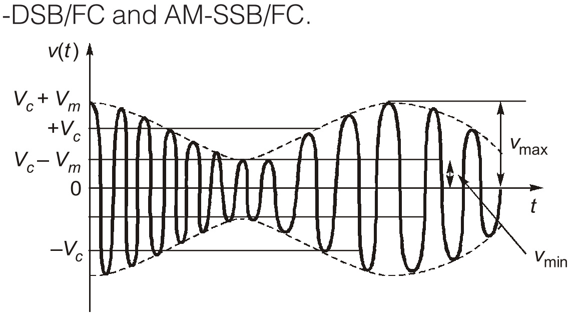
$$v(t) = [V_c + k_a v_m(t)] \cos \omega_c t$$

where, $v_c + k_a v_m(t)$ = New amplitude of carrier signal and is the function of modulating signal

$$v(t) = [V_c + k_a V_m \cos \omega_m t] \cos \omega_c t$$
$$v(t) = V_c [1 + m_a \cos \omega_m t] \cos \omega_c t$$
$$m_a = \frac{k_a V_m}{V_c}$$

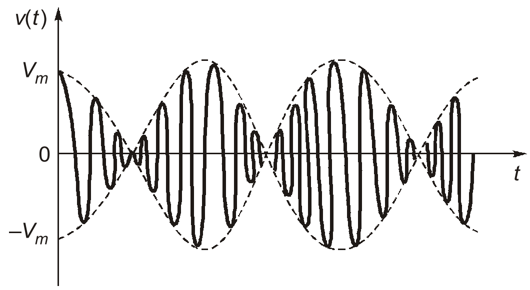
(b) **Waveform for v(t):**

Case-I : AM-DSB/FC and AM-SSB/FC.



for $k_a = 1$

Case-II : AM-DSB/SC and AM-SSB/SC



(c) Modulation Index:

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

It will be recalled that,

$$V_m = mV_c \qquad \text{For } k_a = 1 \qquad \therefore\ m = \frac{V_m}{V_c}$$

and
$$V_m = \frac{V_{max} - V_{min}}{2} \qquad ...(i)$$

and waveform for the AM-DSB/FC shows that,

$$V_c = V_{max} - V_m = V_{max} - \frac{V_{max} - V_{min}}{2}$$

here ∵
$$V_{max} = V_c + V_m$$
$$V_{min} = V_c - V_m$$
$$V_c = \frac{V_{max} + V_{min}}{2} \qquad ...(ii)$$

dividing equation (*i*) by equation (*ii*)

$$m = \frac{V_m}{V_c}$$

$$m = \frac{(V_{max} - V_{min})/2}{(V_{max} + V_{min})/2} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

(d)
$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) = 512.5 \text{ Watt}$$

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2 Angle Modulation

LEVEL 1 Objective Solutions

1. (d)
2. (d)
3. (100)
4. (15.72)
5. (c)
6. (d)
7. (a)
8. (c)
9. (a)
10. (c)
11. (d)

LEVEL 2 Objective Solutions

12. (c)
13. (b)
14. (1.15)
15. (8)
16. (b)
17. (c)
18. (c)
19. (a)
20. (a)
21. (b)

■■■■

LEVEL 3 Conventional Solutions

Solution: 1

(a) $(\Delta f)_{max} = 20$ kHz

(b) $m(t) = 8\sin 500\pi t + 4\sin 1000t$

Solution: 2

(a) When amplitude of modulating signal is doubled,

$$BW_{FM} = 2(\Delta f_{max} + f_m) = 2(200 + 1) \text{ kHz} = 402 \text{ kHz}$$
$$BW_{PM} = 2(\Delta f_{max} + f_m) = 2(20 + 1) \text{ kHz} = 42 \text{ kHz}$$

(b) When the frequency of modulating signal is doubled,

$$BW_{FM} = 2(\Delta f_{max} + f_m) = 2(100 + 2) = 204 \text{ kHz}$$
$$BW_{PM} = 2(\Delta f_{max} + f_m) = 2(20 + 2) \text{ kHz} = 44 \text{ kHz}$$

Solution: 3

(a)
$$Y_{PM}(t) = \cos(2\pi \times 10^6 t + K_p m(t))$$
$$Y_{PM}(t) = \cos(2\pi \times 10^6 t + 4\cos(2\pi \times 10^3\, t))$$

(b) $K_p = 2$ rad/volt

(c) $BW_{PM} = 2(\Delta f_{max} + f_m) = 2(4 + 1) \text{ kHz} = 10 \text{ kHz}$

Solution: 4

(i) Frequency deviation $= k_f m(t) = 40\, A_m \cos 2\pi f_m t$

maximum frequency deviation $= k_f A_m = 40 \times 5 = 200$ Hz

(ii) Modulation index,
$$\beta = \frac{\text{maximum frequency deviation}}{f_m} = 0.2$$

(iii) Instantaneous frequency f_i of *FM* wave

$$f_i = f_c + k_f m(t) = 10^5 + 200 \cos 2\pi \times 10^3 t$$
$$\phi(t) = 2\pi\int f_i dt = 2\pi\int [10^5 + 200\cos 2\pi \times 10^3 t]dt$$
$$= 2\pi \times 10^5 t + 0.2 \sin 2\pi \times 10^3 t$$

Expression for FM wave,
$$s(t) = A_c \cos\{\phi(t)\} = A_c\left[\cos\left\{2\pi \times 10^5 t + 0.2\sin 2\pi \times 10^3 t\right\}\right]$$

Solution: 5

Equation of FM signal,

$$s(t) = A_c \cos[2\pi f_c t + \beta_f \sin 2\pi f_m t] = 4\cos[(50\pi \times 10^6)t + 25 \sin 800\,\pi t]$$

Now modulating frequency is changed as $f'_m = 2$ kHz

$$\beta'_f = \frac{\Delta f_{max}}{f'_m} = 5$$

∴ Now equation of FM signal is

$$s(t) = A_c \cos[2\pi f_c t + \beta'_f \sin 2\pi f'_m\, t] = 4\cos[(50\pi \times 10^6)t + 5 \sin(4000\pi t)]$$

Solution: 6

Equation of FM signal, $s(t) = A_c[\cos 2\pi f_c t + 30 \sin 2\pi f_m t] = 5\cos[36 \times \pi \times 10^6 t + 30 \sin 800\pi t]$

Equation of PM signal, $s(t) = A_c[\cos\{2\pi f_c t + \beta_p \cos 2\pi f_m t\}] = 5\cos[36\,\pi \times 10^6 t + 30 \cos 800\pi t]$

Now, $f'_m = 1.6 \text{ kHz}$

$\therefore$ Now, $$\beta'_f = \frac{\Delta f_{max}}{f'_m} = \frac{12\times 10^3}{1.6\times 10^3} = 7.5$$

Equation of FM signal, $s(t) = 5\cos[36\,\pi \times 10^6 t + 7.5 \sin 3200\pi t]$

$\because$ k_p and A_m are constant, $\beta_p = k_p A_m = 30$

Equation of PM signal, $s(t) = 5\cos[36\pi \times 10^6 t + 30 \cos 3200\pi t]$

Solution: 7

Equation of FM signal, $s(t) = 4.5 \cos[2\pi \times f_c t + \beta_f \sin 2\pi f_m t]$

$= 4.5 \cos[40\pi \times 10^6 t + 22.22 \sin 900\,\pi t]$

Equation of PM signal, $s(t) = A_c \cos[2\pi f_c \text{t} + \beta_p \cos 2\pi f_m t]$

$s(t) = 4.5[\cos\{40\pi \times 10^6 t + 22.22 \cos 900\,\pi t\}]$

Now, $f'_m = 1.8 \text{ kHz}$

Equation of FM signal, $s(t) = 4.5[\cos\{40\pi \times 10^6 t + 5.55 \sin 3600\,\pi t\}]$

Equation of PM signal, $s(t) = 4.5[\cos\{40\pi \times 10^6 t + 22.2 \cos 3600\,\pi t\}]$

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3 Random Variables and Noise

LEVEL 1 Objective Solutions

1.	(b)
2.	(b)
3.	(b)
4.	(b)
5.	(d)
6.	(a)
7.	(c)
8.	(c)
9.	(b)
10.	(a)
11.	(c)

LEVEL 2 Objective Solutions

12.	(b)
13.	(3)
14.	(10)
15.	(a)
16.	(b)
17.	(b)
18.	(b)
19.	(c)
20.	(a)
21.	(b)
22.	(b)
23.	(a)
24.	(c)

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LEVEL 3 Conventional Solutions

Solution: 1

$$\text{Output SNR} = 40 \text{ dB}$$

Solution: 2

$$H(w) = \frac{1}{1+j\omega}$$

(a) $E[x^2(t)] = \frac{1}{\pi}$ and (b) $E[y^2(t)] = \frac{1}{4}$

Solution: 3

(a)
$$K = \frac{2}{3}$$

(b) Mean square value,
$$m_2 = \int_{-\infty}^{\infty} x^2 \rho_X(x)\, dx = 2.4166$$

Solution: 4

(i)
$$P(X < 1 \cap Y < 3) = \int_{-\infty}^{1}\int_{-\infty}^{3} f(x, y)\, dy\, dx = \frac{3}{8}$$

(ii)
$$P(X < 1 \mid Y < 3) = \frac{P(X < 1 \cap Y < 3)}{P(Y < 3)}$$

$$P(Y < 3) = \int_{y=-\infty}^{3}\int_{x=-\infty}^{\infty} f(x, y)\, dx\, dy = \frac{5}{8}$$

$$P(X < 1 \mid Y < 3) = \frac{3}{5}$$

Solution: 5

(i) The correlation function,

$$E[XY] = \frac{1}{2\pi}\int_0^{2\pi} \cos\theta \sin\theta\, d\theta = 0$$

$$\therefore \quad E[XY] = E[X]\cdot E[Y] = 0$$

Thus, X and Y are uncorrelated.

(ii) Two random variables are independent, when,

$$E[XY] = E[X]E[Y] \qquad \text{...(i)}$$

But for the given problem, $E[XY] = E[X] = E[Y] = 0$. Hence, we cannot check the statistical independency of the random variables X and Y using equation (i).

We can check the following equation when equation (i) fails to check the statistical independency of X and Y.

When X and Y are statistically independent,

$$E[X^2Y^2] = E[X^2]\,E[Y^2] \qquad \text{...(ii)}$$

$$E[X^2] = \int_0^{2\pi} f_\theta(\theta)\cdot\cos^2\theta\, d\theta = \frac{1}{2}$$

$$E[Y^2] = \int_0^{2\pi} f_\theta(\theta)\sin^2\theta d\theta = \frac{1}{2}$$

$$E[X^2Y^2] = \frac{1}{2\pi}\int_0^{2\pi}\cos^2\theta\cdot\sin^2\theta d\theta = \frac{1}{8}$$

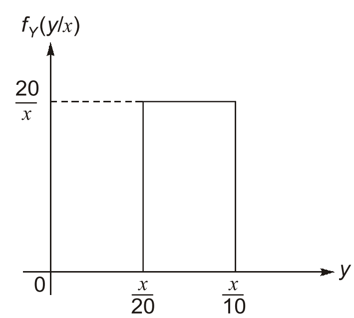
$$E[X^2Y^2] \neq E[X^2]\,E[Y^2]$$

So, the random variables X and Y are statistically dependent.

Solution: 6

The joint probability density function of the random variables X and Y can be written as,

$$f_{XY}(x, y) = f_X(x)\, f_Y(y\,|\,x)$$



$$f_X(x) = \begin{cases} \dfrac{1}{2\times10^4}x\,; & \text{for } 0 \leq x < 200 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

$$f_Y(y) = \frac{20}{x}\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right]$$

Where $u(\tau)$ is an unit step function.

The marginal probability density function can be defined as,

$$f_Y(y) = \int_{x=-\infty}^{\infty} f_{XY}(x,y)\,dx$$

$$= \int_{x=0}^{200}\left(\frac{x}{2\times10^4}\right)\left(\frac{20}{x}\right)\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right]dx$$

$$= \frac{1}{1000}\int_{x=0}^{200}\left[u\left(y-\frac{x}{20}\right)-u\left(y-\frac{x}{10}\right)\right]dx \qquad \text{...(i)}$$

$$\int_{x=0}^{200} u\left(y-\frac{x}{20}\right)dx = -20\int_{\tau=y}^{y-10} u(\tau)\,d\tau = -20[(y-10)u(y-10)-yu(y)] \qquad \text{...(ii)}$$

$$\int_{x=0}^{200} u\left(y-\frac{x}{10}\right)dx = -10\int_{\tau=y}^{y-20} u(\tau)\,d\tau = -10[(y-20)u(y-20)-yu(y)] \qquad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get,

$$f_Y(y) = \frac{1}{1000}\left[20yu(y) - 20(y-10)u(y-10) + 10(y-20)u(y-20) - 10yu(y)\right]$$

$$f_Y(y) = \frac{y}{100}u(y) - \frac{1}{50}(y-10)u(y-10) + \frac{1}{100}(y-20)u(y-20)$$

$f_Y(y)$ also can be expressed as,

$$f_Y(y) = \begin{cases} \frac{y}{100} & ; \quad 0 < y \le 10 \\ \frac{1}{5} - \frac{y}{100} & ; \quad 10 < y \le 20 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

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4 Baseband Pulse Modulation

LEVEL 1 Objective Solutions

1. (b)

2. (0.5)

3. (b)

4. (0.5)

5. (b)

6. (b)

7. (a)

8. (c)

9. (d)

10. (b)

LEVEL 2 Objective Solutions

11. (a)

12. (716.2)

13. (c)

14. (7.305)

15. (c)

16. (c)

17. (b)

18. (d)

19. (d)

20. (b)

■■■■

LEVEL 3 Conventional Solutions

Solution: 1

$$h(t) = u(t) - u(t-T).$$

Solution: 2

Let,

$$m(t) = A_m \sin \omega_m t$$

For no slope overload condition,

$$\frac{\Delta}{T_s} > \left|\frac{dm(t)}{dt}\right|_{max}$$

$$\frac{\Delta}{T_s} > A_m \omega_m \qquad \left[T_s = \frac{1}{f_s} = \text{Sampling period}\right]$$

$$\Rightarrow \qquad A_m < \frac{\Delta}{2\pi f_m T_s}$$

So, maximum allowed signal power

$$P = \frac{A_m^2}{2} = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2}$$

Now, quantization error (Q_e) will lie in the range of $-\Delta$ to $+\Delta$ for DM. Q_e is a uniformly distributed random variable with PDF.

$$f_{Qe}(q_e) = \begin{bmatrix} \frac{1}{2\Delta} & -\Delta \le q_e \le \Delta \\ 0 & \text{otherwise} \end{bmatrix}$$

So, noise power,

$$N_Q = E[q_e^2] = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} q_e^2 \, dq_e = \frac{\Delta^2}{3}$$

This is the noise power for frequency upto f_s. But the LPF at the output will allow only frequencies below $f_m = \frac{\omega_m}{2\pi}$.

So, $$\text{Output noise power} = \frac{f_m \Delta^2}{3f_s}$$

So, $$\text{Maximum output SNR} = \frac{\dfrac{\Delta^2}{8\pi^2 f^2 T_s^2}}{\dfrac{\Delta^2 f_m}{3f_s}} = \frac{3}{8\pi^2}\left(\frac{f_s}{f_m}\right)^3$$

Solution: 3

(a) $$R_b = nf_s = 48 \text{ kbps}$$

(b) $$\text{BW} = \frac{R_b}{2 \times \log_2 M}$$

$$M = 64$$

(c) $$\text{Baud rate} = 8 \text{ kbaud/sec}$$

Solution: 4

Bit rate, $R_b = 768$ kbps

Solution: 5

(i) In a delta modulator the quantization noise is given by the level $\pm \frac{\delta}{2}$

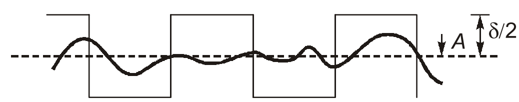
i.e. $$|m(t) - \hat{m}(t)| = \frac{\delta}{2}$$

Now to recognize the signal the amplitude of the signal should be greater than $\frac{\delta}{2}$

i.e. $$A > \frac{\delta}{2}$$

or $$2A > \delta$$

This if this condition is not satisfied then the signal will be lost.



(ii) Now, to avoids slope overload in a delta modulator slope of $m(t)$ should be less then the change in the step of the modulator.

i.e. $$\left|\frac{dm(t)}{dt}\right| < \frac{\delta}{T_s}$$

where δ is the total step size and T_s is the sampling rate

let $$m(t) = A\sin(\omega_m t)$$

∴ $$\frac{d}{dt}\left|A\sin(\omega_m t)\right| < \frac{\delta}{T_s}$$

$$A\omega_n < \frac{\delta}{T_s}$$

$$2\pi f_m \cdot A < \frac{\delta}{T_s}$$

or $$A_{max} = \frac{\delta f_s}{2\pi f_m}$$

(iii) We have $\delta < 2$ A for the signal to be recognisable and $A_{max} = \frac{\delta f_s}{2\pi f_m}$

thus to avoids slope overload

$$\frac{f_s}{\pi f_m} > 1 \text{ if } A > \frac{\delta}{2}$$

∴ $$f_s > \pi f_m$$

$$\approx \quad f_s > 3 f_m$$

Solution: 6

$$55.8 = 1.76 + 6.02R \text{ dB}$$

$$R = 8.976$$

$$R \approx 9 \quad \text{[As } R \text{ has to be an integer]}$$

So minimum number of quantization levels,

$$L_{min} = 2^R = 2^9 = 512$$

■■■■

5 Bandpass Digital Transmitter

LEVEL 1 Objective Solutions

1. (b)
2. (c)
3. (c)
4. (c)
5. (b)
6. (c)
7. (a)
8. (d)
9. (c)

LEVEL 2 Objective Solutions

10. (d)
11. (d)
12. (b)
13. (c)
14. (a)
15. (b)
16. (c)
17. (a)
18. (d)
19. (0.4693)

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LEVEL 3 Conventional Solutions

Solution: 1

$$S_o(T) = \frac{7A^2T}{24}$$

Solution: 2

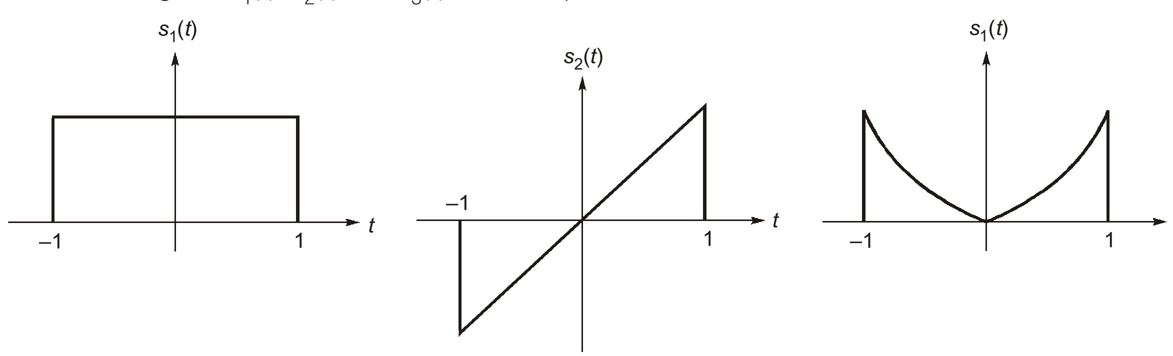
$$P = 36.48 \text{ mW}$$

Solution: 3

$$S_{11} = 2,\ S_{21} = -4,\ S_{22} = 4,\ S_{31} = 3,\ S_{32} = -3 \text{ and } S_{33} = 3.$$
$$u_1(t) = 1\ ;\ 0 \le t \le 1 \text{ and } u_2(t) = -1\ ;\ 1 \le t \le 2$$
$$u_1(t) = 1\ ;\ 2 \le t \le 3.$$

Solution: 4

(i) The three signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ can be represented as shown below.



These three signals are linearly independent. So, three orthonormal basis functions are required. Let, the three orthonormal basis functions are $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$.
Then the given three signals can be represented with respect to $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$ using Gram-Schmidt orthogonalisation process.

To determine $\phi_1(t)$:
$$\phi_1(t) = \frac{s_1(t)}{\|s_1(t)\|}$$
$$\|s_1(t)\| = \sqrt{\text{Energy of } s_1(t)}$$
$$\text{Energy of } s_1(t) = \int_{-1}^{1} dt = 2$$
$$\therefore \quad \|s_1(t)\| = \sqrt{2}$$
$$\therefore \quad \phi_1(t) = \frac{s_1(t)}{\sqrt{2}} = \frac{1}{\sqrt{2}}\ ; \quad -1 \le t \le 1$$

To determine $\phi_2(t)$:
$$\phi_2(t) = \frac{g_2(t)}{||g_2(t)||}$$
$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

Here,
$$s_{21} = \int_{-1}^{1} s_2(t)\phi_1(t)\,dt = \frac{1}{\sqrt{2}}\int_{-1}^{1} t\,dt = 0$$

$$\phi_2(t) = \frac{g_2(t)}{||g_2(t)||} = \frac{s_2(t)}{||s_2(t)||} \qquad (\because s_{21} \text{ is zero})$$

$$||s_2(t)|| = \sqrt{\text{Energy of } s_2(t)}$$

$$\text{Energy of } s_2(t) = \int_{-1}^{1} t^2 dt = \left.\frac{t^3}{3}\right|_{-1}^{1} = \frac{2}{3} \quad ; \quad ||s_2(t)|| = \sqrt{\frac{2}{3}}$$

$\therefore$

$$\phi_2(t) = \frac{s_2(t)}{||s_2(t)||} = \sqrt{\frac{3}{2}}\,t \quad ; \quad -1 < t < 1$$

To determine $\phi_3(t)$:

$$\phi_3(t) = \frac{g_3(t)}{||g_3(t)||}$$

$$g_3(t) = s_3(t) - s_{31}\,\phi_1(t) - s_{32}\,\phi_2(t)$$

$$s_{31} = \int_{-1}^{1} s_3(t)\phi_1(t)dt = \frac{1}{\sqrt{2}}\int_{-1}^{1} t^2 dt = \frac{\sqrt{2}}{3}$$

$$s_{32} = \int_{-1}^{1} s_3(t)\phi_2(t)dt = \sqrt{\frac{3}{2}}\int_{-1}^{1} t^3 dt = 0$$

$\therefore$

$$g_3(t) = s_3(t) - \frac{\sqrt{2}}{3}\frac{1}{\sqrt{2}} = \left(t^2 - \frac{1}{3}\right) \quad ; \quad -1 < t < 1$$

$$||g_3(t)|| = \sqrt{\int_{-1}^{1}\left(t^2 - \frac{1}{3}\right)^2 dt} = \sqrt{\int_{-1}^{1}\left(t^4 + \frac{1}{9} - \frac{2}{3}t^2\right)dt} = \sqrt{\frac{8}{45}}$$

$\therefore$

$$\phi_3(t) = \frac{g_3(t)}{||g_3(t)||} = \sqrt{\frac{45}{8}}\left[t^2 - \frac{1}{3}\right] \quad ; \quad -1 < t < 1$$

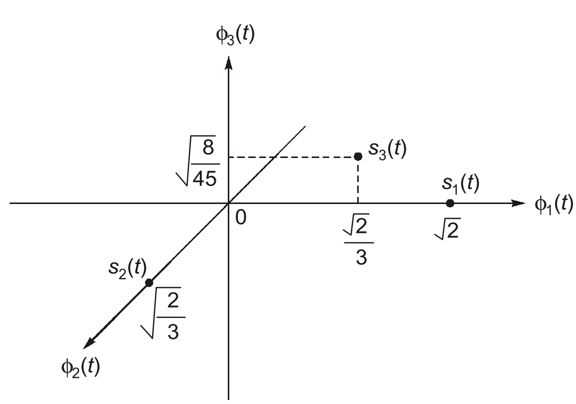
Thus,

$$s_1(t) = \sqrt{2}\,\phi_1(t)$$

$$s_2(t) = \sqrt{\frac{2}{3}}\,\phi_2(t)$$

$$s_3(t) = \frac{\sqrt{2}}{3}\phi_1(t) + \sqrt{\frac{8}{45}}\,\phi_3(t)$$

(ii)



(iii) The Euclidean distance between the signals $s_1(t)$ and $s_3(t)$ is,

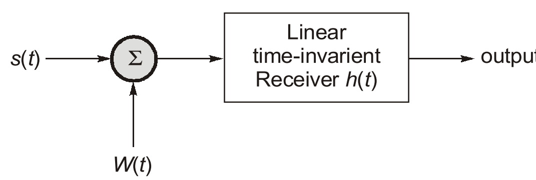
$$d_{13} = \left\|s_1(t) - s_3(t)\right\| = \left\|\sqrt{2}\,\phi_1(t) - \frac{\sqrt{2}}{3}\phi_1(t) - \sqrt{\frac{8}{45}}\,\phi_3(t)\right\|$$

$$= \left\|\frac{2\sqrt{2}}{3}\phi_1(t) - \sqrt{\frac{8}{45}}\,\phi_3(t)\right\|$$

$$= \sqrt{\left(\frac{2\sqrt{2}}{3}\right)^2 + \left(\sqrt{\frac{8}{45}}\right)^2} = \sqrt{\frac{8}{9} + \frac{8}{45}} = \sqrt{\frac{48}{45}} = 1.0328$$

Solution: 5

When a linear filter is designed to maximize the output signal-to-noise ratio for a given input signal it is called as a match filter. The filter is called as match filter because its characterization is matched to that of the signal component in the received signal.



Consider a linear time invariant filter of impulse response $h(t)$ with transfer function $H(f)$, with input $x(t)$ which consists of a signal $s(t)$ and white Gaussian noise $W(t)$ and has an output $y(t)$.
Now, the filter should perform such that the SNR at the output should be maximum.
Let $s_0(t)$ and $n_0(t)$ be the output of the filter which is equal to the output $y(t)$.

$$\therefore \qquad s_0(t) = \int_{-\infty}^{\infty} H(f)\cdot S(f)\cdot \exp(j2\pi ft)\,df$$

The output noise spectral density is equal to the input noise spectral density multiplied by $|H(f)|^2$.

$$\therefore \qquad S_{N_0}(f) = \frac{N_0}{2}|H(f)|^2$$

$$\therefore \qquad N_{out} = \int_{-\infty}^{\infty} S_{N_0}(f)\,df$$

Where N_{out} is the output noise power.

$$\Rightarrow \qquad N_{out} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

By sampling the output at $t = T$, we get,

$$\therefore \qquad SNR_0 = \frac{|s_0(T)|^2}{N_{out}} = \frac{\left|\int_{-\infty}^{\infty} S(f)\cdot H(f)\cdot \exp(j2\pi fT)\,df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2\,df}$$

Now, $$\left|\int_{-\infty}^{\infty} X(f)Y(f)\right|^2 \le \int_{-\infty}^{\infty}|X(f)|^2\,df \int_{-\infty}^{\infty}|Y(f)|^2\,df$$

This is called as Schwarz's inequality. It holds at equality, when,

$$Y(f) = kX^*(f); \quad k \text{ is a constant}$$

Thus our equation of SNR_0 can be written as,

$$SNR_0 \le \frac{\int_{-\infty}^{\infty} |S(f)\exp(j2\pi fT)|^2 df \int_{-\infty}^{\infty} \left|H(f)^2\right| df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

The SNR_0 will be maximum, when the above inequality hods at equality.

i.e., when,

$$H(f) = k[S(f)\exp(j2\pi fT)]^*,$$

$$SNR_0 = (SNR_0)_{max}$$

$$(SNR_0)_{max} = \frac{2E}{N_0}$$

where E = energy of the signal $s(t)$.

So, it is clear that, to get maximum SNR at the output of the filter,

$$H(f) = k[S(f)\exp(j2\pi fT)]^*$$

By taking inverse Fourier transform, we get,

$$h(t)_{opt} = \int_{-\infty}^{\infty} k[S(f)\exp(j2\pi fT)]^* \exp(j2\pi ft)df$$

$$= k\int_{-\infty}^{\infty} S^*(f)\exp(-j2\pi fT)\exp(j2\pi ft)df$$

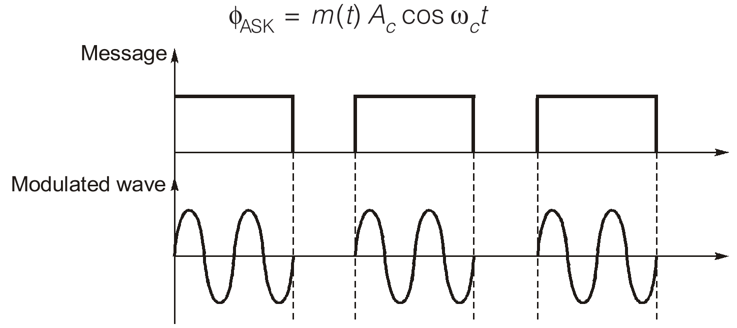
$$= k\int_{-\infty}^{\infty} S^*(f)\exp[-j2\pi f(T-t)]df = k\left[\int_{-\infty}^{\infty} S(f)\exp[j2\pi f(T-t)]df\right]^*$$

$$\therefore \quad h(t)_{opt} = ks^*(T-t)$$

Solution: 6

ASK (Amplitude shift keying): In this scheme, the amplitude of the carrier wave is varied according to the digital data.

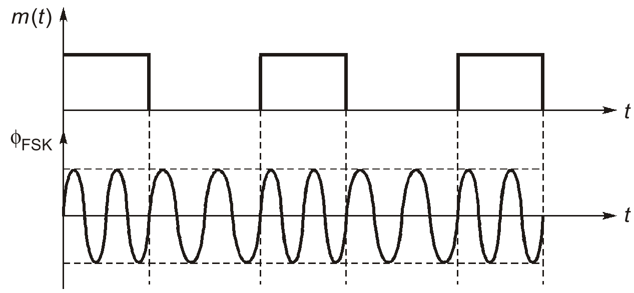
$$\phi_{ASK} = m(t)\, A_c \cos \omega_c t$$



ASK

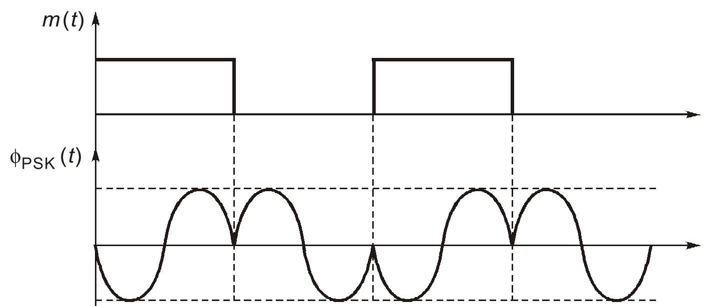
Frequency shift keying (FSK): In this scheme, the frequency of the carrier wave is varied according to the digital data.

$$\phi_{FSK} = A_c \cos[2\pi(f_c + m(t)f_0)t]$$



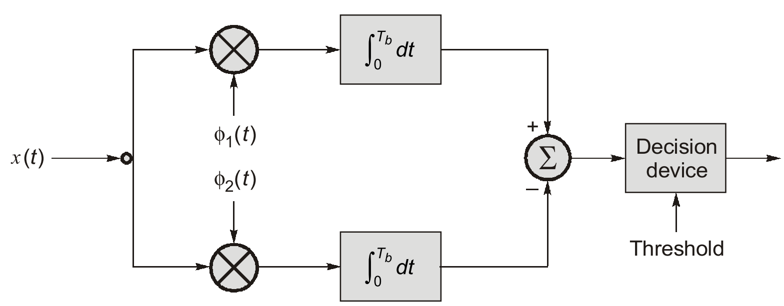
Phase shift keying (PSK): In this scheme, the phase of the carrier wave is varied according to the digital data.

$$\phi_{PSK} = A_c \cos(2\pi f_c t + \pi m(t))$$



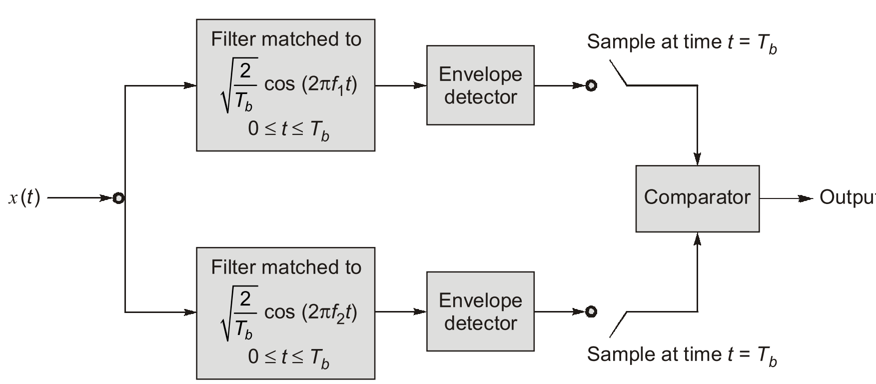
PSK

Coherent detection of FSK:



$$\phi_1(t) = A_c \cos 2\pi f_1 t$$
$$\phi_2(t) = A_c \cos 2\pi f_2 t$$

Non Coherent detection of FSK:



Solution: 7

Taking the inverse Fourier transform of $H(\omega)$, we have

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega$$

The range of integration in the preceding equation can be divided into segments of length $\frac{2\pi}{T}$ as

$$h(t) = \frac{1}{2\pi}\sum_{K=-\infty}^{\infty}\int_{(2K-1)\pi/T}^{(2K+1)\pi/T} H(\omega)e^{j\omega t}d\omega$$

And we can write $h(nT)$ as

$$h(nT) = \frac{1}{2\pi}\sum_{K=-\infty}^{\infty}\int_{(2K-1)\pi/T}^{(2K+1)\pi/T} H(\omega)e^{j\omega nT}d\omega$$

By the change of variable

$$u = \omega - 2\pi(K/T)$$

$$h(nT) = \frac{1}{2\pi}\sum_{K=-\infty}^{\infty}\int_{-\pi/T}^{\pi/T} H\left(u+\frac{2\pi K}{T}\right)e^{j(u+2\pi K/T)nT}du$$

Assuming that the integration and summation can be interchanged we have,

Finally, if Nyquist pulse shaping criterion is satisfied, then

$$h(nT) = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} e^{junT}du = \frac{1}{T}\frac{\sin n\pi}{n\pi} = \begin{cases}\frac{1}{T} & n=0\\ 0 & n\neq 0\end{cases}$$

Which varifies that $h(t)$ with a Fourier transform. $H(\omega)$ satisfying Nyquist criterion.

■■■■

6 Information Theory and Coding

LEVEL 1 Objective Solutions

1. (a)
2. (d)
3. (13.7)
4. (b)
5. (c)
6. (14.4)
7. (c)
8. (c)
9. (c)
10. (c)
11. (d)
12. (b)
13. (a)

LEVEL 2 Objective Solutions

14. (a)
15. (d)
16. (c)
17. (2.46)
18. (b)
19. (a)
20. (a)
21. (b)
22. (c)
23. (d)

■■■■

LEVEL 3 Conventional Solutions

Solution: 1

$$I(x, y) = H(x) + H(y) - H(x, y) = 0.9927 + 0.9984 - 1.9233 = 0.067 \text{ bits/symbol}$$

Solution: 2

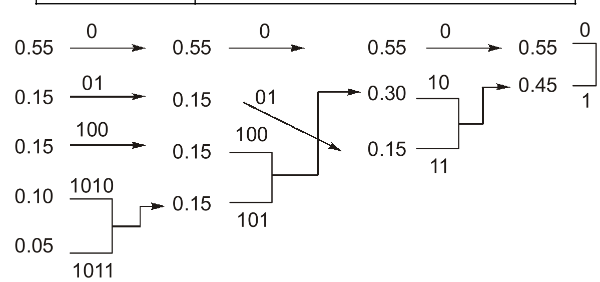
(a)
$$\text{Number of data bits} = k = 4$$
$$\text{Number of parity bits} = n - k = 3$$

(b)
$$C = dG$$

(i) $C = [0111]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = [0111001]$ (ii) $C = [1011]\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = [1011010]$

Solution: 3

Symbol	S_0	S_1	S_2	S_3	S_4
Probability	0.55	0.15	0.15	0.10	0.05



$$L = 1.9 \text{ bits/symbol}$$
$$H = 1.8437 \text{ bits/symbol}$$
$$\%\eta = \frac{H}{L} \times 100 = 97.04\%$$

Solution: 4

(i)
$$P(r_0) = P(r_0|m_0)P(m_0) + P(r_0|m_1)P(m_1) = 0.9(0.5) + 0.2(0.5) = 0.55$$
$$P(r_1) = P(r_1|m_0)P(m_0) + P(r_1|m_1)P(m_1) = 0.1(0.5) + 0.8(0.5) = 0.45$$

(ii) Using Bayes's rule,
$$P(m_0|r_0) = \frac{P(m_0)P(r_0|m_0)}{P(r_0)} = \frac{(0.5)(0.9)}{0.55} = 0.818$$

(iii) Similarly,
$$P(m_1|r_1) = \frac{P(m_1)P(r_1|m_1)}{P(r_1)} = \frac{(0.5)(0.8)}{0.45} = 0.889$$

(iv)
$$P_e = P(r_1|m_0)P(m_0) + P(r_0|m_1)P(m_1) = 0.1(0.5) + 0.2(0.5) = 0.15$$

(v) The probability that the transmitted signal is correctly read at the receiver is
$$P_c = 1 - P_e = 1 - 0.15 = 0.85$$

Solution: 5

$$P(Y = 0) = \frac{9}{32}$$

The probability of transmitted message is '1' if received message is '1' is,

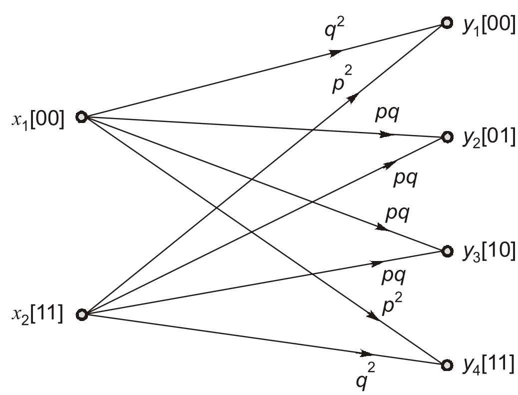
$$P\left(X = 1 / Y = 1\right) = \frac{16}{23}$$

Solution: 6

(i) $C = B \log_2(1 + \text{SNR})$ bits/sec = 1200 kbits/sec

(ii) 37 channels can be accommodated.

Solution: 7



$$H(x, y) = -\sum_{j=1}^{2} \sum_{K=1}^{4} p(x_j, y_k) \log p(x_j, y_k)$$

$$H(x, y) = \left[2\left(\frac{q^2}{2}\log\frac{q^2}{2}\right) + 4\left(\frac{pq}{2}\log\frac{pq}{2}\right) + 2\left(\frac{p^2}{2}\log\frac{p^2}{2}\right)\right]$$

■■■■

7 Multiple Access Techniques and CDMA

LEVEL 1 Objective Solutions

1. (c)

2. (b)

3. (d)

4. (b)

5. (c)

6. (a)

LEVEL 2 Objective Solutions

7. (d)

8. (d)

9. (b)

10. (d)

11. (c)

12. (a)

13. (c)

■■■■

LEVEL 3 Conventional Solutions

Solution: 1

Minimum transmission bandwidth required by the FDM system = 96 kHz

Bit rate of standard 8-bit PCM, $R_b = Nnf_s = 24 \times 8 \times 8 \times 10^3$ bits/sec = 1536 kbits/sec

Minimum transmission bandwidth required by the PCM system = $R_b/2 = 768$ kHz

So, the minimum transmission bandwidth required by standard 8-bit PCM-TDM for sending same 24 message signals is high compared to that of AM-SSB – FDM system.

Solution: 2

(i) Total number of duplex channels for voice = $\frac{50 \times 10^6}{50 \times 10^3} = 1000$

(ii) Total number of duplex channels for control = $\frac{1 \times 10^6}{50 \times 10^3} = 20$

(iii) Total number of cells within a cluster = 5

Voice channels per cell = $\frac{1000}{5} = 200$

Control channels per cell = $\frac{20}{5} = 4$

Total number of channels per cell = 200 + 4 = 204.

Solution: 3

Number of control bits per frame, $C = 5/2 = 2.5$ bits

$$\text{B.W.} = \frac{R_b}{2} = \frac{1}{2}\left[Nnf_s + Cf_s\right] = 250 \text{ kHz}$$

■■■■