2020

RANK Improvement WORKBOOK

Detailed Explanations of Objective & Conventional *Questions*

Electronics Engineering

Signals and Systems

MADE EASY Publications

1 Continuous Time Signals & Systems

LEVEL 1 Objective Solutions

1. (d)

2. (d)

3. (2)

4. (8)

5. (0.303)

6. (d)

7. (c)

8. (a)

9. (b)

10. (d)

11. (b)

12. (c)

LEVEL 2 Objective Solutions

13. (c)

14. (a)

15. (d)

16. (1)

17. (b)

18. (c)

19. (c)

20. (b)

21. (d)

22. (a)

23. (b)

24. (a)

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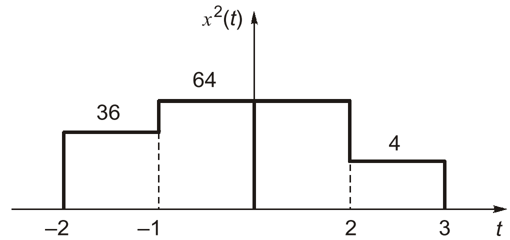
LEVEL 3 Conventional Solutions

Solution : 1

$x^2(t) =$ 

$$\text{Energy of } x(t) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-2}^{-1} 36dt + \int_{1}^{2} 64dt + \int_{2}^{3} 4dt$$

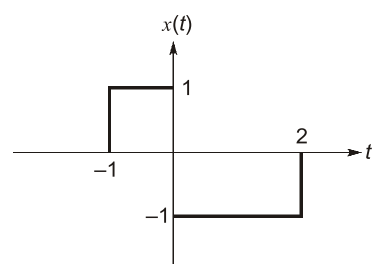
$$= 36 \times 1 + 64 \times 3 + 4 \times 1 = 232 \text{ J}$$

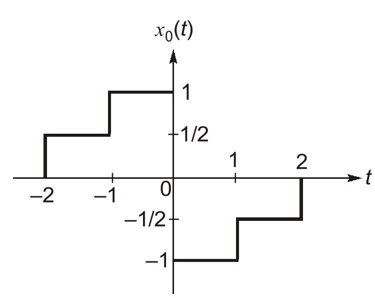
Solution : 2

(i) $h(t) = e^{-2(t-1)}\, u(t-1)$

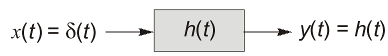
(ii) Yes

Solution : 3





Solution : 4



$\therefore$ $$h(t) = \frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \delta(t)dt$$

$\Rightarrow$ $$h(t) = \left[u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)\right] \times \frac{1}{T}$$

The system is not causal as we can clearly see from equation (i) that for calculation of $y(t)$ at any time t, we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any causal system.

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2 Discrete Time Signals & Sysems

LEVEL 1 Objective Solutions

1.	(b)
2.	(12)
3.	(b)
4.	(c)
5.	(3)
6.	(c)
7.	(1)
8.	(d)
9.	(d)
10.	(b)
11.	(b)

LEVEL 2 Objective Solutions

12.	(a)
13.	(0.5)
14.	(1)
15.	(a)
16.	(b)
17.	(c)
18.	(d)
19.	(b)
20.	(b)
21.	(d)
22.	(a)
23.	(c)
24.	(c)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

(i) Non-linear

(ii) Time variant

Solution : 2

$$h[n] = \frac{3}{2}\delta[n]$$

Solution : 3

(i) No

(b) Yes

Solution : 4

So,

$$\sum_{n=-\infty}^{\infty} h(k) = \sum_{n=0}^{\infty} |a^n| + \sum_{n=-\infty}^{-1} |b^n|$$

$$\sum_{n=-\infty}^{-1} |b^n| = \sum_{n=-\infty}^{-1} |b|^n = \sum_{n=1}^{\infty} |b|^{-n}$$

Solution : 5

$$x[n] = \sin\left[\frac{3\pi}{7}n + \frac{\pi}{4}\right] + \cos\frac{\pi}{4}n$$

$$\Rightarrow \quad x[n+n_0] = \sin\left[\frac{3\pi}{7}(n+n_0) + \frac{\pi}{4}\right] + \cos\left(\frac{\pi}{4}(n+n_0)\right)$$

$$= \sin\left[\frac{3\pi}{7}n + \frac{3\pi}{7}n_0 + \frac{\pi}{4}\right] + \cos\left[\frac{\pi}{4}n + \frac{\pi}{4}n_0\right]$$

The sine function here has a period of $n_0 = 14$, as n_0 can only be an integer. The cosine function here has a period of $n_0 = 8$, as n_0 can only be an integer.

Thus, the fundamental period of $x[n]$ is LCM of 14 and 8, which is 56.

Solution : 6

$$\Rightarrow \quad y(n) = \sum_{k=-\infty}^{\infty} h_2(k)x(n-k)$$

$$\Rightarrow \quad y(n) = \sum_{k=1}^{\infty} x(n-k)$$

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3 Continuous Time Fourier Series

LEVEL 1 Objective Solutions

1. (d)
2. (d)
3. (c)
4. (a)
5. (0.5)
6. (a)
7. (a)
8. (c)
9. (b)
10. (a)
11. (d)
12. (a)
13. (b)

LEVEL 2 Objective Solutions

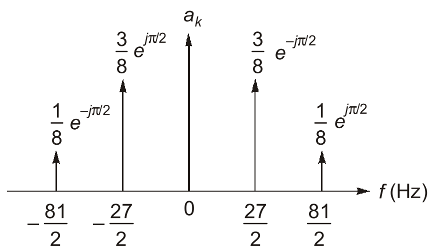
14. (d)
15. (a)
16. (c)
17. (a)
18. (b)
19. (d)
20. (b)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

(i)



(ii) $T = \frac{2}{27}$ sec

Solution : 2

$$x(t) = 4\cos\frac{\pi}{4}t - 8\sin\frac{3\pi}{4}t$$

Solution : 3

(a) Half wave symmetry.

(b) $x(t) = -4\sin 10\pi t + 6\cos 30\pi t + 8\cos 50\pi t + 2\sin 70\pi t$

Solution : 4

Given that $$f(t) = \sum_{n=-\infty}^{\infty} \frac{3}{4+(n\pi)^2} e^{jn\pi t} \quad \text{...(iii)}$$

$$\therefore \quad f_1(t) + f_2(t) = 2\cdot\frac{3}{4+(3\pi)^2}\cos 3\pi t = \frac{6}{4+(3\pi)^2}\cos 3\pi t$$

Comparing with $A\cos 3\pi t$, we have, $A = \frac{6}{4+9\pi^2} = 6.464 \times 10^{-2}$

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4 Fourier Transform and Sampling Theorem

LEVEL 1 Objective Solutions

1.	(a)
2.	(d)
3.	(1)
4.	(–12.56)
5.	(b)
6.	(c)
7.	(7)
8.	(520)
9.	(b)
10.	(c)
11.	(a)
12.	(b)
13.	(c)
14.	(c)

LEVEL 2 Objective Solutions

15.	(d)
16.	(2.41)
17.	(0)
18.	(b)
19.	(a)
20.	(a)
21.	(b)
22.	(c)
23.	(c)
24.	(a)
25.	(a)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

(i) $E_y = \dfrac{1000}{3\pi}$ J

(ii) $\omega_s = 20$ rad/sec

Solution : 2

$$x(t) = \frac{1}{\pi t}[\cos t - 1]$$

Solution : 3

(i) Fourier transform of $x_e(t) = \text{Re}\{X(\omega)\}$

(ii) Fourier transform of $x_0(t) = j\,\text{img}\{X(\omega)\}$

Solution : 4

$$T_s = \frac{1}{f_s} = \frac{1}{5000}\text{sec}$$

$$T_s = 2 \times 10^{-4} \text{ sec}$$

The sampling period given in question is greater than the maximum allowable sampling period. Thus, it cannot be recovered using a low pass filter.

Solution : 5

$$\left.\begin{aligned} C_2 &= \frac{1}{8j - 8\pi}, & C_{-2} &= \frac{1}{-8j - 8\pi} \\ C_3 &= \frac{1+j}{2\sqrt{2}(4 + j6\pi)} & C_{-3} &= \frac{1-j}{2\sqrt{2}(4 - j6\pi)} \end{aligned}\right\}$$

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5 Laplace Transform

LEVEL 1 Objective Solutions

1.	(a)
2.	(d)
3.	(1)
4.	(c)
5.	(c)
6.	(–2)
7.	(b)
8.	(d)
9.	(b)
10.	(a)
11.	(d)

LEVEL 2 Objective Solutions

12.	(c)
13.	(b)
14.	(a)
15.	(c)
16.	(d)
17.	(a)
18.	(1)
19.	(a)
20.	(b)
21.	(a)
22.	(c)
23.	(b)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

$$\frac{2}{s(s+4)}$$

Solution : 2

$$X(s)=\frac{2}{s}+\frac{1}{s^2}-e^{-s}-\frac{e^{-s}}{s}-\frac{2e^{-3s}}{s^2}+\frac{2e^{-4s}}{s^2}$$

Solution : 3

$m = 1, \gamma = 2$ and $k = 2$.

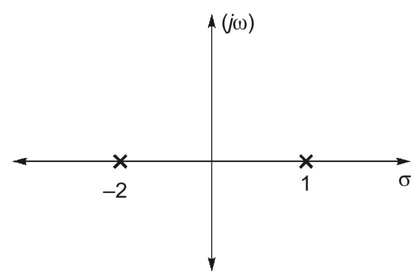
Solution : 4

(i)

$$H(s) = \frac{-1/3}{(s+2)}+\frac{1/3}{(s-1)}$$

(ii)

pole zero plot of the system is as shown in the figure thus



1. for system to be causal ROC of $H(s)$ is $Re(s) > 1$

$\therefore$ $$h(t) = \frac{-1}{3}(e^{-2t}-e^{t})u(t)$$

$\because$ $$\frac{1}{s+a} \overset{\ell}{\longleftrightarrow} e^{-at}\,u(t)$$

2. For system to be stable ROC must contain ($j\omega$) axis

$\therefore$ $$-2 < Re(s) < 1$$

$\therefore$ $$h(t) = -\frac{1}{3}e^{-2t}u(t)-\frac{1}{3}e^{t}u(-t)$$

3. For system to be neither causal nor stable

$$Re(s) < -2$$

$\therefore$ $$h(t) = \frac{1}{3}e^{-2t}u(-t)-\frac{1}{3}e^{t}u(-t)$$

Solution : 5

$$y(t) = L^{-1}[Y(s)] = L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = L^{-1}\left[\frac{1}{(s+1)}-\frac{1}{(s+2)}\right]$$

■■■■

6 Discrete Time Fourier Transform

LEVEL 1 Objective Solutions

1. (a)
2. (c)
3. (c)
4. (b)
5. (2)
6. (d)
7. (0)
8. (b)
9. (b)
10. (a)
11. (c)
12. (b)

LEVEL 2 Objective Solutions

13. (1759.292)
14. (a)
15. (0.25)
16. (0.25)
17. (b)
18. (b)
19. (1.333)
20. (–77.64)
21. (b)
22. (c)
23. (b)
24. (a)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

$$y[n] = \cos\left(\frac{3\pi}{2}n + \frac{11}{12}\pi\right)$$

Solution : 2

(i) $h[n] = \delta[n] + 2\,\delta[n-1] + \delta[n-2]$

(ii) Yes

Solution : 3

(a) 6

(b) 2

(c) 4π

Solution : 4

(i) $$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$\therefore$ $$X(0) = \sum_{n=-\infty}^{\infty} x[n] = 7$$

(ii) Using central ordinate theorem;

$$\int_{-\pi}^{\pi} X(\Omega)\,d\Omega = 2\pi\, x[0] = 0$$

(iii) $$X(\pi) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\pi} = \sum_{n=-\infty}^{\infty} x[n](-1)^n$$

$$= 2 - 1 - 3 - 3 + 2 - 1 + 1 = -3$$

(iv) Using parseval's theorem

$$\int_{\pi}^{\pi} |X(\Omega)|^2\, d\Omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= 2\pi\,[4 + 1 + 9 + 9 + 4 + 1 + 1]$$

$$= 58\,\pi$$

Solution : 5

$\therefore$ $$y[n] = \frac{\sin(n\pi/2)}{n\pi} * x_1[n] = \frac{\sin(n\pi/2)}{n\pi} * [\delta[n] - \delta[n-1]]$$

$\therefore$ $$h[n] = \frac{\sin(n\pi/2)}{n\pi} - \frac{\sin[(n-1)\pi/2]}{(n-1)\pi}$$

■■■■

7 z-Transform

LEVEL 1 Objective Solutions

1.	(a)
2.	(c)
3.	(a)
4.	(b)
5.	(a)
6.	(d)
7.	(2)
8.	(c)
9.	(d)
10.	(b)
11.	(d)
12.	(a)
13.	(a)

LEVEL 2 Objective Solutions

14.	(5)
15.	(d)
16.	(b)
17.	(c)
18.	(11)
19.	(b)
20.	(7)
21.	(d)
22.	(c)
23.	(a)
24.	(a)
25.	(b)
26.	(b)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

$$X(z) = \frac{2z^2}{\left(z-\frac{1}{2}e^{j\pi/3}\right)\left(z-\frac{1}{2}e^{-j\pi/3}\right)}; \quad \text{ROC} = |z| > \frac{1}{2}$$

Solution : 2

$$R_x[n] = \frac{4}{3}\cdot\left(\frac{1}{2}\right)^{|n|}$$

Solution : 3

$$h[n] = \frac{1}{3}[\delta[n-1]+\delta[n]+\delta[n+1]]$$

Solution : 4

(i) $$h(n) = 81\left(\frac{1}{3}\right)^{n+2}u(n+2) - 81\left(\frac{1}{3}\right)^{n+1}u(n+1)$$

(ii) Because system is LTI $\sum_{n=-\infty}^{\infty}|h(n)| < \infty$

Hence system is stable

For $n < -2 \Rightarrow h(n) = 0$

Hence system is right sided and non-causal.

Solution : 5

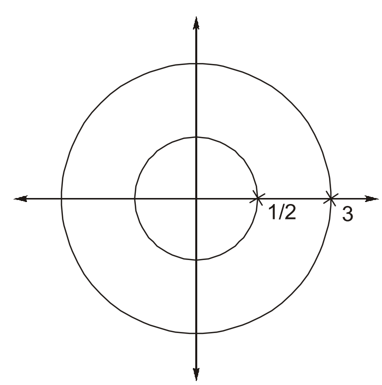
$$H(z) = -\frac{2}{(z^{-1}-2)} - \frac{2}{3\left(z^{-1}-\frac{1}{3}\right)} = -2\left[\frac{1}{(z^{-1}-2)} + \frac{1}{3\left(z^{-1}-\frac{1}{3}\right)}\right]$$

So poles are at $z = 1/2$ and $z = 3$.

(i) When the system is stable

ROC must include unit circle

∴ ROC will be $\frac{1}{2} < |z| < 3$



∴ $$H(z) = -2\left[\frac{1}{(z^{-1}-2)} + \frac{1/3}{\left(z^{-1}-\frac{1}{3}\right)}\right] \qquad \frac{1}{2} < |z| < 3$$

$$H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{2}{(1-3z^{-1})} \qquad \frac{1}{2} < |z| < 3$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 2.3^n u[-n-1]$$

(ii) When the system is causal

ROC must be outside the outermost pole and include ∞ in ROC

So ROC will be $|z| > 3$

$$\therefore \quad H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{2}{\left(1-3z^{-1}\right)} \qquad |z| > 3$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + 2.3^n u[n]$$

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8 Discrete Fourier Transform

LEVEL 1 Objective Solutions

1.	(a)
2.	(4)
3.	(c)
4.	(a)
5.	(b)
6.	(b)
7.	(c)
8.	(b)

LEVEL 2 Objective Solutions

9.	(a)
10.	(700)
11.	(0.2)
12.	(c)
13.	(c)
14.	(b)
15.	(c)
16.	(b)
17.	(a)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

$$X_2[k] = (-1)^k X_2[k]$$

Solution : 2

(i) $y[n] = \{-3, -6, 3, 6\}$

(ii) $X[k] = 1 - e^{-j\pi k};\ 0 \le k \le 3$ and $H[k] = 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k}$

Solution : 3

$y[n] = x[n-2]_5$

So, $y[n] = \{2, 0, 2, 1, 1\}$

Solution : 4

$$X[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & +j \\ 1 & -1 & 1 & -1 \\ 1 & +j & -1 & -j \end{bmatrix} \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = \begin{bmatrix} p+q+r+s \\ p-jq-r+js \\ p-q+r-s \\ p+jq-r-js \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 \\ \beta^2 \\ \gamma^2 \\ \delta^2 \end{bmatrix}$$

Solution : 5

By definition

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}k\cdot n} = \frac{1}{4}\sum_{k=0}^{3} X[k]e^{j\frac{2\pi}{4}kn}$$

$$= \frac{1}{4}\left[1+0+e^{j\frac{2\pi}{4}\times 2n}+0\right] = \frac{1}{4}[1+(-1)^n]$$

Solution : 6

By defintion

$$X[k] = \sum_{n=0}^{9} x[n]e^{-j\frac{2\pi}{10}k\cdot n}$$

(i)

$$X[0] = \sum_{n=0}^{9} x[n] = 2+1+1+0+3+2+0+3+4+6 = 22$$

(ii) $X[5]$

Here $N = 10$,

So

$$X[5] = X\left[\frac{N}{2}\right] = X\left[\frac{10}{2}\right]$$

$$X[5] = \sum_{n=0}^{9} x[n]e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot n} = \sum_{n=0}^{9} x[n](-1)^n$$

$$X[5] = 2 - 1 + 1 - 0 + 3 - 2 + 0 - 3 + 4 - 6 = -2$$

(iii) $\sum_{k=0}^{9} X[k]$

$\because$ $$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{-j\frac{2\pi}{N}\cdot kn}$$

Here $N = 10$

So $$x[0] = \frac{1}{10}\sum_{k=0}^{9} X[k]$$

$$\sum_{k=0}^{9} X[k] = 10\,x[0] = 10 \times 2 = 20$$

(iv) $|X[k]|^2$

According to Parseval's theorem

$$\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2$$

$\Rightarrow$ $$\sum_{k=0}^{N-1} |X[k]|^2 = N\sum_{n=0}^{N-1} |x[n]|^2$$

$$= 10\,[2^2 + 1^2 + 1^2 + 0^2 + 3^2 + 2^2 + 0^2 + 3^2 + 4^2 + 6^2] = 800 \text{ Watt}$$

Solution : 7

$$x(n) = \sum_{k=<N>} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

$\Rightarrow$ $$x(n) = \left(\frac{-1}{2}\right) + \frac{1}{2}e^{j\frac{2\pi}{8}n} + \frac{1}{2}e^{j\frac{14\pi}{8}n}$$

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9 Digital Filters

LEVEL 1 Objective Solutions

1.	(4)
2.	(0.25)
3.	(a)
4.	(a)
5.	(b)
6.	(a)
7.	(c)
8.	(b)

LEVEL 2 Objective Solutions

9.	(c)
10.	(b)
11.	(d)
12.	(a)
13.	(b)
14.	(a)

■■■■

LEVEL 3 Conventional Solutions

Solution : 1

(a) $b = -1$, $b = 0.1$

(b) $\omega_c = 2.92$ rad/sec

Solution : 2

$$H(z) = \frac{2(1+z^{-1})(1+z^{-1})}{63+62z^{-1}+15z^{-2}} = \frac{1+2z^{-1}+z^{-2}}{31.5+31z^{-1}+7.5z^{-2}}$$

Solution : 3

$$H(z) = \frac{0.675z^{-1}}{1-1.347z^{-1}+0.45z^{-2}} = \frac{0.675z}{z^2-1.347z+0.45}$$

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