2020

RANK Improvement WORKBOOK

Answer key and Hint of Objective & Conventional *Questions*

Electronics Engineering

Control Systems

MADE EASY Publications

1 Basics of Control Systems

LEVEL 1 Objective Solutions

1.	**(1)**
2.	**(a)**
3.	**(d)**
4.	**(c)**
5.	**(b)**
6.	**(d)**
7.	**(b)**
8.	**(b)**
9.	**(a)**
10.	**(a)**

LEVEL 2 Objective Solutions

11.	**(c)**
12.	**(a)**
13.	**(c)**
14.	**(d)**
15.	**(c)**
16.	**(a)**
17.	**(a)**
18.	**(100)**
19.	**(9×10^{-3})**
20.	**(b)**

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LEVEL 3 Conventional Solutions

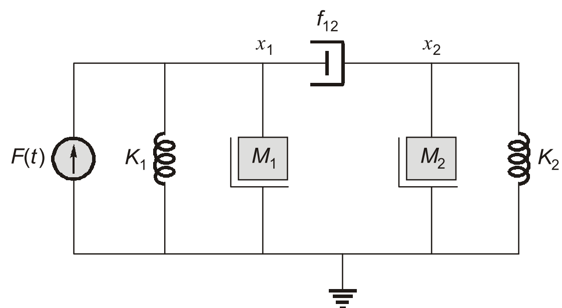
Solution: 1

(i) Differential equations of the mechanical system are:

$$M_1 \ddot{x}_1 + f_{12}(\dot{x}_1 - \dot{x}_2) + K_1 x_1 = F(t)$$

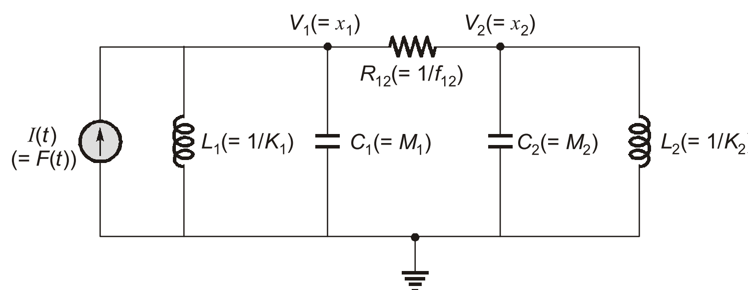
and $$M_2 \ddot{x}_2 - f_{12}(\dot{x}_1 - \dot{x}_2) + K_2 x_2 = 0$$

(ii) Mechanical equivalent representation is shown below:



(iii) Standard force equation:

$M \to C, \quad f \to 1/R, \quad K \to 1/L, \quad F \to I$



Solution: 2

$$\ddot{X} + \frac{f}{M}\dot{X} + \frac{K}{M}X = F \cdot \frac{1}{M}$$

Solution : 3

$$\left| S_H^T \right| = 1.0202$$

Solution : 4

$$K = 12$$

and $$G(s) = \frac{12(s+3)}{s(s+2)(s+4)}$$

Solution : 5

$$\frac{E_0(s)}{E_i(s)} = \frac{1}{(RCs+1)}$$

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2 Block Diagram and Signal Flow Graph

LEVEL 1 Objective Solutions

1.	(d)
2.	(c)
3.	(a)
4.	(c)
5.	(c)
6.	(a)
7.	(a)
8.	(a)
9.	(b)
10.	(d)
11.	(d)
12.	(c)
13.	(b)
14.	(0.117)

LEVEL 2 Objective Solutions

15.	(d)
16.	(b)
17.	(–0.5)
18.	(b)
19.	(a)
20.	(c)
21.	(d)
22.	(a)
23.	(a)
24.	(c)
25.	(b)
26.	(a)

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LEVEL 3 Conventional Solutions

Solution: 1

$$G(s) = \frac{10(10s+1)}{s(11s+1)}$$

and $$H(s) = 1$$

Solution : 2

$$\frac{y_2}{y_1} = \frac{P_1\Delta_1}{\Delta} = \frac{(1+H_4)(1+G_3H_2)}{G_1G_3H_1H_2+(1+H_4)(1+G_1H_1+G_3H_2+G_1G_2G_3H_3)}$$

$$\frac{y_6}{y_1} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{G_1G_3H_1H_2+(1+H_4)(1+G_1H_1+G_3H_2+G_1G_2G_3H_3)}$$

$$\frac{y_7}{y_2} = \frac{G_1G_2G_3G_4+G_1G_5(1+G_3H_2)}{(1+H_4)(1+G_3H_2)}$$

Solution : 3

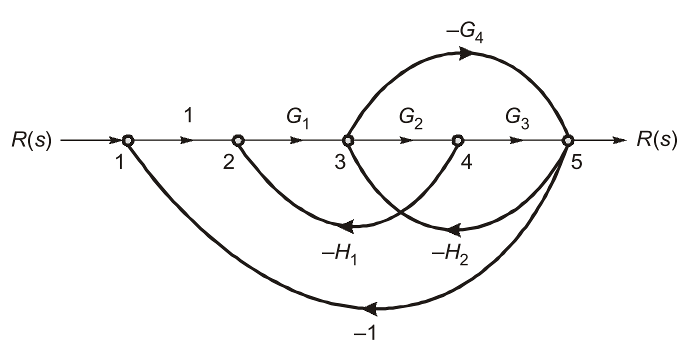
$$\frac{C(s)}{R_1(s)} = \frac{G_1G_2G_3}{1+G_3H_2+G_2H_3+G_1G_2G_3H_1}$$

$$\frac{C(s)}{R_2(s)} = \frac{G_3}{1-\left[\dfrac{-G_1G_2H_1-H_2}{1+G_2H_3}\right]G_3} = \frac{G_3(1+G_2H_3)}{1+G_1G_2G_3H_1+G_3H_2+G_2H_3}$$

Solution : 4

$$\frac{x_5}{x_1} = \frac{G_{12}G_{23}G_{34}G_{45}+G_{12}G_{23}G_{35}(1-G_{44})}{1-(G_{23}G_{35}+G_{23}G_{34}G_{42}+G_{23}G_{34}G_{45}G_{52}+G_{23}+G_{35}+G_{44}) + (G_{23}G_{32}G_{44}+G_{23}G_{35}-G_{52}G_{44})}$$

Solution : 5



$$\frac{C}{R} = \frac{G_1G_2G_3-G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_1G_2G_3-G_1G_4-G_4H_2}$$

Solution : 6

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4+G_4G_5(1+G_2H_2)}{1+G_2H_2+(G_4G_5H_1)(1+G_2H_2)+G_1G_2G_3G_4H_1}$$

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3 Time Response Analysis

LEVEL 1 Objective Solutions

1.	(0.47)
2.	(a)
3.	(38.16)
4.	(1)
5.	(d)
6.	(a)
7.	(b)
8.	(a)
9.	(b)
10.	(b)
11.	(c)
12.	(b)
13.	(c)
14.	(b)
15.	(b)
16.	(c)
17.	(b)
18.	(d)

LEVEL 2 Objective Solutions

19.	(b)
20.	(b)
21.	(13.53)
22.	(12)
23.	(2.8)
24.	(d)
25.	(c)
26.	(c)
27.	(a)
28.	(d)
29.	(d)
30.	(a)
31.	(c)
32.	(d)
33.	(c)
34.	(b)
35.	(c)
36.	(a)

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LEVEL 3 Conventional Solutions

Solution: 1

$$K_D = 1 \text{ and } K_P = 1.23$$

Solution: 2

(i) $$\text{CE} = s^2 + \left(2 + \frac{K}{4}\right)s + K$$

(ii) $$\omega_n = 3.16 \text{ rad/sec and } \xi = 0.71$$

(iii) For critical damping, $$K = 1.37 \text{ or } K = 46.62$$

(iv) $$c(t) = 1 - \frac{e^{-2.24t}}{0.7}\sin[2.22t + 45°]$$

$$t_p = 1.415 \text{ sec and } M_P = 4.21\%$$

Solution: 3

$$K_b = 0.105$$
$$t_p = 1.66 \text{ sec.}$$
$$\%\, M_p = 1.51\%$$
$$\omega_d = 1.897 \text{ rad/sec}$$
$$t_s = 1.58 \text{ sec.}$$

Solution : 4

$$G(s) = \frac{a}{s(s+K)}$$

$$K_p = \infty, K_v = \frac{a}{K} \text{ and } K_a = 0$$

Solution : 5

Total steady state error = 4.54

Solution : 6

(i) $$s^2 + 4s + 18 = 0$$

(ii) $$\omega_n = 4.2426 \text{ rad/sec}$$
$$\xi = 0.4714$$

(iii) for 1st undershoot, $$t = 1.6792 \text{ sec}$$

(iv) $$T = 1.6792 \text{ sec/cycle}$$

(v) Total number of cycles = 1.191

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4 Stability

LEVEL 1 Objective Solutions

1. (b)
2. (2)
3. (b)
4. (b)
5. (c)
6. (d)
7. (c)
8. (b)
9. (d)
10. (b)
11. (c)
12. (d)
13. (b)
14. (b)
15. (c)

LEVEL 2 Objective Solutions

16. (1.41)
17. (1)
18. (2.1)
19. (d)
20. (c)
21. (a)
22. (d)
23. (d)
24. (b)
25. (a)
26. (b)
27. (d)
28. (25)
29. (2)

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LEVEL 3 Conventional Solutions

Solution: 1

$$0 < K < 60 \text{ and } a < \frac{1}{36}\left(54 - K + \frac{360}{K}\right)$$

Solution: 2

$$K = 3.25$$
$$P = 1.36$$

Solution : 3

For stability $K > 0.3$

For 1 pole in RHS of s-plane there should be 1 sign change in first column.

$$0.05 > K > -1$$

For 2 pole there should be 2 sign change

$$0.3 > K > 0.05$$

Solution : 4

The system is unstable and the number of roots with positive real part of the characteristic equation is 2.

Solution : 5

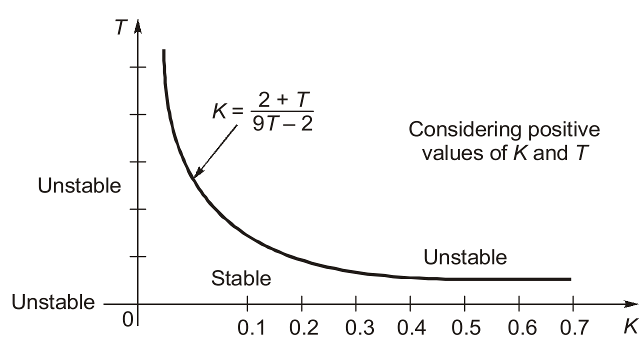
(a) For stability

$$0 < K < 1.49$$

(b) For marginally stable $K = 1.49$

Frequency of sustained oscillation = 0.316 rad/sec

Solution : 6



Solution : 7

Number of $j\omega P = 4$ (repeated)

Number of LHP = 2

As there are repeated roots on imaginary axis, system is unstable.

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5 Root Locus Technique

LEVEL 1 Objective Solutions

1.	(c)
2.	(c)
3.	(b)
4.	(b)
5.	(d)
6.	(b)
7.	(c)
8.	(d)
9.	(a)
10.	(c)
11.	(c)
12.	(c)
13.	(b)
14.	(b)
15.	(d)
16.	(a)
17.	(c)

LEVEL 2 Objective Solutions

18.	(c)
19.	(a)
20.	(d)
21.	(d)
22.	(c)
23.	(c)
24.	(a)
25.	(d)
26.	(c)
27.	(d)
28.	(b)
29.	(b)
30.	(c)
31.	(b)

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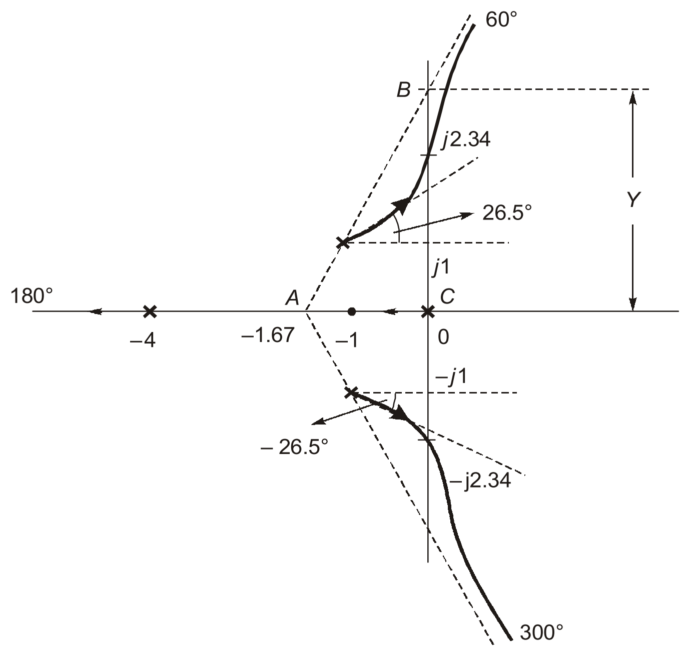
LEVEL 3 Conventional Solutions

Solution: 1



Solution: 2

Angle of Asymptotes: 60°, 180° and 300°

Centroid = (–1, 0)

Break-away or break-in points:

K = 0.234 (valid) at s = –0.422

K = negative (not valid) at s = –1.577

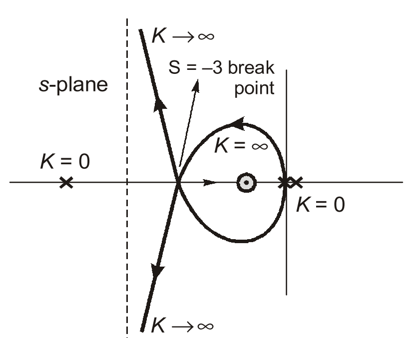
Solution : 3

Break points = –0.46, –2.86

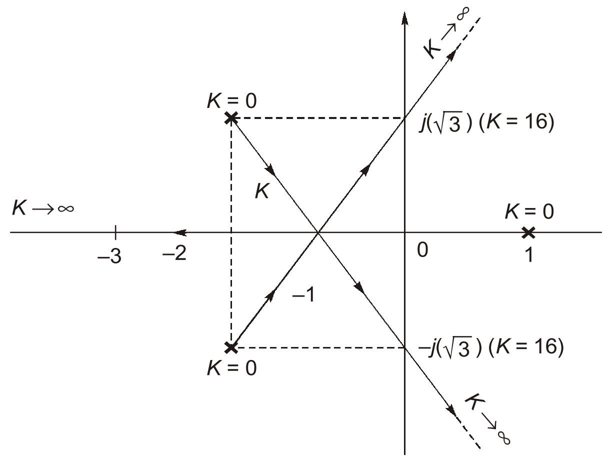
s = –0.46, K = +0.8793 (valid)

s = –2.86, K = –6.064 (non-valid)

Solution : 4



Solution : 5



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6 Frequency Response Analysis

LEVEL 1 Objective Solutions

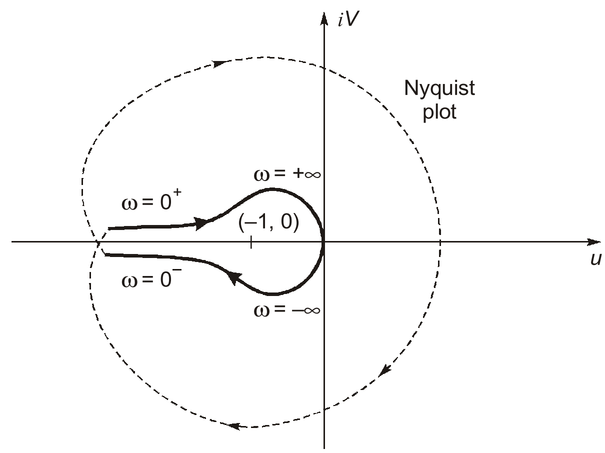
1.	(a)
2.	(10)
3.	(2.50)
4.	(d)
5.	(a)
6.	(d)
7.	(b)
8.	(d)
9.	(c)
10.	(a)
11.	(c)
12.	(a)
13.	(a)
14.	(a)
15.	(b)
16.	(c)
17.	(b)
18.	(c)
19.	(b)
20.	(c)
21.	(c)
22.	(a)

LEVEL 2 Objective Solutions

23.	(c)
24.	(b)
25.	(b)
26.	(0.5)
27.	(d)
28.	(d)
29.	(a)
30.	(c)
31.	(a)
32.	(c)
33.	(c)
34.	(c)
35.	(a)
36.	(c)
37.	(d)
38.	(b)
39.	(b)
40.	(c)
41.	(a)
42.	(b)
43.	(b)

LEVEL 3 Conventional Solutions

Solution: 1



This system is unstable.

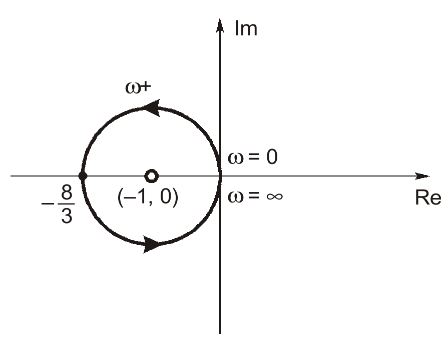
Solution: 2

$$G(s) = \frac{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}{(10s)\left(1+\frac{s}{10}\right)\left(1+\frac{s}{50}\right)}$$

Solution: 3

$$K = 589.66 \text{ and } a = 30.21$$

Solution : 4



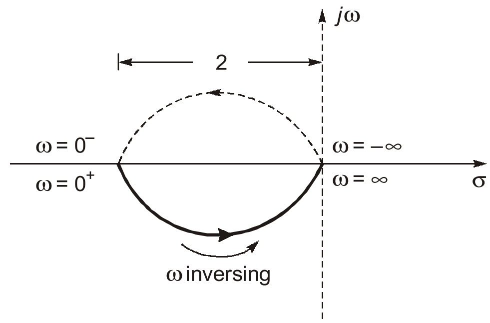
System is stable.

Solution : 5

(i) Phase margin = 55.6°

(ii) $T = 1.08$ sec

Solution : 6



No roots of the characteristic equations are laying on the right hand side of the *s*-plane, therefore, closed-loop system is stable.

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7 Compensators and Controllers

LEVEL 1 Objective Solutions

1.	(d)
2.	(b)
3.	(d)
4.	(c)
5.	(a)
6.	(d)
7.	(a)
8.	(a)
9.	(c)
10.	(d)
11.	(d)
12.	(b)
13.	(c)
14.	(d)
15.	(d)
16.	(d)
17.	(c)
18.	(c)
19.	(d)

LEVEL 2 Objective Solutions

20.	(c)
21.	(d)
22.	(c)
23.	(a)
24.	(a)
25.	(c)
26.	(b)
27.	(d)

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LEVEL 3 Conventional Solutions

Solution: 1

$$G_c(s) = 14.6 + \frac{40}{s} + 2.4s$$

Solution: 2

$$T(s) = 1.31026\frac{(1+0.3018s)}{(1+0.0518s)}$$

Solution : 3

$$G_c(s) = 0.1306(1+1.309s)$$

Solution : 4

$$G_c(s) = 1.119 + \frac{1.1029}{s}$$

Solution : 5

$$G_c(s) = 4.349 + 0.519s$$

Solution : 6

$$G_c(s) = 0.7246$$

Solution : 7

$$K = 11.7$$

and $$K_t = 0.065$$

■■■■

8 State Space Analysis

LEVEL 1 Objective Solutions

1. (–1)
2. (a)
3. (b)
4. (c)
5. (2.5)
6. (a)
7. (b)
8. (a)
9. (a)
10. (b)
11. (a)
12. (b)
13. (a)
14. (c)
15. (d)
16. (d)
17. (c)
18. (b)
19. (d)

LEVEL 2 Objective Solutions

20. (b)
21. (a)
22. (a)
23. (c)
24. (a)
25. (b)
26. (d)
27. (a)
28. (b)
29. (a)
30. (c)
31. (a)
32. (c)
33. (d)
34. (d)
35. (a)
36. (d)
37. (b)
38. (a)
39. (b)
40. (b)

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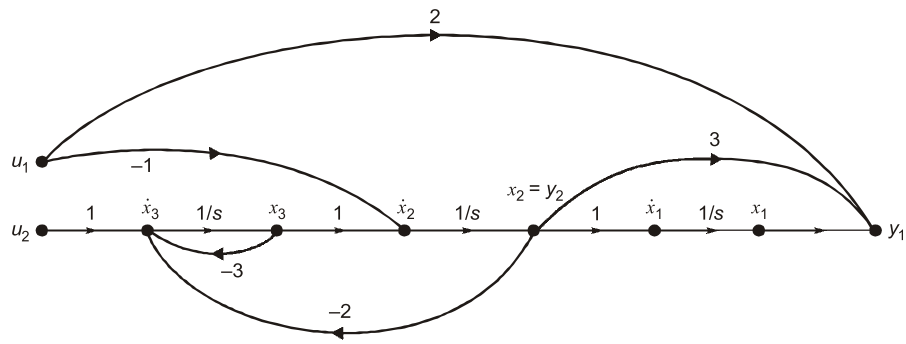
LEVEL 3 Conventional Solutions

Solution: 1



Characteristic roots of system are 0, –1, –2.

Solution: 2

$$\phi(t) = \begin{bmatrix} 3e^{-t} - 2e^{-2t} & 2e^{-t} - 2e^{-2t} \\ -3e^{-t} + 3e^{-2t} & -2e^{-t} + 3e^{-2t} \end{bmatrix}$$

Solution: 3

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and $$O_M = \begin{bmatrix} 4 & 3 & 1 \\ -2 & 1 & 1 \\ -2 & -5 & -1 \end{bmatrix}$$

O_M matrix is non-singular matrix hence the system is observable.
The smallest time constant of system is 1 sec i.e. τ = 1 sec.

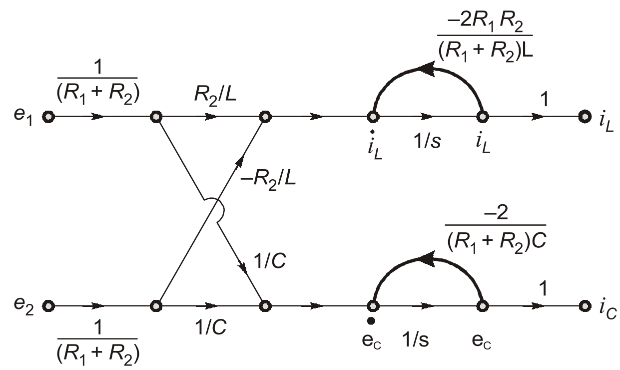
Solution: 4

(i)

$$A = \begin{bmatrix} \dfrac{-2R_1R_2}{(R_1 + R_2)L} & 0 \\ 0 & \dfrac{-2}{(R_1 + R_2)C} \end{bmatrix}$$

and $$B = \begin{bmatrix} \dfrac{R_2}{(R_1 + R_2)L} & \dfrac{-R_2}{(R_1 + R_2)L} \\ \dfrac{1}{(R_1 + R_2)C} & \dfrac{1}{(R_1 + R_2)C} \end{bmatrix}$$

(ii)



Solution : 5

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -10 & -5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Solution : 6

$$\begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} I$$

$$v_0 = [O \quad R] \begin{bmatrix} v_c \\ i_L \end{bmatrix}$$

Solution : 7

(i) CE, $s^3 - 3s^2 - s - 3 = 0$

(ii) Controllability matrix $= \begin{bmatrix} 1 & 2 & 10 \\ 0 & 3 & 9 \\ 1 & 2 & 7 \end{bmatrix}$

(iii) Given system is controllable.

Solution : 8

(i) Controllable form:

State equation,
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -35 & -29 & -18 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

Output equation,

$$Y = [7 \;\; 5 \;\; 3 \;\; 0]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

(ii) Observable form:

State equation,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -35 \\ 1 & 0 & 0 & -29 \\ 0 & 1 & 0 & -18 \\ 0 & 0 & 1 & -5 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \\ 3 \\ 0 \end{bmatrix}[U]$$

Output equation,

$$Y = [0 \;\; 0 \;\; 0 \;\; 1]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Solution : 9

$$x_1(t) = \cos\sqrt{2}t + \frac{1}{\sqrt{2}}\sin\sqrt{2}t$$

$$x_2(t) = -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t$$

$$y = \frac{3}{\sqrt{2}}\sin\sqrt{2}\,t$$

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