2020

RANK Improvement WORKBOOK

Answer key and Hint of Objective & Conventional *Questions*

Electronics Engineering

Electronics Devices & Circuits

MADE EASY Publications

1 Semiconductor Physics

LEVEL 1 Objective Solutions

1.	(b)
2.	(6.25)
3.	(a)
4.	(c)
5.	(a)
6.	(c)
7.	(a)
8.	(a)
9.	(d)
10.	(b)
11.	(b)
12.	(c)
13.	(c)
14.	(a)
15.	(c)
16.	(d)

LEVEL 2 Objective Questions

17.	(b)
18.	(c)
19.	(a)
20.	(c)
21.	(a)
22.	(0.075)
23.	(b)
24.	(5)
25.	(b)
26.	(b)
27.	(c)
28.	(9.36)
29.	(c)
30.	(b)

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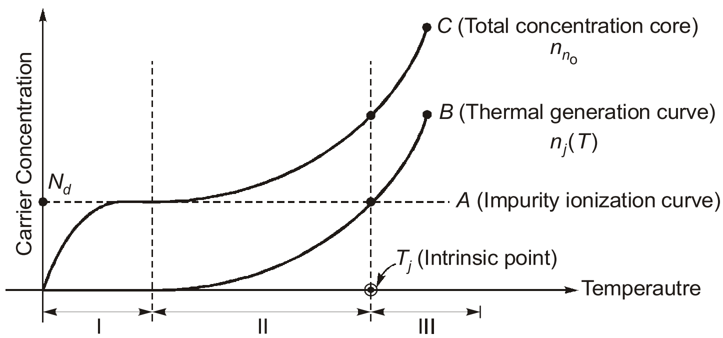
LEVEL 3 Conventional Solutions

Solution: 1



Total carrier concentration (shown by curve-*C* in the above diagram) has 2 types of carrier.

(a) Carrier due to ionized dopant atom (impurity atom).

(b) $n_j(T)$, thermally generated carrier (intrinsic carriers).

The difference between energy of dopant level and that of conduction band in *N*-type, valance band in *p*-type semiconductor is typically very low as compared to band gap energy $E_g(T)$, *T* stands for temperature variation of E_g.

$\Rightarrow$ $(E_d - E_c)$, $(E_a - E_v) << E_g(T)$ [in extrinsic range of temperature]

Where, E_d = Donor level, E_a, acceptor level E_g is also the amount of energy to generate an electron hole pair intrinsic carrier. Due to small energy requirement of impurity ionization, $(E_c - E_d)$ or $(E_a - E_v)$, the temperature required to ionize dopant atoms is very small and typically between 100-150 Kelvin.

The above diagram shows variation of carrier concentration due to increase in temperature rate for an n-type semiconductor.

(i) Impurity ionization carrier (Curve-A).

(ii) Thermally generated carrier (Curve-B) and curve-C is just the sum of two carrier concentration. Show in the above diagram are 3 region, I, II, III. Depending on temperature range.

Region-I (temperature range of partial impurity ionization): It typically stretches upto 150 K. During such small temperature $n_j(T) \approx 0$ and $n_{n_o} \approx N_d^+$ is carrier concentration due to ionized dopant atoms. If N_d be concentration of dopant atom then

$$N_d^+ = N_d\left[1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)\right]^{-1} \quad \text{...(i)}$$

$\left[\frac{1}{2} \text{ term accounts for degenerating of spin.}\right]$

E_F = Fermi level of the n-type specimen.

Region-II (Extrinsic range of temperature): In this range temperature is sufficiently high to ionize impurity (dopant) atoms. So

$$N_d^+ \approx N_d \text{(all impurity atom ionized)}$$

$n_j(T)$ starts to rise but still several orders of magnitude smaller than N_d.

The Region-II ends at that temperature when $n_j(T) = N_d$ at $T = T_j$ (intrinsic temperature)

This particular temperature is called intrinsic point. At this temperature extrinsic behaviour starts to vanish intrinsic carrier concentration takes over the impurity carrier concentration.

$$n_{n_o} = n_i(T) + N_d$$

Region-III (Intrinsic range of temperatures): It start at intrinsic temperature point (T_j) $n_i(T)$ at $T_j = N_d$(by definition given above).

$$\Rightarrow \quad n_i(T_j) = N_d$$

In this region $n_{n_o} \approx n_i(T)$ for temperature slightly above T_j and hence called intrinsic range.

Part-2: T_j by definition is the temperature at which $n_i(T_j) = N_d$ ...(ii)

$$\Rightarrow \because \quad n_i(T) = \sqrt{N_c N_v}\exp\left[-\left(\frac{E_g(T)}{2kT}\right)\right] \quad ...(iii)$$

Where N_c, N_v, $E_g(T)$, K have usual meanings. Now we put $n_i(T_j)$ in equation (ii) and we get,

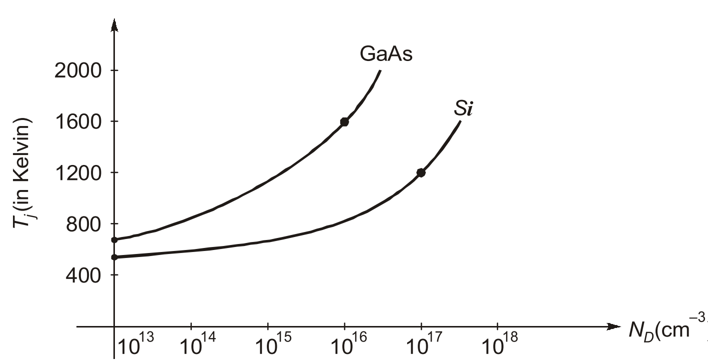
$$\sqrt{N_c N_v}\exp\left[-\frac{E_g(T_j)}{2kT_j}\right] = N_d$$

$$\Rightarrow \quad \exp\left[-\left(\frac{E_g(T_j)}{2kT_j}\right)\right] = \frac{N_d}{\sqrt{N_c N_v}}$$

$$\Rightarrow \quad \frac{-E_g(T_j)}{2kT_j} = ln\frac{N_d}{\sqrt{N_c N_v}}$$

$$\Rightarrow \quad T_j = \frac{E_g(T_j)}{2k\,ln\left[\frac{\sqrt{N_c N_v}}{N_d}\right]}$$

From here, we observe that for a given semiconductor T_j depends on N_d(doping concentration) and $E_g(T_j)$ or band-gap energy at intrinsic temperature point.



Intrinsic temperature Vs Doping

Solution: 2

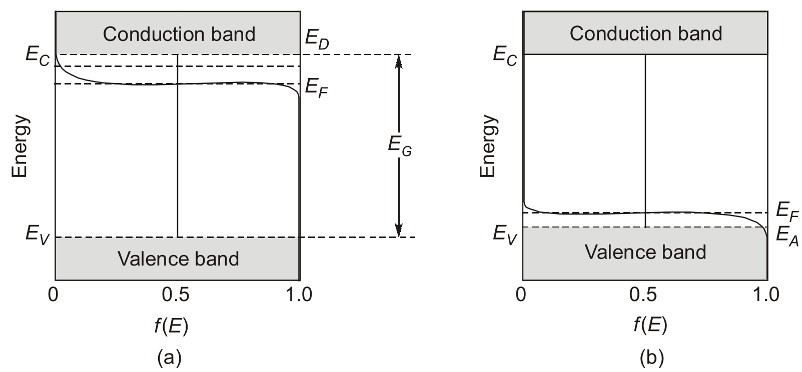
$$p = 2.25 \times 10^5/\text{cm}^3$$

The Fermi level at 300° K for n-type material.

$$\therefore \quad (E_C - E_F) = 0.307 \text{ eV}$$

Thus E_F will be 0.307 *eV* below the bottom of the conduction band, for a donor density 10^{15} atoms/cm^3.

- Concept of Fermi Level (E_F) in extrinsic semiconductor:
  - ⇒ Fermi-level is a measure of probability of occupancy of the allowed energy states.
  - ⇒ In extrinsic semiconductor Fermi level moves closer to the conduction band indicating that many of the energy states in that band are now filled by donor electrons and the number of holes in the valence band are few. This concept is in respect of *n*-type material. While for *p*-type material the Fermi level is nearer to the valence band.
  - ⇒ So, the *CB* and *VB* positions will shift relatively to each other in *n*-type and *p*-type material as shown in figure below:



Positions of Fermi level in (a) *n*-type and (b) *p*-type semiconductors

⇒ As temperature increases the density of electron increases and hence conductivity increases.
⇒ A calculation of exact position of the Fermi level in an n-type semiconductor is,

$$E_F = E_C - kT\,ln\left(\frac{N_C}{N_D}\right)$$

$$\Rightarrow \quad E_C - E_F = kT\,ln\left(\frac{N_C}{N_D}\right) \quad ...(i)$$

For p-type material,

$$\therefore \quad E_F = E_V + kT\,ln\left(\frac{N_V}{N_A}\right)$$

$$\Rightarrow \quad E_F - E_V = kT\,ln\left(\frac{N_V}{N_A}\right) \quad ...(ii)$$

Now, we have to find the Fermi level at 300° K for n-type material.
From equation (i) we have,

$$E_C - E_F = 0.03 ln\left(\frac{2.8\times10^{19}}{10^{15}}\right) \quad \text{(At room temp. } KT \approx 0.03 \text{ eV)}$$

$$\therefore \quad (E_C - E_F) = 0.307 \text{ eV}$$

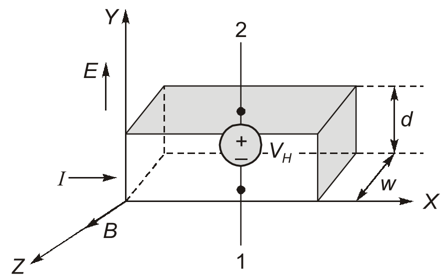
Thus E_F will be 0.307 *eV* below the bottom of the conduction band, for a donor density 10^{15} atoms/cm^3.

Solution: 3

The given phenomenon is Hall Effect.

According to this effect the statements are:

- If a specimen (metal or semiconductor) carrying a current I is placed in a transverse magnetic field B, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon, known as the Hall effect, is used to determine whether a semiconductor is n-type or p-type and to find the carrier concentration.



- Consider a semiconductor specimen bar having volume charge density ρ_v (in C/m^3), width 'w', thickness 'd', cross-sectional area 'A' and developed Hall voltage is "V_H".

Since, $$E = \frac{F}{e}$$

$\therefore$ $$F = eE \quad \text{...(i)}$$

also, $$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{...(ii)}$$

In the equilibrium state,

Force on specimen due to electric field (E) = Force on specimen due to magnetic field (H)

$\Rightarrow$ $$eE = e v_d B \quad \text{[where } v_d = \text{drift velocity]}$$

$\Rightarrow$ $$E = v_d B \quad \text{...(iii)}$$

Also, $$E = \frac{V_H}{d} \quad \text{...(iv)}$$

$\therefore$ $$V_H = B v_d d \quad \text{...(v)}$$

Drift velocity of charge carrier $= v_d = \dfrac{E}{B} = \dfrac{J}{\rho_v}$

Now from equation (v),

$$V_H = \frac{BdJ}{\rho_v} = \frac{BId}{A\rho_v} = \frac{BId}{w \times d \times \rho_v}$$

$\Rightarrow$ $$V_H = \frac{BI}{\rho_v w} \quad \text{...(vi)}$$

This equation (vi) represents the Hall voltage (V_H) developed in a semiconductor bar.

Solution: 4

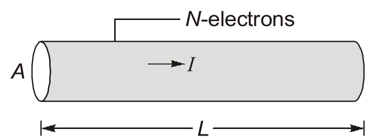
$$n_i = 2.29 \times 10^{21}/\text{m}^3$$
$$v_d = 1.9 \text{ km/sec}$$

Solution: 5

$$n_i = 2.26 \times 10^{12} /\text{m}^3$$
$$\sigma = 3.22 \times 10^{-7} \text{ S}$$

Solution: 6

- **Current density**
- If N electrons are contained in a length L of a conductor as shown in figure below. If T is time taken to traverse distance L, the total number of electrons passing through any cross section of wire in T per unit time is N/T.



(A = cross-sectional area)

Therefore, $$I = \frac{Nq}{T} = \frac{Nqv}{L} \quad \text{...(i)}$$

∴ Current density $$(J) = \frac{I}{A} = \frac{Nqv}{LA} \quad \text{...(ii)}$$

(unit of J = amp/m^2)

Since, $\frac{N}{LA} = n$ (electron concentration in electrons per cubic meter)

∴ $$J = nqv = nev = \rho v \quad \text{...(iii)}$$

where $\rho \equiv ne$ is the **charge density** in Coulombs per cubic meter and v in meters per second.

- The conductivity of a material can be related to the number of charge carriers present in the materials. Now, combining equations (i) and (iii) we get,

$$J = nqv = nq\mu E$$

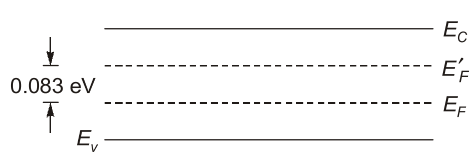
⇒ $$J = \sigma E \quad \text{...(iv)}$$

where, $$\sigma = nq\mu \quad \text{...(v)}$$

where σ is conductivity in (ohm-meter)$^{-1}$

Solution: 7

$$(E_F' - E_F) = 0.083 \text{ eV}$$



Solution: 8

If an electric field is present in addition to the carrier gradient, the current densities will each have a drift component and a diffusion component.

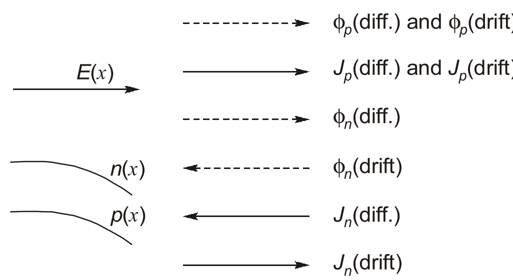
$$J_n(x) = \underbrace{q\mu_n n(x) E(x)}_{\text{drift}} + \underbrace{qD_n \frac{dn(x)}{dx}}_{\text{diffusion}} \quad \text{...(i)}$$

$$J_p(x) = q\mu_p p(x) E(x) - qD_p \frac{dp(x)}{dx} \quad \text{...(ii)}$$

and the total current density is the sum of the contributions due to electrons and holes :

$$J(x) = J_n(x) + J_p(x)$$

The relation between the particle flow and the current of equation (i) by considering a diagram such as shown in figure. In this figure an electric field is assumed to be in the x-direction, along with carrier distributions $n(x)$ and $p(x)$ which decrease with increasing x. Thus the derivatives in equation (i) & (ii) are negative, and diffusion takes place in the x-direction. The resulting electron and hole diffusion currents [J_n(diff.) and J_p(diff.)] are in opposite directions. Holes drift in the direction of the electric field [J_p (drift)], whereas electrons drift in the opposite direction because of their negative charge. The resulting drift current is in the x-direction in each case. Note that the drift and diffusion components of the current are additive for holes when the field is in the direction of decreasing hole concentration, whereas the two components are subtractive for electrons under similar conditions. The total current may be due primarily to the flow of electrons or holes, depending on the relative concentrations and the relative magnitudes and directions of electric field and carrier gradients.



An important result is that minority carriers can contribute significantly to the current through diffusion. Since the drift terms are proportional to carrier concentration, minority carriers seldom provide much drift current. On the other hand, diffusion current is proportional to the gradient of concentration. For example, in n-type material the minority hole concentration 'p' may be many orders of magnitude smaller than the electron concentration 'n', but the gradient (dp/dx) may be significant. As a result, minority carrier currents through diffusion can sometimes be as large as majority carrier currents.

Solution : 9

$$N_{CO} N_{VO} = 1.15 \times 10^{29}$$
$$E_g = 1.25 \text{ eV}$$

Solution : 10

$$N_D = 9.26 \times 10^{14} /\text{cm}^3$$
$$\rho(200°\text{K}) = 2.7 \ \Omega\text{-cm}$$
$$\rho(400°\text{K}) = 9.64 \ \Omega\text{-cm}$$

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2 PN Junction Diode and Special Devices

LEVEL 1 Objective Solutions

1. (a)
2. (b)
3. (12.5)
4. (a)
5. (b)
6. (c)
7. (9.33)
8. (0.45)
9. (0.337)
10. (d)
11. (b)
12. (4)
13. (c)
14. (1.33)
15. (a)
16 (b)
17. (a)
18. (b)
19. (c)
20. (21)
21. (0.526)
22. (d)
23. (b)
24. (d)
25. (c)
26. (1.52)
27. (0.952)
28. (1.822)
29. (0.3)
30. (19.23)
31. (b)
32. (d)

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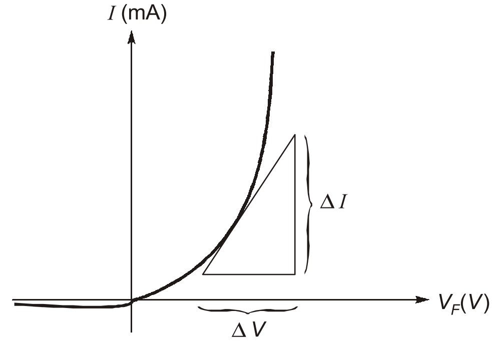
LEVEL 3 Conventional Solutions

Solution: 1

A.C or dynamic resistance (r_f) of a *p-n* junction diode is defined as the reciprocal of the slope of the volt-ampere characteristics.

$$r_f = \frac{\text{Change in voltage}}{\text{Resulting change in current}} = \frac{\Delta V}{\Delta I}$$

A specific change in the voltage and current which may be used to determine the A.C. (or) dynamic resistance for the region of diode characteristics.



Consider the current equation of a *p - n* junction diode

i.e. $$I = I_0\left(e^{\frac{V}{\eta V_T}} - 1\right) \quad \text{...(i)}$$

where, I_0 = reverse current of PN-junction diode

V = applied voltage of PN-junction diode

V_T = thermal voltage

By taking the derivative of the equation (i) w.r.t. the applied voltage, we get,

$$\frac{dI}{dV} = \frac{d}{dV}\left[I_0\left(e^{\frac{V}{\eta V_T}} - 1\right)\right] = I_0\left[\frac{1}{\eta V_T}\cdot e^{V/\eta V_T}\right] = \frac{I_0}{\eta V_T}e^{V/\eta V_T}$$

$$\frac{dI}{dV} = \frac{I + I_0}{\eta V_T}$$

Generally $I >> I_0$ in the vertical slope section of the characteristics.

So, $$\frac{dI}{dV} \approx \frac{I}{\eta V_T}$$

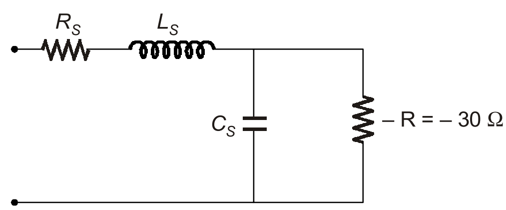
$$\Delta I = \frac{dI}{dV}\Delta V$$

Therefore, $$r_f = \frac{\Delta V}{\Delta I} = \frac{1}{\left(\frac{dI}{dV}\right)} = \frac{\eta V_T}{I}$$

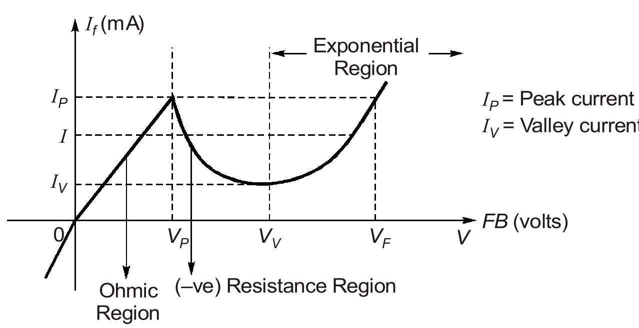
∴ The dynamic resistance varies inversely with the forward current of diode.

Solution : 2

Equivalent circuit:



Tunnel diodes are basically P^+N^+ diode with doping concentration of $1:10^3$. Due to high doping depletion layer is very narrow. Hence charge carrier will be penetrating the depletion layer almost at the speed of light. This behaviour of charge carriers is called as tunneling effect. Tunnel diodes are negative resistance device and are fastest switches. GaAS is popularly used for manufacturing of Tunnel Diodes.
VI characteristics of Tunnel Diode : →



Solution: 3

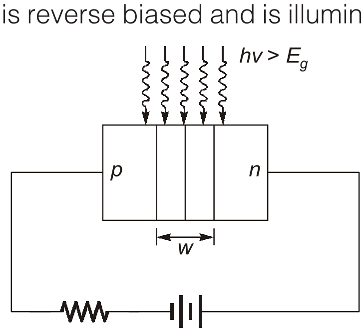
$$V_{D1} = 0.036$$
$$V_{D2} = 4.964 \text{ V}$$

Solution: 4

$$V_0 = 0.85 \text{ V}$$

Solution: 5

Let we have a *p-n* junction which is reverse biased and is illuminated by photon energy $hv > E_g$.



Due to this illumination an added generation rate gap (EHP/cm^3) is participated in current.

- The number of holes created within a diffusion length of transition region on the n-side will be $AL_p \cdot g_{op}$.

Where $L_p \rightarrow$ diffusion length of holes.

$A \rightarrow$ Area of cross-section.

- Similarly $AL_n g_{op}$ electrons are generated per second within L_h of x_{po} and AWg_{op} carriers are generated within W.
- The resulting current due to collection of these optically generated carriers by the junction is

$$I_{op} = qAg_{op}(L_p + L_n + W) \quad \text{...(i)}$$

If $I_0 \rightarrow$ is the reverse saturation current of diode so net current flowing through diode

$$I = I_0[e^{qV/kT} - 1] - I_{op} \quad \text{...(ii)}$$

$\because$ I_{op} is from n to p

When the diode is short circuited

i.e. $V = 0$

i.e short circuit current

$\therefore$ $I_{SC} = -I_{op}$ – sign because this current flow from n to p.

$$I_{SC} = -qAg_{op}(L_p + L_n + W) \quad \text{...(iii)}$$

From equation (ii) $I = I_0[e^{qV/kT} - 1] - I_{op}$

or
$$I = qA\left[\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}\cdot n_p\right](e^{qV/kT} - 1) - qAg_{op}(L_n + L_p + W)$$

When there is an open circuit across the device

i.e. $I = 0$

$V = V_{OC}$

$$\left(e^{qV_{OC}/kT} - 1\right) = \frac{qAg_{op}(L_n + L_p + W)}{qA\left[\frac{L_p}{\tau_p}p_n + \frac{L_n}{\tau_n}n_p\right]}$$

$$e^{qV_{OC}/kT} = \left[1 + \frac{L_p + L_n + W}{(L_p/\tau_p)\,p_n + (L_n/\tau_n)\,n_p}g_{op}\right]$$

$\Rightarrow$
$$V_{OC} = \frac{kT}{q}ln\left[1 + \frac{L_p + L_n + W}{(L_p/\tau_p)\,p_n + (L_n/\tau_n)n_p}g_{op}\right] \quad \text{...(iv)}$$

- When an illuminated junction is operated in 3rd quadrant i.e. voltage is reverse direction as well as current is also.
- Current becomes independent of applied voltage but is proportional to the optical generation rate.
- Such a device provides a useful means of measuring illumination level or of converting time varying optical signal into-electrical signal, so can be used as photo detector.
- From equation (iv)

$$V_{OC} = \frac{kT}{q}ln\left[\frac{L_p + L_n + W}{(L_p/\tau_p)p_n + (L_n/\tau_n)\,n_p}g_{op} + 1\right] \quad \text{...(v)}$$

For a special case of symmetrical junction

$p_n = n_p$, and $\tau_p = \tau_n$

We have $\frac{p_n}{\tau_n} = g_{th}$

$g_{th} \rightarrow$ thermal generation rate
By neglecting the generation within W.
We can rewrite equation (v) as

$$V_{OC} = \frac{kT}{q} ln\left[\frac{g_{op}}{g_{th}} + 1\right]$$

$$V_{OC} \simeq \frac{kT}{q} ln \frac{g_{op}}{g_{th}} \quad \text{for } g_{op} >> g_{th}$$

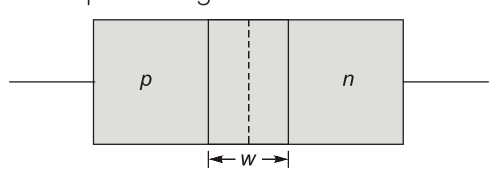
$g_{th} = p_n/\tau_n$ represents the equilibrium thermal generation – recombination rates.

- As minority carriers concentration is increased by optical generation EHP's, the life time becomes shorter and $p_n/\tau_n = g_{th}$ becomes larger making $\frac{g_{op}}{g_{th}}$ constant.
  So V_{OC} can not increased indefinitely with increased generation rate infact limit on V_{OC} is equilibrium contact potential V_0.
- This effect of illuminated junction can be used in photo cell and this is also known photo-voltaic effect.

Solution: 6

Derivation for the contact potential:
Let us consider a p-n junction in open circuited condition

- We know that net current flowing through a open circuited p-n junction is zero.
  Let E_x is the electric field in depletion region.



At equilibrium the drift and diffusion components of hole current will cancel each other i.e.

$$q\left[\mu_p\, p(x)\, E_x - D_p \frac{dp(x)}{dx}\right] = 0$$

$$\Rightarrow \quad \frac{\mu_p}{D_p} E_x = \frac{1}{p(x)} \frac{dp(x)}{dx} \quad \text{...(i)}$$

Electric field can be written in the terms of gradient in potential

$$E_x = -\frac{dV(x)}{dx} \quad \text{...(ii)}$$

$\therefore$ From (i) and (ii)

$$-\frac{\mu_p}{D_p} \frac{dV(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx} \quad \text{...(iii)}$$

From Eienstein relation

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

$$\Rightarrow \quad \frac{\mu_p}{D_p} = \frac{q}{kT} \quad \text{...(iv)}$$

From (iii) and (iv) we get

$$-\frac{q}{kT}\frac{dV(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx} \qquad ...(v)$$

- Here we are interested in the potential on either side of junction V_n and V_p and hole concentration just at the edge of transition region on either side p_p and p_n. Since we have taken one dimensional geometry, p and V can be taken as function of x-only.

Integrating equation (v) we get

$$-\frac{q}{kT}\int_{V_p}^{V_n} dV = \int_{p_p}^{p_n}\frac{1}{p}\,dp$$

$$-\frac{q}{kT}(V_n - V_p) = (\ln p_n - \ln p_p) \qquad ...(vi)$$

The potential difference $V_n - V_p$ is contact potential V_0

$$\therefore \qquad V_0 = \frac{kT}{q}\ln\frac{p_p}{p_n} \qquad ...(vii)$$

$p_p \cong N_A$ concentration of acceptor atoms (doping)

$$p_n = \frac{n_i^2}{N_D}$$

Where $N_D \rightarrow$ donor concentration on n-side

This is according to mass action law

$$\therefore \qquad V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2} \qquad ...(viii)$$

Width of depletion region:

We know that maximum electric field at the junction is

$$E_{max} = -\frac{q}{\epsilon}N_D \cdot x_{n_0} = -\frac{q}{\epsilon}x_{p_0}N_A \qquad ...(ix)$$

$x_{n_0} \rightarrow$ width of depletion region on n-side

$x_{p_0} \rightarrow$ width of depletion region on p-side

also $\qquad x_{n_0} + x_{p_0} = W$

also $\qquad x_{n_0}N_D = x_{p_0}N_A \qquad ...(x)$

$\because$ Electric field is maximum at junction and decreases linearly on either side of junction

So we can take approximately (linear approximation)

$$E_{max} = -\frac{2V_0}{W}$$

V_0 – contact potential

From equation (ix)

$$-\frac{q}{\epsilon}N_D\,x_{n_0} = -\frac{2V_0}{W}$$

$$N_D\,x_{n_0}W = \frac{2V_0\,\epsilon}{q}$$

$$N_D\left[\frac{W}{\left(1+\frac{N_D}{N_A}\right)}\right]W = \frac{2V_0 \in}{q}$$

From equation (x)

$$x_{n0} = \frac{W}{\left(1+\frac{N_D}{N_A}\right)}$$

$$\frac{N_A \cdot N_D}{(N_A+N_D)}W^2 = \frac{2\in}{q}V_0$$

$$W = \sqrt{\frac{2\in}{q}\frac{N_A+N_D}{N_A N_D}\cdot V_0} = \sqrt{\frac{2\in}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_0}$$

Contact potential, $V_0 = 697.32\text{ mV}$

Total depletion width, $W = 0.958836\ \mu\text{m}$

Solution: 7

The Avalanche breakdown can occur at large reverse voltage whereas Zener breakdown occurs at low voltage, the major reason for this is the difference of doping level of Avalanche and Zener. The doping level of Zener is high as compare to the doping level of Avalanche. Since, the depletion width is inversely proportional to the doping level, so the depletion width of Zener is smaller as compare to the depletion width of the Avalanche.

$$W \propto \frac{1}{\sqrt{\text{Doping}}}$$

Also the breakdown voltage is inversely proportional to doping.

$$V_{BR} \propto \frac{1}{\sqrt{\text{Doping}}}$$

Also, the electric field intensity E is

$$E = \frac{V}{W}$$

So, $$E \propto \frac{1}{W}$$

So, due to high doping in Zener, the depletion width is smaller and due to this smaller depletion width a large electric field E exist, at a critical field strength, electrons participating in covalent bonds may be torn from the bonds by the field and accelerated to the n-side of the junction. The electric field required for this type of ionization is on the order of 10^6 V/cm.

For lightly doped junctions (Avalanche) electron tunneling is negligible, and instead, the breakdown mechanism involves the impact ionization of host atoms by energetic carriers.

Required temperature coefficient of the Zener diode,

$$\frac{\Delta V_z}{V_z}\times 100\% \Big/ ^\circ\text{C} = 0.0113/^\circ\text{C}$$

Solution: 8

The term varactor is a shortened form of variable reactor, referring to the voltage variable capacitance of a reverse-biased *p-n* junction. The junction capacitance depends on the applied voltage and the design of the junction. In some cases a junction with fixed reverse bias may be used as a capacitance of a set value. More commonly the varactor diode is designed to exploit the voltage-variable properties of the junction capacitance. For example, a varactor (or a set of varactors) may be used in the tuning stage of a radio receiver to replace the bulky variable plate capacitor. The size of the resulting circuit can be greatly reduced, and its dependability is improved. Other applications of varactors include use in harmonic generation, microwave frequency multiplication and active filters.

If the *p-n* junction is abrupt, the capacitance varies as the square root of the reverse bias V_r in a graded junction, however, the capacitance can usually be written in the form.

$$C_j \propto V_r^{-n}$$

Where $n = \frac{1}{2}$ for step graded or abrupt p-n junction diode

$n = \frac{1}{3}$ for linear graded diode

$n = \frac{1}{2.5}$ for diffused p-n junction diode

$n =$ grading coefficient

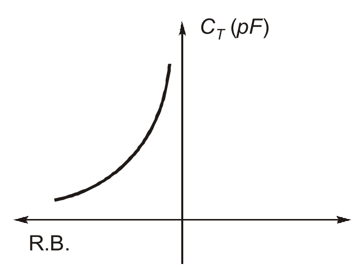
For varactor diode $n = \frac{1}{3}$

So, for varactor diode

$$C_T \propto \frac{1}{\sqrt[3]{R.B.\ \text{voltage}}}$$

$$C_T \propto V^{-1/3}$$

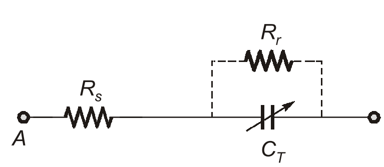
- Varactor diode operates on the principle of transition capacitance (C_T).
- Varactor diode is always operated under reverse bias.
- GaAs material is popularly used for designing of varactor diode.



Characteristic curve of varactor diode

Symbol of varactor diode:

Equivalent circuit:



Where R_s = Ohmic resistance or contact resistance

R_r = Reverse resistance of varactor diode

The tuning frequency of a varactor diode.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Application of varactor diodes:

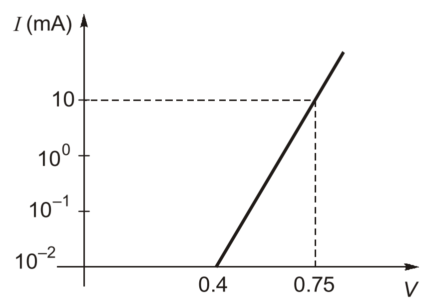
1. In designing of LC resonant tuning circuits.
2. For direct generation of frequency modulation (F.M) by using varactor diode modulator circuit.
3. For self balancing of AC bridges.
4. As a parametric amplifiers (Para-Amp). Para-amp is a microwave power amplifier and it is used in satellite communication.

Solution : 9

$$\eta = 1.94$$

$$\log(I) = \log(I_o) + \frac{17.46}{n} V$$

This is an equation for straight line



Solution : 10

The hole concentration inside n-region is obtained by solving the continuity equation as

$$P_n'(x) = P_n'(0)e^{-x/L_p}$$

The hole diffusion current $I_p(x) = -AqD_p \dfrac{dp(x)}{dx}$

$$\Rightarrow \quad I_{p_n}(x) = -AqD_p\left(\frac{-P_n(0)}{L_p}\right)e^{-x/L_p}$$

$$I_{p_n}(x) = \frac{AqD_p}{L_p} P_{n_0}(e^{-V/V_T} - 1)e^{-x/L_p}$$

At $x = 0$,

$$I_{p_n} = \frac{AqD_pP_{n_0}}{L_p}(e^{V/V_T} - 1)$$

Similarly,

$$I_{n_p}(0) = \frac{AqD_n n_{p_0}}{L_n}(e^{V/V_T} - 1) \quad \text{...(i)}$$

$$I_{p_n}(0) = \frac{AqD_nP_{n_0}}{L_p}(e^{V/V_T} - 1) \quad \text{...(ii)}$$

Dividing equation (i) by equation (ii) gives

$$\frac{I_{p_n}(0)}{I_{n_p}(0)} = \frac{D_p P_{n_o}}{L_p} \times \frac{L_n}{D_p n_{p_o}} = \frac{V_T \mu_p \frac{n_i^2}{N_D}}{V_T \mu_n \frac{n_i^2}{N_A}} \times \frac{L_n}{L_p}$$

$$\Rightarrow \quad \frac{I_{p_n}(0)}{I_{n_p}(0)} = \frac{q \mu_p N_A L_n}{q \mu_n N_D L_p} = \frac{\sigma_p L_n}{\sigma_n L_p}$$

■■■■

3 Bipolar Junction Transistor

LEVEL 1 Objective Solutions

1.	(6.656)
2.	(b)
3.	(b)
4.	(c)
5.	(b)
6.	(b)
7.	(c)
8.	(c)
9.	(d)

LEVEL 2 Objective Solutions

10.	(a)
11.	(a)
12.	(a)
13.	(d)
14.	(b)
15.	(380.28)
16.	(2.718)

■■■■

LEVEL 3 Conventional Solutions

Solution: 1

(i) With increase in base width, the recombination in base region will increase and therefore base current increases and collector current decreases so $\beta = I_C/I_B$ decreases.

(ii) As carrier (minority) life time in base region increase the number of recombinations in base region decreases and more and more carries reach to collector, therefore I_B decreases and I_C increases and as a result $\beta = I_C/I_B$ increases.

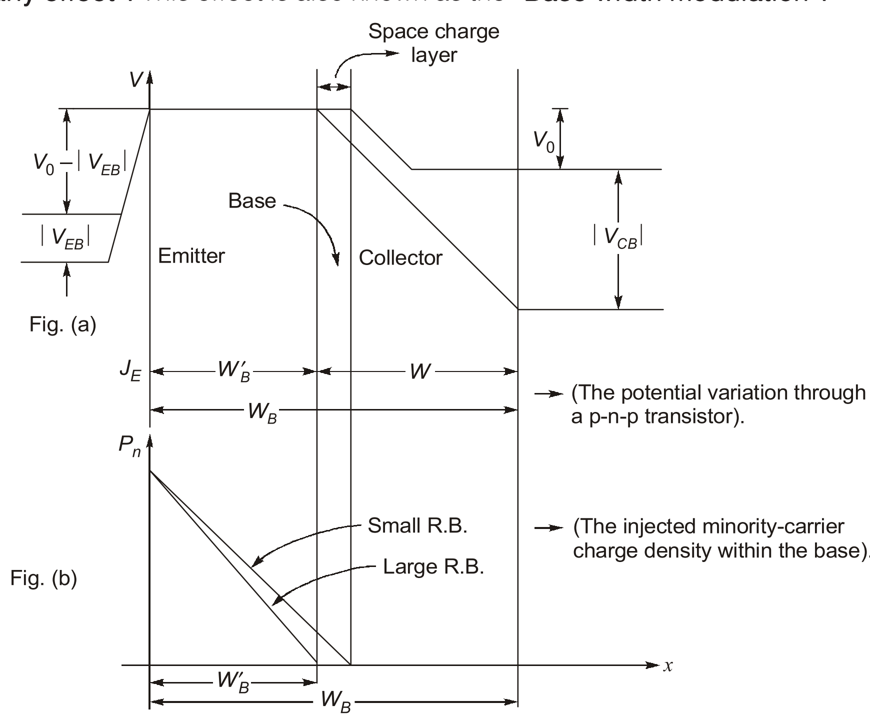
(iii) With increase in temperature, minority carrier life time increases in base region and this tend to increase β while on the otherhand due to increase in temperature, transit time τ_t increases which tend to decrease β, but the effect of increasing lifetime with temperature dominate so β increases with temperature.

(iv) With increase in collector current, β will remain same, as collector current will increase when base current increases, as base current is controlling current on which collector current depends.

(v) As collector voltage increases, β effective base width decreases, so recombination in base region decreases due to this I_B decreases and I_C increases and therefore $\beta = I_C/I_B$ increases.

Solution: 2

- As we know that the width (W) of the depletion region of a diode or a p-n junction increases with the magnitude of the reverse biasing voltage. In the active region, emitter junction is F.B. but the collector junction is R.B. then in the figure below we can see that the barrier width at J_E is negligible compared with the space-charge region width (W) at J_C. As the R.B. voltage increases, the transition region penetrates deeper into the collector and base. Since the doping in the base is ordinarily larger than that of the collector, so the penetration of the depletion region into the base is much smaller than into the collector. If the metallurgical base width is 'W_B' then the effective electrical base width is $W_{eB} = W_B - W$.
- This modulation (small change) of the effective base-width by altering the collector voltage is popularity known as **"Early effect"**. This effect is also known as the **"Base width modulation"**.



- The lowering the value of W'_B with increasing reverse bias collector voltage has the following main consequences:

⇒ There is less chances for recombination with in the base region. Hence 'α' increases with increasing the value of $|V_{CB}|$.

⇒ The concentration gradient of minority carriers p_n is increased within the base as shown in figure (b) above. Note that, $p_n = 0$ at the edge of the base-collector depletion layer.

⇒ For extremely large collector-junction R.B. voltage the effective base width of the transistor i.e. W'_B may be reduced to zero, which results a voltage breakdown in the transistor and this phenomena is called the Punch-through or Reach-through.

⇒ Due to early-effect the transit time is reduced and so the switching-time is reduced and hence transistor becomes faster.

Solution: 3

We know the relation,

[Power transmitted] × [Thermal resistance] = [Temperature difference]

Let,
P_c = Power radiated by collector
T_j = Junction temperature
G_a = Ambient temperature
θ_{JA} = Thermal resistance between the junction and the air

At steady state

$$\Rightarrow \quad P_c \times \theta_{JA} = [T_j - T_A]$$

To prevent thermal runaway, the right hand side of the equation must be greater than the left hand side during any variation of junction temperature.

$$P_c \theta_{JA} < T_j - T_A$$

differentiate with respect to T_j

$$\theta_{JA} \frac{\partial P_c}{\partial T_j} < 1$$

$$\frac{\partial P_c}{\partial T_j} < \frac{1}{\theta_{JA}}$$

■■■■

4 FET and MOSFET Devices

LEVEL 1 Objective Solutions

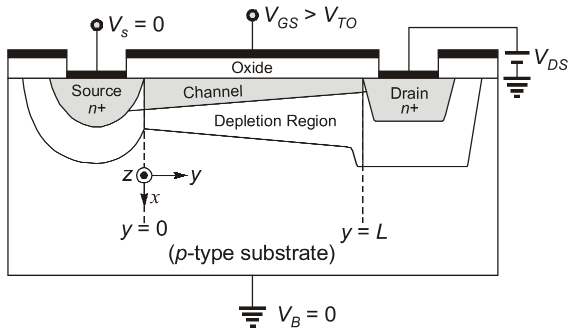
1. (a)
2. (a)
3. (a)
4. (5.07)
5. (b)
6. (b)
7. (b)
8. (b)
9. (a)
10. (0.67)
11. (b)
12. (b)

LEVEL 2 Objective Solutions

13. (28.35)
14. (1.2)
15. (2.25)
16. (1178.31)
17. (a)
18. (–1.32)
19. (b)
20. (b)
21. (a)
22. (d)
23. (a)
24. (b)

LEVEL 3 Conventional Solutions

Solution: 1



⇒ Cross-sectional view of an n-channel transistor, operating in linear region.

Gradual Channel Approximation: In the above diagram, I_D (current flowing in the channel from drain to source) under the applied drain voltage V_{DS}. We assume channel is formed already by applying $V_{GS} > V_{TO}$ (threshold voltage) we first define a co-ordinate system with x direction being perpendicular to the substrate and y is along the channel

⇒ Let $V_c(y)$ be channel voltage with respect to source (which is grounded). We assume V_{TO} independent of y and thus constant (this is assumption not strictly true) we also assume E_y is dominant an E_x, this will allow us to write

$$V_c(y = 0) = V_s = 0 \quad \text{...(i)}$$

$$V_c(y = L) = V_{DS}(V_D) \quad \text{...(ii)}$$

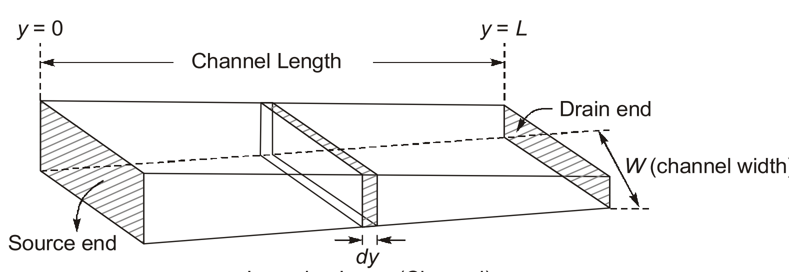
$$[V_{DS} = V_D - V_S = V_D - 0 = V_D]$$

The channel is completed inverted from $y = 0$ to $y = L$ as $V_{GS} \geq V_{TO}$ and so electron flow from source to drain under electric field $E_y(>> E_n)$.

Let $Q_I(y) = -C_{ox}(V_{GS} - V_c(y) - V_{TO})$

$Q_I(y)$ the amount of inversion charge stored in an elemental length dy of channel at a voltage $V_c(y)$ above source.

Next diagram show exclusively the channel or inversion charge layer of the previous diagram.



Inversion Layer (Channel)

Let dR (incremental resistance) in the channel for constant mobility (μ_n) of inversion layer

$$dR = -\frac{dy}{W\mu_n Q_I(y)} \quad \text{[negative sign is to adjst negative } Q_I(y)\text{]}$$

μ_n actually depends on the concentration of carrier and so the above assumption is for simplifying the solution. We further assume that current density is uniform across the length of the channel from $y = 0$ to $y = L$ i.e. I_D is independent of y.

Applying ohm's law for the element dy of the channel

$$dV_c = I_D \cdot d_R = -\frac{I_D}{W\mu_n Q_I(y)} \cdot dy \qquad \text{...(iii)}$$

(i) Can be integrated from $y = 0$ to $y = L$ using boundary condition of equation (i) and equation (ii)

$$-W\mu_n \int_0^{V_{bs}} Q_I(y)V_c = \int_0^L I_D \cdot dy$$

$\because$ I_D is independent of y so $\int_0^L I_D \cdot dy = I_D \cdot L$

$$\Rightarrow \quad I_D \cdot L = -W\mu_n \int_0^{V_{DS}} Q_I(y)dV_c$$

$$Q_I(y) = -C_{ox}(V_{GS} - V_c(y) - V_{To})$$

$$\Rightarrow \quad I_D \cdot L = -W\mu_n(-C_{ox})\int_0^{V_{DS}} (V_{GS} - V_c(y) - V_{To})dV_c$$

$$= W\mu_n C_{ox}\left[\int_0^{V_{DS}} (V_{GS} - V_{To})dV_c \int_0^{V_{DS}} -V_c(y)\,dy_c\right]$$

$$I_D = W\mu_n C_{ox}\left[(V_{GS} - V_{To}) - V_{DS} - \frac{V_{DS}^2}{2}\right]$$

at saturation $(V_{GS} - V_T) = V_{DS}$

$\Rightarrow$ I_{DS} = Saturation drain current

$$I_{DS} = \frac{W\mu_n C_{ox}}{L}\left[\frac{(V_{GS} - V_{To})^2}{2}\right]$$

$$\Rightarrow \quad g_{ms} = \text{Saturation transconductance} \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_{To})$$

Solution: 2

$\because$ We know the expression for threshold voltage is

$$V_T = Q_{ms} - \frac{Q_j}{C_{ox}} - \frac{Q_d}{C_{ox}} + 2Q_F \qquad \text{...(i)}$$

(a) $\Delta Q_{ms} = Q_m - Q_s$ = Metal semiconductor work function difference.

Q_m and Q_s are difference of Fermi-level and vacuum level in metal and semiconductor respectively. Since Fermi-level is a function of doping. So Q_{ms} depends on doping of the substrate.

(b) Q_j = Sum total of oxide trapped charges and the charge at oxide-semiconductor interface.

(c) Q_d = Total depletion charge

Q_d for p-substrate. n-channel device is negative = $-2\sqrt{e\,\epsilon_o E_s N_a Q_{Fp}}$

Q_{Fp} – Fermi-potential of p-type substrate = $kTln\left(\frac{N_a}{n_i}\right)$

N_a = Acceptor doping concentration

$\in_s$ = Relative permittivity of substrate

Q_d for n is a similar manner = $2\sqrt{e \in_o \in_s N_d Q_{Fn}}$

Q_{Fn} Fermi-potential on *n*-substrate = $-kTln\left(\frac{N_d}{n_i}\right)$

N_d = Doron concentration in substrate

$\because$ Q_{Fp}, Q_{Fn} are function of doping level *SU*.

$\Rightarrow$ Q_d is a strong function of doping.

(d) Q_F again the Fermi-potential, a function of doping concentration.

(e) C_{ox} = Oxide capacitance = $\frac{\in_o \in_{ox}}{t_{ox}}$

$\in_{ox}$ = Relative permittivity of oxide

t_{ox} = Oxide thickness

So examining all right hand side of equation (i) we observe that V_T can be controlled by suitably controlling the doping concentration of the substrate and also by varying the oxide thickness. In many application interface charge is deliberately introduced to alter the threshold voltage of a MOS-device (specially depletion mode device).

PART-2

A short channel device is one in which the length of the channel (*L*) is of the same order of the depletion width at source/drain junction

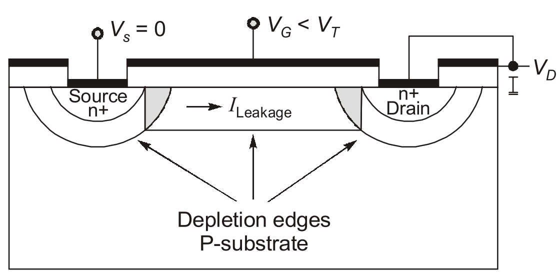


Figure-1
Sub-threshold leakage current in MOS due to DIBL

In long channel MOSFET, when $V_G < V_T$, source is unable to inject electron in the channel due to high source-channel depletion barrier. This sub-threshold source-channel barrier is high enough to stop any leakage current. But in short channel device. The influence of drain electric field is very strong at source junction. This influence leads to reduction in the barrier voltage at source junction and thus electron enter the channel and drain at drain end.

The effect is a subthreshold leakage current and the phenomena is called (DIBL) Drain Induced Barrier Lowering.

Then, one more phenomena which is most important in effecting the device performance in Short Channel Devices is Velocity Saturation.

The velocity of carriers in a semiconductor (*SC*) depends on electric field and doping level and it becomes constant beyond a certain electric field called critical electric field (E_c) as shown in the diagram below, whereas it increase as

$$v_n(\text{velocity}) = \frac{\mu_n E}{\left(1+\frac{E}{E_c}\right)}$$

[Variation of carrier velocity with applied electric field (*E*)]

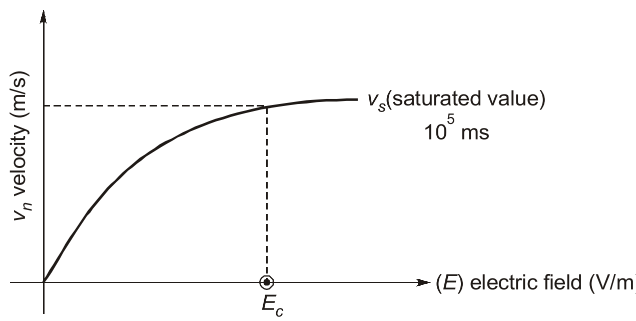


Figure-2: Velocity-Saturaton Effect

Let
μ_n = Mobility of electron
E = Applied electric field
V_{DS} = Drain to source voltage

E_c is typically between $\frac{1\,\text{V}}{\mu\text{m}}$ to $\frac{5\,\text{V}}{\mu\text{m}}$.

In long channel *n*-MOS say $L = 0.25$ μm

$\Rightarrow$ We need $V_{DS} = 2$ Volts to get $E = \frac{V_{DS}}{L} = \frac{2\,\text{V}}{0.25\,\mu\text{m}} = \frac{4\,\text{V}}{\mu\text{m}} > E_c$

In short channel L is much smaller than 0.25 μm and hence $E = \frac{V_{DS}}{L} >> E_c$ for very small voltage and hence velocity saturation is easily seen to occur.

The velocity saturation has pronounced effect on the operation of MOS transistor having short channel.

Drain Current: From the continuity requirement of the velocity saturation diagram in Figure-2.

$$v_s = \left.\frac{\mu_n E}{1+E/E_c}\right|_{E=E_c}$$

$$\Rightarrow \quad v_s = \frac{\mu_n E_c}{2} \Rightarrow E_c = \frac{2v_s}{\mu_n}$$

Drain current expression in presence of velocity saturation becomes

$$I_D = \frac{\mu_n}{\left(1+\frac{V_{DS}}{L\,E_c}\right)} C_{ox}\frac{W}{L}\left[(V_{GS}-V_T)V_{DS}-\frac{V_{DS}^2}{2}\right]$$

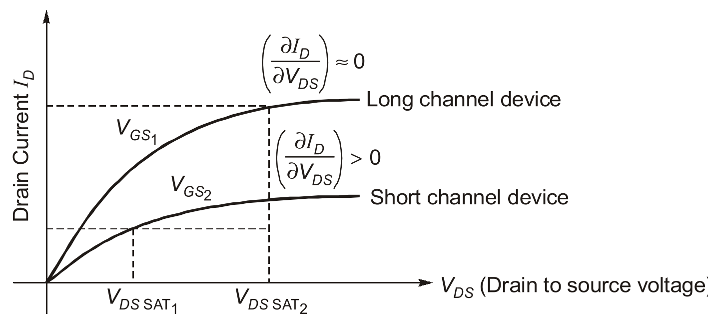
Where symbols have their usual meaning, $\frac{V_{DS}}{L}$ is average field in the channel

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{gs} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]\left[\frac{1}{1+\left(\frac{V_{DS}}{LE_C}\right)}\right]$$

Let $$\frac{1}{1+\frac{V}{LE_c}} = K(V) \quad \Rightarrow \quad \text{If } K(V) \text{ is } 1$$

Velocity saturation is minimum and as V_{DS} increases beyond ($E_C \cdot L$), velocity saturation effect take over.
For short channel devices $K(V)$ is less than 1 and hence current I_{DS} decreases due to velocity saturation.
Also the V_{DS} required for a device = ($E_c \cdot 1$), for saturation to occur.

So $V_{DS\,SAT_1}$ for long channel > $V_{DS\,SAT_2}$ for short channel effect of velocity saturation of device characteristic.



$V_{GS_1} = V_{GS_2}$

Here we observe that short channel saturation current is linearly dependent of V_{DS}

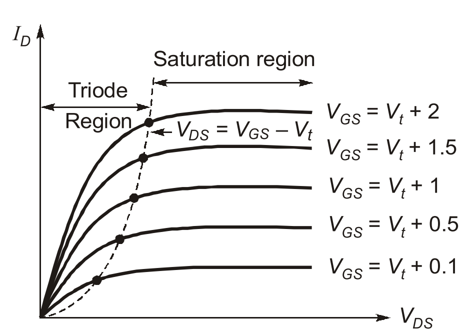
$$\Rightarrow \qquad \frac{\partial I_D}{\partial V_{DS}} > 0 \qquad \left(\text{unlike long channel where } \frac{\partial I_D}{\partial V_{GS}} \approx 0\right)$$

$\Rightarrow$ Output impedance of short channel MOS device reduce due to velocity saturation.

Solution: 3

The n-channel enhancement MOSFET operates in the triode region when V_{GS} is greater than V_t and the drain voltage is lower than the gate by at least V_t volts. In the triode region,

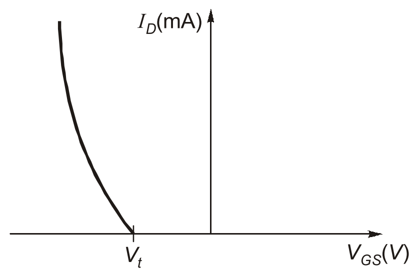
$$i_D = k_n' \frac{\omega}{L}\left[(V_{GS} - V_t)\, V_{DS} - \frac{1}{2}V_{DS}^2\right]$$



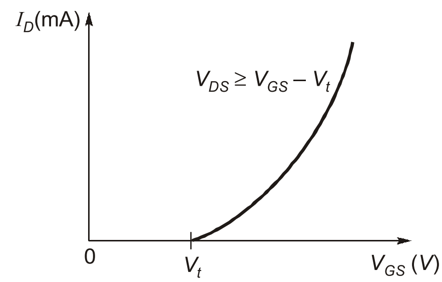
Output characteristics of n-channel MOSFET

The n-channel enhancement MOSFET operates in the saturation region when V_{GS} is greater than 'V_t' and the drain voltage does not, fall below the gate voltage by more than 'V_t' volts.

$$i_D = \frac{1}{2}k'_n \frac{\omega}{L}(V_{GS} - V_t)^2$$



Transfer characteristic of p-channel MOSFET



Transfer characteristics of n-channel MOSFET

Transfer characteristic shows that there will be no current until and unless

$V_{GS} \geq V_t$ for (n-MOS) and $V_{GS} \leq V_t$ for (p-MOS)

Solution: 4

(i) Pinch-off voltage V_P:

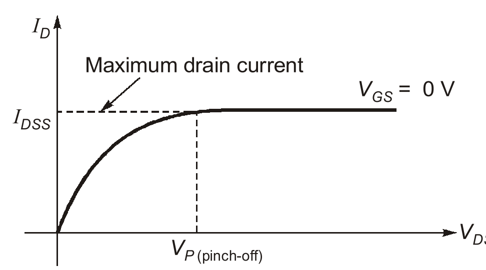
It is the minimum V_{DS} voltage where drain current I_D will be saturated,

$$V_P = \frac{a^2 q V_A}{2\epsilon_0\epsilon_r} \quad \text{(p-channel)}$$

$$V_P = \frac{a^2 q V_D}{2\epsilon_0\epsilon_r} \quad \text{(n-channel)}$$

where a is half-channel height.

Drain characteristics of JFET at $V_{GS} = 0$ V



(ii) Transconductance g_m:

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

Transfer characteristic equation of JFET

$$I_D = I_{DSS}\left[1 - \frac{V_{GS}}{V_P}\right]^2$$

Differentiating with respect to V_{GS},

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = -\frac{2I_{DSS}}{V_P}\left[1 - \frac{V_{GS}}{V_P}\right]$$

Maximum trans-conductance is,

$$g_{m0} = -\frac{2I_{DSS}}{V_P} \quad [V_{GS} = 0\text{ V}]$$

Amplification factor μ :

$$\mu = \frac{\Delta V_{DS}}{\Delta V_{GS}} = \frac{\Delta V_{DS}}{\Delta I_D} \cdot \frac{\Delta I_D}{\Delta V_{GS}}$$

$$\frac{\Delta V_{DS}}{\Delta I_D} = r_d \quad \text{(ac drain resistance)}$$

$$\frac{\Delta I_D}{\Delta V_{GS}} = g_m \quad \text{(Transconductance)}$$

$$\mu = g_m r_d$$

(iii)

$$g_m = 4 \text{ mS}$$
$$\mu = 840$$

Solution: 5

(i) $\left(\frac{W}{L}\right) = 24$

(ii) $I_D = 1.152 \text{ mA}$

(iii) $W = 24.48 \,\mu\text{m}$

$L = 1.02 \,\mu\text{m}$

■■■■