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RPSC Main Exam 2019 : Test Series

Assistant Engineer

Civil Engineering

Compulsory Subject : Paper-I

Test No. 7 | Date of Exam. : 04-08-2019 (9 AM to 12 Noon)

Part-A **[Marks : 40]**

Note: Attempt all the **twenty** questions. Each question carries **2** marks. Answer should not exceed **15** words.

1. Solution:

It is ratio of the moment at the far end to the applied moment at the rotating near end.

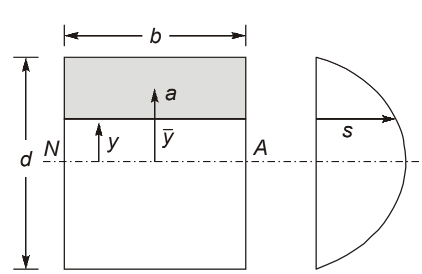
2. Solution:

Stress tensor defines the stresses acting at a point. Two directions are required to define it.

It case of 3D,

$$\text{Stress tensor} = \begin{Bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{zy} & \sigma_{zz} \end{Bmatrix}$$

3. Solution:



4. Solution:

Diameter of strain Mohr's circle,

$$D = 2 \times \text{radius}$$

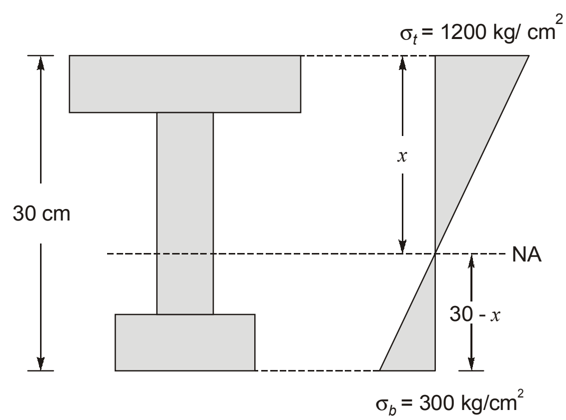
$$= 2\sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= 2 \times \sqrt{0 + \left(\frac{12}{2}\right)^2 \times 10^{-12}}$$

$$= 2 \times 6 \times 10^{-6}$$

$$= 12 \times 10^{-6}$$

5. Solution:



$$\frac{30 - x}{300} = \frac{x}{1200}$$

$$4(30 - x) = x$$

$$120 - 4x = x$$

$$x = 24 \text{ cm}$$

6. Solution:

A stiffener is a plate or an angle section that is connected to the web of a beam or a girder to prevent the out-of-plane deformation of the web plates.

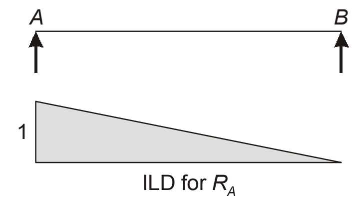
7. Solution:

Its radius of gyration is small about the minor principal axis and the allowable loads are further lowered due to eccentricity of the connection.

8. Solution:

Rivet value (R_v): Rivet value is the minimum of shear strength and bearing strength of rivet in a riveted joint.

9. Solution:



10. Solution:

The redundant forces are chosen as unknowns and additional equations are obtained by considering the geometrical conditions imposed on the deformations of the structure (i.e., compatibility conditions).

11. Solution:

$$D_s = m + r_e - 2j$$
$$= 6 + 3 - 8 = 1$$

12. Solution:

The limitations of plate load test are:

- It has limited depth of influence. It could only give the bearing capacity of soils with depth up to two times the diameter of plate.
- There is scale effect as the size of test plate is smaller than actual foundation.
- To gain access to test position, excavation is carried out which causes significant ground disturbance.

13. Solution:

The soil which has tendency to increase in volume in presence of water and decrease in volume in absence of water are called as expansive soils or swelling soils. Soil containing montmorillonite mineral swell considerably upon imbbing water from outside and shrink upon removal of water. Black cotton soil is one of the best example of expansive soil.

14. Solution:

The following assumptions are made in Darcy's law.

- The soil is saturated.
- The flow through soil is laminar.
- The flow is continuous and steady.
- The total cross-sectional area of soil mass is considered.
- The temperature at the time of testing is 27°C.

15. Solution:

During the installation of sand drains, the clay around the drains may get remoulded, thus reducing the value of co-efficient of consolidation. This is known as smear effect.

16. Solution:

An isobar is a line joining points of equal pressure or stress intensity i.e. it is a contour of equal stress.

17. Solution:

- The main function of wall in framed structure is privacy of resident and effective utilisation of space.
- Walls in a framed building act as lateral load resisting element thereby providing safety to the building against lateral loads like wind and earthquake.

18. Solution:

- Freyssinet system
- Magnel system
- Leonhardt system
- Lee-McCall system
- Gifford-Udall system

19. Solution:

Minimum reinforcement percentage for slab is 0.15% of gross area for mild steel bars and 0.12% of gross area for HYSD bars.

20. Solution:

Two way shear is more critical in case of flat slabs and critical section for this two way shear is at a distance of d/2 from face of column or drop or column capital, as the case may be, where *d* is the effective depth of flat slab.

21. Solution:

Pure bending or simple bending is that in which bending moment is constant along the length.

$\frac{dM}{dx} = 0$ i.e., shear force is zero and M = constant.

Assumptions in Theory of Pure Bending

1. Material of the beam is homogeneous, isotropic and linear elastic in which Hooke's law is valid.
2. The beam is straight before loading.
3. Cross-section of beam is prismatic throughout the length.
4. The plane section before bending remains plane after bending.
5. Every layer of material is free to expand or contract longitudinally and laterally under stress and do not exert pressure upon each other.
6. The value of Young's modulus (E) for the material is same in tension and in compression.
7. The section of the beam is symmetrical in the loading plane.

22. Solution:

Given: Diameter of shaft, D = 125 mm; Angle of twist, $\theta = 1^\circ = \frac{\pi}{180}$ rad ; Length of the shaft, l = 1.5 m = 1.5 × 10^3 mm; Modulus of rigidity, G = 70 GPa = 70 × 10^3 N/mm^2

Let, T = Maximum torque the shaft can transmit

We know, that polar moment of inertia of a solid circular shaft,

$$J = \frac{\pi}{32}\times(D)^4 = \frac{\pi}{32}\times(125)^4 \simeq 24.0\times10^6 \text{ mm}^4$$

and relation for torque transmitted by the shaft,

$$\frac{T}{J} = \frac{G\cdot\theta}{l}$$

$$\frac{T}{24.0\times10^6} = \frac{(70\times10^3)\frac{\pi}{180}}{1.5\times10^3}$$

$\therefore$ $$T = 19.5\times10^6 \text{ Nmm} = 19.5 \text{ kNm}$$

23. Solution:

$$\text{Central deflection, } \Delta = \frac{WL^3}{48\,EI}$$

$$\Rightarrow \quad 14 = \frac{WL^3}{48\,EI}$$

$$\Rightarrow \quad \frac{EI}{L^2} = \frac{200\times1000}{14\times48} = 297.62 \text{ N}$$

Now, when the bar is placed vertical and loaded along its axis then it acts like a strut.

$$\therefore \quad P_E = \frac{\pi^2 EI}{L^2}$$

$$= \pi^2 \times 297.62 \qquad \left(\because \frac{EI}{L^2} = 297.62\right)$$

$$= 2937.38 \text{ N}$$

24. Solution:

Sensitivity is defined as the ratio of the unconfined compressive strength of an undisturbed specimen of the soil to the unconfined compressive strength of a specimen of the same soil after remoulding at unaltered water content.

$$S_t = \frac{(q_u)\text{undisturbed}}{(q_u)\text{remoulded}}$$

It represents degree of disturbance achieved on remoulding.

Soil classification based on sensitivity:

Sensitivity	% Water Content	Consistency
1 - 4	Normal	Gravel, Coarse sand
4 - 8	Sensitive	Sand
8 - 15	Extra Sensitive	Flocculent structure soil
> 15	Quick	Fine clay

25. Solution:

We know that

$$\text{Dry density, } \gamma_d = \frac{G\gamma_w}{1+e} \quad \ldots\text{(i)}$$

$$\text{Also, mass specific gravity, } G_m = \frac{\gamma_d}{\gamma_w} \text{ (for dry soil)}$$

$$\Rightarrow \quad \gamma_d = 1.42\,\gamma_w$$

Put this value in (i)

$$\Rightarrow \quad 1.42\,\gamma_w = \frac{G\gamma_w}{1+e}$$

$$\Rightarrow \quad 1.42 = \frac{2.6}{1+e}$$

$$\therefore \quad e = 0.83$$

26. Solution:

As more than 50% is retained on 75μ IS sieve, the soil is coarse-grained.

Coarse fraction = 55%

Gravel fraction = 40%

Sand fraction = 15%

As more than half the coarse fraction is larger than 4.75 mm sieve, the soil is gravel.

The soil has more than 12% fines, it can be either GM or GC.

Also, I_P = 12 %

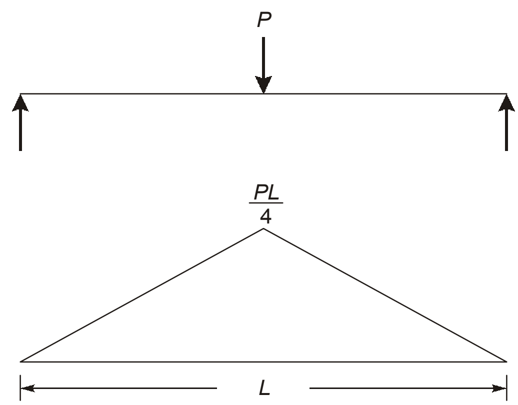
A-line: $I_p = 0.73\,(W_L - 20) = 0.73\,(40 - 20) = 14.6\%$

As soil lies below A line, it will be classified as GM.

27. Solution:

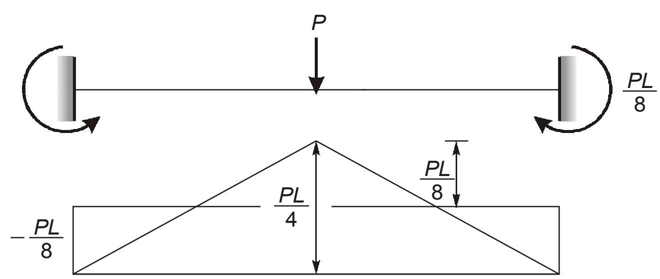
For a given loading the maximum stress and deflection of an indeterminate structure are generally smaller than those of its statically determinate counterpart.

Considering a simply supported beam:



So, maximum moment is $\frac{PL}{4}$

Now consider fixed beam:



So, maximum moment is $\frac{PL}{8}$

Another important reason for selecting a statically indeterminate structure is because it has a tendency to redistribute its load to its redundant supports in cases where faulty design or overloading occurs. In these cases, the structure maintains its stability and collapse is prevented. This is particularly important when sudden lateral loads, such as wind or earthquake, are imposed on the structure.

28. Solution:

Static indeterminacy (D_S): Those structures which can not be analysed by using condition of static equilibrium alone are called indeterminate structures. To analyse these indeterminate structures extra equilibrium condition are required, called compatibility conditions and numbers of compatibility conditions needed to analyse structure is known as degree of indeterminacy.

D_S = Total no. of reaction present (Both internal and external) – No. of available equilibrium equations.

Kinematic Indeterminacy (D_k): It refers to the total no. of available degree of freedom at all joints.

It is equal to total no. of unrestrained displacement component at all joints.

$$D_k = \text{Total degree of freedom at all joints} - \text{degree of freedom restrained by supports}$$

2-*D* Rigid Frames: At each joint there are three degree of freedom (*viz.* Δ_x, Δ_y and θ_z). Hence at all joint there will be $3j$ degree of freedoms. But at supports displacements are not available in the direction of reaction component.

$\therefore$ D_k = unrestrained displacement component

$\Rightarrow$ $D_k = 3j - r_e$ ...members are axially flexible

$D_k = 3j - r_e - m''$ ...m'' member are axially rigid

29. Solution:

1. Prestressed concrete is ideally suited for long span bridge construction. The present trend is to adopt prestressed concrete for long-span cable stayed bridges which are aesthetically superior and economical in comparison with steel bridges.
2. In recent years, prestressed concrete has found novel applications in the construction of a variety of marine structures, such as floating docks, off-shore oil-drilling platforms, etc.
3. It has found extensive applications in the construction of long-span folded plate roofs, aircraft hangers, nuclear containment vessels, etc.
4. It is used in construction of large-capacity liquid-retaining structures.
5. It is also used in construction of rail-road sleepers, poles, television towers and masts.

30. Solution:

Area of steel, $A_{st} = 4 \times \frac{\pi}{4} \times 12^2 = 452.3893 \text{ mm}^2$

$x_{u,\max} = 0.48 \times d = 0.48 \times 350 = 168 \text{ mm}$

Actual depth of neutral axis,

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} . B} = \frac{0.87 \times 415 \times 452.3893}{0.36 \times 20 \times 200}$$

$$x_u = 113.4271 \text{ mm} < x_{u,\max}$$

$\Rightarrow$ Section is under-reinforced.

Moment of Resistance $= 0.36 f_{ck} \cdot B \cdot x_u (d - 0.42 x_u)$

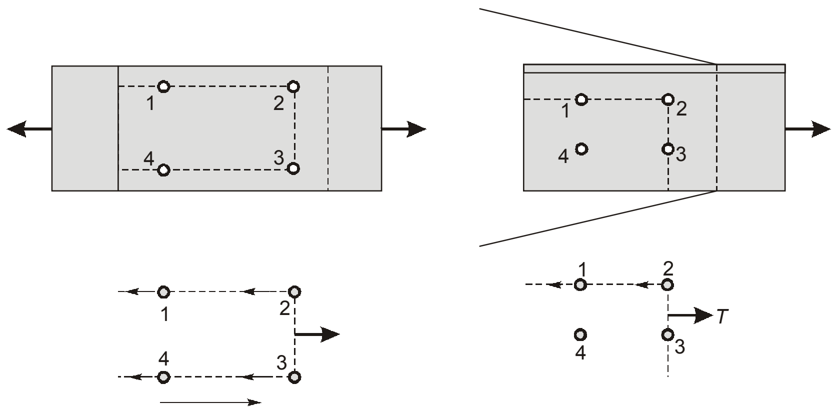
M.R. $= 0.36 \times 20 \times 200 \times 113.4271 (350 - 0.42 \times 113.4271)$

M.R. $= 49386078.8$ N-mm

M.R. $= 49.3861$ kN-m

31. Solution:

- At the connected end of a tension member, failure may occur along a path which involves shear along one plane and tension on the orthogonal plane along the fastener. This type of failure is referred to as block failure.



(a) (b)

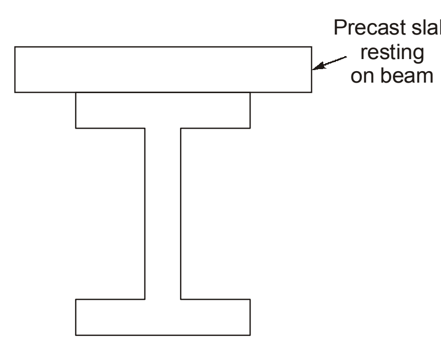
- In **Fig. (a)** failure along planes 1-2 and 3-4 is shear failure and that along plane 2-3 is tension failure.
- In **Fig. (b),** shear failure occurs along 1-2 and tension failure along 2-3.
- For bolted connections, **IS 800 : 2007** recommends the following expression for block shear strength:

$$T_{db} = \begin{cases} \dfrac{A_{vg} f_y}{\sqrt{3}.\gamma_{m0}} + \dfrac{0.9 A_{tn} f_u}{\gamma_{m1}} \\ \dfrac{0.9 A_{vn} f_u}{\sqrt{3}.\gamma_{m1}} + \dfrac{A_{tg} f_y}{\gamma_{m0}} \end{cases} \quad \text{whichever is less}$$

A_{vg} and A_{vn} = Minimum gross and net area in shear (1-2, 3-4 in **Fig. (a)** and 1-2 in **Fig. (b)**)
A_{tg} and A_{tn} = Minimum gross and net area in tension (2-3 in **Fig. (a)** and **(b)**)

32. Solution:

Laterally unsupported beams: Beams which are loaded with bending moment about major axis, and if their compression flange is not restrained against lateral movement, these are called laterally unsupported or laterally unrestrained beam. These beams may not attain material capacity. If the laterally unrestrained length of compression flange is relatively long then a phenomenon known as lateral buckling or lateral torsional buckling of beam may take place, and beam will fail well before it attains its full moment capacity.



Here permissible compressive stress (σ_{ac}) is calculated as

$$\left.\begin{aligned} \sigma_{bc} &= 0.66\sigma_y \\ &= \frac{0.66 f_{cb} \cdot f_y}{\left[(f_{cb})^n + (f_y)^n\right]^{1/n}} \end{aligned}\right\} \text{whichever is less}$$

Where n is imperfection factor and f_{cb} is Euler's critical elastic stress in bending.

33. Solution:

Size of foundation

Column load, $P = 1400$ kN

Let weight of the foundation, P_f = 10% of column load (P) = 0.1 × 1400 kN = 140 kN

∴ Total load, $P_t = 1400 + 140$ kN = 1540 kN

Area of footing required, $A = \frac{1540}{150} = 10.27\text{ m}^2$

Note: Do not use factored load of 1.5 × 1540 kN, here since safe bearing capacity of soil itself takes into account the factor of safety.

Assume $B = 2.5$ m

∴ $D = \frac{A}{B} = \frac{10.27}{2.5} = 4.11\text{ m} = 4.2\text{ m (say)}$

∴ Area provided, $A = 4.2 \times 2.5\text{ m}^2 = 10.5\text{ m}^2 > 10.27\text{ m}^2$ (OK)

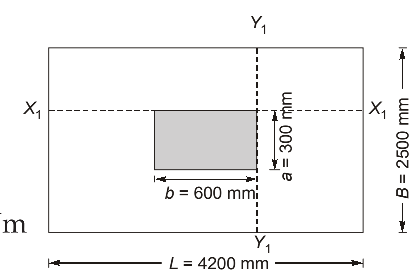
Net soil pressure

$$w_0 = \frac{P}{A} = \frac{1400}{10.5} = 133.33\text{ kN/m}^2$$

$< 150\text{ kN/m}^2$ (= safe bearing capacity of the soil) (OK)

Factored soil pressure, $w_{u0} = 1.5 \times 133.33 = 200\text{ kN/m}^2$

Depth of foundation from bending moment criterion



Moment at section x_1-x_1, $M_{ux} = w_{u0}\left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$

$$= 200 \times \frac{(2.5-0.300)^2}{8} = 121\text{ kNm}$$

Moment at section y_1-y_1, $M_{uy} = w_{u0} \times \left(\frac{L-a}{2}\right)\left(\frac{L-a}{4}\right)$

$$= 200 \times \frac{(4.2-0.6)^2}{8} = 324\text{ kNm}$$

$$Q = 0.36 f_{ck} \times 0.46 \times (1 - 0.42 \times 0.46)$$
$$= 0.36 \times 25 \times 0.46\,(1 - 0.42 \times 0.46)$$
$$= 3.34$$

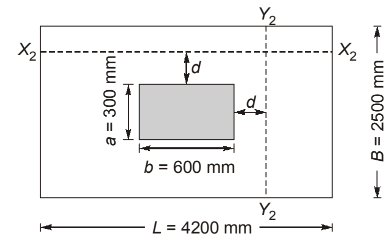
Depth of foundation, $d = \sqrt{\frac{M_{uy}}{QB}} = \sqrt{\frac{324 \times 10^6}{3.34 \times 1000}} = 311.46\text{ mm}$

Take $d = 320$ mm

Check for one way shear

Critical section for one way shear will be at a distance of 'd' from the face of the column

Maximum shear force at section y_2-y_2



$$V_{uy} = w_{u0} \times 1\text{ m} \times \left[\frac{\text{L-a}}{2} - d\right]$$

$$= 200 \times 1\left[\frac{4.2-0.6}{2} - 0.32\right]$$

$$= 296 \text{ kN}$$

Nominal shear stress

$$\tau_{vu} = \frac{V_{uy}}{Bd} = \frac{296 \times 10^3}{1000 \times 320} = 0.925 \text{ N/mm}^2$$

$$k = 1.0 \text{ for d} > 300 \text{ mm}$$

$$k\tau_c = 1 \times 0.29 \text{ N/mm}^2 < \tau_{vu} \;(= 0.925 \text{ N/mm}^2) \text{ Failed}$$

Revised depth of the footing

$$k\tau_c = 0.29 = \frac{V_{uy}}{Bd}$$

$$\therefore \quad \text{d} = \frac{V_{uy}}{B \times 0.29} = \frac{296 \times 10^3}{1000 \times 0.29} = 1020.69 \text{ mm}$$

$$\text{Average of 320 and 1020 mm} = \frac{320 + 1020}{2} = 670 \text{ mm} \approx 700 \text{ mm}$$

Check for d = 700 mm

$$V_{uy} = 200 \times \left[\frac{4.2-0.6}{2} - 0.7\right] = 220 \text{ kN}$$

$$\tau_v = \frac{V_{uy}}{Bd} = \frac{220 \times 10^3}{1000 \times 700} = 0.31 \text{ N/mm}^2$$

Try d = 750 mm

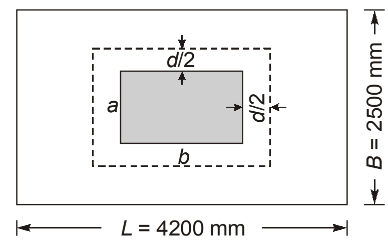
$$V_{uy} = 200 \times \left[\frac{4.2-0.6}{2} - 0.75\right] = 210 \text{ kN}$$

$$\tau_v = \frac{V_{uy}}{Bd} = \frac{210 \times 10^3}{1000 \times 750} = 0.28 \text{ N/mm}^2 < 0.29 \text{ N/mm}^2 \qquad \text{(OK)}$$

$\therefore$ Adopt effective depth, d = 750 mm

Check for punching/two way shear

$$\text{Net punching force} = P_u - w_{u0}\,(a + d)\,(b + d)$$
$$= 1.5 \times 1400 - 200 \times (0.6 + 0.75)(0.3 + 0.75)$$
$$= 1816.5 \text{ kN}$$



$$\text{Punching shear stress} = \tau_{vp\,(dev)} = \frac{\text{Net punching force}}{\text{Perimeter} \times \text{depth}}$$

$$= \frac{1816.5 \times 10^3}{2\left[(a+d)(b+d)\right] \times d}$$

$$= \frac{1816.5 \times 10^3}{2\left[(600+750)(300+750)\right] \times 750} = 0.505 \text{ N/mm}^2$$

$$\tau_{vp} = k_s 0.25\sqrt{f_{ck}}$$

where $k_s = \left(0.5 + \dfrac{300}{600}\right) \not> 1 = 1$

$$\tau_{vp\,(per)} = 1 \times 0.25 \times \sqrt{25} = 1.25 \text{ N/mm}^2 > 0.505 \text{ N/mm}^2 \qquad \text{(OK)}$$

Steel Reinforcement:

Reinforcement required along long direction

$$M_{uy} = 324 \text{ kNm}$$

$$R_y = \frac{M_{uy}}{bd^2} = \frac{324 \times 10^6}{1000 \times 750^2} = 0.576$$

$$\therefore \quad \frac{p_t}{100} = \frac{A_{sty}}{bd} = \frac{f_{ck}}{2 f_y}\left[1 - \sqrt{1 - \frac{4.598 R_y}{f_{ck}}}\right]$$

$$= \frac{25}{2 \times 500}\left[1 - \sqrt{1 - 4.598 \times \frac{0.576}{25}}\right]$$

$$\therefore \quad p_t = 0.136\%$$

$$\therefore \quad A_{sty} = \frac{0.136}{100} \times 1000 \times 750 = 1020 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of } 20\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{1020} = 308 \text{ mm c/c}$$

Provide 20ϕ bars @ 250 m c/c along long direction

$$\therefore \quad A_{sty\ provided} = \frac{1000 \times \frac{\pi}{4} \times 20^2}{250} = 1256.6 \text{ mm}^2 > 1020 \text{ mm}^2 \qquad \text{(OK)}$$

Reinforcement required along short direction

$$M_{ux} = 121 \text{ kNm}$$

$$R_X = \frac{M_{ux}}{bd^2} = \frac{121\times10^6}{1000\times(750-20)^2} = 0.22706$$

$$\therefore \quad p_t = \frac{25}{2\times500}\left[1-\sqrt{1-4.598\times\frac{0.22706}{25}}\right]100$$

$$= 0.05\% < 0.12\% \text{ (minimum reinforcement)}$$

Provided minimum reinforcement of 0.12%

$$\therefore \quad A_{stx} = \frac{0.12}{100}\times1000\times(750-20) = 876 \text{ mm}^2$$

$$\therefore \quad \text{Spacing of } 20\phi \text{ bars} = \frac{1000\times\frac{\pi}{4}\times20^2}{876} = 358.6 \text{ mm c/c but} \not> 300 \text{ mm c/c}$$

$\therefore$ Provide 20ϕ bars @ 250 mmc/c

$$\therefore \quad A_{stx\ provided} = \frac{1000\times\frac{\pi}{4}\times20^2}{250}$$

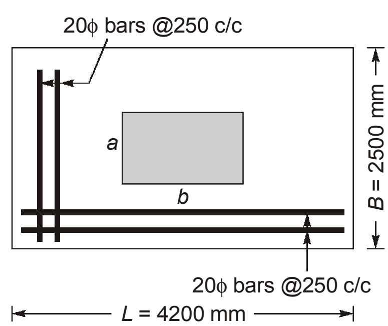
$$= 1256.6 \text{ mm}^2 > 876 \text{ mm}^2 \quad \text{(OK)}$$

$$\text{Development length required } (L_d) = \frac{0.87 f_y\phi}{4\tau_{bd}} = \frac{0.87(500)\phi}{4(1.6\times1.4)}$$

$$= 48.55\phi = 48.55\times20 = 917 \text{ mm}$$

$$\text{Available length} = \left(\frac{2500-300}{2}\right)-75$$

$$= 1025 \text{ mm} > 971 \text{ mm} \quad \text{(OK)}$$



34. Solution:

Gross cross-sectional area of beam $(A) = 250\times400 = 1\times10^5 \text{ mm}^2$

$$\text{Moment of inertia of the beam section } (I) = \frac{250\times400^3}{12} = 13.33\times10^8 \text{ mm}^4$$

Initial prestressing force $(P) = 1150\times360 = 414$ kN

$$m = \frac{E_s}{E_c} = \frac{210}{35} = 6$$

Stress in concrete at the level of steel,

$$f_c = \frac{P}{A} + \frac{Pe}{I}e = \frac{414000}{10^5} + \frac{414000(50)^2}{13.33 \times 10^8}$$

$$= 4.14 + 0.7764 = 4.9164 \text{ N/mm}^2$$

(a) Loss of stress in pre-tensioned beam

(i) Loss of stress due to elastic shortening of concrete

$$= mf_c = 6 \times 4.964 = 29.4984 \text{ N/mm}^2 = 29.5 \text{ N/mm}^2$$

(ii) Loss of stress due to creep of concrete

$$= 45 \times 10^{-6} \times 4.9164 \times 210 \times 10^3 = 46.5 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete

$$= 3 \times 10^{-4} \times 210 \times 10^3 = 63 \text{ N/mm}^2$$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5$ N/mm^2

(v) Loss of stress due to anchorage slip = 0

(vi) Loss of stress due to friction = 0

Total loss of stress = 196.5 N/mm^2

$\therefore$ Percentage loss of stress $= \frac{196.5}{1150} \times 100 = 17.09\%$

(b) Loss of stress in post-tensioned beam

(i) Loss of stress due to elastic shortening of concrete = 0

(ii) Loss of stress due to creep of concrete $= 22 \times 10^{-6} \times 4.9164 \times 210 \times 10^3$

$$= 22.7 \text{ N/mm}^2$$

(iii) Loss of stress due to shrinkage of concrete $= 2.15 \times 10^{-4} \times 210 \times 10^3$

$$= 45.15 \text{ N/mm}^2$$

(iv) Loss of stress due to relaxation $= 0.05 \times 1150 = 57.5$ N/mm^2

(v) Loss of stress due to anchorage slip $= \frac{1.25}{10 \times 1000} \times 210 \times 1000 = 26.25 \text{ N/mm}^2$

(vi) Loss of stress due to friction $= 1150 \times 0.0013 \times 10 = 14.95$ N/mm^2

Total loss of stress = 166.55 N/mm^2

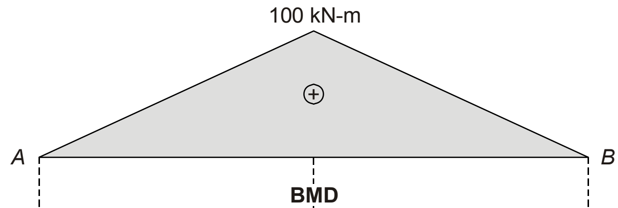
$\therefore$ Percentage loss of stress $= \frac{166.55}{1150} \times 100 = 14.48\%$

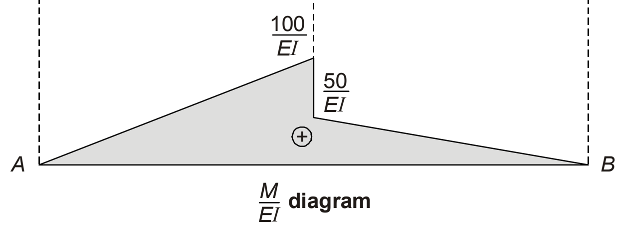
35. Solution:

Reaction in Real beam:

$$R_A = R_B = \frac{50}{2} = 25 \text{ kN}$$

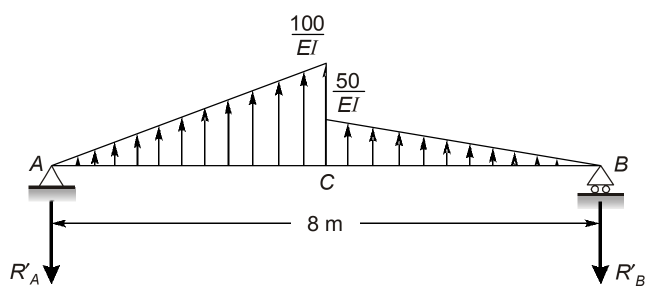
Ordinate of BMD under 50 kN load $= \dfrac{50 \times 8}{4} = 100$ kN-m





Conjugate beam:

Corresponding conjugate beam for given beam will be subjected to M/EI diagram loading.



Reactions in Conjugate beam:

$$R'_A + R'_B = \frac{1}{2} \times 4 \times \frac{100}{EI} + \frac{1}{2} \times 4 \times \frac{50}{EI}$$

$$R'_A + R'_B = \frac{300}{EI} \qquad \ldots(i)$$

Also, $\Sigma M_B = 0$

$$-R'_A \times 8 + \left(\frac{1}{2} \times 4 \times \frac{100}{EI}\right) \times \left(4 + \frac{4}{3}\right) + \left(\frac{1}{2} \times 4 \times \frac{50}{EI}\right) \times \left(4 \times \frac{2}{3}\right) = 0$$

$$-8R'_A + \frac{1066.67}{EI} + \frac{266.67}{EI} = 0$$

$$\therefore \qquad R'_A = \frac{166.67}{EI}$$

From equation (*i*), $\quad R'_B = \dfrac{133.33}{EI}$

(*i*) θ_A in real beam: S.F. at A in conjugate beam $= \theta_A$

$$\therefore \qquad \theta_A = -\frac{166.67}{EI} \quad \text{(radian)}$$

(ii) θ_B in real beam: S.F. at B in conjugate beam = θ_B

$$\therefore \qquad \theta_B = +\frac{133.33}{EI} \quad \text{(radian)}$$

(iii) θ_C in real beam: S.F. at C in conjugate beam = θ_C

$$\therefore \qquad \theta_C = -R'_A + \left(\frac{1}{2}\times 4\times\frac{100}{EI}\right)$$

$$= -\frac{166.67}{EI} + \frac{200}{EI}$$

$$= +\frac{33.33}{EI} \quad \text{(radian)}$$

(iv) Deflection at centre:

$$\Delta_C = \text{Bending moment in conjugate beam at C}$$

$$= -R'_A \times 4 + \left(\frac{1}{2}\times 4\times\frac{100}{EI}\right)\times\frac{4}{3}$$

$$= -\frac{166.67\times 4}{EI} + \frac{200}{EI}\times\frac{4}{3}$$

$$= -\frac{400}{EI} \quad \text{(Downward)}$$

36. Solution:

Length of shaft, $L = 2.5\text{ m} = 2500\text{ mm}$

Dia of steel core, $D_s = 54\text{ mm}$

Outer diameter of aluminium jacket, $D_A = 72\text{ mm}$

Inner diameter of aluminium jacket, $d_A = 54\text{ mm}$

$G_S = 80\text{ GPa}$

$G_A = 27\text{ GPa}$

Let T_S and T_A are torque shared by steel and aluminium shafts respectively. Since both shafts are in parallel combination, hence

$$\therefore \qquad T_A + T_S = T \qquad \text{...(i)}$$

$$\theta_S = \theta_A$$

and

$$\left(\frac{TL}{GI_P}\right)_S = \left(\frac{TL}{GI_P}\right)_A$$

$$\left(\frac{T_S\times 2500}{80\times 10^3\times\frac{\pi}{32}\times 54^4}\right) = \frac{T_A\times 2500}{27\times 10^3\times\frac{\pi}{32}\left(72^4-54^4\right)}$$

$\therefore$ $$T_S = 1.3715\, T_A \qquad \text{...(ii)}$$

On solving eq. (i) and (ii), we get

$$T_A = 0.422\, T \qquad \text{...(iii)}$$

$$T_S = 0.578\, T \qquad \text{...(iv)}$$

Since permissible stresses in steel, $\tau_S = 60$ MPa

$\therefore$ $$T_S = \tau_S (Z_P)_S$$

$$T_S = 60 \times \frac{\pi}{16} D_S^3$$

$$= \frac{60 \times \pi \times 54^3}{16} = 1.855 \times 10^6 \text{ N-mm}$$

From eq. (iv) $$0.578T = 1.855 \times 10^6$$

$\therefore$ $$T = 3.209 \times 10^6 \text{ N-mm}$$

Hence, $$T_A = 1.354 \times 10^6 \text{ N-mm}$$

Angle of twist at end A,

$$\theta_A = \theta_S = \left(\frac{TL}{GI_P}\right)_S$$

$$\theta = \left[\frac{1.855 \times 10^6 \times 2500}{80 \times 10^3 \times \frac{\pi}{32} \times 54^4}\right]$$

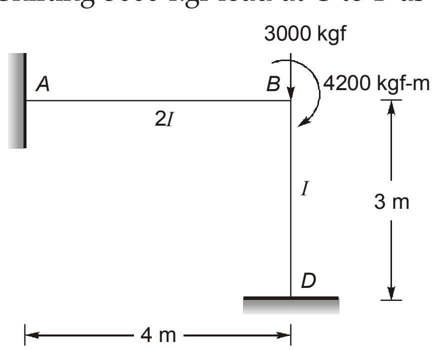
$$\theta = 0.0694 \text{ rad. or } 3.97°$$

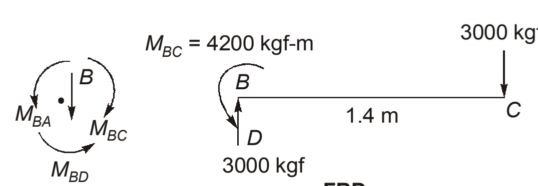
$\therefore$ Value of torque T is 3.209×10^6 Nmm or 3.209 kNm and angle of twist is 3.97°.

37. Solution:

For given frame,

Shifting 3000 kgf load at C to B as $M = 3000 \times 1.4 = 4200$ kgf-m





FBD

Using slope deflection method, fixed end moment

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$= \frac{2E(2I)}{4}(\theta_B) = EI\theta_B \qquad \{\theta_A = 0;\ \delta = 0\}$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\delta}{L}\right)$$

$$= \frac{2E(2I)}{4}(2\theta_B) = 2EI\theta_B$$

$$M_{BD} = \frac{2EI}{L}\left(2\theta_B + \theta_D - \frac{3\delta}{L}\right)$$

$$= \frac{2EI}{3}(2\theta_B) = \frac{4EI\theta_B}{3} \qquad \{\theta_D = 0;\ \delta = 0\}$$

$$M_{DB} = \frac{2EI}{L}\left(2\theta_D + \theta_B - \frac{3\delta}{L}\right) = \frac{2EI}{3}(\theta_B) = \frac{2EI}{3}\theta_B$$

Applying moment equilibrium equation at '*B*'

$$M_{BA} + M_{BD} - 4200 = 0$$

$$\Rightarrow \quad 2EI\theta_B + \frac{4}{3}EI\theta_B - 4200 = 0$$

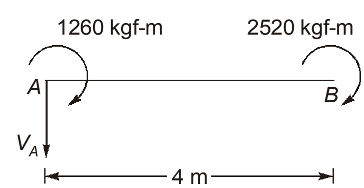
$$\Rightarrow \quad EI\theta_B = 1260 \text{ kgf-m}$$

$$M_{AB} = EI\theta_B = 1260 \text{ kgf-m}$$

$$M_{BA} = 2EI\theta_B = 2 \times (1260) = 2520 \text{ kgf-m}$$

$$M_{BD} = \frac{4EI\theta_B}{3} = \frac{4}{3}\times(1260) = 1680 \text{ kgf-m}$$

$$M_{DB} = \frac{2EI\theta_B}{3} = \frac{2}{3}\times(1260) = 840 \text{ kgf-m}$$

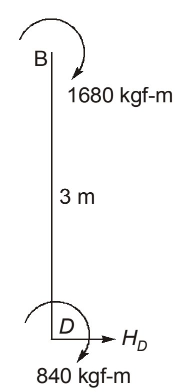


For member *AB*

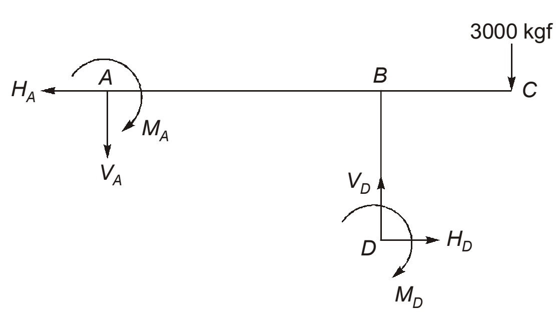
Using $M_B = 0 \Rightarrow V_A \times 4 = 1260 + 2520 \Rightarrow V_A = 945$ kgf (↓)

For member *BD*

Using $M_B = 0 \Rightarrow 1680 + 840 = H_D \times 3 \Rightarrow H_D = 840$ kgf (→)



Applying equilibrium equations on frame



$$\Sigma F_y = 0 \quad \Rightarrow \quad V_A + V_D = 3000$$

$$\Rightarrow \quad -945 + V_D = 3000 \quad \Rightarrow \quad V_D = 3945 \text{ kgf-m } (\uparrow)$$

$$\Rightarrow \quad \Sigma F_X = 0 \quad \Rightarrow \quad H_A = H_D \quad \Rightarrow \quad H_A = 840 \text{ kgf } (\leftarrow)$$

So support reactions

at A, H_A = 840 kgf (←); V_A = 945 kgf (↓); M_A = 1260 kgf-m (↻)
at D, H_D =840 kgf (→); V_D = 3945 kgf (↑); M_D = 840 kgf-m (↻)

38. Solution:

The knowledge of subsoil conditions at a site is a prerequisite for safe and economical design of substructure elements. The field and laboratory studies carried out for obtaining the necessary information about the subsoil characteristics including the position of ground water table, are termed as soil investigation or exploration.

It is generally carried out in two stages, namely, preliminary and detailed.

Preliminary exploration consists of the geological study of the site and the site reconnaissance. During preliminary investigations, geophysical methods and tests with cone penetrometers and sounding rods can be very useful.

Detailed investigation follows the preliminary investigation and is normally carried out to determine the nature, sequence and thickness of various subsoil layers, their lateral variation, physical properties and the position of ground water table. Boring and detailed sampling are usually undertaken to obtain this information. Various in situ tests also form a part of the detailed investigation programme.

SPT: The test employs a split spoon sampler which consists of a driving shoe, a split-barrel of circular cross-section which is longitudinally split into two parts and coupling. **IS 2131 :1981** gives the standard procedure for carrying out test. It is briefly stated below:

(a) The **borehole** is advanced to the required depth and the bottom cleaned.

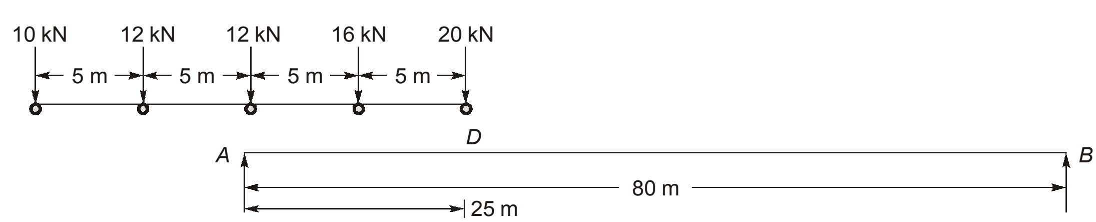
(b) The **split-spoon sampler**, attached to standard drill rods of required length is lowered into the borehole and rested at the bottom.

(c) The **split-spoon sampler** is driven into the soil for a distance of 450 mm by blows of a drop hammer of 65 kg falling vertically and freely from a height of 750 mm. The number of blows required to penetrate every 150 mm is recorded while driving the sampler. The number of blows required for the last 300 mm of penetration is added together and recorded as the N value at that particular depth of borehole. The number of blows required to effect the first 150 mm of penetration, called the seating drive, is disregarded.

(d) The **split-spoon sampler** is then withdrawn. The soil sample collected inside the split barrel is carefully collected so as to preserve the natural moisture content and transported to the laboratory for tests.

39. Solution:

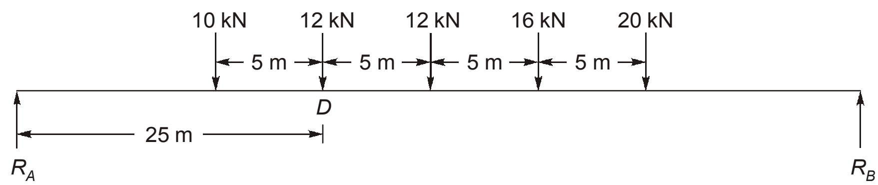
(i) Let AB be the beam and D the given section,



Let us allow the loads to cross the given section one after another and find average loads on AD and BD. This calculation is shown as;

Load Crossing the section *D*	Average load on *AD*	Average load on *BD*	Remarks
20 kN	$\frac{50}{25}$	$\frac{20}{55}$	$(AD)_L > (BD)_L$
16 kN	$\frac{34}{25}$	$\frac{36}{55}$	$(AD)_L > (BD)_L$
12 kN	$\frac{22}{25}$	$\frac{48}{55}$	$(AD)_L > (BD)_L$
12 kN	$\frac{10}{25}$	$\frac{60}{55}$	$(AD)_L < (BD)_L$

Hence, for maximum bending moment at *D*, the arrangement of load will be given as,



$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_B \times 80 = 10 \times 20 + 12 \times 25 + 12 \times 30 + 16 \times 35 + 20 \times 40$$

$$\Rightarrow \quad R_B = 27.75 \text{ kN } (\uparrow)$$

$$\Sigma V = 0$$

$$R_A + R_B = 70$$

$$\Rightarrow \quad R_A = 42.25 \text{ kN } (\uparrow)$$

$$\therefore \quad M_{\max,\ D} = R_A \times 25 - 10 \times 5$$

$$= 42.25 \times 25 - 50 = 1006.25 \text{ kNm}$$

(ii) Absolute maximum bending moment:

$$\text{Total load} = 70 \text{ kN}$$

Taking moment about trailing load of 10 kN,

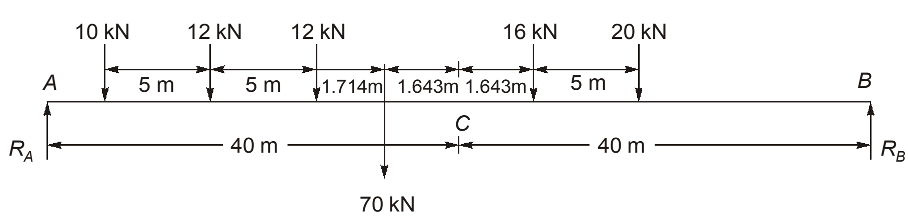
$$70 \times \bar{x} = 12 \times 5 + 12 \times 10 + 16 \times 15 + 20 \times 20$$

$$\Rightarrow \quad \bar{x} = 11.714 \text{ m from trailing 10 kN load}$$

It is obvious absolute maximum bending moment will occur under load 16 kN, because 16 kN load in the load system is bigger than 12 kN load which is nearer to the resultant. Hence, for this condition the load system should be so placed on the span that resultant of all wheel loads 70 kN and 16 kN load are equidistant from middle point of girder.

$\therefore$ Distance between resultant load and 16 kN = 15 – 11.714 = 3.286 m

Hence, for the above condition, 16 kN should be placed 1.643 m. From centre on right side of girder.



Taking, $\Sigma M_A = 0$

$R_B \times 80 = 20 \times 46.643 + 16 \times 41.643 + 12 \times 36.643 + 12 \times 31.643 + 10 \times 26.643$

$\Rightarrow \quad R_B = 33.56 \text{ kN } (\uparrow)$

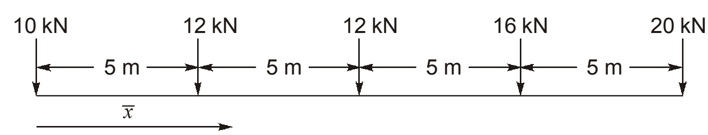
M_{max} under load 16 kN at 46.643 from left end of girder is given as

$= 33.56 \times (40 - 1.643) - 20 \times 5$

$M_{abs\ max} = 1187.26 \text{ kNm}$

Alternate Method

For finding absolute bending moment by influence line method:



Taking moment about trailing load of 10 kN

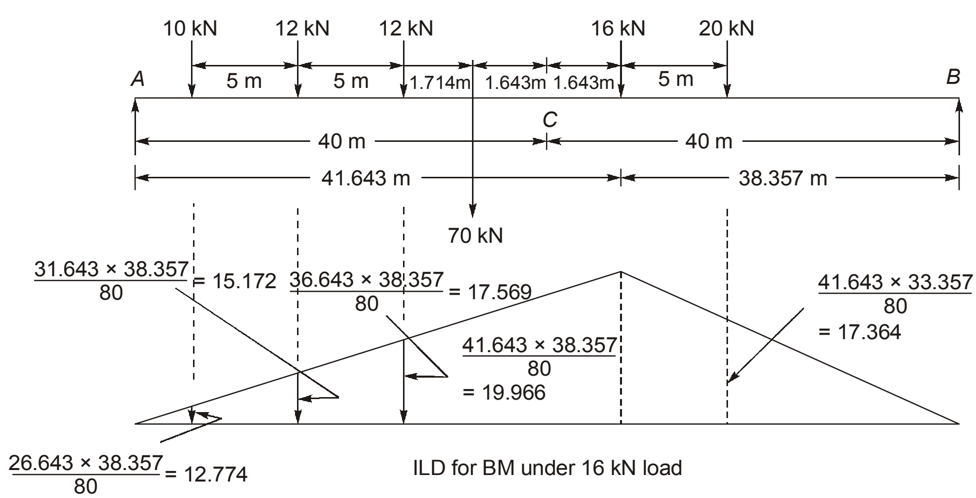
$(10 + 12 + 12 + 16 + 20)\bar{x} = 12 \times 5 + 12 \times 10 + 16 \times 15 + 20 \times 20$

$\bar{x} = 11.714$ m from trailing 10 kN load

It is obvious absolute maximum bending moment will occur under load 16 kN, because 16 kN load in the load system isn biggest load and this load is also near to the resultant load.

Hence load positions should be such that resultant of all wheel loads i.e., 70 kN and 16kN load are equidistant from middle point of girder.

Hence load position are:



ILD for BM under 16 kN load

$M_{abs\ max} = 12.774 \times 10 + 15.172 \times 12 + 17.569 \times 12 + 16 \times 19.966 + 20 \times 17.364$

$= 1187.368 \text{ kNm}$