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RPSC Main Exam 2019 : Test Series

Assistant Engineer

Civil Engineering

Compulsory Subject : Paper-I

Test No. 3 | Date of Exam. : 07-07-2019 (9 AM to 12 Noon)

Part-A

1. Solution:

Endurance limit is the stress level below which even large number of stress cycle cannot produce fatigue failure.

2. Solution:

$$E = 3k(1 - 2\mu)$$

3. Solution:

It is the circumferential stress that occurs in cylinders and acts normal to the cylinder axis and also normal to the radius vector. Hoop stress is tangential to the surface on which it acts.

4. Solution:

$$\text{Maximum shear stress} = 1.5\,\tau_{avg}$$

Average shear stress, $\tau_{avg} = \dfrac{S}{bd} = \dfrac{60\times10^3}{100\times d}$

and $\tau_{max} = 4\text{ N/mm}^2$

$\therefore$ $4 = 1.5\times\dfrac{60\times10^3}{100\times d}$

$\Rightarrow$ $d = 225\text{ mm}$

5. Solution:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\text{Maximum strain} = \left(80 - \frac{40}{4}\right)\frac{1}{E} = \frac{70}{E}$$

$$\therefore \text{ One dimensional stress} = 70 \text{ N/mm}^2$$

6. Solution:

It is a short length of an angle section used at a joint to connect the outstanding leg of a member, to the Gusset plate and thereby reducing the length of the joint.

7. Solution:

Grillage foundation consists of a number of layers of beams, usually laid at right angles to each other and used to disperse heavy point loads from the super structure to an acceptable ground bearing pressure.

8. Solution:

The only difference is in the effective length, which is increased by 10% in case of battened column whereas in case of laced column it is increased by 5%.

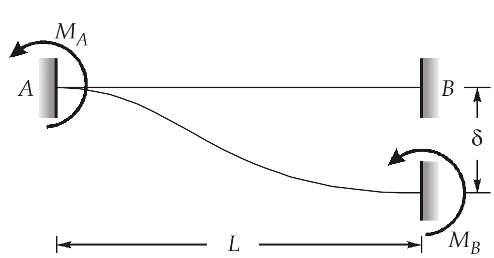
9. Solution:

The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

10. Solution:

The work done by one load on the displacement due to a second load is equal to the work done by the second load on the displacement due to the first.

11. Solution:



$$M_A = M_B = \frac{6EI\delta}{L^2}$$

12. Solution:

Shrinkage Ratio: It is defined as the ratio of volume change which is expressed as percentage of dry soil volume to the corresponding change in water content i.e.

$$\text{Shrinkage ratio} = \frac{(V_1 - V_2)/V_d}{w_1 - w_2} \times 100$$

where,

V_1 = Vol. of soil at water content w_1.

V_2 = Vol. of soil at water content w_2.

V_d = Vol. of dry soil.

13. Solution:

Design Loads

Loads to be considered in the design of masonry structures are:

1. Dead load of walls, columns, floors and roofs as per **IS 875 (Part-I)**.
2. Live load on floors and roof as per **IS 875 (Part-II)**.
3. Wind loads on walls and sloping roofs as per **IS 875 (Part-III)**.
4. Seismic loads as per **IS 1893 (Part-I)** and snow loads as per **IS 875 (Part-IV).**

14. Solution:

Over consolidation ratio (OCR), is defined as the ratio of preconsolidation stress applied to a soil sample in the past to the current effective stress applied on the sample.

15. Solution:

It is a type of deep foundation in the shape of a hollow prismatic box which is built above the ground level and then sunk into the ground at the required depth as a single unit. It is a water tight chamber which is used for laying foundations under water like rivers, lakes etc.

16. Solution:

Anchor Piles are the piles that are required to resist lateral loads with or without being braced depending on circumstances and an ordinary or standard house piles are required to carry a vertical load.

17. Solution:

The temporary structure erected to support the concrete in its required form, till it hardens and becomes self-supporting is known as formwork, centering or shuttering.

18. Solution:

Cover: The following minimum cover is required for:

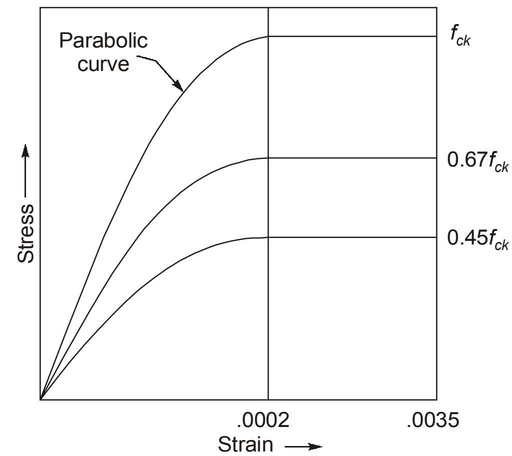
(a) Beams - 25 mm

(b) Slabs - 15 mm

(c) Columns - 40 mm

(d) Foundation-50 mm

19. Solution:



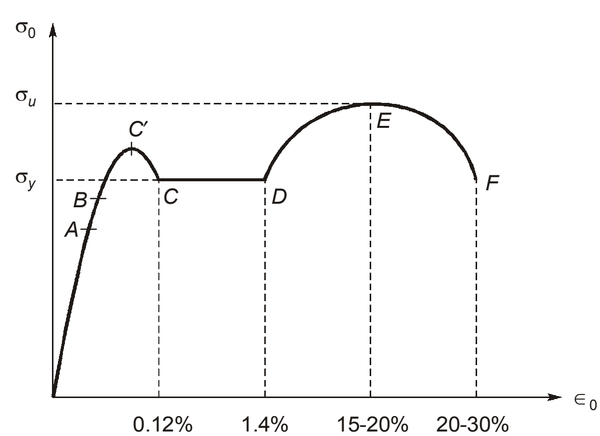
Stress-Strain Curve for Concrete

20. Solution:

Best section for RCC beam: Usually slab is casted monolithically with beams and thus a T-section is best suited as the beam section since it provides large compression area (in the form of large flange width) which reduces the overall depth of beam required.

Part-B

21. Solution:



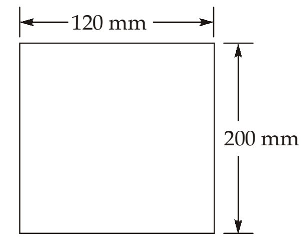
Stress-strain Curve for Mild Steel in Tension

A → Proportional Limit	B → Elastic Limit
C' → Upper Yield Point	C → Lower Yield Point/Actual Yield Point
E → Ultimate point	F → Fracture point
σ_u → Ultimate stress/Tenacity	σ_y → Yield stress
CD → Yield Plateau	
DE → Strain hardening	EF → Strain Softening/Necking Region

22. Solution:

Given: Maximum bending stress, σ_{max} = 7 N/mm^2; Width of section, b = 120 mm; depth of section, d = 200 mm; length of beam, l = 4 m

Let 'w' kN/m be the safe *udl*.



Maximum B.M. in simply supported beam due to *udl*

$$M = \frac{w \cdot l^2}{8} \text{ kN-m}$$

Section Modulus,
$$Z = \frac{b.d^2}{6} = \frac{120 \times 200^2}{6} \text{ mm}^3$$
$$= 8 \times 10^5 \text{ mm}^3$$

We know,
$$M = \sigma_{max} \cdot Z$$
$$\frac{w \times 10^3}{10^3} \times \frac{(4 \times 10^3)^2}{8} = 7 \times 8 \times 10^5$$
$$w = 2.8 \text{ kN/m}$$

23. Solution:

Following are the theories of failure:

1. Maximum Principal Stress Theory/Rankine's Theory
2. Maximum Principal Strain Theory/Saint Venant's Theory
3. Maximum Shear Stress Theory/Tresca's or Guest-Coloumb's Theory
4. Maximum Strain Energy Theory/Belt Reimi-Heigh's Theory
5. Maximum Shear Strain Energy Theory/Distortion Energy Theory/Von Mises-Hankee Theory
6. Mohr's Theory

(i) **Mild steel** is a ductile Material and **Distortion Energy theory** is most suited.

Distortion Energy Theory: By this theory the material will fail when distortion energy per unit volume of a material will exceed the distortion energy per unit volume for a material in case of simple tensile test.

For 3D loading, failure condition:

$$\left[\sigma_{P_1}^2 + \sigma_{P_2}^2 + \sigma_{P_3}^2 - \sigma_{P_1}\sigma_{P_2} - \sigma_{P_2}\sigma_{P_3} - \sigma_{P_1}\sigma_{P_3}\right] > \left(\frac{\sigma_y}{FOS}\right)^2$$

(ii) **Concrete** is a brittle material and Rankine's Theory is suitable for brittle material.

Rankine's Theory: According to this theory material will fail when the value of Maximum Principal stress reaches the maximum elastic stress in uniaxial loading test.

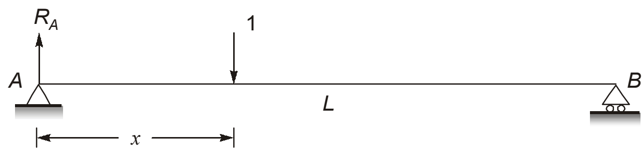
Failure condition, $\left[\sigma_{P_1} \geq \frac{\sigma_y}{FOS}\right]$.

24. Solution:

Influence line : Influence line is a representation of variation of stress function like support reactions, shear force, bending moment at particular cross-section when concentrated unit load moves over span of a structure such as beam, girder, arch or truss from one end to another end.

It is used to analyse the effect to moving loads on the structure. It can also be determined using Muller-Breslau principle.

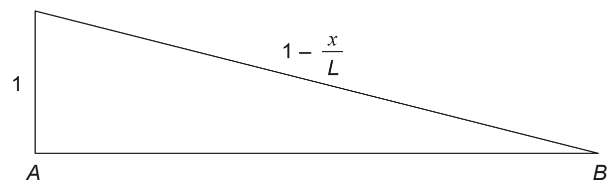
Example:



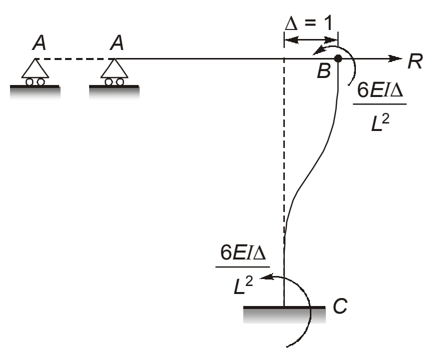
For a simple supported beam, influence line of vertical reactions at 'A', can be found out using unit load at a distance 'x'.

For which $$R_A = \frac{L-x}{L} = 1 - \frac{x}{L}$$

ILD for R_A



25. Solution:



For given frame, let stiffness matrix be k

where, $$k = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

For k_{11} and k_{12},
Providing unit horizontal deflection at 'B'

$$\Sigma M_C = 0$$

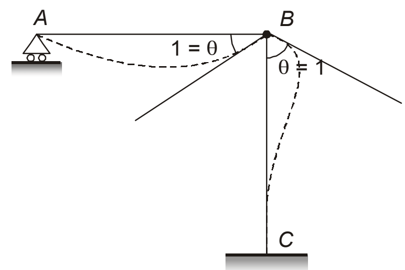
$$R_C \times L = \frac{6EI}{L^2} + \frac{6EI}{L^2}$$

$$\Rightarrow \quad R_C = \frac{12EI}{L^3} \Rightarrow k_{11} = \frac{12EI}{L^3}$$

$$k_{12} = k_{21} = \frac{6EI}{L^2} \quad \text{(Moment induced at '}B\text{')}$$

For k_{22}

Providing unit rotation at B



$$k_{22} = \frac{4EI}{L} + \frac{3EI}{L} = \frac{7EI}{L}$$

$$\text{Stiffness matrix, } k = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{7EI}{L} \end{bmatrix}$$

26. Solution:

For sand,

All around pressure, $\sigma_3 = 350 \text{ kN/m}^2$

Angle of shear resistance, $\phi = 38°$

Using,

Principal stress, $\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right)$

$\Rightarrow$ $\sigma_1 = 350\tan^2\left(45° + \frac{38°}{2}\right)$

$\Rightarrow$ $\sigma_1 = 1471.31 \text{ kN/m}^2$

Deviator stress, $\sigma_d = \sigma_1 - \sigma_3$

$= 1471.31 - 350$

$= 1121.31 \text{ kN/m}^2$

27. Solution:

For given soil,

Length of drainage path $d = \frac{2}{2} = 1 \text{ m}$

Coefficient of consolidation, $C_v = 2 \times 10^{-4} \text{ cm}^2/\text{sec}$

$= 2 \times 10^{-8} \text{ m}^2/\text{sec}$

For half of ultimate settlement, degree of consolidation,

$U = 0.5$

For $U = 0.5$, time factor, $T_v = \frac{\pi}{4} \times U^2 = \frac{\pi}{4} \times 0.5^2 = 0.196$

Using, $$T_v = \frac{C_v t}{d^2}$$

$$\Rightarrow \quad 0.196 = \frac{2\times 10^{-8}\times t}{(1)^2}$$

$\Rightarrow$ Time of settlement, t = 9800000 sec = 113.43 days

28. Solution:

For soil,

Minimum dry density, $$\rho_{d,min} = \frac{1550}{10\times 10\times 10} = 1.55 \text{ gm/cc}$$

Maximum dry density, $$\rho_{d,max} = \frac{1550}{10\times 10\times (10-2)} = 1.9375 \text{ gm/cc}$$

Dry density in natural state, $$\rho_d = \frac{\rho_{bulk}}{1+w} = \frac{1.9}{1+0.12} = 1.696 \text{ gm/cc}$$

Relative density, $$I_D = \frac{\frac{1}{\rho_{min}} - \frac{1}{\rho}}{\frac{1}{\rho_{min}} - \frac{1}{\rho_{max}}} = \frac{\frac{1}{1.55} - \frac{1}{1.696}}{\frac{1}{1.55} - \frac{1}{1.9375}} = 0.43$$

29. Solution:

The volume of loosely deposited damp sand is much more than the volume of same sand in loose and dry state. This increase in the volume of damp sand is called as bulking. In the damped state, a sort of cohesion develops between the sand particles due to capillary water. This cohesion prevents the sand particles from taking a stable compact position. The effect of bulking of sand is more pronounced when moisture content is about 4 to 5% and the resultant increase in the volume of sand is about 20 to 30%.

30. Solution:

Equation of parabola with origin at a support is given by,

$$y = \frac{4ex}{L^2}(L-x)$$

$$\text{Slope} = \frac{dy}{dx} = \frac{4e}{L^2}(L-2x)$$

$$\text{Slope at support } (x = 0 \text{ and } x = L) = \pm\frac{4e}{L}$$

Total eccentricity at support and midspan = 50 + 75 = 125 mm

$$\therefore \quad \text{Slope at support } (x = 0) = \frac{4\times 0.125}{10} = 0.05$$

Net change in slope between supports,

$$\alpha = 2 \times 0.05 = 0.10 \text{ radian}$$

$$\text{Initial prestress, } P_1 = 1000 \times 500 = 500 \times 10^3 \text{ N}$$

If P_2 is prestress at the other end,

$$P_2 = P_1 e^{-(\alpha\mu + KL)}$$

$$\mu\alpha + KL = 0.55 \times 0.10 + 0.0015 \times 10 = 0.07$$

$$\therefore \quad P_2 = 500 \times 10^3 \, e^{-0.07} = 466.2 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Percentage loss} = \left(\frac{500 - 466.2}{500}\right) \times 100 = 6.7\%$$

Alternatively, relation between P_1 and P_2 is given by

$$P_2 = P_1 (1 - \mu\alpha - KL)$$

$$\Rightarrow \quad P_2 = 500 \times 10^3 (1 - 0.07) = 465 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Percentage loss} = \left(\frac{500 - 465}{500}\right) \times 100 = 7\%$$

31. Solution:

Following are components of plate girder,

1. **Web plate:** These plate are used effectively to resist shear force on the girder.
2. **Flange plate :** There are plates effectively used to resist bending forces on the girder.
3. **Stiffners:** These are provided to reduce possibility of diagonal and vertical web buckling.
   These are further classified as:
   (i) Horizontal stiffner
   (ii) Vertical stiffner
   (iii) Bearing stiffner

32. Solution:

In case of tension member, stress is constant which is independent of the cross sectional area. It means for a given load we can determine the required area directly as given below,

$$\text{Required area,} \quad A_{req} = \frac{\text{Applied load}}{\text{Permissible tensile stress}}$$

while in compression member, permissible stress σ_{ac} depends upon slenderness ratio (λ) and so on the area itself. Hence, we have to assume area first and then we check whether the applied force is less than the permissible force or not.

$$P < \sigma_{at} \times \text{Area}_{\text{assumed}}$$

Hence, in case of compression member indirect method of design is used.

Part-C

33. Solution:

A stair may be defined as series of steps suitably arranged for the purpose of connecting different floors of a building at different elevations. It may also be defined as an arrangement of treads, risers, stringers, newel posts, hand rails and baluster, so designed and constructed as to provide an easy and quick access to the different floors rendering comfort and safety to the users. The enclosure containing the complete stair way is termed as staircase.

A staircase without any waist slab is called a tread riser staircase. In this type of staircase only riser and tread are provided so they are called **tread riser staircase**. They are also known as straight staircase.

Design steps for stairs:

(i) Effective Span of Stairs: Stair slab may be divided into two categories, depending upon the direction in which the stair slab spans.

(a) Stair slab spanning horizontally/transversely

(b) Stair slab spanning longitudinally(along the incline)

Stair slab spanning horizontally: In this category, the slab is supported on each side by side wall or stringer beam on one side and beam on the other side. Some times as in the case of straight stairs, the slab may also be supported on both sides by two side walls. In such a case the effective span L is the horizontal distance between centre to centre of supports.

Stair slab spanning longitudinally: In this category, the slab is supported at bottom and top of the flight and remain unsupported on the sides. The effective span of such stairs, without stringer beams, should be taken as follows:

(a) Where supported at top and bottom risers by beams spanning parallel with the risers, the distance between centre to centre of beams.

(b) Where spanning on the edge of a landing slab, which spans parallel, with the risers, a distance equal to the going of stairs plus at each end either half the width of the landing or one meter whichever is smaller.

(c) Where the landing slab spans in the same direction as the stairs, they shall be considered as acting together to form a single slab and the span determined as the distance c/c of the supporting beams or walls, the going being measured horizontally.

(ii) Live load: IS : 875 - 1987 (Part-II) give the loads for staircases. For stairs in residential buildings, office buildings, hospital wards, hostels, etc. where there is no possibility of over crowding, the live load may be taken to be 3000 N/m^2 subject to a minimum of 1300 N concentrated load at the unsupported end of each step for stairs constructed out of structurally independent cantilever step. For other public buildings liable to be over crowded, the live load may be taken by 5000 N/m^2.

(iii) Distribution of loading on stairs:

(a) In case of stairs with open walls, where spans partly crossing at right angles occur, the load on areas common to any two such spans may be taken as one-half in each direction.

(b) Where flights or landings are built into walls at a distance not less than 110 mm and are designed to span in the direction of the flight, a 150 mm strip may be deducted from the loaded area and the effective breadth of the section increased by 75 mm for the purposes of design.

(iv) Estimation of dead weight:

(a) Dead weight of waist slab: The dead weight w′ per unit area is first calculated at right angles to the slope. The corresponding load per unit horizontal area is then obtained by increasing w′ by the ratio $\frac{\sqrt{R^2+T^2}}{T}$ where R = rise and T = tread. Thus if t = thickness of waist slab in mm, then

$$w' = \frac{t\times1\times1}{1000}\times 25000 = 25t\ \text{N/m}^2 \text{ of inclined area}$$

Hence dead weight w_1 per unit horizontal area is given by

$$w_1 = w'\times\frac{\sqrt{R^2+T^2}}{T} = 25t\times\frac{\sqrt{R^2+T^2}}{T}$$

$$w_1 = 25t\sqrt{1+(R/T)^2}$$

(b) Dead weight of steps: The dead weight of steps is calculated by treating the steps to be equivalent horizontal slab of thickness equal to half the rise $\left(\frac{R}{2}\right)$. Thus if w_2 is the weight of step per unit horizontal area, we have

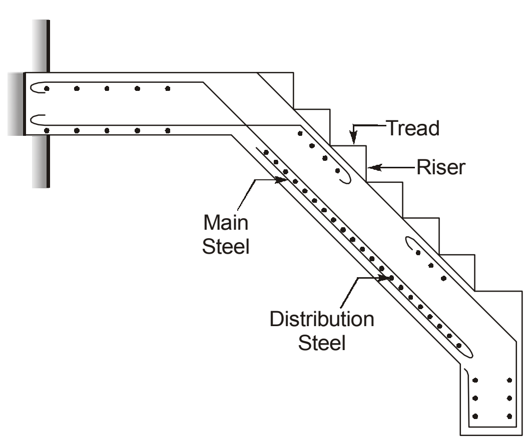
$$w_2 = \frac{R}{2\times1000}\times1\times1\times25000 = 12.5\ R\ \text{N/m}^2$$

Where R is rise in mm.

$$\text{Total } w = (w_1 + w_2)$$

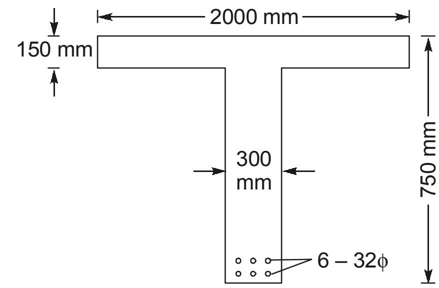
per unit horizontal area

(v) Depth of the Section: The depth of the section shall be taken as the minimum thickness perpendicular to the soffit of the stair case. Once the depth of the section is finalized, area of reinforcement can be known too along with spacing.



34. Solution:

Given: Flange width, b = 2000 mm; Span, l_0 = 9 m = 9000 mm; Flange thickness, D_f = 150 mm; Overall depth, D = 750 mm; Rib width, b_w = 300 mm



$$A_{st} = 6 \times \frac{\pi}{4} \times 32^2 = 4825.486 \text{ mm}^2$$

Effective width of flange for an isolated T-beam is given as

$$b_f = \frac{l_0}{\left(\frac{l_0}{b}\right) + 4} + b_w = \frac{9000}{\frac{9000}{2000} + 4} + 300$$

$\Rightarrow$ $$b_f = 1358.82 \text{ mm} < b$$

Now to calculate the moment of resistance of the beam, we need to calculate the actual depth of neutral axis.

Assuming that the NA lies in the flange.

Force of compression, $C = 0.36 f_{ck} b_f x_u$

Force of tension, $T = 0.87 f_y A_{st}$

But $C = T$

$\therefore$ $0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$

$\Rightarrow$ $$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f} = \frac{0.87 \times 500 \times 4825.486}{0.36 \times 25 \times 1358.82}$$

$\Rightarrow$ $x_u = 171.64$ mm

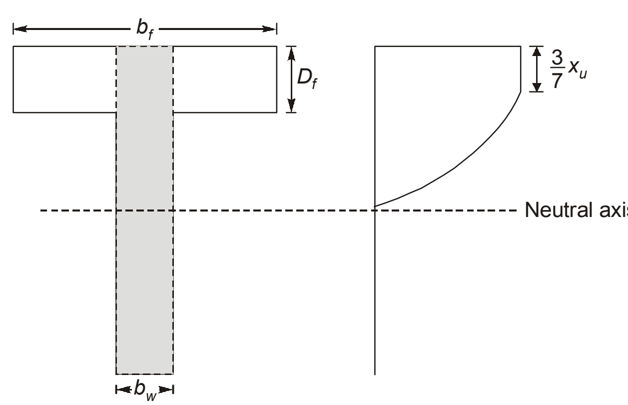
$\therefore$ $x_u > D_f$

Hence our assumption is wrong.

$\therefore$ NA lies in the web.

As we can see from the stress diagram, depth of rectangular portion of stress diagram

$$= \frac{3}{7} x_u = \frac{3}{7} \times 171.64 = 73.56 \text{ mm}$$



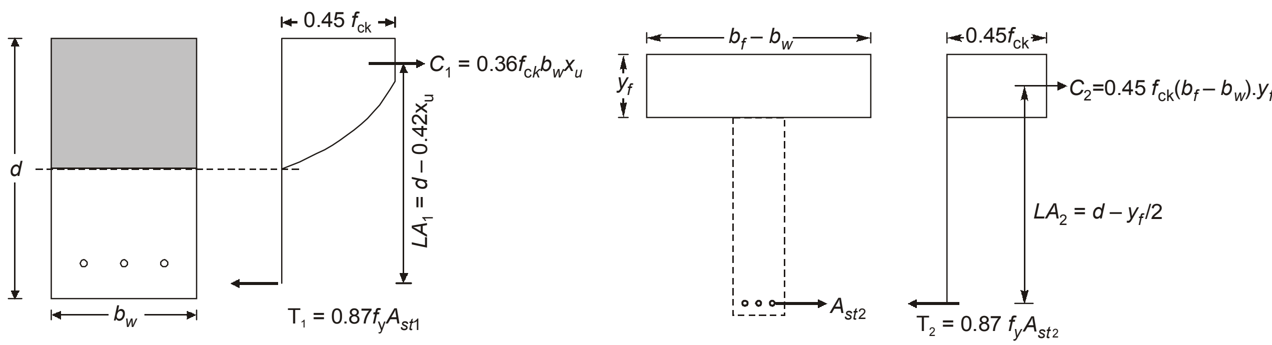
$\therefore$ $D_f > \frac{3}{7} x_u$

It means that depth of flange is greater than the rectangular portion of stress diagram.

In this case it is assumed that depth of rectangular portion of the stress block is equal to y_f

where, $y_f = 0.15x + 0.65D_f \not> D_f$

$\therefore$ The given T-beam can be shown as



$$\therefore \quad C = C_1 + C_2$$
$$= 0.36 f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) y_f$$
$$T = T_1 + T_2$$
$$= 0.87 f_y A_{st1} + 0.87 f_y A_{st2} = 0.87 f_y A_{st}$$

But $C = T$

$$\therefore \quad 0.36 f_{ck} b_w x_u + 0.45 f_{ck}(b_f - b_w) y_f = 0.87 f_y A_{st}$$
$$\Rightarrow \quad 0.36 \times 25 \times 300 \times x_u + 0.45 \times 25 \times (1358.82 - 300) y_f$$
$$= 0.87 \times 500 \times 4825.486$$
$$\Rightarrow \quad 2700 x_u + 11911.725 \times (0.15 x_u + 0.65 \times 150)$$
$$= 2099{,}086.41$$

Solving $x_u = 209$ mm

$\therefore$ $x_u > D_f$ (OK)

and $\frac{3}{7} x_u = \frac{3}{7} \times 209 = 89.57\ mm$

$\therefore$ $D_f > \frac{3}{7} x_u$ (OK)

$\therefore$ Moment of the resistance of the beam with respect to concrete is given by

$$M_R = C_1 \times L_1 + C_2 \times L_2$$
$$= 0.36 f_{ck} b_w x_u \times (d - 0.42 x_u) + 0.45 f_{ck}(b_f - b_w) y_f (d - y_f/2)$$

$\therefore$ Effective depth, $d = D - 50$

$$= 750 - 50 = 700 \text{ mm}$$
$$y_f = 0.15 x_u + 0.65 D_f$$
$$= 0.15 \times 209 + 0.65 \times 150 = 128.85 \text{ mm}$$
$$\therefore \quad M_R = 0.36 \times 25 \times 300 \times 209 \times (700 - 0.42 \times 209)$$
$$+ 0.45 \times 25 \times (1358.82 - 300) \times (0.15 \times 209$$
$$+ 0.65 \times 150) \times (700 - 64.425)$$
$$= 345{,}475{,}746 + 975{,}496{,}886.4$$

$$= 132{,}097{,}2632 \text{ N-mm}$$
$$= 1320.973 \text{ kN-m}$$

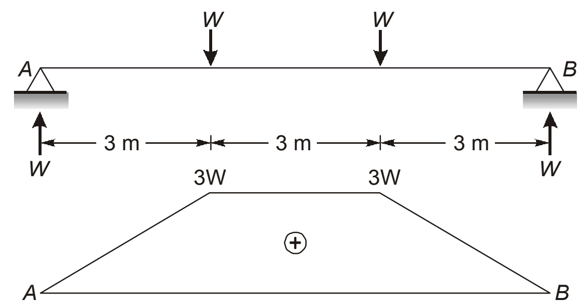
Now, to calculate the magnitude of point load, equating the bending moment to moment of resistance

$$W_d = (2 \times 0.15 + 0.3 \times 0.60) \times 25 = 12 \text{ kN/m}$$

$$\frac{M_u}{1.5} = M_w = 880.65 \text{ kN-m}$$

$$880.65 = \frac{W_d l^2}{8} + \frac{Wl}{3} = \frac{12 \times 9^2}{8} + \frac{Wl}{3}$$

$$W = 253.05 \text{ kN}$$



35. Solution:

For loading diagram:

Point *A*:

At point A there will be an upward reaction as SF is positive.

$\therefore$ $$R_A = 19.5 \text{ t}$$

Portion *AC*:

Between *A* and *C*, SFD is an Inclined line. Hence portion AB will be subjected to UDL.

$$\text{Intensity of loading} = \text{Slope of SFD}$$

$$w_1 = \frac{-16.5 - 19.5}{6 - 0} = -6 \text{ t/m}$$

Negative sign indicate downward loading.

Point *C*:

There is a drastic change in SF at *C* from –16.5t to 12t, hence there is a support at C and a reaction on it.

$$R_C = 16.5 + 12$$
$$R_C = 28.5 \text{ t}$$

Portion *CD*:

In portion *CD*, SFD consists incline line, hence portion *CD* will be subjected to UDL.

Intensity of UDL, $$w_2 = \frac{0 - 12}{2 - 0} = -6 \text{ t/m}$$

Negative sign indicate downward loading.

End Supports:

At support *A*,

$$\Sigma M_A = (28.5 \times 6) - (6 \times 8 \times 4) = 171 - 192 = -21 \text{ t-m}$$

$\therefore$ Reactive moment at *A* will be $M_A = 21$ kNm

Hence support *A* will be fixed or built-up.

For bending moment diagram:

Portion AC:

$$M_x\ (x \text{ from } A) = +R_A x - M_A - \frac{6x^2}{2}$$

$[0 \le x \le 6]$

$$M_x = 19.5x - 21 - 3x^2$$

(Parabolic)

at $x = 0$, $\quad M_A = -21$ t-m

at $x = 6$, $\quad M_C = 19.5 \times 6 - 21 - 3 \times 6^2$

$$M_C = -12 \text{ t-m}$$

For M_{max}, $\quad \frac{dM}{dx} = 0$

$$19.5 - 6x = 0$$

$$x = \frac{19.5}{6} = 3.25 \text{ m from A}$$

$\therefore \quad M_{max}\ (x = 3.25 \text{ m}) = 19.5 \times 3.25 - 21 - 3 \times 3.25^2 = 10.687$ t-m

For bending moment to be zero

$$M_x = 0$$

$$19.5x - 3x^2 - 21 = 0$$

$$3x^2 - 19.5x + 21 = 0$$

On solving $\quad x = 5.14$ m and $x = 1.36$ m from A

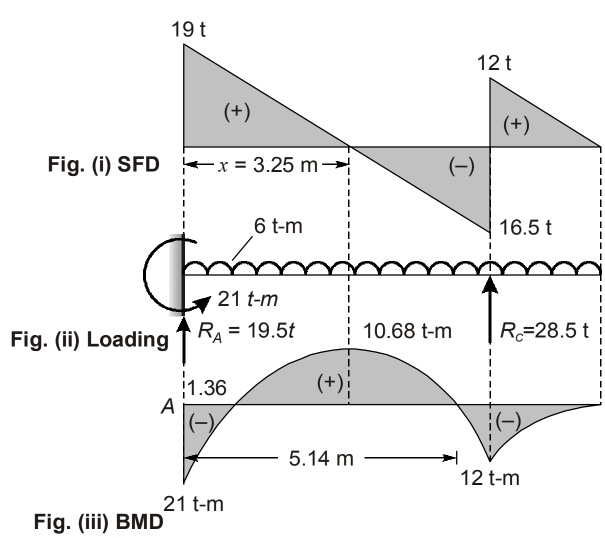
Portion CD:

$$M_x\ (x \text{ from } D) = -\frac{6x^2}{2}\ [0 \le x \le 2]$$

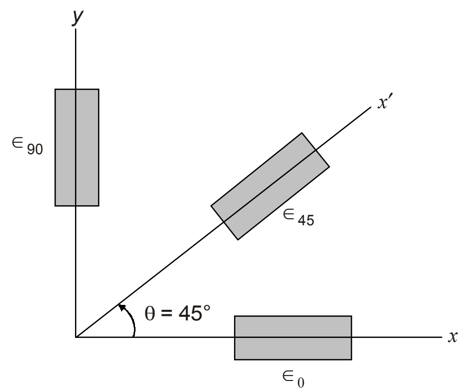
$$M_x = -3x^2$$

at $x = 0$, $\quad M_D = 0$

at $x = 2$, $\quad M_C = -3 \times 2^2 = -12$ t-m



36. Solution:



Let ϕ_{xy} be the shear strain in xy plane

Normal strain in x' direction is given by

$$\in_x' = \frac{\in_x + \in_y}{2} + \frac{\in_x - \in_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Here, $\theta = 45$, $\in_x = \in_0$ and $\in_y = \in_{90}$

Hence, $$\in_{45} = \frac{\in_0 + \in_{90}}{2} + \frac{\in_0 - \in_{90}}{2}\cos 90° + \frac{\phi_{xy}}{2}\sin 90$$

$\therefore$ $$\phi_{xy} = 2\in_{45} - (\in_0 + \in_{90})$$

$$\phi_{xy} = 2 \times 500 \times 10^{-6} - (600 \times 10^{-6} + 200 \times 10^{-6})$$
$$= 1000 \times 10^{-6} - 800 \times 10^{-6} = 200 \times 10^{-6} \text{ (radian)}$$

Now we have, $\in_x = \in_0$, $\in_y = \in_{90}$ and ϕ_{xy}

Principal strains can be calculated as

$$\in_1/\in_2 = \frac{\in_x + \in_y}{2} \pm \sqrt{\left(\frac{\in_x - \in_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$\in_1/\in_2 = \frac{\in_0 + \in_{90}}{2} \pm \sqrt{\left(\frac{\in_0 - \in_{90}}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$\in_1/\in_2 = \frac{(600+200)\times 10^{-6}}{2} \pm \sqrt{\left(\frac{(600-200)\times 10^{-6}}{2}\right)^2 + \left(\frac{200\times 10^{-6}}{2}\right)^2}$$

$\therefore$ $$\in_1/\in_2 = 400 \times 10^{-6} \pm 223.61 \times 10^{-6}$$

$$\in_1 = 623.61 \times 10^{-6}$$

and $$\in_2 = 176.39 \times 10^{-6}$$

Thus principal stresses will be given as

$$\sigma_1 = \frac{E}{1-\mu^2}[\in_1 + \mu \in_2] = \frac{2\times10^5}{1-0.3^2}\left(623.61\times10^{-6} + 0.3\times176.39\times10^{-6}\right) = 148.69 \text{ MPa}$$

and
$$\sigma_2 = \frac{E}{1-\mu^2}[\in_2 + \mu \in_1] = \frac{2\times10^5}{1-0.3^2}\left(176.39\times10^{-6} + 0.3\times623\times10^{-6}\right) = 79.88 \text{ MPa}$$

The **direction of principal plane** will be given as

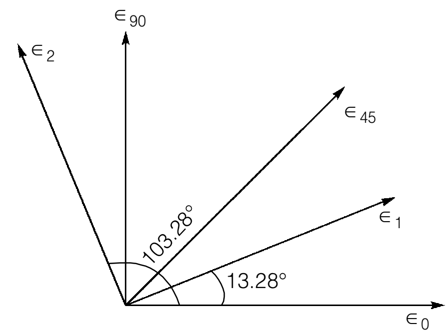
$$\tan 2\theta_P = \frac{\phi_{xy}}{\in_x - \in_y} \Rightarrow \frac{\phi_{xy}}{\in_0 - \in_{90}}$$

$$\tan 2\theta_P = \frac{200\times10^{-6}}{(600-200)\times10^{-6}}$$

$$\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta_{P_1} = 13.28°$$

and
$$\theta_{P_2} = \theta_{P_1} + 90° \Rightarrow 103.28°$$



Putting θ_{P_1} in

$$\in_x' = \frac{\in_x + \in_y}{2} + \left(\frac{\in_x - \in_y}{2}\right)\cos 2\theta + \frac{\phi_{xy}\sin 2\theta}{2}$$

and $\in_x' = \in_1 = 623.61 \times 10^{-6}$

Hence major principal plane will be inclined at 13.28° with $\in_0$ and minor principal plane will be inclined at 103.28° with $\in_0$, in anticlockwise direction.

37. Solution:

(i) Lateral pressure will be earth pressure at rest,

$$K_0 = 1 - \sin\phi = 1 - \sin 28° = 0.53$$

Total thrust,
$$P_0 = \frac{1}{2}\times K_0 \cdot \gamma_d \cdot Z^2$$

$$P_0 = \frac{1}{2}\times 0.53\times 16\times 5^2 = 106 \text{ kN/m}$$

Point of application from bottom, $Z_0 = \frac{5}{3} = 1.67$ m

(ii) Lateral pressure will be active earth pressure,

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 28°}{1+\sin 28°} = 0.36$$

Total thrust,
$$P_A = \frac{1}{2}\cdot K_a \cdot \gamma_d \cdot Z^2$$

$$P_A = \frac{1}{2}\times 0.36\times 16\times 5^2 = 72 \text{ kN/m}$$

Point of application from bottom, $Z_A = \frac{5}{3} = 1.67$ m

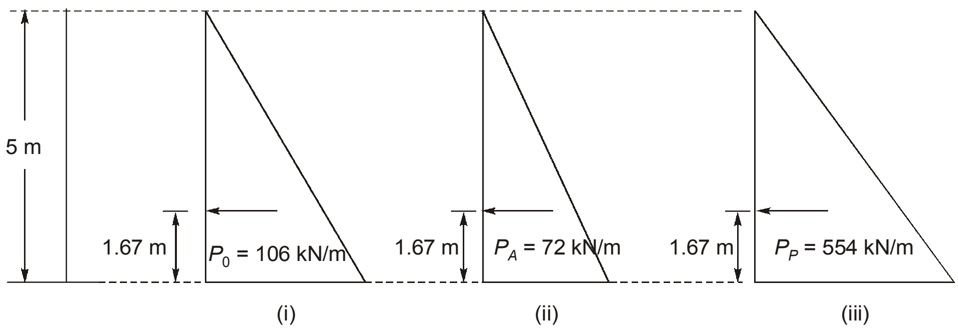
(iii) Lateral pressure will be passive earth pressure,

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1+\sin 28^\circ}{1-\sin 28^\circ} = 2.77$$

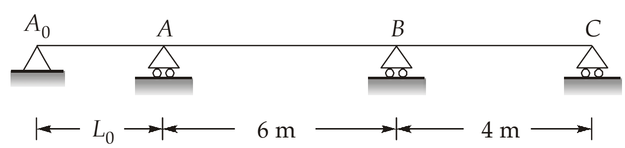
Total thrust,
$$P_p = \frac{1}{2}\cdot K_p \cdot \gamma_d \cdot Z^2$$

$$P_p = \frac{1}{2}\times 2.77\times 16\times 5^2 = 554 \text{ kN/m}$$

Point of application from bottom, $Z_p = \dfrac{5}{3} = 1.67$ m



38. Solution:



To handle fixed end case at end A, end A is made continuous support, by adding imaginary beam A_0A of span L_0 and flexural rigidity $EI_0 \to \infty$.

Support B is 10 mm below mid support A,

Hence, $h_B = -10 \text{ mm} = -0.010 \text{ m}$

$E = 200 \text{ kN/mm}^2 = 200 \times 10^6 \text{ kN/m}^2$

$I = 1 \times 10^8 \text{ mm}^4 = 1 \times 10^{-4} \text{ m}^4$

The three moment equation for span A_0A and AB is

$$M_0\left(\frac{L_0}{\infty}\right) + 2M_A\left(\frac{L_0}{\infty} + \frac{6}{2I}\right) + M_B\left(\frac{6}{2I}\right) = -0 - 0 + 0 + \frac{6\times 200\times 10^6(-0.010)}{6}$$

$$\Rightarrow \quad 6M_A + 3M_B = -200 \times 10^4 \times I$$

Substituting the value of I, we get

$$6M_A + 3M_B = -200 \quad \text{...(i)}$$

Consider spans AB and BC and since B sinks by 10 mm, support A and C are 10 mm above the position of mid support B.

Hence, $h_A = h_C = 10 \text{ mm} = 0.010 \text{ m}$

Therefore, the three moment equation for these two spans is:

$$M_A\left(\frac{6}{2I}\right)+2M_B\left(\frac{6}{2I}+\frac{4}{I}\right)+M_C\left(\frac{4}{I}\right) = -0-0+\left(\frac{6\times200\times10^6\times0.01}{6}\right)+\left(\frac{6\times200\times10^6\times0.010}{4}\right)$$

Noting that $M_C = 0$ and multiplying throughout by I, we get

$$3M_A + 14M_B = 6\times200\times10^6\times0.01\left(\frac{1}{6}+\frac{1}{4}\right)\times I$$

$$\Rightarrow \quad 3M_A + 14M_B = 6\times200\times10^6\times0.01\left(\frac{1}{6}+\frac{1}{4}\right)\times1\times10^{-4}$$

$$\Rightarrow \quad 3M_A + 14M_B = 500 \qquad \text{...(ii)}$$

Solving equations (i) and (iii), we get

$$25M_B = 1200$$

$$\Rightarrow \quad M_B = 48 \text{ kNm}$$

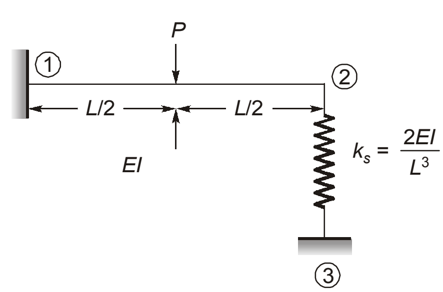
Substituting in equation (i), we get

$$6M_A + 3\times48 = -200$$

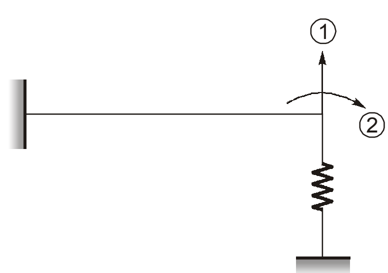
$$M_A = -57.333 \text{ kNm}$$

Thus, $M_A = -57.333$ kNm, $M_B = 48$ kNm, $M_C = 0$

39. Solution:



The degree of freedom is 2. As there are two independent displacement components at node 2 of fixed end beam at one end resting on elastic spring support at another end.



The stiffness matrix with reference to the chosen coordinates may be developed as;

$$\begin{bmatrix}\Delta_1\\ \Delta_2\end{bmatrix} = \begin{bmatrix}\Delta\\ \theta\end{bmatrix}\text{(Displacement components)}$$

$$\Rightarrow \qquad k_{ij} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$\therefore \qquad k_{11} = \frac{12EI}{L^3} \qquad k_{21} = \frac{6EI}{L^2}$$

$$k_{12} = \frac{6EI}{L^2} \qquad k_{22} = \frac{4EI}{L}$$

$$\therefore \qquad [k] = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

Given, k_s = Stiffness of spring = $\dfrac{2EI}{L^3}$

$$\therefore \qquad [k_M] = \text{Modified stiffness matrix} = \begin{bmatrix} \left(\frac{12EI}{L^3} + \frac{2EI}{L^3}\right) & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

$$\Rightarrow \qquad [k_s] = \begin{bmatrix} \frac{14EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

We know, $$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = [k_M]^{-1} \{[P_M] - [P']\} \qquad \text{...(i)}$$

$[P_M]$ = No external loads at nodes.

$$\Rightarrow \qquad [P'] = \begin{bmatrix} P_1' \\ P_2' \end{bmatrix}$$

Matrix is determined by considering the span. AB as fixed. Determining the forces generated at nodes due to central point load P on beam AB.

$$\therefore \qquad \begin{bmatrix} P_1' \\ P_2' \end{bmatrix} = \begin{bmatrix} \frac{P}{2} \\ \frac{PL}{8} \end{bmatrix}$$

Put these value in equation (i);

$$\Rightarrow \qquad \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} \frac{14EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}^{-1} \left\{ 0 - \begin{bmatrix} \frac{P}{2} \\ \frac{PL}{8} \end{bmatrix} \right\}$$

$$\Rightarrow \quad \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \frac{-1}{\left\{\frac{56(EI)^2}{L^4} - \frac{36(EI)^2}{L^4}\right\}} \begin{bmatrix} \frac{4EI}{L} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{14EI}{L^3} \end{bmatrix} \begin{bmatrix} \frac{P}{2} \\ \frac{PL}{8} \end{bmatrix}$$

$$= \frac{-L^4}{20(EI)^2} \begin{bmatrix} \frac{4PEI}{2L} - \frac{3EI \times P}{4L} \\ \frac{-3EIP}{L^2} + \frac{7PEI}{4L^2} \end{bmatrix} = \frac{-L^4}{20(EI)^2} \begin{bmatrix} \frac{5PEI}{4L} \\ \frac{-5PEI}{4L^2} \end{bmatrix}$$

$$\therefore \quad \Delta_1 = \frac{-L^4}{20(EI)^2} \times \frac{5PEI}{4L} = \frac{-PL^3}{16EI}$$

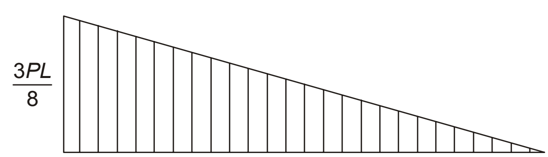
$$\Delta_2 = \frac{-L^4}{20(EI)^2} \times \left(\frac{-5}{4}\right) \times \left(\frac{PEI}{L^2}\right) = \frac{PL^2}{16EI}$$

$\therefore$ Let R be reaction is spring.

Hence, $$R = k_s \Delta_1 = -\frac{2EI}{L^3} \times \frac{PL^3}{16EI} = \frac{-P}{8} (\downarrow)$$

$\therefore$ BM at support A $$= R \times L - \frac{PL}{2}$$

$$= \frac{P}{8} \times L - \frac{PL}{2} = \left(\frac{PL - 4PL}{8}\right) = \frac{-3PL}{8}$$



Bending moment diagram