2020

MADE EASY

WORKBOOK

Detailed Explanations of Try Yourself Questions

Electrical Engineering

Signals and Systems

MADE EASY Publications

1 Introduction

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

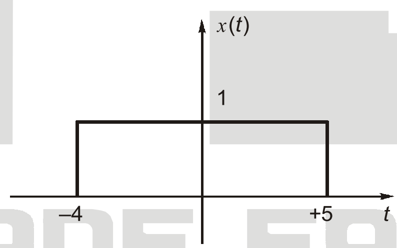
As we know even part of any signal is $\frac{x(t)+x(-t)}{2}$

So if we use the above formulae we can determine that even part of $x(t)$ is represented as signal given in option (a).

T2 : Solution

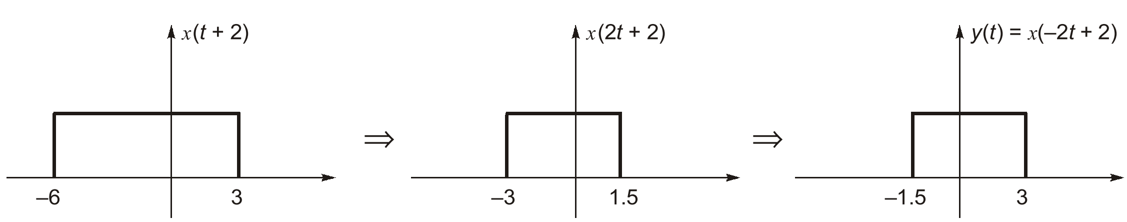
(b)

The signal $x(t) = u(-t+5) - u(-t-4)$



Now we need to find $y(t) = x(-2t+2)$

To find $y(t)$ first of all we will find $x(t+2)$ i.e. left shift signal $x(t)$ by 2, then we will find $x(2t+2)$ i.e. scaling the signal $x(t+2)$ by factor of 2 and then we will find $x(-2t+2)$ i.e. time reversal of signal $x(2t+2)$



T3 : Solution

(a)

The expression of $x(t)$ is $x(t) = \sum_{k=-\infty}^{\infty} \delta(t-4k) - \delta(t-4k-1)$.

So $x(t)$ is a subtraction of two signals each periodic with period 4. So $x(t)$ is periodic with period 4.

T4 : Solution

The signal is, $x(t) = 3e^{-t}u(t)$

Now, energy of signal will be $E_x = \int_0^{\infty} [3e^{-t}]^2 dt = 4.5$

T5 : Solution

The signal $x(t)$ is $(t-1)^2$

We need to find $\int_{-\infty}^{\infty} x(t)\,\delta(t-1)dt = x(1)$

So, $x(1) = (1-1)^2 = 0$

So answer is '0'.

■■■■

2 Fourier Series

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

Given that Fourier series coefficient of $x(t)$ is a_k

So, $$x(t) \xrightarrow{\text{F.S.}} a_k$$

Now, real part of $x(t)$ is $\dfrac{x(t)+x^*(t)}{2}$

and if $$x(t) \xrightarrow{\text{F.S.}} a_k$$

then $$x^*(t) \xrightarrow{\text{F.S.}} a^*_{-k}$$

So real part of $x(t)$, $$\frac{x(t)+x^*(t)}{2} \xrightarrow{\text{F.S.}} \frac{a_k + a^*_{-k}}{2}$$

T2 : Solution

(c)

We know that Fourier series cannot be defined if the signal is non periodic. From the given options, signal given in option (c) is non periodic. So answer is (c).

T3 : Solution

(d)

For real periodic signal the Fourier series coefficients (c_k) are conjugate symmetric ($c_k = c^*_{-k}$) which means even magnitude and odd phase. So option (c).

T4 : Solution

(d)

The signal $x(t)$ is such that $x(t) = -x\left(t + \frac{T_0}{2}\right)$, so the signal has half wave symmetry. So the signal will have only odd harmonics, and since signal is even it will have only cosine terms.

T5 : Solution

Power of signals is $\sum_{-\infty}^{\infty} |C_n|^2 \Rightarrow \sum_{-2}^{2} |C_n|^2$

So power is $= \sum_{-2}^{2} |C_n|^2 = (2)^2 + (8)^2 + (8)^2 + (2)^2 = 136$

3 Fourier Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

The Fourier transform is $X(\omega) = u(\omega) - u(\omega - 2)$, we know that

- If signal is real then $X(\omega)$ is conjugate symmetric.
- If signal is imaginary then $X(\omega)$ is conjugate anti-symmetric

The given $X(\omega)$ is neither conjugate symmetric nor conjugate anti-symmetric.

So $x(t)$ is complex signal.

T2 : Solution

(d)

Given that $x(t)$ is real

$$x(t) = 0 \text{ for } t \leq 0$$

and
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \text{Re}[X(\omega)]\, e^{j\omega t} d\omega = |t|\, e^{-|t|}$$

We know that if
$$x(t) \xrightarrow{\text{F.T.}} X(\omega)$$

then Even $[x(t)]$,
$$\frac{x(t) + x(-t)}{2} \xrightarrow{\text{F.T.}} \frac{X(\omega) + X^*(-\omega)}{2}$$

So Fourier transform of even part of $x(t)$ is real part of $X(\omega)$. So inverse Fourier transform of Re$[X(\omega)]$ will be even part of $x(t)$.

So even part of $x(t)$ is $|t|\, e^{-|t|}$

So,
$$\frac{x(t) + x(-t)}{2} = |t|\, e^{-|t|}$$

and
$$x(t) = 0 \text{ for } t < 0$$

So,
$$x(t) = 2te^{-t}\, u(t)$$

T3 : Solution

(a)

We know that if $e^{-2t}u(t) \xrightarrow{\text{F.T.}} \dfrac{1}{j\omega+2}$

and $e^{2t}u(t) \xrightarrow{\text{F.T.}} \dfrac{1}{2-j\omega}$

T4 : Solution

(c)

Given Fourier transform of $e^{-t}u(t)$ is $\dfrac{1}{1+j2\pi f}$.

Now Fourier transform of $\dfrac{1}{1+j2\pi t}$ will be $e^{-f}u(f)$ using the duality theorem.

T5 : Solution

(a)

Fourier transform is $G(\omega) = \dfrac{\omega^2+21}{\omega^2+9}$

So, $$G(\omega) = \frac{\omega^2}{\omega^2+9} + \frac{21}{\omega^2+9} = 1 + \frac{12}{\omega^2+9}$$

As we know that Fourier transform of $e^{-a|t|}$ is $\dfrac{2a}{a^2+\omega^2}$

So $$g(t) = \delta(t) + 2\exp(-3|t|)$$

T6 : Solution

(d)

Given that the signal is conjugate symmetric, that is $x(t) = x^*(-t)$

Let $x(t) \xrightarrow{\text{F.T.}} X(\omega)$

then $x^*(-t) \xrightarrow{\text{F.T.}} X^*(\omega)$

Since $x(t) = x^*(-t) \Rightarrow X(\omega) = X^*(\omega)$

So $X(\omega)$ is real.

T7 : Solution

(a)

Given that $$x(t) = \frac{2a}{a^2+t^2}$$

Since $$e^{-a|t|} \xrightarrow{\text{F.T.}} \frac{2a}{a^2+\omega^2}$$

then (using duality) $$\frac{2a}{a^2+t^2} \xrightarrow{\text{F.T.}} 2\pi\, e^{-a|\omega|}$$

T8 : Solution

Given that $x(t) = 5\,Sa(2t)$ (where Sa is sampling functions)

Using the definition of sampling function, we get

$$x(t) = \frac{5\sin(2\pi t)}{2\pi t}$$

The Fourier transform of $x(t)$ will be $\frac{5}{2}\text{rect}\,(\omega/4\pi)$

We need to find $\int_{-\infty}^{\infty} X(\omega)dt$

Since, $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{+j\omega t}d\omega$$

$$2\pi\, x(0) = \int_{-\infty}^{\infty} X(\omega)\,d\omega$$

So, $$\int_{-\infty}^{\infty} X(\omega)d\omega = 2\pi\, x(0) = 31.4$$

T9 : Solution

(c)

Properties of distortionless system are:

- Magnitude should be constant w.r.t. frequency.
- Phase should depend linearly on frequency.

Only function given in option (c) follow the given conditions.

T10 : Solution

(b)

Given $h(t) = \cos t\, u(t)$

Then system will be causal, unstable and dynamic.

T11 : Solution

(a)

The signal $x(t) = (2 + e^{-3t})\, u(t)$ then final value i.e. $x(\infty)$ will be 2.

T12 : Solution

(d)

Differentiation $H(s) = s = j\omega$

So magnitude vary linearly w.r.t. frequency and has constant phase.

■■■■

4 Laplace Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

To determine the Laplace transform of $e^{-3t}\cos 2t$, we know that

$$\cos 2t \xrightarrow{\text{L.T.}} \frac{s}{s^2+4}$$

$$e^{-3t}\cos 2t \xrightarrow{\text{L.T.}} \frac{(s+3)}{(s+3)^2+4}$$

$$\int_0^{\tau} e^{-3\tau}\cos 2\tau\, d\tau \xrightarrow{\text{L.T.}} \frac{(s+3)}{s[(s+3)^2+4]}$$

T2 : Solution

(a)

We know that,

$$u(t) \xrightarrow{\text{L.T.}} \frac{1}{s}$$

$$e^{-t}u(t) \xrightarrow{\text{L.T.}} \frac{1}{s+1}$$

$$e^{-(t+1)}u(t+1) \xrightarrow{\text{L.T.}} \frac{e^{s}}{(s+1)}$$

$$e^{-t}u(t+1) \xrightarrow{\text{L.T.}} \frac{e^{s+1}}{(s+1)} \qquad \text{Re}\{s\} > -1$$

T3 : Solution

(a)

Given that Laplace transform of $f(t)$ is $\left(\frac{\omega}{s^2+\omega^2}\right)$ then the signal is sinusoidal and we cannot find the value of signal at $t = \infty$.

T4 : Solution

(b)

Given that
$$H(s) = \frac{1}{s^2(s-2)}, \text{ then}$$
$$h(t) = (t * e^{2t})\, u(t)$$

T5 : Solution

(d)

Given that signal is right sided and system is causal and stable then poles of the system should be in left hand side of the imaginary axis, so option (d) is correct.

T6 : Solution

(d)

Given,
$$H(s) = \frac{k(s^2+\omega_0^2)}{s^2+\left(\frac{\omega_0}{Q}\right)s+\omega_0^2}$$

So value of $H(s)$ at $s \to \infty$ is k
and value of $H(s)$ at $s \to 0$ is k.
So the filter is a band stop filter or notch filter.

■■■■

5 Discrete Time System

Detailed Explanation of Try Yourself Questions

T1 : Solution

To find the value of $\int_{-\infty}^{\infty} e^{-t}\delta(2t-1)dt \Rightarrow \frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta\left(t-\frac{1}{2}\right)dt$

$$= \frac{e^{-1/2}}{2}$$

So, value of integral is 0.303.

T2 : Solution

(a)

Given that $y(n) = \sum_{K=0}^{\infty} 3^k\, x(n-k)$ and

We know that, $y(n) = \sum_{-\infty}^{\infty} h(k)\, x(n-k)$

So comparing the above two relation, we get

$$h(n) = 3^n\, u(n)$$

T3 : Solution

(a)

Given that

$$x_1(n) \xrightarrow{\text{System}} y_1(n)$$

$$\delta(n) + \delta(n-1) + \delta(n-2) \xrightarrow{\text{System}} \cos(n)$$

and

$$x_2(n) \xrightarrow{\text{System}} y_2(n)$$

$$\delta(n-1) + \delta(n-2) \xrightarrow{\text{System}} \sin(n)$$

So impulse response can be found as follows:

Since, $x_1(n) - x_2(n) = \delta(n)$

So impulse response will be $y_1(n) - y_2(n)$

So, $h(n) = \cos(n) - \sin(n)$

T4 : Solution

Given bandwidth (B) of the filter is 3 MHz, lower cut-off frequency is 2 MHz.

So upper cut-off frequency (f_H) will be 5 MHz.

The sampling frequency (f_s) is $\frac{2f_H}{k}$ (where f_H: upper cut-off frequency; k = integer part of f_H/B)

So, $\frac{f_H}{B} = \frac{5}{3} = 1.66$, So, $k = 1.0$

So, $f_s = \frac{2f_H}{k} = 10$ MHz

■■■■

6 Z-Transform

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

Let z-transform of $x(n)$ is $X(z)$

So, $$x(n) \xrightarrow{\text{Z.T.}} X(z)$$

and $$\left(\frac{1}{2}\right)^n x(n) \xrightarrow{\text{Z.T.}} X(2z)$$

Thus, $$\left(\frac{1}{2}\right)^{n+1} x(n) \xrightarrow{\text{Z.T.}} \frac{1}{2} X(2z)$$

T2 : Solution

(d)

Given that $x(n)$ is non zero only for finite values of n, thus ROC of $X(z)$ will be entire z-plane.

T3 : Solution

(d)

Given that $$x(n) = 2^n u(n) - 4^n u(-n-1)$$

So the ROC will be $(|z| > 2) \cap (|z| < 4)$

Thus, ROC is $2 < |z| < 4$.

T4 : Solution

(a)

Given that, $$X(z) = \frac{18}{(1-2z^{-1})^2 (1+z^{-1})} = \frac{2}{(1+z^{-1})} + \frac{4}{(1-2z^{-1})} + \frac{12}{(1-2z^{-1})^2}$$

$$= 2(-1)^n u(n) + 4(2)^n u(n) + 12n(2)^n u(n)$$

T5 : Solution

(a)
Given that the system has $h(n) = 0$ for $n < 0$ and system is not necessarily stable then $H(z)$ should have degree of numerator less than degree of denominators.
Only (I) and (III) satisfy the required condition.

T6 : Solution

(d)
Given that $x(n)$ is right sided and real, $X(z)$ has two poles, two zeros at origin and one pole at $e^{j\pi/2}$, $X(1) = 1$. Since $x(n)$ is real so poles of $X(z)$ should be in conjugate pairs so other pole will be at $e^{-j\pi/2}$.

So, $$X(z) = \frac{k\,z^2}{(z - e^{-j\pi/2})\,(z - e^{+j\pi/2})} = \frac{k\,z^2}{z^2 + 1}$$

Since, $X(1) = 1$ so, $k = 2$

So, $$X(z) = \frac{2z^2}{z^2+1} \text{ and } |z| > 1$$

T7 : Solution

(a)
Signal is $a^{nT}\,u(n)$ then the z-transform will be

$$\frac{1}{1 - a^T z^{-1}} = \left(\frac{z}{z - a^T}\right)$$

T8 : Solution

(b)

Given $$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

and $$y(n) - \frac{1}{3}y(n-1) = x(n)$$

So, $$Y(z) - \frac{1}{3}Y(z)\,z^{-1} = X(z)$$

$\Rightarrow$ $$Y(z) = \frac{X(z)}{\left(1 - \frac{1}{3}z^{-1}\right)} = \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)} = -\frac{2}{\left(1 - \frac{1}{3}z^{-1}\right)} + \frac{3}{\left(1 - \frac{1}{2}z^{-1}\right)}$$

So, $$y(n) = \left[3\left(\frac{1}{2}\right)^n u(n) - 2\left(\frac{1}{3}\right)^n u(n)\right]$$

■■■■

7 Miscellaneous

Detailed Explanation of Try Yourself Questions

T1 : Solution

(c)

$\because$ $$\bar{k} = DFT\{[a\ b\ c\ d] \otimes [a\ b\ c\ d]\}$$

$$= [1\ \ 3\ \ 5\ \ 8] \cdot [1\ \ 3\ \ 5\ \ 8]$$

$$= [1\ \ 9\ \ 25\ \ 64]$$

T2 : Solution

(a)

By the definition of DFT

$$X(K) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \text{; } k = 0 \text{ to } N-1$$

$\Rightarrow$ $$X(K) = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi}{4}kn}$$

$$x[n] = \left\{ \underset{\uparrow}{\frac{1}{3}}, \frac{1}{3}, \frac{1}{3}, 0 \right\}$$

$\therefore$ $$X(K) = \frac{1}{3} + \frac{1}{3} e^{-j\frac{\pi}{2}k} + \frac{1}{3} e^{-j\pi k} + 0$$

$\therefore$ $$X[0] = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$X[1] = \frac{1}{3} + \frac{1}{3} e^{-j\frac{\pi}{2}} + \frac{1}{3} e^{-j\pi} = \frac{1}{3} - \frac{j}{3} + \frac{1}{3}(-1) = \frac{-j}{3}$$

$$X[2] = \frac{1}{3} + \frac{1}{3}e^{-j\pi} + \frac{1}{3}e^{-j2\pi} = \frac{1}{3}$$

$$X[3] = \frac{1}{3} + \frac{1}{3}e^{-j\frac{3\pi}{2}} + \frac{1}{3}e^{-j3\pi} = \frac{1}{3} + \frac{j}{3} - \frac{1}{3} = \frac{j}{3}$$

$\therefore$ $$X[k] = \left\{1, \frac{-j}{3}, \frac{1}{3}, \frac{j}{3}\right\}$$

T3 : Solution

(b)

Number of complex multiplication is

$$= \frac{N}{2}\log_2 N$$

$$= \frac{1024}{2}\log_2 1024 = 5120$$

$\therefore$ $$\text{time} = 5120 \times 10^{-6} = 5.12 \text{ msec}$$

T4 : Solution

(c)

Time shifting property of DFT is given as $x[n+M]_N = X(k)e^{\frac{j2\pi kM}{N}}$, where N is the cycle period.

T5 : Solution

$\because$ $x[n]$ is real

$\therefore$ $$X[k] = X^*[N-k]$$

$$N = 8$$

$\therefore$ $$a = X[2]$$

$$= X^*[8-2]$$

$$= X^*[6] = -2$$

T6 : Solution

(b)

Given $x(n)$ signal is real and 8-point DFT is {5, 1 – 3j, 0, 3 – 4j, 3 + 4j, γ, α, β}, we need to find α, β

We know that when signal is real then DFT is conjugate symmetric $X(k) = X(-k)^*$ and DFT will be periodic with period 8, that is $X(k) = X(8-k)$

So $$\alpha = X(2)^* = 0$$

$$\beta = X(1)^* = 1 + 3j$$

■■■■