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UPSC ESE 2019

Main Exam Detailed Solutions

Electrical Engineering

PAPER-II

EXAM DATE : 30-06-2019 | 2:00 PM to 5:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/ error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

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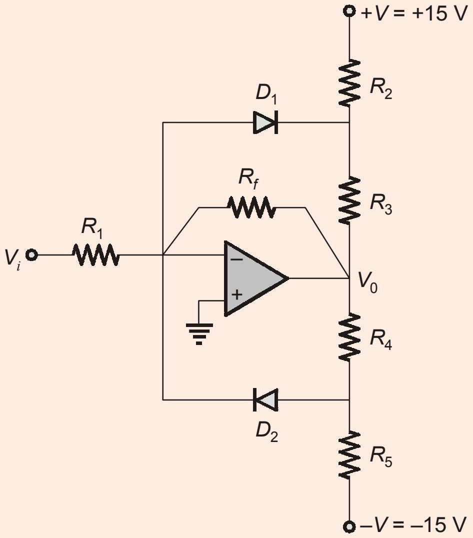
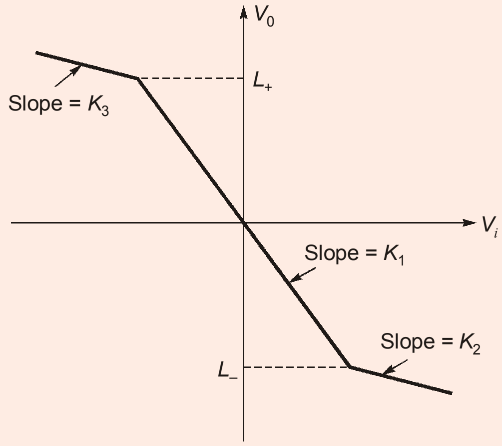
Electrical Engineering Paper Analysis
ESE 2019 Main Examination

Sl.	Subjects	Total Marks
1.	Analog Electronics	32
2.	Digital Electronics	32
3.	Systems & Signal Processing	52
4.	Control Systems	84
5.	Electrical Machines	104
6.	Power Electronics	84
7.	Power Systems	92
	Total	**480**

Scroll down for detailed solutions

1. (a) Determine the values of slope K_1, K_2, K_3 and the voltages L_+ and L_- for the amplifier and its transfer characteristics shown in the figure given below:



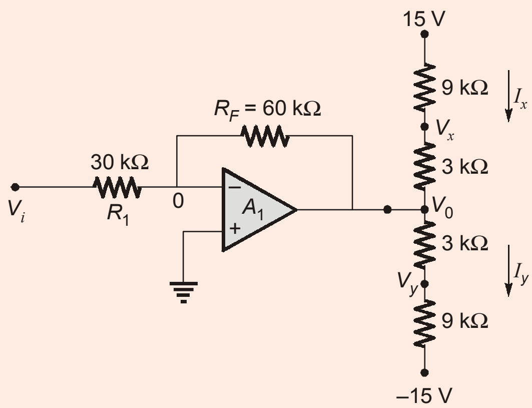


(R_1 = 30 kΩ, $R_2 = R_5$ = 9 kΩ, $R_3 = R_4$ = 3 kΩ, R_f = 60 kΩ).
The diodes may be assumed to be ideal.

[12 Marks]

Solution:

Assume D_1 and D_2 off



Op-amp A, is inverting amplifier,

$$V_0 = -\frac{R_F}{R_1}V_i = -\frac{60k}{30k}V_i = -2\ V_i$$

Case-1:

$$I_x = \frac{15 - V_0}{9k + 3k} = \frac{15 - (-2V_i)}{12k} = \frac{15 + 2V_i}{12k}$$

$$V_x = 15 - 9k \times I_x$$

$$= 15 - 9k \times \left(\frac{15 + 2V_i}{12k}\right)$$

$$V_x = 15 - \frac{3}{4}(15 + 2V_i)$$

When, $V_x < 0$ D_1 ON

Keep, $V_x = 0$ (boundary) for slope change

$$0 = 15 - \frac{3}{4}(15 + 2V_i)$$

$$15 = \frac{3}{4}(15 + 2V_i)$$

$$\frac{60}{3} = (15 + 2V_i)$$

$$2V_i = 5$$

$$V_i = 2.5 \text{ then } L = -2V_i = -5 \text{ V}$$

Similarly, $I_y = \frac{-2V_i - (-15)}{12k} = \frac{15 - 2V_i}{12k}$

$$V_y = I_y \times 9k - 15$$

$$= \frac{(15 - 2V_i)}{12k} \times 9k - 15$$

When, $V_y > 0$, D_2 ON

Keep, $V_y = 0$ (Boundary) for slope change

$$0 = (15 - 2V_i)\frac{3}{4} - 15$$

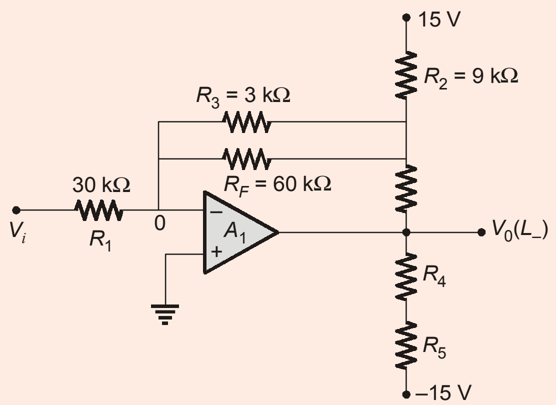
$$15 = (15 - 2V_i)\frac{3}{4}$$

$$\frac{60}{3} = (15 - 2V_i)$$

$$-2V_i = 5$$

$$V_i = -2.5 \text{ then } L_t = -2V_i = +5 \text{ V}$$

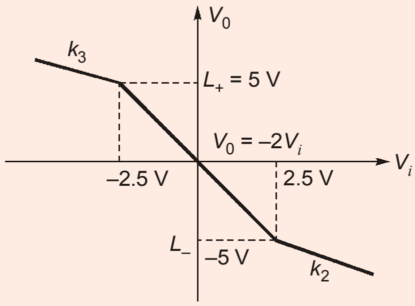
when D_1 on



$$= -\frac{60k \parallel 3k}{30k} V_i = -\frac{2}{21} V_i$$

Similarly, when D_2 on $V_0(L_+) = +\frac{60k \parallel 3k}{30k} V_i = \frac{2}{21} V_i$

$$k_3(\text{slope}) = \frac{2}{21}$$



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- **ESE 2019 Mains Test Series:** Q.6(b) (ii) of Test-13

End of Solution

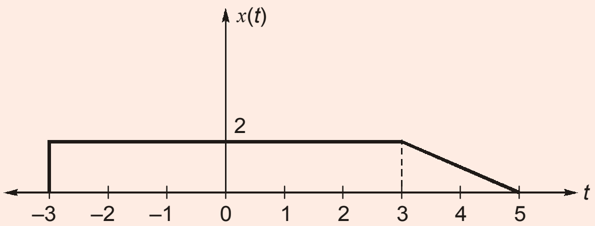
1. (b) Determine the total energy and average power of the following signal:

$$x(t) = \begin{cases} 2 & -3 \le t \le 3 \\ 5-t & 3 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

[12 Marks]

Solution:

The given signal is energy signal, so energy is finite and power should be zero.



$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-3}^{3} (2)^2 dt + \int_{3}^{5} (5-t)^2 dt$$

$$= 4(t)\Big|_{-3}^{3} + \int_{3}^{5} (25 + t^2 - 10t)\, dt$$

$$= 26.67 \text{ units}$$

End of Solution

1. (c) Show the permissible area for the poles of a second order system which must simultaneously meet the following criteria:
(i) Maximum percent overshoot ≤ 5%
(ii) Settling time for 2% criterion ≤ 500 ms

[12 Marks]

Solution:

$$M_P \le 0.05$$

$$e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 0.05$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = ln\ 0.05$$

$$\xi = 0.69$$

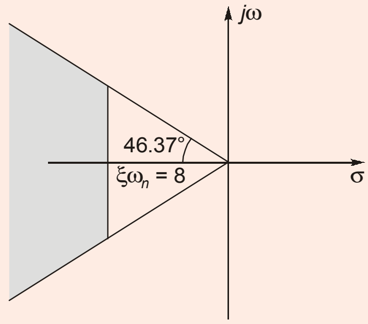
$$\Rightarrow \quad \theta = \cos^{-1}(0.69) = 46.37°$$

$$\therefore \quad \xi \ge 0.69$$

$$t_s \le 500 \text{ ms}$$

$$\frac{4}{\xi\omega_n} \le 500 \text{ ms}$$

$$\xi\,\omega_n \ge 8$$



End of Solution

1. (d) A 1000 VA, 440/220 V single-phase two-winding transformer is connected as autotransformer to supply a load at 440 V from a supply voltage of 660 V ac mains. Draw the schematic diagram of the autotransformer with proper labelling. If the full load unity power factor (pf) efficiency of the two-winding transformer is 96.2%, what will be the full load efficiency of the autotransformer at 0.85 pf lagging? Also find the maximum primary and secondary currents of the autotransformer.

[12 Marks]

Solution:

Given, 1000 VA, 440/220

$$I_1 = \frac{1000}{220} = 4.545 \text{ A}$$

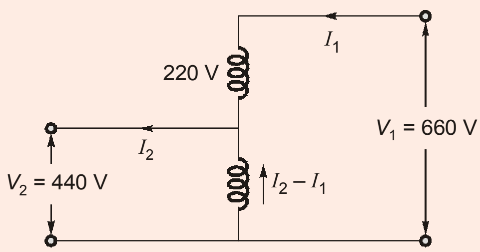
$$\Rightarrow \quad I_1 = 4.545 \text{ A}$$

$$\frac{-I_1 + I_2}{I_1} = \frac{V_1 - V_2}{V_1} = \frac{220}{440}$$

$$\Rightarrow \quad \frac{I_2}{I_1} - 1 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{I_2}{I_1} = 1 + \frac{1}{2}$$

$$\Rightarrow \quad I_2 = \frac{3}{2} I_1$$



$$\Rightarrow \qquad I_2 = \frac{3}{2} \times 4.545 = 6.8175 \text{ A}$$

As two winding transformer,

$$\eta_{TW} = \frac{1000}{1000 + P_L} = 0.962$$

$$\Rightarrow \qquad P_L = \frac{1000}{0.962} - 1000 = 39.50 \text{ W}$$

Auto transformer raitng = 3000 VA

$$\eta_{auto} = \frac{3000 \times 0.85}{3000 \times 0.85 + 39.5} = 0.9847 \text{ or } 98.47\%$$

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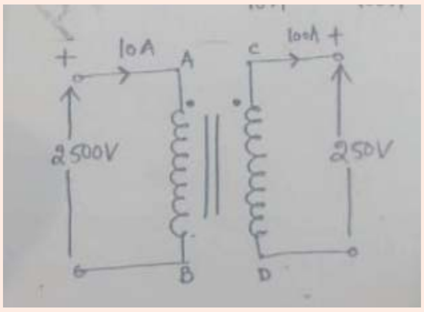
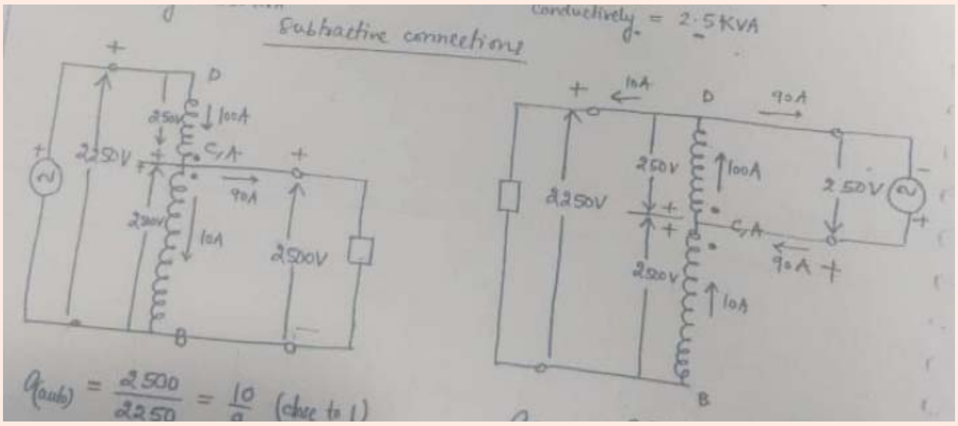
- **MADE EASY Classnotes**





- **Theory Book:** Electrical Machines Ex,. 2.19 (Page No. 51)

End of Solution

1. (e) The reverse recovery time of a diode is t_{rr} = 6 μs, and the rate of fall of the diode current di/dt = 10 A/μs. If the softness factor SF = 0.5.
   (i) Find the storage charge Q_{RR},
   (ii) Find the peak reverse current I_{RR}, and
   (iii) Draw the labelled reverse recovery characteristics.

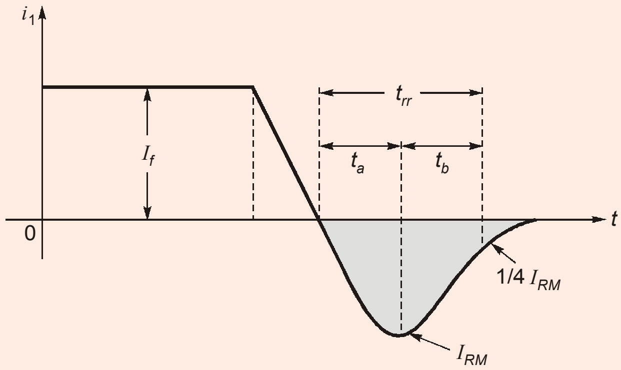
[12 Marks]

Solution:

(i)
$$SF = \frac{t_b}{t_a} = 0.5 = \frac{1}{2}$$
$$t_a = 2t_b$$
$$t_{rr} = t_a + t_b = 6$$
$$3t_b = 6$$
$$t_b = 2\ \mu s;\quad t_a = 4\ \mu s$$
$$\frac{di}{dt} = \frac{I_{RM}}{t_a}$$
$$I_{RM} = 4 \times 10 = 40\ A$$
$$Q_{RR} = \frac{1}{2} \times I_{RM} \times t_{rr} = \frac{1}{2} \times 40 \times 6 = 120\ \mu C$$

(ii)
$$\frac{di}{dt} = \frac{I_{RM}}{t_a}$$
$$I_{RM} = 4 \times 10 = 40\ A$$

(iii)



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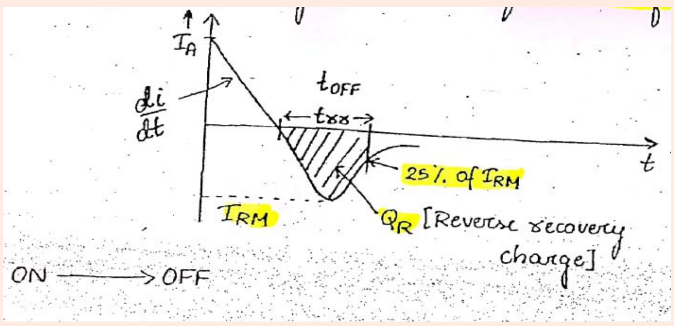
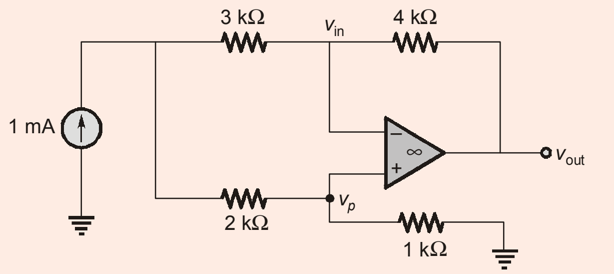
- **MADE EASY Classnotes**



- **Mains Work Book :** Q.8, Page 153

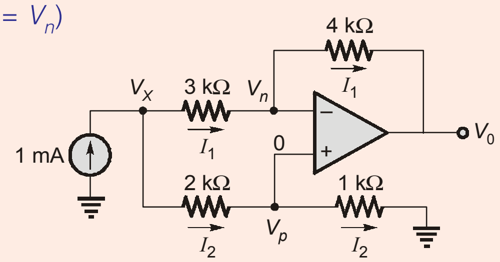
End of Solution

2. (a) Determine the value of v_p, v_n and v_{out} in the circuit given below which uses an ideal operational amplifier. Find a resistance R that, when connected in parallel with the 1 mA source, will cause v_{out} to drop to half its value when R is not present.



[20 Marks]

Solution:



$$\frac{V_X - V_n}{3\text{ k}\Omega} = I_1 \qquad (\text{where } V_P = V_n)$$

$$\frac{V_X - V_p}{2\text{k}\Omega} = I_2$$

$$I_1 \times 3\text{ k}\Omega = I_2 \times 2\text{ k}\Omega$$

$$\frac{I_1}{I_2} = \frac{2\text{ k}\Omega}{3\text{ k}\Omega} = \frac{2}{3}$$

$$I_1 + I_2 = 1\text{ mA}$$

$$I_1 = \frac{2}{5}\text{ mA}$$

and
$$I_2 = \frac{3}{5}\text{mA}$$

$$V_P = I_1 \times 1\text{ k}\Omega$$

$$= \frac{3}{5} \times 1\text{ k}\Omega = \frac{3}{5}\text{V}$$

$$V_p = V_n = \frac{3}{5}\text{V}$$

$$V_0 = V_n - 4\text{ k} \times I_1$$

$$= \frac{3}{5} - 4\text{k} \times \frac{2}{5}\text{mA} = \frac{3}{5} - \frac{8}{5} = \frac{-5}{5} = -1\text{ V}$$

$$V_0 = -1\text{ V}$$

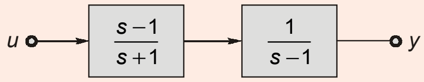
$$V_p = \frac{3}{5}\text{ V}; \qquad V_n = \frac{3}{5}\text{ V}$$

Similarly, If R is connected in parallel to 1 mA current source where

$$V_0 = \frac{-1\text{ V}}{2} = -0.5\text{ V}$$

End of Solution

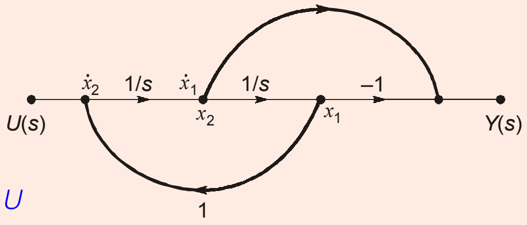
2. (b) Check the controllability and observability of the system shown in the figure given below. u is the input and y is the output.



[20 Marks]

Solution:

$$\text{T.F.} = \frac{s-1}{s^2-1}$$



State model:

Controllable form,
$$X^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = [-1 \quad 1]\ X + [0]\ U$$

$$Q_C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad |Q_C| \neq 0 \quad \therefore \text{ Controllable}$$

$$Q_0 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}; \quad |Q_0| = 0 \quad \therefore \text{ Unobservable}$$

Statement:

Observable form,
$$X^0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} X + \begin{bmatrix} -1 \\ 1 \end{bmatrix} U$$

$$Y = [0 \quad 1]\ X + [0]\ U$$

$$Q_C = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}; \quad |Q_C| = 0 \quad \therefore \text{ Uncontrollable}$$

$$Q_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad |Q_0| \neq 0 \quad \therefore \text{ Observable}$$

Conclusion: If pole-zero cancellation occurs then the system cannot be both controllable and observable.

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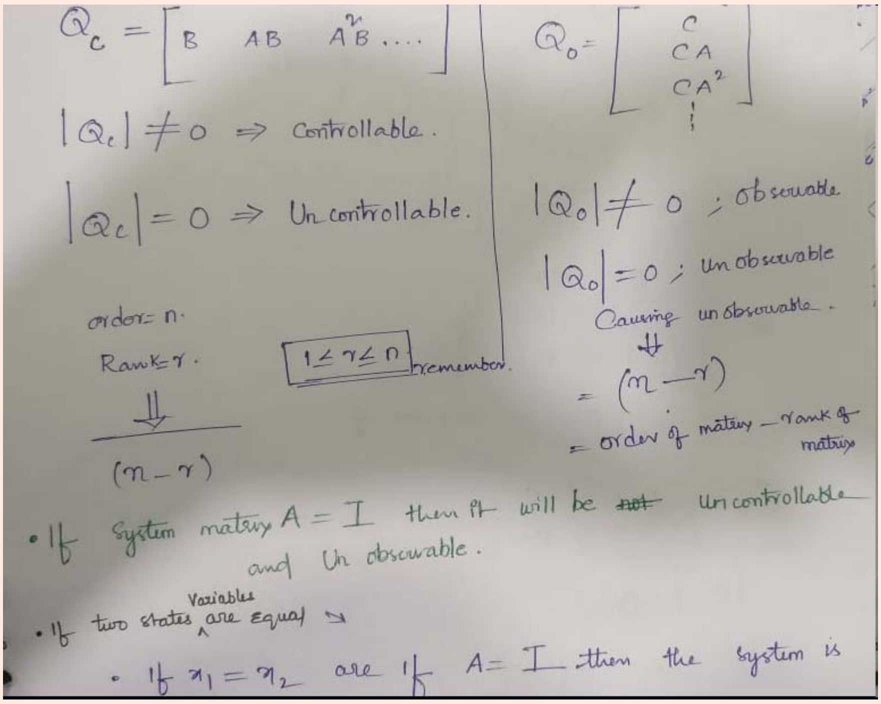
- **MADE EASY Classnotes**



$Q_c = [B \quad AB \quad A^2B \ldots]$

$|Q_c| \neq 0 \Rightarrow$ Controllable.

$|Q_c| = 0 \Rightarrow$ Uncontrollable.

order = n.

Rank = r.

$1 \le r \le n$ remember.

⇓

$(n - r)$

$Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix}$

$|Q_0| \neq 0$; observable

$|Q_0| = 0$; unobservable

Causing unobservable.

⇓

$= (n - r)$

= order of matrix − rank of matrix

• If system matrix A = I then it will be ~~not~~ uncontrollable and unobservable.

• If two states variables are equal ↘

• If $x_1 = x_2$ are if A = I then the system is

End of Solution

2. (c) A single-phase thyristor controlled bridge rectifier is supplying a dc load of 1 kW. A 1.5 kVA isolation transformer with a source side voltage rating of 120 V at 50 Hz is used. It has total leakage reactance of 8% based on its rating. The source voltage of nominally 115 V is in the range of ±10%. Assuming load current is nearly constant, find:

(i) The minimum turns ratio of the transformer, if the dc load voltage is to be regulated at constant value of 100 V,

(ii) The reduction in average load voltage due to commutation, and

(iii) The value of firing angle α when the source voltage is 115 + 10% V.

[20 Marks]

Solution:

$$P_0 = 1000 \text{ W}$$

$$Z_b = \frac{\text{kV}^2}{\text{MVA}} = \frac{120^2}{1500} = 9.6\ \Omega$$

$$X_{Ls} = 0.08 \times 9.6$$

$$\omega L_s = 0.768\ \Omega$$

$$L_s = 2.445 \text{ mH}$$

(i) Let 'a' be the turns ratio,

$$a = \frac{V_P}{V_s}$$

$$P_0 = 100 \times I_0 = 1000$$

$$I_0 = 10 \text{ A}$$

For minimum turns ratio $\alpha = 0$,

and $V_P = 115 \times 0.9 = 103.5$ V

$$V_0 = \frac{2\sqrt{2}}{\pi} V_s - \Delta V_0$$

$$V_0 = \frac{0.9 V_P}{a} - \left(\frac{4f L_s I_0}{a^2}\right)$$

$$\frac{0.9 \times 103.5}{a} - \frac{4 \times 50 \times 2.445 \times 10^{-3} \times 10}{a^2} = 100$$

$$\frac{93.15}{a} - \frac{4.89}{a^2} = 100$$

$$100a^2 - 93.15a + 4.89 = 0$$

$$a = 0.876, 0.0558$$

Possible value, $a = 0.876$

(ii)
$$\Delta V_{d0} = \frac{4fL_s I_0}{a^2}$$

$$= \frac{(4 \times 2.445 \times 10^{-3} \times 10 \times 50)}{0.876^2} = \frac{4.89}{0.876^2}$$

$$\Delta V_{d0} = 6.372 \text{ V}$$

(iii)
$$V_P = 115 \times 1.1 = 126.5 \text{ V}$$

$$V_0 = \frac{0.9}{a} V_P \cos\alpha - \Delta V_0$$

$$\frac{0.9}{0.876} \times 126.5 \cos\alpha - 6.372 = 100$$

$$\cos\alpha = 0.8154$$

$$\alpha = 35.07°$$

End of Solution

3. (a) A salient-pole synchronous motor (with negligible armature resistance and $X_d = 23.2\,\Omega$ and $X_q = 14.5\,\Omega$/phase) can support a maximum load of 563 kW without field excitation.
This motor is now excited with nominal field current and the motor is loaded with a load torque of 3.82 kN-m. If the motor draws armature current at 0.8 power factor (leading), determine excitation emf and corresponding power angle (δ).

[12 Marks]

Solution:

According to given condition there will be only reluctance power and for it to maximum $\delta = 45°$.

$$\therefore \quad \frac{563\times10^3}{3} = \frac{V^2}{2\times23.2\times14.5}(23.2-14.5)$$

On solving, $V_{ph} = 3809.57$ V

Now, $P' = T \times \omega_s$

$$= 3.82\times10^3\times\frac{2\pi\times1000}{60}$$

Note: Here any data pertaining to speed is not given but it is impossible to solve the question without speed hence assume $N_s = 1000$ rpm.

$\therefore \quad P' = 400.02$ kW

Now, $P' = \sqrt{3}VI\cos\phi$

$$\Rightarrow \quad 400.02\times10^3 = \sqrt{3}\times6.6\times1000\times I_a\times0.8$$

$$\Rightarrow \quad I_a = 43.73$$

$$\tan\Psi = \frac{V\sin\phi + I_aX_q}{V\cos\phi - I_aR_a}$$

$$\Psi = 43.77°$$

$$\therefore \quad \delta = \Psi - \phi = 43.77 - 36.86 = 6.91°$$

$$E = V\cos\delta - I_dX_d$$

$$= 3809\cos(6.91°) - 30.25(23.2)$$

$$= 3080.25 \text{ V}$$

End of Solution

3. (b) (i) Fourier transform of a periodic signal is given as,

$$X(j\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$$

Determine the fundamental angular frequency and the Fourier series coefficients. Then determine the corresponding time signal.

(ii) Determine the Laplace transform and the ROC for the signal

$$x(t) = e^{at}u(t-k)$$

[20 Marks]

Solution:

(i) Given Fourier transform of a periodic signal as,

$$X(j\omega) = j\delta\left(\omega - \frac{\pi}{3}\right) + 2\delta\left(\omega - \frac{\pi}{7}\right)$$

$$\delta(t) \xrightleftharpoons{\text{F.T.}} 1$$

$$1 \rightleftharpoons 2\pi\,\delta(\omega)$$

According to frequency shifting property,

$$e^{j\omega_0 t}\,x(t) \rightleftharpoons X(\omega-\omega_0)$$

$$e^{j\left(\frac{\pi}{3}\right)t} \rightleftharpoons 2\pi\delta\left(\omega-\frac{\pi}{3}\right)$$

$$je^{j\left(\frac{\pi}{3}\right)t} \rightleftharpoons j2\pi\delta\left(\omega-\frac{\pi}{3}\right)$$

Similarly, $$2e^{j\left(\frac{\pi}{7}\right)t} \rightleftharpoons 4\pi\delta\left(\omega-\frac{\pi}{7}\right)$$

$$x(t) \rightleftharpoons X(j\omega)$$

$$x(t) = \frac{j}{2\pi}e^{j\left(\frac{\pi}{3}\right)t}+\frac{1}{\pi}e^{j\left(\frac{\pi}{7}\right)t} \quad ...(1)$$

Fundamental angular frequency,

$$\omega_0 = GCD\left\{\frac{\pi}{3},\frac{\pi}{7}\right\} = \frac{\pi}{21}\text{ rad/s}$$

$x(t)$ can be represented as,

$$x(t) = \sum_{k=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad ...(2)$$

By comparing (1) and (2),

$$x(t) = \frac{j}{2\pi}e^{j7\omega_0 t}+\frac{1}{\pi}e^{j3\omega_0 t}$$

So Fourier series coefficients are,

$$C_3 = \frac{1}{\pi}, \qquad C_7 = \frac{j}{2\pi}$$

(ii) Given signals, $x(t) = e^{at}\,u(t-k)$

$$x(t) = e^{a(t-k+k)}\cdot u(t-k)$$

$$= e^{ak}[e^{a(t-k)}\cdot u(t-k)]$$

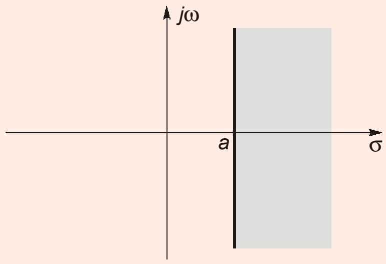
By using time shifting property,

$$x(t-t_0) = X(s)\cdot e^{-st_0} \quad \text{(No change in ROC)}$$

$$X(s) = e^{ak}\cdot\left[\frac{1}{s-a}\cdot e^{-ks}\right] = e^{k(a-s)}\cdot\frac{1}{s-a}$$

ROC of the signal is Re{s} > a

Right side of the right most pole.



End of Solution

3. (c) A Buck-Boost converter is operating at 20 kHz with Inductor L = 50 μH. The output capacitor C is sufficiently large and source voltage V_d = 15 V. The output is to be regulated at 10 V and the converter is supplying a load of 10 W. Find:

(i) The duty ratio D, and

(ii) Maximum value of Inductor current. [20 Marks]

Solution:

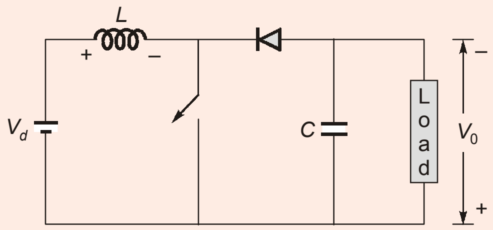
(i) Initially the mode of conduction is assume to be continuous,

$$V_0 = \frac{\alpha}{1-\alpha} V_d$$

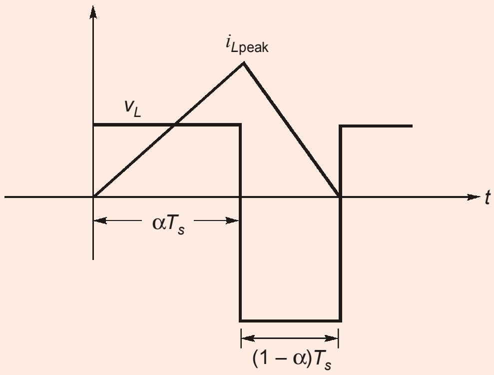
$$10 = \left(\frac{\alpha}{1-\alpha}\right) 15$$

$$\left(\frac{\alpha}{1-\alpha}\right) = 1.5$$

$$\alpha = 0.4$$



At the boundary condition,



$$I_{OB,max} = \frac{V_0}{2fL} = \frac{10}{2\times 20\times 10^3 \times 50\times 10^{-6}} = 5\text{ A}$$

$$I_{OB} = I_{OB,max}(1-\alpha)^2 = 5(1-0.4)^2 = 1.8\text{ A}$$

$$P_0 = V_0 I_0$$

$$10 = 10 \times I_0$$

$$I_0 = 1\text{ A}$$

Since the output current $I_0 = 1$ A, is less than I_{OB}.

∴ It is discontinuous mode.

$$\alpha_1 = \frac{V_0}{V_d}\sqrt{\frac{I_0}{I_{OB,max}}} = \frac{10}{15}\sqrt{\frac{1}{5}} = 0.298$$

(ii) Maximum inductor current,

$$I_{LB,max} = \frac{V_0}{2fL} = \frac{10}{2\times 20\times 10^3\times 50\times 10^{-6}} = 5 \text{ A}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(a) of Test-15

End of Solution

4. (a) The open-loop transfer function of a unity feedback system is given by

$$G(s)H(s) = \frac{K}{(s+20)(s^2-2s+1)}$$

Use Nyquist stability criteria to find the range of K for closed-loop stability.

[20 Marks]

Solution:

$$G(s)H(s) = \frac{k}{(s+20)(s-1)^2}$$

$$G(j\omega)H(j\omega) = \frac{k}{(20+j\omega)(-1+j\omega)^2}$$

$$|GH| = \frac{k}{\sqrt{400+\omega^2}\,(\omega^2+1)}$$

ω	|GH|	∠GH
0	$\frac{k}{20}$	–360°
∞	0	–270°

$$\angle GH = -360° + 2\tan^{-1}\omega - \tan^{-1}\frac{\omega}{20}$$

$$\angle GH = 90°$$

$$2\tan^{-1}\omega - \tan^{-1}\frac{\omega}{20} = 450°$$

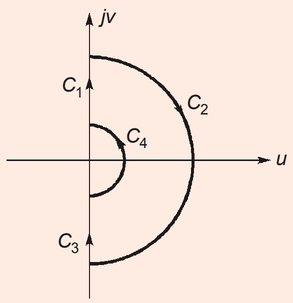
$$\tan^{-1}\left(\frac{2\omega}{1-\omega^2}\right) - \tan^{-1}\frac{\omega}{20} = 450°$$

$$\tan^{-1}\left(\frac{40\omega-\omega+\omega^3}{20-20\omega^2+2\omega^2}\right) = 450°$$

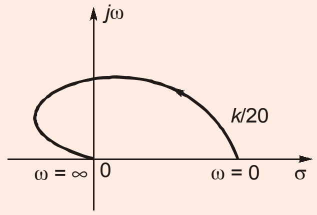
$$\frac{39\omega+\omega^3}{20-18\omega^2} = \tan 90°$$

$$20 - \omega^2 \times 18 = 0$$

$$\omega = \sqrt{\frac{20}{18}} = 1.054 \text{ rad/sec}$$



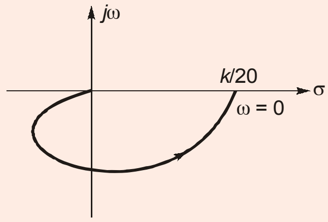
Contour:



C_1: $s = j\omega$

C_3: $s = -j\omega$

Inverse polar plot:



C_2: $S = \lim_{R\to\infty} Re^{j\theta}, \quad \theta \to \left(\frac{\pi}{2} \to \frac{\pi}{2}\right)$

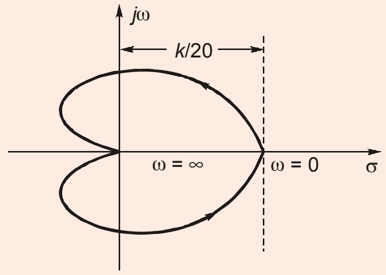
$$GH = \lim_{R\to\infty} \frac{k}{R^3 e^{j3\theta}} = \lim_{R\to\infty} \frac{k}{R^3} e^{-j3\theta}$$

$$= 0; \quad -3\theta \to \left(\frac{3\pi}{2} \to \frac{-3\pi}{2}\right)$$

C_4: $S = \lim_{R\to 0} Re^{j\theta}, \quad \theta \to \left(-\frac{\pi}{2} \to \frac{\pi}{2}\right)$

$$GH = \lim_{R\to\infty} \frac{k}{(Re^{j\theta}+20)\,(Re^{j\theta}+1)^2} = \frac{k}{20} e^{-j2\pi}$$

Nyquist plot:



$N = P - Z$

$P = 2$

$N = 2 - Z$

For any value of k there is no encirclement of $(-1 + j0)$.

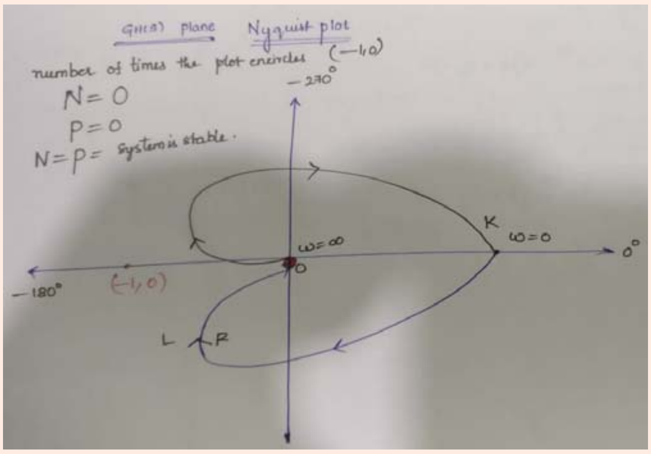
$\therefore$ $N = 0$

$Z = 2$

$\therefore$ For all values of k system in unstable.

MADE EASY Source

- **MADE EASY Classnotes**



- **Mains Work Book :** Q. 47, Page No. 70

End of Solution

4. (b) Draw and elaborate (with appropriate mathematical justification) the graphical locus of induction motor (voltage, current and power) for a complete range of slip from approximate equivalent circuit model. Justify its circular nature for naming it as circle diagram of induction motor.
Also, state and explain with the help of the circle diagram, how to obtain rotor/stator copper losses, torque and slip at any arbitrary point on circle diagram.

[20 Marks]

Solution:

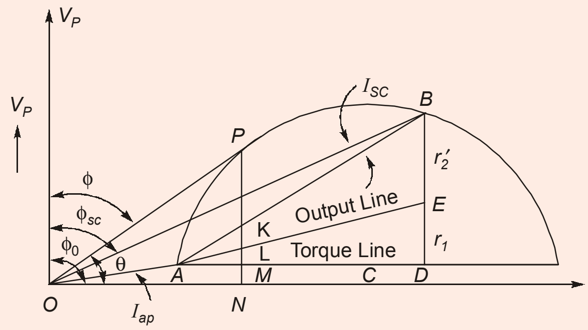
As load varies, the slip changes and hence as seen from the equivalent circuit, the winding impedance changes, viz. the reactance remaining same, the resistance varies. It can be shown that the locus of current for different loading conditions is a semicircle, the diameter of which is given by

$$\frac{V_p}{x_2} = \frac{V_p}{x_2 + x_2'}$$

Knowing the diameter line, the semicircle can be drawn locating two points on the semicircle, viz, no-load current I_{op} and blocked rotor current I_{sc} that corresponds to the rated applied voltage to the stator.

$$I_{sc} = \frac{I_p V_p}{C_{sc}}$$

Figure shows the circle diagram for a 3-phase induction motor. Draw a vertical line from the origin 'O' representing the axis of phase voltage, V_p. Draw vectors OA and OB with magnitudes I_{op} and I_{sc} lagging the phase voltage vector V_p an angle ϕ_o and ϕ_{sc}, respectively. Join AB forming chord on the semicircle. Draw the perpendicular bisector of the line AB and let it cut the horizontal line drawn from A to C. With 'C' as centre and CA as radius, draw a semicircle which forms the locus of the load current.



Circle diagram of 3 phase induction motor

Predetermination of characteristics from the circle diagram : Draw a perpendicular from '*B*' to meet the diameter at '*D*'. Divide *BD* in the following ratio,

$$\frac{BE}{ED} = \frac{\text{Rotor current loss / phase}}{\text{Stator current loss / phase}} = \frac{I_2'^2 r_2'}{I_2'^2 r_1}$$

and join *AE*.

For a given loading condition, let the input stator current per phase be '*OP*' at a power factor angle ϕ, lagging. Draw a perpendicular from '*P*' to meet the horizontal line drawn from origin '*O*' at '*N*'.

Then, the power input/phase

$$= V_P \times \text{OP} \cos\phi = V_P \times NP$$

As V_p is constant, *NP* represents the power input to certain scale (which can be calculated from the current scale).

Accordingly,the power input = NP
the power output = KP
rotor current loss = LK
stator current loss = ML
no-load losses = NM

From the above,

$$\text{the percentage efficiency} = \frac{KP}{NP} \times 100\,\%$$

$$\text{the torque} = LP \text{ synchronous watts/phase}$$

MADE EASY Source

- **Theory Book:** Electrical Machines (Page No. 241)

End of Solution

4. (c) Draw the wiring diagram showing currents for power and relaying circuit used for protecting a transformer of the rating 25 MVA, 220 Y/13.8 Δ kV, *X* = 10%. The transformer has a short-term overload capacity of 30 MVA. You are required to use CTs with common turns ratios such as 50/5 A, 100/5 A, 150/5 A, ..., 1000/5 A, 1200/5 A. If needed, auxiliary CT of adequate turns ratio may be used.

[20 Marks]

Solution:

Power transformer (over load of 30 MVA)

$$S = \sqrt{3}V_1 I_1 = \sqrt{3}V_2 I_2$$

$$30 \times 10^6 = \sqrt{3} \times 220 \times 10^3 \times I_1$$

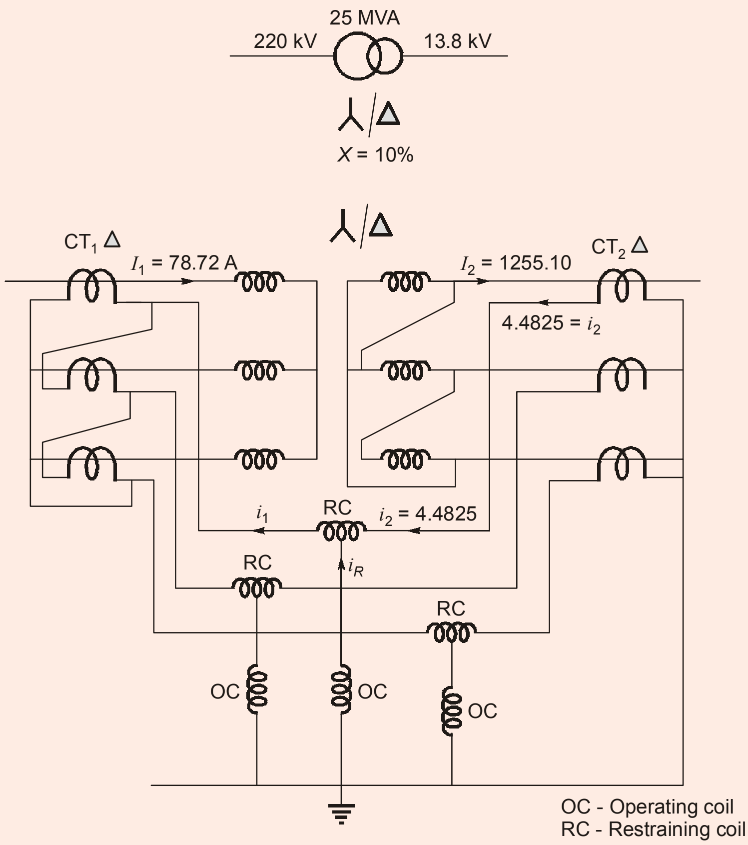
$$= \sqrt{3} \times 13.8 \times 10^3 \times I_2$$

$$I_1 = 78.72 \text{ A}, \qquad I_2 = 1255.10 \text{ A}$$

From the CT ranges given,

For CT_2 select: 1400 A/5 A

Pilot wire [Secondary of CT_2] $\Rightarrow i_2 \Rightarrow \dfrac{1255.10}{(1400/5)} = 4.4825$ A



Case-(i):

If, $\quad i_1 = 4.4825 \Rightarrow i_R = i_1 - i_2 = 0$

CT_2 secondary current $\Rightarrow \dfrac{4.4825}{\sqrt{3}} = 2.5879$ A

CT_1 primary current $= I_1 = 78.72$ A

Required CT ratio is $\dfrac{78.72}{2.5879} = \dfrac{78.72}{\left(\dfrac{4.4825}{\sqrt{3}}\right)}$ A

Case-(ii):

The above rating is not available. So we have to use 100 A/5 A CT on the primary side of power transformer.

With CT_1 100 A/5 A

$$CT_1 \text{ secondary current } = \frac{78.72}{100/5} = 3.936 \text{ A}$$

Pilot wire current, $$i_1 = 3.936 \times \sqrt{3} = 6.81735 \text{ A}$$

$$i_R = i_1 - i_2$$

$\Rightarrow$ 6.81735 – 4.4825 = 2.3348 A

For avoiding relay maloperation: % difference relay has to be used with slope of

$$k\left(\frac{i_1 + i_2}{2}\right) \geq i_R$$

$$k \geq \frac{i_R}{\left(\frac{i_1 + i_2}{2}\right)}$$

$$k \geq \frac{2.3348}{\left(\frac{6.818735 + 4.4825}{2}\right)}$$

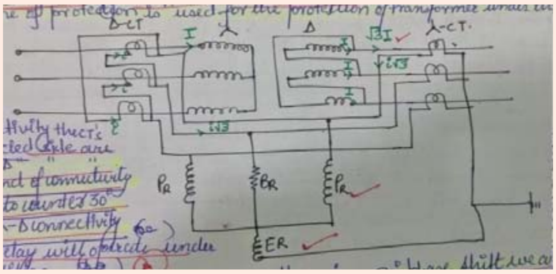
$$k \geq 0.4132$$

$$k \geq 41.32\%$$

Minimum % setting required for % differential relay.

MADE EASY Source

- **MADE EASY Classnotes**



- **Theory Book:** Electrical Machines (Page No. 305)

End of Solution

5. (a) The transfer function of a linear system is given by

$$G(s) = \frac{10}{(s+1)(s+2)}$$

The sinusoidal steady-state response of the system to an input is given by

$$1 + \sin(t - 60°) + 5\sin(2t - 45°)$$

Determine the input.

[12 Marks]

Solution:

Transfer function of system is,

$$G(s) = \frac{10}{(s+1)(s+2)}$$

$$A\sin(\omega_1 t + \phi_{input}) \rightarrow A|H(\omega_1)|\sin(\omega_1 t + \phi_{in} + \phi_H)$$

$$\phi_H = \angle H(\omega_1)$$

$$G(j\omega) = \frac{10}{(1+j\omega)(2+j\omega)}$$

At $\omega = 0$, $$G(j0) = \frac{10}{2} = 5$$

$$G_0 = 5\angle 0°$$

At $\omega = 1$ rad/s,

$$G(j1) = \frac{10}{(1+j1)(2+j1)}$$

$$G_1\angle\phi_1 = 3.16\angle -71.56°$$

At $\omega = 2$ rad/s,

$$G(j2) = \frac{10}{(1+j2)(2+j2)}$$

$$G_2\angle\phi_2 = 1.58\angle -108.43°$$

$$\text{Input} = \frac{1}{|G_0|} + \frac{1}{|G_1|}\sin(t - 60° - \phi_1) + \frac{5}{|G_2|}\sin(2t - 45° + \phi_2)$$

$$= \frac{1}{5} + \frac{1}{3.16}\sin(t - 60° + 71.56°) + \frac{5}{1.58}\sin(2t - 45° + 108.43°)$$

$$\text{Input} = 0.2 + 0.31\sin(t - 11.56°) + 3.16\sin(2t + 63.43°)$$

End of Solution

5. (b) Draw phasor diagram of an over-excited salient-pole synchronous motor having armature resistance R_a, d and q-axis reactances X_d and X_q respectively. Also prove, for lagging power factor,

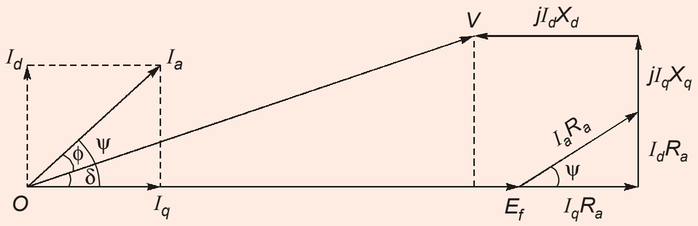
$$\tan(\phi - \delta) = \frac{V_t \sin\phi - I_a X_q}{V_t \cos\phi - I_a R_a}$$

where V_t is the terminal voltage applied to motor, ϕ being the power factor angle, δ is power angle and I_a is armature current.

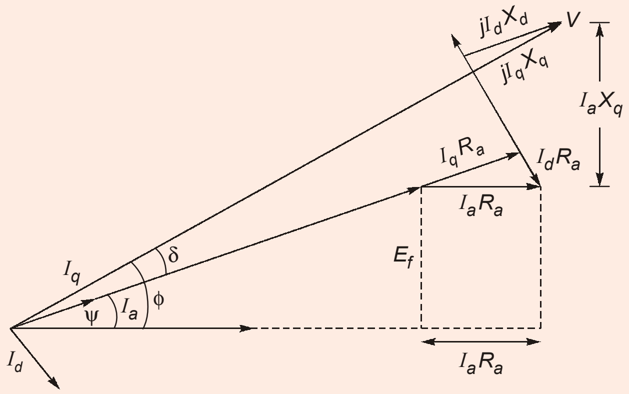
[12 Marks]

Solution:

Phasor diagram of over excited salient pole synchronous motor:



For lagging power factor:



From above figure:

$$\tan\psi = \frac{V_t \sin\phi - I_aX_q}{V_t \cos\phi - I_aR_a}$$

End of Solution

5. (c) A flyback converter has the following circuit parameters:

$$V_s = 24\text{ V}, \quad \frac{N_1}{N_2} = 3$$
$$L_m = 500\ \mu\text{H}, \quad R = 5\ \Omega$$
$$C = 200\ \mu\text{F}, \quad f = 25\text{ kHz}$$
$$V_0 = 10\text{ V}$$

Find:

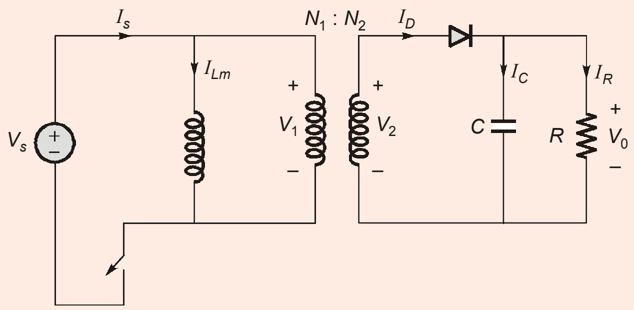
(i) The average magnetizing current and

(ii) The critical value of magnetizing inductor.

[12 Marks]

Solution:

(i)



$$V_0 = V_s\left(\frac{\alpha}{1-\alpha}\right)\frac{N_2}{N_1}$$

$$10 = 24\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{3}$$

$$\left(\frac{1}{\alpha}-1\right) = 0.8$$

$$\alpha = 0.555$$

$$V_0 I_0 = V_s I_s$$

$$I_0 = \frac{V_0}{R} = \frac{10}{5} = 2\text{ A}$$

$$I_{Lm} = \frac{I_s}{\alpha} = \frac{10\times 2}{24}\times\frac{1}{0.555} = 1.501\text{ A}$$

(ii)

$$I_{L,\min} = I_{Lm} - \frac{\Delta I_{Lm}}{2} = 0$$

$$\frac{\alpha V_s}{2fL_{mc}} = 1.501$$

$$L_{mc} = 177.48\ \mu\text{H}$$

MADE EASY Source

- **ESE 2019 Prelims Test Series:** Q.100 of Test-22

End of Solution

5. (d) A 220 kV three-phase transmission line is 90 km long. The resistance is 0.1 Ω/km and the inductance is 1.0 mH/km. Use the short transmission line model to find:
   (i) Voltage at the sending end, and
   (ii) Voltage regulation at the sending end. [12 Marks]

Solution:

Note: Data regarding load is not given hence.

Assume: P_L = 10 MW at unity power factor

Given, $V_R = 220$ kV

$$Z = [0.1 + j(2\pi \times 50 \times 1 \times 10^{-3})] \times 90$$

$$= 29.672\angle 72.34^\circ\ \Omega$$

$$I_R = \frac{10\times10^6}{\sqrt{3}\times220\times10^3} = 26.24 \text{ A}$$

$$\bar{V}_s = \bar{V}_R + \bar{I}\,Z_R$$

$$= \frac{220}{\sqrt{3}}\times10^3 + 26.24\times29.672\angle72.34^\circ$$

$$\bar{V}_s = 127.2554\angle0.334 \text{ kV phase}$$

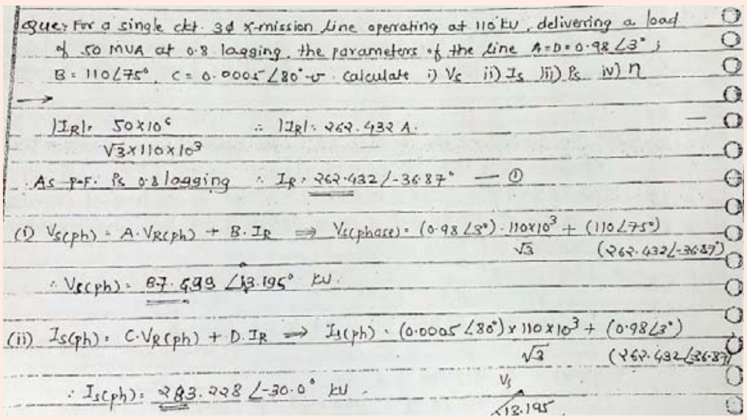
$$\left|\vec{V}_{sL}\right| = 127.255\times\sqrt{3} = 220.412 \text{ kV line}$$

(ii) Voltage regulation,

$$|VR| = \frac{|V_s|-|V_R|}{|V_R|} = \left(\frac{220.412-220}{220}\right)\times100 = 0.187\%$$

MADE EASY Source

- **MADE EASY Classnotes**



Que: For a single ckt. 3ϕ X-mission line operating at 110 kV, delivering a load of 50 MVA at 0.8 lagging. the parameters of the line A = D = 0.98∠3°; B = 110∠75°, C = 0.0005∠80° ℧. Calculate i) V_s ii) I_s iii) P_s iv) η

→

$|I_R| = \frac{50\times10^6}{\sqrt{3}\times110\times10^3}$ ∴ $|I_R| = 262.432$ A.

As p.f. is 0.8 lagging ∴ $I_R = 262.432\angle-36.87^\circ$ — ①

(i) $V_{s(ph)} = A\cdot V_{R(ph)} + B\cdot I_R$ ⟹ $V_{s(phase)} = (0.98\angle3^\circ)\cdot\frac{110\times10^3}{\sqrt{3}} + (110\angle75^\circ)(262.432\angle-36.87^\circ)$

∴ $V_{s(ph)} = 87.499\angle13.195^\circ$ kV.

(ii) $I_{s(ph)} = C\cdot V_{R(ph)} + D\cdot I_R$ ⟹ $I_{s(ph)} = (0.0005\angle80^\circ)\times\frac{110\times10^3}{\sqrt{3}} + (0.98\angle3^\circ)(262.432\angle-36.87^\circ)$

∴ $I_{s(ph)} = 283.228\angle-30.0^\circ$ kV

- **Theory Book:** Power Systems (Page No. 23)

End of Solution

5. (e) A 12 bit dual-slope ADC utilizes a 1 MHz clock and has V_{ref} = 10 V. Its analog input voltage is in the range of 0 to –10 V. Find out the time required to convert an input signal equal to the full scale value. Also find the integrator time constant if the peak voltage reached at the output of the integrator is 10 V.

[12 Marks]

Solution:

$$T_1 = 2^{N+1}\,T_c$$

Here, f_c = 1 MHz

$$T_c = \frac{1}{10^6}\text{ sec} = 10^{-6}\text{ sec}$$

Here, N = 12

∴ $T_1 = 2^{13}\times10^{-6} = 8192\times10^{-6}$

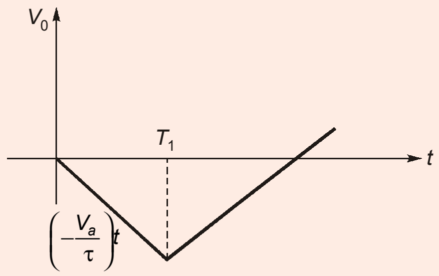
$= 0.008192$ sec

Also, $$\frac{V_a T_1}{\tau} = 10 \text{ V}$$

$$\Rightarrow \quad \frac{10 \times 2^n \times T_{clk}}{\tau} = 10$$

$$\tau = 2^{12} \times 10^{-6}$$

$$\tau = 4.096 \text{ msec}$$



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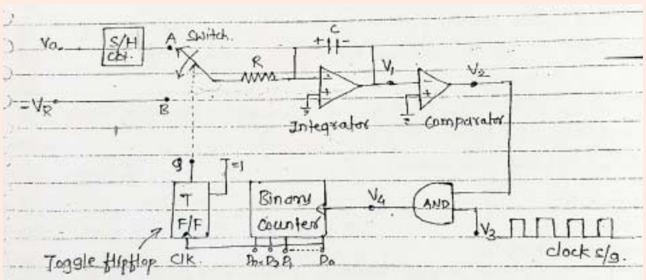
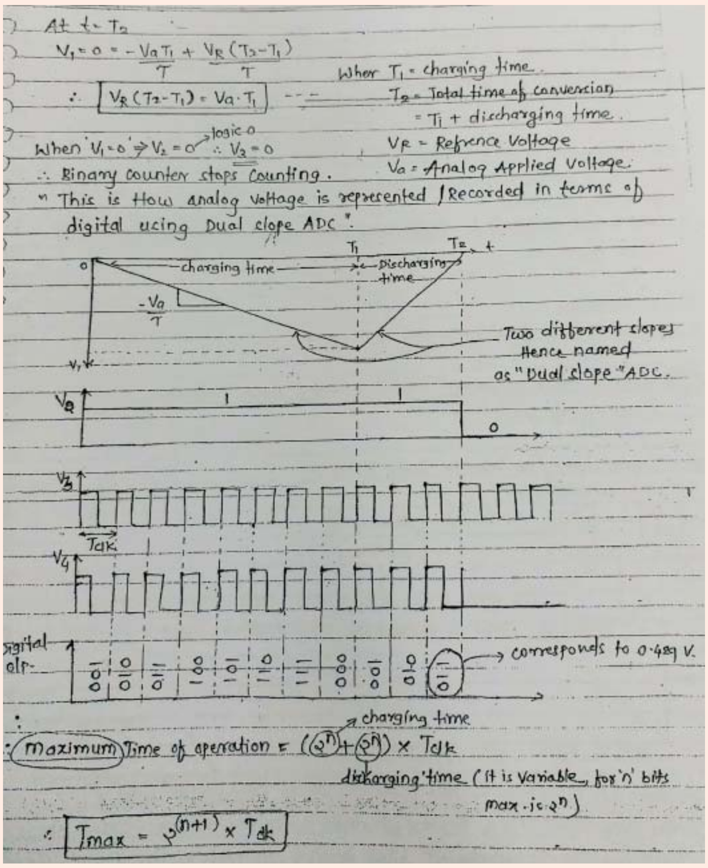
- **MADE EASY Classnotes**





- **Mains Work Book:** Q. 34, Page 98

End of Solution

6. (a) A 50 Hz, 4-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of 7.5 MJ/MVA. Find:
   (i) Rotor acceleration, if the input to the generator is suddenly raised to 400 MW for an electrical load of 350 MW.
   (ii) The speed of rotor is rpm, if the rotor acceleration calculated in part (i) is constant for a period of 10 cycles.
   (iii) The change in torque angle δ in elect. degrees.

[20 Marks]

Solution:

Given: $P = 4$, $G = 500$ MVA, 22 kV, $H = 7.5$ MJ/MVA

(i) $$P_{accelerating} = 400 - 350 = 50 \text{ MW}$$

By using swing equation,

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\alpha = \frac{P_a}{M}$$

$$M = \frac{GH}{180f} = \frac{500\times7.5}{180\times50} = 0.4166 \text{ MJ/elec.degree}$$

$$\alpha = \frac{50}{0.4166} = 120 \text{ elec.degree/s}^2$$

(ii) $$\alpha = \frac{120}{2} = \text{mechanical degree/s}^2$$

$$= 60 \text{ mech. degree/s}^2$$

$$\alpha = \frac{60\times60}{360} = 10$$

Change in speed $= \Delta N = \alpha t$

$$= 10 \times t$$

$$t = 10 \times 0.2 = 2 \text{ rpm}$$

∴ Speed at the end of 10 cycles

$$= N + \Delta N = 1500 + 2 = 1502 \text{ rpm}$$

(iii) New torque angle, $\delta = \delta_0 + \Delta\delta$

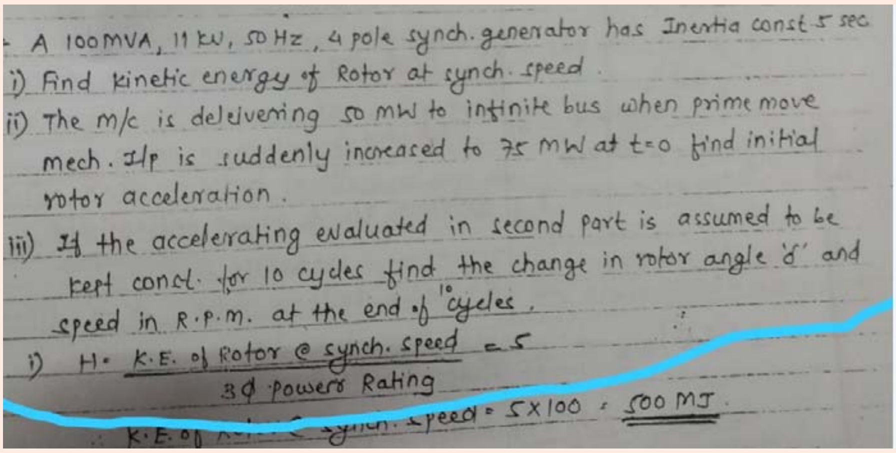
Change in torque angle,

$$\Delta\delta = \frac{1}{2}\cdot\alpha\cdot t^2 = \frac{1}{2}\times10\times(0.2)^2$$

$$= 0.2°$$

MADE EASY Source

- **MADE EASY Classnotes**



A 100MVA, 11 kV, 50 Hz, 4 pole synch. generator has Inertia const 5 sec

i) Find kinetic energy of Rotor at synch. speed.

ii) The m/c is delivering 50 MW to infinite bus when prime move mech. I/p is suddenly increased to 75 MW at t=0 find initial rotor acceleration.

iii) If the accelerating evaluated in second part is assumed to be kept const. for 10 cycles find the change in rotor angle 'δ' and speed in R.P.M. at the end of 10 cycles.

i) H = K.E. of Rotor @ synch. speed / 3φ Powers Rating = 5

K.E. of Rotor @ synch. speed = 5 × 100 = 500 MJ

- **Theory Book:** Power System (Q. 41, Page No. 199)

End of Solution

6. (b) The full bridge inverter is used to produce a 50 Hz voltage across a series RL load using Bipolar PWM. The dc input to the bridge is 200 V, the frequency modulation m_f is 21 and amplitude modulation m_a is 0.8. The load has resistance of R = 10 Ω and inductance L = 20 mH. Find:

(i) The amplitude of fundamental voltage and current and

(ii) Total harmonic distortion in load current.

Assume harmonics (> 25[th] order) are insignificant and normalized voltage is

m_a = 1		0.9	0.8	0.7	0.6	0.5
$n = 1$	1.0	0.9	0.8	0.7	0.6	0.5
$n = m_f$	0.6	0.71	0.82	0.92	1.01	1.15
$n = m_f \pm 2$	0.32	0.27	0.22	0.17	0.13	0.09

[12 Marks]

Solution:

(i)

$$V_1 = m_a V_{dc}$$

$$V_1 = 0.8 \times 200 = 160 \text{ V}$$

$$Z_1 = l + j\omega L$$

$$= 10 + g100\pi \times 20 \times 10^{-3}$$

$$= 11.81\angle 32.142° \ \Omega$$

$$I_1 = \frac{V_1}{Z_1} = \frac{160}{11.81\angle 32.142°}$$

$$= 13.548\angle -32.142°$$

(ii) For bipolar PWM,

$$f_n = (jm_f \pm k)\ f_L\ ; \qquad V_{21} = 0.82 \times 200 = 164 \text{ V}$$
$$f_n = (j21 \pm k) \times 50$$
$$f_{21} = 21 \times 50 = 1050 \text{ Hz}; \qquad V_{23} = 0.22 \times 200 = 44 \text{ V}$$
$$f_{23} = 23 \times 50 = 1150 \text{ Hz}$$
$$f_{19} = 19 \times 50 = 950 \text{ Hz}; \qquad V_{19} = 0.22 \times 200 = 44 \text{ V}$$
$$Z_n = R + j\omega_n L$$
$$= 10 + j2\pi f_n = 0.02$$
$$= (10 + 0.04\pi f_n)\ \Omega$$

n	f_n(Hz)	V_n(max)	$Z_n(\Omega)$	I_n(max) A
1	50	160	11.81∠32.14°	13.55∠–32.14°
19	950	44	119.79∠85.2°	0.3673∠–85.2°
21	1050	164	132.32∠85.66°	1.239∠–85.66°
23	1150	44	144.86∠86.04°	0.304∠–86.04°

$$\text{THD} = \frac{\sqrt{\sum_{n+1} I_n^2}}{I_L} = \frac{\sqrt{0.3673^2 + 1.239^2 + 0.304^2}}{13.55} = 0.0979$$

$$\text{THD} = 9.79\%$$

MADE EASY Source

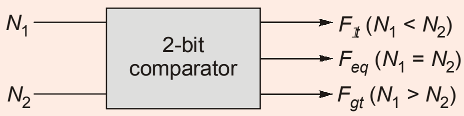
- **Main Work Book :** Q. 33, Page 144

End of Solution

6. (c) Design a circuit that takes as input two 2-bit numbers, N_1 and N_2 for comparison and generates three outputs:
$N_1 = N_2$, $N_1 < N_2$ and $N_1 > N_2$. These three binary outputs are represented by F_{eq}, F_{lt} and F_{gt} respectively. Realize the outputs in Sum of Products (SoP) form.

[20 Marks]

Solution:



N_1 and N_2 are two inputs for which the output of the comparator is specified by three binary variables. That indicate whether,

$$N_1 > N_2,\ N_1 = N_2 \text{ or } N_1 < N_2$$

If, $$N_1 > N_2$$

$$F_{gt} = \sum (m_4, m_8, m_9, m_{12}, m_{13}, m_{14})$$

$$= A_1\bar{B}_1 + (A_1 \odot B_1)\ A_0\bar{B}_0$$

If, $N_1 < N_2$

$$F_{lt} = \sum(m_1, m_2, m_3, m_6, m_7, m_{11})$$

$$= \bar{A}_1 B_1 + (A_1 \odot B_1)\,\bar{A}_0 B_0$$

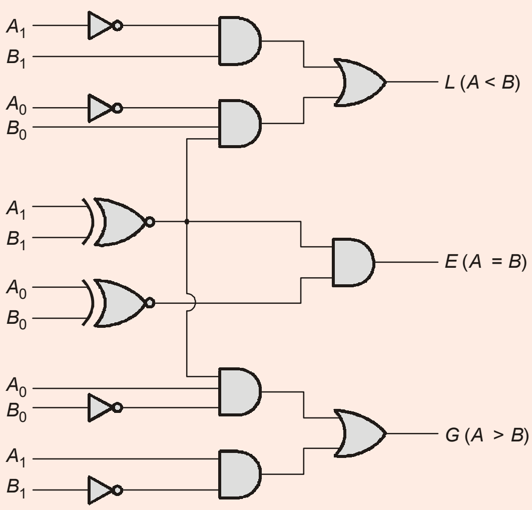
If, $N_1 = N_2$

$$F_{eq} = \sum(m_0, m_5, m_{10}, m_{15})$$

$$= (A_1 \odot B_1)\,(A_0 \odot B_0)$$

Minterm	A_1	A_0	B_1	B_0	F_{gt}	F_{eq}	F_{lt}
m_0	0	0	0	0	0	1	0
m_1	0	0	0	1	0	0	1
m_2	0	0	1	0	0	0	1
m_3	0	0	1	1	0	0	1
m_4	0	1	0	0	1	0	0
m_5	0	1	0	1	0	1	0
m_6	0	1	1	0	0	0	1
m_7	0	1	1	1	0	0	1
m_8	1	0	0	0	1	0	0
m_9	1	0	0	1	1	0	0
m_{10}	1	0	1	0	0	1	0
m_{11}	1	0	1	1	0	0	1
m_{12}	1	1	0	0	1	0	0
m_{13}	1	1	0	1	1	0	0
m_{14}	1	1	1	0	1	0	0
m_{15}	1	1	1	1	0	1	0

Logic circuit:



End of Solution

7. (a) For a causal system specified by the transfer function,

$$H(z) = \frac{z}{z-0.5}$$

Determine the zero state response to the input,

$$r(k) = (0.8)^k\, u(k) + (2)^{k+1}\, u\{-(k+1)\}$$

[20 Marks]

Solution:

Given, $H(z) = \dfrac{z}{z-0.5}$

Input given as, $r(k) = (0.8)^k\, u(k) + (2)^{k+1}\, u\{-(k+1)\}$

As we know, $Y(z) = R(z)\, H(z)$ ROC of $Y(z)$: Common ROC of $r(k)$ and $h(k)$

$$R(z) = \frac{1}{1-0.8z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$H(z) = \frac{z}{z-0.5}; \quad \text{ROC: } |z| > 0.5$$

$R(z)$ ROC: $|z| > 0.8$ and $|z| < 2$

Common ROC: $0.8 < |z| < 2$

$$Y(z) = H(z) \cdot X(z)$$

$$= \left[\frac{z}{z-0.5}\right]\left[\frac{1}{1-0.8z^{-1}} - \frac{2}{1-2z^{-1}}\right]$$

$$= \left(\frac{z}{z-0.5}\right)\left(\frac{z}{z-0.8} - \frac{2z}{z-2}\right)$$

$$Y(z) = \frac{z^2}{(z-0.5)(z-0.8)} - \frac{2z^2}{(z-0.5)(z-2)}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-0.5)(z-0.8)} - \frac{2z}{(z-0.5)(z-2)}$$

After performing partial fraction,

$$Y(z) = \frac{-\frac{5}{3}z}{z-0.5} + \frac{\frac{8}{3}z}{z-0.8} + \frac{\frac{2}{3}z}{z-0.5} - \frac{\frac{8}{3}z}{z-2}$$

$$= \frac{-z}{z-0.5} + \frac{\frac{8}{3}z}{z-0.8} - \frac{\frac{8}{3}z}{z-2}$$

$$y(k) = -(0.5)^k u(k) + \frac{8}{3}(0.8)^k u(k) + \frac{8}{3}(2)^k u(-k-1)$$

End of Solution

7. (b) An unbalanced 2-ϕ, 1000 V, 50 Hz induction motor has unequal winding impedances, $Z_a = 3 + j2.7$ and $Z_b = 7 + j3\ \Omega$. This motor is supplied by Scott-connected transformer combination from a 3-phase, 11 kV system. Calculate phase currents I_a and I_b of the motor and line currents on 3-phase supply side.

[20 Marks]

Solution:

$$\frac{N_1}{N_2} = \frac{11\times 1000}{1000} = 11$$

$$\therefore \qquad \frac{\sqrt{3}}{2}\frac{N_1}{N_2} = \frac{11\sqrt{3}}{2} = 9.526$$

Motor currents are:

$$I_a = \frac{1000}{3+j2.7} \quad \text{and} \quad I_b = \frac{1000}{7+j3}$$

$$I_a = 247.76\angle{-41.98^\circ}\text{ A}$$

$$I_b = 131.31\angle{-23.198^\circ}\text{ A}$$

Now currents on 3-phase side:

$$I_A = \frac{2}{\sqrt{3}}\left(\frac{N_2}{N_1}\right)\vec{I}_a = \frac{2}{\sqrt{3}}\times\frac{1}{11}\times 247.76\angle-41.98^\circ\text{ A}$$

$$I_A = 26.008\angle{-41.98^\circ}\text{ A}$$

$$\vec{I}_{BC} = \frac{N_2}{N_1}\vec{I}_b$$

$$\vec{I}_{BC} = \frac{131.31}{11}\angle-23.198^\circ = 11.937\angle{-23.198^\circ}\text{ A}$$

$$\vec{I}_B = \vec{I}_{BC} - \frac{\vec{I}_A}{2}$$

$$\vec{I}_C = -\vec{I}_{BC} - \frac{\vec{I}_A}{2}$$

$$\Rightarrow \qquad \vec{I}_B = 11.937\angle-23.198^\circ - \frac{26.008}{2}\angle-41.98^\circ$$

$$= 4.203\angle 71.91^\circ$$

$$\vec{I}_C = -\vec{I}_{BC} - \frac{\vec{I}_A}{2} = 24.607\angle 147.005^\circ\text{ A}$$

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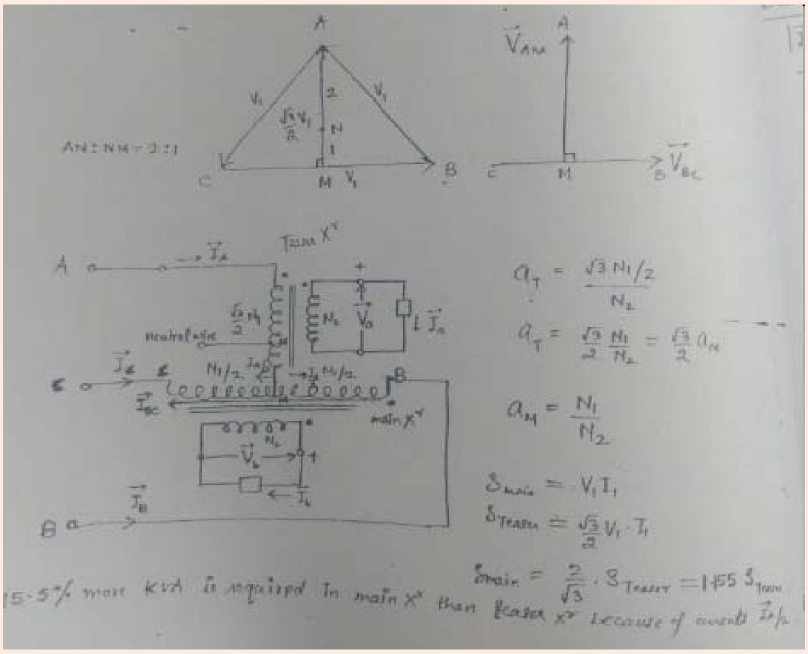
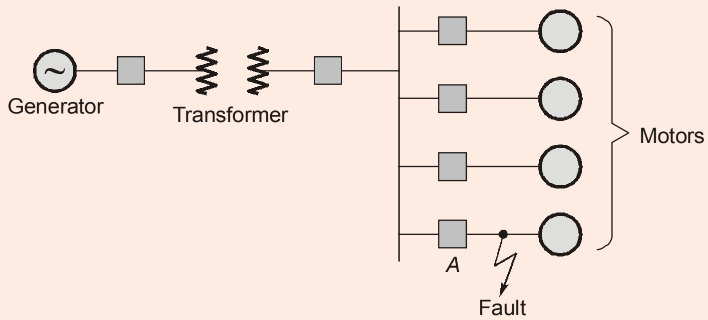


- **ESE 2019 Mains Test Series:** Q.2(a) of Test-4

End of Solution

7. (c) A 25 MVA, 13.8 kV generator with $X_d'' = 15\%$ is connected through a 25 MVA, 13.8/6.9 kV transformer with leakage reactance of 10% to a bus which supplies four identical motors as shown in the figure. The sub-transient reactance X_d'' of each motor is 20% on base of 5 MVA, 6.9 kV. Find:



(i) The sub-transient current in the fault.
(ii) The sub-transient current in breaker A.

[20 Marks]

Solution:

(i) Let, Base MVA = 25 MVA

For generator base voltage = 13.8 kV

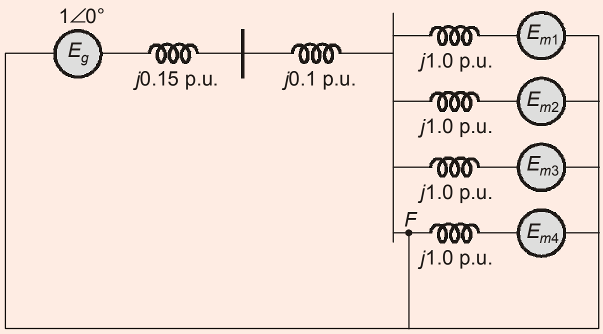
For motors base voltage = 6.9 kV

Generator G: X''_{dg} p.u. = $j0.15$ p.u.

Each motor, $X''_{dm}\text{ p.u.} = j0.2 \times \frac{25}{5} = j1.0\text{ p.u.}$

Transformer, $X_{T\text{ p.u.}}$ = j0.10 p.u.

Reactance diagram:



$$X_{eq\text{ p.u.}} = \frac{\frac{j1.0}{4} \times 0.25}{\frac{j1.0}{4} + j0.25} = j0.125\text{ p.u.}$$

The p.u. subtransient fault current,

$$I_{f\text{p.u.}} = \frac{1.0}{j0.125} = -j8\text{ p.u.}$$

Base current,

$$I_b \text{ in 6.9 kV circuit} = \frac{(\text{MVA})_b \times 10^3}{\sqrt{3} \times (\text{kV})_b} = \frac{25 \times 10^3}{\sqrt{3} \times 6.9} = 2091.84\text{ A}$$

$$I_f = I_{f\text{p.u.}} \times I_b$$
$$= 8 \times 2091.84 = 16734.79$$

(ii) For subtransient current in breaker A,

$$|I''_{CA}| = \frac{1\angle 0^\circ}{j0.25} + \frac{3 \times 1\angle 0^\circ}{j1.0} = 7.0\text{ p.u.}$$

$$|I''_{CA}| = 7 \times 2091.84 = 14642.88\text{ A}$$

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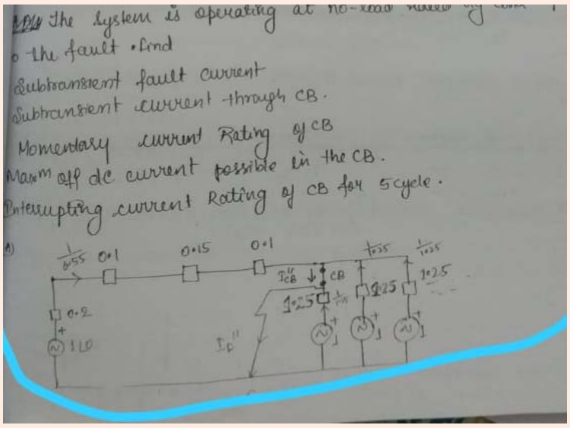
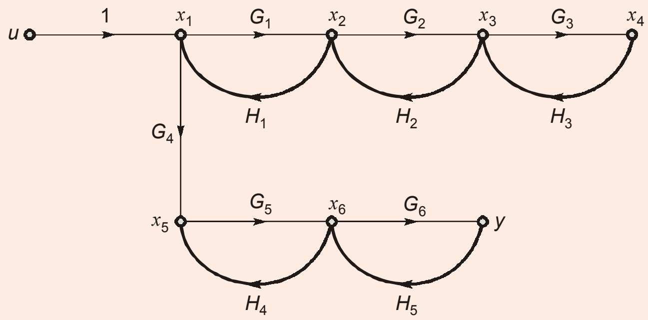


- **ESE 2019 Mains Test Series:** Q.7(b) of Test-11

End of Solution

8. (a) Find the transfer function $Y(s)/U(s)$ using Mason's Gain formula. Also find $X_5(s)/U(s)$.



[20 Marks]

Solution:

$$\frac{Y}{U} = \frac{G_4G_5G_6[1-(G_2H_2+G_3H_3)]}{\begin{aligned}&1-[G_1H_1+G_2H_2+G_3H_3+G_5H_4+G_6H_5]\\&+[G_1H_1\times G_3H_3+G_1H_1\times G_5H_4+G_1H_1\times G_6H_5+G_2H_2\times G_5H_4\\&+G_2H_2\times G_6H_5+G_3H_3\times G_5H_4+G_3H_3\times G_6H_5]\\&-[G_1H_1\times G_3H_3\times G_5H_4+G_1H_1\times G_3H_3\times G_6H_5]\end{aligned}}$$

$$\frac{X_5}{U} = \frac{G_4[1-(G_2H_2+G_3H_3+G_6H_5)+(G_2H_2\times G_6H_5)+(G_3H_3\times G_6H_5)]}{\begin{aligned}&1-[G_1H_1+G_2H_2+G_3H_3+G_5H_4+G_6H_5]\\&+[G_1H_1\times G_3H_3+G_1H_1\times G_5H_4+G_1H_1\times G_6H_5+G_2H_2\times G_5H_4\\&+G_2H_2\times G_6H_5+G_3H_3\times G_5H_4+G_3H_3\times G_6H_5]\\&-[G_1H_1\times G_3H_3\times G_5H_4+G_1H_1\times G_3H_3\times G_6H_5]\end{aligned}}$$

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End of Solution

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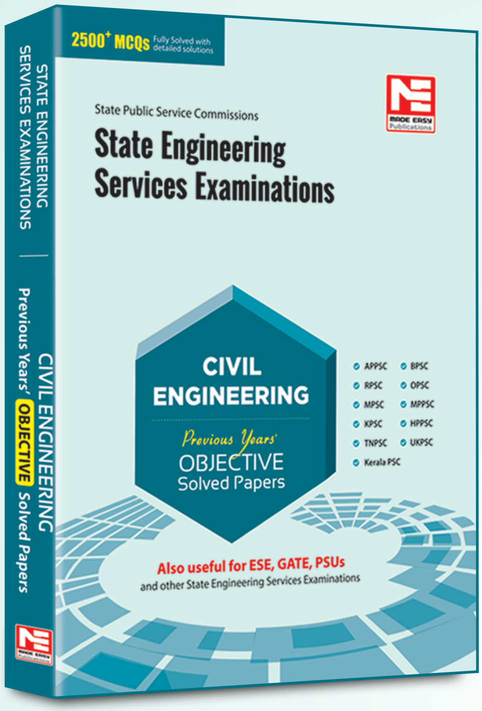

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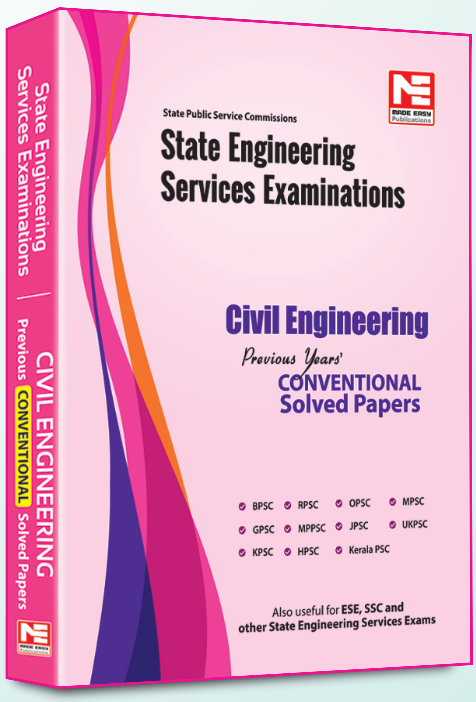

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8. (b) A 440 V, 50 HZ, 6-pole Y-connected induction motor has following parameters per phase referred to the stator:

$$R_s = R_r' = 0.3\ \Omega,\ X_s = X_r' = 1.0\ \Omega \text{ and } X_m = 40\ \Omega$$

The nominal full load slip is 0.05.
The motor is to be braked by plugging from its initial full load condition. Determine initial braking torque without braking resistor (R_B).
Also find the value of R_B so that braking current is limited to 1.5 times the full load current. What will be the corresponding breaking torque as a ratio of full load torque?
Note: Assume braking resistor R_B is connected to rotor circuit.

[20 Marks]

Solution:

(i) Given;

$$R_S = R_r' = 0.3\ \Omega$$
$$X_S = X_r' = 1.0\ \Omega$$
$$s_{fL} = 0.05$$

Full load current,

$$I_{rf'} = \frac{440/\sqrt{3}}{\sqrt{\left(0.3 + \frac{0.3}{0.05}\right)^2 + (1+1)^2}} = 38.43\ \text{A}$$

Full load torque,

$$T_{fL} = \frac{3 \times (38.43)^2 \times 0.3}{0.05 \times 104.72} = 253.85\ \text{Nm}$$

In plugging, slip, $s_b = 2 - s_f = 2 - 0.05 = 1.95$
Initial braking current,

$$I_r' = \frac{440/\sqrt{3}}{\sqrt{\left(0.3 + \frac{0.3}{1.95}\right)^2 + (1+1)^2}} = 123.86\ \text{A}$$

Initial braking torque,

$$T = \frac{3 \times (123.86)^2 \times 1}{104.72 \times 1.95} = 225.38\ \text{Nm}$$

(ii) With an external resistance R_e in rotor,

$$I_r' = \frac{V}{\sqrt{\left(R_s + \left(\frac{R_r' + R_e'}{2 - s}\right)\right)^2 + (X_s + X_r')^2}}$$

or

$$1.5 \times 38.43 = \frac{440/\sqrt{3}}{\sqrt{\left(0.3 + \left(\frac{0.3 + R_e'}{1.95}\right)\right)^2 + 4}}$$

$$\Rightarrow \left(0.3+\left(\frac{0.3+R'_e}{1.95}\right)\right)^2+4 = 19.42$$

$$\frac{0.3+R'_e}{1.95} = 3.6268$$

$$R'_e = 6.772\ \Omega$$

As stator to turns ratio is not given hence,

$$R'_e = R_e = 6.772\ \Omega$$

$$T = \frac{3\times(1.5\times 38.43)^2\times 6.772}{1.95\times 104.72} = 330.595 \text{ Nm}$$

$$\frac{T}{T_f} = \frac{330.595}{253.85} = 1.302$$

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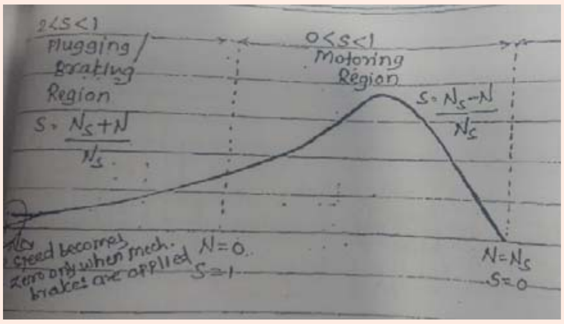
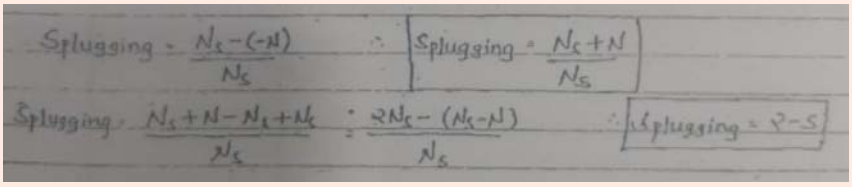




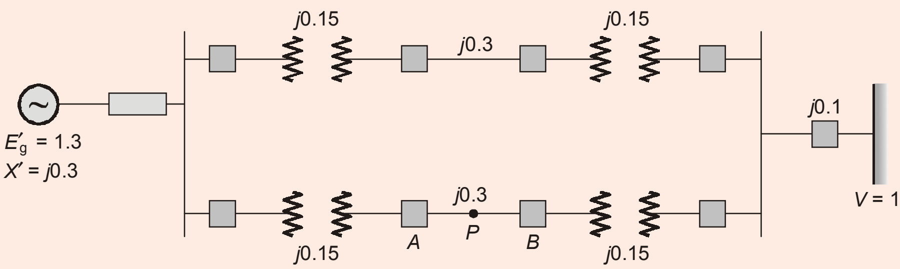
End of Solution

8. (c) A generator is connected by a double line to an infinite bus, the voltage of which is V = 1 pu as shown in the figure. Per unit values of reactances and voltages are indicated in the figure. A three-phase short-circuit occur at the point P. The circuit breakers A and B open simultaneously and remain open. The mechanical power supplied to the generator before the fault is P_m = 1 pu.

(i) Determine the electrical powers P_{e1}, P_{e2} and P_{e3} before, during and post the fault.

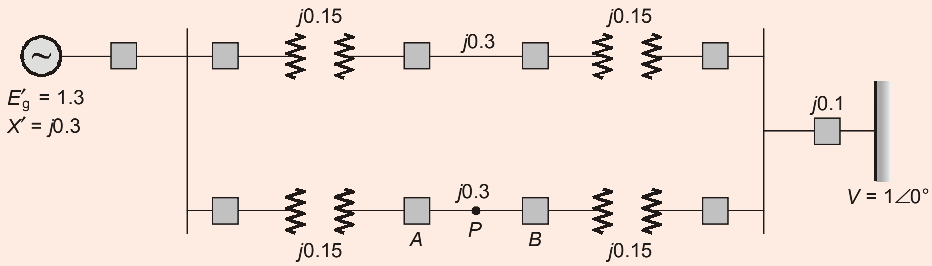
(ii) Draw on the same graph, power angle curves for P_{e1}, P_{e2} and P_{e3}.

(iii) Calculate the angles δ_0, δ_1 and δ_{max} where δ_0 is the initial power angle, δ_1 is the post fault power angle and δ_{max} is the maximum power angle.



[20 Marks]

Solution:

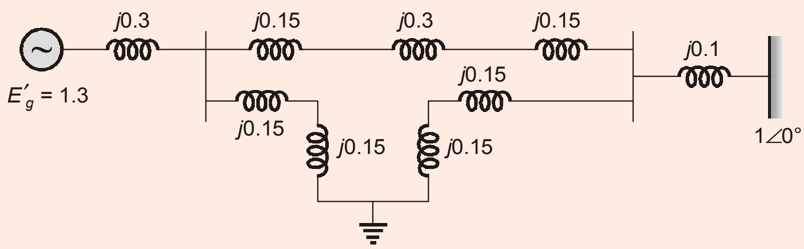


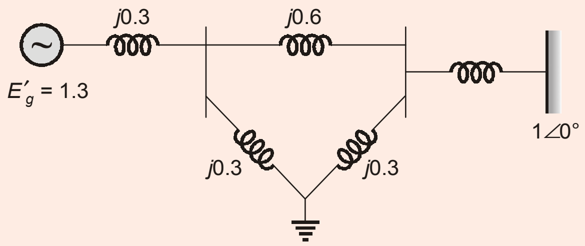
Prefault condition:

$$X = j0.3 + j0.1 + \left(\frac{j0.15 + j0.3 + j0.15}{2}\right) = j0.7\text{ p.u.}$$

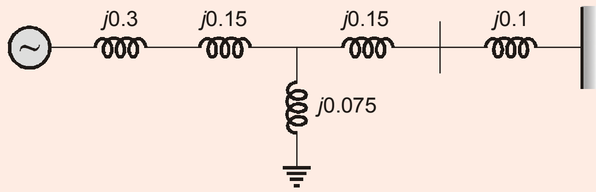
Hence, $$P_{e1\max} = \frac{1.3 \times 1}{0.7} = 1.8571\text{ p.u.}$$

During fault:





Applying Δ to ⅄ transformation,



Again ⅄ to Δ,

$$X = j0.45 + j0.25 + \frac{j0.45 \times 0.25}{j0.075} = j2.2 \text{ p.u.}$$

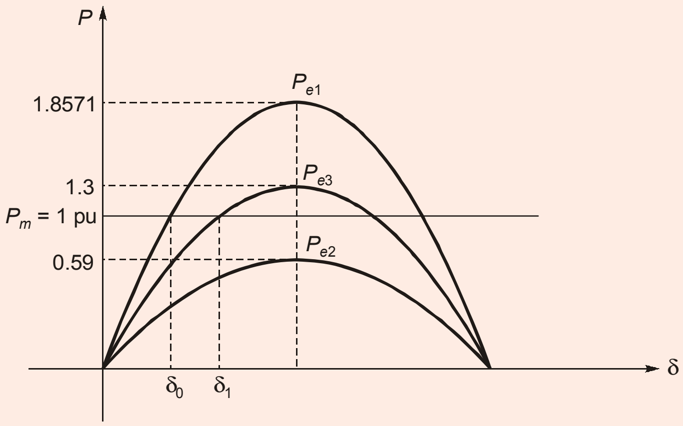
$$\therefore \quad P_{e2\max} = \frac{1.3 \times 1}{2.2} = 0.59 \text{ p.u.}$$

Post fault:

One line should be isolated,

$$X = j0.3 + j0.1 + j0.15 + j0.3 + j0.15 = j1.0 \text{ p.u.}$$

$$P_{e3\max} = \frac{1.3 \times 1}{1} = 1.3 \text{ p.u.}$$



For δ_0:

$$P_{e1} = P_m$$

$$\Rightarrow \quad 1 = 1.8571 \sin\delta_0$$

$$\Rightarrow \quad \delta_0 = \sin^{-1}\left[\frac{1}{1.8571}\right] = 32.58°$$

For δ_1:

$$1 = 1.3 \sin\delta_2$$

$$\delta_1 = \sin^{-1}\left(\frac{1}{1.3}\right) = 50.284°$$

$$\delta_{max} = 180° - 50.284° = 129.715°$$

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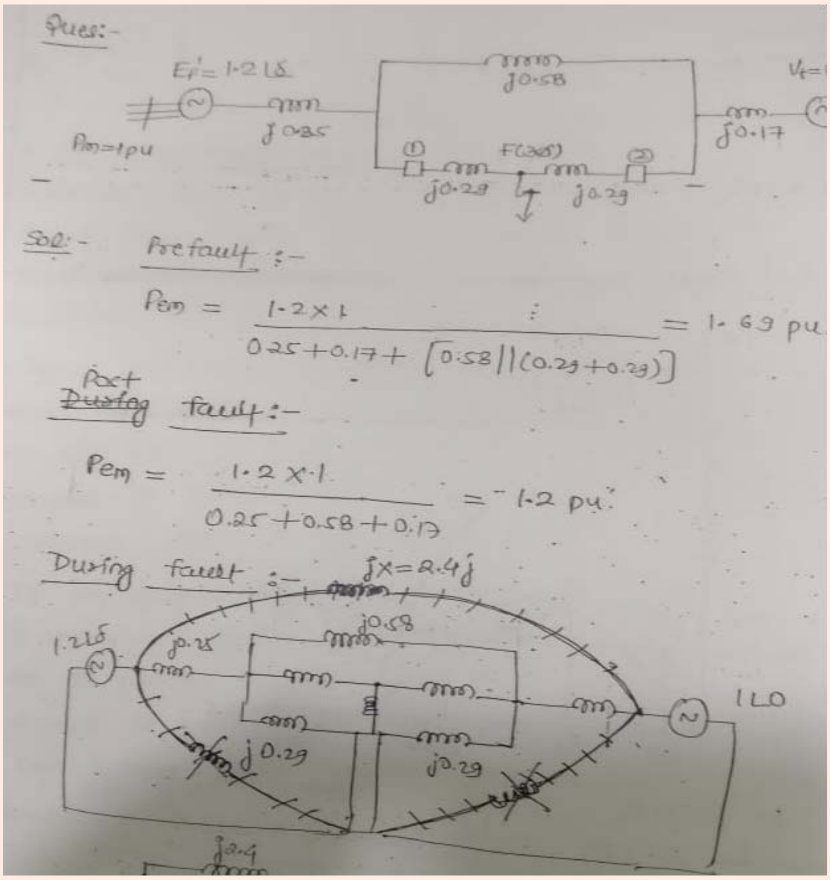

End of Solution

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