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UPSC ESE 2019

Main Exam Detailed Solutions

Electrical Engineering

PAPER-I

EXAM DATE : 30-06-2019 | 9:00 AM to 12:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/ error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

Corporate Office : 44-A/1, Kalu Sarai, Near Hauz khas metro station, New Delhi-110016

011-45124612, 9958995830 www.madeeasy.in

Electrical Engineering Paper Analysis
ESE 2019 Main Examination

Sl.	Subjects	Total Marks
1.	Electric Circuits	52
2.	Electromagnetic Fields	44
3.	Electrical Materials	72
4.	Engineering Mathematics	72
5.	Basic Electronics Engineering	84
6.	Computer Fundamental	72
7.	Electrical and Electronic Measurements	84
	Total	**480**

Scroll down for detailed solutions

1. (a) Determine the eigen vectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Show that those eigen vectors are linearly independent. [12 Marks]

Solution:

Characteristic equation of A is

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$(-2 - \lambda)\,[-\lambda + \lambda^2 - 12] - 2\,(-2\lambda - 6) - 3\,(-4 + 1 - \lambda) = 0 \qquad \text{... (i)}$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

By trial: If $\lambda = -3$, then $-27 + 9 + 63 - 45 = 0$

So, $(\lambda + 3)$ is one factor of equation (i).

The remaining factors are obtained on dividing (i) by $\lambda + 3$.

−3	1	1	−21	−45
	0	−3	6	45
	1	−2	−15	0

$$(\lambda + 3)\,(\lambda^2 - 2\lambda - 15) = 0$$

$$(\lambda + 3)\,(\lambda - 5)\,(\lambda + 3) = 0$$

$$\therefore \qquad \lambda = 5,\ -3,\ -3$$

To find the eigen vectors for corresponding eigen values, we will consider the matrix equation:

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{... (ii)}$$

On putting $\lambda = 5$ in equation (ii),

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We have, $-7x + 2y - 3z = 0$

$$2x - 4y - 6z = 0$$

$$\frac{x}{-12-12} = \frac{y}{-6-42} = \frac{z}{28-4} = k$$

$$\frac{x}{-24} = \frac{y}{-48} = \frac{z}{24} = k$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1} = k$$

$$x = k,\ y = 2k;\ z = -k$$

Hence, the eigen vector,

$$x_1 = \begin{bmatrix} k \\ 2k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Put $\lambda = -3$ in equation (ii), it becomes

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

We have,
$$x + 2y - 3z = 0$$
$$2x + 4y - 6z = 0$$
$$-x - 2y + 3z = 0$$

Here first, second and third equations are same.

Let, $x = k_1;\ y = k_2;\ z = \frac{1}{3}(k_1 + 2k_2)$

Hence, the eigen vector is $\begin{bmatrix} k_1 \\ k_2 \\ \frac{1}{3}(k_1 + 2k_2) \end{bmatrix}$

Let $k_1 = 0;\ k_2 = 3;$ Hence $X_2 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

Since the matrix is non-symmetric. The corresponding eigen vectors X_2 and X_3 must be linearly independent.

This can be done by choosing $k_1 = 3,\ k_2 = 0$ and hence $X_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$.

Hence,
$$X_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix};\qquad X_2 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix};\qquad X_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$|X_1\ X_2\ X_3| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 0 \\ -1 & 2 & 1 \end{vmatrix} = 1(3) + 3(4 + 3) = 3 + 21 = 24$$

As determinant of vectors is not equal to zero. Therefore, these eigen vectors are linearly independent.

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- **Source: Made Easy Mains Workbook, Q. 9 (Page 107)**

End of Solution

1. (b) Discuss superconductivity, superconducting materials and their applications.
[12 Marks]

Solution:

Superconductivity: A state of material in which it has zero resistivity, is called super conductivity. At the state of super conductivity material shows diamagnetism property. Mercury (Hg) was first discovered super conductor material and observed that the resistivity of mercury vanished completely below 4.2 K, the transition from normal to super conductivity occurring over a very narrow range of temperature of the order of 0.05 K. The temperature at which superconductivity appears is called the critical temperature or transition temperature (TC). Superconductors are materials which show super conductivity under certain conditions of temperature and magnetic field. The state in which the superconductor does not show superconductivity is called the normal state. Some superconducting materials are shown in table here:

Element	T_C(K)	Alloy or Compound	T_C(K)
Al	1.19	*Nb – Ti*	8.0
La	5.9	*Nb – Zr*	11.0
Nb	9.2	$Nb_3\,Sn$	18.3
Pb	7.18	$Nb_3\,Ge$	22.5
Sn	3.72	$V_3\,Si$	17.0
Hg	4.2	*Nb N*	17.3
Ta	4.48		
Tc	8.22		
V	5.13		

Applications of Superconductor: Superconductor can be used for generation of strong magnetic field. It may also be used for the design of modulators, rectifiers, used for detection of modulated high frequency waves. These materials are used in electronic switching devices called "Cryotrons". Superconducting magnets find applications in the following areas:

⇒ Magnets for nuclear fusion.
⇒ Generator and motors.
⇒ Magnetically Leviated Transportation.
⇒ Magnets for high-energy physics.
⇒ Superconducting magnets for energy storage.
⇒ Magnetic resonance imaging (MRI) and other applications (in medicine).

MADE EASY Source

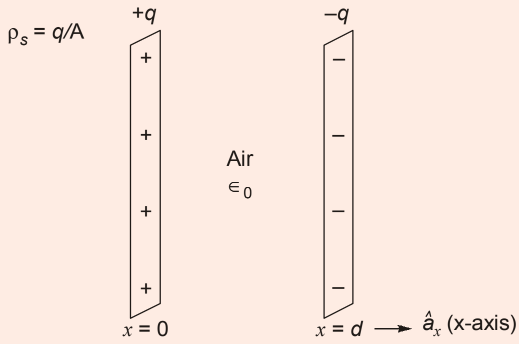
- **ESE 2019 Mains Test Series :** Q.1(e) of Test-5

End of Solution

1. (c) Find the force with which the plates of a parallel-plate capacitor attract each other. Also determine the pressure on the surface of the plate due to the field.

[12 Marks]

Solution:



$$\rho_s = \frac{q}{A} = \text{Surface charge density (C/m}^2\text{)}$$

A - Area of the plate.

Electric field intensity due to single plate having ρ_s (C/m^2) surface charge density is

$$\vec{E} = \frac{\rho_s}{2\in_0}\hat{a}_x \text{ for } x > 0 \quad \text{... (i)}$$

Force on ϕ charge due to $\vec{E}$ is $\vec{F} = Q\vec{E}$ ... (ii)

Force on $-q$ charge due to $\vec{E}$ is $\vec{F} = -q\vec{E}$ ... (iii)

Put (i) in (iii)

$$\vec{F} = -q\frac{\rho_s}{2\in_0}\hat{a}_x \quad \text{where, } \rho_s = \frac{q}{A}$$

$$= -q\left(\frac{q}{2A\in_0}\right)\hat{a}_x$$

$$\vec{F} = \frac{-q^2}{2A\in_0}\hat{a}_x \quad \text{... (iv)}$$

Force on $-q$ charge plate is towards $+q$ charge plate indicating both plates attract each other. Pressure on the surface of the plate

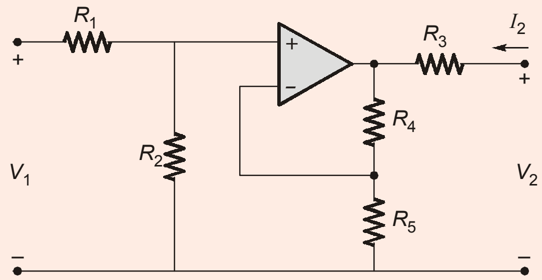
$$= P = \frac{F}{A} = \frac{q^2}{2\in_0 A^2} \text{ (N/m}^2\text{)}$$

MADE EASY Source

- **Theory Book:** EMT (Page No. 71 & 72)

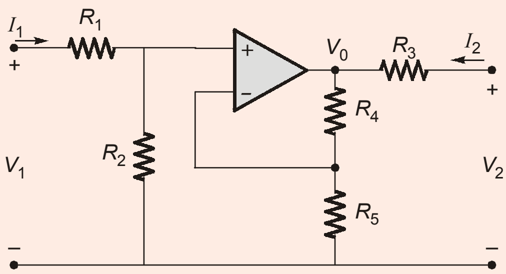
End of Solution

1. (d) Find the Z-parameter of the network shown in the figure. Is the network reciprocal? If so why? Assume the operational amplifier is ideal.



[12 Marks]

Solution:



$$V_1 = I_1(R_1 + R_2) + 0I_2$$

$$\therefore \quad z_{11} = (R_1 + R_2)$$

$$z_{12} = 0$$

$$V_0 = \left(1+\frac{R_4}{R_5}\right)\times\frac{R_2}{R_1+R_2}\times V_1$$

$$= \left(1+\frac{R_4}{R_5}\right)\times\frac{R_2}{R_1+R_2}\times(R_1+R_2)I_1$$

$$= \left(1+\frac{R_4}{R_5}\right)\times R_2 I_1 \quad \text{... (i)}$$

$$I_2 = (V_2 - V_0)\times\frac{1}{R_3}$$

$$I_2R_3 = V_2 - V_0$$

$$V_0 = V_2 - I_2R_3 \quad \text{... (ii)}$$

Put equation (ii) in equation (i)

$$V_2 - I_2R_3 = \left(1+\frac{R_4}{R_5}\right)\times R_2 I_1$$

$$V_2 = \left(1+\frac{R_4}{R_5}\right)\times R_2 I_1 + I_2R_3$$

$$z_{21} = \left(R_2 + \frac{R_2R_4}{R_5}\right)$$

and $$z_{22} = R_3$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} (R_1 + R_2) & 0 \\ \left(R_2 + \frac{R_2R_4}{R_5}\right) & R_3 \end{bmatrix}$$

For a reciprocal network,

$$z_{12} = z_{21}$$

But for above network,

$$z_{12} \neq z_{21}$$

Hence it is not reciprocal network.

End of Solution

1. (e) Volume charge density is the same as the divergence of the electric flux density. Using Gauss's law, derive equations to prove it.

[12 Marks]

Solution:

Gauss law states that the total electric flux crossing any closed surface is equal to the total charge enclosed by that closed surface.

ψ crossing closed surface $= Q_{enclosed}$ ... (i)

given $\vec{D}$ electric flux density then electric flux crossing closed surface is

ψ crossing closed surface $= \oiint \vec{D}\cdot d\vec{S}$ ... (ii)

Put (ii) in (i)

$$\Rightarrow \quad \oiint \vec{D}\cdot d\vec{S} = Q_{enclosed}. \quad \text{... (iii)}$$

Inside a closed surface volume is present. Total charge in a volume inside closed surface is

$$Q_{enc} = \iiint \rho_v dv \quad \text{... (iv)}$$

Put (iv) in (iii)

$$\oiint \vec{D}\cdot d\vec{S} = \iiint \rho_v dv \quad \text{... (v)}$$

from divergence theorem $\oiint \vec{D}\cdot d\vec{S} = \iiint \left(\vec{\nabla}\cdot\vec{D}\right) dv$ ... (vi)

Put (vi) in (v),

$$\iiint (\vec{\nabla}\cdot\vec{D})\, dv = \iiint \rho_v dv$$

Compare both sides,

$$\vec{\nabla}\cdot\vec{D} = \rho_v \quad \text{... (viii)}$$

equation (viii) says divergence of electric flux density is equal to volume charge density.

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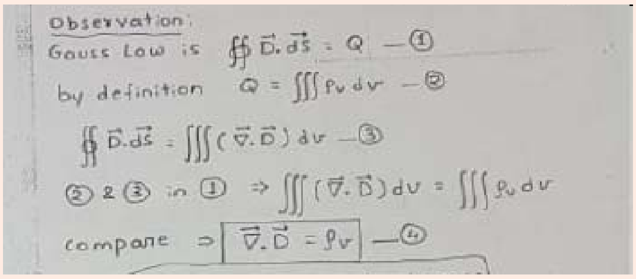
- **MADE EASY Classnotes**



- **Theory Book:** EMT (Page No. 33)

End of Solution

2. (a) Find the difference between the values of $\int_c \phi\, d\vec{r}, \phi(x,y) = x^2y + 2y$ from (1, 1, 0) to (2, 4, 0) along the curve $y = x^2$, $z = 0$ and along the straight line joining (1, 1, 0) and (2, 4, 0). Hence evaluate $\int_c (\nabla\cdot\vec{f})\, d\vec{r}$, where $\vec{f} = \frac{1}{4}x^4y\hat{i} + y^2\hat{j} + xy\hat{k}$ along the curve which is a parabola $y = x^2$, $z = 0$ from (1, 1, 0) to (2, 4, 0)

[20 Marks]

Solution:

Given that, $\phi(x, y) = x^3y + 2y$

Along the curve $y = x^2$:

$$= \int_{(1,1,0)}^{(2,4,0)} (x^3y + 2y)(dx\hat{i} + dy\hat{j})$$

As, $y = x^2$, $dy = 2xdx$

$$= \int_1^2 \left(x^3(x^2) + 2(x^2)\right)(dx\hat{i} + 2xdx\hat{j})$$

$$= \int_1^2 (x^5 + 2x^2)(dx\hat{i} + 2xdx\hat{j})$$

$$= \int_1^2 (x^5 + 2x^2)dx\hat{i} + (2x^6 + 4x^3)\hat{j}$$

$$= \left(\frac{x^6}{6} + \frac{2x^3}{3}\right)_1^2 \hat{i} + \left(\frac{2x^7}{7} + x^4\right)_1^2 \hat{j}$$

$$= \left[\frac{16}{6} + \frac{16}{3} - \frac{1}{6} - \frac{2}{3}\right]\hat{i} + \left[\frac{256}{7} + 16 - \frac{2}{7} - 1\right]\hat{j}$$

$$= 15.16\hat{i} + 51.28\hat{j}$$

Along the straight line from (1, 1, 0) to (2, 4, 0)

$$\text{Slope of line} = \frac{4-1}{2-1} = \frac{3}{1} = 3$$

$$(y-1) = 3(x-1)$$

$$y - 1 = 3x - 3$$

$$y = 3x - 2$$

$$\therefore \quad dy = 3dx$$

$$= \int_{(1,1,0)}^{(2,4,0)} \left(x^3 y \times 2y\right) \times \left(dx\hat{i} + dy\hat{j}\right)$$

$$= \int_1^2 \left[x^3(3x-2) + 2(3x-2)\right]\left[dx\hat{i} + 3dx\hat{j}\right]$$

$$= \int_1^2 \left[3x^4 - 2x^3 + 6x - 4\right]dx\left[\hat{i} + 3\hat{j}\right]$$

$$= \int_1^2 \left[\frac{3x^5}{5} - \frac{2x^4}{4} + \frac{6x^2}{2} - 4x\right]_1^2 \left[\hat{i} + 3\hat{j}\right]$$

$$= \left[\left(\frac{3\times 32}{5} - \frac{32}{4} + \frac{6\times 4}{2} - 8\right) - \left(\frac{3}{5} - \frac{2}{4} + \frac{6}{2} - 4\right)\right]\left[\hat{i} + 3\hat{j}\right]$$

$$= \left[\left(\frac{96}{5} - \frac{32}{4} + 12 - 8\right) - \left(\frac{3}{5} - \frac{2}{4} - 1\right)\right]\left[\hat{i} + 3\hat{j}\right]$$

$$= 16.1\hat{i} + 48.3\hat{j}$$

The difference between the values

$$= \left|15.61\hat{i} + 51.28\hat{j}\right| - \left|16.1\hat{i} + 48.3\hat{j}\right|$$

$$= 53.60 - 50.91 = 2.69$$

$$\int\left(\nabla \cdot \vec{f}\right)d\vec{r} = \int\left(x^3 y + 2y\right) \times \left(dx\hat{i} + dy\hat{j}\right)$$

Along the curve $y = x^2$ from (1, 1, 0) to (2, 4, 0)

$$= 15.16\hat{i} + 51.28\hat{j}$$

End of Solution

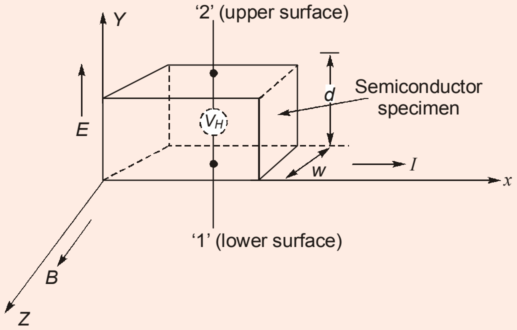
2. (b) (i) State Hall effect and discuss the applications of Hall effect.

(ii) A flat silver strip of width 1.5 cm and thickness 1.5 mm carries a current of 150 A. A magnetic field of 2.0 Tesla is applied perpendicular to the flat face of the strip. The emf developed across the width of the strip is measured to be 17.9 μV (Hall effect). Estimated the number density of the free electrons in the metal.

[12 + 8 = 20 Marks]

Solution:

(i) **Hall Effect:** The **Hall effect** has played a decisive role in revealing the mechanism of conduction in semiconductors. "If a specimen (metal or semiconductor) carrying a current '*I*' is placed in a **transverse magnetic field 'B'**, an electric field '*E*' is induced in the direction perpendicular to both *I* and *B*. This phenomenon is known as the **"Hall effect".** In figure *I* is in (+)ve X-direction and *B* is in (+)ve Z-direction; a force will be exerted in the (–)ve Y-direction on the charge carriers.



The current (I) may be due to holes (if specimen is p-type) moving from left to right or to free electrons (if specimen is *n*-type) moving from right to left in the specimen. Hence independently of whether the carriers are holes or electrons, they will be forced downward **toward side '1'.**

If the semiconductor is **n-type material**, then all free electrons will accumulate on **side '1'** and so the lower surface becomes (–)vely charged with respect to **side '2'**. Hence a potential, called the **Hall voltage** (V_H) appears between surfaces '1' and '2'. If the polarity of **"V_H"** is (+)ve at terminal '2', then as explained above, the carrier must be electrons and so the specimen is **n-type.**

On the other hand, if terminal '1' becomes charged positively with respect to terminal '2', the semiconductor bar must be **p-type.**

Applications of Hall Effect:

It is used to determine whether a semiconductor is **n-type** or **p-type.**

It is also used to determine:

- the carrier concentration.
- the mobility.
- the drift velocity.
- the conductivity of specimen.
- the field 'B'.

It is also used in manufacturing of **magnetic-flux meter** and **Hall-effect multiplier.**

(ii)
$$V_H = \frac{IB}{nte}$$

I = current across the conductor length in ampere

V_H = Hall voltage in volts

B = Magnetic field in Tesla

n = charge carrier density of the carrier electrons per m^3

t = Thickness of the conductor in metre

e = 1.6×10^{-19} C

$$V_H = \frac{IB}{nte}$$

$$17.9 \times 10^{-6} = \frac{150 \times 2}{n \times 1.5 \times 10^{-3} \times 1.6 \times 10^{-19}}$$

$$n = 6.983 \times 10^{28}/\text{m}^3$$

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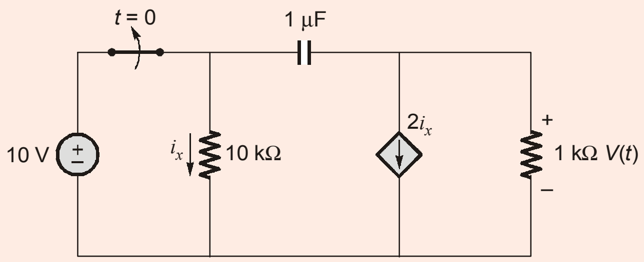
- **ESE 2019 Mains Test Series:** Q.4(a) of Test-12

End of Solution

2. (c) For the circuit shown in figure



(i) Find the expression of $V(t)$, the voltage across 1 kΩ resistor when the switch is opened at time, $t = 0$.

(ii) Sketch $V(t)$ with respect to time (t) and marks the time constant τ.

[20 Marks]

Solution:

For $t < 0$,

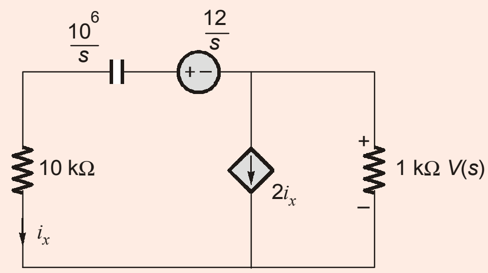
Initially switch is closed,

$$V_C(0^-) = 10000\, i_x + 1000\,(2i_x)$$

$$= 12000 i_x$$

$$= 12000 \times \frac{10}{10000} = 12 \text{ V}$$

Converting circuit in laplace domain,



$$3i_x = -V(t)$$

$$10ki_x + 3ki_x + \frac{10^6}{s} i_x = \frac{12}{s}$$

$$s(13 \times 10^3\, i_x) + 10^6\, i_x = 12$$

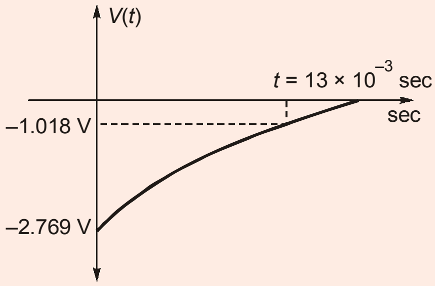
$$i_x = \frac{12}{13\times10^6 s+10^6} = \frac{\frac{12}{13}\times10^{-3}}{s+\frac{10^3}{13}}$$

$$i_x(t) = 0.923 \times 10^{-3}\ e^{-76.92}$$

$$V(t) = -3i_x(t)$$

$$= -2.769\ e^{-76.92t}\ \text{V}$$

At $t = 0$, $\quad V(0) = -2.769$



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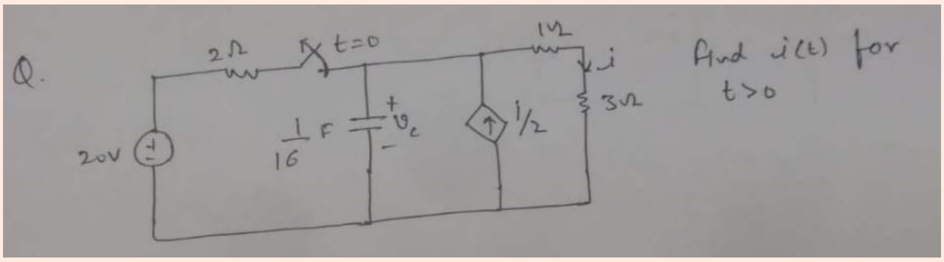


End of Solution

3. (a) A string of flexible wire stretched on a sitar has its ends fixed at $x = 0$ and $x = 20$. Initially at $t = 0$, the string is at rest and takes the shape as defined by $h(x) = \mu(20x - x^2)$, μ being a constant, and then it is released to vibrate. Formulate this boundary value problem and solve that to find the displacement at any point x at an instant t. The solution, to be obtained, should involve definite constants not the arbitrary ones.

[20 Marks]

Solution:

The above equation is $\dfrac{\partial^2 u}{\partial x^2} = a^2 \dfrac{\partial^2 u}{\partial t^2}$ ... (i)

The boundary conditions are:

(i) $u(0, t) = 0$

(ii) $u(l, t) = 0$

(iii) $\dfrac{\partial u}{\partial t}(x, 0) = 0$

After applying the above boundary conditions in the solution of equation we get,

$$u(x,\ t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi a t}{l}\right) \quad \text{... (ii)}$$

Now, $$u(x,\ 0) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{... (iii)}$$

Given that, $$u(x,\ 0) = \mu(20x - x^2) \quad \text{... (iv)}$$

From (iii) and (iv),

$$\mu(20x - x^2) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{l}\right)$$

which is in the form of half-range Fourier sine series.

$$B_n = \frac{2}{l}\int_0^l f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$= \frac{2}{l}\int_0^l \mu\left(20x - x^2\right)\cdot\sin\left(\frac{n\pi x}{l}\right)\cdot dx$$

$$B_n = \frac{2\mu}{l}\left[-\frac{l\left(20x - x^2\right)}{n\pi}\cos\left(\frac{n\pi x}{l}\right) - \frac{l^2}{n^2\pi^2}(20 - 2x)\sin\frac{n\pi x}{l} + \frac{2l^3}{n^3\pi^3}\cos\left(\frac{n\pi x}{l}\right)\right]_0^l$$

$$B_n = \frac{2\mu}{l}\left[-\frac{l\left(20l - l^2\right)}{n\pi}(-1)^n + \frac{2l^3}{n^3\pi^3}(-1)^n - \left(0 + 0 + \frac{2l^3}{n^3\pi^3}\right)\right]$$

Given that $l = 20$

$$= \frac{2\mu}{20}\left[\frac{2(20)^3}{n^3\pi^3}(-1)^n - \frac{2(20)^3}{n^3\pi^3}\right]$$

$$= \frac{2\mu}{20}\left[\frac{2(20)^3}{n^3\pi^3}\left\{(-1)^n - 1\right\}\right] = \frac{1600\mu}{n^3\pi^3}\left\{(-1)^n - 1\right\}$$

$$B_n = \begin{cases} \dfrac{-3200\mu}{n^3\pi^3}. & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases}$$

Substituting the value of B_n is equation (ii), we get

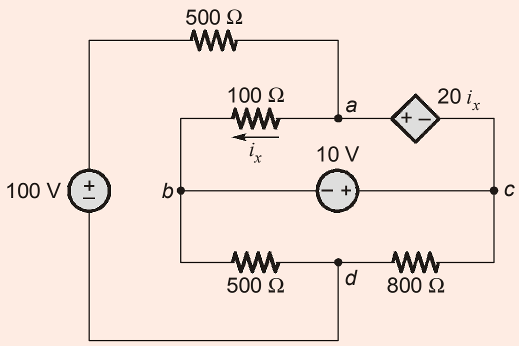
$$u(x,\ t) = \sum_{n=1,3,5}^{\infty} -\frac{3200\mu}{n^3\pi^3}\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi a t}{l}\right)$$

Since $l = 20$,

$$u(x,\ t) = \sum_{n=1,3,5}^{\infty} -\frac{3200\mu}{n^3\pi^3}\sin\left(\frac{n\pi x}{20}\right)\cos\left(\frac{n\pi a t}{20}\right)$$

End of Solution

3. (b) For the circuit shown in the figure,

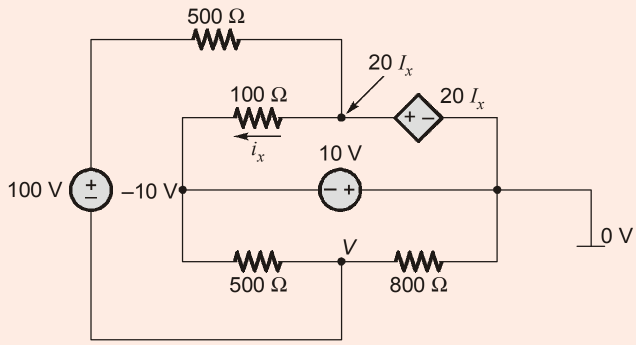


(i) Find the node voltages.

(ii) Power absorbed by the 800 Ω resistor.

[20 Marks]

Solution:



$$\frac{V+100-20I_x}{500}+\frac{V+10}{500}+\frac{V}{800}=0$$

$$8V + 800 - 160I_x + 8V + 80 + 5V = 0$$

$$21\ V - 160I_x = -880 \quad \text{... (i)}$$

Write KCL equation for I_x

$$\frac{20I_x+10}{100} = I_x$$

$$I_x = \frac{1}{8}\ \text{A} \quad \text{... (ii)}$$

Substituting (ii) in (i),

$$21V-160\left(\frac{1}{8}\right) = -880$$

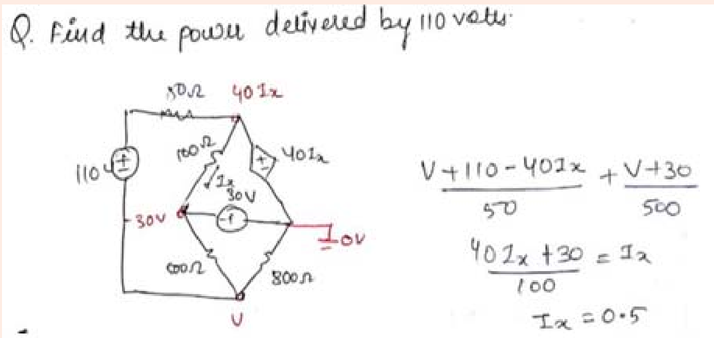
$$V = -40.95\ \text{V}$$

Node voltages: –10 V, –40.95 V, $20\left(\frac{1}{8}\right) \Rightarrow 2.5\ \text{V}$

power consumed by 800 Ω $\Rightarrow \frac{V^2}{R} \Rightarrow \frac{(-40.95)^2}{800} \Rightarrow 2.096\ \text{W}$

MADE EASY Source

- **MADE EASY Classnotes**



End of Solution

3. (c) List properties of ceramic materials and write their applications in technology.

[20 Marks]

Solution:

Ceramics: Ceramics are hard, strong, dense and brittle

- High temperature stability
- Generally inorganic materials
- They are non-metallic oxides, nitrides, carbides
- Generally crystalline materials (only exception is amorphous glass)

  Ex.: CdS, MgO, ZnO, garnets, $BaTiO_3$, cement, ferrites, SiC, TiO_2 (Rutile), ZnS, Quartz.

Types of Ceramics:

(a) **Porcelain (ϵ_r < 12):** It is used for low and high voltage applications. It is used as insulator in transmission and distribution system, plugs and sockets.

(b) **Steatite (ϵ_r < 12):** It is used in high frequency application.

(c) **Alumina (ϵ_r < 12):** Al_2O_3

- It is used in high temperature application.
- It is used in circuit breakers and resistance cores.

(d) **Titanate (ϵ_r > 12):** High dielectric constant.

- It is used in capacitors.

Another Classification of Ceramics:

(a) ϵ_r < 12: Ceramics having relative permittivity (ϵ_r < 12) used as insulators.

Ex.: Porcelain, steatite, alumina

(b) ϵ_r > 12: This ceramics used in capacitor applications due to high dielectric constant.

Ex.: TiO_2 (Kutile)

MADE EASY Source

- **Theory Book:** MaterialScience(Page No. 118)

End of Solution

4. (a) (i) An electrostatic field in xy-plane is given by $\phi(x, y) = 3x^2y - y^3$. Find the stream function ψ such that the complex potential $\omega = \phi + i\psi$ is an analytic function.

(ii) Find three Laurent's series expansions of the function $f(z) = \dfrac{1}{3z - z^2 - 2}$ and specify the regions in which those expansions are valid.

[12 Marks]

Solution:

(i)
$$\phi(x, y) = 3x^2y - y^3$$

Given complex function $\omega = \phi + i\psi$ is an analytic function.

C – R equations are,

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

We know, that,
$$d\psi = \frac{\partial \psi}{\partial x}dx + \left(\frac{\partial \phi}{\partial x}\right)dy$$

$$= -(3x^2 - 3y^2)dx + 6xydy$$

$$= -3x^2dx + 3y^2dx + 6xydy$$

$$d\psi = -d(x^3) + 3d(xy^2)$$

$$\therefore \quad \psi = \int -d(x^3) + 3\int d(xy^2) + c$$

$$\psi = -x^3 + 3xy^2 + c$$

$\therefore$ Stream function, $\psi = -x^3 + 3xy^2 + c$

(ii)
$$f(z) = \frac{1}{3z - z^2 - 2} = \frac{-1}{(z-1)(z-2)} = -\frac{1}{z-2} + \frac{1}{z-1} \quad \text{... (i)}$$

$$= \frac{1}{2}\left(1 - \frac{z}{2}\right)^{-1} - (1-z)^{-1}$$

(a) For region $|z|<1$:

For $|z|<1$, both $\left|\frac{z}{2}\right|$ and $|z|$ are less than 1.

$$f(z) = \frac{1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + ...\right) - \left(1 + z + z^2 + z^3 + ...\right)$$

$$= -\left(\frac{1}{2} + \frac{3z}{4} + \frac{7z^2}{8} + \frac{15z^3}{16} + ...\right)$$

(b) For region $1<|z|<2$:

We can write equation (i) as

$$f(z) = \frac{1}{2}\frac{1}{1-\frac{z}{2}} + \frac{1}{z\left(1-z^{-1}\right)}$$

and both $\left|\frac{z}{2}\right|$ and $\left|z^{-1}\right|$ are less than 1.

$$f(z) = \frac{1}{2}\left(1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+....\right)+\frac{1}{z}\left(1+z^{-1}+z^{-2}+z^{-3}+....\right)$$

$$= z^{-4}+z^{-3}+z^{-2}+z^{-1}+\frac{1}{2}+\frac{1}{4}z+\frac{1}{8}z^2+\frac{1}{16}z^3+....$$

(c) For region $|z|>2$:

We write equation (i) as

$$f(z) = \frac{-1}{z\left(1-2z^{-1}\right)}+\frac{1}{z\left(1-z^{-1}\right)}$$

$$= -z^{-1}\,(1 + 2z^{-1} + 4z^{-2} + 8z^{-3} + ...)$$
$$+z^{-1}\,(1 + z^{-1} + z^{-2} + z^{-3} + ...)$$
$$= .....\ -7z^{-4} - 3z^{-3} - z^{-2} - .....$$

$\therefore$ For all three regions $|z| < 1$, $1 < |z| < 2$, $|z| > 2$ expansion are valid.

MADE EASY Source

- **MADE EASY Mains Workbook Q. 48 (Page 133)**
- **ESE 2019 Mains Test Series:** Test - 8, Q. 8 (b)

End of Solution

4. (b) What are magnetic materials? Give classification of magnetic materials and name some materials in each class.

[20 Marks]

Solution:

Magnetic Materials: The materials which can be magnetized are called magnetic materials. All materials show some magnetic effect. In many substances the effects are so weak that the materials are often considered to be nonmagnetic. However, a vacuum is the only truly nonmagnetic medium. The response of a material at electronic, atomic, molecular and microscopic level to a magnetic field constitutes magnetic properties. Many characteristics of the magnetic materials are similar to the characteristics of the dielectric materials. Atoms and molecules give magnetic dipole moments similar to electric dipole moments. Some magnetic materials exhibit spontaneous magnetization just like spontaneous polarization in dielectrics. The study of magnetic materials can be done parallel to the study of dielectric materials. The difference is that individual electric charges of one sign do exist, whereas, magnetic monopole does not occur. While the electric field is due to fundamental charges, the magnetic field is always associated with an electric current flowing in a loop.

Classification of Magnetic Materials:

A material possesses magnetic properties on account of

1. motion of charges and
2. permanent magnetic dipoles or moments of the atom or electrons.

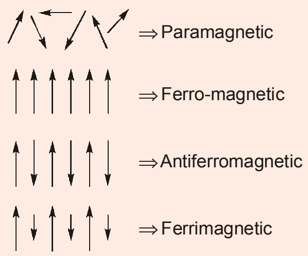
Hence those materials which lack permanent magnetic dipoles are called **diamagnetic**. Those atoms which possess permanent magnetic dipoles may be **paramagnetic, ferromagnetic, antiferromagnetic or ferrimagnetic** depending on the interaction between individual dipoles.

Paramagnetic: In this material, the interaction between adjacent dipole moments of atoms is negligible or zero.

Ferromagnetic: The dipole moment of neighbouring atoms are aligned in a particular direction (parallel) to each other and there is a strong magnetic field.

Antiferromagnetic: The permanent dipole moment of neighbouring atoms are aligned opposite to each other (antiparallel) so that there is no magnetization.

Ferrimagnetic: he dipole moments of neighbouring atoms are aligned antiparallel like antiferromagnetic. But these magnitudes are unequal so that there is large net magnetization like ferromagnetic materials. Figure indicates dipole arrangements for different magnetic behaviours.



Different magnetic behaviour of materials
(arrangement of dipole moments of spins)

Table summarizes features of different magnetic behaviour of materials.

Type	Susceptibility x_m	x_m Vs T relation	Examples
1. Diamagnetic	$\sim -10^{-6}$ (negative)	Independent	Atoms of solids having closed shells and some metal Au, Ge, etc.
2. Paramagnetic	$\sim -10^{-5}$ (positive)	$x_m = \frac{C}{T}$ Curie-law or $x_m = \frac{C}{T-\theta}$ Curie-Weiss law	Atoms possessing odd number of electrons, ionic crystals, etc. $MnSO_4$, Fe_2O_3 Cr_2O_3, $FeCl_2$ etc.
3. Ferromagnetic	Very large and positive	$(x_m \to \infty)$	Iron, cobalt, nickel, godolinium
4. Antiferromagnetic	Small and positive	x_m decreases with temperature	Salts and oxides of transition metals, e.g. NiO, MnF_2
5. Ferrimagnetic	Large and positive	$x_m \to \infty$	Ferrites, e.g. FeO_4

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(e) (ii) of Test-7

End of Solution

4. (c) (i) Show that in a source-free region ($J = 0$, $\rho_v = 0$), Maxwell's equations can be reduced to two. Identify the two all-embracing equations.

(ii) Determine the total charge for the following:

(a) on a line $0 \le x \le 5$ m if $\rho_L = 12x^2$ milii C/m

(b) on a cylinder $\rho = 3$, $0 \le z \le 4$ m if $\rho_s = \rho z^2$ nC/m^2

[10+10 = 20 Marks]

Solution:

(i) Let the medium is homogeneous, Isotropic, source free $\left(\vec{J} = 0; \rho_v = 0\right)$ then Maxwell equation are:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial \mu \vec{H}}{\partial t} \Rightarrow \vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{... (i)}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = 0 + \frac{\partial (\in \vec{E})}{\partial t} \Rightarrow \vec{\nabla} \times \vec{H} = \in \frac{\partial \vec{E}}{\partial t} \quad \text{... (ii)}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_V \Rightarrow \vec{\nabla} \cdot \in \vec{E} = 0 \Rightarrow \in (\vec{\nabla} \cdot \vec{E}) = 0 \Rightarrow \vec{\nabla} \cdot \vec{E} = 0 \quad \text{... (iii)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot \mu \vec{H} = 0 \Rightarrow \mu(\vec{\nabla} \cdot \vec{H}) = 0 \Rightarrow \vec{\nabla} \cdot \vec{H} = 0 \quad \text{... (iv)}$$

Take curl of equation (i) both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\mu \frac{\partial \vec{H}}{\partial t}\right)$$

$$(\vec{\nabla} \cdot \vec{E})\vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla})\vec{E} = -\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) \quad \text{... (v)}$$

Put (ii), (iii) in (v),

$$0 - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\in \frac{\partial \vec{E}}{\partial t}\right) \quad \text{... (vi)}$$

Equation (vi) is called electric wave equation. Take curl of equation (ii) both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \times \left(\in \frac{\partial \vec{E}}{\partial t}\right)$$

$$(\vec{\nabla} \cdot \vec{H})\vec{\nabla} - (\vec{\nabla} \cdot \vec{\nabla})\vec{H} = \in \frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{E}\right) \quad \text{... (vii)}$$

Put (i), (iv) in (vii),

$$0 - \nabla^2 \vec{H} = \in \frac{\partial}{\partial t}\left(-\mu \frac{\partial \vec{H}}{\partial t}\right)$$

$$\Rightarrow \quad \nabla^2 \vec{H} = \mu \in \frac{\partial^2 \vec{H}}{\partial t^2} \quad \text{... (viii)}$$

Equation (viii) is called magnetic wave equation.

Equation (vi) and (viii) are similar equations. Solving equations (vi) and (viii) we get solutions of electric and magnetic field of EM waves. Equation (vi) and (viii) are called wave equations or Helmholtz equations.

(ii) (a)

$$Q = \int \rho_l dl = \int_{x=0}^{5} 12x^2\left(10^{-3}\right) = 12\times10^{-3}\int_{x=0}^{5} x^2 dx$$

$$= 12\times10^{-3}\left(\frac{x^3}{3}\right)_{x=0}^{5} = 12\times10^{-3}\left(\frac{5^3}{3}\right) = 4\times10^{-3}(125)$$

$$= 0.5 \text{ coulomb}$$

(b) On a cylinder $\rho = 3;\ 0 \le z \le 4$ m if $\rho_s = \rho z^2$ nC/m^2

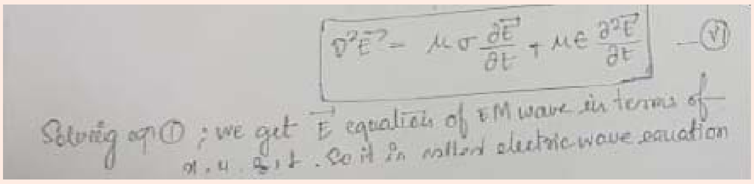
$$Q = \iint \rho_s ds = \iint \rho z^2\left(\times10^{-9}\right)\rho\, d\phi\, dz\Big|_{\rho=3 \text{ surface}}$$

$$= (3)^2 \times 10^{-9} \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{4} z^2 dz = 9\times10^{-9}(\phi)_{\phi=0}^{2\pi}\left(\frac{z^2}{3}\right)_{z=0}^{4}$$

$$= 18\pi\times10^{-9}\left(\frac{4^3}{3}\right) = 1.206\ \mu\text{C}.$$

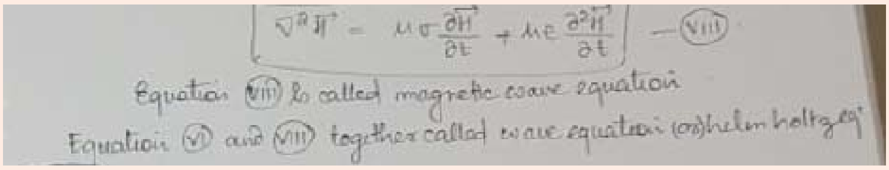
MADE EASY Source

- **MADE EASY Classnotes**



$$\boxed{\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{E}}{\partial t}}$$ — (vi)

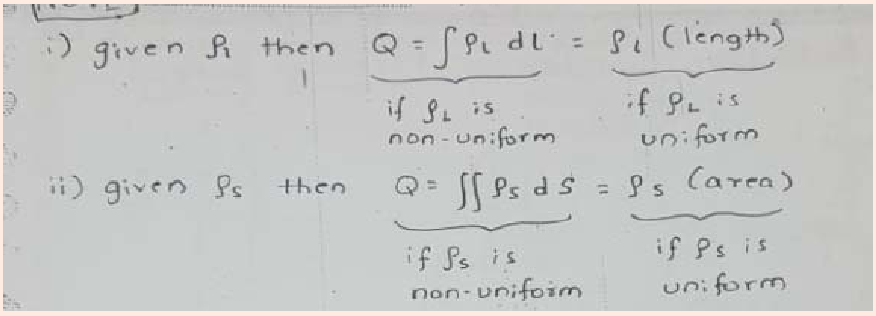
Solving eqn ① ; we get $\vec{E}$ equation of EM wave in terms of σ, μ, ε, t. So it is called electric wave equation



$$\boxed{\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t}}$$ — (viii)

Equation (viii) is called magnetic wave equation

Equation (vi) and (viii) together called wave equation (or) helmholtz eqn



i) given ρ_L then $\underbrace{Q = \int \rho_L\, dl}_{\text{if } \rho_L \text{ is non-uniform}} = \underbrace{\rho_L\,(\text{length})}_{\text{if } \rho_L \text{ is uniform}}$

ii) given ρ_s then $\underbrace{Q = \iint \rho_s\, dS}_{\text{if } \rho_s \text{ is non-uniform}} = \underbrace{\rho_s\,(\text{area})}_{\text{if } \rho_s \text{ is uniform}}$

- **Mains Work Book :** Q. 26 (Page No. 66)

End of Solution

5. (a) (i) Determine the possible base of the number in the operation mentioned below:

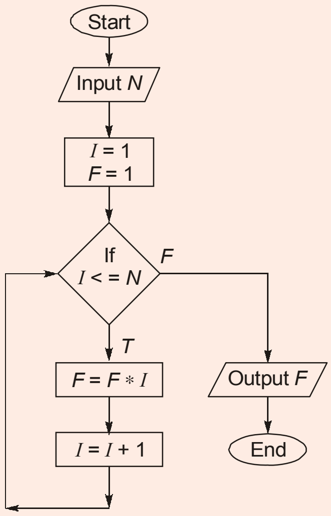
$$23 + 44 + 14 + 32 = 223$$

(ii) Find the minimal sum of products (SOP) expression of the following boolean function $f(a, b, c, d)$

$$f(a, b, c, d) = abc + abd + a'bc' + cd + bd'$$

Where a', b', c' and d' are complements of variables a, b, c and d respectively.

(iii)



What computation is performed by the given flowchart? Perform dry run and write down problem statement.

[4 + 4 + 4 = 12 Marks]

Solution:

(i) Assuming the base of numbers be: b

We can write

$(23)_b + (44)_b + (14)_b + (32)_b = (223)_b$

$(2b^1 + 3) + (4b^1 + 4) + (b^1 + 4) + (3b^1 + 2) = 2b^2 + 2b + 3$

On solving above expression,

$$10b + 13 = 2b^2 + 2b + 3$$

or $\quad 2b^2 - 10b + 2b - 10 = 0$

$b(2b - 10) + 1\,(2b - 10) = 0$

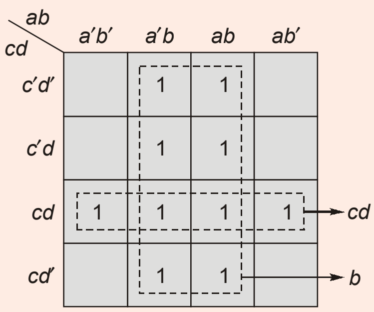
$$b = -1$$
$$b = 5$$

$\therefore$ Acceptable value of base,

$$b = 5$$

(ii) For SOP expression of given boolean function,

$$\begin{aligned} f(a, b, c, d) &= abc + abd + a'bc' + cd + bd' \\ &= abc(d + d') + abd(c + c') + a'bc'(d + d') \\ &\quad + cd(a + a')\,(b + b') + bd'(a + a')\,(c + c') \\ &= abcd + abcd' + abc'd + a'bc'd + a'bc'd' \\ &\quad + ab'cd + a'bcd + abcd' + abc'd' + a'bcd' + a'bc'd' \end{aligned}$$



$$f = b + cd$$

(iii) Given flow chart corresponds to computation of factorial of any given number N.

Step-1 : Initialize input variable N, I and F given values 1 to I and F both.

Step-2 : Initialize loop by checking whether I value is less than equal to N. If yes go to step-3 else to step-6.

Step-3 : Perform multiplying operation $F = F \times I$.

Step-4 : Perform increment in value of I by 1.

Step-5 : Go back to loop at step-2.

Step-6 : Output value

Assuming a value $N = 3$

Step-1 : $I = 1,\ F = 1$

Step-2 : $(1 < = 3)$ True

Step-3 : $F = 1 \times 1 = 1$

Step-4 : $I = 1 + 1 = 1$

Step-2 : $(2 < = 3)$ True

Step-3 : $F = 2 \times 1 = 2$

Step-4 : $I = 2 + 1 = 3$

Step-2 : $(3 < = 3)$ = True

Step-3 : $F = 2 \times 3 = 6$

Step-4 : $I = 3 + 1 = 4$

Step-2 : $(4 < = 3)$ = False

Step-6 : Output : 6

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.2(c) of Test-1
- **ESE 2019 Mains Test Series:** Q.3(c) of Test-1

End of Solution

5. (b) A $4\frac{1}{2}$ digit and a $3\frac{1}{2}$ digit voltmeter on 10 V range are used for voltage measurements.

(i) Find the resolution of each meter.

(ii) How would the reading 0.7582 be displayed on these two meters?

[12 Marks]

Solution:

(i) $4\frac{1}{2}$ digit voltmeter is on 10 V range.

| 1 | 0 | .0 | 0 | 0 |

In 10 V range the minimum voltage can be measured by $4\frac{1}{2}$ digit voltmeter is 00.001 V i.e. 1 mV.

$\therefore$ Resolution = 1 mV for $4\frac{1}{2}$ digit voltmeter.

$3\frac{1}{2}$ digit voltmeter, is on 10 V range.

| 1 | 0 | .0 | 0 |

In 10 V range,

the minimum voltage can be measured by $3\frac{1}{2}$ digit voltmeter is 00.01 V i.e. 10 mV.

$\therefore$ Resolution = 10 mV for $3\frac{1}{2}$ digit voltmeter.

(ii) In $4\frac{1}{2}$ digit voltmeter 0.7582 is displayed on 10 V range as | 0 | 0 | .7 | 5 | 8 |.

In $3\frac{1}{2}$ digit voltmeter 0.7582 is displayed on 10 V range as | 0 | 0 | .7 | 5 |.

MADE EASY Source
- **ESE 2019 Mains Test Series:** Q.5(e) of Test-9
- **Theory Book:** Measurement Ex. 10.3 (Page No. 284)

End of Solution

5. (c) A 1000/5 A, 50 Hz current transformer at its rated load of 50 VA has an iron loss of 0.5 W and a magnetizing current of 8 A. Calculate the ratio error and phase angle when rated output is supplied to a meter whose resistance is 0.4 Ω and inductance is 0.7 mH.

[12 Marks]

Solution:

Nominal ratio, $k_n = \frac{1000}{5} = 200$

In the absence of any other data. The turns ratio is taken equal to the nominal ratio n = 200. Neglecting the burden of the secondary winding, the total burden of the secondary circuit is equal to the burden of the meter.

$\therefore$Burden of secondary circuit = 50 VA.

Voltage across primary winding E_P

$$E_P = \frac{VA}{I_P} = \frac{50}{I_P} = \frac{50}{1000} = 0.05 \text{ V}$$

Loss component of current,

$$I_e = \frac{\text{Iron loss}}{E_P} = \frac{0.5}{0.05} = 10 \text{ A}$$

Magnetizing current, $I_m = 8$ A

Secondary circuit phase angle,

$$\delta = \tan^{-1}\frac{X}{R} = \tan^{-1}\left[\frac{2\pi \times 0.7 \times 10^{-3} \times 50}{0.4}\right]$$

$$\delta = 28.8°$$

$$\cos\delta = \cos(28.8) = 0.876$$

$$\sin\delta = \sin(28.8) = 0.481$$

$$\text{Actual ratio},\ R = n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s} = 200 + \frac{10 \times 0.876 + 8 \times 0.481}{5}$$

$$R = 202.52$$

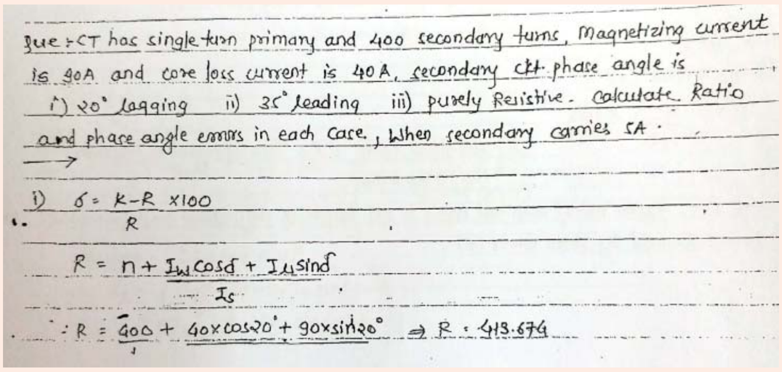
$$\text{Ratio error} = \frac{k_n - R}{R} \times 100 = \frac{200 - 202.52}{202.52} \times 100 = -1.244\%$$

Phase angle,

$$\theta = \frac{180}{\pi}\left[\frac{8 \times 0.876 - 10 \times 0.481}{200 \times 5}\right] = 0.12°$$

MADE EASY Source

- **MADE EASY Classnotes**



Que :- CT has single-turn primary and 400 secondary turns, Magnetizing current is 90A and core loss current is 40A, secondary ckt phase angle is i) 20° lagging ii) 35° leading iii) purely Resistive. Calculate Ratio and phase angle errors in each case, when secondary carries 5A.

→

i) $\delta = \frac{K-R}{R} \times 100$

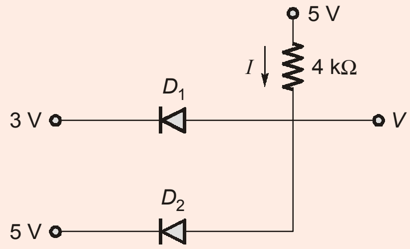
$R = n + \frac{I_w \cos\delta + I_\mu \sin\delta}{I_s}$

$\therefore R = 400 + \frac{40 \times \cos 20° + 90 \times \sin 20°}{5} \Rightarrow R = 413.674$

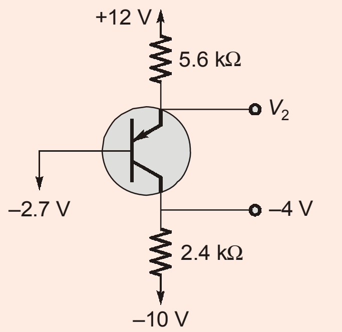
- **Mains Work Book:** Q. 34 (Page No. 120)

End of Solution

5. (d)



(i) For the circuit shown in above figure using ideal diodes, find the values of the voltage V and current I indicated in the figure.

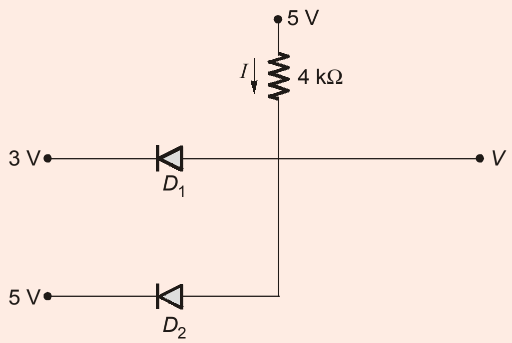


(ii) For the circuits in the above figure assume that the transistor have very large β. Some measurements have been made on these circuits. The results are indicated in the figure. Find the values of the voltage V_2.

[6 + 6 = 12 Marks]

Solution:

(i)



For the given circuit as all diodes are ideal.

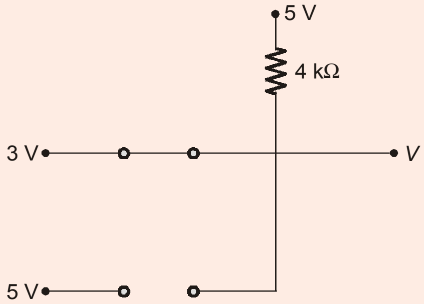
Current through diode D_2

As potential is same at both ends, So $I_{D2} = 0$.

Current through diode D_1

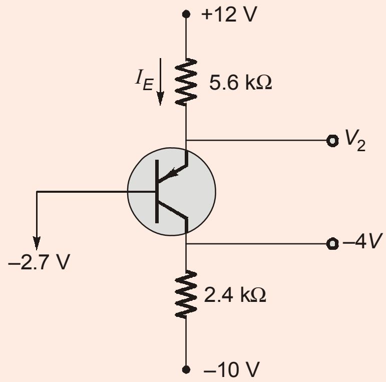
$$I_{D1} = \frac{5-3}{4\,\text{k}\Omega} = \frac{2}{4\,\text{k}\Omega} = 0.5\,\text{mA}$$

when D_1 conducts circuit becomes



∴Voltage at given terminal = 3V

(ii)



For the given p-n-p transistor. β is very large.

$$I_E \; ; \; I_C$$

Collector current is given by

$$I_C = \frac{-4-(-10)}{2.4\,\text{k}\Omega} = 2.5\,\text{mA} \simeq I_E$$

Apply KVL, $V_2 = -5.6\ kI_E + 12\ \text{V}$

$= 12 - (5.6 \times 2.5) = -2\ \text{V}$

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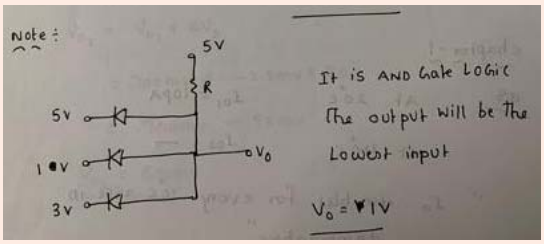

- **ESE 2019 Prelims Test Series:** Q.3 of Test-12
- **ESE 2019 Mains Test Series:** Q.1(d) of Test-14

End of Solution

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5. (e) (i) Bipolar junction transistors (BJTs) are considered "normally off" devices, because their natural state with no signal applied to the base is no conduction between emitter and collector, like an open switch. Are junction field effect transistors (JFETs) considered the same? Why or why not? Justify your answer.

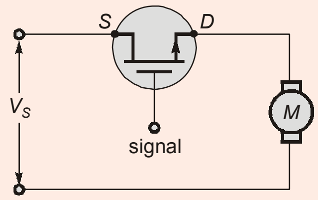
(ii) How an *n*-channel enhancement mode MOSFET can be used to switch a motor on and off? Justify your answer.

[6 + 6 = 12 Marks]

Solution:

(i) Normally there is a channel between drain and source, JFET operates as a ON switch during normal condition (absence of gate signal). When a reverse bias voltage is applied to the gate terminal, then the width of drain to source voltage starts decreasing and at a particular voltage pinch off region is created and JFET stops conducting. In BJT, if no current is flowing through base terminal than there will be no current between emitter and collector and it works as a normally off device. BJT will starts operating when there is a base current is the base terminal.

(ii) If the absence of signal, MOSFET normally operates as off switch. When we apply the signal to the gate terminal, a channel is formed between drain and source and MOSFET becomes ON and motor will starts operating.



End of Solution

6. (a) (i) What are the different types of registers generally contained in the central processing unit? Explain functions of each register used in computer systems.

(ii) List the steps involved in interrupt drivers *I*/O with a suitable pseudo code/ flowchart from the view of in *I*/O module.

[10 + 10 = 20 Marks]

Solution:

(i) Registers are one of the three components of central processing unit which include: Registers, ALU and control unit. For execution registers is a internal storage elements of the CPU, which may include following essential kinds:

1. PC : → Holds the instruction address
2. IR : → Holds the opcode
3. ACC : → Holds the ALU I/P and ALU O/P.
4. TR : → Holds the ALU 2nd operation
5. MAR : → Holds the memory address connected to address lines of a system bus.
6. MBR : → Holds the memory content, connected to data lines of a system bus.
   MDR
7. SP : → Holds the top of stack address
8. Flag register : → Holds the status of an instruction
9. GPR : → Holds the data
   (General purpose register)

(ii) Following steps are followed in interrupt driver I/O when interrupt goes high.

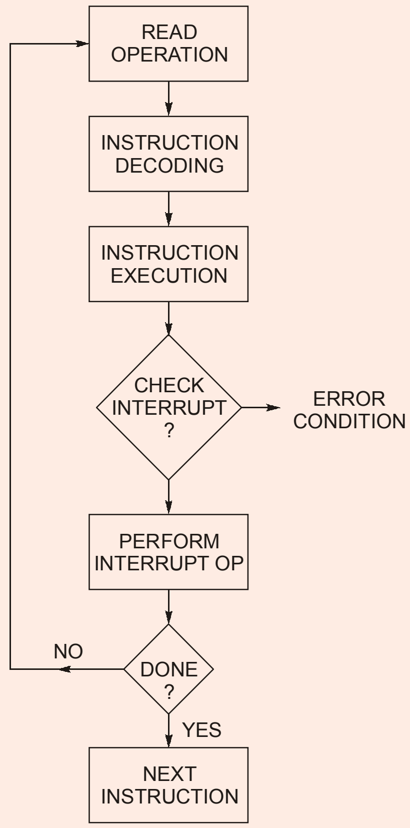
(a) The microprocessor completes the execution of the instruction OUT.

(b) It senses the line INTR high and that interrupt is enabled.

(c) The microprocessor disables the interrupt, stops execution and sends out a control signal $\overline{\text{INTA}}$ (interrupt ac knowledge).

(d) The $\overline{\text{INTA}}$ (active low) enables the tristate buffer and the instruction EFH is placed on the data bus.

(e) The microprocessor series the address of next instruction on the stack and program is transferred to memory location which should also jump instruction to transfer the program to service routine.



End of Solution

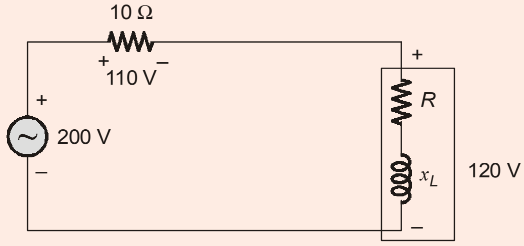
6. (b) In a single-phase power measurement test by three-voltmeter method, the following readings were obtained. Across AC mains is 200 V; across the non-inductive resistance of 10 Ω is 110 V; across the load consisting of resistance (R) and inductance (L) is 120 V.

(i) Calculate the power supplied to the load.

(ii) Calculate the inductive reactance (X_L) and resistance (R) of the load.

[20 Marks]

Solution:



$$(200)^2 = \left(\sqrt{(110)^2 + (120)^2 + 2 \times 110 \times 120 \cos\phi}\right)^2$$

$$40000 = 12100 + 14400 + 26400 \cos\phi$$

$$26400 \cos\phi = 13500$$

$$\cos\phi = \frac{13500}{26400} = 0.511$$

$$\text{Power supplied to the load} = VI\cos\phi$$

$$= 120 \times \frac{110}{10} \times 0.511 = 674.52$$

$$\text{Power supplied by load} = I^2R$$

$$(11)^2R = 674.52$$

$$R = 5.574\ \Omega$$

$$V_R = 61.32$$

$$V_x = \sqrt{(120)^2 - (61.32)^2} = 103.15$$

$$IX_L = 103.15$$

$$X_L = 9.37\ \Omega$$

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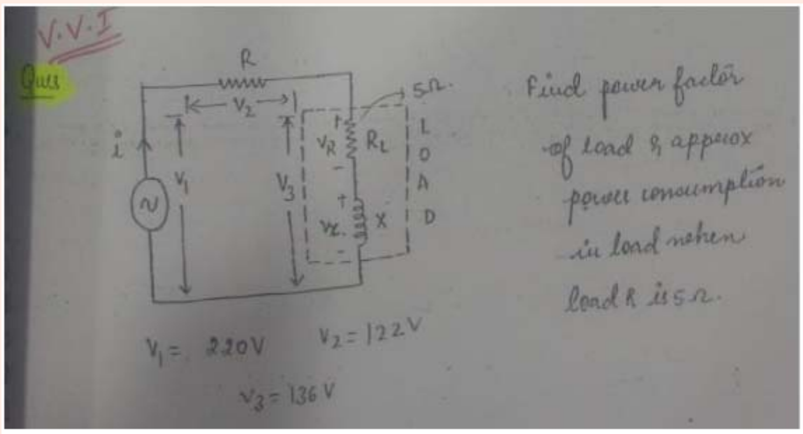
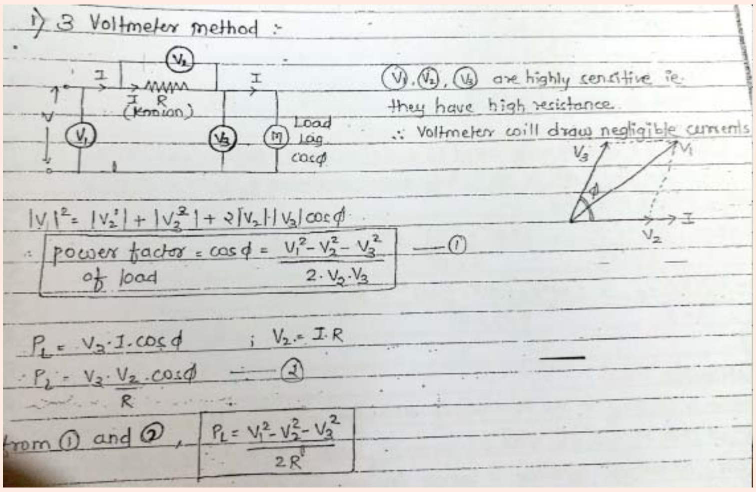




i) 3 Voltmeter method :-

V_1, V_2, V_3 are highly sensitive ie. they have high resistance.

∴ Voltmeter will draw negligible currents

$|V_1|^2 = |V_2^2| + |V_3^2| + 2|V_2||V_3|\cos\phi$

∴ $\boxed{\text{power factor of load} = \cos\phi = \dfrac{V_1^2 - V_2^2 - V_3^2}{2 \cdot V_2 \cdot V_3}}$ — (1)

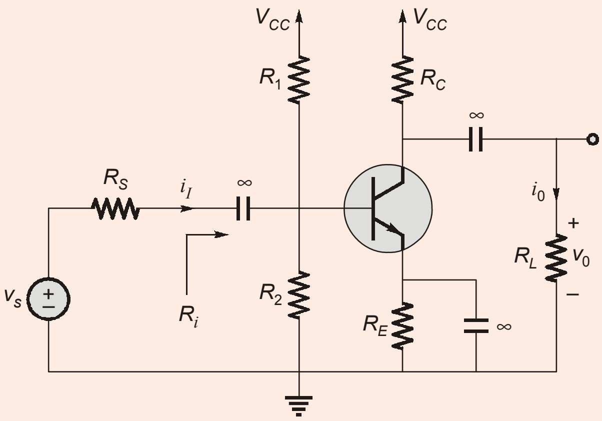
$P_L = V_3 \cdot I \cdot \cos\phi$; $V_2 = I \cdot R$

∴ $P_L = V_3 \cdot \dfrac{V_2}{R} \cdot \cos\phi$ — (2)

from (1) and (2), $\boxed{P_L = \dfrac{V_1^2 - V_2^2 - V_3^2}{2R}}$

End of Solution

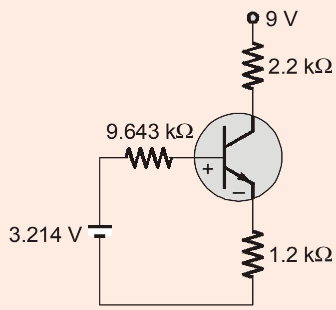
6. (c)



For the common-emitter amplifier shown in the figure, let V_{CC} = 9 V, R_1 = 27 kΩ, R_2 = 15 kΩ, R_E = 1.2 kΩ and R_C = 2.2 kΩ. The transistor has β = 100 and V_A = 100 V (V_A = Early voltage). Calculate the dc bias current I_E. If the amplifier operates between a source for which R_S = 10 kΩ and a load of 2 kΩ (R_L), replace the transistor with its hybrid-π model, and find the values of R_i and voltage gain v_0/v_s. Assume V_{BE} = 0.7 V, V_T (thermal voltage) = 25 mV.

[20 Marks]

Solution:



$$V_{th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{15}{15+27} \times 9$$

$$V_{th} = 3.214 \text{ V}$$

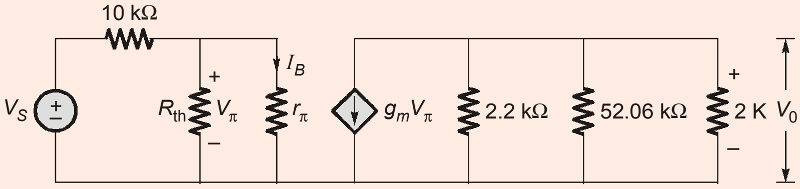
$$R_{th} = R_1 \parallel R_2 = 27 \parallel 15$$

$$= 9.643 \text{ k}\Omega$$

$$-3.215 + 9.643 I_B + 0.7 + 1.2 I_E = 0$$

$$I_B(9.643 + 1.2 \times 101) = 2.514$$

$$I_B = 19.214\ \mu\text{A}$$
$$I_E = 1.941\ \text{mA}$$
$$I_C = 1.921\ \text{mA}$$
$$r_0 = \frac{V_A}{I_C} = \frac{100}{1.921} = 52.06\ \text{k}\Omega$$
$$r_e = \frac{V_T}{I_E} = \frac{25}{1.941} = 12.88\ \Omega$$
$$r_\pi = \beta r_e = 1.288\ \text{k}\Omega$$



$$V_0 = -g_mV_\pi (2.2 \,||\, 52.06 \,||\, 2)$$
$$= \frac{-1 \times 1.0269}{12.88 \times 10^{-3}} \times V_\pi = -79.732\ V_\pi$$
$$V_0 = -79.732 \times \left(\frac{R_i}{R_i + R_s}\right) V_s$$
$$R_i = R_{th} \,||\, r_\pi = 9.643 \,||\, 1.288$$
$$R_i = 1.1362\ \text{k}\Omega$$
$$V_0 = -79.732 \times \frac{1.136}{1.136 + 10} \times V_S$$
$$\frac{V_0}{V_S} = -8.134$$
$$A_V = -8.134$$

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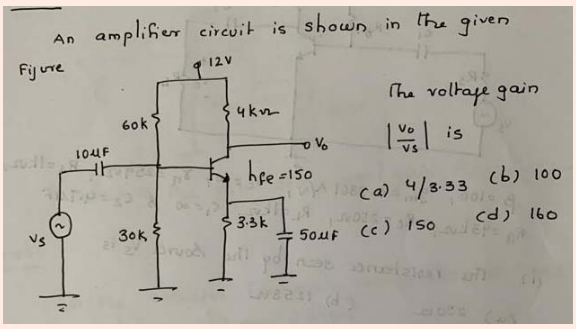


- **ESE 2019 Mains Test Series:** Q.6(b) of Test-6

End of Solution

7. (a) (i) Write a program in any programming language to find highest common factor (HCF) of two positive integer numbers.

(ii) In virtual memory based system, suppose we have an average of one page fault after every 10,000,000 instructions. A normal instruction takes 4 ns (4 nano seconds), and a page fault causes the instructions to take an additional 10 milli seconds. What is the average instruction time, taking page faults into account?

[10 + 10 = 20 Marks]

Solution:

(i) C progam to find highest common factor (*H*(*f*)) of two positive integer.

```
# include <stdio.h>
        int main {
        int a, b, x, y, t, hcf ;
        printf ("Enter two integer\n");
        scanf ("%d %d", & x, &y);
        a = x;
        b = y;
        while (b! = 0) {
        t = b;
        b = a % b;
        a = t;
}
        hcf = a;
print f ("Highest common factor %d and %d is %d \n", x, y, hcf);
return 0;
}
```

(ii) For given virtual memory system,

Instruction execution time without fault = 4 nsec

Page fault rate for instruction = $\dfrac{1}{10,000,000}$

Extra delay time due fault of instruction = 10 msecond = 10^{-2} second

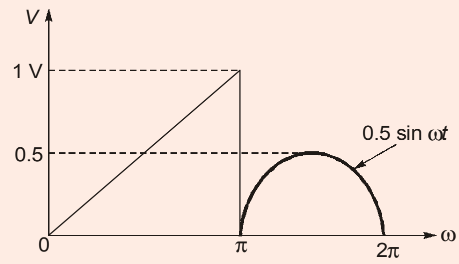
Average instruction time,

$$4 \times 10^{-9} + \frac{1}{10,000,000} \times 10^{-2}\,\text{sec} = 5 \text{ nsec}$$

End of Solution

7. (b) Calculate the reading of

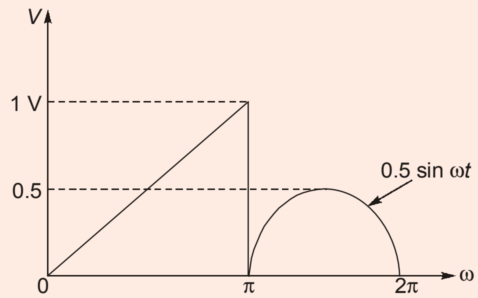
(i) Moving coil voltmeter (ii) Moving iron voltmeter



when these voltmeters are measuring the voltage of the waveform shown in the figure.

[20 Marks]

Solution:



(i) Moving coil voltmeter reads average value of waveform

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} V(t)\cdot d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\frac{1}{\pi}(\omega t)\,d(\omega t) + \int_{\pi}^{2\pi}\frac{1}{2}\sin(\omega t - \pi)\,d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{\pi}\left(\frac{\omega t^2}{\pi}\right)_0^{\pi} + \frac{1}{2}(-\cos(\omega t - \pi))_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{\pi}\left[\frac{\pi^2}{2}\right] + \frac{1}{2}(\cos 0 - \cos\pi)\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2} + 1\right] = \frac{2.57}{2\pi}\text{Volt} = 0.41 \text{ volt}$$

∴ Moving coil voltmeter reads 0.41 Volt.

(ii) Moving iron voltmeter reads RMS value of waveform

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{T}[V(t)]^2\cdot dt}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}\left(\frac{1}{\pi}\omega t\right)^2 \cdot d(\omega t) + \int_{\pi}^{2\pi}\left[\frac{1}{2}\sin(\omega t - \pi)\right]^2 d(\omega t)\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\left(\frac{(\omega t)^3}{3\pi^2}\right)_0^{\pi} + \int_4^{2\pi}\frac{1}{4}\left(\frac{1-\cos 2(\omega t - \pi)}{2}\right)\cdot d(\omega t)\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\frac{\pi^2}{3\pi^2} + \frac{\pi}{8} - \frac{1}{8}\left(\frac{\sin 2(\omega t - \pi)}{2}\right)_0^{2\pi}\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\frac{\pi}{3} + \frac{\pi}{8}\right]} = \sqrt{\frac{1}{2}\left(\frac{1}{3} + \frac{1}{8}\right)}$$

$$V_{RMS} = 0.478 \text{ volt}$$

Moving iron voltmeter reads 0.478 volt.

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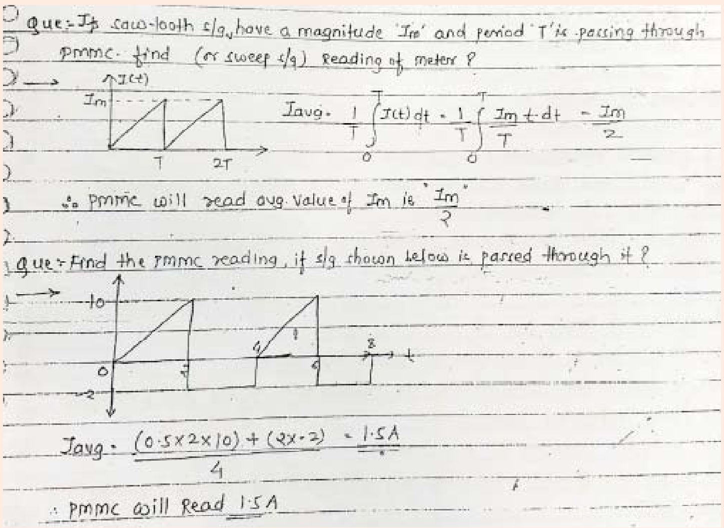
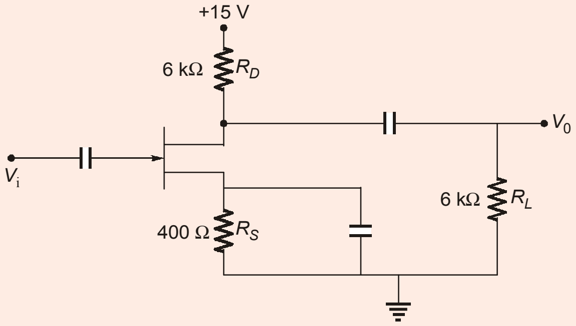


- **ESE 2019 Mains Test Series:** Q.5(b) of Test-3

End of Solution

7. (c)



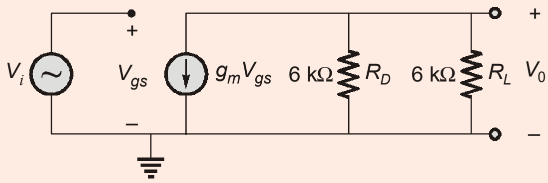
(i) In the given JFET as amplifier shown in the figure, the drain current changes from 4 mA to 6 mA when the gate voltage is changed from –3.8 V to –3.5 V in the amplifier circuit. Calculate the voltage gain of the amplifier.

(ii) Explain the Barkhausen criterion for an oscillator circuit. How will the oscillator circuit's performance be affected if the Barkhausen criterion falls below 1, or goes much above 1?

[12 + 8 = 20 Marks]

Solution:

(i) Transe conductance $= g_m = \dfrac{\Delta I_D}{\Delta V_{gs}} = \dfrac{6\text{ mA} - 4\text{ mA}}{-3.5 + 3.8} = \dfrac{1}{150}\text{s}$



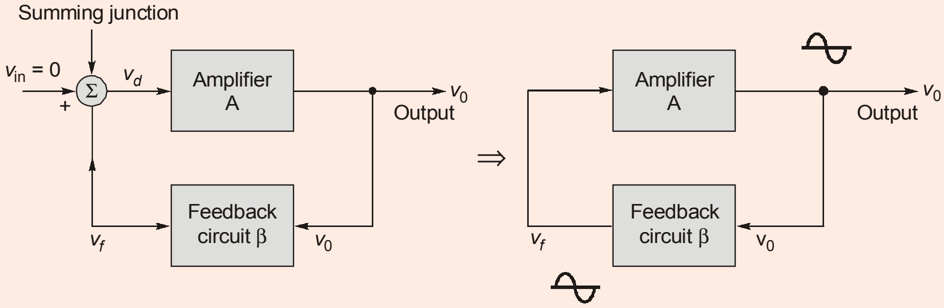
R_S is short circuited through by pass capacitor

Voltage gain, $A_V = -g_m[R_D / / R_L]$

$$= -\frac{1}{150}[6\text{ k} / / 6\text{ k}]$$

$A_V = -20$

(ii) An oscillator is a type of feedback amplifier in which part of the output is feedback to the input via a feedback circuit. If the signal feedback is of proper magnitude and phase, the circuit produces alternating currents or voltages. To visualize the requirements of an oscillator, consider the block diagram of figure. This diagram looks identical to that of the feedback amplifiers. However, here the input voltage is zero (v_{in} = 0). Also the feedback is positive because most oscillators use positive feedback.



In the block diagram of figure,

$$v_d = v_f + v_{in}$$
$$v_0 = A\, v_d$$
$$v_f = \beta v_0$$

using these relationships, the following equation is obtained:

$$\frac{v_0}{v_{in}} = \frac{A}{1 - A\beta}$$

However, $v_{in} = 0$ and $v_0 \neq 0$ implies that,

$$A\beta = 1 \quad \text{...(i)}$$

Expressed in polar form

$$\boxed{A\beta = 1\angle 0^\circ \text{ or } 360^\circ} \quad \text{...(ii)}$$

Equation (ii) gives two requirements for oscillation:

- the magnitude of the loop gain Aβ must be at least 1, and
- the total phase shift of the loop gain Aβ must be equal to 0° or 360°.

The condition given by equation (2) is known as ***Barkhausen criterion.***

In figure if the amplifier causes a phase shift of 180°, the feedback circuit must provide an additional phase shift of 180° so that the total phase shift around the loop is 360°. The type of waveform generated by an oscillator depends on the components in the circuit hence may be sinusoidal, square or triangular. In addition the frequency of oscillation is determined by the components in the feedback circuit.

For perfect oscillation $|A\beta| = 1$. But due to non-linearity present in the circuit $|A\beta|$ becomes less than one. So, value of $|A\beta|$ in set slightly larger then unity and with time is comes very close to unity. If the value of $|A\beta|$ is set very higher than unity then the amplitude increases with time and system becomes unstable.

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- **Theory Book:** Analog Electronics (Page No. 297)

End of Solution

8. (a) (i) Write a program in any programming language of sum of the following series up to N terms.:

$$\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + \lfloor 5 + ... + \lfloor N$$

where $\lfloor N =$ factorial of N.

(ii) A company wishes to transmit numerous 10,000 bytes files between two computer systems connected through a computer network. Each byte consists of 8 data bits, no parity bits used in this application. Error is not a factor for transmission. What is the overhead percentage if a single file is sent asynchronously? What is the overhead percentage if the file is sent synchronously using blocks with 1000 data bytes in each block and each block is accompanied by 15 special bytes? Assume in asynchronous communication 1 start bit and 2 stop bits sent with each data byte. Comment, which transmission method is most efficient.

[10 + 10 = 20 Marks]

Solution:

(i) Program to find sum of series of $\lfloor 1 + \lfloor 2 .... \lfloor N$

```
# include <stdio.h>
int main ( )
{
int num, i, j, fact, sum = 0; //variables defining
printf ("enter the last number of series:\n");
scanf ("%d", & num); // last number of series
for (i = 1; i< = num, i++) // loop for finding factorial and sum
{
      fact = 1 ;
      if (i! = num)
      printf ("%d! + ", i);
      else
      printf ("%d! = ", i);
for (j = 1, j < = i, j++)
      fact = fact * j;
      sum = sum + fact ;
}
      print f ("%d", sum);
      return 0;
}
```

(ii) Given,

No. of files to be transferred = 10000 bytes

Each byte = 8 bits

For asynchronous file transfer:

1 + start bit + 2 stop bit extra for each data byte

$\therefore$ Total overhead for asynchronous = (1 + 2) × (10000) = 30000

$\therefore$ Total size in bits for asynchronous transfer including overhead

$$= 80\text{ k} + 30\text{ k}$$
$$= 110\text{ kbits}$$

$$\text{Overhead percentage} = \frac{30}{110} \times 100 = 27.27\%$$

$$\text{Transfer efficiency} = \frac{80}{110} \times 100 = 72.72\%$$

In case of synchronous file transfer,

15 special bytes + 1000 data bytes

$$= (15 \times 8)\text{ special bits} + 1000 \times 8\text{ data bits}$$
$$= 120\text{ spec} + 8000\text{ data}$$
$$\text{total size} = 8120\text{ bits}$$

$$\text{Overhead percentage} = \frac{120}{8120} \times 100 = 1.478\%$$

$$\text{Transfer efficiency} = \frac{8000}{8120} \times 100 = 98.52\%$$

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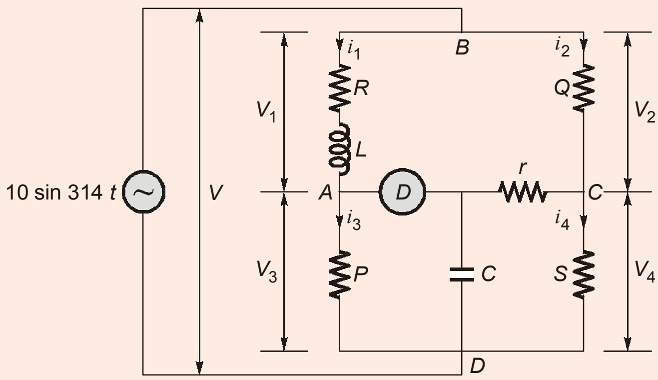
End of Solution

8. (b)



The bridge network shown in the figure measures the inductance (L) and resistance (R) if the impedance between A and B is unknown.

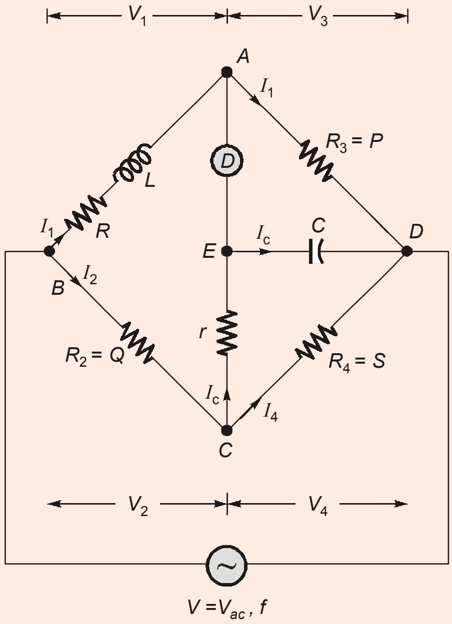
(i) Find R and L if bridge balance is obtained for $Q = S = 1000\ \Omega$, $P = 100\ \Omega$, $r = 50\ \Omega$, $C = 1\ \mu\text{F}$.

(ii) Draw the vector diagram showing the voltage and current at every point of the network.

[20 Marks]

Solution:

Given circuit is Anderson's bridge and can be redrawn as



L_1 : self inductance to be measured

R_1 : resistance of self inductor

r, R_4, R_2, R_3 : known non inductive resistances

C : fixed standard capacitor

$$I_1 = I_3$$

and $$I_2 = I_C + I_4$$

We know, by using KVL, $$I_1R_3 = I_C \times \frac{1}{j\omega C}$$

$$I_C = I_1 j\omega C R_3$$

writing the balance equations for bridge

$$I_1(R_1 + j\omega L_1) = I_2R_2 + I_c r \quad \text{...(i)}$$

and $$I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_c)R_4 \quad \text{...(ii)}$$

Substituting value of I_C in above equation (i) and (ii),

$$I_1(R_1 + j\omega L_1) = I_2R_2 + I_1\, j\omega CR_3 r$$

or $$I_1(R_1 + j\omega L_1 - j\omega CR_3 r) = I_2R_2 \quad \text{...(iii)}$$

and $$j\omega CR_3 I_1\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1 j\omega CR_3)R_4$$

$$I_1(j\omega CR_3 r + j\omega CR_3R_4 + R_3) = I_2R_4 \quad \text{...(iv)}$$

Using equation (iii) and (iv),

$$I_1(R_1 + j\omega L_1 - j\omega CR_3 r) = I_1\left(j\omega CR_3R_2 + j\omega\frac{CR_2R_3 r}{R_4} + \frac{R_2R_3}{R_4}\right)$$

Equating real and imaginary parts,

$$R_1 = \frac{R_2 R_3}{R_4} \quad \text{...(v)}$$

$$L_1 = C\frac{R_3}{R_4}[r(R_4 + R_2 + R_2 R_4)] \quad \text{...(vi)}$$

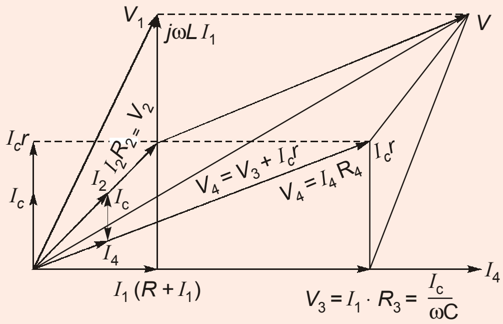
(i)
$$R = \frac{PQ}{S} = \frac{100 \times 1000}{1000} = 100\ \Omega$$

$$L = \frac{CQ}{S}[r(S+P)+PS]$$

$$= \frac{10^{-6} \times 1000}{1000}[50(1000+100)+100000)]$$

$$= 0.155 \text{ H}$$

(ii) The phasor diagram can be drawn as follows



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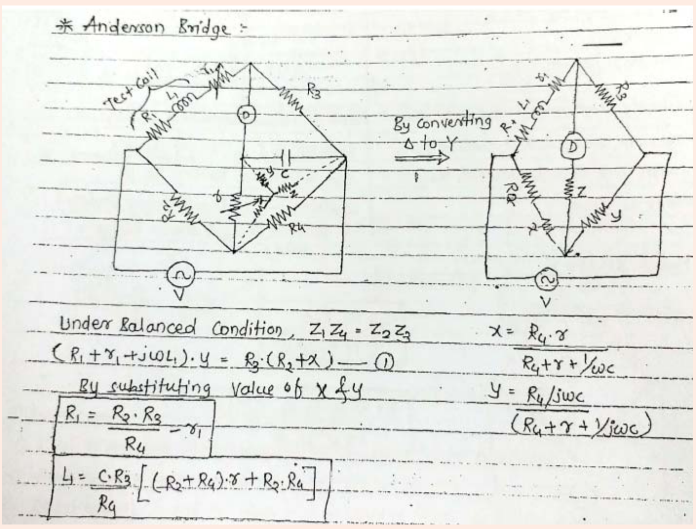
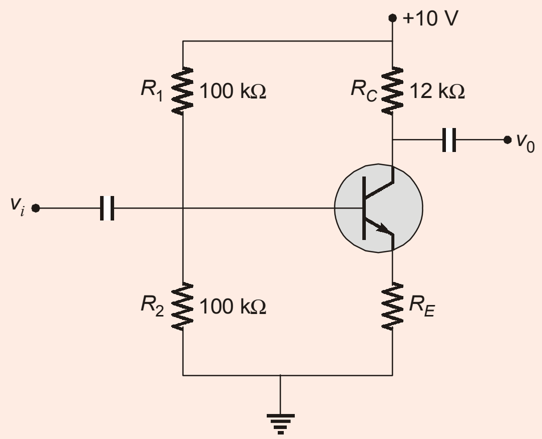
- **MADE EASY Classnotes**



End of Solution

8. (c)



(i) Common emitter (CE) amplifier shown in the figure has voltage gain 400 when $R_E = 0$. Stability is brought through negative feedback by adding resistor R_E. Find the value of resistor R_E using feedback concepts so that final voltage gain is equal to 200.

(ii) Not all "zener" diodes breakdown in the exact same manner. Some operate on the principle of zener breakdown, while other operate on the principle of avalanche breakdown. How do the temperature coefficients of these two zener diode types compare? Are you able to discern whether a zener diode uses one principle or the other just from its breakdown voltage rating? Justify your answer.

[10 + 10 = 20 Marks]

Solution:

(i) Feedback factor,

$$\beta = \frac{V_f}{V_0} = -\frac{R_E}{R_C}$$

$$A_{Vf} = \frac{A}{1+A\beta}$$

$$A = -400$$

$$A_{Vf} = -200$$

$$-200 = \frac{-400}{1+\left(400\dfrac{R_E}{R_C}\right)}$$

$$1+\left(400\frac{R_E}{R_C}\right) = 2$$

$$R_E = \frac{R_C}{400} = \frac{12\times10^3}{400} = 30\ \Omega$$

(ii) Temperature coefficient of a zener diode may be positive or negative depending upon the breakdown or zener voltage, v_z.

For lower breakdown voltages (less than 5 V), breakdown takes place due to tunneling effect and the phenomenon is Zener Breakdown. While for higher breakdown voltages (exceeding 8 V), Avalanche Breakdown occurs due to impact ionization. In between both mechanism contribute to breakdown.

Despite the fact that Avalanche and Zener constitute two types of diodes, the name Zener is commonly applied to both types of diodes.

Low voltage Zener diodes have negative temperature coefficient since Zener breakdown takes place more easily with increase in temperature (tunneling occurs earlier due to increase in thermal energy).

High voltage Zener diodes have positive temperature coefficient because breakdown takes place due to Avalanche effect and increase in thermal vibration reduces kinetic energy gain, thus increasing breakdown voltage.

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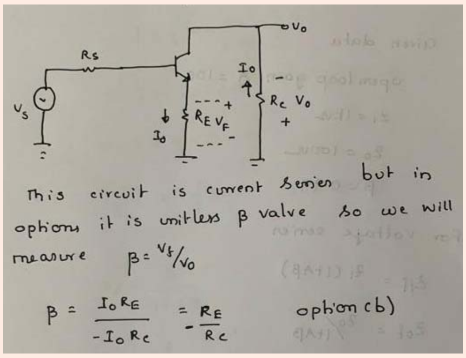

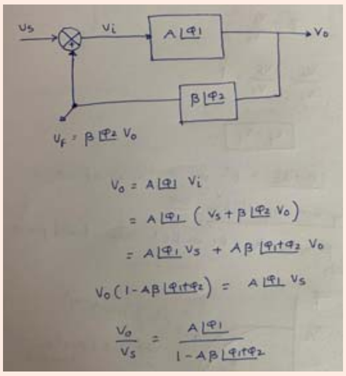

- **ESE 2019 Mains Test Series:** Q.1(c) of Test-12

End of Solution

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