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UPSC ESE 2019

Main Exam Detailed Solutions

Mechanical Engineering

PAPER-II

EXAM DATE : 30-06-2019 | 2:00 PM to 5:00 PM

MADE EASY has taken due care in making solutions. If you find any discrepency/error/typo or want to contest the solution given by us, kindly send your suggested answer with detailed explanations at info@madeeasy.in

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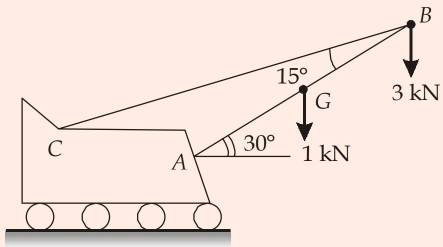
Mechanical Engineering Paper-II Analysis
ESE 2019 Main Examination

Sl.	Subjects	Total Marks
1.	Engineering Mechanics and SOM	74
2.	Engineering Materials	52
3.	Mechanisms and Machines	94
4.	Design of Machine Elements	72
5.	Manufacturing Engineering	66
6.	Industrial and Maintenance Engineering	50
7.	Mechatronics and Robotics	72
	Total	**480**

Scroll down for detailed solutions

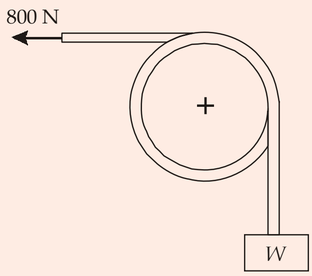
SECTION-A

1. (a) (i) A 10 m boom *AB* weighs 1 kN. The distance of centre of gravity is 5 m from *A*. For the position shown in the figure given below, determine the tension *T* in the cable and reaction at *A*:



[6 Marks]

(ii) A rope making $1\frac{1}{4}$ turns around a stationary horizontal drum is used to support a 4 weight as shown in the figure given below. If the coefficient of friction is 0.3, what range of weight can be supported by exerting an 800 N force at the end of the rope?



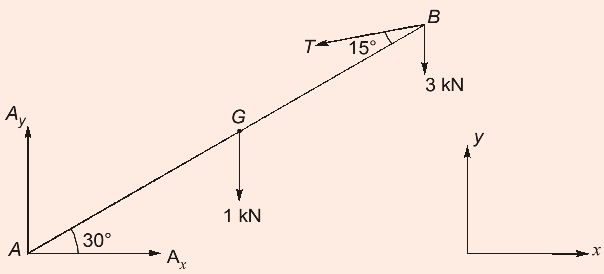
[6 Marks]

Solution:

(i)

Given AB = 10 m
Weight of AB = 1 kN
Distance of center of gravity of AB from A = 5 m
Tension = ?
Reaction at A = ?

Drawing *FBD* of *AB*



Taking moment about A,

$1 \times \cos 30° \times 5 + 3 \times \cos 30° \times 10 - T \times \sin 15° \times 10 = 0$

$\Rightarrow \quad T = 11.71 \text{ kN}$

Balancing forces in x direction

$$A_x = T\cos(30° - 15°) = 11.312 \text{ kN}$$

Balancing forces in y direction

$$A_y = 1 + 3 + T\sin(30° - 15°) = 7.031 \text{ kN}$$

$$\text{Reaction at } A,\ R_A = \sqrt{A_x^2 + A_y^2} = 13.319 \text{ kN}$$

$$R_A \text{ makes an angle } \theta \text{ with horizontal}, \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = 31.863°$$

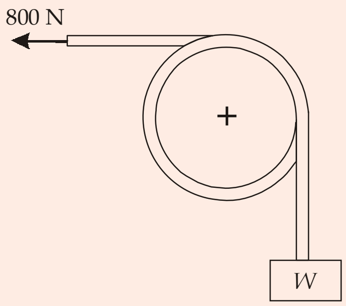
(ii)

Given, Turns rope making around the drum = $1\frac{1}{4}$ turns

Coefficient of friction, $\mu = 0.3$

Angle made by rope around the drum,

$$\theta = \frac{5}{4} \times 2\pi = \frac{5\pi}{2}$$



We can get maximum and minimum value of W by consider $W > 800$ N and $W < 800$ N respectively.

For maximum value

$$\frac{W_{max}}{800} = e^{\mu\theta}$$

$$\Rightarrow \quad W_{max} = e^{0.3 \times \frac{5\pi}{2}} \times 800 = 8440.579 \text{ N}$$

For minimum value

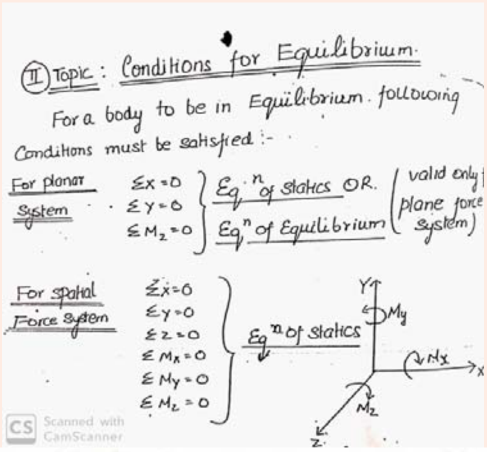
$$\frac{800}{W_{min}} = e^{\mu\theta}$$

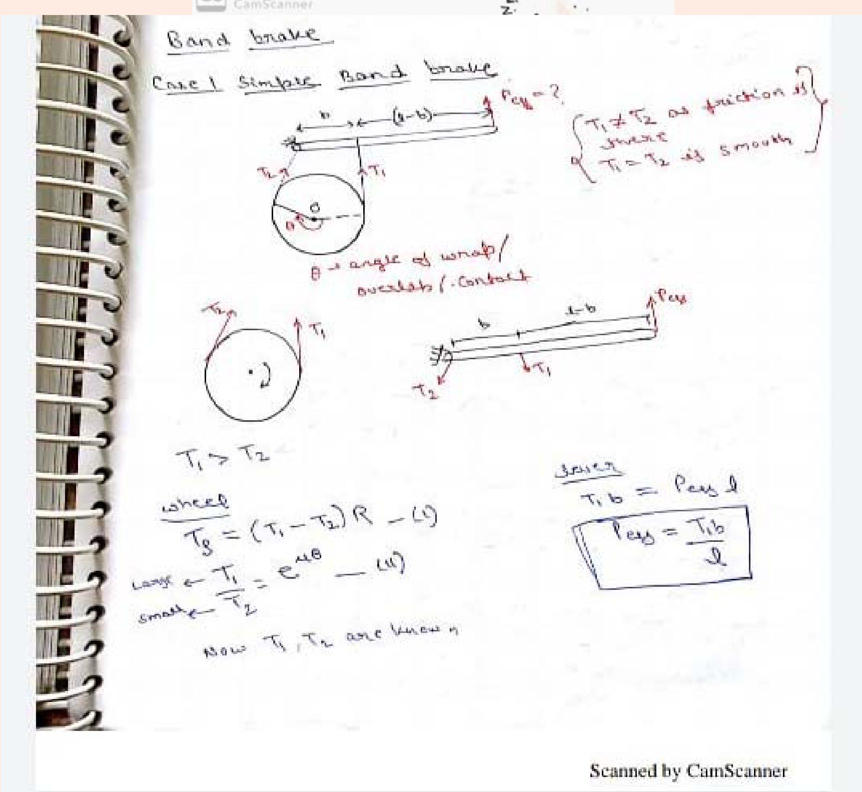
$$\Rightarrow \qquad W_{min} = \frac{800}{e^{0.3 \times \frac{5\pi}{2}}} = 75.824 \text{ N}$$

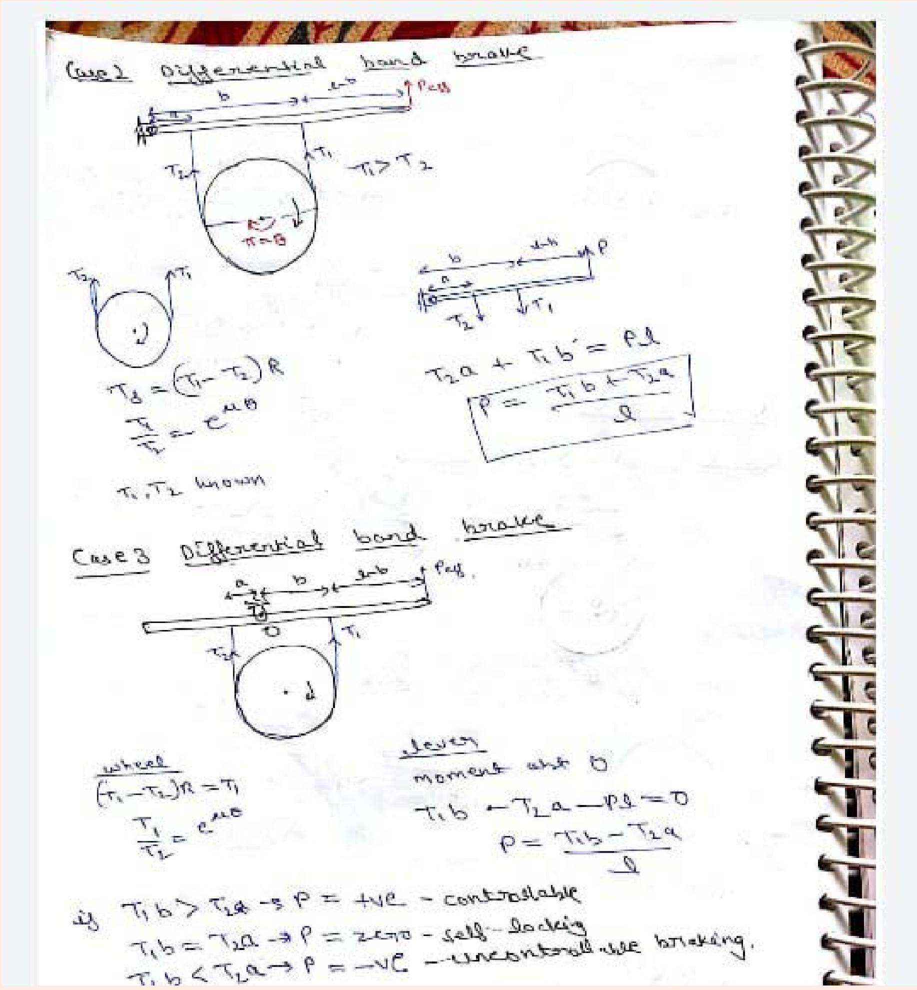
So range of W is from 75.824N to 8440.579N

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(c) of Test-5
- **MADE EASY Classnotes**







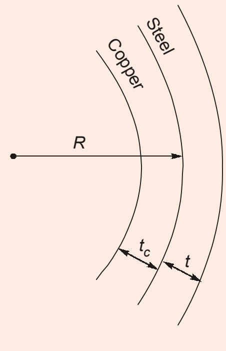
End of Solution

1. (b) A steel tube of 100 mm internal diameter and 10 mm wall thickness in a plant is lined internally with well-fitted copper sleeve of 2 mm wall thickness. If the composite tube is initially unstressed, calculate the hoop stress set up assumed to be uniform throughout the wall thickness, in a unit length of each part of the tube due to an increase in temperature of 100°C.

For steel, E = 208 GPa, α = 11×10^{-6}/°C

For copper, E = 104 GPa, α = 18×10^{-6}°C [12 Marks]

Solution:



Given, internal diameter of steel tube, d = 100 mm

Steel tube wall thickness, t = 10 mm

Copper sleeve wall thickens, t_c = 2 mm

Increase in temperature, ΔT = 100°C

Hoop stress is uniform throughout the thickness

E_S = 208 GPa

$\alpha_S = 11 \times 10^{-6}$/°C

E_C = 104 GPa

$$\alpha_C = 18 \times 10^{-6}/°C$$

As $\alpha_C > \alpha_S$, sleeve will try to expand more then tube hence pressure (P) will be acted between them.

$$R = \frac{100}{2} = 50 \text{ mm}$$

The change in radius at interface will be same for both copper sleeve and steel tube.

For copper tube

$$\frac{(dR)_C}{R} = \alpha_C \Delta T - \frac{PR}{t_c E_C} \quad .....(1)$$

For steel tube

$$\frac{(dR)_C}{R} = \alpha_s \Delta T + \frac{PR}{tE_S} \quad .....(2)$$

From (1) and (2)

$$\alpha_C \Delta T - \frac{PR}{t_c E_C} = \alpha_S \Delta T + \frac{PR}{tE_S}$$

$$\Rightarrow \quad (\alpha_C - \alpha_S)\,\Delta T = PR\left(\frac{1}{t_c E_C} + \frac{1}{tE_S}\right)$$

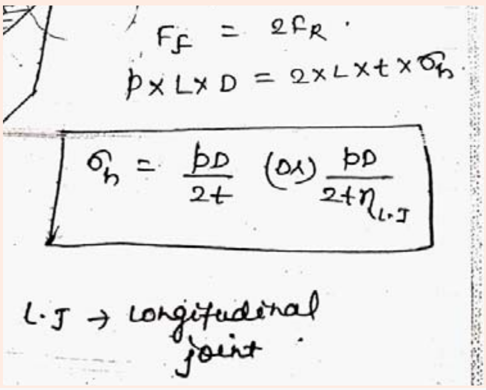
$$\Rightarrow \quad P = 2.647 \text{ MPa}$$

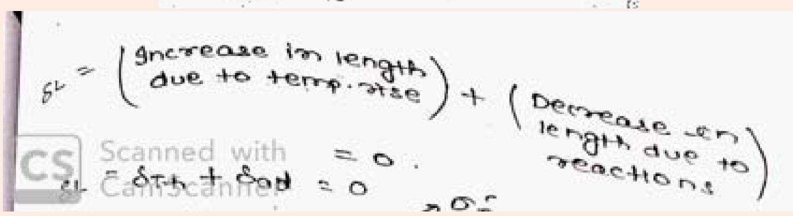
Hoop stress in steel tube, $\sigma_S = \frac{PR}{t} = 13.235$ MPa

Hoop stress in copper sleeve, $\sigma_C = \frac{PR}{t_C} = 66.175$ MPa

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.8(b) of Test-5
- **MADE EASY Classnotes**





End of Solution

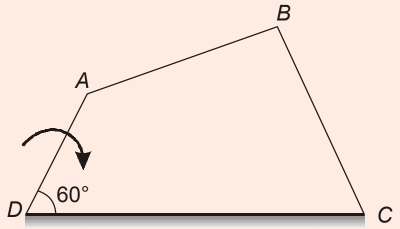
1. (c) (i) What is kinematic pair? How are kinematic pairs classified? Explain.

[6 Marks]

(ii) A four-bar mechanism has the following dimensions:

DA = 200 mm, CB = AB = 300 mm, DC = 500 mm

The link DC is fixed and the angle ADC is 60°. The driving link DA rotates uniformly at a speed of 100 r.p.m. clockwise and constant driving torque has the magnitude of 50 N-m. Determine the velocity of point B and angular velocity of the driven link CB. If the efficiency is 70%, calculate also the resisting torque:



[6 Marks]

Solution:

(i)

Kinematic pair: The connection between the two link is a joint or pair and this pair will be a kinematic pair if the relative motion between the links is a constrained motion (completely constrained motion or successfully constrained motion)

Kinematic pair can be classified according to

1. Nature of contact
2. Nature of mechanical constraint
3. Nature of relative motion

1. According to the nature of contact.

(a) Lower pair : When pair/joint having surface or area contact between the members. Example: Nut turning on a screw, shaft rotation in a bearing, all pairs of slider crank mechanism and universal joint.

(b) Higher pair: When pair/joint having line or point contact between members. Example: Wheel rolling on a surface, cam and follower pair, ball and roller bearings.

(c) Wrapping pair: When one link is wrapped over other link. Example: Belt and pulley, rope and pulley.

2. According to the nature of mechanical constraint.

(a) Closed pair/self closed pair: When the elements of pair are held mechanically. Generally all the lower pair and some of the higher pair are closed pairs.

(b) Force closed pair/ unclosed pair/forcefully closed pair: When the elements of a pair form contact either due to force of gravity or by some spring action, then it is known as force closed pair.

Example: cam and follower pair.

3. According to the nature of relative motion:

(a) Sliding pair: If the two links having a translatory/sliding motion to each other. Example: A rectangular rod in a rectangular hole in a prism is a sliding pair.

(b) Turning pair: If the two links having rotary/turning motion relative to each other then they form turning pair.
Example: Circular shaft inside a bearing forms turning pair pin joint.

(c) Rolling pair: When the links of pair have pure rolling motion.
Example: A rolling wheel on a flat surface in a ball bearing, the ball and the shaft constitute one rolling pair.

(d) Screw pair (helical pair): When links of pair having a turning as well as sliding motion between them.
Example: Lead screw and the nut of a lathe form screw pair.

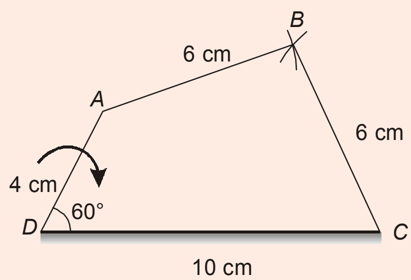
(e) Spherical pair: When one link in the form of sphere turns inside a fixed link then it forms spherical pair.
Example: The ball inside the socket forms spherical pair.

(ii)

As per given information :–
Configuration diagram :–
Assuming scale is

$$50 \text{ mm} = 1 \text{ cm}$$
$$DA = 200 \text{ mm after scaling} = 4 \text{ cm}$$
$$CB = 300 \text{ mm after scaling} = 6 \text{ cm}$$
$$DC = 500 \text{ mm after scaling} = 10 \text{ cm}$$

Configuration diagram:



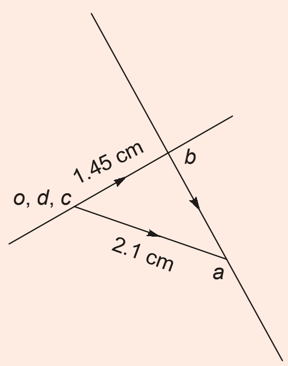
Far velocity polygon

$$V_{AD} = \frac{2\pi N}{60} \times AD = \frac{2\pi \times 100}{60} \times 0.2 = 2.094 \text{ m/s}$$

Assuming scale: $1 \text{ m/s} = 1 \text{ cm}$

$$da = 2.094 \text{ m/s after scaling } da = 2.094 \text{ cm} \approx 2.1 \text{ cm}$$

Velocity polygon :



From velocity polygon

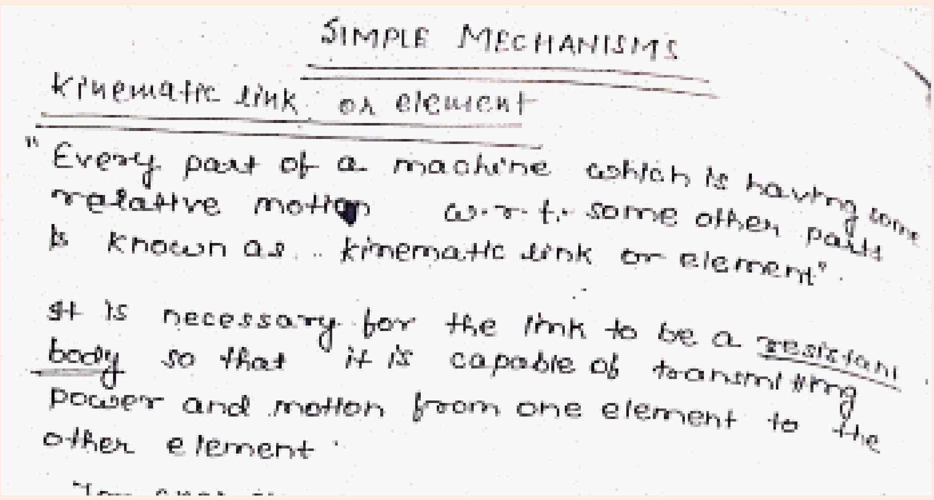
$$db = 1.45 \text{ cm after descaling}$$
$$db = 1.45 \times 1 \text{ m/s} = 1.45 \text{ m/s}$$
$$\mathbf{V_B = 1.45 \text{ m/s}}$$
$$V_{BC} = \omega_{BC} \times BC$$
$$1.45 = \omega_{BC} \times 0.3 \Rightarrow \omega_{BC} = 4.833 \text{ rad/s}$$

Point	With respect to	Procedure
A	*D*	Line $\perp^{ar}$ to *DA* link
B	*A*	Line $\perp^{ar}$ to *AB* link
B	*C*	Line $\perp^{ar}$ to *BC* link

$$\text{input torque} = 50 \text{ Nm.}$$
$$\omega_{input} = \frac{2\pi \times 100}{60} = 10.4719 \text{ rad/s}$$
$$\text{Power input, } P = T_{input} \times \omega_{input} = 50 \times 10.4719 \text{ Watt}$$
$$P_{input} = 523.5987 \text{ watt}$$
$$\eta_{efficiency} = \frac{\text{Power output}}{\text{Power inpout}}$$
$$T_{output} \times \omega_{output} = 0.7 \times 523.5987$$
$$T_{output} \text{ 'or' resisting torque} = \frac{0.7 \times 523.5987}{4.833}$$
$$\mathbf{T_{output} = 75.836 \text{ Nm}}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.1(c) of Test-11
- **ESE 2019 Mains Test Series:** Q.1(b) of Test-2
- **ESE 2019 Mains Workbook:** Q. 12 from Theory of Machines discussed in Class
- **MADE EASY Classnotes**

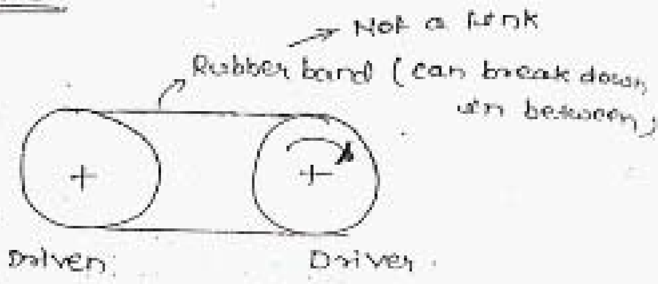

SIMPLE MECHANISMS

Kinematic link or element

"Every part of a machine which is having some relative motion w.r.t. some other parts is known as kinematic link or element".

It is necessary for the link to be a resistant body so that it is capable of transmitting power and motion from one element to the other element.

For example



Types of links

(1) Rigid link

Deformations are negligible.
For example :- crank, connecting Rod, piston cylinder.

(2) Flexible link

Deformations are there but are in permissible limit.

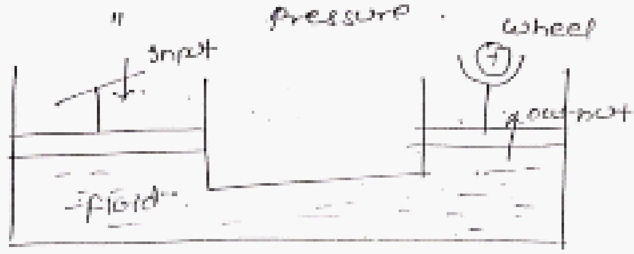
For example

Belt drives, chain drives, Rope drive.

Fluid links

when the power is transmitted because of fluid pressure.

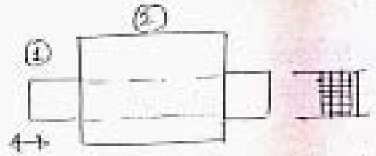
Eg. – Hydraulic crane
" lift
" Jack
" Ram
" Pressure

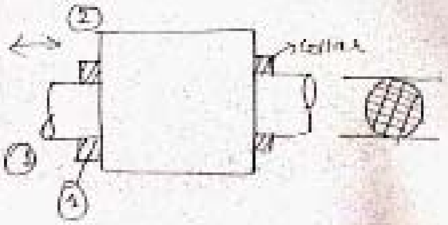


Different types of Relative motions

To analyze the relative motion
↓
(system) is having two links
(link 1, link 2)

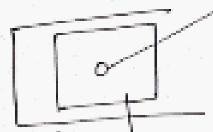
(i) Completely constraint motion → constraint motion
(only one relative motion)



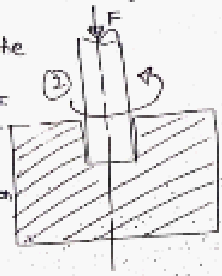


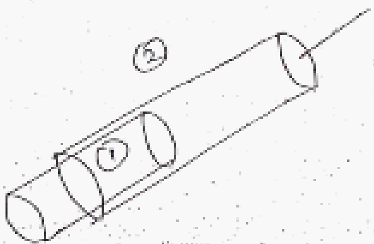
(ii) Constraint motion
↓ mean
Desired motion
↓
for unique and input these must be unique output.

Eg:-



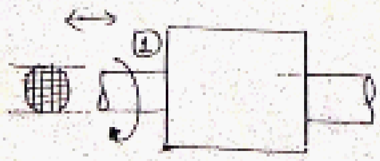
Here the heavy load F is resp. enable for the restriction of up-down motion.





Syringe (Here the person using syringe make the motion constraint).

Incompletely constraint motion



Both reciprocating & rotating motion

Kinematic pair

" The connection between the two links is always a joint or a pair but the pair will also be a kinematic pair if the relative motion b/w the links is a constraint motion".

(A) On the basis of the type of Relative motion

- Turning pair : (Revolute Pair) (Pin joint)

  Relative motion is pure turning.

  Eg:- knee joint in our body.

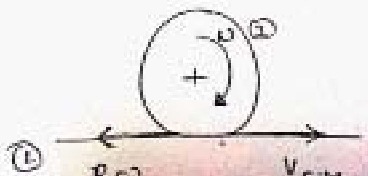
- Sliding pair : (Prismatic Pair)

  Relative motion is pure sliding

  Eg :- piston in cylinder.

- Rolling pair

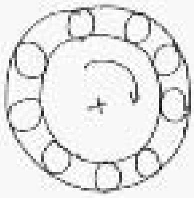
  Relative motion is pure rolling.
  means
  Rolling without slipping.



Condition for pure Rolling

$$V_{c \cdot M} = R\omega$$

Ball Bearing



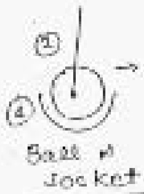
Between inner/outer race & balls → Rolling pair.
But b/w outer & inner race → Turning pair.

Screw pair

when the relative motion is over the threads.

Eg :- Nut & bolt.

Spherical pair



→ 3D rotation
↓
spherical motion.

Another eg :- our neck movement.

(B) According to type of contact

lower pair :- Surface contact
Higher pair :- line/point "

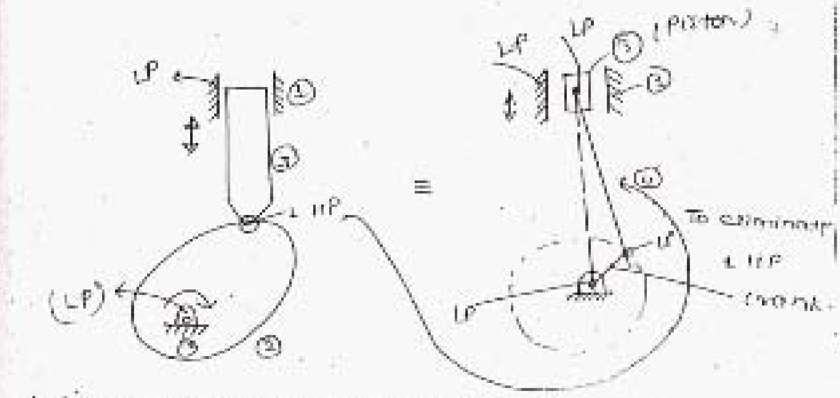
wrapping pair :- when one link is wrapped over other link.
↓
close to higher pair

Eg :- Belt & pulley, Rope & pulley.

1 Higher pair ≡ 2 Lower pair

[1 HP ≡ 2 LP]

For example

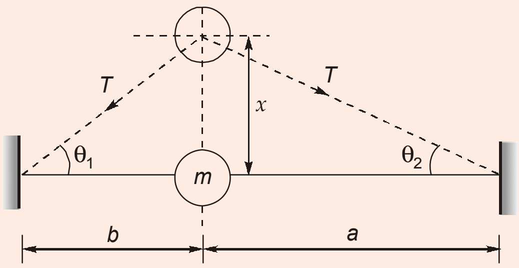


c) According to the type of closure

- Self closed pair (closed pair)
  ↳ permanent contact
  eg:- Piston in cylinder.
- Force closed pair (open pair)
  ↓
  Forceful contact
  For eg:-
  Higher pair in cam & follower.
  Automatic clutch opening system
  Door closer

End of Solution

1. (d) (i) The tension T in the spring as shown in the figure given below can be assumed to be constant for small displacements. Determine the natural frequency of the vertical vibrations of the spring and also show that the period of vibration is greatest when $a = b$:



[6 Marks]

(ii) A vibrating system has the following constants:

W = 19.62 kg, K = 8 kg/cm, C = 0.08 kg-s/cm

Determine–

(1) damping factor;

(2) natural frequency of damped oscillations;

(3) logarithmic decrement.

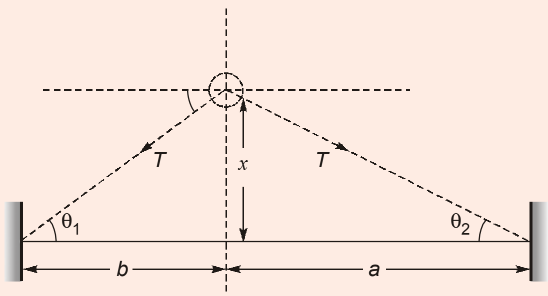
Here, W = Weight of mass K = Spring stiffness C = Damping coefficient

[2 × 3 = 6 Marks]

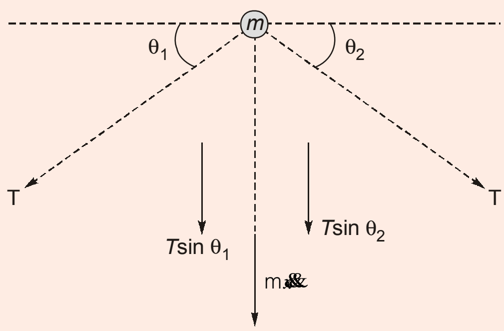
Solution:

(i)

As per given data



Free body diagram of mass after small displacement x



Thus the equation of motion will be

$$T\sin\theta_1 + T\sin\theta_2 + m\ddot{x} = 0$$

$$m\ddot{x} + T(\sin\theta_1 + \sin\theta_2) = 0 \quad .....(1)$$

From above configuration diagram for small θ

We can write

$$\sin\theta_1 \approx \tan\theta_1 = \frac{x}{b} = \theta_1$$

$$\sin\theta_2 \approx \tan\theta_2 = \frac{x}{a} = \theta_2$$

From equation (1)

$$m\ddot{x} + T(\theta_1 + \theta_2) = 0$$

$$m\ddot{x} + T\left(\frac{x}{b} + \frac{x}{a}\right) = 0$$

$$\ddot{x} + \frac{T}{m}\left(\frac{1}{b} + \frac{1}{a}\right) = 0 \quad .....(2)$$

equation (2) is the equation of a simple harmonic motion and is analogous to

$$\ddot{x} + \omega_n^2 x = 0 \quad .....(3)$$

From equation (2) & (3)

$$\omega_n^2 = \frac{T}{m}\left(\frac{1}{b} + \frac{1}{a}\right)$$

$$\omega_n = \sqrt{\frac{T}{m}\left(\frac{1}{b} + \frac{1}{a}\right)} \text{ rad/s}$$

$$\text{Period of vibration, } T_P = \frac{2\pi}{\omega_n}$$

$$T_P = \frac{2\pi}{\sqrt{\frac{T}{m}\left(\frac{a+b}{ab}\right)}}$$

$$T_P = 2\pi\sqrt{\frac{m(ab)}{T(a+b)}}$$

For period of vibration is greatest

$$T_P = 2\pi\sqrt{\frac{m}{T}} \times \sqrt{\frac{(ab)}{(a+b)}} = \text{constant}\sqrt{\frac{ab}{a+b}}$$

$\because \{a+b = \text{constant}\}$

$\{a+b = l$

$$T_b = \frac{\text{costant}\sqrt{a(l-a)}}{\sqrt{l}}$$

For maximum

$$y = a\,(l - a) \text{ should be maximum}$$

$$\frac{dy}{da} = l - 2a = 0$$

$$a = \frac{l}{2}$$

$$a = \frac{a+b}{2}$$

$$2a = a+b \Rightarrow \quad a = b \text{ (proved)}$$

(ii)

As per given data

For the vibrating system,

$$W = 19.62 \text{ kg} = 19.62 \times 9.81 = 192.4722 \text{ N}$$

$$k = \frac{8\text{kg}}{\text{cm}} = \frac{8\times 9.81}{10^{-2}}\frac{\text{N}}{\text{m}} = 7848\frac{\text{N}}{\text{m}}$$

$$c = 0.08 \text{ kg-s/cm} = \frac{0.08\times 9.81}{10^{-2}} = 78.48 \text{ N-s/m}$$

(i) $$2\xi\omega_n = \frac{c}{m}$$

$$\xi = \frac{c}{2m\omega_n}$$

$$\xi = \frac{c}{2\sqrt{km}} = \frac{78.48}{2\sqrt{7848\times 19.62}} = 0.1$$

$$\xi = 0.1$$

(ii) Natural frequency $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{7848}{19.62}} = 20$ rad/s

Natural frequency of damped vibration $= \omega_d = \sqrt{1-\xi^2}\,(\omega_n)$

$$\omega_d = \sqrt{1-0.1^2}\times 20$$

$$\omega_d = 19.899 \text{ rad/s}$$

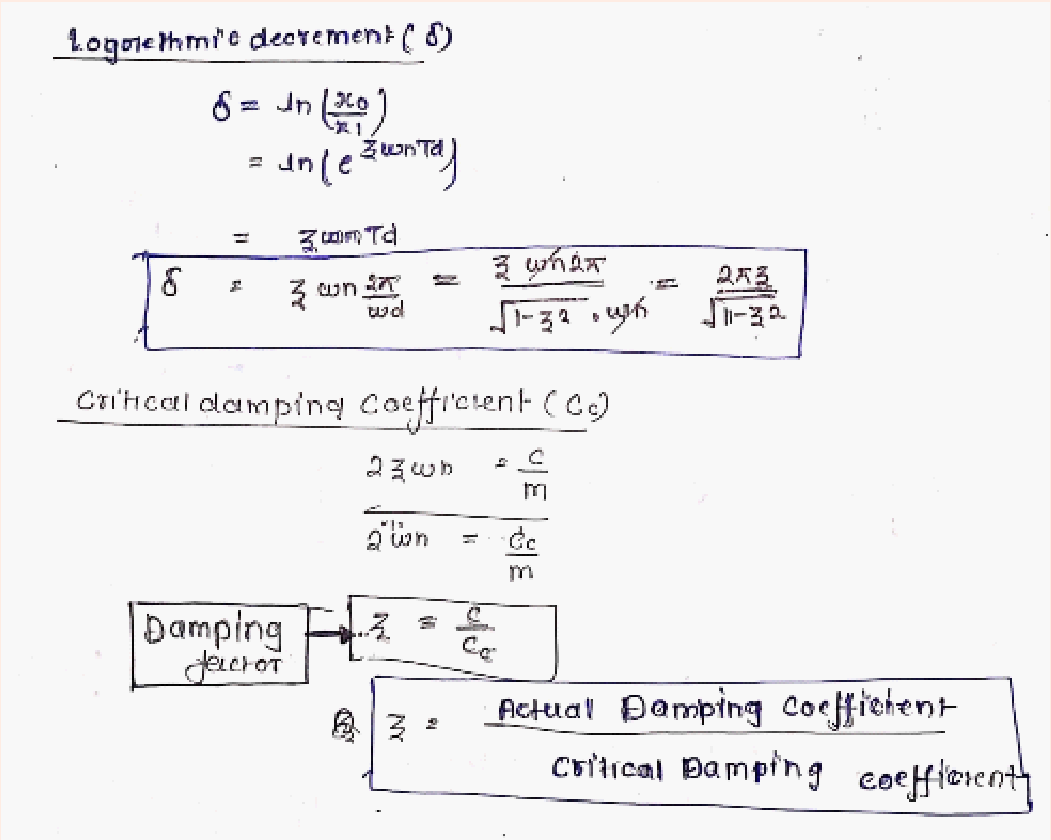
(iii) logarithmic decrement (δ)

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2\pi\times 0.1}{\sqrt{1-0.1^2}} = 0.6314$$

$$\delta = 0.6314$$

MADE EASY Source

- **ESE Mains 2019 Workbook:** Q.69 of TOM (Page no. 66) discussed in Class
- **MADE EASY Classnotes**



Logarithmic decrement (δ)

$$\delta = \ln\left(\frac{x_0}{x_1}\right) = \ln\left(e^{\zeta \omega_n T_d}\right) = \zeta \omega_n T_d$$

$$\delta = \zeta \omega_n \frac{2\pi}{\omega_d} = \frac{\zeta \omega_n 2\pi}{\sqrt{1-\zeta^2}\cdot \omega_n} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Critical damping coefficient (C_c)

$$2\zeta\omega_n = \frac{c}{m}$$

$$2\omega_n = \frac{C_c}{m}$$

Damping factor → $\zeta = \frac{c}{C_c}$

$$\zeta = \frac{\text{Actual Damping Coefficient}}{\text{Critical Damping coefficient}}$$

End of Solution

1. (e) Differentiate between 'shaft' and 'axle'. A solid shaft of diameter d is used in power transmission. Due to the modification of existing transmission system, the solid shaft is required to be replaced by a hollow shaft of the same material and equally strong in torsion. The weight of the hollow shaft per unit length is to be half of the solid shaft. Determine the outer diameter of the hollow shaft in terms of d.

[12 Marks]

Solution:

Shaft	Axle
Shafts are rotating members which are subjected to bending moments, twisting moments and axial loads.	Axles are rotating or non rotating members which are subjected to only bending moments and axial load due to members supported by it
It does transmit torque or power.	It does not transmit torque or power.
It does bend and twist.	It does not twist, it only bends.

For equally strong in torsion polar modulus should be equal so

$$(Z_P)_{\text{solid shaft}} = (Z_P)_{\text{hollow shaft}}$$

$$\Rightarrow \quad \frac{\pi d^3}{16} = \frac{\pi\left(d_o^4 - d_i^4\right)}{16 d_0}$$

(Here, d_o and d_i are outer and inner diameter respectively)

$$\Rightarrow \quad d^3 = \frac{d_o^4 - d_i^4}{d_o} \qquad .....(1)$$

As given weight of the hollow shaft is half of the solid shaft for a unit length, So

$$\frac{1}{2}\left(\rho\frac{\pi d^2}{4}\right) = \rho\frac{\pi\left(d_o^2 - d_i^2\right)}{4}$$

$$\Rightarrow \quad \frac{d^2}{2} = d_o^2 - d_i^2 \qquad .....(2)$$

From eq^n (1) and (2) we get

$$d^3 d_o = d_o^4 - \left(d_o^2 - \frac{d^2}{2}\right)^2$$

$$d^3 d_o = d_o^4 - d_o^4 - \frac{d^4}{4} + d_o^2 d^2$$

$$d_o^2 d^2 - d^3 d_o - \frac{d^4}{4} = 0$$

$$\Rightarrow \quad d_o = \frac{d^3 \pm \sqrt{d^6 + d^6}}{2d^2}$$

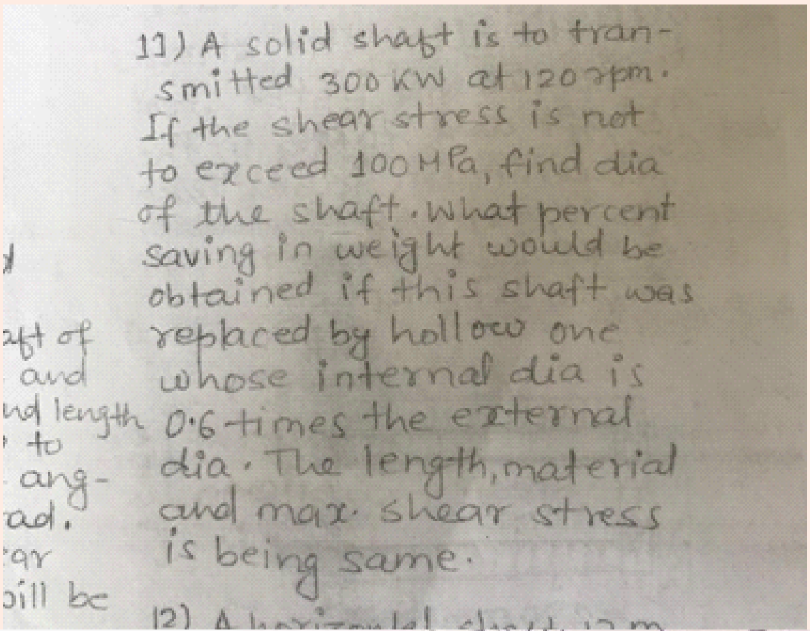
$$d_o = \left(\frac{1+\sqrt{2}}{2}\right)d$$

$$d_o = \left(\frac{1-\sqrt{2}}{2}\right)d \qquad \text{(not possible)}$$

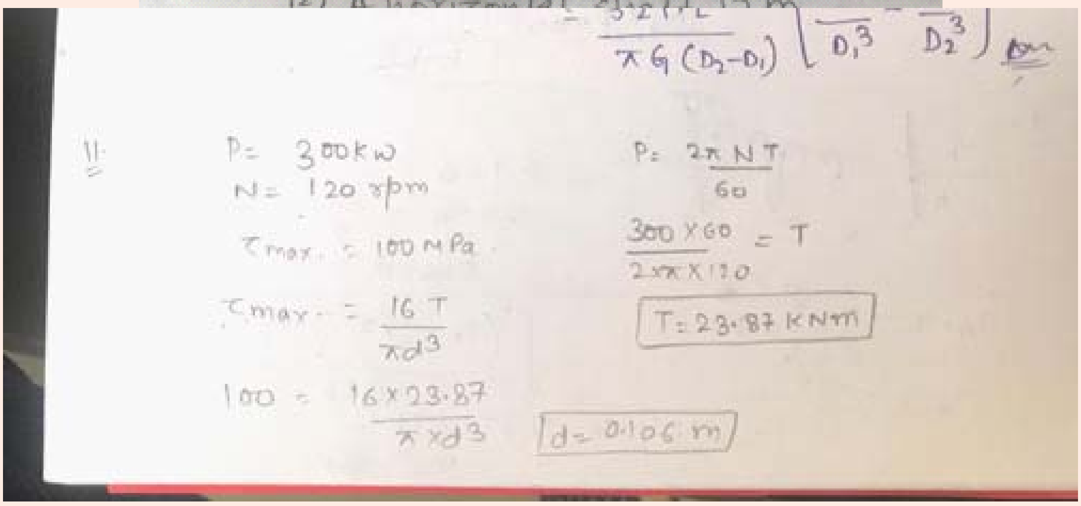
So outer diameter of hollow shaft will be 1.207 d.

MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.29 of SOM (Solved Que.) discussed in Class
- **MADE EASY Classnotes**



11) A solid shaft is to transmitted 300 kW at 120 rpm. If the shear stress is not to exceed 100 MPa, find dia of the shaft. What percent saving in weight would be obtained if this shaft was replaced by hollow one whose internal dia is 0·6 times the external dia. The length, material and max. shear stress is being same.



11.

$P = 300\text{ kW}$

$N = 120\text{ rpm}$

$\tau_{max} = 100\text{ MPa}$

$\tau_{max} = \dfrac{16T}{\pi d^3}$

$100 = \dfrac{16 \times 23.87}{\pi \times d^3}$ $\quad$ $d = 0.106\text{ m}$

$P = \dfrac{2\pi NT}{60}$

$\dfrac{300 \times 60}{2\pi \times 120} = T$

$T = 23.87\text{ kNm}$

$d_o = 0.106$, $d_i = 0.6 \times 0.106$

$T = 23.87$ kN-m

$\tau_{max} = 100$ MPa

$\frac{d_i}{d_o} = 0.6$

$$\frac{16T}{\pi d_o^3}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = 100$$

$$\frac{16 \times 23.87}{\pi (d_o)^3}\left[1 - (0.6)^4\right] = 10$$

$d_o = 0.111$ m
$d_i = 0.067$ m

$$\% \text{ Saving in wt.} = \frac{W_S - W_H}{W_S} \times 100\%$$

$$= \frac{V_S - V_H}{V_S} \times 100\%$$

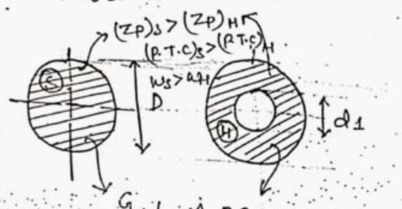
$$= \frac{A_S - A_H}{A_S} \times 100\%$$

$$= \frac{d^2 - (d_o^2 - d_i^2)}{d^2} \times 100\%$$

$= 30.29\%$ Ans

$l = 12$ m

(8) Determine the ratio of power transmission capacities of solid and circular shaft rotating at same R.P.M. Assume diameters, length and materials are same.



Area ↑ → MOI ↑ (IInd moment of area)

Drawback of solid → wt. is more, wt. ∝ Area.

$$\frac{(P)_S}{(P)_H} = \frac{(T\omega)_S}{(T\omega)_H} = \frac{T_S}{T_H}$$

$$= \frac{\left(\frac{\pi}{16} d^3 \tau_{per}\right)_S}{\left[\frac{\pi}{16} D^3 (1 - K^4)\tau_{per}\right]_H} = \frac{1}{1 - K^4}$$

(9) $\dfrac{\theta_B}{\theta_A} = \dfrac{(\phi L/R)_B}{(\phi L/R)_A} = \dfrac{\phi_B}{\phi_A} \times \dfrac{L_B}{L_A} = 2 \times \dfrac{1}{2} = 1.$

$$\boxed{\theta_B = \theta_A = \theta}$$

Analysis of Torsion eqn

$$\boxed{\underbrace{\frac{T_R}{J}}_{A} = \underbrace{\frac{\tau_{max}}{R}}_{B} = \underbrace{\frac{G\theta}{L}}_{C}}$$

Case 1 :-

$$(A) = (B)$$

$$\frac{T_R}{J} = \frac{\tau_{max}}{R}$$

$$\tau_{max} = \frac{T_R R}{J}$$

$$\Rightarrow \tau_{max} = \frac{T}{(J/R)} = \frac{T}{Z_p}$$

$$\boxed{\tau_{max} = \frac{T}{Z_p}} \quad \text{— (I)}$$

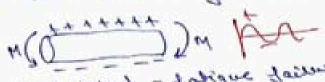
- τ_{max} -
- independent of material and length of shaft

$$\boxed{Z_p = \frac{J}{R} = \text{Polar section modulus.}}$$

	A	I	J	Z_{NA}	Z_p
d	$\frac{\pi}{4}d^2$	$\frac{\pi}{64}d^4$	$\frac{\pi}{32}d^4$	$\frac{\pi}{32}d^3$	$\frac{\pi}{16}d^3$
d, D, $\boxed{K = \frac{d}{D} < 1}$	$\frac{\pi}{4}D^2(1-K^2)$	$\frac{\pi}{64}D^4(1-K^4)$	$\frac{\pi}{32}D^4(1-K^4)$	$\frac{\pi}{32}D^3(1-K^4)$	$\frac{\pi}{16}D^3(1-K^4)$

11th Aug

Axle	Shaft
Always bending; No power transmit ⇓ No torque ⇓ No twisting	– power transmission ⇓ Always torque ⇓ Always twisting
Axle are design by bending only	– but also bending comes



if rotated – fatigue failure

Hence practical shaft are designed by "fatigue" + "Rigidity design" + "strength design"

End of Solution

2. (a) The turbine rotor of a ship has a mass of 3000 kg. It has a radius of gyration of 0.45 m and a speed of 2000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect on the ship—
   (i) when the ship is steering to the left on a curve of 100 m radius at a speed of 30 km/hr;
   (ii) when the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

[20 Marks]

Solution:

As per given data

mass, $m = 3000$ kg

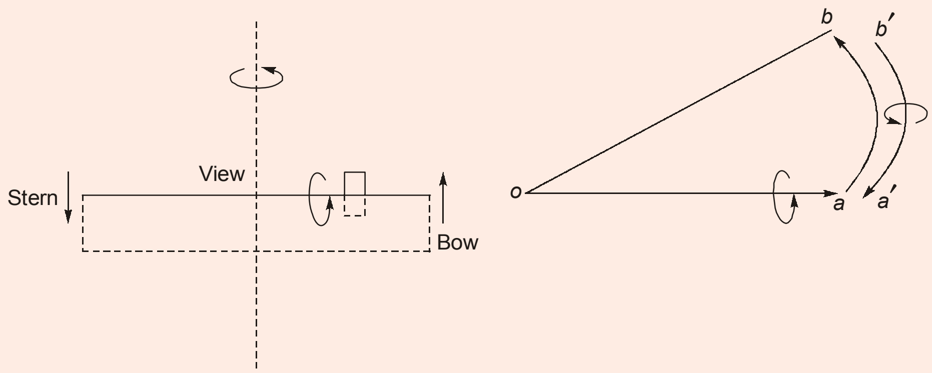
Radius of gyration, $k = 0.45$ m

Speed, $N = 2000$ rpm

$$\Rightarrow \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.439 \text{ rad/s}$$

$$I = mk^2 = 3000 \times (0.45)^2$$

$$\omega = 209.439 \text{ rad/s}$$



(i) Radius of curve, $R = 100$ m

$$\text{Speed, } V = 30 \text{ km/hr} \Rightarrow V = \frac{30 \times 1000}{3600} \text{ m/s}$$

$$V = 8.333 \text{ m/s}$$

$$\omega_P = \frac{V}{R} = \frac{8.333}{100} = 0.08333 \text{ rad/s}$$

$$\text{Gyroscopic couple} = I\omega\omega_p$$

$$= 607.5 \times 209.439 \times 0.0833$$

$$\text{Gyroscopic couple} = 10602.425 \text{ Nm}$$

The effect is to lower the stern and raise the bow

(ii) The pitching motion is simple harmonic

The period of pitching = 40 seconds

Total angular displacement between two extreme

Position = 12°

So, $$\phi = \frac{\text{total angular displacment}}{2}$$

$$\phi = \frac{12°}{2} = 6°$$

The ship pitches 6° above and 6° below the normal position.

$$\phi = 6° \Rightarrow \phi = 6 \times \frac{\pi}{180} = 0.1047 \text{ rad}$$

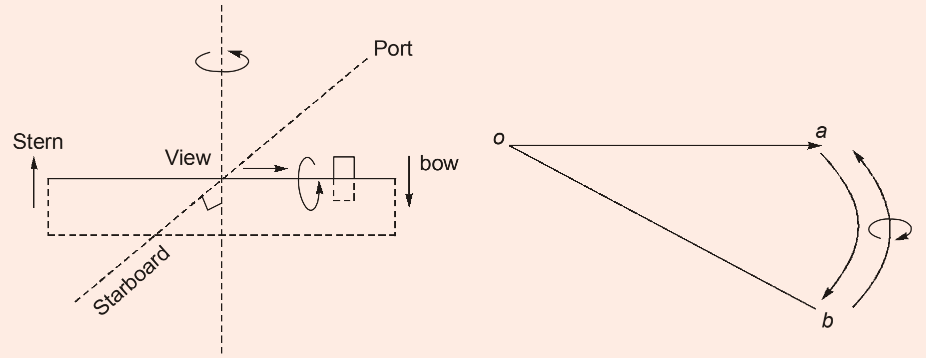
$$T = 40 \text{ sec.}$$

∴ $$\omega_O = \frac{2\pi}{T} = \frac{2\pi}{40} = 0.1570 \text{ rad/s}$$

$$\omega_P = \phi.\ \omega_O = 0.1047 \times 0.1570 = 0.01644 \text{ rad/s}$$

Gyroscopic couple = $I\omega\omega_P$ = 607.5 × 209.439 × 0.01644

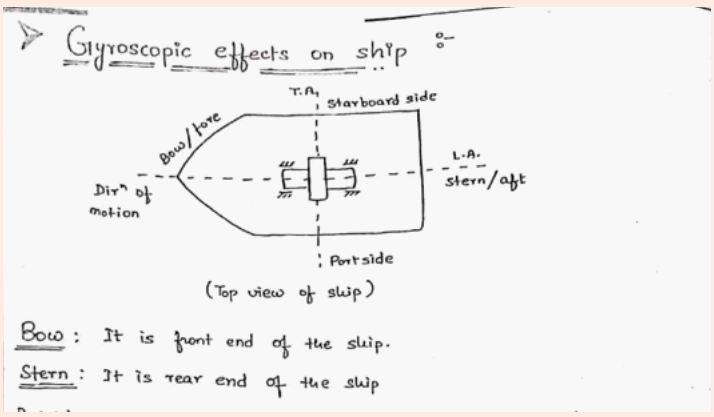
Gyroscopic couple = 2091.730 Nm



As the bow falling during pitching the ship would turn towards left or port side.

MADE EASY Source

- **ESE Mains 2019 Workbook:** Q.56 (Page no. 77) of TOM discussed in Class
- **ESE Prelims 2019 Workbook:** Q.22 (Gyroscope chapter) of TOM discussed in Class
- **MADE EASY Classnotes**



Portside : It is left hand side of ship when looking from stern.

Starboard side : It is right hand side of ship when looking from stern.

Steering :- It is taking turn towards right or left when moving forward.

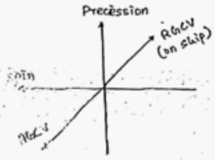
Pitching → It is up and down motion of bow of ship about transverse axis.

Rolling → It is angular oscillational of ship about longitudinal axis.

→ Rotor is spinning in CW sense as viewed from stern,

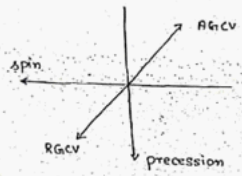
i). Steering

a). towards left



Effect : Bow will be raised
Stern " " depressed

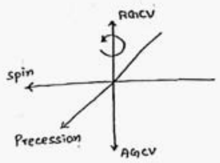
b). towards right



Effect : Bow will be depressed
Stern —"— raised.

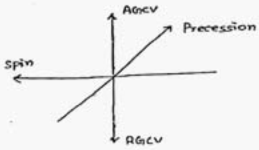
ii). Pitching

a). Bow is going down



Effect : Bow will turn towards portside
Stern will turn towards starboard side.
'or' ship will rotate in ACW sense as viewed from top.

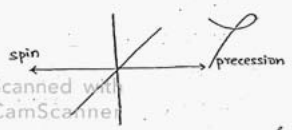
b). Bow is going up



Effect : Bow will turn towards starboard side.
Stern will turn towards portside.

(iii). Rolling

a). Ship is rolling in ACW sense as viewed from stern.



As spin vector and precession vector are coming out parallel, so actually it is not precession vector, that is why there are no Gyroscopic effects on the ship under rolling.

End of Solution

2. (b) A steel cantilever of length 2 m of circular cross-section, 50 mm in diameter, carries uniformly distributed load of intensity w. What is the maximum value of w so that deflection at free end is not to exceed 1 mm? Find out the slope at free end. Take E = 200 GPa.

[20 Marks]

Solution:

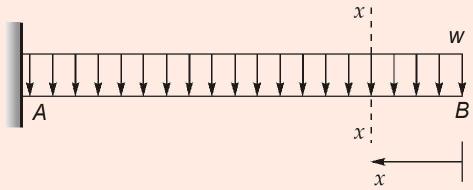
Given length, $L = 2$ m

Diameter, $d = 50$ mm

$\Rightarrow$ $r = 25$ mm

Deflection, $\delta < 1$ mm

$E = 200$ GPa



Moment at x, $M_x = \dfrac{-wx^2}{2}$

According to double integration

$$\frac{EId^2y}{dx^2} = M_x$$

$$\frac{EId^2y}{dx^2} = \frac{-wx^2}{2}$$

$$\Rightarrow \quad \frac{EIdy}{dx} = \frac{-wx^3}{6} + c_3$$

At $x = L$, $\dfrac{dy}{dx} = 0$

So we get $0 = \dfrac{-wL^3}{6} + c_3$

$$\Rightarrow \quad \frac{EIdy}{dx} = \frac{-wx^3}{6} + \frac{wL^3}{6} \quad \text{...(1)}$$

$$\Rightarrow \quad EIy = \frac{-wx^4}{24} + \frac{wL^3x}{6} + c_4$$

At $x = L$, $y = 0$

So we get $0 = \dfrac{-wL^4}{24} + \dfrac{wL^4}{6} + c_4$

$$c_4 = \frac{-wL^4}{8}$$

$$\Rightarrow \quad EIy = \frac{-wx^4}{24} + \frac{wL^3x}{6} - \frac{wL^4}{8}$$

At free end,

$$y = \frac{-wL^4}{8EI}$$

So for maximum value of w,

$$\frac{wL^4}{8EI} = 1 \text{ mm}$$

$$\Rightarrow \frac{w(2)^4}{8\times200\times10^9\times\frac{\pi}{4}(0.025)^4} = 10^{-3}$$

$$\Rightarrow \qquad w = 30.6796 \text{ N/m}$$

Using eq. (1)

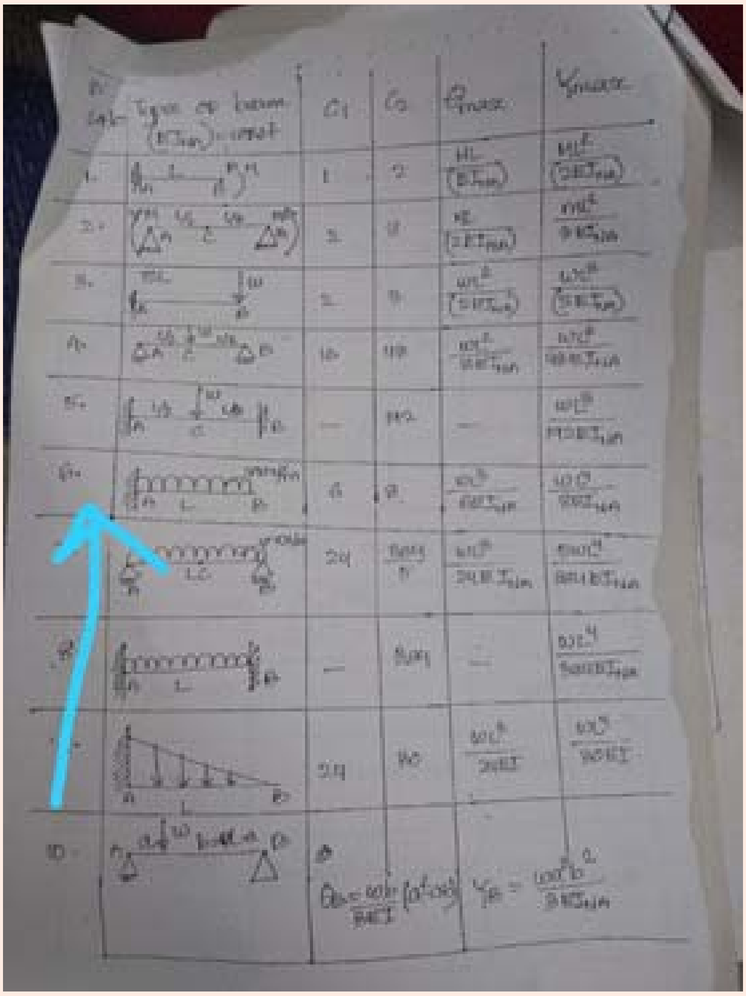
$$\frac{EIdy}{dx} = \frac{-wx^3}{6}+\frac{wL^3}{6}$$

Slope at free end,

$$\theta_B = \frac{wL^3}{6EI} = \frac{30.6796\times2^3}{6\times200\times10^9\times\frac{\pi}{4}(0.025)^4} = 6.667\times10^{-4} \text{ rad}$$

MADE EASY Source

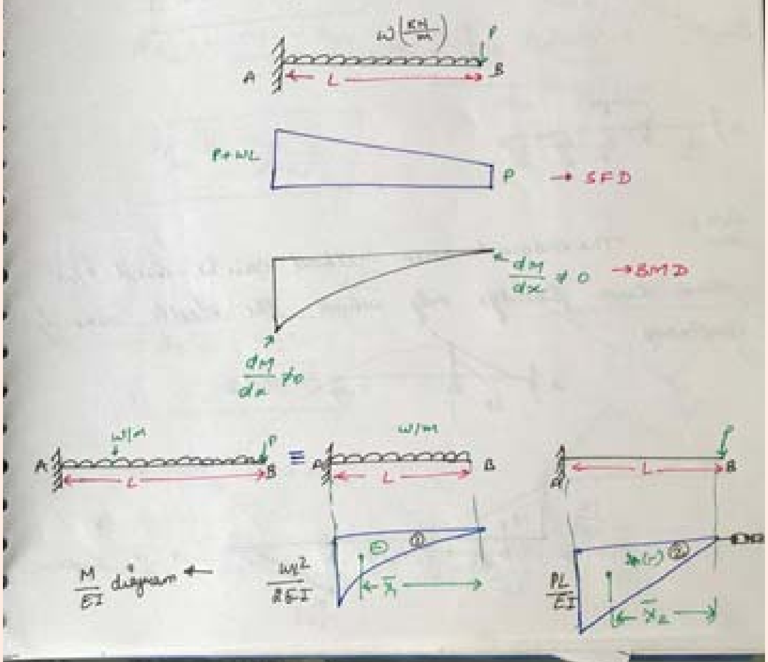
- **MADE EASY Classnotes**



$\bar{x} = \frac{3}{4} b$

$\bar{y} = \frac{3}{10} h$

Que. 11 Determine slope & deflection at B



$$A_1 = -\frac{1}{3} \times L \times \frac{wL^2}{2EI} = -\frac{wL^3}{6EI}$$

$$A_2 = -\frac{1}{2} \times L \times \frac{PL}{EI} = \frac{PL^2}{2EI}$$

$$\bar{x}_1 = \frac{3}{4} L \; ; \; \bar{x}_2 = \frac{2}{3} L$$

$$\theta_B - \theta_A^{\,0} = A_1 + A_2 = \boxed{-\frac{wL^3}{6EI} - \frac{PL^2}{2EI}}$$

$$\Delta_B = \Delta_A^{\,0} + \theta_A^{\,0} AB + \Delta_{B/A}$$

$$= A_1\bar{x}_1 + A_2\bar{x}_2 \Rightarrow \boxed{\frac{-wL^4}{8EI} - \frac{PL^3}{3EI}}$$

→ Do this by conjugate beam

End of Solution

2. (c) A thick cylinder is subjected to both internal and external pressure. The internal diameter of the cylinder is 200 mm and the external diameter is 250 mm. If the maximum permissible stress is 30 N/mm^2 and the external pressure is 8 N/mm^2, determine the intensity of internal radial pressure.

[20 Marks]

Solution:

Given a thick cylinder is subjected to both internal, P_i and external pressure P_e.

Internal diameter, d_1 = 200 mm

r_1 = 100 mm

External diameter, d_2 = 250 mm

r_2 = 125 mm

Maximum permissible stress, σ_{per} = 30 N/mm^2

P_e = 8 N/mm^2

For internal pressure, hoop stress can be written as

$$(\sigma_h)_i = \frac{P_i R_1^2}{(r_2^2 - r_1^2)} + \frac{P_i R_1^2 r_2^2}{(r_2^2 - r_1^2) r^2}$$

For external pressure, hoop stress can be written as

$$(\sigma_h)_e = \frac{-P_e r_2^2}{(r_2^2 - r_1^2)} - \frac{P_e r_1^2 r_2^2}{(r_2^2 - r_1^2) r^2}$$

(Negative sign as hoop stress for external pressure is compressive in nature).

For combined external and internal pressure, hoop stress can be written as,

$$\sigma_h = \frac{P_i r_1^2 - P_e r_2^2}{(r_2^2 - r_1^2)} + \frac{(P_i - P_e) r_1^2 r_2^2}{(r_2^2 - r_1^2) r^2}$$

$$\Rightarrow \quad \sigma_h = \frac{P_i \times 100^2 - 8 \times 125^2}{(125^2 - 100^2)} + \frac{(P_i - 8) 100^2 \times 125^2}{(125^2 - 100^2) \times r^2}$$

$$\Rightarrow \quad \sigma_h = \left(\frac{P_i - 12.5}{0.5625}\right) + \frac{(P_i - 8)}{3.6 \times 10^{-5} \times r^2}$$

If σ_h is equal to σ_{per} then

$$30 = \frac{(P_i - 12.5)}{0.5625} + \frac{(P_i - 8)}{3.6 \times 10^{-5} \times (100)^2}$$

(maximum hoop stress at $r = r_1$)

P_i = 16.341 N/mm^2

So, maximum value of P_i can be 16.341 N/mm^2

MADE EASY Source ____________________

- **ESE 2019 Mains Test Series:** Q.1(b) of Test-4
- **ESE 2019 Mains Workbook:** Q.34 of SOM (Solved Que.) discussed in Class

- **MADE EASY Classnotes:** Mains Batch (Case-III)

Case III :- Internal pr. 'P_r' and external Pr. 'P_R'

Boundary cond^n

at $x = R_i$, $P_x = P_r$ and $x = R_o$, $P_x = P_R$

$$\therefore \; P_r = \frac{B}{R_i^2} - A \quad \text{and} \quad P_R = \frac{B}{R_o^2} - A$$

$$\therefore \; B = (P_r - P_R)\cdot\frac{R_i^2 R_o^2}{R_o^2 - R_i^2} \quad \text{and} \quad A = \frac{P_r R_i^2 - P_R R_o^2}{R_o^2 - R_i^2}$$

In this case also max. Hoop stress will occur at $x = R_i$

$$\sigma_{h_{max}} = \frac{P_r(R_o^2 - R_i^2) - 2P_R \cdot R_i^2}{R_o^2 - R_i^2}$$

Case IV :- Solid circular shaft subjected to external radial pr. 'P'.

$$P_x = \frac{B}{x^2} - A, \quad \sigma_{h_x} = \frac{B}{x^2} + A.$$

now at $x = 0$, $P_x = \infty$, which is not possible $\therefore$ $B = 0$.

hence $P_x = -A = P = -\sigma_{h_x}$

It means intensity of radial pr is constant everywhere and its value is equal to 'P'. Also the intensity of hoop stress is const. everywhere and is compressive.

End of Solution

3. (a) A horizontal gas engine running at 200 r.p.m. has a bore of 200 mm and a stroke of 400 mm. The connecting rod is 900 mm long and the reciprocating parts weigh 20 kg. When the crank has turned through an angle of 30° from the inner dead centre, the gas pressures on the cover and the crank sides are 500 kN/m^2 and 60 kN/m^2 respectively. The diameter of the piston rod is 40 mm. Determine—

(i) Turning moment on the crankshaft;

(ii) Thrust on the bearings;

(iii) Acceleration of the flywheel which has a mass of 8 kg and radius of gyration of 600 mm while the power of the engine is 22 kW.

[20 Marks]

Solution:

As per given data

Reciprocating mass, $m = 20$ kg.

Engine speed, $N = 200 \text{ rpm} \Rightarrow \omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 200}{60} = 20.943 \text{ rad/s}$

Bore diameter, $D = 200 \text{ mm} = 0.2 \text{ m}$

Stroke length, $2r = 400 \text{ mm} = 0.4 \text{ m}$

Crank radius, $r = \dfrac{0.4}{2} = 0.2 \text{ m}$

Connecting rod length, $l = 900 \text{ mm} = 0.9 \text{ m}$

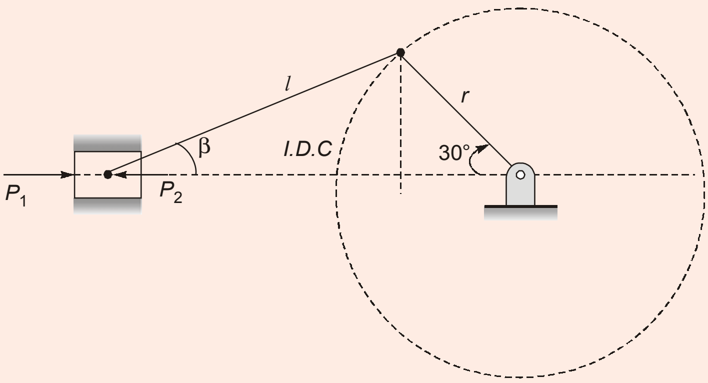
$\theta = 30°$ from I.D.C

$$n = \frac{l}{r} = \frac{0.9}{0.2} = 4.5$$

Gas pressure, $p_1 = 500 \text{ kN/m}^2$

Gas pressure, $p_2 = 60 \text{ kN/m}^2$

diameter of piston rod, $d = 40$ mm



$\omega = 20.943$ rad/s

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 30°}{4.5} = 0.1111$$

$\beta = 6.379°$

Net pressure force $F_P = (P_1 A_1 - P_2 A_2)$

$$F_P = 500 \times 10^3 \times \frac{\pi}{4} \times D^2 - 60 \times 10^3 \times \frac{\pi}{4}\left(D^2 - d^2\right)$$

$$F_P = 500 \times 10^3 \times \frac{\pi}{4} \times 0.2^2 - 60 \times 10^3 \times \frac{\pi}{4}\left(0.2^2 - 0.04^2\right)$$

$F_P = 1389\,0.4059\ N$

Inertia force,, $F_I = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$F_I = 20 \times 0.2 \times 20.943^2 \left(\cos 30° + \frac{\cos 60°}{4.5}\right)$$

$F_I = 1714.324$ N

Piston effort, $F = F_P - F_I$

$F = 13898.4059 - 1714.324$

$F = 12184.0814\ N$

(i) Turning moment, $T = \frac{F}{\cos\beta}\sin(\theta + \beta) \times r$

$$T = \frac{12184.0814}{\cos 6.379°}\sin(30° + 6.379°) \times 0.2$$

Turning moment, $T = 1454.338$ Nm

(ii) Thrust on the bearings, F_r

$$F_r = \frac{F}{\cos\beta}\cos(\theta + \beta)$$

$$= \frac{12184.0814}{\cos 6.379°}\cos(30° + 6.379°)$$

$F_r = 9870.653\ N$

(iii) Power, $P = 22$ kW

Radius of gyration, $k = 600$ mm $= 0.6$ m

mass of flywheel = 8 kg.

acceleration torque = (Turning moment torque – Resisting torque)

$P = T \times \omega$

$22000 = T \times 20.943$

$T = 1050.470$ Nm

Acceleration torque = 1454.338 – 1050.470 Nm

$I.d = 403.867$ Nm

$M.K^2.\alpha = 403.867$

$8 \times 0.6^2.\alpha = 403.867$

acceleration of flywheel, $\alpha = 140.231$ rad/s^2

MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.33 of TOM(Page no 73) discussed in Class

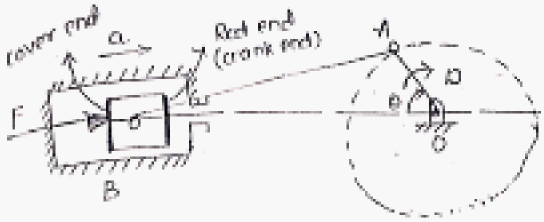
- **MADE EASY Classnotes**

Dynamic analysis of single slider crank mechanism

effective driving force to drive the piston

PISTON EFFORT (F)

(we will always calculate from cover end to crank end).



Approach

Gas pressure force

P_1, P_2 ⇒ pressure of gas at the cover end and crank end of piston.

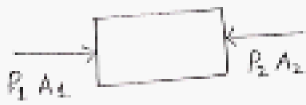
A_1, A_2 ⇒ cross-sectional area of piston exposed to gas enviorment at cover end & crank end side.

D → Piston dia

d → Piston rod dia

$A_1 = \frac{\pi}{4} D^2$ (Area of piston)

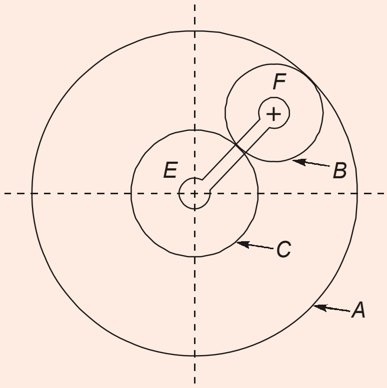
$A_2 = \frac{\pi}{4} (D^2 - d^2)$ (Area of piston rod)



$$\boxed{F_{gas} = P_1 A_1 - P_2 A_2}$$

End of Solution

3. (b) An epicyclic gear consists of three gears A, B and C as shown in the figure given below. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 20 r.p.m. If the gear A is fixed, determine the speed of gears B and C:



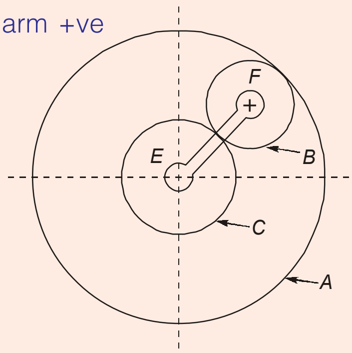
[20 Marks]

Solution:

As per given information

teeth on gear, A, $Z_A = 72$, arm rotate = 20 rpm

teeth on gear C, $Z_C = 32$ assuming direction of arm +ve



from configuration diagram,

$$r_c + 2r_B = r_A$$

$$\frac{mZ_C}{2} + mZ_B = \frac{mZ_A}{2}$$

$$Z_C + 2Z_B = Z_A$$

$$32 + 2Z_B = 72$$

$$Z_B = 20$$

Action	arm(EF)	Gear (c)	Gear B	Gear A
arm EF is fixed+1	0	+1	$\frac{-1\times 32}{20}$	$\frac{-1\times 32}{20}\times\frac{20}{72}$
arm EF is fixed +x rev	0	$+x$	$-\frac{32}{20}x$	$\frac{-32}{72}x$
add y	y	$y+x$	$y-\frac{32}{20}x$	$y-\frac{32}{72}x$

arm speed, y = 20 rpm

Gear A is fixed $\Rightarrow$ $y - \frac{32}{72}x = 0$

$$y = \frac{32}{72}x$$

$$\Rightarrow \qquad x = \frac{20\times72}{32} = 45 \text{ rpm}$$

Speed of gear, $B = y - \frac{32}{20}x = 20 - \frac{32}{20}\times45$

Speed of gear, B = – 52 rpm

Speed of gear, $C = y + x = 20 + 45 = 65$ rpm

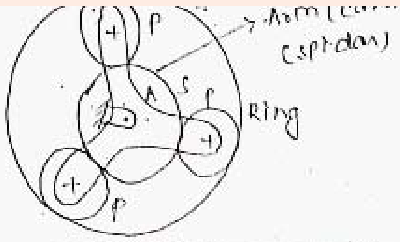
MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.24 of TOM (Page no 71) discussed in Class
- **MADE EASY Classnotes**

$T_D = \frac{252}{3.5} \Rightarrow T_D = 72$

$T_s + 2T_p = 72$

$N_D = 0,\ N_S = +5N_A$.



Arm (A)	S (T_s)	T_p (P)	P(72)
0	$+x$	$-x\frac{T_s}{T_p}$	$-x\frac{T_s}{T_p}\cdot\frac{T_p}{72}$
y	$y+x$	$y-x\frac{T_s}{T_p}$	$y-x\frac{T_s}{72}$

$y + x = 5y \Rightarrow x = 4y$

$N_D = 0 \Rightarrow y - x\frac{T_s}{72} = 0$

$y\left(1-\frac{T_s}{18}\right) = 0 \Rightarrow T_s = 18$.

$\therefore T_p = \frac{72-T_s}{2}$

End of Solution

3. (c) A single-cylinder reciprocating engine has a speed of 300 r.p.m., stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If two-thirds of the reciprocating parts and all the revolving parts are to be balanced, find—

(i) the balance mass required at a radius of 300 mm;

(ii) the residual unbalanced force when the crank has rotated 60° from top dead centre.

[10 Marks]

Solution:

As per given data

speed of engine, $N = 300$ rpm

$$\Rightarrow \quad \omega = \frac{2\pi \times 300}{60} = 10\pi \text{ rad/s}$$

Stroke length, $2r = 300$ mm $= 0.3$ m

Crank radius, $r = 0.15$ m

mass of reciprocating part, $m = 50$ kg

Mass of revolving part, $m_p = 37$ kg at 150 mm radius

$$c = \frac{2}{3}rd \text{ of the reciprocating parts}$$

$$r_c = 300 \text{ mm} = 0.3 \text{ m}$$

(i) mass to be balanced at the crank pin, $m = cm + m_p$

$$= \frac{2}{3} \times 50 + 37 = 70.333 \text{ kg}$$

$$m_c r_c = mr$$

$$m_c \times 0.3 = 70.333 \times 0.15$$

$$m_c = 35.166 \text{ kg}$$

The balance mass required at a radius of 300 mm

$$m_c = 35.166 \text{ kg}$$

(ii) Residual unbalanced force (at $60° = \theta$) from top dead centre,

$$= \sqrt{\left[(1-c)mr\omega^2 \cos\theta\right]^2 + \left(cmr\omega^2 \sin\theta\right)^2}$$

$$= \sqrt{\left[\left(1-\frac{2}{3}\right)50 \times 0.15 \times (10\pi)^2 \cos 60°\right]^2 + \left[\frac{2}{3} \times 50 \times 0.15 \times (10\pi)^2 \sin 60°\right]^2}$$

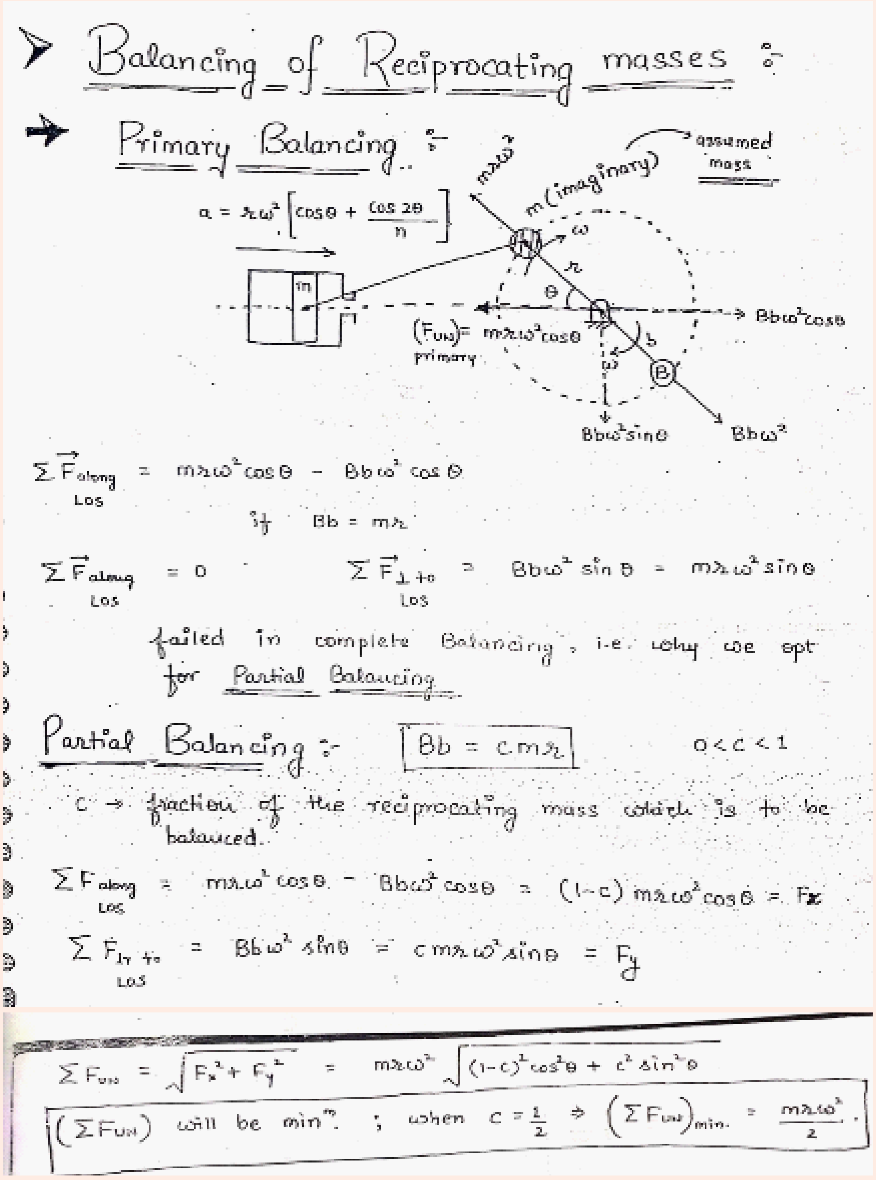
$$= \sqrt{1522017.047 + 18264204.57}$$

$$= 4448.170 \text{ N}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.3(b) of Test-11
- **ESE 2019 Mains Workbook:** Q.51 of TOM (Page no. 77) discussed in Class

- **MADE EASY Classnotes**



Balancing of Reciprocating masses :

Primary Balancing :

$a = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$\sum \vec{F}_{along\ LOS} = mr\omega^2\cos\theta - Bb\omega^2\cos\theta$

if $Bb = mr$

$\sum \vec{F}_{along\ LOS} = 0$ $\quad$ $\sum \vec{F}_{\perp\ to\ LOS} = Bb\omega^2\sin\theta = mr\omega^2\sin\theta$

failed in complete Balancing, i.e. why we opt for Partial Balancing

Partial Balancing : $\boxed{Bb = cmr}$ $\quad 0 < c < 1$

c → fraction of the reciprocating mass which is to be balanced.

$\sum F_{along\ LOS} = mr\omega^2\cos\theta - Bb\omega^2\cos\theta = (1-c)\,mr\omega^2\cos\theta = F_x$

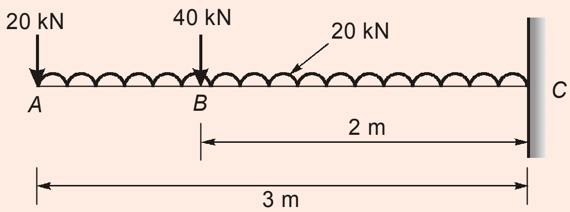
$\sum F_{\perp\ to\ LOS} = Bb\omega^2\sin\theta = cmr\omega^2\sin\theta = F_y$

$\sum F_{un} = \sqrt{F_x^2 + F_y^2} = mr\omega^2\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$

$(\sum F_{un})$ will be min^m ; when $c = \frac{1}{2}$ ⇒ $(\sum F_{un})_{min.} = \frac{mr\omega^2}{2}$.

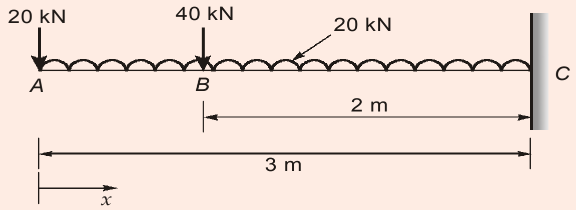
End of Solution

3\. (d) Draw the shear force and bending moment diagram for the cantilever beam as shown in the figure below:

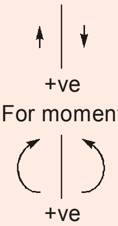


[10 Marks]

Solution:



Sign convection for shear



For shear force diagram,

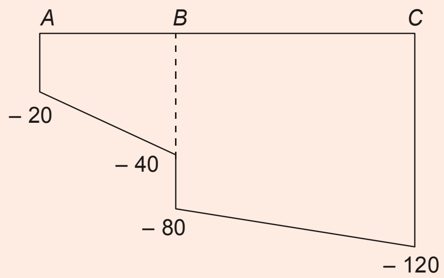
For section *AB*

$$V = -20 - 20x$$
$$V_A = -20 \text{ kN}$$
$$V_B = -20 - 20 \times 1 = -40 \text{ kN}$$

For section *BC*,

$$V = -60 - 20x$$
$$V_B = -80 \text{ kN}$$
$$V_C = -120 \text{ kN}$$

SFD:



Now for bending moment diagram

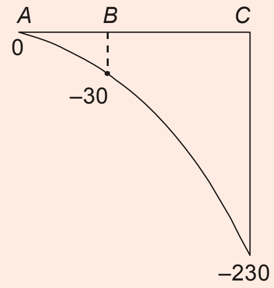
For section *AB*

$$M_x = -20x - 20 \times \frac{x^2}{2}$$
$$= -20x - 10x^2$$
$$M_A = 0$$
$$M_B = -30 \text{ kNm}$$

For section *BC*,

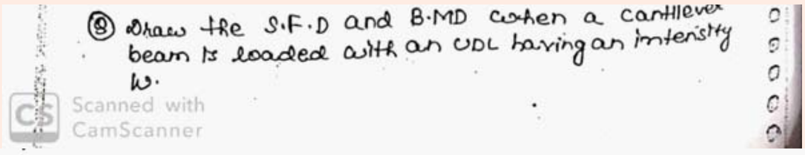
$$M_x = -20 \times \frac{x^2}{2} - 40(x-1)$$
$$= 40 - 60x - 10x^2$$
$$M_B = -30 \text{ kNm}$$
$$M_C = -230 \text{ kNm}$$

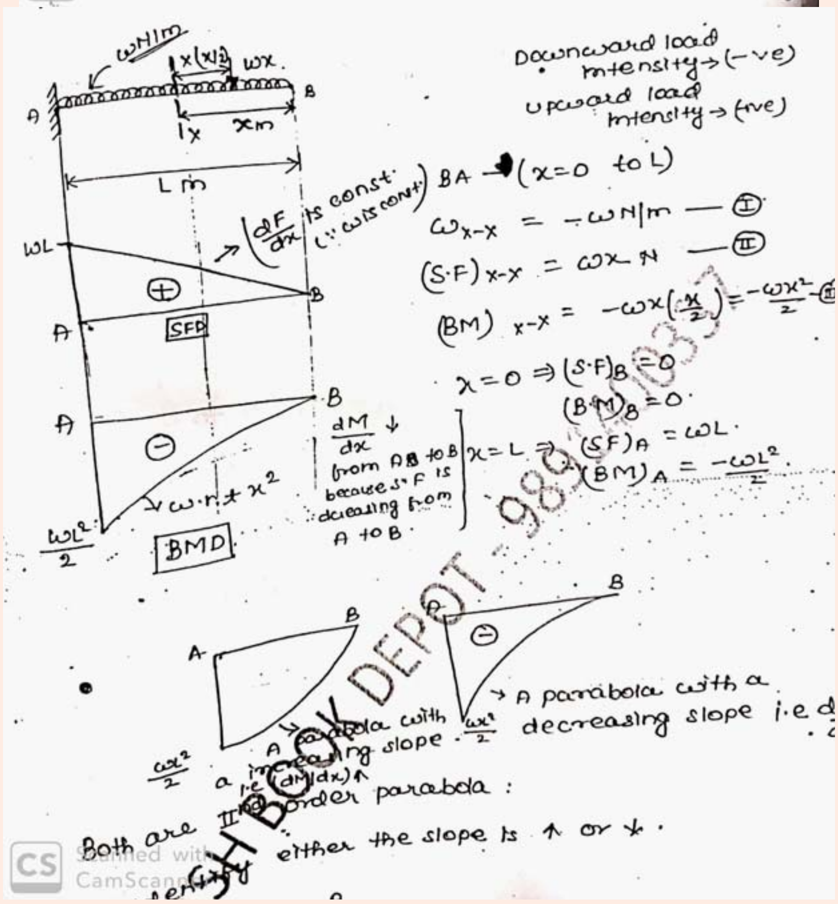
BMD:



MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.4 (b) of Test-11
- **ESE 2019 Mains Workbook:** Q.22 Unsolved, Q.5 and Q.7 Solved Question of SOM
- **MADE EASY Classnotes**





End of Solution

4. (a) (i) Describe angular contact bearings and taper roller bearings with the help of neat sketches. Also, cite at least two advantages and two disadvantages of each.

[8 Marks]

(ii) A pair of spur gears with 20° full-depth involute teeth consists of a 20 teeth pinion meshing with a 41 teeth gear. The module is 3 mm while the face width is 40 mm. The material for both the pinion and the gear is steel having an ultimate tensile strength of 660 N/mm^2. The gears are heat-treated to a surface hardness of 400 BHN. The pinion rotates at 1500 r.p.m. and the service factor is 2.0. Assume that the velocity factor accounts for the dynamic load and the factor of safety is 1.5. Determine the rated power that the gears can transmit. Assume a Lewis form factor of 0.32.

[12 Marks]

Solution:

(i)

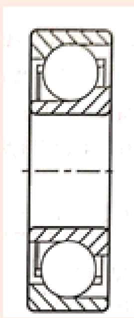
Angular contact bearing: In angular contact bearing, the grooves in inner and outer races are so shaped, that the line of reaction at the contact between balls and races makes an angle with the axis of the bearing. This reaction has two components-radial and axial. Therefore, angular contact bearing can take radial and thrust loads. Angular contact bearings are often used in pairs, either side by side or at the opposite ends of the shaft, in order to take the thrust load in both directions. These bearings are assembled with a specific magnitude of pre-load.

Angular contact bearings offer following advantages:

- Angular contact bearing can take both, radial and thrust loads.
- In angular contact bearing, one side of the groove in the outer race is cut away to permit the insertion of more number of balls than that of deep groove ball bearing. This permits the bearing to carry relatively large axial and radial loads. Therefore, the load carrying capacity of angular contact bearing is more than that of deep groove ball bearing.

The disadvantages of angular contact bearings are as follows:

- Two bearings are required to take thrust load in both directions.
- Angular contact bearing must be mounted without axial play.
- Angular 'contact bearing requires initial pre-loading



Angular contact bearing

Taper roller bearing: The taper roller bearing consists of rolling elements in the form of a frustum of cone. They are arranged in such a way that the axes of individual rolling elements intersect in a common apex point on the axis of the bearing. In kinematics' analysis, this is the essential requirement for pure rolling motion between conical surfaces. In taper roller bearing, the line of resultant reaction through the rolling elements makes an angle with the axis of the bearing. Therefore, taper roller bearing can carry both radial and axial loads. In fact, the presence of either component results in the other, acting on the bearing. In other words, a taper roller bearing subjected to pure radial load induces thrust component and vice versa. Therefore, taper roller bearings are always used in pair to balance the thrust component.

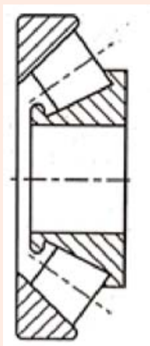
Taper roller bearing has separable construction. The outer ring is called 'cup' and the inner ring called 'cone'. The cup is separable from the remainder assembly of the bearing elements including the rollers, cage, and the cone.

Taper roller bearing offers following advantages:

- Taper roller bearing can take heavy radial and thrust loads.
- Taper roller bearing has more rigidity.
- Taper roller bearing can be easily assembled and disassembled due to separable parts.

The disadvantages of taper roller bearing are as follows:

- It is necessary to use two taper roller bearings on the shaft to balance the axial force.
- It is necessary to adjust the axial position of the bearing with pre-load. This is essential to coincide the apex of the cone with the common apex of the rolling elements.
- Taper roller bearing cannot tolerate misalignment between the axes of the shaft and the housing bore.
- Taper roller bearings are costly.



Taper roller bearing

(ii)

Given: $\phi = 20°$, $z_p = 20$, $z_g = 41$, $m = 3$ mm, $b = 40$ mm, $\sigma_{ut} = 660$ N/mm^2
BHN = 400, $N_p = 1500$ rpm, $C_s = 2$, $F(s) = 1.5$, [Lewis form factor] $y = 0.32$

Beam strength:

Since the same material is used for the pinion and the gear, the pinion is weaker than the gear.

According to Earle Buckingham, Endurance limit stress [bending stress] of gear tooth is approximately one-third of the ultimate tensile strength of the material.

So, $\sigma_b = S_e = \dfrac{S_{ut}}{3}$

$$\sigma_b = \frac{660}{3} = 220 \text{ N/mm}^2$$

Beam strength,

$$S_b = \sigma_b \times m \times b \times Y$$
$$= 220 \times 3 \times 40 \times 0.32 = 8448 \text{ N}$$

Wear strength:

$$S_w = d_p \times k \times Q \times b$$
$$d_p = m \times z_p = 3 \times 20 = 60 \text{ mm}$$
$$Q = \frac{2z_g}{z_g + z_p} = \frac{2 \times 41}{41 + 20}$$
$$Q = 1.344$$

where, $k = 0.16\left(\dfrac{BHN}{100}\right)^2$

This equation is applicable only when both gears are made of steel with a 20° pressure angle.

$$k = 0.16\left(\frac{400}{100}\right)^2 = 2.56$$
$$S_w = 60 \times 2.56 \times 1.344 \times 40 = 8257.536 \text{ N}$$

Effective load:

$$P_{\text{eff}} = \frac{P_t \times C_s}{C_v}$$

Speed of gear, $V = \dfrac{\pi d_p \times N_p}{60} = \dfrac{\pi \times 60 \times 1500}{60 \times 1000} = 4.7123$ m/s

As, speed, $V < 10$ m/s

$$C_v = \frac{3}{3+V} = 0.3889 = 0.39$$
$$P_{\text{eff}} = \frac{C_s \times P_t}{C_v} = \frac{2 \times P_t}{0.39}$$
$$P_{\text{eff}} = 5.142\, P_t$$

As wear strength is lower than beam strength, the wear strength is the criterion of design

So, $S_w = P_{\text{eff}} \times F(s)$

$$S_w = P_{\text{eff}} \times 1.5$$
$$8257.536 = 5.142 \times P_t \times 1.5$$
$$P_t = 1070.683 \text{ N}$$

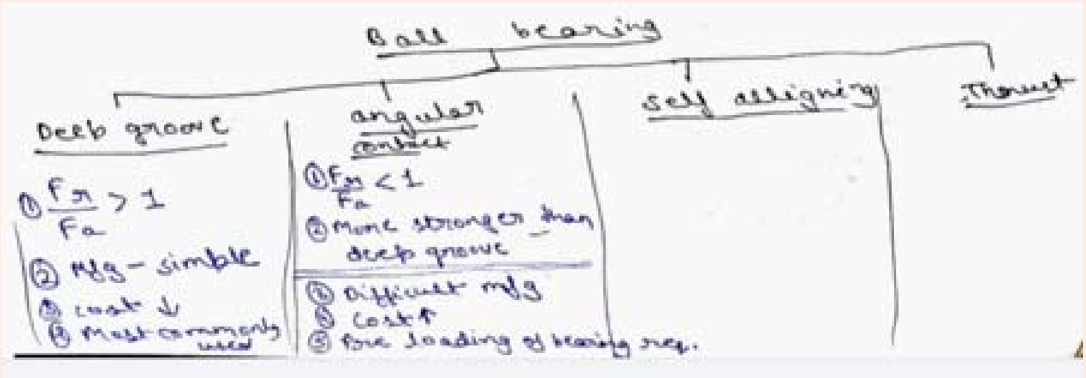
Rated power,

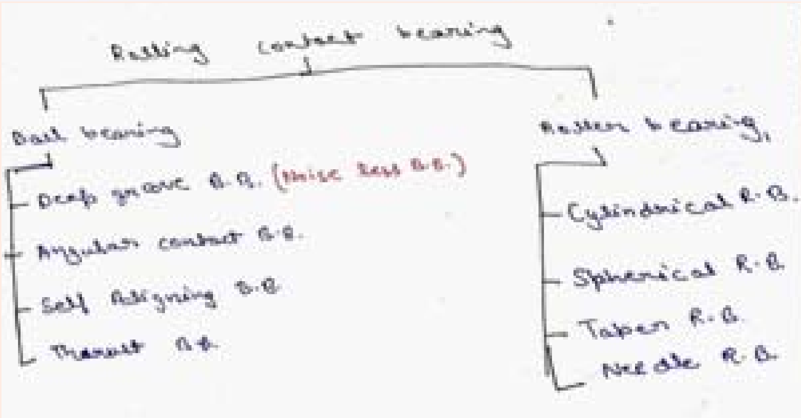
$$\text{Torque, } M_t = P_t \times \frac{d_p}{2} = 1070.683 \times 30 = 32120.49 \text{ Nmm}$$

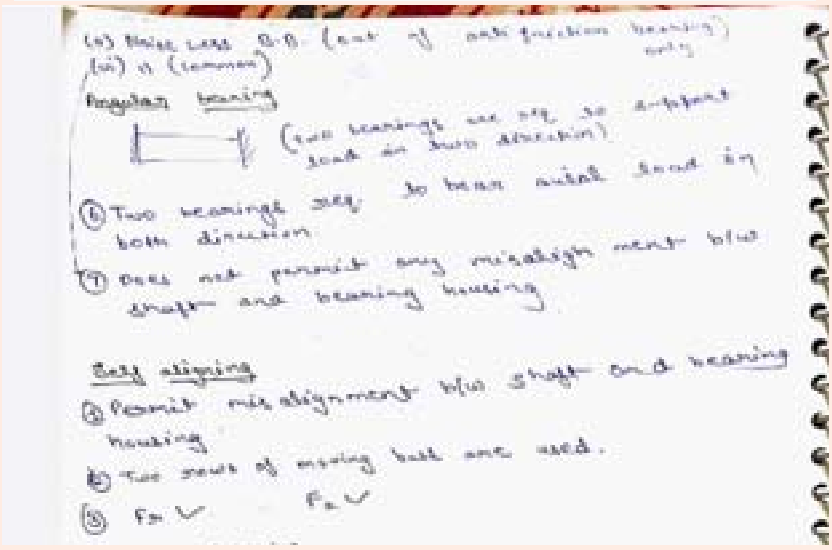
$$\text{Power, } P = \frac{2\pi \times N_p \times M_t}{60 \times 10^6} = 5.045 \text{ kW}$$

MADE EASY Source ___

- **ESE 2019 Mains Workbook:** Q.23 of Machine Design (Solved) 'Source for Q. 4 (a) (ii)'
- **MADE EASY Classnotes**







Thrust b. bearing

F_r ✗ F_a ✓

that's why preferred for only vertical shaft

Roller

① Cylindrical R.B.

F_r ✓ F_a ✗

horizontal shaft
– maximum radial load bearing capacity

② Spherical R.B.
– permit misalignment b/w shaft and bearing housing
– 2 row of moving roller are used

F_r ✓ F_a ✓

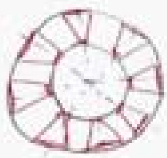
③ Taper Rolling bearing

→ maximum load bearing capacity
→ Fatigue and Impact loading

$\frac{F_r}{F_a} > 1$

eg. axle of Bus, Truck

→ manufacturing is very difficult
→ cost maximum
→ pre loading is required
→ Taper's Roller are very sensitive to assemble in races hence adjacent nut or tightening nut are required
→ Radial space requires more



→ always used in pair.
→ don't permit misalignment

④ Needle R.B.

$\frac{L}{D} > 1$ F_r ✓ F_a ✓

– needle rollers are used where radial space is constraint.
... capacity in a given radial

Following data are given for spur gear ? –

No. of teeth on pinion, z_1 = 30

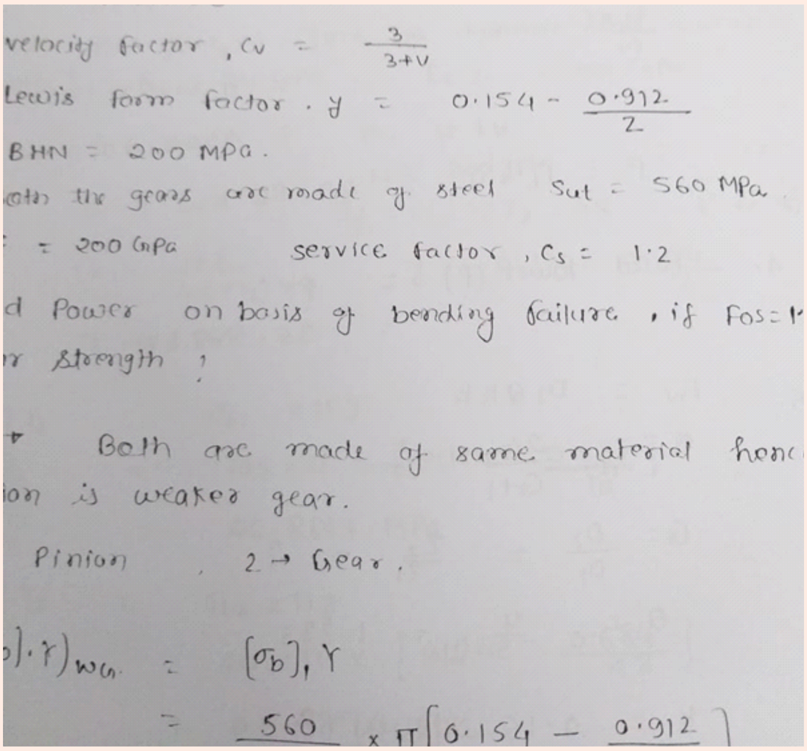
No. of teeth on gear, z_2 = 60

Speed of pinion, N_1 = 1440 rpm

Pressure angle ϕ = 20° full depth

module, m = 3 mm

face width, b = 33 mm



velocity factor, $C_v = \frac{3}{3+V}$

Lewis form factor, $y = 0.154 - \frac{0.912}{Z}$

BHN = 200 MPa.

oth the gears are made of steel S_{ut} = 560 MPa.

: = 200 GPa service factor, C_s = 1.2

d Power on basis of bending failure, if FOS = 1·

r strength ?

Both are made of same material hence

ion is weaker gear.

Pinion , 2 → Gear.

$]\cdot Y)_{wea} = [\sigma_b], Y$

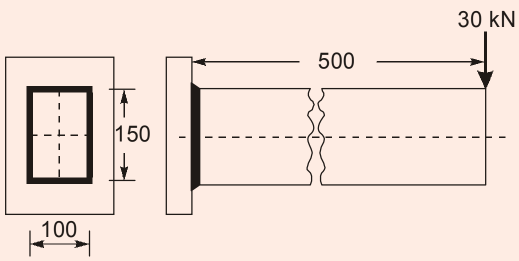
$= \frac{560}{\quad} \times \pi\left[0.154 - \frac{0.912}{\quad}\right]$

End of Solution

4. (b) What advantages do the welded joints offer in comparison to riveted joints? Neatly sketch the basic symbols used to specify the following types of weld:

(i) Fillet
(ii) Square butt
(iii) Single V-butt
(iv) Spot
(v) Seam
(vi) Projection

A beam of rectangular cross-section is welded to a support by means of fillet welds as shown in the figure given below. Determine the size of the welds if the permissible shear stress is 80 N/mm^2:

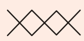


[20 Marks]

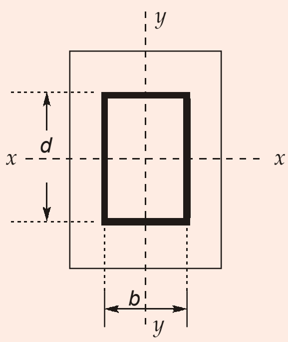
Solution:

Welded joints offer following advantages compared with riveted joints:

- Riveted joints require additional cover plates, gusset plates, straps, clip angles, and a large number of rivets, which increase the weight. Since there are no such additional parts, welded assembly results in light weight construction.
- Due to the elimination of these components, the cost of welded assembly is lower than that of riveted joints.
- The design of welded assemblies can be easily and economically modified to meet the changing product requirements. Alterations and additions can be easily made in the existing structure by welding.
- Welded assemblies are tight and leak proof as compared with riveted assemblies.
- The production time is less for welded assemblies.
- When two parts are joined by the riveting method, holes are drilled in the parts to accommodate the rivets. The holes reduce the cross-sectional area of the members and result in stress concentration. There is no such problem in welded connections.
- Welded structure has smooth and pleasant appearance. The projection of rivet head adversely affects the appearance of the riveted.
- The strengths of the welded joint is high. Very often, the strength of the weld is more than the strength of the plates that are joined together.

Type of weld	Symbol
Fillet	
Square Butt	
Single V-Butt	
Spot	
Seam	
Projection	

Given: P = 30 kN, τ_{perm} = 80 N/mm^2



Due to force P = 30 kN, there will be primary shear stress and bending stress. Assume t is the throat of the weld.

1. Primary shear stress:

A = Total area of the horizontal and vertical weld

$= 2[100t + 150t]$

$A = 500t\ \text{mm}^2$

So, primary shear stress is,

$$\tau = \frac{P}{A} = \frac{30 \times 10^3}{500 \times t} = \frac{60}{t}\ \text{N/mm}^2$$

2. Bending stress:

I_{x1} = moment of inertia of horizontal welds about x-x axis

$$= 2 \times \left[\frac{bt^3}{12} + \left(\frac{d}{2}\right)^2 \times b \times t\right]$$

Neglecting t^3 term

$$= 2 \times \frac{d^2}{4} \times b \times t = \frac{d^2}{2} \times b \times t\ \text{mm}^4$$

I_{x2} = moment of inertia of vertical welds about y-y axis

$$= \left[\frac{t \times d^3}{12}\right] \times 2$$

$$I_{x2} = \frac{t \times d^3}{6}\ \text{mm}^4$$

$$I = I_{x1} + I_{x2} = \frac{d^2}{2} \times b \times t + \frac{t \times d^3}{6}$$

$$= \frac{150^2}{2} \times 100 \times t + \frac{t \times (150)^3}{6} \quad [\text{where, } b = 100\ \text{mm}, d = 150\ \text{mm}]$$

$I = 1687500t\ \text{mm}^4$

Bending moment, $M_b = 30 \times 10^3 \times 500$ Nmm

$M_b = 15 \times 10^6$ Nmm

$$\sigma = \frac{M_b \times y}{I} = \frac{15 \times 10^6 \times d}{2 \times I} = \frac{15 \times 10^6 \times 150}{2 \times 1687500t} = \frac{666.66}{t}\ \text{N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} \le \tau_{perm}$$

where,

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{666.66}{2t}\right)^2 + \left(\frac{60}{t}\right)^2}$$

$$\tau_{max} = \frac{338.687}{t}\ \text{N/mm}^2$$

So,

$$\tau_{max} \le \tau_{per}$$

$$\frac{338.687}{t} = 80$$

$t = 4.233$ mm

We know that,

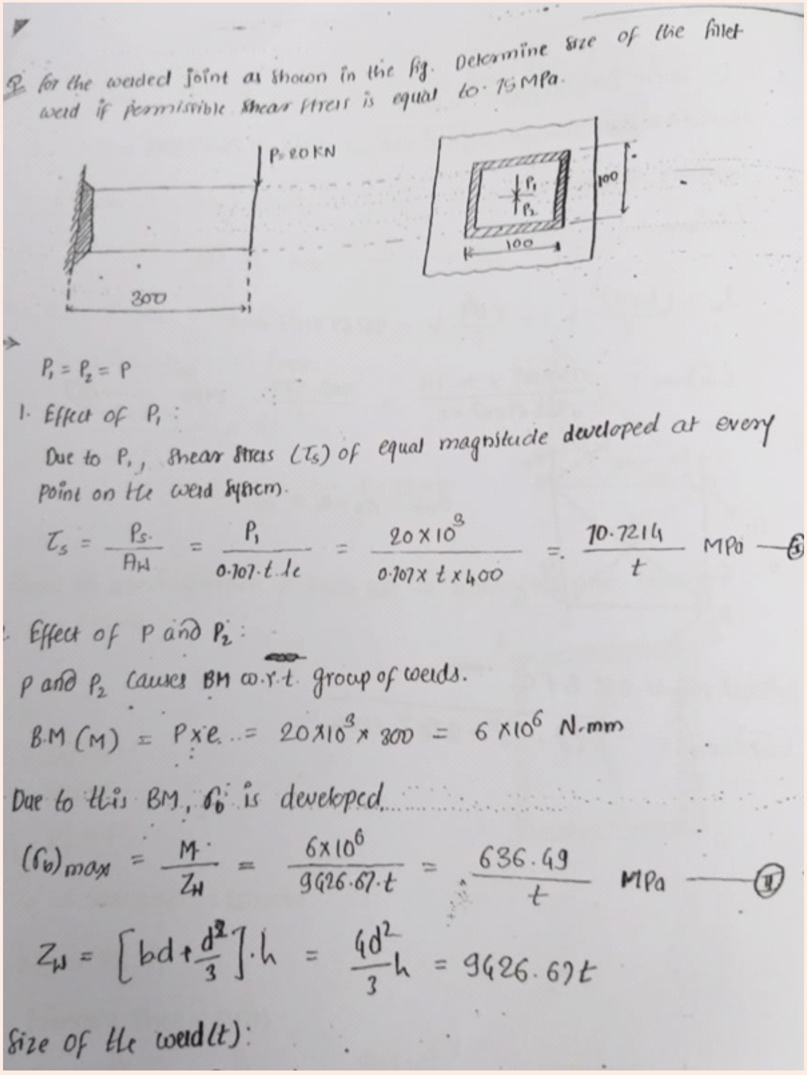
$$t = 0.707 \times h \qquad \text{(where } h \text{ is the leg of weld)}$$

$$h = 5.988$$

$$h = 6 \text{ mm}$$

MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.14 of Machine Design (Solved Que) discussed in Class
- **MADE EASY Classnotes**



Q. For the welded joint as shown in the fig. Determine size of the fillet weld if permissible shear stress is equal to 75 MPa.

$P_1 = P_2 = P$

1. Effect of P_1:

Due to P_1, shear stress (τ_s) of equal magnitude developed at every point on the weld system.

$$\tau_s = \frac{P_s}{A_W} = \frac{P_1}{0.707 \cdot t \cdot l_e} = \frac{20 \times 10^3}{0.707 \times t \times 400} = \frac{70.7214}{t} \text{ MPa} \quad \text{— (I)}$$

Effect of P and P_2:

P and P_2 causes BM w.r.t. group of welds.

$$B.M\,(M) = P \times e = 20 \times 10^3 \times 300 = 6 \times 10^6 \text{ N.mm}$$

Due to this BM, σ_b is developed

$$(\sigma_b)_{max} = \frac{M}{Z_W} = \frac{6 \times 10^6}{9426.67 \cdot t} = \frac{636.49}{t} \text{ MPa} \quad \text{— (II)}$$

$$Z_W = \left[bd + \frac{d^2}{3}\right] \cdot h = \frac{4d^2}{3} h = 9426.67t$$

Size of the weld (t):

End of Solution

4. (c) A shaft is subjected to a maximum torque of 10 kN-m and a maximum bending moment of 7.5 kN-m at a particular section. If the allowable equivalent stress in simple tension is 160 MN/m^2, find the diameter of the shaft according to (i) maximum shear stress theory, (ii) strain energy theory and (iii) shear strain energy theory. Take Poisson's ratio as 0.24.

[20 Marks]

Solution:

Given:

$$T_{max} = 10 \text{ kNm}$$
$$M_{max} = 7.5 \text{ kNm}$$
$$\sigma_{allowable} = 160 \text{ MN/m}^2$$
$$\mu = 0.24$$

Let d is the diameter of shaft,

So,
$$T_{max} = \frac{\pi}{16}\tau \times d^3$$
$$\tau = \frac{16T_{max}}{\pi d^3} = \frac{16\times 10\times 10^3}{\pi d^3}$$
$$\tau = \frac{50.929\times 10^3}{d^3} \text{ N/m}^2$$
$$\sigma = \frac{32M_{max}}{\pi d^3} = \frac{32\times 7.5\times 10^3}{\pi d^3}$$
$$= \frac{76.394\times 10^3}{d^3} \text{ N/m}^2$$

Principal stresses,
$$\sigma_{1,\,2} = \frac{1}{2}\left[\sigma \pm \sqrt{\sigma^2 + 4\tau^2}\right]$$
$$\sigma_{1,\,2} = \frac{1}{2}\left[\frac{76.394\times 10^3}{d^3} \pm \sqrt{\left(\frac{76.394\times 10^3}{d^3}\right)^2 + 4\times\left(\frac{50.929\times 10^3}{d^3}\right)^2}\right]$$
$$= \frac{1}{2}\left[\frac{76.394\times 10^3 \pm 127.3228\times 10^3}{d^3}\right]$$
$$\sigma_1 = \frac{101.858\times 10^3}{d^3} \text{ N/m}^2$$
$$\sigma_2 = \frac{-25.464\times 10^3}{d^3} \text{ N/m}^2$$

(i) Maximum shear stress theory

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_{allowable}}{2}$$
$$\frac{(101.858 + 25.464)\times 10^3}{d^3} = 160 \times 10^6$$
$$d = 0.092667 \text{ m} = 92.667 \text{ mm}$$

(ii) Strain energy theory,

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 \leq (\sigma_{allowable})^2$$
$$\left(\frac{101.858\times 10^3}{d^3}\right)^2 + \left(\frac{-25.464\times 10^3}{d^3}\right)^2 - 2\times 0.24\times\frac{101.858\times(-25.464)\times 10^6}{d^6} = \left(160\times 10^6\right)^2$$

$$\frac{1.2268\times10^{10}}{d^6} = (160 \times 10^6)^2$$

$$d = 0.08846 \text{ m}$$

$$d = 88.46 \text{ mm}$$

(iii) Shear strain energy theory:

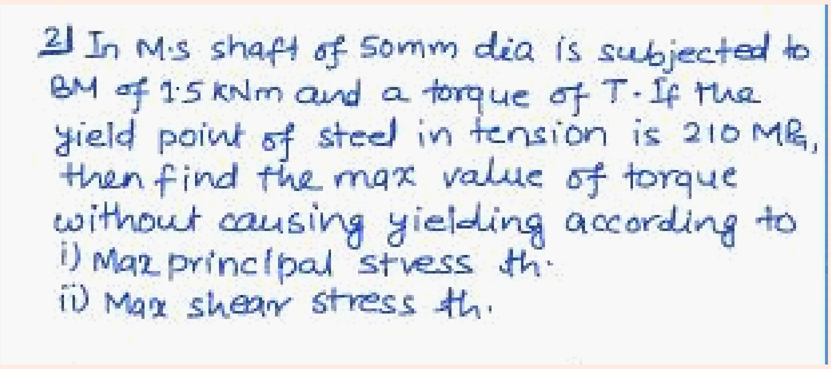
$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \le 2 \times (\sigma_{allowable})^2$$

$$\left(\frac{101.858\times10^3 + 25.464\times10^3}{d^3}\right)^2 + \left(\frac{101.858\times10^3}{d^3}\right)^2 + \left(\frac{-25.464\times10^3}{d^3}\right)^2 = 160 \times 10^6$$

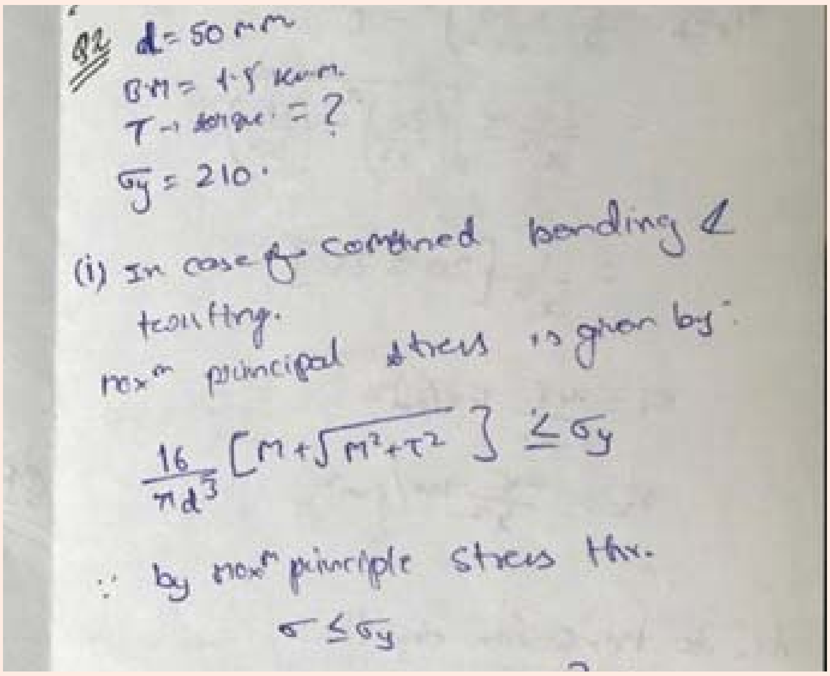
$$d = 0.09001 \text{ m} = 90.013 \text{ mm}$$

MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.27 of Strength of Material (Solved Que) discussed in Class
- **MADE EASY Classnotes**



21 In M.S shaft of 50mm dia is subjected to BM of 1.5 kNm and a torque of T. If the yield point of steel in tension is 210 MPa, then find the max value of torque without causing yielding according to
i) Max principal stress th.
ii) Max shear stress th.

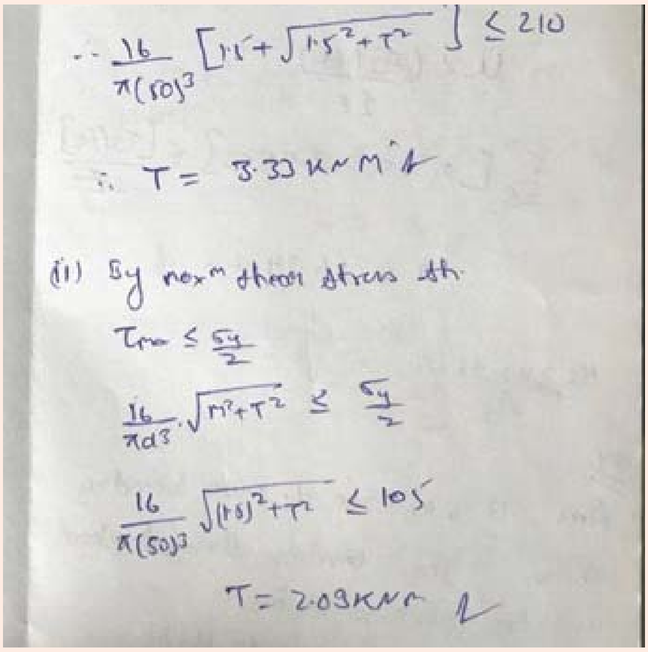


$d = 50$ mm
B.M = 1.5 kN-m
T → torque = ?
$\sigma_y = 210$.

(i) In case of combined bending & torsion.
max^m principal stress is given by:

$$\frac{16}{\pi d^3}\left[M + \sqrt{M^2+T^2}\right] \le \sigma_y$$

∵ by max^m principle stress thr.
$\sigma \le \sigma_y$



End of Solution

SECTION-B

5. (a) Describe the following microconstituents of iron-carbon alloys in relation to the phases present, arrangement of phases and their relative mechanical properties:
   - (i) Spheroidite
   - (ii) Pearlite
   - (iii) Bainite
   - (iv) Martensite

[12 Marks]

Solution:

i. **Spheroidite**: If pearlite is heated just below the eutectoid temperature (say 700°C) and held at this temperature for a day or so, the cementite lamelle in pearlite get transformed to spherical shape. The structure is called "spheroidite".
This structure is less conducive to stress concentration because of spherical grains, as compared to cementite (lamelle structure). This, spheroidite is more tough but less hard as compared to pearlite.

ii. **Pearlite**: Pearlite is phase mixture of a-iron and Fe_3C. Pearlite is having plate like structure of α-iron and Fe_3C.

$$\gamma-\text{Iron} \xrightleftharpoons[0.8\%C]{725°C} \underbrace{\alpha-\text{Iron}+Fe_3C}_{\text{Pearlite}}$$

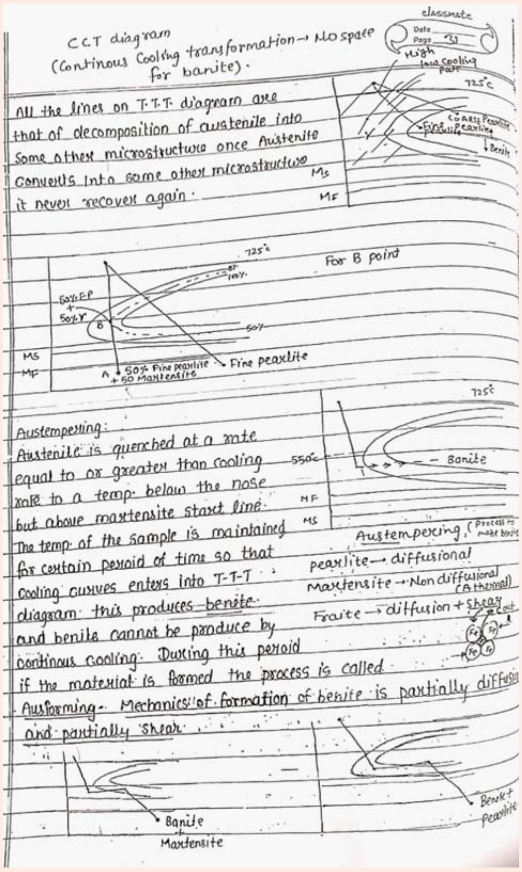
Pearlite formation takes place primarily by process of diffusion. Pearlite is having good thoughness and ductility. Pearlite has less brittleness and hardness in comparison of bainite and martensite.

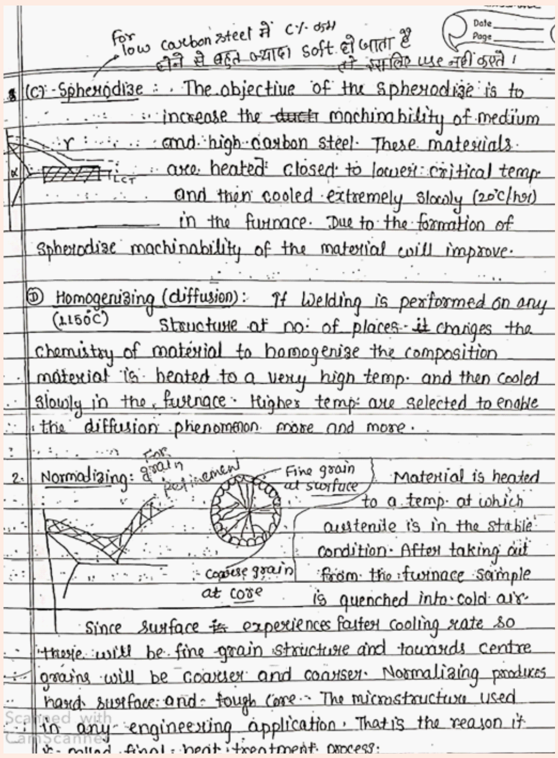
iii. **Bainite**: Austenite sample is quenced at a rate greater than or equal to critical cooling rate to a temperature below the nose of TTT but above the martensite start line. This temperature is maintained for substantial period of time so that transformation line enters into TTT curve, this produces bainite. The microstructure of bainite consists of ferrite and cementite phase, and this diffusional processes are involved in its formation. Bainite forms as needles or plates, depending on the temperature of the transformation. Bainite is having high hardness but less ductility and thoughness.

iv. **Martensite**: Any cooling rate greater than or equal to critical cooling rate will freeze carbon at its location and the microstructure will appear like mechanical mixture of ferrite and cementite or colloidal solution of cementite into ferrite or submicroscopic cementite spread in the microstructure of ferrite. Martensite is having body centered tetragonal structure (BCT). It is a non-equilibrium single -phase structure that results from a diffusionless transformation of austenite. Since the martensite transformation does not involve diffusion, it occurs almost instantaneously. Martensite is hardest and most brittle phase of iron.

MADE EASY Source

- **Theory Book (2019 Edition):** Material Science (Page No. 93)
- **MADE EASY Classnotes**


CCT diagram
(Continous Cooling transformation → No space for banite).
All the lines on T.T.T. diagram are that of decomposition of austenite into some other microstructure once Austenite converts into some other microstructure it never recover again.
Austempering:
Austenite is quenched at a rate equal to or greater than cooling rate to a temp. below the nose but above martensite start line. The temp. of the sample is maintained for certain peroid of time so that cooling curves enters into T-T-T diagram this produces benite. and benite cannot be produce by continous cooling. During this peroid if the material is formed the process is called Ausforming. Mechanics of formation of benite is partially diffusion and partially shear.

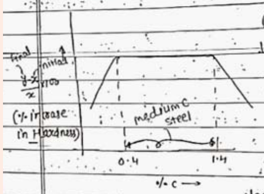

For low carbon steel में C% कम होने से बहुत ज्यादा soft हो जाता है इसलिए use नहीं करते।
(C) Spherodize: The objective of the spherodize is to increase the ~~duct~~ machinability of medium and high carbon steel. These materials are heated closed to lower critical temp. and then cooled extremely slowly (20°C/hr) in the furnace. Due to the formation of spherodize machinability of the material will improve.
(D) Homogenizing (diffusion): (1150°C) If welding is performed on any structure at no. of places ~~it~~ changes the chemistry of material to homogenize the composition material is heated to a very high temp. and then cooled slowly in the furnace. Higher temp. are selected to enable the diffusion phenomenon more and more.
2. Normalizing: For grain refinement. Material is heated to a temp. at which austenite is in the stable condition. After taking out from the furnace sample is quenched into cold air. Since surface ~~is~~ experiences faster cooling rate so there will be fine grain structure and towards centre grains will be coarser and coarser. Normalizing produces hard surface and tough core. The microstructure used in any engineering application. That is the reason it is called final heat treatment process.

greater than cooling rate so that entire microstructure converts into martensite.

Objective of Hardening is to produce martensite.

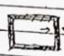
Quenching Media:

When sample is quenched into water it forms a thin layer of vapour blanket all around the specimen. This decreases the cooling rate. Addition of salt suppress the tendency of vapour blanket. So fastest cooling will be provided by brine solution. But uniform cooling provided by the oil baths. But the cooling rated offer low. Oil bath are used in case of alloy steels.



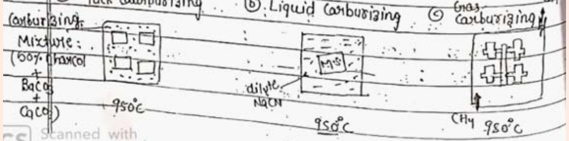
Only eutectoid composition produces 100% martensite and the moment we are deviating from eutectoid composition we will get pro-eutectoid phase and these phases doesn't contribute to the formation of martensite. That is the reason low carbon steels and very high carbon steels do not respond to heat treatment process. But in majority of engineering applications we use low carbon steel and their strength is increased by case Hardening.

Case Hardening: (for ferrous alloys)

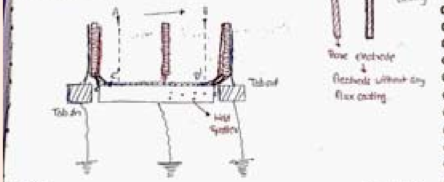


1. Carburizing: (low C steel)

(a) Pack carburizing (b) Liquid carburizing (c) Gas carburizing



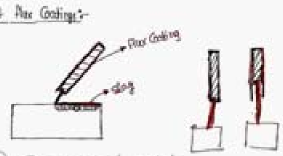
Arc Blow:- [only in DC]



- Deflection of electric arc from its intended path due to deflection of magnetic flux lines towards the w/p is arc blow.
- Due to arc blow heat concentration on the w/p at the beginning and end will be decreased and weld spatter will be produced.

Remedies:- Provide some extra material at the beginning and end of the w/p known as 'Tab in' and 'Tab out'.
- Use small arc length.
- Reduce the intensity of welding current.
- Provide the flux coating around the electrode.

Function of Flux Coating:-



- Flux coating material will act as deoxidizers.
- By forming the slag liquid metal can be protected from the atmospheric gases.
- It will increase the strength of the joint by adding alloying element.
- It will control the viscosity of liquid metal and H.T. rate in the weld pool.
- By reducing the arc blow stability of the arc can be increased.
- By reducing heat transfer losses heat concentration on the w/p can will be increased.

Flux Coating Materials:-

i) De-oxidizing Element:-
Graphite, Alumina, Ferro Silicon and Ferro Magnese

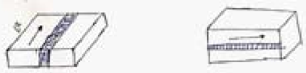
ii) Slag formation Compounds:-
Iron oxide, Silicon dioxide, Titanium oxide, Silica flour and Calcium fluoride.

iii) Arc Stabilizer:-
Sodium oxide, Calcium oxide, potassium silicate.

iv) Alloying Elements:-
Cr, Ni, Co and V

Welding Techniques based on position:-

i) Flat position Welding (F):-   ii) Horizontal (H):-



End of Solution

5. (b) In an orthogonal cutting operation, the cutting speed is 2.5 m/s, rake angle is 6° and the width of cut is 10 mm. The undeformed chip thickness is 0.2 mm. 13.36 grams of steel chips with total length of 50 cm are obtained. The tool post dynamometer gives cutting and thrust forces as 1134 N and 453.6 N respectively. Find—

(i) shear plane angle;

(ii) friction energy at tool-chip interface as percentage of total energy;

(iii) specific cutting energy.

Assume density of steel = 7.8 grams/cm^3.

[12 Marks]

Solution:

Given: Cutting speed, $V = 2.5$ m/s

Rake angle, $\alpha = 6°$

Width of cut, $b = 10$ mm $= 1$ cm

Uncut chip thickness, $t = 0.2$ mm

Length, $l = 50$ cm

Mass of chips, $m = 13.36$ gram

Cutting force, $F_c = 1134$ N

Thrust force, $F_t = 453.6$ N

Density, $\rho = 7.8$ gram/cm^3

Actual chip thickness in this question should be calculated by the mass of chips produced.

Now, $m = \rho \times (b \times l \times t_c)$

$13.36 = 7.8 \times (1 \times 50 \times t_c)$ [where t_c is chip thickness]

Chip thickness, $t_c = 0.034256$ cm

$t_c = 0.34256$ mm

$$\text{Chip thickness ratio, } r = \frac{t}{t_c} = \frac{0.2}{0.34256} = 0.58383$$

$r \simeq 0.584$

(i) If shear plane angle is ϕ.

$$\tan\phi = \frac{r\cos\alpha}{(1 - r\sin\alpha)} = \frac{0.584\cos 6°}{1-(0.584\sin 6°)}$$

$\tan\phi = 0.61856$

$\phi = 31.74°$

(ii) Total energy $= F_c \cdot V$

$= 1134 \times 2.5 = 2835$ Watt

Friction force, $F = F_c\sin\alpha + F_t\cos\alpha$

$= 1134 \sin 6° + 453.6 \times \cos 6°$

$= 569.65$ N

Frictional power $= F \cdot V_c$

$$= F\frac{V\sin\phi}{\cos(\phi-\alpha)} = \frac{569.65\times 2.5\times \sin 31.74^\circ}{\cos(31.74-6)^\circ} = 831.710 \text{ Watt}$$

$$\text{Frictional power\%} = \frac{F\cdot V_c}{F_c\cdot V}\times 100 = \left(\frac{831.71}{2835}\right)\times 100 = 29.337\%$$

(iii) Specific cutting energy,
$$e = \frac{\text{Power (in Watt)}}{\text{Material removal rate (in mm}^3\text{/s)}}$$

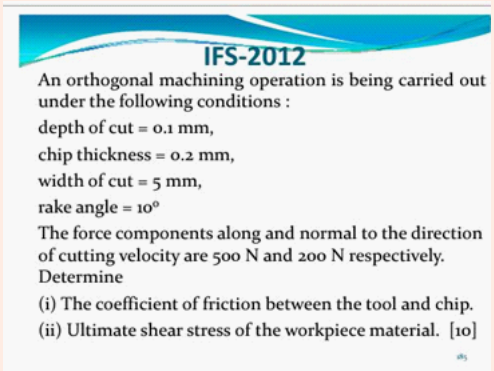
$$= \frac{F_c\cdot V}{(fd)\cdot V} = \frac{F_c}{(1000)fd} = \frac{F_c}{(1000)bt}$$

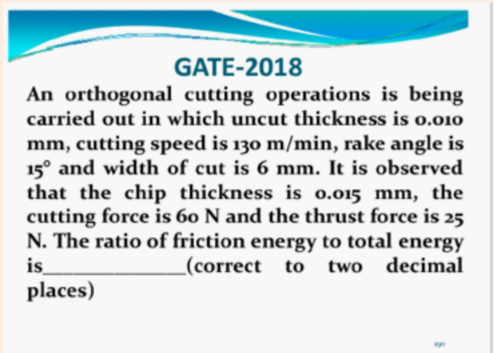
$$e = \frac{1134}{(1000)\times 10\times 0.2}$$

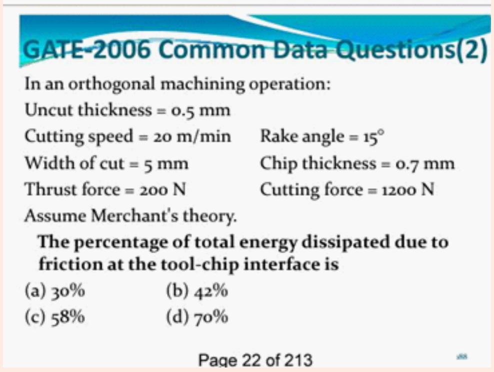
Specific cutting energy, $e = 0.567 \text{ J/mm}^3$

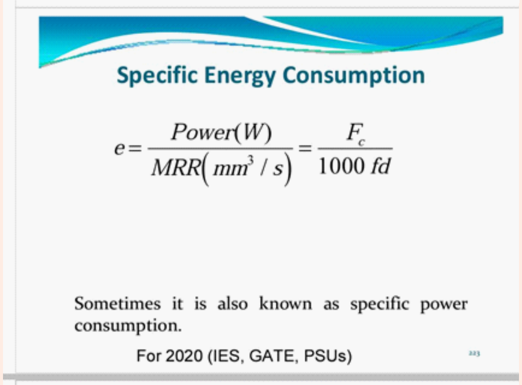
MADE EASY Source

- **MADE EASY Classnotes**









End of Solution

5. (c) Describe four tests of flexibility that an automated manufacturing system should satisfy to qualify as being flexible. Also list the application areas where FMS technology is successfully employed.

[12 Marks]

Solution:

Tests of flexibility: For a manufacturing system for becoming flexible there are several criteria. Four important test of flexibility in an automated manufacturing system are as follows:

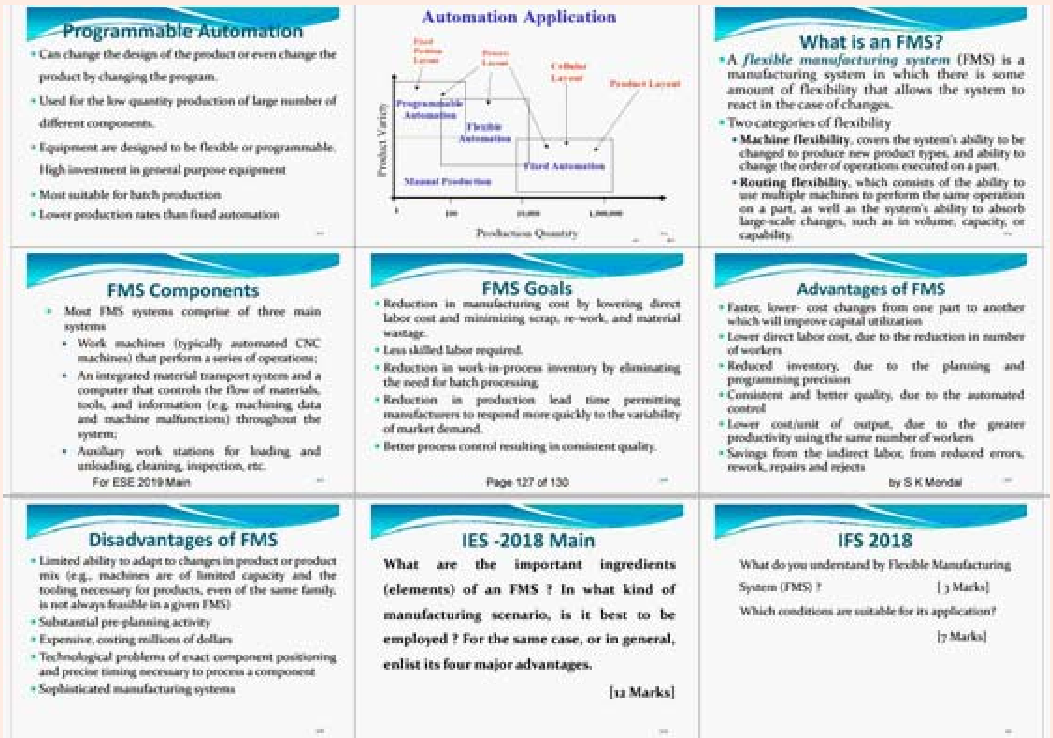
1. **Part variety test**: It is to check the ability of a system to process different part styles in a non-batch mode. It is done for checking machine flexibility or production flexibility.
2. **Schedule change test**: It shows the ability of a system to accept to the changes in production schedule readily, these changes may be either part mix or in production quantity. It is used for mix flexibility, volume flexibility and expansion flexibility.
3. **Error recovery test**: It shows the ability of a system that how well it recover from equipment malfunctions and breakdowns, so that the production system is less disrupted. It is used for check routing flexibility.
4. **New part test**: It tells us that how easily a new part design can be introduced into the existing product mix. It tells us about product flexibility.

Application of FMS:

1. Metal cutting machining
2. Metal forming
3. Assembly
4. Joining -welding, gluing
5. Surface treatment
6. Inspection and testing

MADE EASY Source

- **Theory Book (2019 Edition):** Page no 257 of Production Engineering
- **MADE EASY Classnotes**



5. (d) Describe at least five main functions carried out by coating on electrode in electric arc welding process. Also, list the constituents of coating and their purpose.

[12 Marks]

Solution:

Functions of flux coating:

1. Flux coating material may act as deoxidizers.
2. Flux coating material by forming the slag, protect liquid metal from the atmospheric gases.
3. Flux coating material increases the strength of the joint by adding alloying element.
4. Flux coating material control the viscosity of liquid metal and heat transfer rate in the weld pool.
5. Flux coating material by reducing the arc blow increases the stability of the arc.
6. Flux coating material by reducing the heat transfer losses increases the heat concentration on the workpiece.

Flux coating materials:

i. **De-oxidizing material:** Graphite, Alumina, Ferro silicon and Ferro manganese.
ii. **Slag formation compounds:** Iron oxide, Silicon dioxide, Titanium oxide, Silica flour and Calcium fluoride.
iii. **Arc stabilizer:** Sodium oxide, Calcium oxide, Potassium silicate.
iv. **Alloying elements:** Chromium, Nickel, Cobalt and Vanadium.

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5 (c) of Test-7

End of Solution

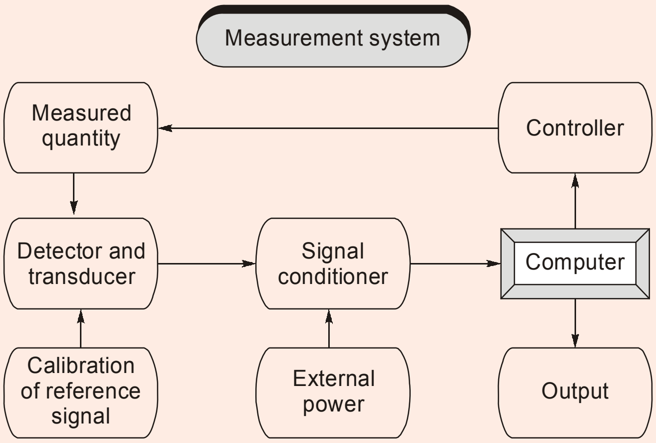
5. (e) Explain the distinction between the following using block diagrams and examples:
(i) Measurement systems and Control systems
(ii) Open-loop systems and Closed-loop systems

[12 Marks]

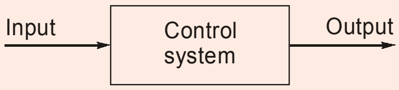
Solution:

(i) **Measurement systems:** Measurement systems have wide applications such as measurement of electrical and physical quantities like current, voltage, power, temperature, pressure, displacement etc.

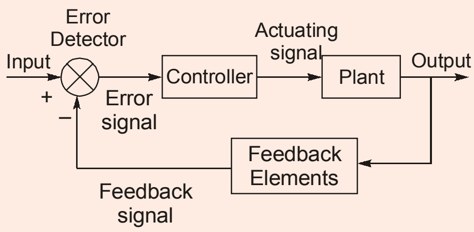
The reason or measurement arises when one wants to generate data for design or when one wants to propose a theory based on a set of measurement and instrumentation for commerce.



Control system: A control system is a system, which provides the desired response by controlling the output. A control system is a system, which provides the desired response by controlling the output. The following figure shows the simple block diagram of a control system.

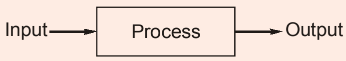


Here, the control system is represented by a single block. The output is controlled by varying input, this input is varied with some mechanism.

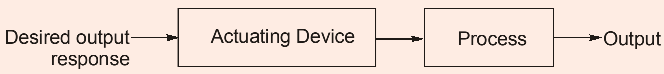


(ii) Open loop Systems:

An open loop control system utilizes an actuating device to control the process directly without using feedback channel. A component or process to be controlled can be represented by a block diagram as shown in figure. The block can be investigated one at a time, the output of one block will be input to another block and we can use cause and effect reasoning. This will become complicated when it is connected in a closed loop system because of interaction between different blocks.



Process to be controlled



Open loop control system (Non feedback)

Advantages:

1. Simple construction
2. Low cost

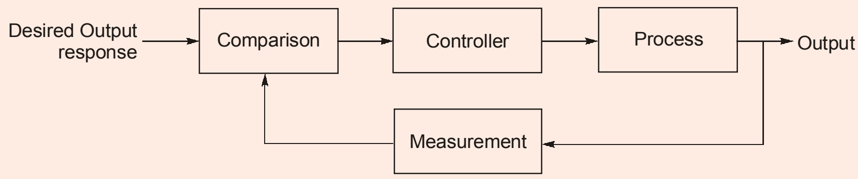
3. Maintenance is easy
4. Stability is good

Disadvantages

1 Less Accuracy
2. Reliability is less
3. The change in output cannot be modified

Closed Loop Systems:

A closed loop control system uses a measurement of the output and feedback of this signal to compare it with the desired output. The system can be easily spotted by seeing its block relation, it does not always maintain only relation from one block head to another block tail, it also has relation from different blocks as shown in figure of closed loop feedback control system. Since the increasing complexity of the system is under control and the optimum performance of feedback system has grown it in past few decades.



Closed loop control system (feedback system)

Advantages:

1. Closed loop control is more accurate, even in the case of non-linearity.
2. Highly accurate because the error of output can be modified by feedback signal.
3. Range of Bandwidth is large.
4. Lesser sensitive to disturbance and noise.
5. Lesser sensitive to the characteristics and parameter variations.

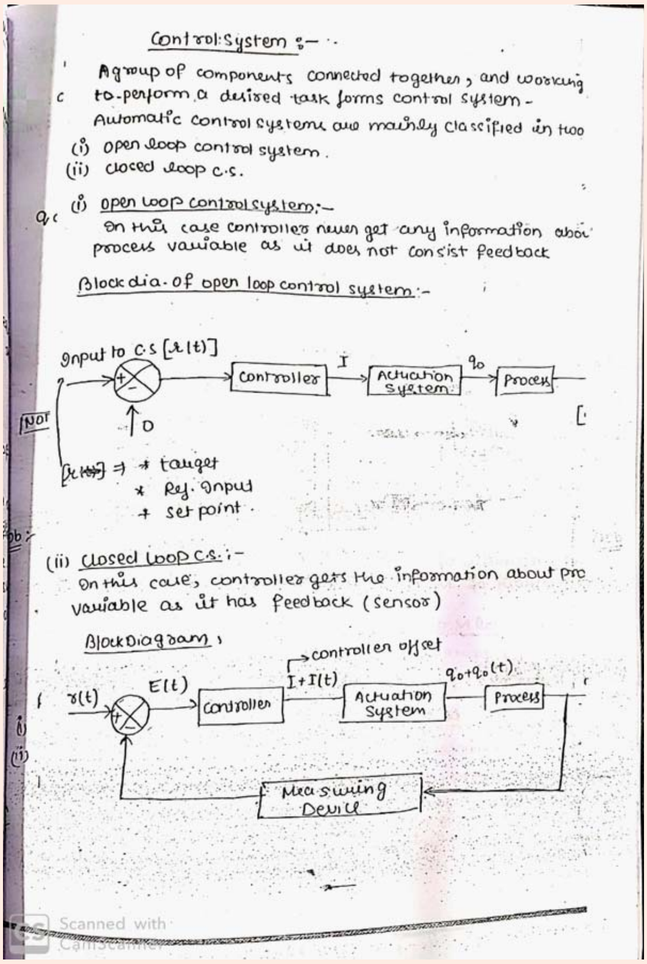
Disadvantages:

1. The cost is high.
2. Design is complicated.
3. More maintenance required, which further increases the cost.
4. Overall gain reduces due to feedback.

MADE EASY Source

- **ESE 2019 Mains Workbook:** Q.10 of Mechatronics (Unsolved Que) discussed in Class
- **Theory Book (2019 Edition):** Page No. (348 to 349) of Mechatronics

- **MADE EASY Classnotes**


Control System :-
A group of components connected together, and working to perform a desired task forms control system.
Automatic control systems are mainly classified in two
(i) open loop control system.
(ii) closed loop c.s.
(i) open loop control system :-
In this case controller never get any information about process variable as it does not consist feedback
Block dia. of open loop control system :-

(ii) Closed Loop C.S. :-
On this case, controller gets the information about process variable as it has feedback (sensor)
Block Diagram,

End of Solution

6. (a) (i) In an open die forging, a strip 150 mm wide, 4 mm long and 10 mm thick is compressed in plane strain such that the dimension 400 remains same. The yield strength of material in uniaxial compression is equal to 200 N/mm2. Find the minimum, average and maximum die pressures at the beginning of plastic deformation if the coefficient of friction on the interface between the die and the material is equal to 0.1.

[10 Marks]

(ii) For a product, the purchase prices are given below:

Sl. No.	Order Quantity (Q_i)	Unit Prices (Rs.)
1	$Q_1 < 500$	10.00
2	$500 \leq Q_2 < 750$	9.25
3	$Q_3 \geq 750$	8.75

Determine the optimum purchase quantity if the annual demand of the product is 2400 units. The cost of ordering is 100 and the inventory carrying charge is 24% of the purchase price per year.

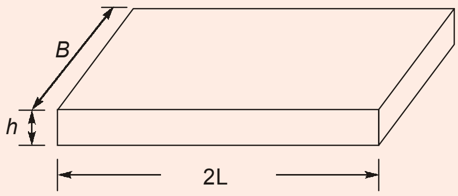
[10 Marks]

Solution:

(i)

As given in question that this is plain strain condition. In plane strain condition width remains same. So width of rectangular bar is 400 mm.

In forging length increases and height decreases.



So, total length, (2L) = 150 mm

$$L = 75 \text{ mm}$$

Given: $\mu = 0.1$, $\sigma_y = 200$ MPa, $h = 10$ mm

Sticking zone length,

$$x_s = L - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$$

$$x_s = 75 - \frac{10}{2\times0.1}\ln\left(\frac{1}{2\times0.1}\right) = -5.472 \text{ mm}$$

Hence there is no sticking. Only sliding is there in this question.

Pressure distribution in sliding region is given by,

$$P = 2ke^{(2\mu/h)(L-x)} \quad \text{...(1)}$$

Where x is distance from centre and pressure variation is symmetric about centerline. Maximum value of pressure will be at centre and minimum will be at corner points.

For maximum pressure, $x = 0$

$$P_{max} = 2ke^{(2\mu/h)\times L} \quad \text{...(2)}$$

(I) Using Von-Mises theory:

Where, $$k = \frac{\sigma_y}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ MPa}$$

From eq. (2) for maximum pressure, $$P_{max} = \left(2\times\frac{200}{\sqrt{3}}\right)e^{\left(\frac{2\times0.1\times75}{10}\right)}$$

$$P_{max} = 1035 \text{ MPa}$$

For minimum pressure putting, x = L in eq. (1),

$$P_{min} = 2ke^{\left(\frac{2\mu}{h}\right)(L-L)}$$

$$= 2k = \frac{2\times 200}{\sqrt{3}}$$

$$P_{min} = \frac{400}{\sqrt{3}} = 230.94 \text{ MPa}$$

For mean pressure, $P_{mean} = \dfrac{2\int_0^L \left[2ke^{\frac{2\mu}{h}(L-x)}\cdot Bdx\right]}{B\times(2L)}$

$$P_{mean} = \left(\frac{4k}{2L}\right)\int_0^{75} e^{\frac{2\times 0.1}{10}(75-x)}\cdot dx = \frac{4\times 200}{2\times 75\times\sqrt{3}}\times 174.0844$$

$$P_{mean} = 536.04 \text{ MPa}$$

Using Tresca theory:

$$k = \frac{\sigma_y}{2} = \frac{200}{2} = 100 \text{ MPa}$$

For maximum pressure, from equation (2)

$$P_{max} = 2ke^{(2\mu/h)\times L} = 2\times 100\times e^{\left(\frac{2\times 0.1\times 75}{10}\right)} = 896.34 \text{ MPa}$$

For minimum pressure, $P_{mean} = \dfrac{2\int_0^L \left[2ke^{\frac{2\mu}{h}(L-x)}\cdot Bdx\right]}{B\times(2L)}$

$$P_{mean} = \left(\frac{4k}{2L}\right)\int_0^{75} e^{\frac{2\times 0.1}{10}(75-x)}\cdot dx = \frac{4\times 100}{2\times 75}\times 174.0844$$

$$P_{mean} = 464.225 \text{ MPa}$$

(ii)

Given: D = 2400 units, C_o = Rs. 100/order, C_h = 24% of unit cost

Let, C = unit price

Case 1: $Q < 500$, C = Rs. 10/unit

$$EOQ,\ Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2\times 2400\times 100}{0.24\times 10}} = 447.21 \text{ units}$$

$Q^* < Q$ [So, EOQ is in feasible region]

Total cost per year, $(TC)_1 = \sqrt{2DC_oC_h} + D\times C = \sqrt{2\times 2400\times 100\times 0.24\times 10} + 2400\times 10$

= Rs. 25073.31

Case 2: $500 \le Q < 750$, C = Rs. 9.25/unit

$$EOQ,\ Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times 2400\times 100}{0.24\times 9.25}} = 464.99 \text{ unit}$$

∴ EOQ is not in feasible region.

So, Q = 500 units

So, minimum total cost per year, $(TC)_2 = \left(\dfrac{D}{Q}\right)\times C_o + \dfrac{Q}{2}\times C_h + D\times C$

$$= \left(\frac{2400}{500}\right) \times 100 + \left(\frac{500}{2}\right) \times 0.24 \times 9.25 + (9.25 \times 2400)$$

= Rs. 23235

Case 3: $Q \geq 750$ units, C = Rs. 8.75

$$\text{EOQ, } Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 2400 \times 100}{0.24 \times 8.75}} = 478.09 \text{ unit}$$

Q* is not in feasible region.

$$\text{Minimum total cost per year, } (TC)_3 = \left(\frac{D}{Q}\right) \times C_o + \left(\frac{Q}{2}\right) \times C_h + D \times C$$

$$= \left(\frac{2400}{750}\right) \times 100 + \left(\frac{750}{2}\right) \times 0.24 \times 8.75 + (2400 \times 8.75)$$

= Rs. 22107.5

$(TC)_3 < (TC)_2 < (TC)_1$. Hence minimum cost is for case 3.

Result: Hence total cost per year is minimum for order size of 750 units.

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(c) of Test-15 (Source for Q.6 (a) (ii))
- **MADE EASY Classnotes (Mains Batch):** Source for Q.6 (a)

IES – 2005 Conventional

A strip of lead with initial dimensions 24 mm x 24 mm x 150 mm is forged between two flat dies to a final size of 6 mm x 96 mm x 150 mm. If the coefficient of friction is 0.25, determine the maximum forging force. The average yield stress of lead in tension is 7 N/mm²

Solution: $h = 6$ mm, $2L = 96$ mm, $\mu = 0.25$

$$x_s = L - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right) = 48 - \frac{6}{2 \times 0.25}\ln\left(\frac{1}{2 \times 0.25}\right) = 39.68\,mm$$

$$F_{total} = 2 \times \int_0^{x_s}\left[P_s + \frac{2K}{h}(x_s - x)\right] B.\,dx + 2 \times \int_{x_s}^{L} 2K\,e^{\frac{2\mu}{h}(L-x)}\,B.\,dx$$

Applying Von-Mises theory $K = \frac{\sigma_o}{\sqrt{3}} = 4.04\,N/mm^2$

$$P_s = \frac{K}{\mu} = 16.16\,N/mm^2$$

or

or $$F = 2 \times \int_0^{39.68}\left\{16.16 + \frac{2 \times 0.25}{6}(39.68 - x)\right\} \cdot 150\,.\,dx + 2 \times \int_{39.68}^{48}(2 \times 4.04)\,e^{\frac{2 \times 0.25}{6}(48-x)} \cdot 150\,.\,dx$$

$$= 510\,kN + 29.10\,kN = 539\,kN\,(Von - Mises)$$

Applying Tresca's Theory, $K = \frac{\sigma_o}{2} = 3.5\,N/mm^2$; $P_s = \frac{K}{\mu} = \frac{3.5}{0.25} = 14\,N/mm^2$

$$F = 2 \times \int_0^{39.68}\left\{14 + \frac{2 \times 3.5}{6}(39.68 - x)\right\} \cdot 150 \cdot dx + 2\int_{39.68}^{48}(2 \times 3.5)\,e^{\frac{2 \times 0.25}{6}(48-x)} \cdot 150 \cdot dx$$

$$442\,kN + 25\,kN = 467\,kN\,(Tresca's)$$

Practice Problem-1

A strip of metal with initial dimensions 24 mm x 24 mm x 150 mm is forged between two flat dies to a final size of 6 mm x 96 mm x 150 mm. If the coefficient of friction is 0.05, determine the maximum forging force. Take the average yield strength in tension is 7 N/mm²

Given: $2L = 96$ mm; $L = 48$ mm; $h = 6$ mm; $B = 150$ mm; $\mu = 0.05$

$$x_s = L - \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right) \qquad K = 4.04\text{ N/mm}^2$$

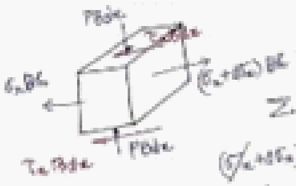
$$x_s = -90.155\,mm$$

Since x_s came negative so there will be no sticking only sliding will take place.

Analysis of forging:

- Open die Forging (Slab Method of Analysis)
  1. Rectangular bar forging
  2. Axis symmetrical forging.
  } Assumption to be read from notes. (Written before analysis)

(1) Rectangular Bar FORGING:



- As the system is in equilibrium, the element must be in equilibrium therefore Net Resultant force in x-dir will be zero.

$\Sigma F_x = 0$;

$(\sigma_x + d\sigma_x)Bh - \sigma_x Bh - 2\tau B dx = 0$

$d\sigma_x Bh - 2\tau B dx = 0 \Rightarrow \boxed{\frac{d\sigma_x}{dx} - \frac{2\tau_x}{h} = 0}$ —(1)

Here in eq(1) three variables are present; σ_x, τ_x and x. We have to reduce it in 2 variables by applying conditions;

- For ductile material there are two theories of plasticity
  1. von-Mises theory
  2. Tresca's theory

$\sigma_2 = 0$ (Plane strain cond.)

$\sigma_2 = \mu(\sigma_1 + \sigma_3)$

From Fig $\sigma_1 = \sigma_x$, $\sigma_3 = -P$ $\Rightarrow$ $\sigma_2 = \frac{1}{2}(\sigma_x - P)$

Acc. to von-Mises: $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2\sigma_0^2$

$\left(\sigma_x - \frac{1}{2}(\sigma_x - P)\right)^2 + \left(\frac{1}{2}(\sigma_x - P) + P\right)^2 + (-P-\sigma_x)^2 = 2\sigma_0^2$

$\frac{(\sigma_x+P)^2}{4} + \frac{(\sigma_x+P)^2}{4} + (P+\sigma_x)^2 = 2\sigma_0^2$

$(\sigma_x+P)^2 \times \frac{6}{4} = 2 \times \sigma_0^2 \Rightarrow (\sigma_x+P)^2 = \frac{4}{3}\sigma_0^2 \Rightarrow (\sigma_x+P) = \frac{2}{\sqrt{3}}\sigma_0$

$\boxed{\sigma_x + P = 2K}$ where $\left(K = \frac{\sigma_0}{\sqrt{3}}\right) \rightarrow$ Flow shear stress

Acc. to Tresca: $\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_0}{2} \Rightarrow \boxed{\sigma_x + P = \sigma_0}$ Also; $\boxed{\sigma_x + P = 2K}$ —(2) where $K = \frac{\sigma_0}{2}$ → Flow shear stress

Now, differentiate (2) eq^n w.r.t. x

$\frac{d\sigma_x}{dx} + \frac{dP}{dx} = 0$; $\boxed{\frac{d\sigma_x}{dx} = -\frac{dP}{dx}}$ —(3)

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Condition 1: Considering sliding friction all over the surface. ($\tau_x = \mu P$)

From eq^n ① and ③;

$$\frac{d\sigma_x}{dx} - \frac{2\tau_x}{h} = 0 \Rightarrow -\frac{dP}{dx} - \frac{2\mu P}{h} = 0$$

$$\int \frac{dP}{P} = -\frac{2\mu}{h}\int dx \Rightarrow \boxed{\ln P = -\frac{2\mu}{h}x + C} \quad \text{——(4)}$$

At $x = L$, $\sigma_x = 0$ (because no material for resistance) $\rightarrow \sigma_x + P = 2K$, $\boxed{P = 2K}$

$$\ln(2K) = -\frac{2\mu}{h}L + C \Rightarrow C = \ln(2K) + \frac{2\mu}{h}L \leftarrow \text{Substituting the value in eqn 4}$$

$$\ln P = -\frac{2\mu}{h}x + \ln(2K) + \frac{2\mu}{h}L \Rightarrow \ln\left(\frac{P}{2K}\right) = \frac{2\mu}{h}(L - x)$$

$$\boxed{P = 2K e^{\frac{2\mu}{h}(L-x)}} \quad \text{——(5) (E-1)}$$

Pressure distribution is exponential decreasing

At $x = 0$, $P_{max} = 2K e^{\frac{2\mu}{h}L}$

At $x = L$; $P_{min} = 2K$

$$F = 2\int_0^L P\underbrace{B\,dx}_{\text{Area of strip}} = 2\int_0^L 2Ke^{\frac{2\mu}{h}(L-x)}\cdot B\,dx = 4BK\int_0^L e^{\frac{2\mu}{h}(L-x)}\,dx$$

$$= 4KB\left[\frac{e^{\frac{2\mu}{h}(L-x)}}{-\frac{2\mu}{h}}\right]_0^L = \frac{4KBh}{-2\mu}\left[e^{\frac{2\mu}{h}[L-L]} - e^{\frac{2\mu}{h}[L-0]}\right]$$

$$\boxed{F = \frac{2KBh}{\mu}\left[e^{\frac{2\mu}{h}L} - 1\right]}$$

{Questions are asked with proof and then putting values}

Condition 2: Considering sticking friction all over the surface ($\tau_x = K$)

From eq^n ① and ③;

$$\frac{d\sigma_x}{dx} - \frac{2\tau_x}{h} = 0 \Rightarrow -\frac{dP}{dx} - \frac{2K}{h} = 0$$

$$\int dP = -\frac{2K}{h}\int dx\;; \quad \boxed{P = -\frac{2K}{h}x + C} \quad \text{——⑥}$$

In some situation discount is offered on unit purchase price of inventory for large quantity purchase and these discount take the form of price break. As discount is offered on unit purchase price so in order to determine the best order size, we consider purchasing cost alongwith ordering and holding cost. In these problem first we find feasible EOQ and then total cost is computed at EOQ and the next higher order size having price break, wherever the total cost is minimum gives the best order size.

ES-08

Annual demand is 8000 units and ordering cost is Rs 1800 per order. Holding cost is 10% of unit price of inventory and items can be purchased in the lot given below. Determine the best order size :-

Lot size	Unit price (Rs/unit)
1 – 999	220
1000 – 1499	200
1500 – 1999	190
2000 & above	185

Sol^n — $D = 8000$ units, $C_0 = $ Rs 1800 /order

$C_h = 10\%$ of C

$$EOQ = \sqrt{\frac{2DC_0}{C_h}}$$

Starting from the lowest unit price, searching for feasible EOQ.

$C = $ Rs 185/unit

$$q^* = \sqrt{\frac{2 \times 8000 \times 1800}{185 \times 0.1}} = 1247.7 \text{ units/order.}$$

q^* is not feasible.

For $c = $ Rs 185/unit.

$Q \geq 2000$.

End of Solution

6. (b) What is a eutectoid reaction? Explain the development of microstructure in iron-carbon alloys of hypoeutectoid, eutectoid and hypereutectoid compositions when they are cooled from high temperature with the help of neatly labelled diagrams indicating the phases present.

[20 Marks]

Solution:

Eutectoid reaction is:

$$\text{Solid} \underset{\text{Heating}}{\overset{\text{Cooling}}{\rightleftharpoons}} \text{Solid}_1 + \text{Solid}_2 \text{ (eutectoid mixture)}$$

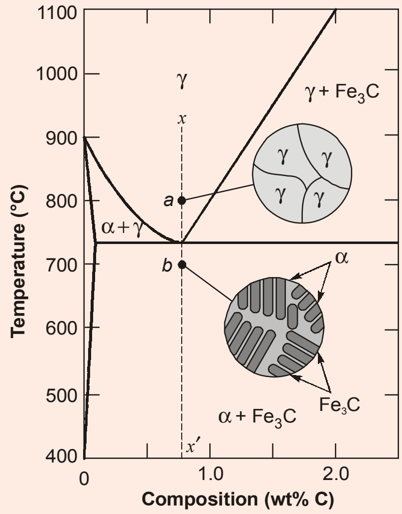
The eutectoid reaction for (0.8% C, 727°C)

$$\gamma - \text{Iron} \underset{0.8\%C}{\overset{727°C}{\rightleftharpoons}} \underbrace{\alpha - \text{Iron} + Fe_3C}_{\text{Pearlite}}$$

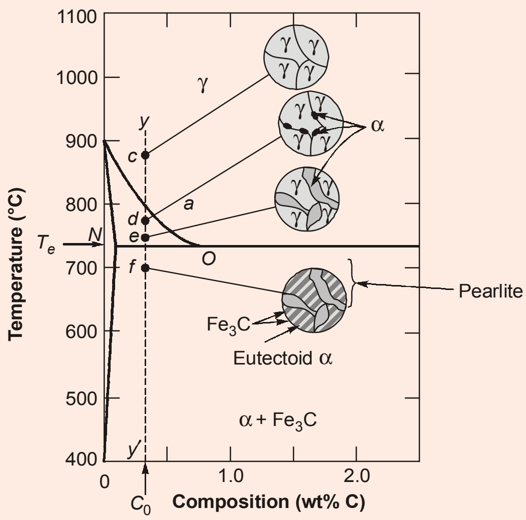
Development Of Microstructures In Iron-Carbon Alloys:

1. Eutectoid composition (0.8% C): An alloy of eutectoid composition (0.8 wt% C) as it is cooled from a temperature within the phase region (Point a), say, 800°C, and moving down the vertical line xx'. At point a, the alloy is entirely in the austenite

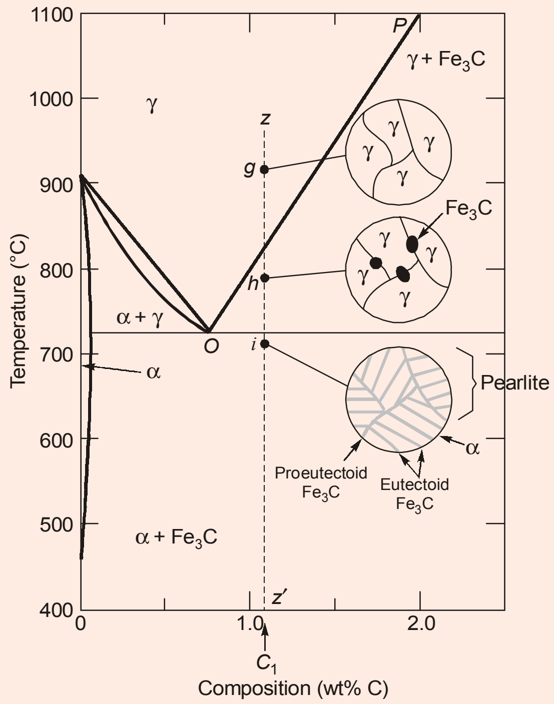
phase. After cooling through the eutectoid temperature, the microstructure for this eutectoid steel (Point b) consists of alternating layers or lamellae of the two phases (α and Fe_3C) also known as pearlite.



2. **Hypoeutectoid Alloys (<0.8%C):** A steel sample in hypo eutectoid steel range is cooled along yy′. At *point c*, the alloy is entirely in the austenite phase. The moment temperature decreases to *point d*, ferrite start appearing in the microstructure. This ferrite that appears before eutectoid temperature is called pro- eutectoid ferrite (pro α). As temperature decreases mass fraction of pro α will increase and at *point e* there will be canals of pro a at the grain boundary. As the temperature is lowered just below the eutectoid, to *point f*, all the γ phase that was present at temperature T_e(and having the eutectoid composition) will transform to pearlite. There will be virtually no change in the a phase that existed at *point e* in crossing the eutectoid temperature. It will normally be present as a continuous matrix phase surrounding the isolated pearlite colonies. It should also be noted that two micro constituents are present, i.e., proeutectoid ferrite and pearlite which will appear in all hypoeutectoid iron–carbon alloys that are slowly cooled to a temperature below the eutectoid.

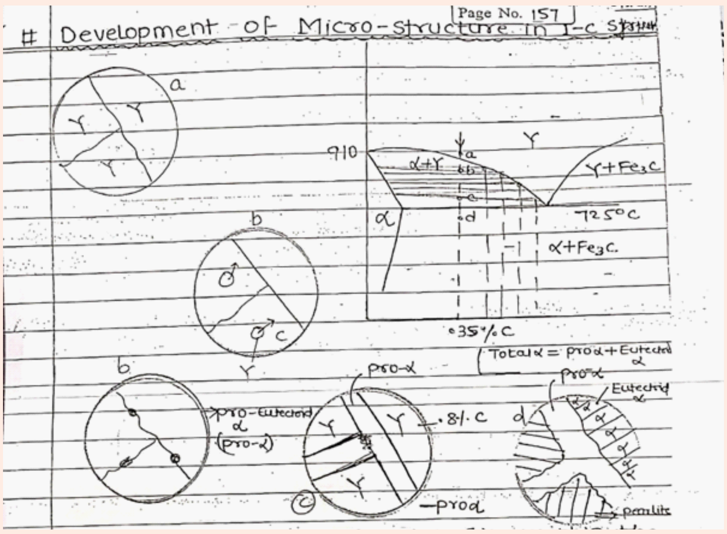


3. **Hypereutectoid Alloys (0.8% to 2.14% C)**: Analogous transformations and microstructures result for hypereutectoid alloys, those containing between 0.8 and 2.14 wt% C, which are cooled from temperatures within the γ phase field. Before eutectoid temperature proeutectoid cementite is formed . In the final structure (Point i) proeutectoid cementite and pearlite will appear.



MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.6(c) (ii) of Test-15
- **Theory Book (2019 Edition):** Material Science (Page No. 55)
- **MADE EASY Classnotes**



At point A shown in the Fig within the grain boundary it will all be austenite with carbon uniformaly Distributed in the micro=structure. The moment temp. is slightly decreases at point B, ferrite will be form in the grain and Hence different crystal structure it will be push towards grain boundary. This ferrite that appears before the eutectoid temp. is called pro-eutectoid ferrite. when the Sample cooled slowly from point B to C following conclusion can be made using the lever Rule.

End of Solution

6. (c) (i) Describe, with neat sketches, the working principle of—

(1) linear variable differential transformer (LVDT);

(2) Hall effect sensor

[5 + 5 Marks]

(ii) A measurement system consists of a cylindrical load cell of diameter 2.5 mm. The material of the cell is steel with modulus of elasticity, E = 210 GPa and Poisson's ratio, $\gamma = 0.3$. This carries four strain gauges each with gauge factor 2.1. Two of them are mounted longitudinal and other two are transverse. The resistances of the gauges are 120Ω. This load cell is required to yield a voltage through the bridge of strain gauges with bias 10V. If the maximum load sustained by the cell is – 2500 N, what is the corresponding voltage across the bridge?

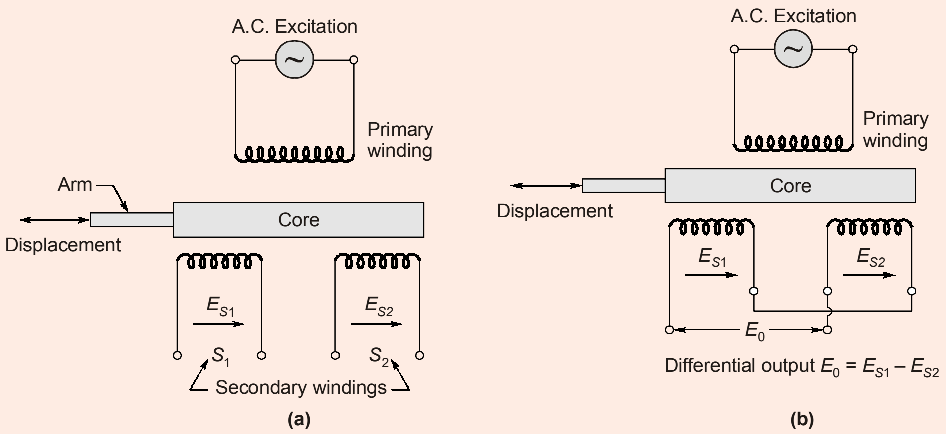
[10 Marks]

Solution:

(i)

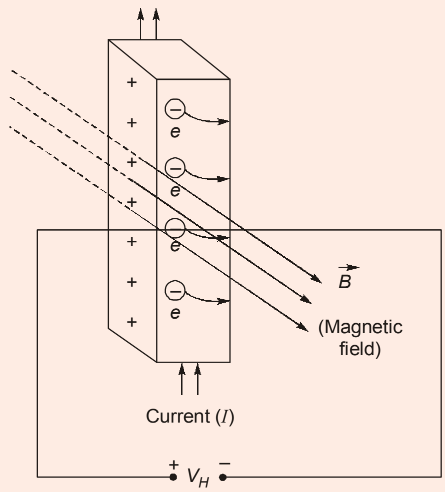
1. **Linear variable differential transformer (LVDT):**

- Acronym for Linear Variable Differential Transformer, a common type of electromechanical transducer that can convert the rectilinear motion of an object to which it is coupled mechanically into a corresponding electrical signal.
- LVDT linear position sensors are readily available that can measure movements as small as a few millionths of an inch up to several inches, but are also capable of measuring positions up to ±20 inches (±0.5 m).
- A rotary variable differential transformer (RVDT) is a type of electrical transformer used for measuring angular displacement.
- Working principal of LVDT is based on mutual induction. The dislocation is a nonelectrical energy that is changed into an electrical energy.



2. **Hall Effect Sensor**: Principle: When a beam of charged particles passes through a magnetic field, forces act on the particles and the beam is deflected from the straight line path.

   If a current (I) Carrying conductor of thickness ('t') is placed inside a magnetic field of (B); Then voltage will be developed across the crystal, which is given as hall voltage (VH).



$\Rightarrow$ Hall voltage $V_H = K_H \dfrac{I \cdot B}{t}$

'K_H' is the Half-coefficient.

$t \rightarrow$ Thickness of object (m)

$I \rightarrow$ Current through conductor (A)

$B \rightarrow$ Magnetic flux density Wb/m^2

Applications:

- Used to measure position, displacement
- Used in proximity sensor
- Used in Brushless DC-motor

(ii)

Given: Poisson's ratio, $\mu = 0.3$, V_s = (supplied voltage) = 10V

E = 210 GPa, Gauge factor, G = 2.1

Resistance, R = 120 Ω, Load, P = 2500 N (compressive)

Cylindrical diameter = 2.5 mm

Stress due to load P,

$$\sigma = \frac{P}{A} = \frac{2500}{\frac{\pi}{4}(2.5\times10^{-3})^2\times10^6} = 509.295 \text{ MPa}$$

Strain,
$$\varepsilon = \frac{\sigma}{E} = \frac{509.295}{210\times10^3} = 2.425\times10^{-3}$$

$$\text{Gauge factor, } G = \frac{\Delta R}{R/\varepsilon}$$

$$\frac{\Delta R}{R} = G\times\varepsilon = 2.1\times2.425\times10^{-3} = 5.0929\times10^{-3}$$

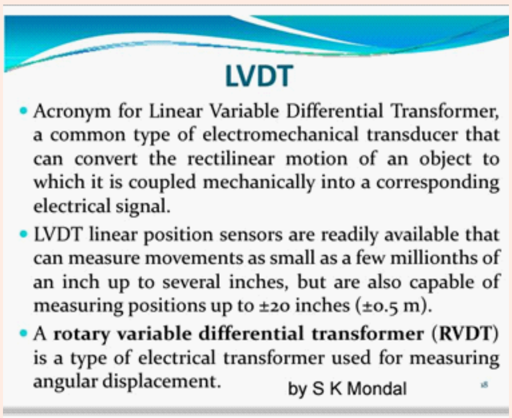
Output voltage across bridge is given by

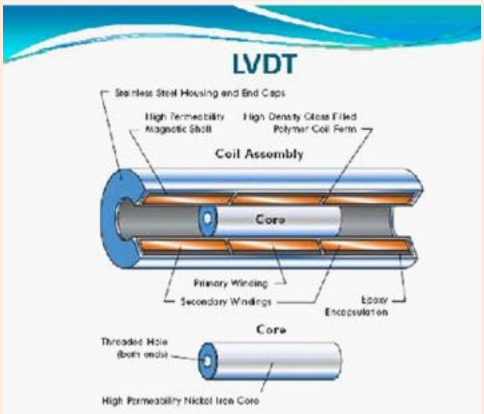
$$V_o = (1+\mu)\frac{dR}{R}\cdot\frac{V_s}{2} = (1+0.3)\times5.0929\times10^{-3}\times\frac{10}{2}$$

$$V_o = 0.0331 \text{ V}$$

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.7(b) of Test-15 (Source for Q.6 (a) (ii))
- **Theory Book (2019 Edition):** Mechatronics (Page No. 327)
- **MADE EASY Classnotes**





End of Solution

7. (a) (i) On the basis of microstructure, briefly explain why gray iron is brittle and weak in tension. Compare gray and malleable cast irons with respect to (1) composition and heat treatment, (2) microstructure and (3) mechanical properties. [10 Marks]

(ii) Cite three sources of internal residual stresses in metal components. What are two possible adverse consequences of these stresses? Describe the following heat treatment procedures for steels and for each, the intended final microstructure:

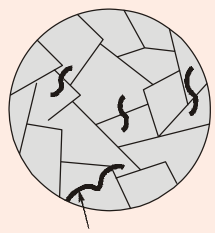
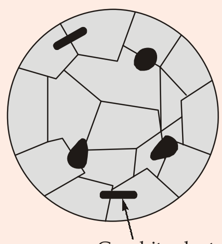
Full annealing, Normalizing, Tempering and Quenching [10 Marks]

Solution:

(i) Gray cast iron is brittle and weak in tension as a consequence of its microstructure for most of the cast iron, the graphite exists in the form of flakes(similiar to corn flakes) which are normally surrounded by an α-ferrite or pearlite matrix. Because of these graphic flakes a fractured surfaces takes on a gray appearance hence its name. The tips of the graphite flakes are sharp and pointed and may serve as points of stress concentration when an external tensile stress is applied.

Cast Iron: Cast iron are defined as the iron-carbon alloys that contain carbon between 2.1% and 6.67%. However commercial cast irons normally contain less than 4.5% of carbon. They can be easily melted because of the high percent of carbon and can be castable to required shape because of their fluidity.

Gray cast iron microstructure consists of graphite flakes dispersed throughout the metal matrix. The tips of the graphite flakes are sharp and pointed, and may act as the sites for crack initiation, when an external tensile stress is applied.

	Gray cast iron	Malleable cast iron
(i) Composition and heat treatment	C - 2.5 – 4% Si - 1.3% Mn - 0.7 – 0.8% No special heat treatment required.	C - 2.3 – 2.7% Si - 1–1.7% Mn - <0.55% Requires initial rapid cooling for formation of white cast iron, later, white cast iron is held at 800-900°C for more graphite formation, which leads to formation of malleable cast iron.
(ii)	 Graphite flakes	 Graphite clusters
(iii) Mechanical characteristics	→Weak in tension, but excellent compressive strength. →Good machinability as graphite flakes offer lubrication and chip breaking. →Very good damping characteristics as graphite flakes absorb energy. →Good resistance to adhesive wear.	→Increased ductility →Higher corrosion resistance →Excellent impact strength →Higher tensile strength
(iv) Application	As damper machine base materials, engine blocks of automobiles	General engineering equipments and others including. connecting rods, transmission gears and other heavy duty services.

(ii)

The main reason of internal residual stresses can be:

1. **Mechanical load**: Plastic deformation process such as machining and grinding.
2. **Thermal load**: Non-uniform cooling of a component that was processed or fabricated at an elevated temperature. Such as weld or a casting.
3. **Phase change**: A phase transformation that is induced upon cooling where in parent and product phases have different densities such as austenite to martensite.

Two possible adverse consequences of residual stresses are:

- The residual stresses whether tensile or compressive predominantly affect the soundness, dimensional stability and mechanical performance of the weld joint.

- Effects of residual stresses are mostly observed either near the last stage of weld thermal cycle or after sometime of welding in form of :
  Cracks : hot cracking, lamellar tearing, cold cracking
  Distortion : longitudinal , angular and transverse
  Reduction in mechanical performance of weld joint.
- **Full Annealing:** Metal is heated above the upper critical temperature and held there until the temperature of the workpiece is uniform throughout, and finally cooling the workpiece at a slowly controlled rate in furnace so that the temperature of the surface and that of the centre of the workpiece is approximately the same. Resulting microstructure is coarse pearlite.
- **Normalising**: Heat the steel from 30°C to 50°C above its upper critical temperature held about fifteen minutes and then allowed to cool down in still air. Homogeneous structure provides a higher yield point, ultimate tensile strength and impact strength with lower ductility to steels. Resulting microstructure is fine pearlite. It produces hard surface and tough core.
- **Tempering**: Tempering is the process of heating martensitic steel at a temperature below the eutectoid transformation temperature to make it softer and more ductile. During the tempering process, Martensite transforms to a structure containing iron carbide particles in a matrix of ferrite. Resulting microstructure is tempered martensite.
- **Quenching :** Quenching is heat treatment process where material is cooled at a rapid rate from elevated temperature to produce Martensite phase.

Comparative cooling rates of quenching media:

Brine	1.20 to 1.30
Water	1
Water + NaOH or KOH	< 1
Oil	0.40 to 0.50
Forced air	0.03
Still air	0.02

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(d) of Test-15 (Source for Q.7 (a)(i))
- **Theory Book (2019 Edition):** Material Science (Page No. 85) (Source for Q.7 (a)(ii))

- **MADE EASY Classnotes**

1. Grey cast iron

- Carbon = 3 to 3.5%;
- The grey colour is due to the fact that the carbon is present in the form of free graphite.
- It has a low tensile strength, high compressive strength and no ductility.
- It can be easily machined.
- A very good property of grey cast iron is that the free graphite in its structure acts as a lubricant. Due to this reason, it is very suitable for those parts where sliding action is desired.

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Contd...

- The grey iron castings are widely used for machine tool bodies, automotive cylinder blocks, heads, housings, fly-wheels, pipes and pipe fittings and agricultural implements.
- The grey cast iron is designated by the alphabets 'FG' followed by a figure indicating the minimum tensile strength in MPa or N/mm^2. For example, 'FG 150' means grey cast iron with 150 MPa or N/mm^2 as minimum tensile strength.

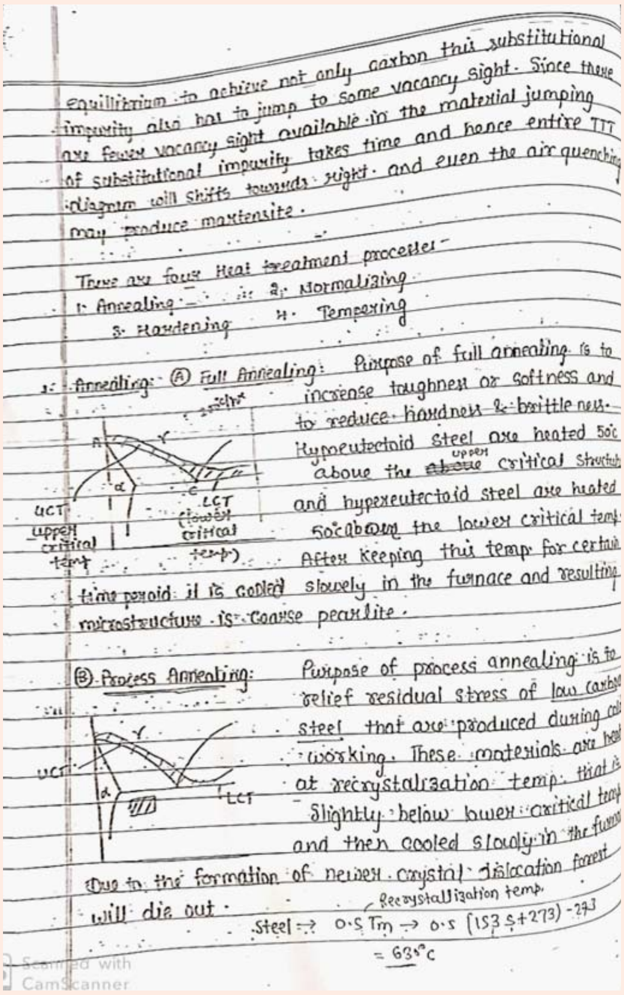
5. Malleable cast iron

- The malleable iron is a cast iron-carbon alloy which solidifies in the as-cast condition in a graphite free structure, *i.e.* total carbon content is present in its combined form as cementite (Fe_3C).
- It is ductile and may be bent without breaking or fracturing the section.
- The tensile strength of the malleable cast iron is usually higher than that of grey cast iron and has excellent machining qualities.

Contd...

- It is used for machine parts for which the steel forgings would be too expensive and in which the metal should have a fair degree of accuracy, *e.g.* hubs of wagon wheels, small fittings for railway rolling stock, brake supports, parts of agricultural machinery, pipe fittings, door hinges, locks etc.
- In order to obtain malleable iron castings, it is first cast into moulds of white cast iron. Then by a suitable heat treatment (*i.e.* annealing), the combined carbon of the white cast iron is separated into nodules of graphite.
- There are two process:
  1. Black-heart process,
  2. White-heart process

Contd...



equilibrium to achieve not only carbon this substitutional impurity also has to jump to some vacancy sight. Since there are fewer vacancy sight available in the material jumping of substitutional impurity takes time and hence entire TTT diagram will shifts towards right. and even the air quenching may produce martensite.

There are four Heat treatment processes-
1. Annealing 2. Normalizing
3. Hardening 4. Tempering

1. Annealing: (A) Full Annealing: Purpose of full annealing is to increase toughness or softness and to reduce hardness & brittleness. Hypoeutectoid steel are heated 50°C above the upper critical structure and hypereutectoid steel are heated 50°C above the lower critical temp. After keeping this temp for certain time period it is cooled slowely in the furnace and resulting microstructure is coarse pearlite.

(B) Process Annealing: Purpose of process annealing is to relief residual stress of low carbon steel that are produced during cold working. These materials are heated at recrystalization temp. that is slightly below lower critical temp and then cooled slowly in the furnace. Due to the formation of newer crystal dislocation forest will die out.

Recrystallization temp.
Steel → 0.5 Tm → 0.5 (1535+273) - 273
= 635°C

End of Solution

7. (b) (i) Why is unilateral tolerance preferred over bilateral tolerance? Find the limits of tolerance and allowance for a 25 mm H_8d_9 shaft and hole pair. The 25 mm shaft lies in the 18-30 diameter step. The fundamental tolerance can be computed using $i = 0.45\sqrt[3]{D} + 0.001D$ μm. For H_8 hole, the fundamental tolerance is 40i. The fundamental deviation for the shaft can be computed using $-16D^{0.44}$ μm. What type of fit is given by H_8d_9?

List the causes of getting primary texture and secondary texture in machined components. Further, list the three main methods of assessment of surface texture.

[10 Marks]

(ii) Five jobs are to be processed on three machines. The processing time (in hours) is given in the following table. Find the optimal schedule so that the total elapsed time is minimized. Also, find the idle time on each machine:

		Jobs				
		J_1	J_2	J_3	J_4	J_5
Machines	M_1	8	10	6	7	11
	M_2	5	6	2	3	4
	M_3	4	9	8	6	5

[10 Marks]

Solution:

(i)

Unilateral tolerance are preferred over bilateral tolerance due to following reasons:

1. Unilateral tolerance system is preferred when precision fits are required. Unilateral system is easy and simpler in comparison of bilateral system for determining deviations.
2. In unilateral tolerance system, go gauge ends can be standardized as the holes of different grades have the same lower limit and all the shafts have same upper limit.

Given shaft and hole pair dimension = 25 mm H_8d_9.

$$D = \sqrt{18 \times 30} = 23.238 \text{ mm}$$

$$i = 0.45\sqrt[3]{D} + 0.001D = 0.45(23.238)^{1/3} + 0.001 \times 23.238$$

$$i = 1.307 \ \mu\text{m}$$

Fundamental tolerance for H_8 hole = $25i$

$$= 25 \times 1.307 = 32.675 \ \mu\text{m}$$

$$\simeq 0.0327 \text{ mm}$$

Fundamental tolerance for d_9 shaft = 40i

$$= 40 \times 1.307 = 52.28 \ \mu\text{m}$$

$$\simeq 0.0523 \text{ mm}$$

Fundamental deviation for the shaft = $-16D^{0.44}$ = $-$ 63.86 μm $\simeq$ $-$0.0639 mm

Lower limit of hole = 25 mm

Upper limit of hole = 25 + 0.0327 = 25.0327 mm

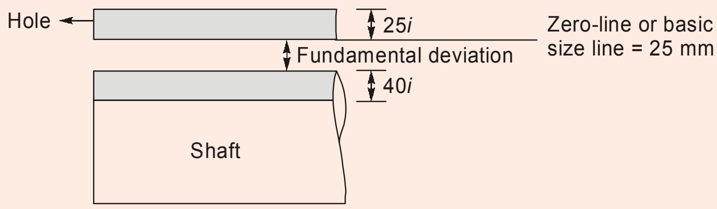
Upper limit of shaft = 25 – 0.0639 = 24.9361 mm

Lower limit of shaft = 25 – 0.0639 – 0.0327 = 24.9034 mm

Type of fit: Lower limit of hole is more than upper limit of shaft, so this is a case of clearance fit.

Allowance = Lower limit of hole – Upper limit of shaft

= 25 – 24.9361 = 0.0639 mm



Primary Texture (Roughness):

- **Roughness height:** is the parameter with which generally the surface finish is indicated. It is specified either as arithmetic average value or the root mean square value.
- **Roughness width**: is the distance parallel to the nominal part surface within which the peaks and valleys, which constitutes the predominant pattern of the roughness.
- **Roughness width cut-off**: is the maximum width of the surface that is included in the calculation of the roughness height.
- **Secondary Texture (Waviness):** refers to those surface irregularities that have a greater spacing than that of roughness width.
- Determined by the height of the waviness and its width.
- The greater the width, the smoother is the surface and thus is more desirable.
- **Methods of measuring surface Texture :** There are a number of useful techniques for measuring surface texture (roughness).
- Observation and touch
- Stylus based equipment
- Interferometry

(ii)

Minimum of machine, $M_1 = 6$

Maximum of machine, $M_2 = 6$

Minimum of machine, $M_3 = 4$

For converting n jobs on 3 machines to n jobs on 2 machines one of the following condition must be satisfied.

(I) (Minimum of M_1) $\geq$ (Maximum of M_2)

(II) (Minimum of M_3) $\geq$ (Maximum of M_2)

Here Ist condition is satisfied. Hence it can be converted into n jobs on 2 machines.

Machine	Jobs				
	J_1	J_2	J_3	J_4	J_5
$X = M_1 + M_2$	13	16	8	10	15
$Y = M_2 + M_3$	9	15	10	9	9

Hence optimum sequence of job as per Johnson's algorithm:

$$J_3 - J_2 - J_5 - J_1 - J_4$$

Jobs	Machine M_1		Machine M_2		Machine M_3	
	In	Out	In	Out	In	Out
J_3	0	6	6	8	8	16
J_2	6	16	16	22	22	31
J_5	16	27	27	31	31	36
J_1	27	35	35	40	40	44
J_4	35	42	42	45	45	51

Hence make span time = 51 hours

Idle time on machine M_1 = 51 – (8 + 10 + 6 + 7 + 11)

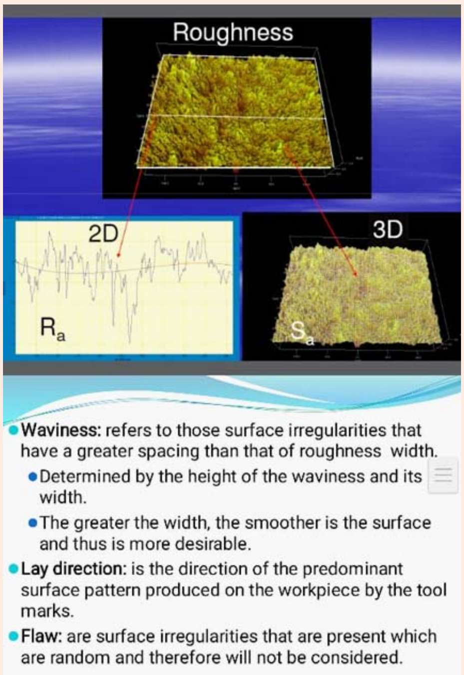
= 51 – 42 = 9 hours

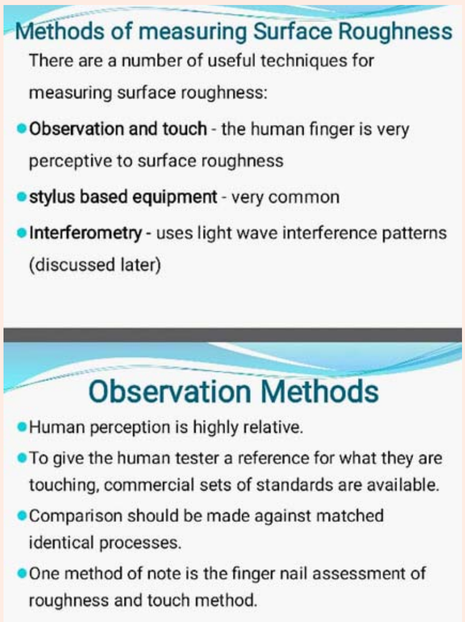
Idle time on machine M_2 = 51 – (5 + 6 + 2 + 3 + 4) = 51 – 20 = 31 hours

Idle time on machine M_3 = 51 – (4 + 9 + 8 + 6 + 5) = 51 – 32 = 19 hours

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.1 (c) of Test-5(Source for Q.7 (b) (i))
- **ESE 2019 Mains Test Series:** Q.6 (c) of Test-7 (Source for Q.7 (b) (ii))
- **MADE EASY Classnotes**





IES-2015 Conventional

Determine the fundamental deviation and tolerances and the limits of size for hole and shaft pair in the fit: 25 mm H8d9. The diameter steps are 18 mm and 30 mm. The fundamental deviation for d shaft is given as $-16D^{0.44}$. The tolerance unit is,

$$i = 0.45\sqrt[3]{D} + 0.001D$$

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The tolerance grade for number 8 quality is 25i and for 9 quality is 40i.

[10 Marks]

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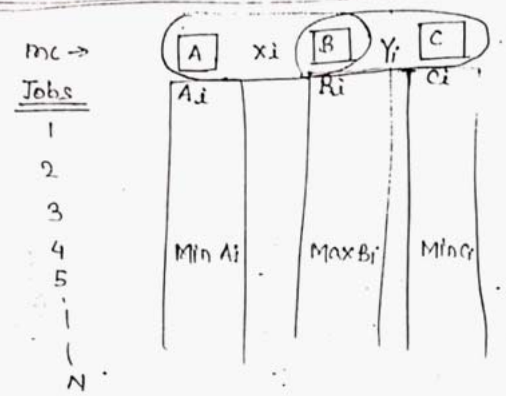
N - no of Jobs on three machines



By combining A & B ; xi machine is obtained
" " B & C ; Yi " " "

Condition to be satisfied :-

(1) Min. $A_i \geq$ Max. B_i.

(2) Min. $C_i \geq$ Max. B_i

$$X_i = A_i + B_i$$

$$Y_i = B_i + C_i$$

(Q) obtain optimum sequence for the following set of jobs to be processed on three machines. Also find make span time and idle time for each machine

Jobs	mc-A	mc-B	mc-C
1	3	4	6
2	8	3	7
3	7	2	5
4	4	5	11

End of Solution

7. (c) (i) What are natural and forced responses of a dynamic system? Derive the expression for dynamic natural response of a spring-mass system.

[10 Marks]

(ii) A vector $25i + 10j + 20k$ is translated by 8 units in X and 5 units in Y directions. Subsequent to this the vector is rotated by 60° about Z-axis and 30° about X-axis. Determine the final form of the vector.

[10 Marks]

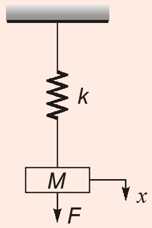
Solution:

(i)

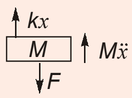
Natural responses: The natural response is the response or output of the system when the input are zero it is due to initial conditions only.

Forced responses: The forced response is the response or output of the system when the input are present.

Consider a spring mass system on which the force F acts. Displacement x is positive in the direction shown and zero position is taken to be at the point where the spring and mass are in static equilibrium.



Free body diagram,



From equation of motion

$$F = M\ddot{x} + kx$$

$$F = M\frac{d^2x}{dt^2} + kx \qquad \text{...(1)}$$

Applying Laplace transformation both sides in equation (1)

$$F(s) = Ms^2\, X(s) + kX(s)$$

$$H(s) = \frac{X(s)}{F(s)} = \frac{1/M}{s^2 + \frac{k}{M}} = \frac{1}{M\sqrt{\frac{k}{M}}}\frac{\sqrt{\frac{k}{M}}}{\left(s^2 + \left(\frac{k}{M}\right)\right)}$$

By taking inverse Laplace transformation

we get natural response of the system as

$$h(t) = \frac{1}{\sqrt{kM}} \sin\left(\sqrt{\frac{k}{M}} t\right)$$

$$h(t) = \frac{1}{\sqrt{kM}} \sin(\omega_n t)$$

(ii)

Given position vector, $P = 25\hat{i} + 10\hat{j} + 20\hat{k}$.

As mention in question there are three transformation. So, we need to find three transformation matrix to get final form of the position vector.

For 8 units translation in x and 5 units translation in y, we will get

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For rotation by 60° about z-axis

$$T_2 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For rotation by 30° about x-axis

$$T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ & 0 \\ 0 & \sin 30^\circ & \cos 30^\circ & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If P_F is final form of vector then we can write,

$$P_F = T_3 \times T_2 \times T_1 \times P$$

$$P_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 10 \\ 20 \\ 1 \end{bmatrix}$$

Multiplying T_1 and P first we get

$$\Rightarrow \qquad P_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 33 \\ 15 \\ 20 \\ 1 \end{bmatrix}$$

Now multiplying last two matrices

$$P_F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3.5096 \\ 36.0788 \\ 20 \\ 1 \end{bmatrix}$$

$$\Rightarrow \qquad P_F = \begin{bmatrix} 3.5096 \\ 21.2451 \\ 35.3599 \\ 1 \end{bmatrix}$$

Final form of vector = $3.5096\hat{i} + 21.2451\hat{j} + 35.3599\hat{k}$

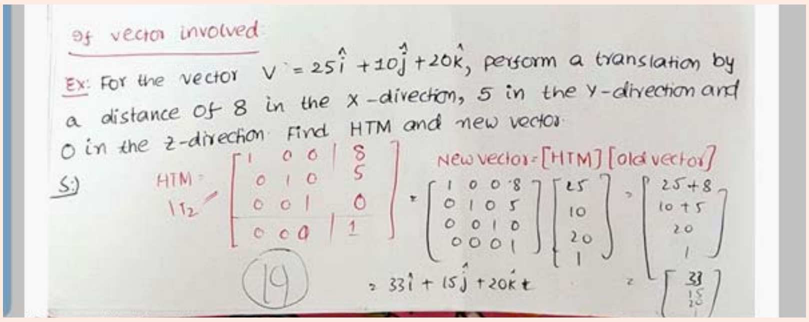
MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.1(e) of Test-8 and Q.2 (a) of Test-9
- **MADE EASY Classnotes**

Example-5

In this case, assume the same point $p(7, 3, 1)^T$, attached to F_{noa} is subjected to the same transformations, but the transformations are performed in a different order. Find the coordinates of the point relative to the reference frame at the conclusion of transformations.

1. A rotation of 90° about the z-axis,
2. Followed by a translation of [4, -3, 7],
3. Followed by a rotation of 90° about the y-axis.


of vector involved
Ex: For the vector $V = 25\hat{i} + 10\hat{j} + 20\hat{k}$, perform a translation by a distance of 8 in the x-direction, 5 in the y-direction and 0 in the z-direction. Find HTM and new vector.

S:) HTM = $\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

New vector = [HTM] [old vector]

$= \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 25 \\ 10 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 25+8 \\ 10+5 \\ 20 \\ 1 \end{bmatrix} = \begin{bmatrix} 33 \\ 15 \\ 20 \\ 1 \end{bmatrix}$

$= 33\hat{i} + 15\hat{j} + 20\hat{k}$

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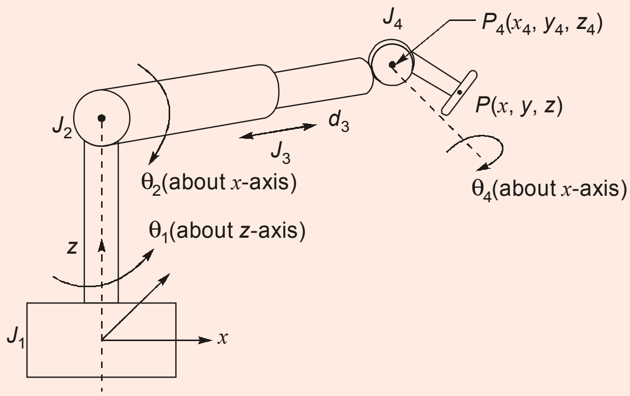
End of Solution

8\. (a) (i) Explain briefly the following:

(1) Four configurations of Robot
(2) Work volume
(3) Spatial resolution
(4) Accuracy
(5) Repeatability

[10 Marks]

(ii) A 4 d-o-f manipulator of Maker Robot type is shown in the figure given below. Prepare a D-H parameter table for this configuration. Define the position of end wrist P in terms of joint lengths and angles:



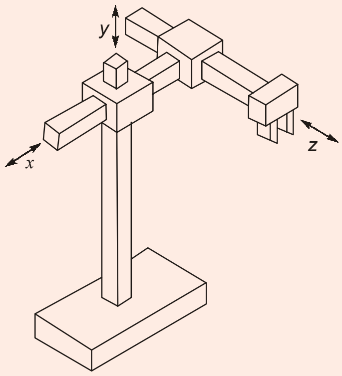
[10 Marks]

Solution:

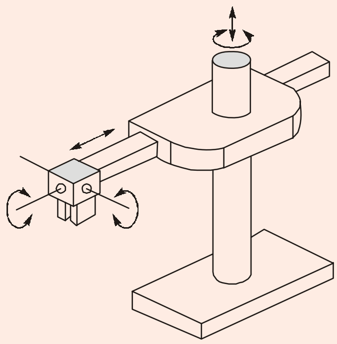
(i)

1. Four configurations of Robot

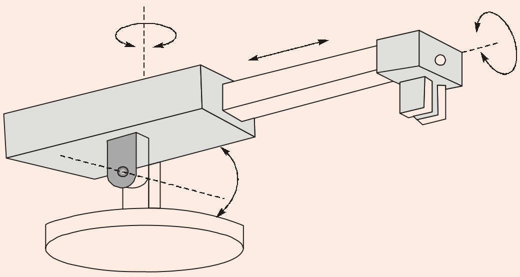
- **Cartesian co-ordinates Robots :** It's a robot whose arm has three prismatic joints, whose axes are coincident with a Cartesian coordinator. It is used for pick and place work, assembly operations, handling machine tools and arc welding.



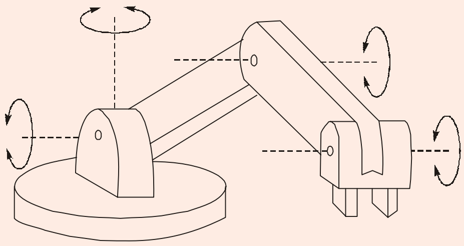
- **Cylindrical Coordinate Robots:** It's a robot whose axes form a cylindrical coordinate system. It is used for assembly operations, handling at machine tools, spot welding, and handling at die casting machines. It has two prismatic and one revolute joint.



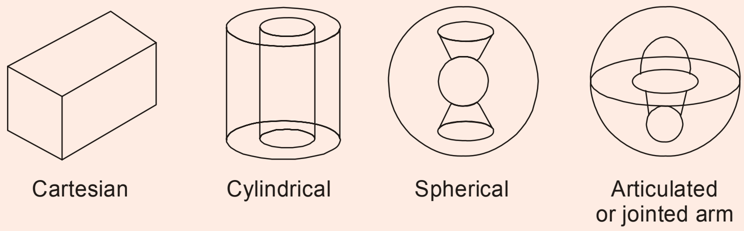
- **Spherical or Polar Coordinate Robots:** It's a robot whose axes form a polar coordinate system. Used for machining, welding, spray painting etc. It has two revolute and one prismatic joint.



- **Jointed Arm Robot:** It's a robot whose arm, has at least three, rotary joints. It is used for assembly operations, die casting, fettling machines, gas welding, arc welding and spray painting. It needs smaller workspace.



2. **Work volume**: Depending on the configuration and the size of the links and wrist joints, robots can reach a collection of points that constitute a work volume. The shape of the work volume for each robot is uniquely related to its design.



work volume for common robot configurations

3. **Spatial resolution:** Spatial resolution is defined as the smallest increment of motion achieved by robotics manipulator at its tool or end effector. The spatial resolution of a robot is the smallest increment of movement into which the robot can divide its work volume.

   Spatial resolution depends on two factors: The system's control resolution and the robot's mechanical inaccuracies.

4. **Accuracy:**

   Robot Accuracy : A measure of difference between the actually attained position/ path of a robot end effector the input position/path commanded by the robot controller.

   Absolute Accuracy: A measure of difference between the actually attained position/ path of a robot end effector and the input position/path (in absolute world coordinate frame) commanded by the robot controller.

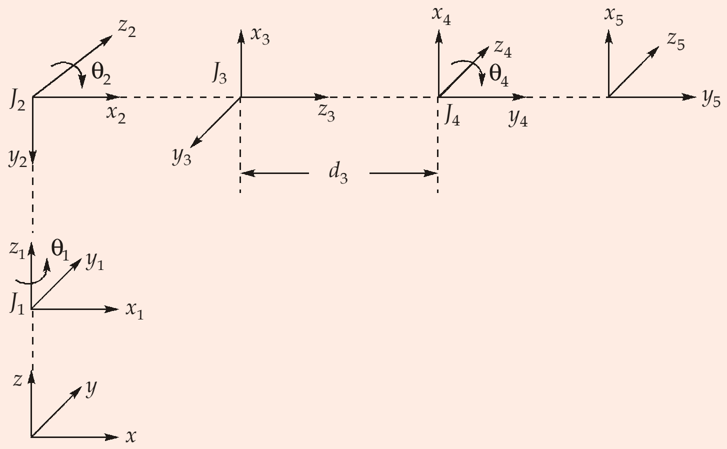
   Relative Accuracy : A measure of difference between the actually attained position/ path of a robot end effector and the input position/path (relative to an intermediate reference frame) commanded by the robot controller.

5. **Repeatability**: Repeatability is the ability of the robot to reposition itself to a position to which it was previously commanded or trained. Repeatability describes the positional error of the tool tip when it is automatically returned to a position previously taught. Repeatability and accuracy are similar. But repeatability and accuracy refer to two different aspects of the robot's precision. Accuracy relates to the robot's capacity to be programmed to achieve a given target point. The actual programmed point will probably be different from the target point due to limitations of control resolution. Repeatability refers to the robot's ability to return to programmed point

when commanded to do so. Repeatability will generally always be better than accuracy exclusive of drift.

(ii)

We need to assign frames to each joint first to prepare the D-H parameter table:



D-H parameter table

	θ	d	α	a
J_1	θ_1	0	$-90°$	0
J_2	$-90° + \theta_2$	0	$-90°$	0
J_3	0	d_3	$+90°$	0
J_4	θ_4	0	0	0

Please note that the D-H parameters may vary according to the frame you decide.

$$\text{Transformation matrix} = \begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & ac\theta \\ s\theta & c\theta c\alpha & -c\theta s\theta & as\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1(\theta_1) = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2(\theta_2) = \begin{bmatrix} c(\theta_2 - 90) & 0 & -s(\theta_2 - 90) & 0 \\ s(\theta_2 - 90) & 0 & c(\theta_2 - 90) & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_3(d_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_4(\theta_4) = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall transformation matrix $^{0}T_4 = {}^{0}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_4$

Position of end wrist P can be found using $^{0}T_4$.

$$^{0}T_1 \times {}^{1}T_2 = {}^{0}T_2 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} s_2 & 0 & c_2 & 0 \\ -c_2 & 0 & s_2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1s_2 & s_1 & c_1c_2 & 0 \\ s_1s_2 & -c_1 & s_1c_2 & 0 \\ c_2 & 0 & -s_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_2 \times {}^{2}T_3 = {}^{0}T_3 = \begin{bmatrix} c_1s_2 & s_1 & c_1c_2 & 0 \\ s_1s_2 & -c_1 & s_1c_2 & 0 \\ c_2 & 0 & -s_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

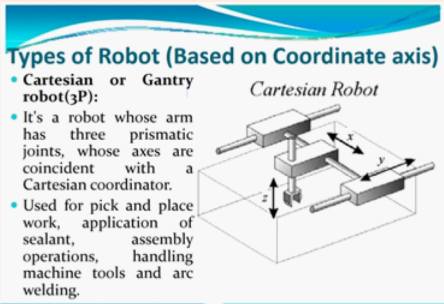
$$^{0}T_3 = \begin{bmatrix} c_1s_2 & c_1c_2 & -s_1 & d_3c_1c_2 \\ s_1s_2 & s_1c_2 & c_1 & d_3s_1c_2 \\ c_2 & -s_2 & 0 & -s_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

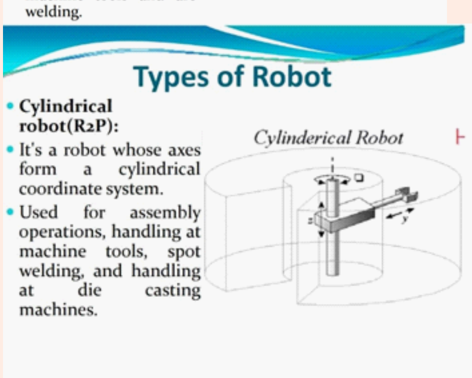
$$^{0}T_3 \times {}^{3}T_4 = {}^{0}T_4 = \begin{bmatrix} c_1s_2 & c_1s_2 & -s_1 & d_3c_1c_2 \\ s_1s_2 & -s_1c_1 & c_1 & d_3c_1c_2 \\ c_2 & -s_2 & 0 & -s_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

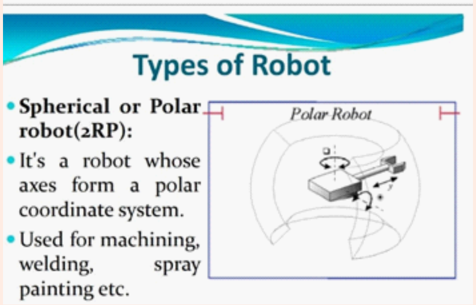
$$^{0}T_4 = \begin{bmatrix} c_1s_2c_4 + c_1c_2s_4 & c_1s_2(-s_4) + c_1c_2s_4 & -s_1 & d_3c_1c_2 \\ s_1s_2c_4 + s_1c_2s_4 & s_1s_2(-s_4) + s_1c_2s_4 & c_1 & d_3s_1c_2 \\ c_1c_4 - s_2s_4 & -s_4c_2 - s_2c_4 & 0 & -s_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

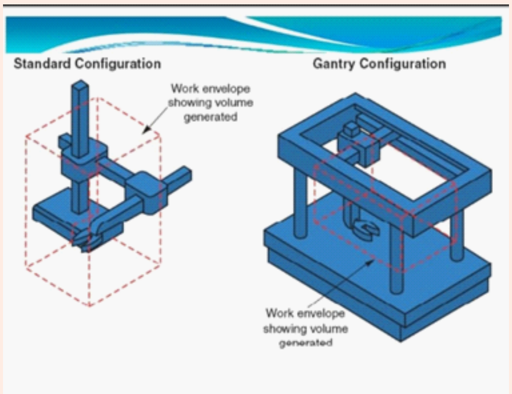
MADE EASY Source

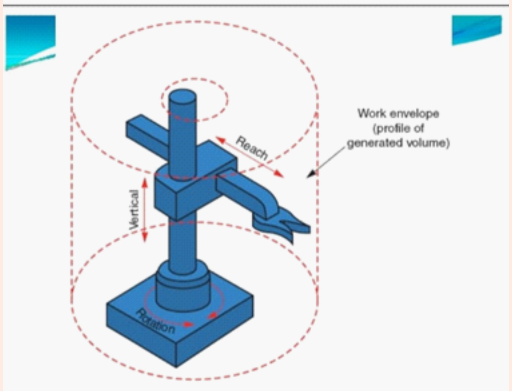
- **ESE 2019 Mains Test Series:** Q.3 (c) of Test-8 (Source for Q.8 (a) (ii))
- **MADE EASY Classnotes**

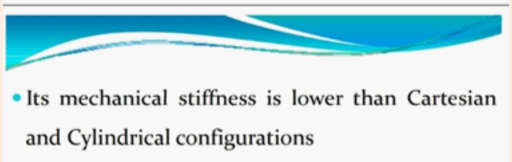


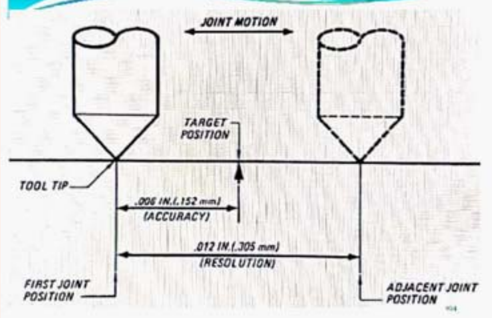












Robot Repeatability

- Repeatability is the ability of the robot to reposition itself to a position to which it was previously commanded or trained. Repeatability describes the positional error of the tool tip when it is automatically returned to a position previously taught.
- Repeatability and accuracy are similar. But repeatability and accuracy refer to two different aspects of the robot's precision. Accuracy relates to the robot's capacity to be programmed to achieve a given target point. The actual programmed point will probably be different from the target point due to limitations of control resolution. Repeatability refers to the robot's ability to return to

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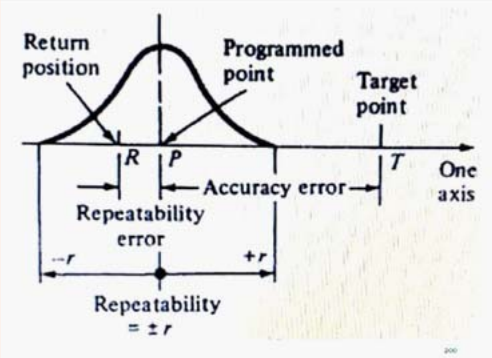


Resolution

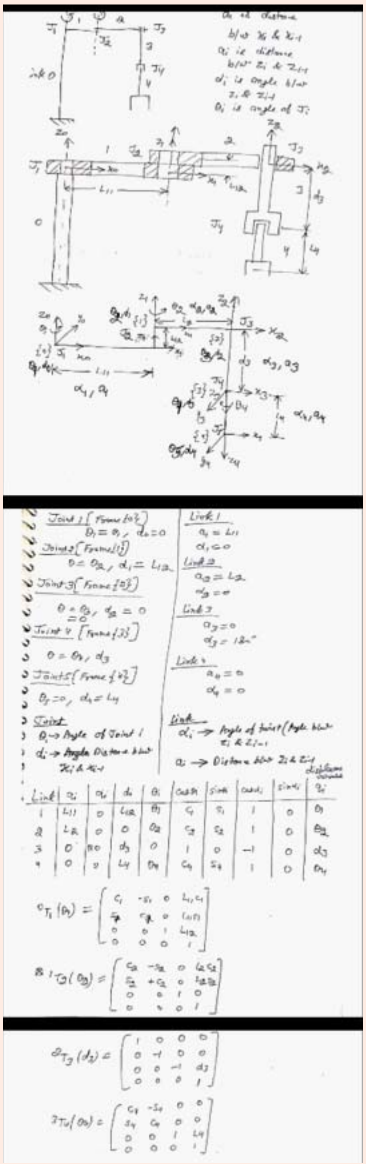
- Control resolution is the smallest increment that the robot arm can move by means of digital to analog converter commands for position feedback control.
- If a linear robot arm is to move a distance 'd' by 'n' bit digital to analog converter through the controller then the linear resolution will be $d/2^n$.
- For the rotary motion one has to divide the angular range by 2^n to obtain the angular resolution. Angular resolution multiplied by the arm length gives the control resolution.

Accuracy & Precision

- **Accuracy** - The ability of a measurement to match the actual (true) value of the quantity being measured. The expected ability for a system to discriminate between two settings. Smaller the bias more accurate the data.
- **Precision** - The precision of an instrument indicates its ability to reproduce a certain reading with a given accuracy 'OR' it is the degree of agreement between repeated results.
- Precision data have small dispersion (spread or scatter) but may be far from the true value.
- A measurement can be accurate but not precise, precise but not accurate, neither, or both.
- A measurement system is called valid if it is both accurate and precise.

Repeatability

- It is the ability of a measuring system to reproduce output readings when the same input is applied to it consecutively, under the same conditions, and in the same direction.
- Imperfections in mechanical systems can mean that during a Mechanical cycle, a process does not stop at the same location, or move through the same spot each time. The variation range is referred to as repeatability.



End of Solution

8. (b) (i) Draw the "bathtub curve" and indicate various failure regions. List the major causes of failure in mechanical components/system. Draw the flowchart for failure modes and effects analysis (FMEA).

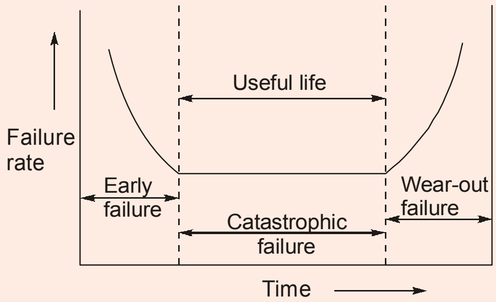
[10 Marks]

(ii) Explain the mechanism of metal removal in die-sinking EDM. State the three main advantages of electron-beam machining (EBM).

[10 Marks]

Solution:

(i) Phase of failure/Bathtub curve:



Early failure: These failure occur at the beginning due to deflective design, manufacturing, or assembly work warranty is based on the concept of early failure.

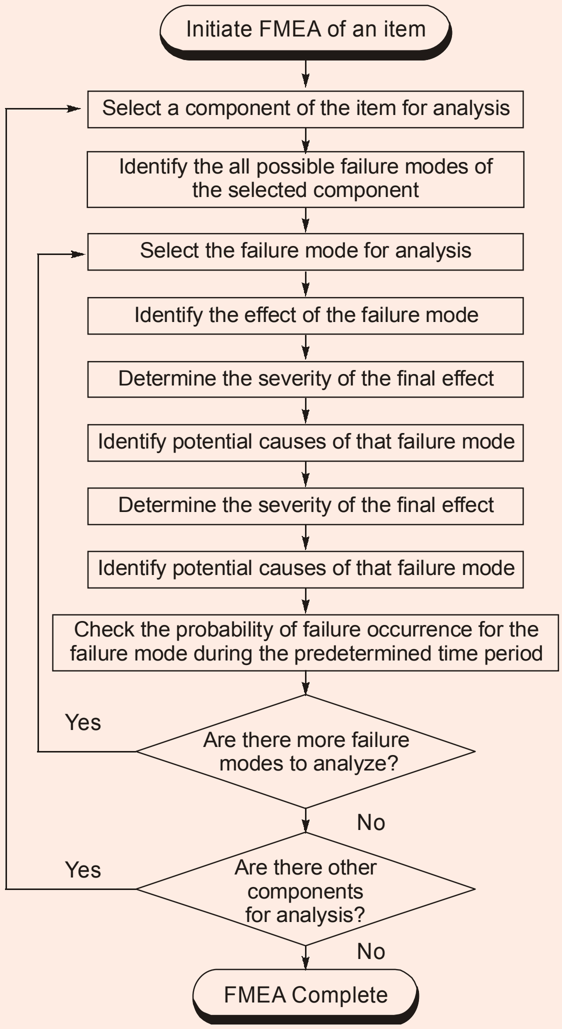
Catastrophic failure: These failure occur during actual working of product, they occur randomly and suddenly. These are caused due to sudden stress accumulation beyond the designed limit.

Wearout failure: It is a typical aging problem and product is more likely to fail due to wear and tear. Proper care and maintenance can delay the failure.

Major causes of failure in mechanical systems component:

1. Excessive. deflection (e.g. in beam), angular misalignment (e.g. in shaft), excessive elongation (e.g. belt)
2. Creep failure.
3. Corrosion or deterioration of material
4. Fatigue failure
5. Wear
6. Thermal shock
7. Brittle and ductile failure
8. Buckling of component
9. Impact load
10. Material bond failure.

Flow chart for failure modes and effects analysis (FMEA):



(ii) Mechanism of metal removal in die-sinking EDM:

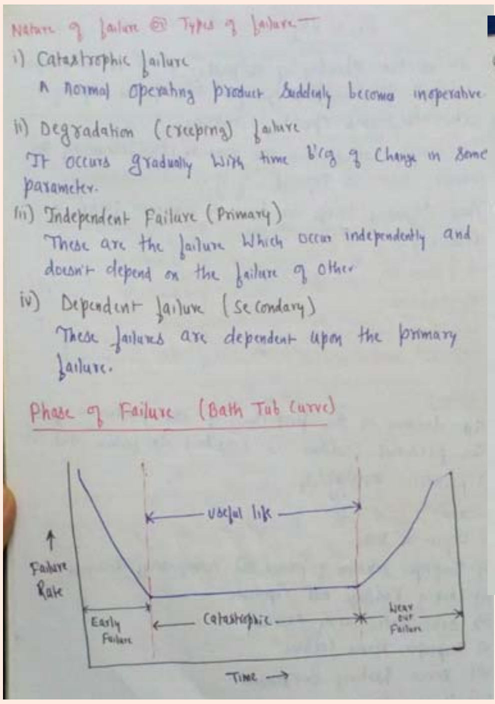
- Mechanics of material removal - melting and evaporation aided by cavitation.
- The process is based on melting temperature, not hardness, so some very hard materials can be machined this way.
- The arc that jumps heats the metal, and about 1 to 10% of the molten metal goes into the fluid. The melted metal then recast layer is about 1 to 30 μm thick, and is generally hard and rough.
- The electrode workpiece gap is in the range of 10 μm to 100 μm.
- Uses Voltage of 60 to 300 V to give a transient arc lasting from 0.1 μs to 8 ms.
- Typical cycle time is 20 ms or less, up to millions of cycles may be required for completion of the part.
- Rotating the wire in an orbital direction will,
- Increase accuracy in form and surface finish
- Decrease electrode wear
- Surface finish obtained 0.25 μm

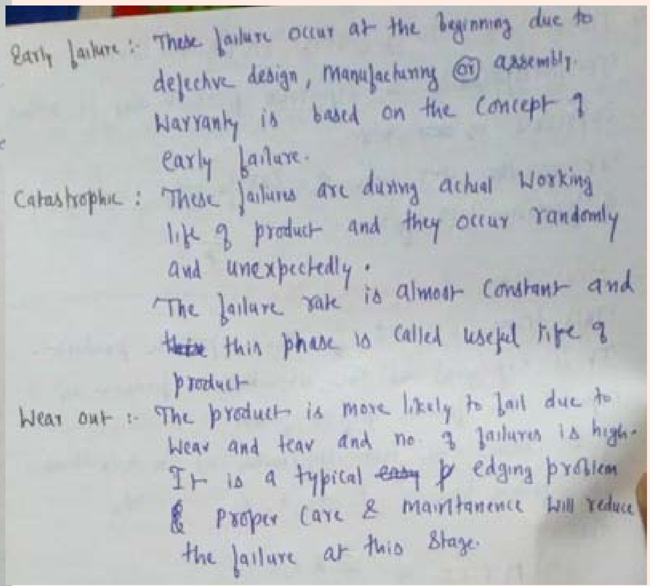
Advantages of electron-beam machining:

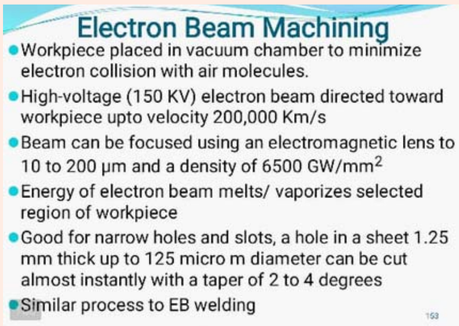
1. EBM can be used for very accurate cutting of a wide variety of metals. It can also be used for producing very small hole in any shape.
2. Surface finish is better and kerf width is narrower than that for other thermal processes.
3. Highly reactive material can be machined easily because machining is done under vacuum.
4. It can machine any material irrespective of its hardness and other mechanical properties.

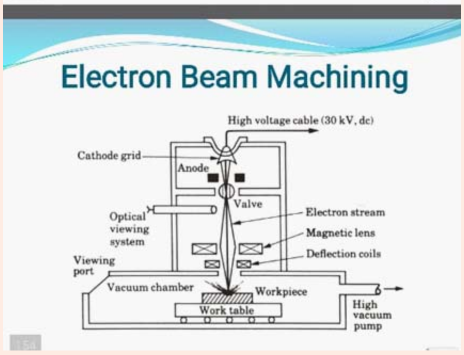
MADE EASY Source

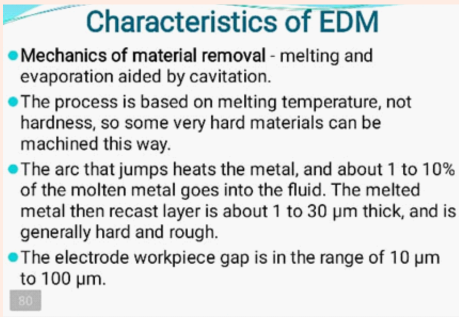
- **Theory Book (2019 Edition):** Production Engg. (Page No. 273) (Source for Q.8 (b) (ii))
- **MADE EASY Classnotes**

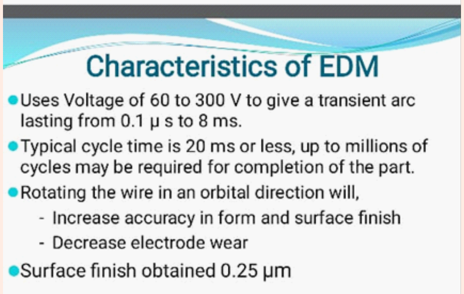

Nature of failure (or) Types of failure
i) Catastrophic failure
A normal operating product suddenly become inoperative
ii) Degradation (creeping) failure
It occurs gradually with time b'cz of change in some parameter.
iii) Independent Failure (Primary)
These are the failure which occur independently and doesn't depend on the failure of other
iv) Dependent failure (Secondary)
These failures are dependent upon the primary failure.
Phase of Failure (Bath Tub Curve)


Early failure :- These failure occur at the beginning due to defective design, manufacturing (or) assembly. Warranty is based on the concept of early failure.
Catastrophic : These failures are during actual working life of product and they occur randomly and unexpectedly. The failure rate is almost constant and this phase is called useful life of product
Wear out :- The product is more likely to fail due to wear and tear and no. of failures is high. It is a typical ~~easy~~ p edging problem & proper care & maintenance will reduce the failure at this stage.









End of Solution

8. (c) A manufacturer of patient medicines is proposed to prepare a production plan for medicines A and B. There are sufficient ingredients available to make 20000 bottles of medicine A and 40000 bottles of medicine B. However, there are only 45000 bottles into which either of the medicines can be filled. Further, it takes three hours to prepare enough material to fill 1000 bottles of medicine A and one hour to prepare enough material to fill 1000 bottles of medicine B, and there are 66 hours available for this operation. The profit is Rs. 8 per bottle for medicine A and Rs. 7 per bottle for medicine B.

(i) Formulate this problem as linear programming problem.

(ii) How does the manufacturer schedule his production in order to maximize profit? Use graphical method.

[20 Marks]

Solution:

Let x_1 bottle of medicine A and x_2 bottles of medicine *B* are produced.

Given constraints: $x_1 \leq 20000$

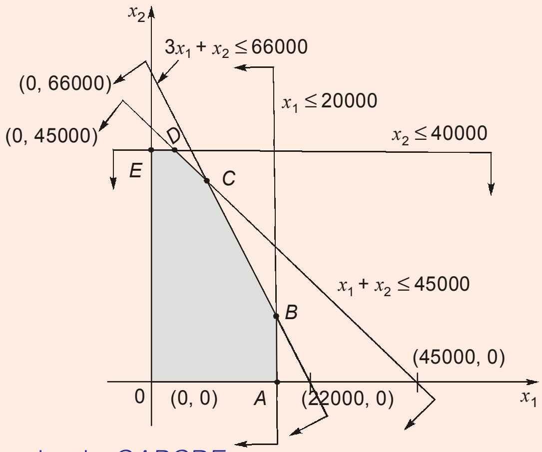
$x_2 \leq 40000$

Number of bottle constraint, $x_1 + x_2 \leq 45000$

Time constraint, $\frac{3x_1}{1000} + \frac{x_2}{1000} \leq 66$

$3x_1 + x_2 \leq 66000$

Objective function, Maximize, $Z = 8x_1 + 7x_2$



Hence feasible region is *OABCDE*.

Feasible region corner points are O(0, 0), A(20000, 0), E(0, 40000)

For point *D*,

$$x_2 = 40000$$
$$x_1 + x_2 = 45000$$
$$x_1 = 5000$$

D(5000, 40000)

For point *C*,

$$x_1 + x_2 = 45000 \quad ...(1)$$
$$3x_1 + x_2 = 66000 \quad ...(2)$$

Solving eq. (1) and (2)

$$x_1 = 10500$$
$$x_2 = 34500$$

C(10500, 34500)

For point *B*,

$$x_1 = 20000$$
$$3x_1 + x_2 = 66000$$
$$x_2 = 6000$$

B(20000, 6000)

Feasible region co-ordinates (x_1, x_2)	$Z_{max} = 8x_1 + 7x_2$ (in Rs.)
$O(0,0)$	$Z_{max} = 0$
$A(20000, 0)$	$Z_{max} = 160000$
$B(20000, 6000)$	$Z_{max} = 8 \times 20000 + 7 \times 6000 = 202000$
$C(10500, 34500)$	$Z_{max} = 10500 \times 8 + 7 \times 34500 = 325500$
$D(5000, 40000)$	$Z_{max} = 5000 \times 8 + 7 \times 40000 = 320000$
$E(0, 40000)$	$Z_{max} = 0 \times 8 + 40000 \times 7 = 280000$

Hence maximum profit is Rs. 325500.

Optimum quantity of medicine A = 10500 bottles

Optimum quantity of medicine B = 34500 bottles

MADE EASY Source

- **ESE 2019 Mains Test Series:** Q.5(e) of Test-7

End of Solution