



MADE EASY

India's Best Institute for IES, GATE & PSUs

ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-5: Analog Circuits + Materials Science

Electronic Devices & Circuits-1 + Advanced Electronics Topics-1

Analog and Digital Communication Systems-2

Name : SHUBHAM YADAV

Roll No :

E	C	1	9	M	B	D	L	A	3	3	4
---	---	---	---	---	---	---	---	---	---	---	---

Test Centres

Delhi Bhopal Noida Jaipur Indore
Lucknow Pune Kolkata Bhubaneswar Patna
Hyderabad

Student's Signature

Shubham

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	48
Q.2	44
Q.3	-
Q.4	-
Section-B	
Q.5	37
Q.6	-
Q.7	44
Q.8	14
Total Marks Obtained	187

Signature of Evaluator

Shubham

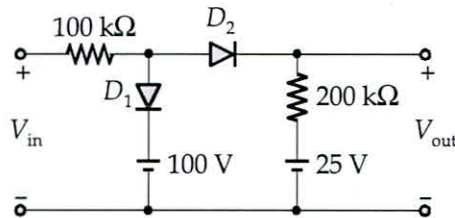
Cross Checked by

Shubham

$$\frac{200}{3} + \frac{25}{3} = \frac{225}{3}$$

Section A : Analog Circuits + Materials Science

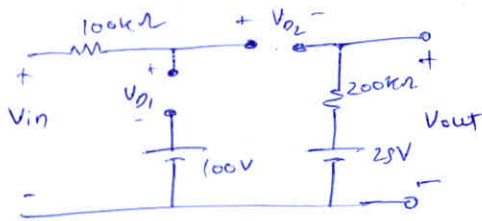
(a) Consider the circuit shown in the figure below:



By assuming that the diodes are ideal, develop the transfer characteristic curve of the above circuit.

Assume both the diodes are off initially

[12 marks]



From above circuit we obtain

$$V_{D1} = V_{in} - 100 \quad \text{--- (1)}$$

$$V_{D2} = V_{in} - 25 \quad \text{--- (2)}$$

$$V_{out} = 25 \text{ volts}$$

For both the diodes to remain off, voltage across them must be negative

$$V_{in} - 100 < 0 \quad \& \quad V_{in} - 25 < 0$$

$$\boxed{V_{in} < 100 \text{ volts}} \quad \& \quad \boxed{V_{in} < 25 \text{ volts}}$$

Hence for both diodes to remain off $V_{in} < 25 \text{ volts}$ &

$$\boxed{V_{out} = 25 \text{ volts}} \quad \text{--- (3)}$$

For $25 < V_{in} < 100 \text{ volts}$

\Rightarrow $D1$ will remain off & $D2$ will become on.

Hence ~~from~~ $I = \frac{V_{in} - 25}{300} \text{ mA}$

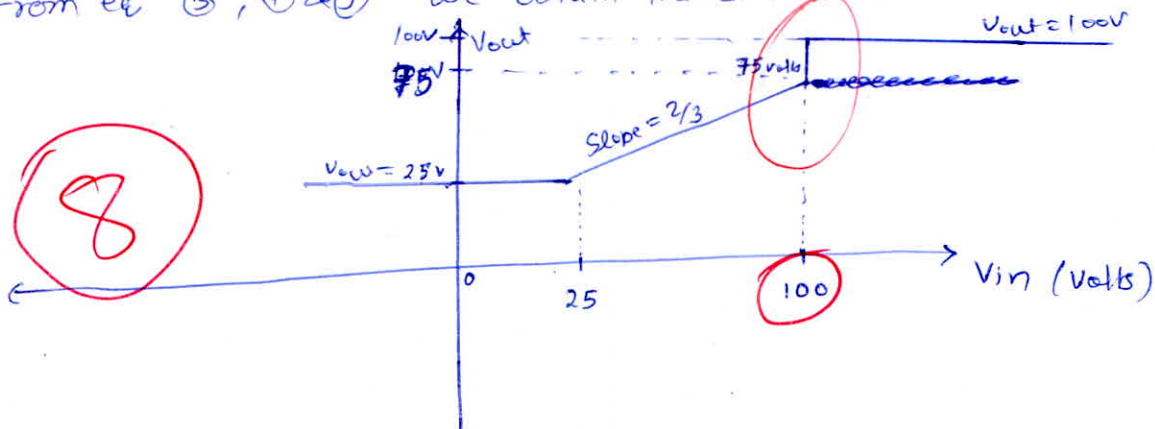
$$V_{out} = I \cdot 200 + 25$$

$$= \frac{(V_{in} - 25) \cdot 2}{3} + 25 = \frac{2V_{in}}{3} + \frac{25}{3} \text{ volts}$$

$$\boxed{V_{out} = \frac{2}{3} \cdot V_{in} + \frac{25}{3} \text{ volts}} \quad \text{--- (4)}$$

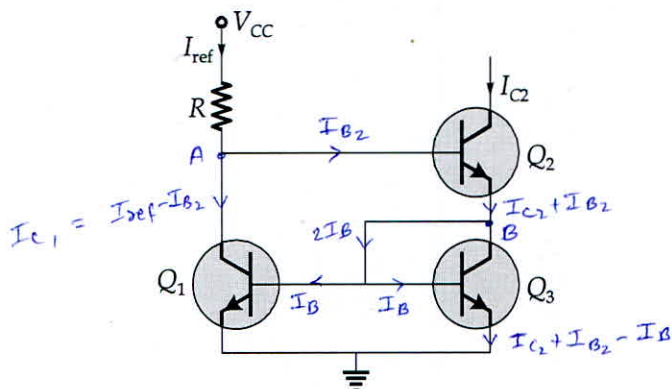
For $V_{in} = 100 \text{ Volts} \Rightarrow V_{out} = 100 \text{ Volts}$ - (5)

From eq (3), (4) & (5) we obtain the sketch as



Transfer characteristics.

Q.1 (b) Consider the Wilson current mirror circuit as shown in the figure below:



Assume that the three transistors to be matched with $V_{BE1} = V_{BE3}$ and $\beta_1 = \beta_2 = \beta_3 = \beta$. Derive an expression for I_{C2} in terms of I_{ref} .

[12 marks]

Given that 3 Transistors are matched with $V_{BE1} = V_{BE2}$

For Transistor Q_2

$$I_{C2} = \beta I_{B2} \quad \text{--- (1)}$$

$$I_{E2} = I_{C2} + I_{B2} = (\beta + 1) I_{B2}$$

Using KCL at Node A

$$I_{C1} = I_{ref} - I_{B2} \quad \text{--- (2)}$$

For Transistor Q_1

$$I_{C1} = \beta (I_B)$$

$$I_{ref} - I_{B2} = \beta (I_B) \quad \text{--- (3)}$$

From Tx Q₃

$$(I_{C2} + I_{B2} - 2I_B) = \beta I_B$$

$$I_{C2} + I_{B2} = (\beta + 2) I_B$$

$$I_B = \frac{I_{C2} + I_{B2}}{\beta + 2} \quad - (4)$$

From eq (3) & (4)

$$I_{ref} - I_{B2} = \beta \left[\frac{I_{C2} + I_{B2}}{\beta + 2} \right]$$

$$I_{ref} = \frac{\beta I_{C2}}{\beta + 2} + I_{B2} \left[1 + \frac{\beta}{\beta + 2} \right] \quad - (5)$$

Using eq (1) & (5) we obtain

$$I_{ref} = \frac{\beta I_{C2}}{\beta + 2} + \frac{I_{C2}}{\beta} \left[\frac{2(\beta + 1)}{\beta + 2} \right]$$

$$I_{ref} = I_{C2} \left[\frac{\beta}{\beta + 2} + \frac{2(\beta + 1)}{\beta(\beta + 2)} \right]$$

$$I_{ref} = I_{C2} \left[\frac{\beta^2 + 2\beta + 2}{\beta(\beta + 2)} \right]$$

$$\Rightarrow I_{C2} = I_{ref} \left[\frac{\beta(\beta + 2)}{1 + (\beta + 1)^2} \right]$$

$$I_{C2} = I_{ref} \left[\frac{1}{\frac{(\beta + 1)^2}{\beta(\beta + 1)} + \frac{1}{\beta(\beta + 2)}} \right]$$

Assuming β to be very large

$$I_{C2} = I_{ref} \left[\frac{1}{1 + \frac{1}{\beta(\beta + 2)}} \right]$$

- Q.1 (c) A long narrow rod (having cubic structure) has an atomic density of 5×10^{28} atoms/m³. Each atom has a polarizability of 10^{-40} F-m². Calculate the internal electric field in the rod when an external axial field of 1 V/m is applied.

[12 marks]

$$N = 5 \times 10^{28} \text{ atoms/m}^3$$

$$\alpha = 10^{-40} \text{ F-m}^2$$

$$E = 1 \text{ V/m}$$

For cubic structure $\gamma = 1/3$

Internal Electric field is given by

$$\vec{E}_i = \vec{E} + \gamma \frac{\vec{P}}{\epsilon_0}$$

$$\vec{E}_i = \vec{E} + \frac{\vec{P}}{3\epsilon_0}$$

where polarization is given as

$$\vec{P} = N \alpha \vec{E}_i$$

$$\vec{E}_i = \vec{E} + \frac{N \alpha \vec{E}_i}{3\epsilon_0}$$

$$\vec{E}_i \left[1 - \frac{N \alpha}{3\epsilon_0} \right] = \vec{E}$$

$$\vec{E}_i = \frac{\vec{E}}{1 - \frac{N \alpha}{3\epsilon_0}}$$

$$\vec{E}_i = \frac{1}{1 - \frac{5 \times 10^{28} \times 10^{-40}}{3 \times 8.854 \times 10^{-12}}}$$

$$\vec{E}_i = \frac{1}{1 - \frac{5}{3 \times 8.854}}$$

$$\vec{E}_i = 1.231 \text{ V/m}$$

11

(d) Explain Silsbee's rule for superconductors. Also give some applications of superconductors.

[12 marks]

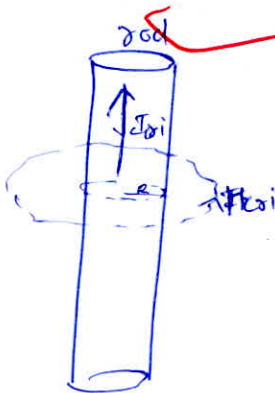
Silsbee's Rule

Silsbee's Rule states that the superconductivity of a super conductor can be destroyed its own Magnetic field when the current flowing through the super conductor reaches a critical value.

$$H_{c1} \cdot 2\pi R = I_{c1}$$

where H_{c1} = Critical value of magnetic field intensity above which the superconductivity of material gets destroyed.

I_{c1} = Critical value of current flowing through superconductor beyond which its super conductivity gets destroyed.



$$I_{c1} = \int H_{c1} \cdot dl$$

$$I_{c1} = H_{c1} \cdot 2\pi R$$

where R is the radius of superconductor.

Super conductors: Superconductors are materials which shows zero resistance to the flow of current. Practically most of the super conductors exist at very low temperatures below the room temperatures which results in the

limitations to the use of superconductors. So most of the applications of superconductor are at low temp. which are as follows:

- i) Superconductors are used to obtain very strong magnetic fields which is used in various processes like lifting & transporting heavy weights, reducing friction for very high speed trains etc.
- ii) Superconductors are used to obtain very high transmission speeds for faster communication.
- iii) Superconductors are used in supercomputers for faster processing & calculations.

10

(e) Write short notes on the following nanomaterials:

- (i) Quantum dots
- (ii) Carbon nanotubes

[6 + 6 marks]

Quantum dots

⇒ Its size range from 1 nm to 60 nm.

⇒ It is a 1-dimensional structure.

⇒ It has a wide range of applications ranging

formation of 1-dimensional colour display to

changing the characteristics of materials at quantum level.

3

Carbon Nano tubes

⇒ Carbon Nanotubes is single molecule having very large length.

⇒ It is a 2-dimensional structure

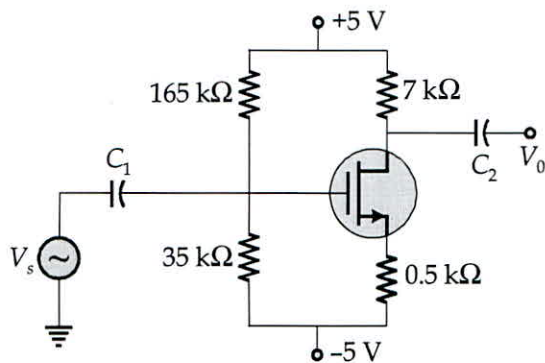
⇒ Its strength to weight ratio is very high.

⇒ It has a wide range of application ranging from forming very high strength fibres, making new semiconductors etc.

- ⇒ Carbon Nanotubes change their properties ~~which~~ with the different levels of ~~carbon~~ doping conc. of impurities, ranging from becoming very good conductor to an insulator.
- ⇒ It ~~can~~ only consist of Carbon-Carbon bonds.

5

(i) Consider the common source transistor circuit shown in the figure below:



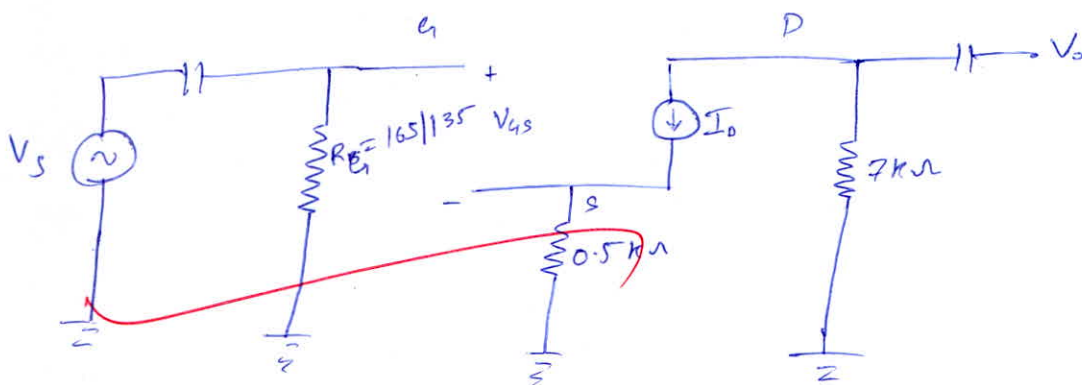
The transistor parameters are $V_{TN} = 0.8 \text{ V}$, $K_n = \frac{\mu_n C_{ox} W}{2L} = 1 \text{ mA/V}^2$ and $\lambda = 0$.

Calculate the value of small signal voltage gain V_0/V_s of the circuit.

(ii) A differential amplifier has input voltages $V_1 = 1 \text{ mV}$ and $V_2 = 3 \text{ mV}$. The amplifier has differential gain $A_d = 5 \times 10^3$ and CMRR = 1000. Calculate the output voltage of the amplifier.

[15 + 5 marks]

The small signal Model of above ckt is



$$R_G = 165 || 35 = 28.875 \text{ k}\Omega$$

$$V_G = \frac{5 \times 35 + (165)(-5)}{200} = -3.25 \text{ V}$$

$$I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{GS})$$

here $\lambda = 0$, $K_n = 1 \text{ mA/V}^2$, $V_{TN} = 0.8$

$$\text{Hence } I_D = 1 (V_{GS} - 0.8)^2 \quad \text{--- (1)}$$

Also $V_{GS} = V_G - V_S$
 $V_{GS} = -3.25 - \frac{I_D}{2}$

4

$$I_D = 2(V_{GS} + 3.25) \quad \text{--- (2)}$$

From eq (1) & (2)

$$2(V_{GS} + 3.25) = (V_{GS} - 0.8)^2$$

$$2V_{GS} + 6.5 - V_{GS}^2 + 0.64 - 1.6V_{GS}$$

$$V_{GS}^2 - 3.6V_{GS} - 5.86 = 0$$

$$V_{GS} = 4.816 \text{ volts or } V_{GS} = -1.2166$$

For n-channel MOS, ~~V_{GS}~~ V_{GS} must be +ve

~~$$V_{GS} = 4.816 \text{ volts}$$~~

$$V_{GS} = -1.2166$$

Hence ~~$I_D = 16.13 \text{ Amp}$~~

$$I_D = 2(-1.2166 + 3.25)$$

$$I_D = 4.0668 \text{ A}$$

$$V_S = 2.0334 \text{ volt}$$

$$V_1 = 1 \text{ mV} \quad V_2 = 3 \text{ mV}$$

$$A_d = 5 \times 10^3$$

$$\text{CMRR} = 1000$$

$$\text{CMRR} = \frac{A_d}{A_c}$$

$$A_c = \frac{A_d}{\text{CMRR}} = 5$$

Common Mode Gain = 5

$$V_d = V_2 - V_1 = 3 - 1 = 2 \text{ mV}$$

$$V_c = 1 \text{ mV}$$

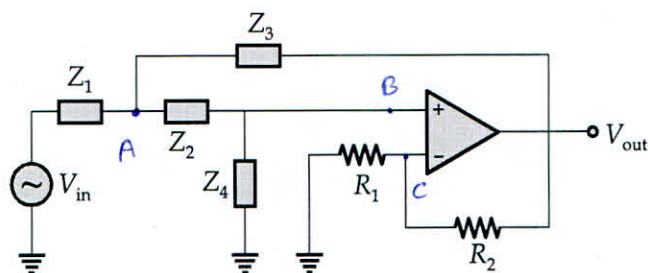
$$\text{Output Voltage} = A_c V_c + A_d V_d$$

$$= 5 \times 10^{-3} + 5 \times 10^3 \times (2 \times 10^{-3})$$

$$= 0.005 + 10$$

$$V_{\text{out}} = 10.005 \text{ volts}$$

Q.2 (b) Consider the circuit shown in the figure below:



The figure represents a second order active filter system.

- (i) Derive an expression for V_{out}/V_{in} .
 (ii) If each of the impedance elements Z_1 through Z_4 are replaced by a resistor of value R , then find the value of V_{out}/V_{in} .

[20 marks]

Sol 2(b) i)

Calculating voltage at Node C in terms of O/P voltage. Using voltage division:

$$V_C = V_{out} \left[\frac{R_1}{R_1 + R_2} \right]$$

For ideal opamp. $V_B = V_C$

$$V_B = \frac{V_{out} \cdot R_1}{R_1 + R_2} \quad \text{--- (1)}$$

Using KCL at Node A

$$\frac{V_{in} - V_A}{Z_1} = \frac{V_A - V_{out}}{Z_3} + \frac{V_A - V_B}{Z_2} \quad \text{--- (2)}$$

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{Z_3} + \frac{V_B}{Z_2} = V_A \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] \quad \text{--- (2)}$$

Also

$$V_B = V_A \cdot \frac{Z_4}{Z_2 + Z_4} \quad \text{--- (3)}$$

$$\Rightarrow V_A = V_B \left(\frac{Z_2 + Z_4}{Z_4} \right) \quad \text{--- (3)}$$

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{Z_3} + \frac{V_B}{Z_2} = V_B \left(\frac{Z_2 + Z_4}{Z_4} \right) \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] \quad \text{--- (4)}$$

Using eq (1) & (4)

$$\frac{V_{in}}{Z_1} + \frac{V_{out}}{Z_3} + \frac{V_{out} R_1}{(R_1 + R_2) Z_2} = \frac{V_{out} R_1}{R_1 + R_2} \left(\frac{Z_2 + Z_4}{Z_4} \right) \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right]$$

$$\frac{V_{in}}{Z_1} = V_{out} \left[\frac{R_1}{R_1 + R_2} \cdot \left(1 + \frac{Z_2}{Z_4} \right) \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - \frac{1}{Z_3} - \frac{R_1}{(R_1 + R_2) Z_2} \right]$$

$$\frac{V_{in}}{Z_1} = V_{out} \left[\frac{R_1}{R_1 + R_2} \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - \frac{R_1}{(R_1 + R_2) Z_2} - \frac{1}{Z_3} + \frac{R_1 Z_2}{R_1 + R_2} \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] \right]$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{Z_1 \left[\frac{R_1}{R_1 + R_2} \left(1 + \frac{Z_2}{Z_4} \right) \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right) - \frac{1}{Z_3} - \frac{R_1}{(R_1 + R_2) Z_2} \right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{Z_1 \left[R_1 \left[1 + \frac{Z_2}{Z_4} \right] \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - \frac{R_1 + R_2}{Z_3} - \frac{R_1}{Z_2} \right]}$$

Ans

If $Z_1 = Z_2 = Z_3 = Z_4 = R$ then

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R \left[R_1 \left[1 + \frac{R}{R} \right] \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R} \right] - \frac{R_1 + R_2}{R} - \frac{R_1}{R} \right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R \left[R_1 \left[2 \right] \left[\frac{3}{R} \right] - \frac{2R_1}{R} - \frac{R_2}{R} \right]}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R \left[\frac{6R_1}{R} - \frac{2R_1}{R} - \frac{R_2}{R} \right]} = \frac{R_1 + R_2}{[4R_1 - R_2]}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{[4R_1 - R_2]}}$$

18

- Q.2 (c) (i) For a dielectric, establish an expression for the relationship between the polarizability and permittivity. How does this relation lead to Clausius-Mossotti equation?
- (ii) When an NaCl crystal is subjected to an electric field of 1000 V/m, the resulting polarization is 4.3×10^{-8} C/m². Calculate the relative permittivity of NaCl.

[15 + 5 marks]

Suppose a small dipole has polarization \vec{p} & there are N no. of dipoles present in the material then,

$$\vec{p} = \alpha \vec{E}_i \quad \text{where } \alpha = \text{polarizability}$$

\vec{E}_i internal electric field

for N No. of dipoles

$$\vec{P} = N\vec{p} = N\alpha \vec{E}_i \quad \text{--- (1)}$$

Net polarization ~~is~~ ~~in~~ ~~internal~~ ~~electric~~ ~~field~~ in a material is given by

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\epsilon_0 \epsilon_r \vec{E} = \epsilon_0 \vec{E} + \vec{P}$$

$$\boxed{\vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E}}$$

$$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}} \quad \text{--- (2)}$$

From eq ① & ②

$$N \alpha E = \epsilon_0 \chi_e \vec{E}$$

$$\alpha = \frac{\epsilon_0 \chi_e}{N} \Rightarrow \boxed{\alpha = \frac{\epsilon_0 (\epsilon_r - 1)}{N}}$$

Net Electric field in a material is given by

$$\vec{E}_i = \vec{E} + \frac{\gamma P}{\epsilon_0} ; \text{ where } \gamma = \frac{1}{3} \left\{ \begin{array}{l} \text{for cubic} \\ \text{crystals} \end{array} \right.$$

$$\vec{E}_i = \vec{E} + \frac{N \alpha E_i}{3 \epsilon_0}$$

$$E_i \left[1 - \frac{N \alpha}{3 \epsilon_0} \right] = \vec{E} \Rightarrow \boxed{\vec{E}_i = \frac{\vec{E}}{1 - \frac{N \alpha}{3 \epsilon_0}}} \quad \text{--- (3)}$$

Put value of E_i in eq ①

$$\vec{P} = N \alpha E_i$$

$$\epsilon_0 (\epsilon_r - 1) E = \frac{N \alpha E}{\left(1 - \frac{N \alpha}{3 \epsilon_0} \right)}$$

$$\boxed{\epsilon_r - 1 = \frac{N \alpha / \epsilon_0}{1 - \frac{N \alpha}{3 \epsilon_0}}} \quad \text{--- (4)}$$

Add 3 both sides, we obtain

$$\boxed{\epsilon_r + 2 = \frac{3}{1 - \frac{N \alpha}{3 \epsilon_0}}} \quad \text{--- (5)}$$

Divide eq (4) by (5)

$$\boxed{\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N \alpha}{3 \epsilon_0}} \Rightarrow \text{C-M equation}$$

17

$$\Rightarrow E = 1000 \text{ V/m} \quad \vec{P} = 4.3 \times 10^{-8} \text{ C/m}^2$$

$$\vec{P} = \epsilon_0 (\epsilon_r - 1) E$$

~~$$4.3 \times 10^{-8} = \epsilon_0 (\epsilon_r - 1) \times 1000$$~~

~~$$-1 + \epsilon_r = \frac{\vec{P}}{\epsilon_0 E}$$~~

$$\epsilon_r = 1 + \frac{\vec{P}}{\epsilon_0 E}$$

$$\epsilon_r = 1 + \frac{4.3 \times 10^{-8}}{8.854 \times 10^{-12} \times 1000}$$

$$\epsilon_r = 1 + \frac{4.3 \times 10^{-8}}{8.854 \times 10^{-9}}$$

$$\epsilon_r = 1 + \frac{43}{8.854}$$

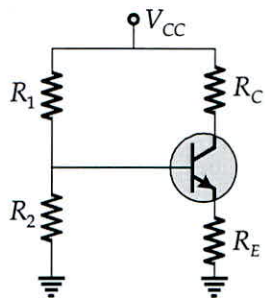
$$\epsilon_r = 5.856$$

5

Relative Permittivity of NaCl is, $\epsilon_r = 5.856$

Ans

(a) Consider the voltage divider biasing circuit shown in the figure below:



For this circuit,

- (i) Derive an expression for stability factor S [i.e., the variation of I_C w.r.t. I_{CO}].
- (ii) Derive an expression for stability factor S' [i.e., the variation of I_C w.r.t. V_{BE}].
- (iii) Derive a relation between S and S' .

[20 marks]

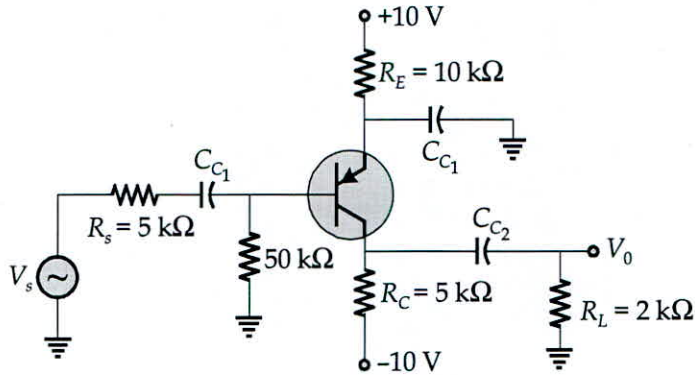
- (b) What are the types of cubic crystal structure? Derive the atomic packing factor of all the cubic crystal structures.

[20 marks]

- Q.3 (c) Electron drift mobility in indium (In) has been measured to be $6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$. At room temperature (27°C), the resistivity of In is $8.37 \times 10^{-8} \Omega \text{ m}$ and its atomic mass and density are $114.82 \text{ g mol}^{-1}$ and 7.31 g cm^{-3} respectively.
- (i) Based on the resistivity value, determine the effective number of free electrons donated by each In atom in the crystal.
 - (ii) If the mean speed of conduction electrons in In is $1.74 \times 10^8 \text{ cm s}^{-1}$, what is the mean free path?
 - (iii) Calculate the thermal conductivity of In at room temperature.

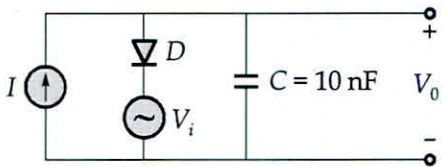
[20 marks]

- Q.4 (a) Consider a $p-n-p$ transistor shown in the figure below. The transistor has $V_{EB(on)} = 0.7 \text{ V}$, $\beta = 150$ and $V_A = \infty$. Draw a neat and labelled graph for DC and AC load line. Mark the Q -point on the graph.



[20 marks]

Q.4 (b) Consider the circuit shown in the figure below:



I is DC current and V_i is a sinusoidal signal with small amplitude and frequency of 100 kHz. Thus for small signal input and output voltages V_i and V_0 , calculate:

- (i) Phase angle difference between V_i and V_0 .
- (ii) The value of DC current I for which the phase shift between V_i and V_0 is -45° .
(Assume $V_T = 25 \text{ mV}$)
- (iii) The range of phase shift that is achieved as I is varied over the range of 0.1 to 10 times of the value obtained in part (ii).

[20 marks]

- 4 (c) (i) What do you understand by magnetic hysteresis? Differentiate between hard and soft magnetic materials?
- (ii) In a magnetic material, the field strength is found to be 10^6 A/m. If the magnetic susceptibility of the material is 0.5×10^{-5} , calculate the intensity of the magnetization and the magnetic flux density in the material.

[12 + 8 marks]



Section B : Electronic Devices & Circuits-1 + Advanced Electronics Topics-1 + Analog and Digital Communication Systems-2

(a) With neat diagrams, explain the Local Oxidation of Silicon (LOCOS) isolation technique used in IC fabrication.

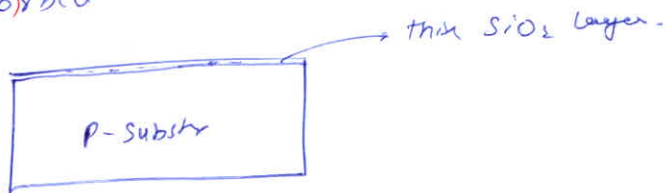
[12 marks]

LOCOS

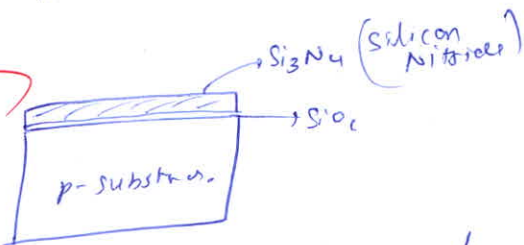
LOCOS stands for Local Oxidation of Silicon substrate.

⇒ Procedure

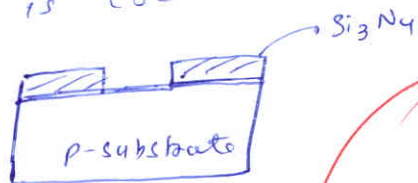
→ A p-type substrate is first oxidized (thermal oxidation) to obtain a thin layer of SiO_2 over the surface which act as shock absorber.



→ Now a layer of Si_3N_4 is deposited over SiO_2 layer.

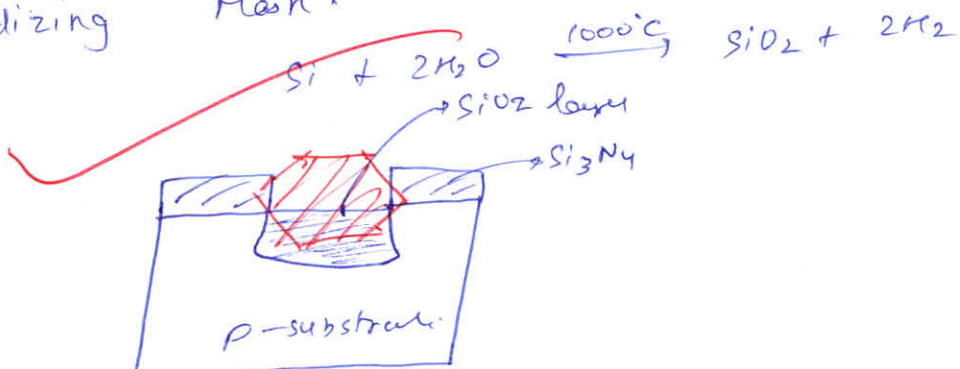


→ Photo litho graphy is used to remove a section of Si_3N_4 layer i.e a hole is created

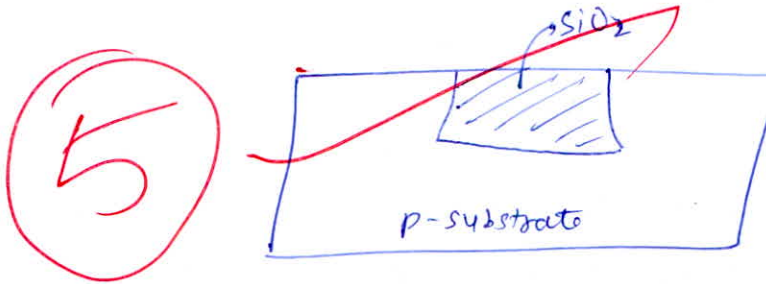


Now thermal oxidation of the above sample is done. Here Silicon Nitride act as an

Antioxidizing Mask.



Now again photo lithography is used to
~~obtain~~ @ remove the Si_3N_4 layer
 and obtain the locally oxidized substrate



- Q.5 (b) (i) The oxide removal rate and the removal rate of a layer underneath the oxide (called a stop layer) are r and $0.1r$ respectively. To remove $1 \mu\text{m}$ of oxide and a $0.01 \mu\text{m}$ stop layer, the total removal time is 5.5 minutes. Find the oxide removal rate (r).
- (ii) Calculate the Al average etch rate and etch rate uniformity on a 200 mm diameter silicon wafer, assuming the etch rates at the center, left, right, top and bottom of the wafer are 750, 812, 765, 743 and 798 nm/min respectively.

[6 + 6 marks]

q-(b)(i)

$$\text{Oxide removal rate} = r$$

$$\text{Layer removal rate} = 0.1r$$

$$\text{Net length of oxide removed} = 1 \mu\text{m}$$

$$\text{Time taken} = 5.5 \text{ min} = 330 \text{ sec}$$

(1)

$$\text{Oxide removal rate } r = \frac{\text{Oxide removed}}{\text{per unit time}}$$

$$= \frac{1 \mu\text{m}}{5.5 \text{ min}}$$

$$r = 0.1818 \mu\text{m/Minute}$$

or

$$r = 3.03 \text{ nm/sec}$$

$$(ii) \text{ Al Avg etch rate} = \frac{750 + 812 + 765 + 743 + 798}{5}$$

$$\textcircled{3} = \frac{3868}{5}$$

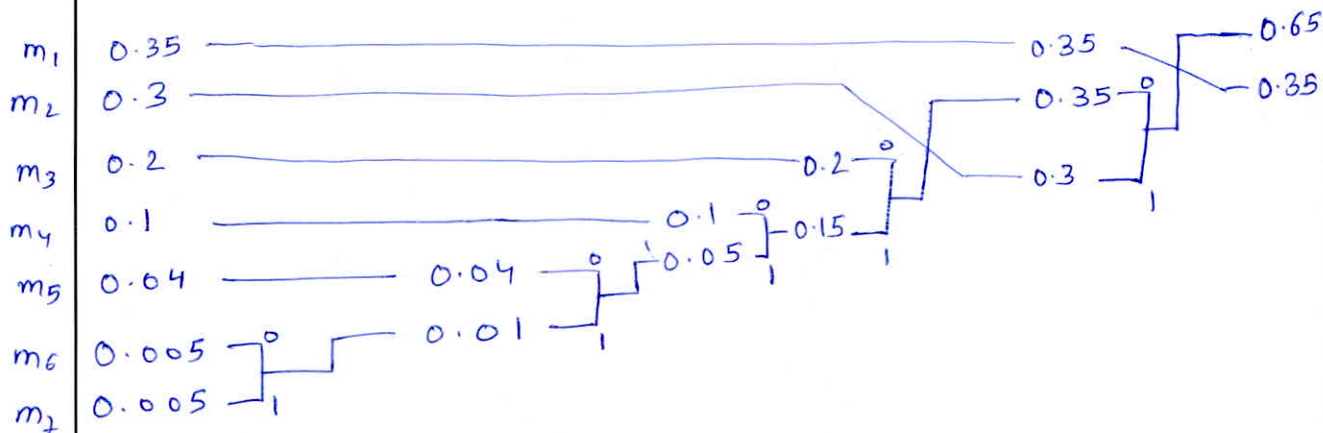
$$\text{Avg Etch rate} = 773.6 \text{ nm/min}$$

Etch rate uniformly?

Q.5 (c) A source emits seven symbols with probabilities 0.35, 0.3, 0.2, 0.1, 0.04, 0.005, 0.005. Give Huffman coding for these symbols and calculate average bits of information and average binary digits of information per symbol.

[12 marks]

$$p(m) = \{ 0.35, 0.3, 0.2, 0.1, 0.04, 0.005, 0.005 \}$$



$$m_1 = 1$$

$$m_2 = 01$$

$$m_3 = 000$$

$$m_4 = 0010$$

$$m_5 = 00110$$

$$m_6 = 001110$$

$$m_7 = 001111$$

$$\begin{aligned} \text{Avg bit per symbol} &= \sum_{i=1}^7 p_i \cdot l_i \\ &= 0.35 \times 1 + 0.3 \times 2 + 0.2 \times 3 + 0.1 \times 4 \\ &\quad + 0.04 \times 5 + 0.005 \times 6 + 0.005 \times 6 \end{aligned}$$

$$\bar{L} = 2.21 \text{ bits/symbol}$$

$$\begin{aligned} \text{Avg info bit} &= \text{Entropy} = - \sum_{i=1}^7 p(x_i) \log_2 p(x_i) \\ &= - \left[0.35 \log_2 0.35 + 0.3 \log_2 0.3 \right. \\ &\quad + 0.2 \log_2 0.2 + 0.1 \log_2 0.1 \\ &\quad + 0.04 \log_2 0.04 + 0.005 \log_2 0.005 \\ &\quad \left. + 0.005 \log_2 0.005 \right] \end{aligned}$$

$$H(x) = 2.1099 \text{ bits / symbol}$$

Efficiency.

$$\eta = \frac{2.1099}{2.21} \times 100 = 95.47 \%$$



- d) The distribution (with respect to energy) of electron concentration in the conduction band is given by density of allowed quantum states times the probability that state being occupied by an electron. i.e., $n(E) = g_c(E) f(E)$ where, $g_c(E)$ = Density of allowed states, $f(E)$ = probability of state being occupied. Assuming that Boltzmann approximation in a semiconductor is valid, calculate the ratio of $n(E)$ at $E = E_c + 4kT$ to that at $E = E_c + (kT/2)$. Here, k = Boltzmann constant, E_c = edge of the conduction band and T = temperature in $^{\circ}\text{K}$.

[12 marks]

$$n(E) = g_c(E) f(E)$$

$$n(E) = g_c(E) \cdot \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

$$n(E) \approx g_c(E) \cdot e^{-\left(\frac{E - E_F}{kT}\right)}$$

$$n(E_1) = g_c(E_1) \cdot e^{-\left(\frac{E_c + 4kT - E_F}{kT}\right)} \quad \text{--- (1)}$$

$$\text{also } \frac{N_c}{g_c(E)} = 2 \left(\frac{2\pi m_n kT}{h^2} \right)^{3/2}$$

$$n(E_2) = g_c(E_2) \cdot e^{-\left(\frac{E_c + kT/2 - E_F}{kT}\right)} \quad \text{--- (2)}$$

Dividing eq ① by ②

$$\frac{n(E_1)}{n(E_2)} = \frac{g(E_1)}{g(E_2)} \cdot \frac{e^{-\left(\frac{E_1 + 4kT - E_F}{kT}\right)}}{e^{-\left(\frac{E_2 + \frac{3kT}{2} - E_F}{kT}\right)}}$$

$$\frac{n(E_1)}{n(E_2)} = \frac{g(E_1)}{g(E_2)} \cdot e^{\left(\frac{3kT}{2kT}\right)}$$

$$\frac{n(E_1)}{n(E_2)} = \frac{g(E_1)}{g(E_2)} \cdot e^{-3/2}$$

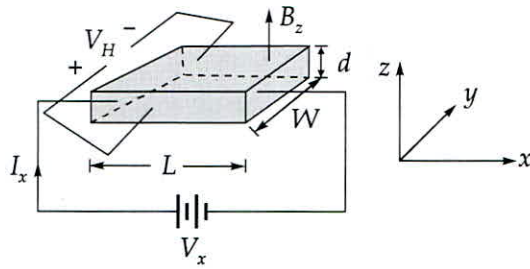
6

Also $g(E_1) \approx g(E_2)$ (remains const.)

$$\frac{n(E_1)}{n(E_2)} = e^{-3.5} \times 2.72$$

$$\frac{n(E_1)}{n(E_2)} = \frac{0.03019}{0.6854}$$

e) Consider a silicon Hall effect device which is used for the experiment as shown below:



The device has dimensions $d = 5 \times 10^{-3}$ cm, $W = 5 \times 10^{-2}$ cm and $L = 0.5$ cm. The electrical parameters measured as the result of the experiment are $I_x = 0.5$ mA, $V_x = 1.25$ V and $B_z = 6.5 \times 10^{-2}$ T. If the induced Hall electric field is $E_{Hy} = -16.5$ mV/cm, then determine:

- (i) Hall voltage (V_H)
- (ii) The type of semiconductor
- (iii) The majority carrier concentration

[12 marks]

$$E_{Hy} = \frac{V_H}{W}$$

$$V_H = (-16.5) \times 10^{-3} \times 5 \times 10^{-2}$$

$$V_H = -82.5 \times 10^{-5} \text{ Volts}$$

$$V_H = -0.825 \text{ mV}$$

ii) V_H is -ve & small. Hence it is n-type semiconductor.

9

$$V_H = \frac{R_H I B}{d}; \quad R_H = \frac{1}{eP}$$

$$-0.825 \times 10^{-3} = \frac{1}{eP} \times \frac{0.5 \times 10^{-3} \times 6.5 \times 10^{-2}}{5 \times 10^{-5}}$$

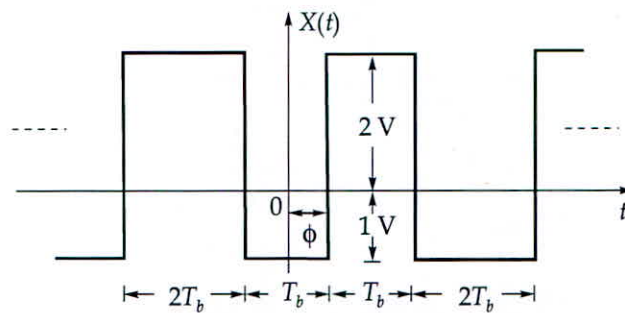
$$P = \frac{0.5 \times 6.5 \times 10^{-2}}{(-0.825) \times (-1.6 \times 10^{-19}) \times 10^{-5}}$$

$$P = \frac{0.5 \times 6.5}{0.825 \times 1.6} \times 10^{22}$$

Majority Carrier Conc. P

$$P = 2.462 \times 10^{22} \text{ /m}^3$$

Q.6 (a) Consider the random binary wave shown below:



In this binary wave, logic-1 is represented with positive rectangular pulse and logic-0 is represented with negative rectangular pulse, both with different amplitudes. ϕ is an independent random variable uniformly distributed in the range $[0, T_b]$, where T_b is the bit duration. Determine and sketch the auto-correlation function of $X(t)$. Assume that logic-1 and logic-0 are occurring with equal probability.

[20 marks]

- Q.6 (b)** A 1 cm long bar of n -type Ge has a cross section of 1 mm \times 1 mm. The resistivity of material is 20 Ω -cm and the lifetime of the carriers is 100 microseconds. (Assume $\mu_n = 3800$ cm²/V-s, $\mu_p = 1800$ cm²/V-s and intrinsic carrier concentration $n_i = 2.5 \times 10^{13}$ /cm³).
- (i) Calculate the resistance of the bar.
 - (ii) Calculate the donor concentration.
 - (iii) Calculate the resistance of the bar when it is illuminated such that excess electron-hole pairs are generated at a rate of 10^{15} cm⁻³ s⁻¹, uniformly all over the bar.

[20 marks]

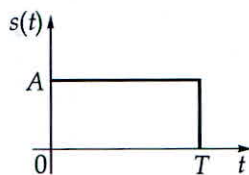
- (i) Binary data (equiprobable bits) with a rate of 1 Mbps is transmitted through an AWGN channel using different modulation schemes. The two sided power spectral density of the channel noise is 0.5×10^{-11} W/Hz and the carrier signal used in the transmitters is $5\cos(2\pi f_c t)$ mV. In each case of different modulation schemes, the signals are received by their respective correlator receivers with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.
- (ii) Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t)n(t)dt \quad \text{and} \quad n_2 = \int_0^T s_2(t)n(t)dt$$

Prove that n_1 and n_2 are orthogonal.

[15 + 5 marks]

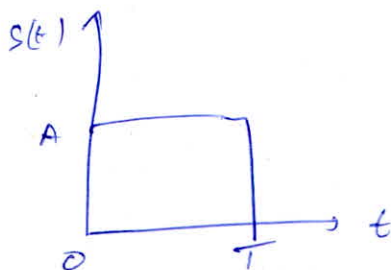
Q.7 (a) Consider the signal shown in the figure below:



This signal is passed through a channel and applied to a filter matched to the signal $s(t)$ at the receiving end. If the channel is not ideal, but has an impulse response

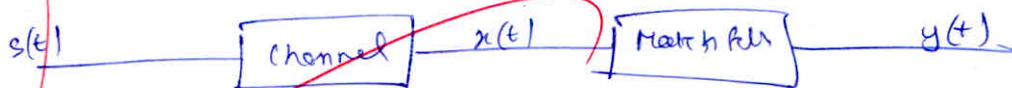
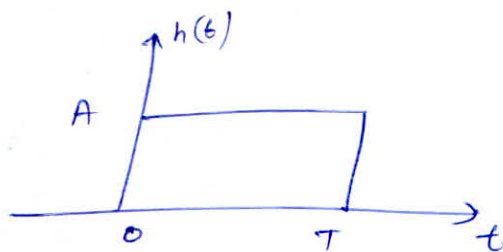
$$c(t) = \delta(t) + \frac{1}{2}\delta\left(t - \frac{T}{2}\right),$$

[20 marks]



Matched filter response

$$h(t) = s(T-t)$$



$$x(t) = s(t) * c(t)$$

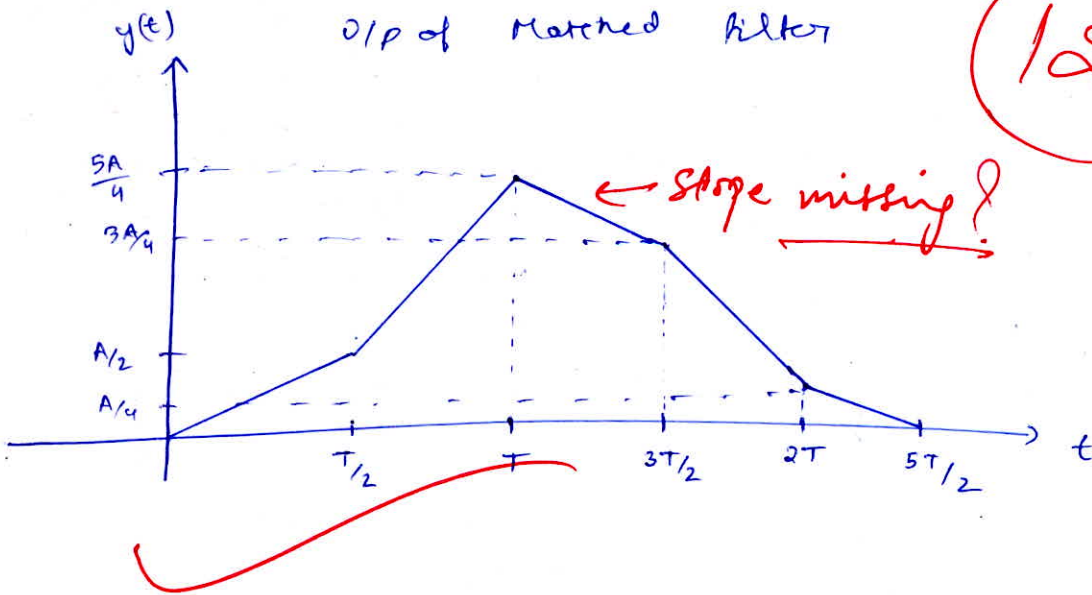
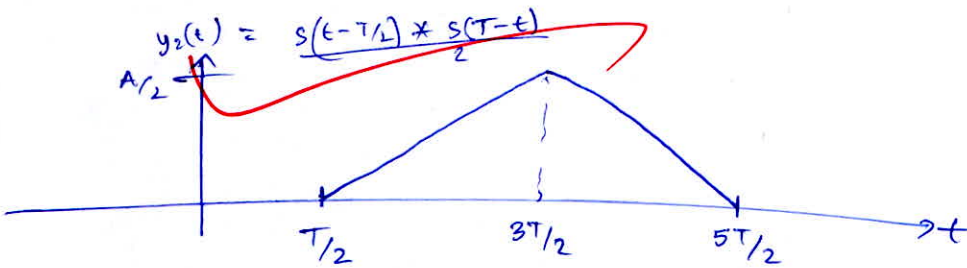
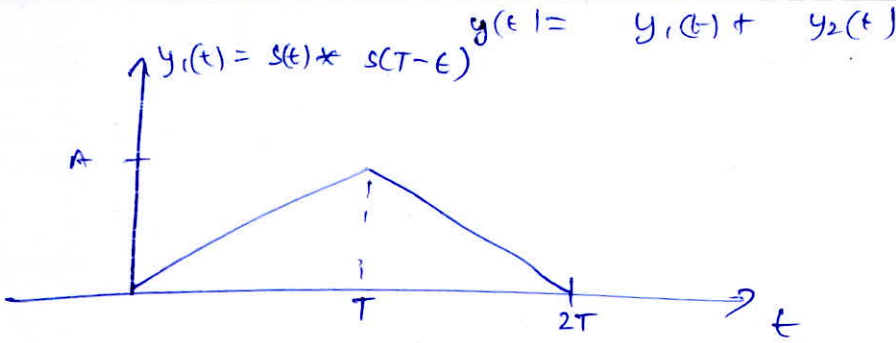
$$= s(t) * \left[\delta(t) + \frac{1}{2} \delta\left(t - \frac{T}{2}\right) \right]$$

$$x(t) = s(t) + \frac{s(t - T/2)}{2}$$

$$y(t) = x(t) * h(t)$$

$$= \left[s(t) + \frac{s(t - T/2)}{2} \right] * s(T-t)$$

$$= s(t) * s(T-t) + \frac{s(t - T/2) * s(T-t)}{2}$$



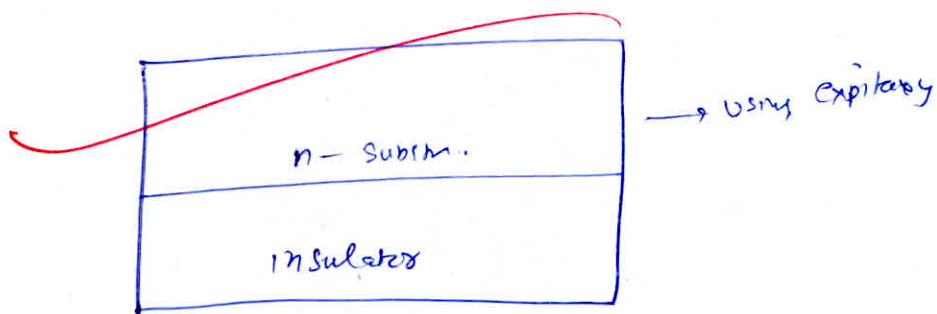
Q.7 (b) Explain the basic steps involved in the fabrication of a CMOS transistor using silicon on sapphire (SOS) process.

[20 marks]

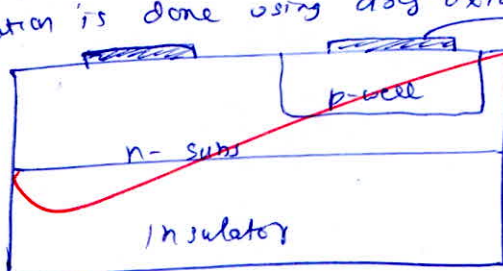
(SOS) is also known as Silicon on Insulated technology

① ⇒ First Sapphire Insulator is taken

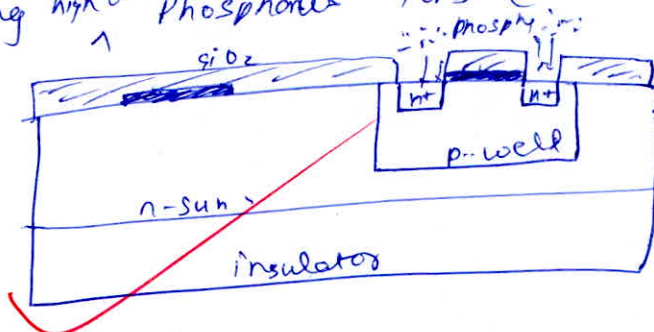
② ⇒ An Epitaxy layer of p-substrate is deposited over its surface.



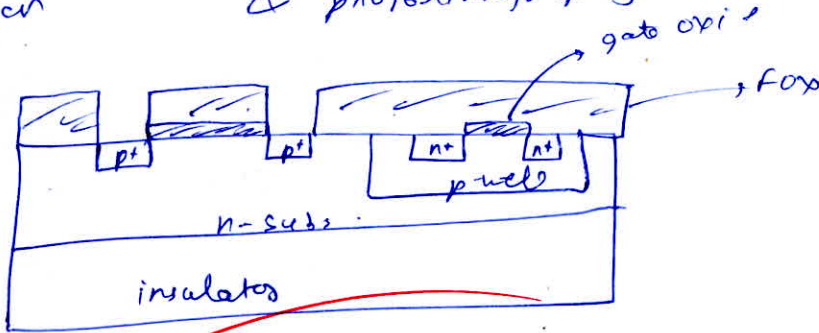
③ ⇒ P-well formation is done into n-substrate
Gate formation is done using dry oxidation gate oxide



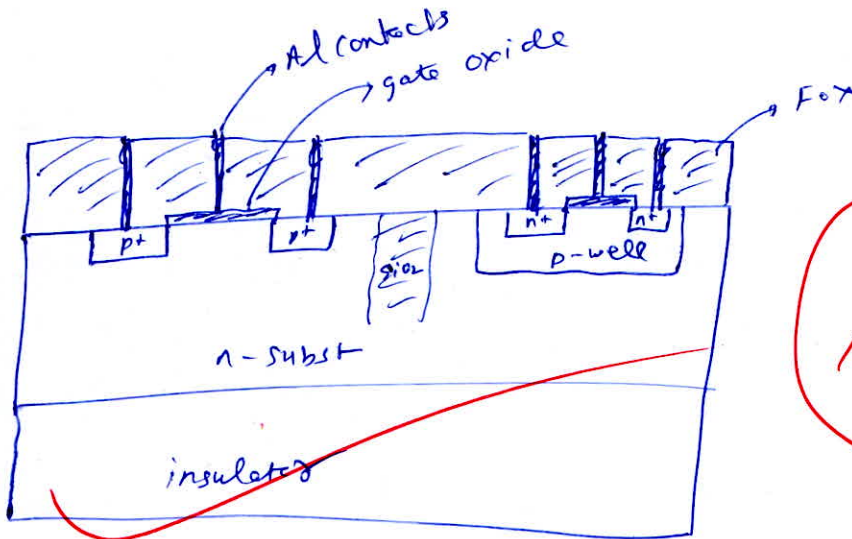
⇒ ~~Next~~ Now n^+ source & drain formation is done using high density Phosphorus ions (phosphine gas) using photolithography



Now p^+ source & drain diffusion is done using boron & photolithography -



Now metallization & Contact formation is done. Locod is also done to avoid CMOS Latch-up.



12

$$R = \frac{\rho l}{A} \quad \sigma = \frac{1}{R \frac{l}{A}}$$

- Q.7 (c) A p -type lightly doped semiconductor has electron mobility μ_n , hole mobility μ_p , intrinsic carrier concentration n_i and the acceptor impurity concentration N_A .
- Derive an expression for the hole concentration ' p ' in terms of n_i , μ_n and μ_p , such that the conductivity of the semiconductor is minimum.
 - Derive an expression for the minimum conductivity of the semiconductor.
 - If $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$, $\mu_n = 1300 \text{ cm}^2/\text{V-sec}$ and $\mu_p = 500 \text{ cm}^2/\text{V-sec}$, then calculate the value of minimum conductivity.
 - If there is 100% ionization of doping atoms, then calculate the value of acceptor impurity concentration (N_A).

[20 marks]

Charge Neutrality equation

$$N_D + p = N_A + n$$

$$p = N_A + n \quad \text{--- (1)}$$

$$\{N_D = 0\}$$

Using Mass Action Law

$$pn = n_i^2$$

$$n = \frac{n_i^2}{p} \quad \text{--- (2)}$$

From eq (1) & (2)

$$p = N_A + \frac{n_i^2}{p}$$

$$p^2 - N_A p - n_i^2 = 0$$

$$p = \frac{N_A \pm \sqrt{N_A^2 + 4n_i^2}}{2}$$

$$p = \frac{N_A \pm \sqrt{N_A^2 + 4n_i^2}}{2}$$

$$\sigma = p\mu_p e + n e \mu_n = p\mu_p e + \frac{n_i^2}{p} e \mu_n$$

$$\frac{d\sigma}{dp} = \mu_p e - \frac{n_i^2}{p^2} e \mu_n$$

For Min Conductivity

$$\mu_p e = \frac{n_i^2}{p^2} e \mu_n$$

$$p = n_i \sqrt{\frac{\mu_n}{\mu_p}}$$

$$\sigma_{\min} = p_{\min} \mu_p e + n_{\min} e \mu_n$$

$$= n_i \sqrt{\frac{\mu_n}{\mu_p}} \cdot e \mu_p + \frac{n_i^2}{p} e \mu_n$$

$$= n_i \sqrt{\frac{\mu_n}{\mu_p}} \cdot \mu_p \cdot e + \frac{n_i^2}{n_i \sqrt{\frac{\mu_n}{\mu_p}}} \cdot e \mu_n$$

$$= n_i \sqrt{\mu_n \mu_p} e + n_i \sqrt{\mu_n \mu_p} e$$

$$\sigma_{\min} = 2 n_i \sqrt{\mu_n \mu_p} \cdot e$$

$$\sigma_{\min} = 2 \times 1.5 \times 10^{10} \sqrt{1300 \times 300} \times 1.6 \times 10^{-19}$$

$$\sigma_{\min} = 3.8698 \times 10^{-6} \text{ } \Omega^{-1} \text{ m}^{-1}$$

107

$$N_A = n_i \sqrt{\frac{\mu_n}{\mu_p}} - n_i$$

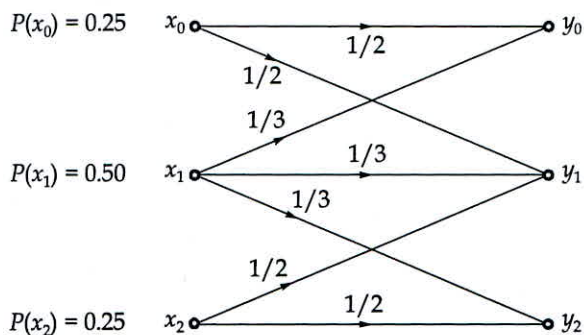
$$16 = n_i \left[\sqrt{\frac{1300}{500}} - 1 \right]$$

$$= 1.5 \times 10^{10} \times 0.612$$

$$N_A = 0.9186 \times 10^{10} \text{ atoms/m}^3$$

1.488

Consider the discrete memoryless channel shown below:



Determine the mutual information $I(X; Y)$.

[20 marks]

$$I(X, Y) = H(X) + H(Y) - H(X, Y)$$

$$H(X) = -\sum_{i=1}^3 p(x_i) \ln p(x_i)$$

$$H(Y) = -\sum_{i=1}^3 p(y_i) \ln p(y_i)$$

$$H(X, Y) = -\sum_{i=1, j=1}^{3, 3} p(x_i, y_j) \ln (p(x_i, y_j))$$

3

$$H(X) = 0.25 \log 0.25 + 0.5 \log 0.5 + 0.25 \log 0.25$$

$$p(y_0) = 0.25 \times \frac{1}{2} + 0.5 \times \frac{1}{3}$$

$$p(y_1) = 0.5 \times \frac{1}{2} + 0.25 \times \frac{1}{2} + 0.25 \times \frac{1}{2}$$

$$p(y_2) = \frac{1}{2} \times 0.25 + 0 + 0.5 \times \frac{1}{3}$$

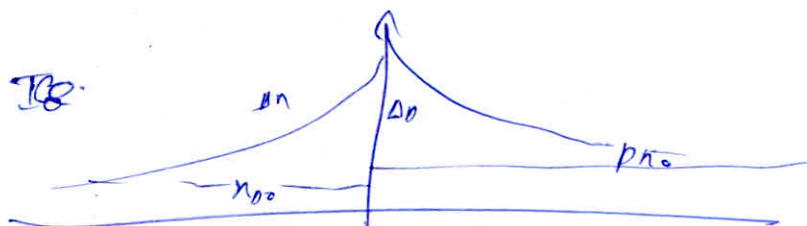
$$= 0.125 + 0.1667$$

- Q.8 (b) For a boron diffusion in silicon at 1000°C , the surface concentration is maintained at 10^{19} cm^{-3} and the diffusion time is 1 hour. Assume that the diffusivity (D) of Boron in Silicon at 1000°C is $2 \times 10^{-14} \text{ cm}^2/\text{s}$. Determine:
- (i) The total number of dopant atoms per unit area of semiconductor.
 - (ii) The distance of the location from the surface where the dopant concentration reaches 10^{15} cm^{-3} . Assume that $\text{erfc}^{-1}(10^{-4}) = 2.75$.
 - (iii) The gradient of the diffusion profile at the surface.
 - (iv) The gradient of the diffusion profile at the distance from the surface obtained in part (ii).

[20 marks]

- Q.8 (c) (i) Find the expression for reverse saturation current I_0 in a $p-n$ junction diode in terms of intrinsic carrier concentration n_i .
- (ii) Find an expression for the reverse saturation current in terms of the conductivity of the device and prove that, $I_0 = AV_T \frac{b\sigma_i^2}{(1+b)^2} \left[\frac{1}{L_p\sigma_n} + \frac{1}{L_n\sigma_p} \right]$ where, $b = \frac{\mu_n}{\mu_p}$

[20 marks]



$$p = p_{n0} e^{V/V_T}$$

$$p = \Delta p + p_{n0}$$

$$\Delta p = p_{n0} (e^{V/V_T} - 1) \quad \text{--- (1)}$$

$$\text{Similarly } \Delta n = n_{p0} (e^{V/V_T} - 1) \quad \text{--- (2)}$$

Net Diffusion current

$$I = A q D_p \frac{\partial p}{\partial x} + A q D_n \frac{\partial n}{\partial x}$$

$$I = A e \frac{D_p}{L_p} \cdot \frac{n_i^2}{N_D} + A e \frac{D_n}{L_n} \frac{n_i^2}{N_A}$$

$$I = A e n_i^2 \left[\frac{D_p}{L_p} \frac{1}{N_D} + \frac{D_n}{L_n} \frac{1}{N_A} \right] \quad \text{--- (3)}$$

$$\sigma_n = N_D q \mu_n$$

$$N_D = \frac{\sigma_n}{q \mu_n} \rightarrow \textcircled{4} \quad N_A = \frac{\sigma_p}{q \mu_p} \quad \textcircled{5}$$

for $\textcircled{3}$, $\textcircled{4}$ & $\textcircled{5}$

$$I = \textcircled{Ae n_i^2} \left[\frac{D_p q \mu_n}{L_p \sigma_n} + \frac{D_n q \mu_p}{L_n \sigma_p} \right]$$

$$I_c = Ae \frac{\left(\frac{\mu_n}{\mu_p} \right)^2 \sigma_i}{1 + \left(\frac{\mu_n}{\mu_p} \right)^2} \left[\frac{1}{L_p \sigma_n} + \frac{1}{L_n \sigma_p} \right]$$

$\textcircled{11}$

Space for Rough Work

Space for Rough Work
