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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-5: Analog Circuits + Materials Science
Electronic Devices & Circuits-1 + Advanced Electronics Topics-1
Analog and Digital Communication Systems-2

Name : Nidhi gupta

Roll No : | E | C | 1 | 9 | M | B | D | L | A | 6 | 0 | 6 |

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Student's Signature

Nidhi gupta

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

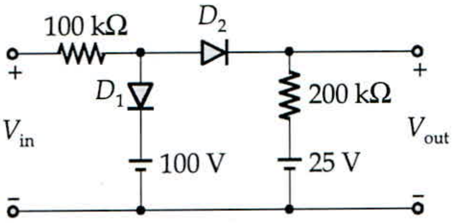
Question No.	Marks Obtained
Section-A	
Q.1	39
Q.2	—
Q.3	50
Q.4	—
Section-B	
Q.5	39
Q.6	—
Q.7	38
Q.8	22
Total Marks Obtained	188

Signature of Evaluator ....

Cross Checked by ....

Section A : Analog Circuits + Materials Science

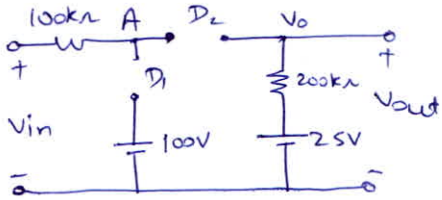
1 (a) Consider the circuit shown in the figure below:



By assuming that the diodes are ideal, develop the transfer characteristic curve of the above circuit.

[12 marks]

Solu: Initial assume diode D_1 & D_2 be off, circuit will become



$V_A = V_{in}$

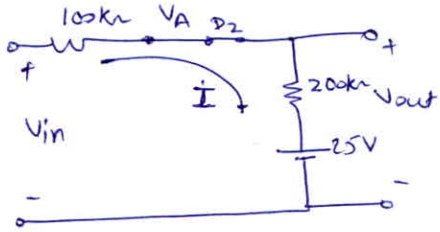
& $V_{out} = 25V$

D_1 remains in off if

$V_A < 100V$ i.e. $V_{in} < 100V$

D_2 remains off if : $V_A < V_o \Rightarrow V_{in} < 25V$

→ If $V_{in} > 25V$; D_2 will conduct, D_1 will off



$$I = \frac{V_{in} - 25}{300} \text{ mA}$$

$$\therefore V_{out} = 25 + 200\left(\frac{V_{in} - 25}{300}\right)$$

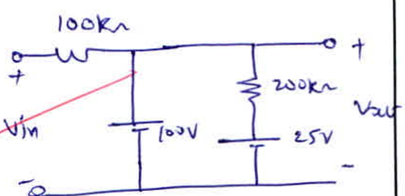
$$= \frac{2V_{in}}{3} + \frac{25}{3}$$

$V_A = V_{out}$

So, for diode D_1 to conduct $V_A > 100V$

$$\Rightarrow \frac{2V_{in}}{3} + \frac{25}{3} > 100 \Rightarrow V_{in} > 137.5V$$

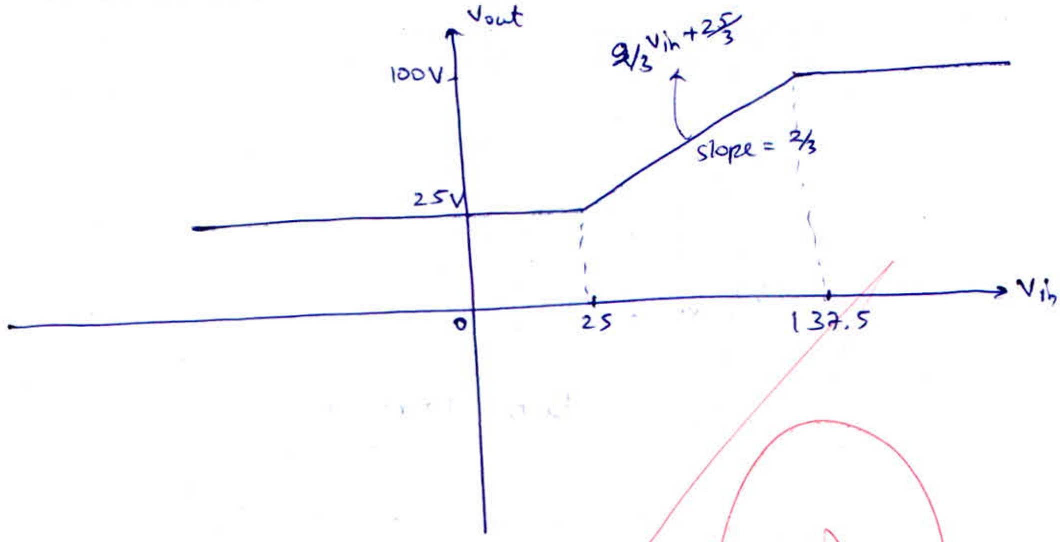
→ If $V_{in} > 137.5V$; D_1 & D_2 will conduct



$V_{out} = 100V$

overall transfer characteristics of the given network ;

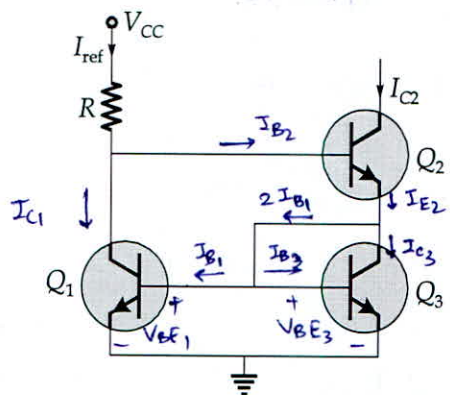
$$V_o = \begin{cases} 25V & ; V_{in} < 25V \\ \frac{2V_{in}}{3} + \frac{25}{3} & ; 25 < V_{in} < 137.5V \\ 100V & ; V_{in} > 137.5V \end{cases}$$



Transfer characteristic plot.

11

Q.1 (b) Consider the Wilson current mirror circuit as shown in the figure below:



Assume that the three transistors to be matched with $V_{BE1} = V_{BE3}$ and $\beta_1 = \beta_2 = \beta_3 = \beta$. Derive an expression for I_{C2} in terms of I_{ref}.

[12 marks]

Soln: from the circuit :

$$I_{E_2} = 2I_{B_1} + I_{C_3}$$

also, $I_{B_1} = \frac{I_{C_1}}{\beta}$ & $I_{B_1} = I_{B_3} = \frac{I_{C_3}}{\beta} = \frac{I_{C_1}}{\beta}$ (Given, transistors are matched)

$$\Rightarrow I_{E_2} = \left(1 + \frac{2}{\beta}\right) I_{C_1} \quad \text{—— (1)}$$

Now, $I_{Ref} = I_{C_1} + I_{B_2} = I_{C_1} + \frac{I_{C_2}}{\beta}$

$$I_{Ref} = \frac{I_{E_2}}{\left(1 + \frac{2}{\beta}\right)} + \frac{I_{C_2}}{\beta} \quad \text{(from eq. (1))}$$

as we know that $I_{E_2} = \left(1 + \frac{1}{\beta}\right) I_{C_2}$

$\therefore \quad I_{Ref} =$

$$\Rightarrow \quad I_{Ref} = \frac{\left(1+\frac{1}{\beta}\right)}{\left(1+\frac{2}{\beta}\right)} I_{C_2} + \frac{I_{C_2}}{\beta}$$

$$= I_{C_2}\left(\frac{\beta+1}{2+\beta} + \frac{1}{\beta}\right)$$

$$= I_{C_2}\left(\frac{\beta^2+\beta+2+\beta}{\beta(\beta+2)}\right)$$

$$I_{Ref} = I_{C_2}\left(\frac{\beta^2+2\beta+2}{\beta(\beta+2)}\right)$$

$$\therefore \quad I_{C_2} = \left[\frac{\beta(\beta+2)}{2+2\beta+\beta^2}\right] \cdot I_{Ref}$$

Q.1 (c) A long narrow rod (having cubic structure) has an atomic density of 5×10^{28} atoms/m^3. Each atom has a polarizability of 10^{-40} F-m^2. Calculate the internal electric field in the rod when an external axial field of 1 V/m is applied.

[12 marks]

Soln:

Given, Atomic density, $N = 5 \times 10^{28}$ atoms/m^3

polarizability, $\alpha = 10^{-40}$ F-m^2

external field, $E = 1$ V/m

As we know that,

Polarization, $P = N \alpha E_i$

also $E_i = E + \frac{\gamma P}{\epsilon_0}$

given cubic structure, $\gamma = \frac{1}{3}$

$\Rightarrow E_i = E + \frac{\gamma N \alpha E_i}{\epsilon_0}$

$E_i = \frac{E}{1 - \frac{\gamma N \alpha}{\epsilon_0}}$

$= \frac{1}{1 - \frac{1}{3} \times \frac{5 \times 10^{28} \times 10^{-40}}{8.85 \times 10^{-12}}}$

$E_i = 1.232$ V/m

internal electric field in the rod is

$\boxed{E_i = 1.232 \text{ V/m}}$

1 (d) Explain Silsbee's rule for superconductors. Also give some applications of superconductors.

[12 marks]

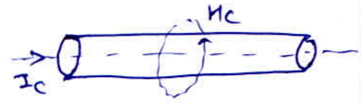
Silsbee's rule :

A superconducting wire carrying a current generate its magnetic field, if the magnetic field generated is equal to the critical magnetic field, superconducting nature will destroy. Thus, field created due to current can itself destroy the superconducting nature.

critical field

$$H_c \times 2\pi r = I_c$$

$$H_c = \frac{I_c}{2\pi r}$$



Application of superconductors :

① magnetically leviated transportation

② magnetic resonance imaging (MRI), ~~As a magne~~

③ As a magnet in generator & motors

④ Switching devices Ex: cryotron.

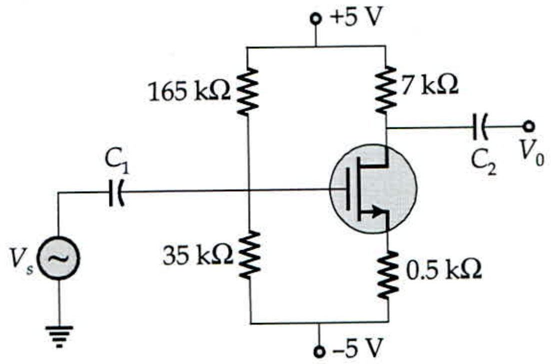
1 (e) Write short notes on the following nanomaterials:

(i) Quantum dots

(ii) Carbon nanotubes

[6 + 6 marks]

2 (a) **(i)** Consider the common source transistor circuit shown in the figure below:



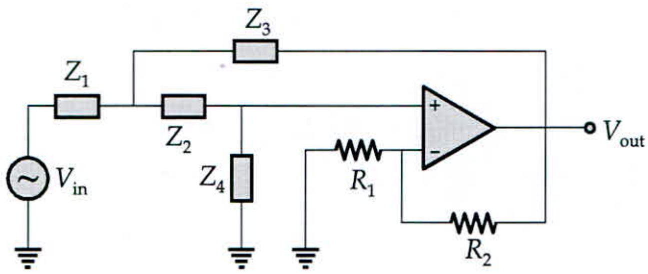
The transistor parameters are $V_{TN} = 0.8$ V, $K_n = \dfrac{\mu_n C_{ox} W}{2L} = 1\ \text{mA/V}^2$ and $\lambda = 0$.

Calculate the value of small signal voltage gain V_0/V_s of the circuit.

(ii) A differential amplifier has input voltages $V_1 = 1$ mV and $V_2 = 3$ mV. The amplifier has differential gain $A_d = 5 \times 10^3$ and CMRR = 1000. Calculate the output voltage of the amplifier.

[15 + 5 marks]

Q.2 (b) Consider the circuit shown in the figure below:



The figure represents a second order active filter system.

(i) Derive an expression for V_{out}/V_{in}.

(ii) If each of the impedance elements Z_1 through Z_4 are replaced by a resistor of value R, then find the value of V_{out}/V_{in}.

[20 marks]

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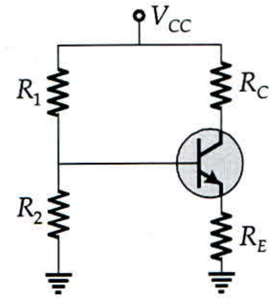
Q.2 (c) **(i)** For a dielectric, establish an expression for the relationship between the polarizability and permittivity. How does this relation lead to Clausius-Mossotti equation?

(ii) When an NaCl crystal is subjected to an electric field of 1000 V/m, the resulting polarization is 4.3×10^{-8} C/m^2. Calculate the relative permittivity of NaCl.

[15 + 5 marks]

.3 (a) Consider the voltage divider biasing circuit shown in the figure below:



For this circuit,

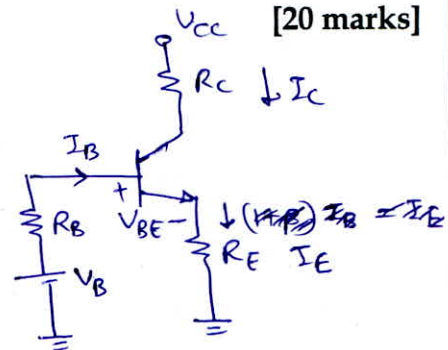
(i) Derive an expression for stability factor S [i.e., the variation of I_C w.r.t. I_{CO}].

(ii) Derive an expression for stability factor S' [i.e., the variation of I_C w.r.t. V_{BE}].

(iii) Derive a relation between S and S'.

[20 marks]

Soln: Self bias circuit, can be rearranged as



$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_B = \frac{R_2}{R_1+R_2} \cdot V_{CC}$$

(i) By KVL in base loop:

$$-V_B + I_B R_B + V_{BE} + R_E I_E = 0$$

$\Rightarrow \quad \because \; I_E = I_C + I_B$

$$V_{BE} - V_B + I_B(R_B + R_E) + R_E I_C = 0$$

differentiate above equation with respect to I_C, we get

$$\Rightarrow \quad 0 + \frac{\partial I_B}{\partial I_C}(R_B + R_E) + R_E = 0 \quad \Rightarrow \quad \frac{\partial I_B}{\partial I_C} = -\frac{R_E}{R_B + R_E} \quad \text{—(1)}$$

Now, we know that

$$I_C = \beta I_B + (1+\beta) I_{CO}$$

differentiate above eq. w.r.t. I_C, we get

$$1 = \beta \frac{\partial I_B}{\partial I_C} + (1+\beta)\frac{\partial I_{CO}}{\partial I_C} \quad \left\{ \text{consider } \beta \text{ constant} \right\}$$

$$\Rightarrow \quad \frac{\partial I_{CO}}{\partial I_C} = \frac{1 - \beta \frac{\partial I_B}{\partial I_C}}{1+\beta}$$

$\therefore$ stability factor, $S = \dfrac{\partial I_C}{\partial I_{CO}} = \dfrac{1+\beta}{1 - \beta \frac{\partial I_B}{\partial I_C}}$

from eq. (1)

$$S = \frac{1+\beta}{1+\beta\frac{R_E}{R_B+R_E}} = \frac{(1+\beta)(R_B+R_E)}{R_B+(1+\beta)R_E}$$

$$\boxed{S = \frac{(1+\beta)\left(1+\frac{R_E}{R_B}\right)}{\left(1+(1+\beta)\frac{R_E}{R_B}\right)}}$$ Ans

(ii) From KVL @ base :

$$-V_B + I_B R_B + V_{BE} + R_E(I_B+I_C) = 0$$

substitute $I_B = \frac{I_C - (1+\beta)I_{CO}}{\beta}$ in above equation

$$\Rightarrow -V_B + (R_B+R_E)\left(\frac{I_C-(1+\beta)I_{CO}}{\beta}\right) + R_E I_C + V_{BE} = 0$$

$$\Rightarrow -V_B + V_{BE} + I_C\left(\frac{R_B+R_E}{\beta} + R_E\right) - \frac{(R_B+R_E)}{\beta}(1+\beta)I_{CO} = 0$$

differentiate above equation w.r.t. I_C, considering β & I_{CO} constant.

$$\Rightarrow 0 + \frac{\partial V_{BE}}{\partial I_C} + \left(\frac{R_B+R_E}{\beta} + R_E\right) - 0 = 0$$

$$\Rightarrow \frac{\partial V_{BE}}{\partial I_C} = -\left(\frac{R_B+R_E}{\beta} + R_E\right)$$

∴ stability factor S'

$$S' = \frac{\partial I_C}{\partial V_{BE}} = \frac{-1}{R_E + \frac{R_B+R_E}{\beta}}$$

$$\boxed{S' = \frac{-\beta}{R_B+(1+\beta)R_E}}$$

(iii)

$$\frac{S'}{S} = \frac{\frac{-\beta}{R_B+(1+\beta)R_E}}{\frac{(1+\beta)(R_B+R_E)}{R_B+(1+\beta)R_E}} = \frac{-\beta}{(1+\beta)(R_B+R_E)}$$

$$\therefore \quad S = -\left(\frac{1+\beta}{\beta}\right)(R_B + R_E)\; S'$$

Q.3 (b) What are the types of cubic crystal structure? Derive the atomic packing factor of all the cubic crystal structures.

[20 marks]

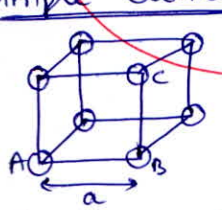
Soln:- Three types of cubic crystal structure :

① Simple cubic : ~~All~~ atoms are present at each corner of the lattice point. (cube)

② Body centred (BCC) : 8 atoms are at corner & one atom at the centre of the cube.

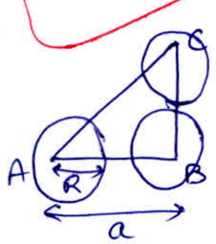
③ Face centred (FCC) : 8 atoms are at each corner & 6 atoms at the centre of each faces.

→ Simple cubic :



Effective number of atoms in a unit cell

$$N = 8 \times \frac{1}{8} = 1$$



$$R = \frac{a}{2}$$

where R : radius of sphere (atom)

a : lattice parameter

$$\text{Volume of sphere (atom)} = \frac{4}{3}\pi R^3$$

$$\text{Volume of unit cell} = a^3$$

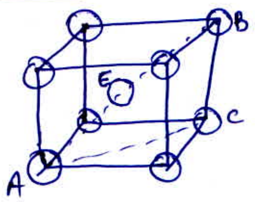
Atomic packing factor, $APF = \dfrac{\text{Sum of atomic volume in unit cell}}{\text{volume in unit cell.}}$

$$\Rightarrow \quad APF = \frac{N \times \frac{4}{3}\pi R^3}{a^3}$$

$$= 1 \times \frac{4}{3}\pi\left(\frac{a/2}{a}\right)^3 = \frac{\pi}{6}$$

$$\boxed{APF = 0.523 \text{ (or) } 52.3\%.}$$

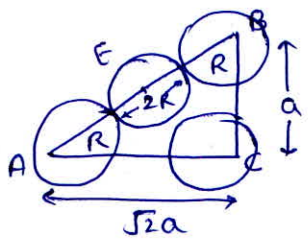
→ **BCC :**



Effective no. of atoms in a cell

$$N = 8 \times \frac{1}{8} + 1 = 2$$

In triangle ABC



$$AB = R + 2R + R = 4R$$

$$BC = a$$

$$AC = \sqrt{2}\,a$$

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow \quad 16R^2 = a^2 + 2a^2$$

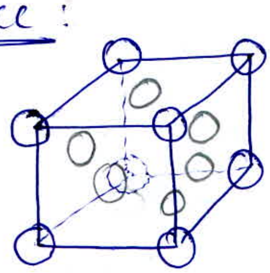
$$\boxed{R = \frac{\sqrt{3}}{4}a}$$

$$APF = \frac{N \times \frac{4}{3}\pi R^3}{a^3} = 2 \times \frac{4}{3}\pi\left(\frac{\sqrt{3}/4\, a}{a}\right)^3$$

$$= 2 \times \frac{4}{3}\pi \times \frac{3\sqrt{3}}{64}$$

BCC, $\boxed{APF = 0.68 \text{ (or) } 68\%.}$

→ **FCC :**

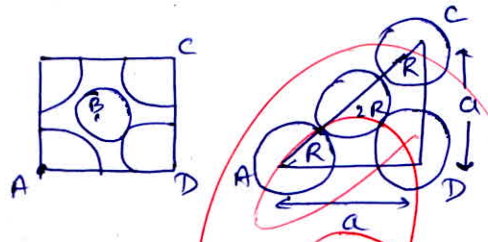


Effective no. of atoms in a unit cell

$$N = 8 \times \frac{1}{8} + 6 \times \frac{1}{2}$$

$$= 4$$

Front view :



In ΔACD

$AC = 4R$

$AD = a$

$DC = a$

$AC^2 = AD^2 + DC^2$

$(4R)^2 = a^2 + a^2$

$R = \frac{\sqrt{2}a}{4} = \frac{a}{2\sqrt{2}}$

$APF = \frac{N \times \frac{4}{3}\pi R^3}{a^3}$

$= 4 \times \frac{4}{3}\pi \left(\frac{a/2\sqrt{2}}{a}\right)^3$

$= 4 \times \frac{4}{3} \times \pi \times \frac{1}{8 \times 2\sqrt{2}}$

$= \frac{\pi}{3\sqrt{2}}$

For FCC,

APF = 0.74 (or 74%)

Q.3 (c) Electron drift mobility in indium (In) has been measured to be 6 $cm^2 V^{-1} s^{-1}$. At room temperature (27°C), the resistivity of In is 8.37×10^{-8} Ω m and its atomic mass and density are 114.82 $g mol^{-1}$ and 7.31 $g cm^{-3}$ respectively.

(i) Based on the resistivity value, determine the effective number of free electrons donated by each In atom in the crystal.

(ii) If the mean speed of conduction electrons in In is 1.74×10^8 $cm s^{-1}$, what is the mean free path?

(iii) Calculate the thermal conductivity of In at room temperature.

[20 marks]

Soln.

Given

mobility, $\mu_{n} = 6\ cm^2/V\text{-}sec = 6\times10^{-4}\ m^2/V\text{-}sec$

Temperature, $T = 300\ °K$

resistivity, $\rho = 8.37\times10^{-8}\ \Omega\text{-}m$

(i) we know that,

resistivity, $\rho = \frac{1}{ne\mu}$

$\Rightarrow \quad n = \frac{1}{\rho e \mu}$

effective number of free e^- donated by each In atom in crystal

$$n = \frac{1}{8.37\times10^{-8}\times1.6\times10^{-19}\times6\times10^{-4}}$$

$$\boxed{n = 1.244\times10^{29}} \text{ electrons}/m^3$$

(ii) mean speed of conduction electrons, $\bar{c} = 1.74\times10^{8}\ cm/sec$

$\bar{c} = 1.74\times10^{6}\ m/sec$

Also, Relaxation time of electron, $\tau = \frac{m\mu}{e}$

$$\tau = \frac{9.1\times10^{-31}\times6\times10^{-4}}{1.6\times10^{-19}}$$

$$\tau = 3.4125\times10^{-15}\ sec$$

∴ mean free path,

$$\lambda = \bar{c}.\tau$$

$$= 1.74\times10^{6}\times3.4125\times10^{-15}\ m$$

$$= 5.938\times10^{-9}\ m$$

$\Rightarrow$ $\boxed{\lambda = 5.938 \text{ nm}}$

(iii) Thermal conductivity of In at room temp.

$$K = L \sigma T$$

where L = Lorentz number $= 2.44 \times 10^{-8}$ $(V/K)^2$

σ = conductivity $= \frac{1}{\rho}$

$= \frac{1}{8.37 \times 10^{-8}}$ $\mho/m$

$T = 300°K$

$\Rightarrow$

$$K = 2.44 \times 10^{-8} \times \frac{1}{8.37 \times 10^{-8}} \times 300$$

$\boxed{K = 87.455}$ $J/m^2 sec$

6

3

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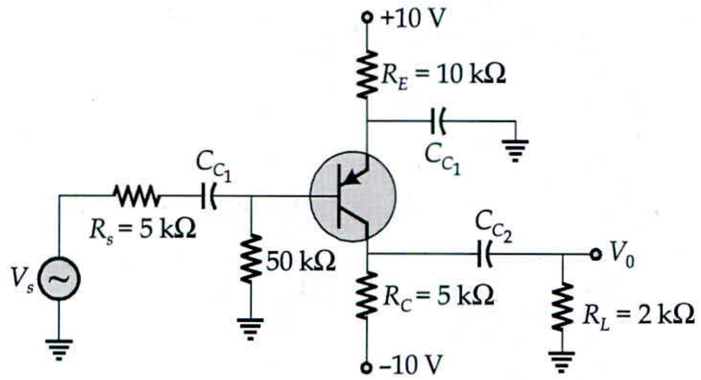
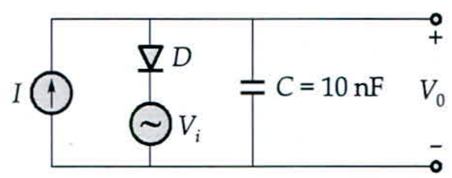
Q.4 (a) Consider a *p-n-p* transistor shown in the figure below. The transistor has $V_{EB(on)} = 0.7$ V, $\beta = 150$ and $V_A = \infty$. Draw a neat and labelled graph for DC and AC load line. Mark the *Q*-point on the graph.



[20 marks]

Q.4 (b) Consider the circuit shown in the figure below:



I is DC current and V_i is a sinusoidal signal with small amplitude and frequency of 100 kHz. Thus for small signal input and output voltages V_i and V_0, calculate:

(i) Phase angle difference between V_i and V_0.

(ii) The value of DC current I for which the phase shift between V_i and V_0 is –45°. (Assume V_T = 25 mV)

(iii) The range of phase shift that is achieved as I is varied over the range of 0.1 to 10 times of the value obtained in part (ii).

[20 marks]

Q.4 (c) (i) What do you understand by magnetic hysteresis? Differentiate between hard and soft magnetic materials?

(ii) In a magnetic material, the field strength is found to be 10^6 A/m. If the magnetic susceptibility of the material is 0.5×10^{-5}, calculate the intensity of the magnetization and the magnetic flux density in the material.

[12 + 8 marks]

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Section B : Electronic Devices & Circuits-1 + Advanced Electronics Topics-1 + Analog and Digital Communication Systems-2

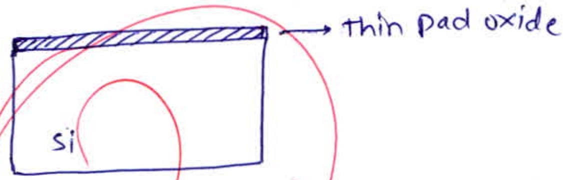
.5 (a) With neat diagrams, explain the Local Oxidation of Silicon (LOCOS) isolation technique used in IC fabrication.

[12 marks]

Sol: LOCOS : local oxidation of silicon.

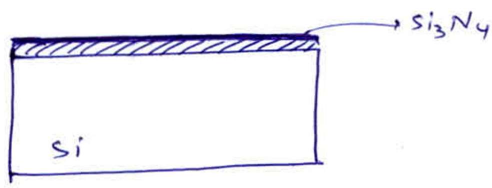
- used for growing SiO_2
- used in IC fabrication for ~~ga~~ oxide isolation

① Take the substrate & grow ~~th~~ thin pad oxide :

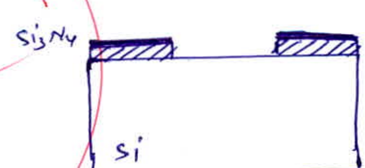


Grow a thin pad oxide , to reduce the stren developed by deposition of Si_3N_4.

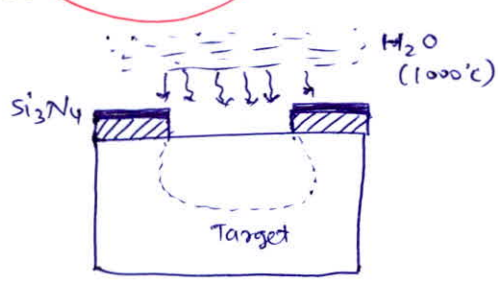
② Deposition of Si_3N_4 :



③ Patterning of Si_3N_4 using lithography : To open window for target area.

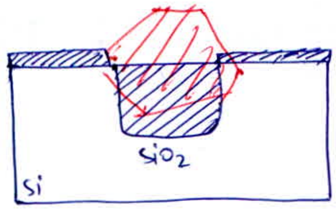


④ Thermal oxidation :



- Si_3N_4 acts a mask for SiO_2

(5)



• LOCOS process has problem of ~~beak~~ bird beak formation during the thermal oxidation step.

Q.5 (b) **(i)** The oxide removal rate and the removal rate of a layer underneath the oxide (called a stop layer) are r and $0.1r$ respectively. To remove 1 μm of oxide and a 0.01 μm stop layer, the total removal time is 5.5 minutes. Find the oxide removal rate (r).

(ii) Calculate the Al average etch rate and etch rate uniformity on a 200 mm diameter silicon wafer, assuming the etch rates at the center, left, right, top and bottom of the wafer are 750, 812, 765, 743 and 798 nm/min respectively.

[6 + 6 marks]

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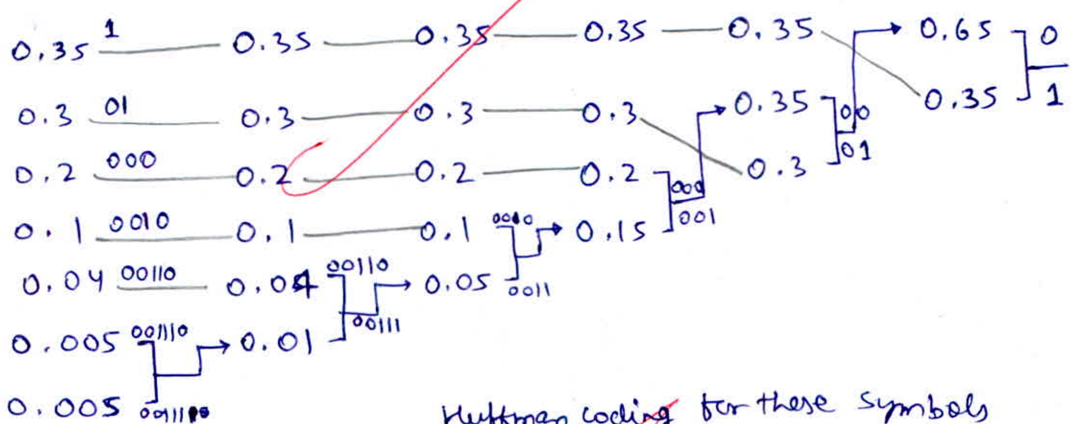
Q.5 (c) A source emits seven symbols with probabilities 0.35, 0.3, 0.2, 0.1, 0.04, 0.005, 0.005. Give Huffman coding for these symbols and calculate average bits of information and average binary digits of information per symbol.

[12 marks]

Sol:

- Arrange the given probabilities in decending order.



Huffman coding for these symbols

Probability (P_i)	code	Code length (n_i)	$n_i P_i$
0.35	1	1	0.35
0.3	01	2	0.6
0.2	000	3	0.6
0.1	0010	4	0.4
0.04	00110	5	0.20
0.005	001110	6	0.030
0.005	001111	6	0.030
		$\sum n_i P_i =$	2.21

Average bit of information

$$H = \sum_{i=} P_i \log_2 \frac{1}{P_i} \quad \text{bits/symbol}$$

$$= 0.35 \log_2 \frac{1}{0.35} + 0.3 \log_2 \frac{1}{0.3} + 0.2 \log_2 \frac{1}{0.2} + 0.1 \log_2 \frac{1}{0.1}$$

$$+ 0.04 \log_2 \frac{1}{0.04} + 2 \times 0.005 \log_2 \frac{1}{0.005}$$

$$= 2.109$$

$$\boxed{H = 2.11 \text{ bits/symbol}}$$

avg. binary digits of information per symbol

$$L = \sum n_i P_i$$

$$\boxed{L = 2.21 \text{ bits/symbol}}$$

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Q.5 (d) The distribution (with respect to energy) of electron concentration in the conduction band is given by density of allowed quantum states times the probability that state being occupied by an electron. i.e., $n(E) = g_C(E)\,f(E)$
where, $g_C(E)$ = Density of allowed states, $f(E)$ = probability of state being occupied.
Assuming that Boltzmann approximation in a semiconductor is valid, calculate the ratio of $n(E)$ at $E = E_C + 4kT$ to that at $E = E_C + (kT/2)$. Here, k = Boltzmann constant, E_C = edge of the conduction band and T = temperature in °K.

[12 marks]

Soln:-

Density of allowed states, $g_c(E) = 4\pi\left(\frac{m_e T}{h}\right)^{3/2}(1.6\times10^{-19})^{3/2}(E-E_c)^{1/2}$

$$g_c(E) = A.\,(E-E_c)^{1/2}$$

& fermi distribution function, $f(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$

for $E-E_F >> 3kT$; $f(E) \simeq e^{-\frac{(E-E_F)}{kT}}$

at $E = E_c+4kT$

$$n_1(E) = A.\,(E-E_c)^{1/2}.\,e^{-\frac{(E-E_F)}{kT}}$$

$$= A.\,(E_c+4kT-E_c)^{1/2}.\,e^{-\frac{(E_c+4kT-E_F)}{kT}}$$

$$n_1(E) = A\,(4kT)^{1/2}\,e^{-\left(\frac{E_c+4kT-E_F}{kT}\right)}$$

$$= A\,(4kT)^{1/2}\,e^{-\frac{(E_c-E_F)}{kT}}.\,e^{-4} \quad —(1)$$

at $E = E_c + \frac{kT}{2}$

$$n_2(E) = A\left(E_c + \frac{kT}{2} - E_c\right)^{1/2} e^{-\frac{\left(E_c + \frac{kT}{2} - E_F\right)}{kT}}$$

$$= A\left(\frac{kT}{2}\right)^{1/2} e^{-\frac{(E_c - E_F)}{kT}} \cdot e^{-1/2} \quad \text{--- (2)}$$

from (1) & (2)

$$\frac{n_1(E)}{n_2(E)} = \frac{A(4kT)^{1/2} e^{-\frac{(E_c-E_F)}{kT}} \cdot e^{-4}}{A\left(\frac{kT}{2}\right)^{1/2} e^{-\frac{(E_c-E_F)}{kT}} \cdot e^{-1/2}}$$

$$= 2\sqrt{2}\, e^{-4+1/2}$$

$$\boxed{\frac{n_1(E)}{n_2(E)} = 0.0854}$$

ratio of state being occupied by an electron in two energy levels.

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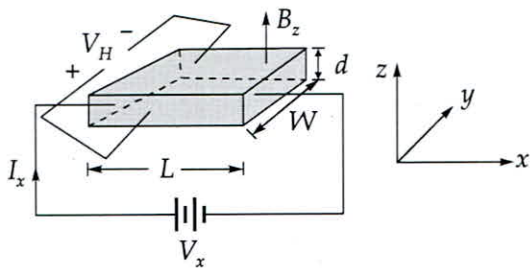
5 (e) Consider a silicon Hall effect device which is used for the experiment as shown below:



The device has dimensions $d = 5 \times 10^{-3}$ cm, $W = 5 \times 10^{-2}$ cm and $L = 0.5$ cm. The electrical parameters measured as the result of the experiment are $I_x = 0.5$ mA, $V_x = 1.25$ V and $B_z = 6.5 \times 10^{-2}$ T. If the induced Hall electric field is $E_{Hy} = -16.5$ mV/cm, then determine:

(i) Hall voltage (V_H)

(ii) The type of semiconductor

(iii) The majority carrier concentration

[12 marks]

Sol:

(i) Hall electric field, $E_{Hy} = -16.5\text{mV/cm}$

Hall voltage $(V_H) = |E_{Hy} \times W|$

$= 16.5 \times 10^{-3} \times 5 \times 10^{-2}$

$= 82.5 \times 10^{-5}$ V

$\boxed{V_H = 8.25\ \mu V}$

(ii) As we know that, hall coefficient

$$R_H = \frac{E_H}{J_x \cdot B_z}$$

$$J_x = \frac{I_x}{A} = \frac{I_x}{dW} = \frac{0.5 \times 10^{-3}}{5\times10^{-3} \times 5\times10^{-2}} = 2\ A/cm^2$$

$$\therefore R_H = \frac{-16.5\times10^{-3}\times10^{-2}}{2 \times 6.5\times10^{-2}} = -0.1269\times10^{-2}\ m^3/A\text{-sec}$$

−ve sign indicate n type semiconductor.

(iii) $$\sigma = \frac{J_x}{E_x} = \frac{I_x \times L}{W d \times V_x} = \frac{0.5\times10^{-3}\times0.5}{5\times10^{-2}\times5\times10^{-3}\times1.25}$$

$= 0.8\ ℧/cm = 0.8\times10^{2}\ ℧/m$

majority carrier concentration, $$n = \frac{1}{eR_H}$$

$$= \frac{1}{1.6\times10^{-19}\times 0.1269\times10^{-2}}\ /m^3$$

$\boxed{n = 4.925 \times 10^{21}\ /m^3}$

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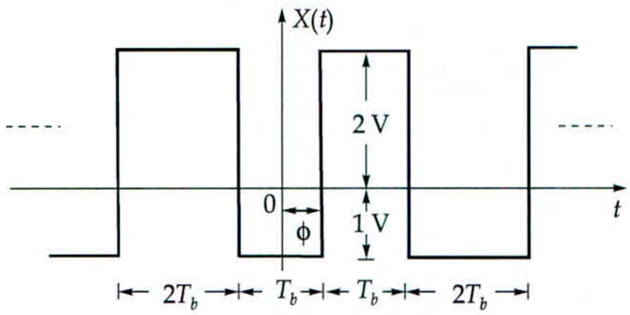
Q.6 (a) Consider the random binary wave shown below:



In this binary wave, logic-1 is represented with positive rectangular pulse and logic-0 is represented with negative rectangular pulse, both with different amplitudes. ϕ is an independent random variable uniformly distributed in the range $[0, T_b]$, where T_b is the bit duration. Determine and sketch the auto-correlation function of $X(t)$. Assume that logic-1 and logic-0 are occurring with equal probability.

[20 marks]

Do not write in this margin

Q.6 (b) A 1 cm long bar of n-type Ge has a cross section of 1 mm × 1 mm. The resistivity of material is 20 Ω-cm and the lifetime of the carriers is 100 microseconds.

(Assume μ_n = 3800 cm²/V-s, μ_p = 1800 cm²/V-s and intrinsic carrier concentration $n_i = 2.5 \times 10^{13}/\text{cm}^3$).

(i) Calculate the resistance of the bar.

(ii) Calculate the donor concentration.

(iii) Calculate the resistance of the bar when it is illuminated such that excess electron-hole pairs are generated at a rate of 10^{15} cm^{-3} s^{-1}, uniformly all over the bar.

[20 marks]

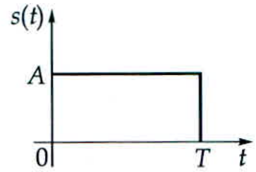
5 (c) (i) Binary data (equiprobable bits) with a rate of 1 Mbps is transmitted through an AWGN channel using different modulation schemes. The two sided power spectral density of the channel noise is 0.5×10^{-11} W/Hz and the carrier signal used in the transmitters is $5\cos(2\pi f_c t)$ mV. In each case of different modulation schemes, the signals are received by their respective correlator receivers with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.

(ii) Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t)n(t)\,dt \text{ and } n_2 = \int_0^T s_2(t)n(t)\,dt$$

Prove that n_1 and n_2 are orthogonal. **[15 + 5 marks]**

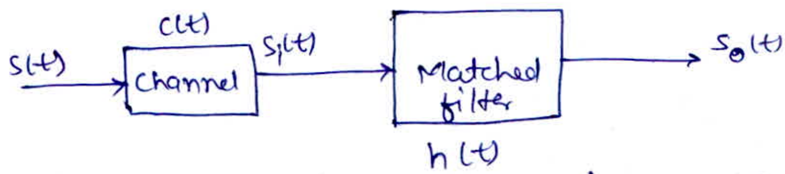
Q.7 (a) Consider the signal shown in the figure below:



This signal is passed through a channel and applied to a filter matched to the signal $s(t)$ at the receiving end. If the channel is not ideal, but has an impulse response $c(t) = \delta(t) + \frac{1}{2}\delta\left(t - \frac{T}{2}\right)$, then determine and sketch the output of the matched filter.

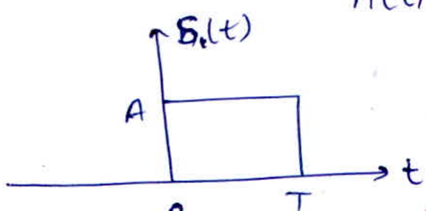
[20 marks]

Soln:



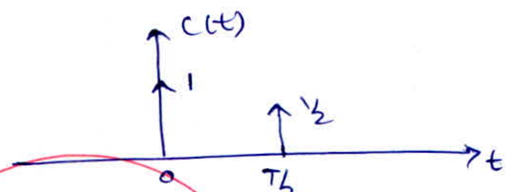
we know that, matched filter impulse response

$h(t) = s_i(T_1 - t)$ = ~~s(t)~~



channel impulse response.

$c(t) = \delta(t) + \frac{1}{2}\delta(t - T/2)$



$\therefore \quad s_i(t) = s(t) * c(t)$

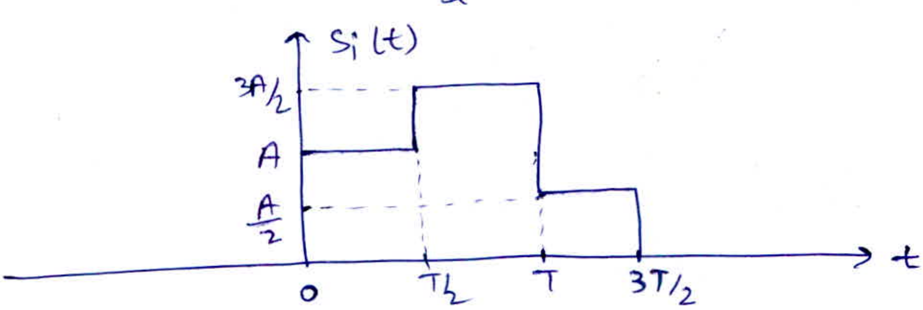
$= s(t) * \left(\delta(t) + \frac{1}{2}\delta(t - T/2)\right)$

$= s(t) * \delta(t) + \frac{1}{2} s(t) * \delta(t - T/2)$

$= s(t) + \frac{1}{2} s(t - T/2)$

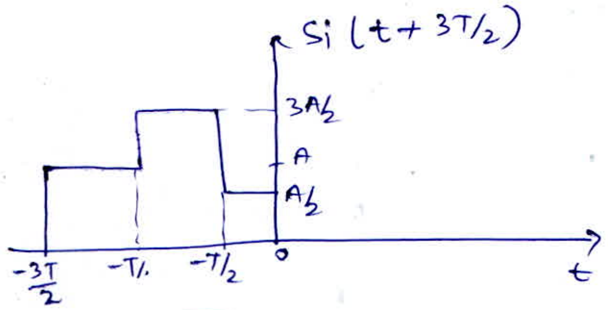
$\{\because$ ~~δ(t)*~~ $\delta(t - t_0) * x(t) = x(t - t_0)\}$

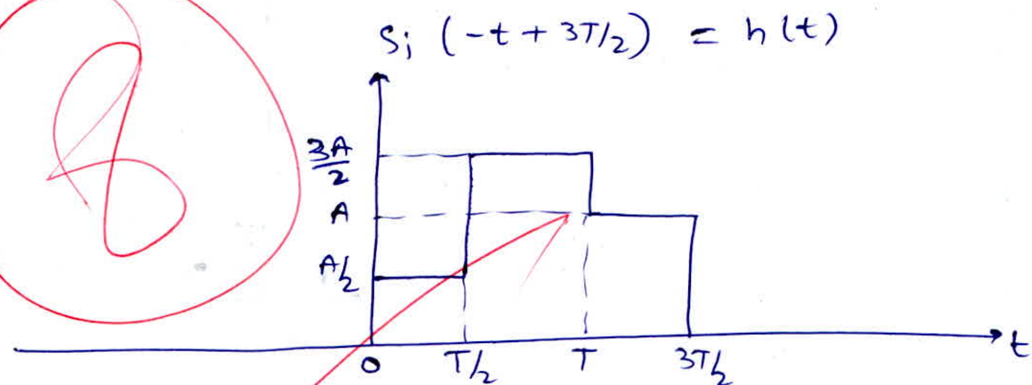
$\Rightarrow \quad s_i(t) = s(t) + \frac{1}{2} s(t - T/2)$



$\therefore \quad h(t) = s_i(T_1 - t)$

where $\quad T_1 = \frac{3T}{2}$





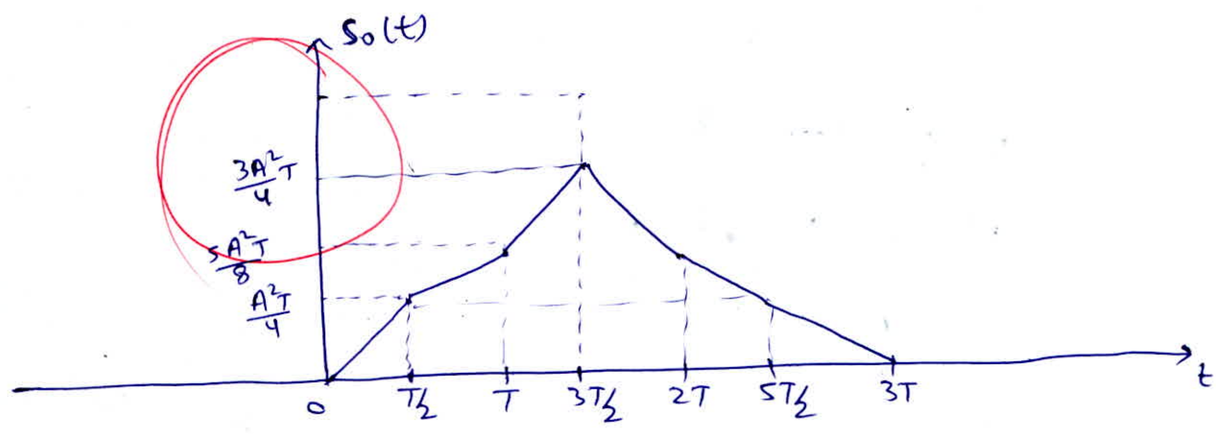
$\therefore$ output of matched filter, $s_o(t) = s_i(t) * h(t)$

$$s_i(t) = A\,u(t) + \frac{A}{2}u(t - T/2) - A\,u(t - T) - A/2\, u(t - 3T/2)$$

$$h(t) = A/2\, u(t) + A\,u(t - T/2) - \frac{A}{2}u(t - T) - A\,u(t - \tfrac{3T}{2})$$

$$\therefore \quad s_o(t) = \left[A u(t) + \frac{A}{2}u(t-T/2) - A u(t-T) - A/2\, u(t-3T/2)\right] * \left[A/2\, u(t) + A u(t-T/2) - A/2\, u(t-T) - A u(t-3T/2)\right]$$

$$= \frac{A^2}{2} r(t) + \frac{A^2}{4} r(t - T/2) - \frac{A^2}{2} r(t-T) - \frac{3A^2}{2} r(t - 3T/2) - \frac{A^2}{2} r(t - 2T) + \frac{5A^2}{4} r(t - 5T/2) + \frac{A^2}{2} r(t - 3T)$$



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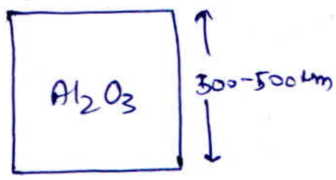
Q.7 (b) Explain the basic steps involved in the fabrication of a CMOS transistor using silicon on sapphire (SOS) process.

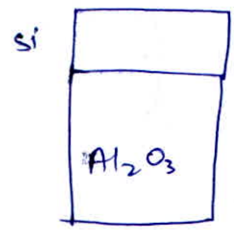
[20 marks]

Soln:

(i) Take sapphire (Al_2O_3)



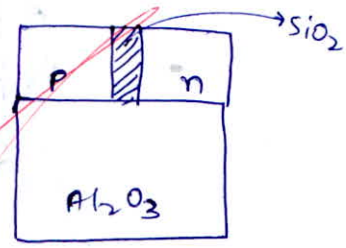
(ii) ~~Perform~~ silicon ~~of~~ epitaxy



(iii) ~~Gate~~ oxide isolation using LOCOS

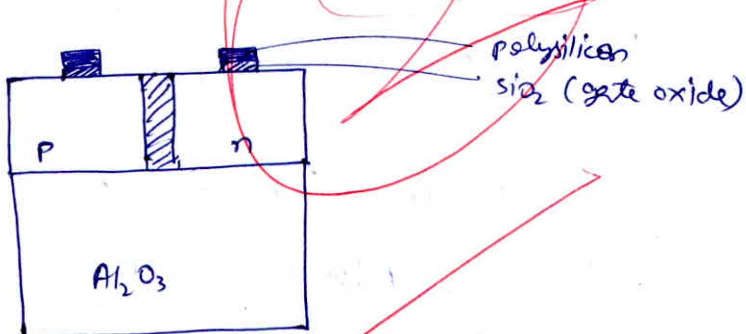
(iv) Formation of p-well. (diffusion)

(v) Formation of n-well (diffusion)

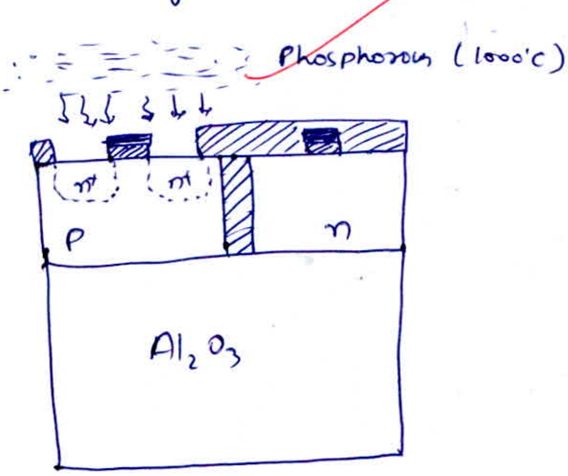


~~(vi) open window for NMOS – source, drain & gate.~~

(vii) Formation of gate & gate oxide for both PMOS & NMOS.



(viii) ~~Formation~~ of source & drain diffusion of NMOS.



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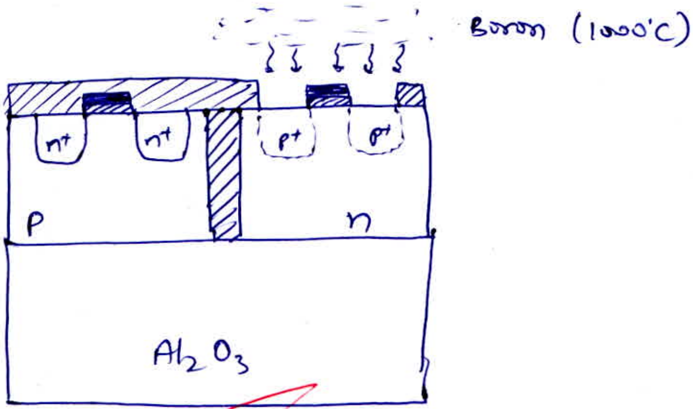
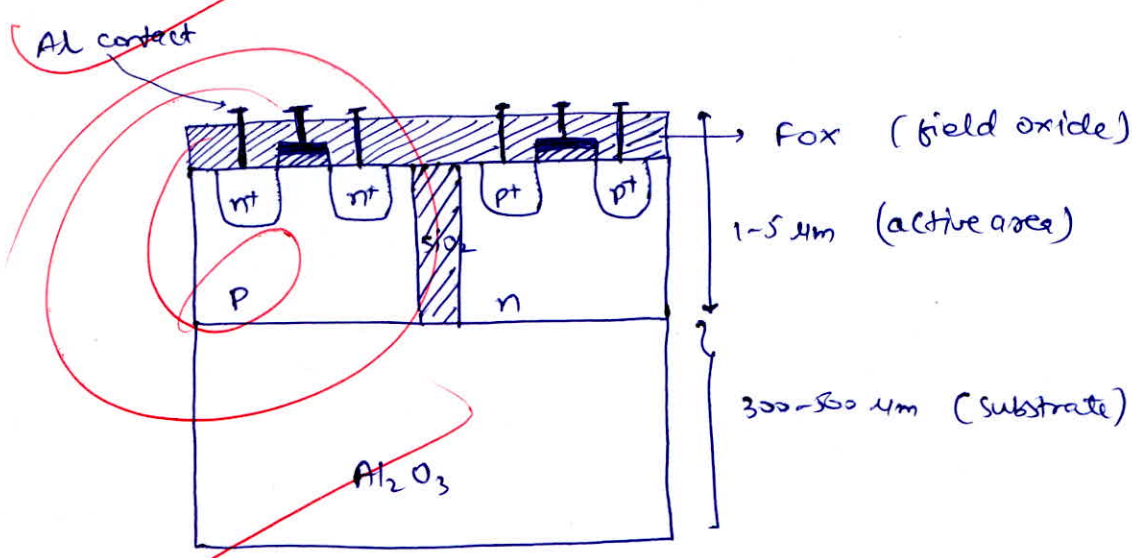
(ix) source & drain diffusion of PMOS:

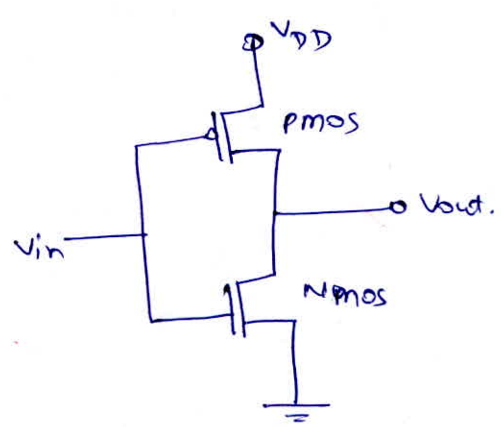


(x) Metallisation :



Fox : to avoid the connection between aluminium contacts.

CMOS:



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Q.7 (c) A p-type lightly doped semiconductor has electron mobility μ_n, hole mobility μ_p, intrinsic carrier concentration n_i and the acceptor impurity concentration N_A.

(i) Derive an expression for the hole concentration 'p' in terms of n_i, μ_n and μ_p, such that the conductivity of the semiconductor is minimum.

(ii) Derive an expression for the minimum conductivity of the semiconductor.

(iii) If $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_n = 1300$ cm^2/V-sec and $\mu_p = 500$ cm^2/V-sec, then calculate the value of minimum conductivity.

(iv) If there is 100% ionization of doping atoms, then calculate the value of acceptor impurity concentration (N_A).

[20 marks]

Soln:

(i) conductivity, $\sigma_i = (n\mu_n + p\mu_p)q$

by mass action law, $np = n_i^2 \Rightarrow n = \frac{n_i^2}{p}$

$$\Rightarrow \quad \sigma_i = \left(n_i^2 \frac{\mu_n}{p} + p\mu_p\right)q.$$

differentiate above eq, w.r.t. p.

$$\frac{d\sigma_i}{dp} = \left(-\frac{\mu_n n_i^2}{p^2} + \mu_p\right)q$$

for minimum conductivity, $\frac{d\sigma_i}{dp} = 0$

$$\Rightarrow \quad \frac{\mu_n n_i^2}{p^2} = \mu_p \Rightarrow p^2 = n_i^2 \frac{\mu_n}{\mu_p}$$

$$\boxed{p = n_i\sqrt{\frac{\mu_n}{\mu_p}}}$$

∴ For minimum conductivity, hole concentration, $\boxed{p = n_i \sqrt{\frac{\mu_n}{\mu_p}}}$

(ii) Put the value of p in σ_i expression, we get

$$\sigma_{i\,min} = \left[\frac{n_i^2 \mu_n}{n_i \sqrt{\frac{\mu_n}{\mu_p}}} + n_i \sqrt{\frac{\mu_n}{\mu_p}}\, \mu_p\right] q$$

$$\boxed{\sigma_{i\,min} = 2 n_i q \sqrt{\mu_n \mu_p}}$$

(iii) $n_i = 1.5 \times 10^{10}\ cm^{-3}$

$\mu_n = 1300\ cm^2/V\text{-}sec$

& $\mu_p = 500\ cm^2/V\text{-}sec$

⇒ $\sigma_{i\,min} = 2 \times 1.5 \times 10^{10} \times 1.6 \times 10^{-19}\left[\sqrt{1300 \times 500}\right]$

$$\boxed{\sigma_{i\,min} = 3.869 \times 10^{-6}\ \mho/cm}$$

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(iv) using charge balance equation,

$N_A + n = N_D + p$

→ p-type, $N_D = 0$,

$N_A + n = p$

At minimum conductivity, $p = n_i \sqrt{\frac{\mu_n}{\mu_p}}$

& $n = \frac{n_i^2}{p} = n_i \sqrt{\frac{\mu_p}{\mu_n}}$

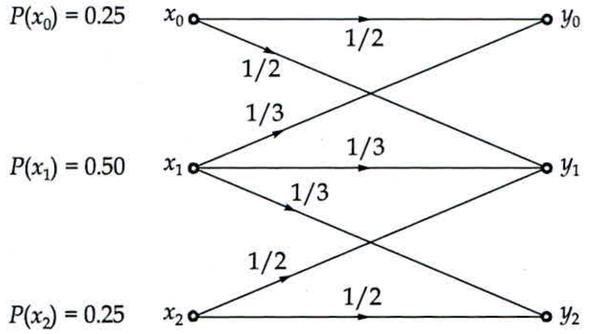
⇒ $N_A = n_i \sqrt{\frac{\mu_n}{\mu_p}} - n_i \sqrt{\frac{\mu_p}{\mu_n}}$

$= 1.5 \times 10^{10}\left(\sqrt{\frac{1300}{500}} - \sqrt{\frac{500}{1300}}\right)$

$$\boxed{N_A = 1.488 \times 10^{10}\ cm^{-3}}$$

Acceptor impurity concentration at minimum conductivity.

(a) Consider the discrete memoryless channel shown below:



Determine the mutual information $I(X\,;\,Y)$.

[20 marks]

Solⁿ

From the channel diagram:

$$P\left[\frac{Y}{X}\right] = \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_0 & y_1 & y_2 \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array}$$

& $P[X] = [0.25 \quad 0.5 \quad 0.25]$

Joint probability matrix

$$P(X,Y) = P(Y/X) \cdot P(X]_d$$

$$= \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

$$P(X,Y) = \begin{bmatrix} \frac{1}{8} & \frac{1}{8} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

$$\therefore \quad P\left(\frac{X}{Y}\right] = \frac{P(X,Y)}{P(Y]_d} = \begin{bmatrix} 3/7 & 3/10 & 0 \\ 4/7 & 2/5 & 4/7 \\ 0 & 3/10 & 3/7 \end{bmatrix}$$

$$P(Y) = P(Y/X) \cdot P(X]$$

$$= \left[7/24 \quad \frac{5}{12} \quad \frac{7}{24} \right]$$

Do not write in this

→ $H(x) = \sum_i P(x_i) \log_2 \frac{1}{P(x_i)}$

$= 2 \times 0.25 \log_2 \frac{1}{0.25} + 0.5 \log_2 \frac{1}{0.5}$

$= \frac{1}{2} \log_2 4 + \frac{1}{2} \log_2 2 = 1.5$ bits/symbol

1

→ $H\left(\frac{X}{Y}\right) = \sum_{i,j} P(x_i, y_j) \log\left(\frac{1}{P\left(\frac{x_i}{y_j}\right)}\right)$

$= 2 \times \frac{3}{7} \log_2 \frac{1}{3/7} + 2 \times \frac{4}{7} \log_2 \frac{1}{4/7} + 2 \times \frac{3}{10} \log_2 \frac{10}{3}$

$+ \frac{2}{5} \log_2 5/2$

$H(X/Y) = 3.5414$ bits/symbol

mutual information, $I(X;Y) = H(X) - H(X/Y)$

$= 1.5 - 3.5414$

$= -2.0414$ bits

Mutual information never be a negative quantity

Q.8 (b) For a boron diffusion in silicon at 1000°C, the surface concentration is maintained at 10^{19} cm^{-3} and the diffusion time is 1 hour. Assume that the diffusivity (D) of Boron in Silicon at 1000°C is 2×10^{-14} cm^2/s. Determine:

(i) The total number of dopant atoms per unit area of semiconductor.

(ii) The distance of the location from the surface where the dopant concentration reaches 10^{15} cm^{-3}. Assume that $\text{erfc}^{-1}(10^{-4}) = 2.75$.

(iii) The gradient of the diffusion profile at the surface.

(iv) The gradient of the diffusion profile at the distance from the surface obtained in part (ii).

[20 marks]

Solⁿ:

Surface concⁿ, $N_S = 10^{19}\ cm^{-3}$

diffusion time, $t = 1\ hr = 3600\ sec$

Diffusivity, $D = 2 \times 10^{-14}\ cm^2/sec$

(i) Total no. of dopant atoms per unit area of semiconductor

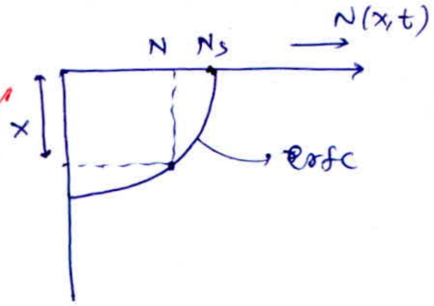
Dose, $S = 2N_S\sqrt{\dfrac{Dt}{\pi}}$

$= 2 \times 10^{19}\sqrt{\dfrac{2 \times 10^{-14} \times 3600}{\pi}}$

$\boxed{\text{Dose} = 9.575 \times 10^{13}\ \text{atoms}/cm^2}$

(ii) ~~Difo~~ Dopant profile,

$N(x,t) = N_S\, \text{erfc}\left(\dfrac{x}{2\sqrt{Dt}}\right)$



at $N = 10^{15}\ cm^{-3}$, $x = ?$

$\Rightarrow 10^{15} = 10^{19}\, \text{erfc}\left(\dfrac{x}{2\sqrt{Dt}}\right)$

$\Rightarrow \dfrac{x}{2\sqrt{Dt}} = \text{erfc}^{-1}(10^{-4}) = 2.75$

$x = 2\sqrt{2 \times 10^{-4} \times 3600} \times 2.75$

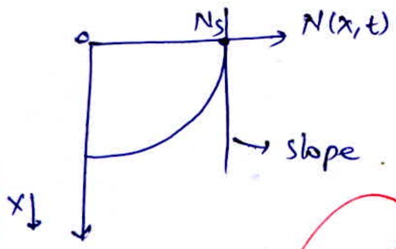
$\boxed{x = 4.667\ cm}$

location from the surface where dopant concⁿ is $10^{15}\ cm^{-3}$

(iii

(iii) Gradient of diffusion profile at surface.

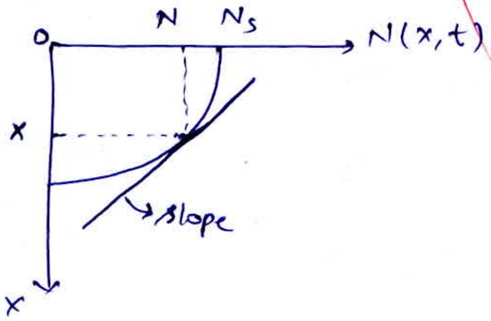
= Slope at surface



= Ns

= 10^{19} cm^{-4}

(iv) Gradient at ~~diff~~ distance $x = 4.667$ cm

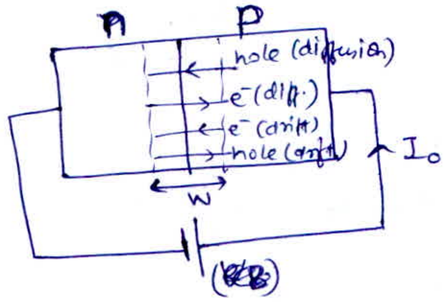


Q.8 (c) **(i)** Find the expression for reverse saturation current I_0 in a *p-n* junction diode in terms of intrinsic carrier concentration n_i.

(ii) Find an expression for the reverse saturation current in terms of the conductivity of the device and prove that, $I_0 = AV_T \frac{b\sigma_i^2}{(1+b)^2}\left[\frac{1}{L_p\sigma_n}+\frac{1}{L_n\sigma_p}\right]$ where, $b = \frac{\mu_n}{\mu_p}$

[20 marks]

Soln: (i)



consider a pn junction diode in forward bias, current through the junction is mainly due to the diffusion.

$$I = I_p(0) + I_n(0)$$

$I_p(0)$ = diffusion current due to hole

$I_n(0)$ = diffusion current due to electron.

As we know that,

$$J_{p\,diffusion}(x) = -D_p q \frac{dP(x)}{dx}$$

where $P(x) = P_{no} + \Delta p\, e^{-x/L_p}$

$$\Rightarrow J_{p\,diffusion}(x) = +D_p q \cdot \Delta p \cdot \frac{1}{L_p} e^{-x/L_p}$$

$$= \frac{D_p q}{L_p} e^{-x/L_p}\left[P_{no}(e^{V/V_T}-1)\right]$$

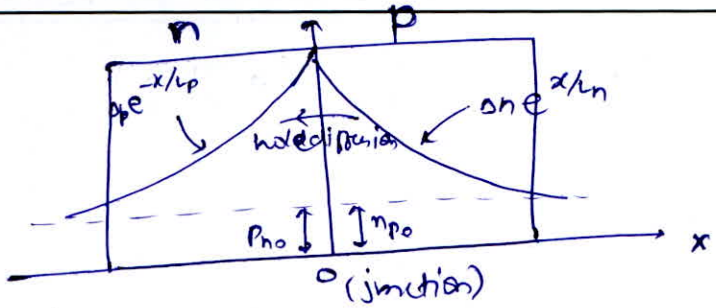
from law of junction, $\Delta p = P_{no}(e^{V/V_T}-1)$

where; P_{no} = hole concⁿ in n-side

V = applied voltage.

at $x = 0$

$$\Rightarrow J_{p\,diffusion}(0) = \frac{D_p q}{L_p} P_{no}(e^{V/V_T}-1) \quad \text{——— (1)}$$



similarly, $J_{n,diffusion}(x) = D_n q \cdot \frac{dn(x)}{dx}$

where $n(x) = n_{p_0} + \Delta n\, e^{+x/L_n}$

$= n_{p_0} + n_{p_0} e^{x/L_n} (e^{V/V_T} - 1)$

$\Rightarrow \quad J_{n,diffusion}(x) = \frac{D_n q}{L_n} \cdot n_{p_0} e^{x/L_n} (e^{V/V_T} - 1)$

at $x = 0$ (junction)

$J_{n,diffusion}(0) = \frac{D_n q}{L_n} \cdot n_{p_0} (e^{V/V_T} - 1)$ — (2)

from (1) & (2),

Total current density at junction ($x = 0$)

$J(0) = J_{p,diffusion}(0) + J_{n,diffusion}(0)$

$J(0) = \frac{D_p q}{L_p} p_{n_0} (e^{V/V_T} - 1) + \frac{D_n q}{L_n} n_{p_0} (e^{V/V_T} - 1)$

→ current, $I = J(0) \times$ Area of junction

$I = \left(\frac{D_p q}{L_p} p_{n_0} + \frac{D_n q}{L_n} n_{p_0}\right)(e^{V/V_T} - 1) \times A$

Now, donor concentration (n-side) $= N_D$

∴ hole concentration on n-side, $p_{n_0} = \frac{n_i^2}{N_D}$

→ Acceptor concentration (p-side) $= N_A$

e^- concentration on p-side, $n_{p_0} = \frac{n_i^2}{N_A}$

where n_i = intrinsic carrier concentration.

Substituting values of p_{n_0} & n_{p_0} in expression of I, we get

$I = A\left(\frac{D_p q}{L_p} \cdot \frac{n_i^2}{N_D} + \frac{D_n q}{L_n} \cdot \frac{n_i^2}{N_A}\right)(e^{V/V_T} - 1)$

$$I = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)\left(e^{V/V_T} - 1\right)$$

$$I = I_0 \left(e^{V/V_T} - 1\right)$$

where I_0 = reverse saturation current

$$\boxed{I_0 = Aq n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)}$$

(ii) → ~~I_0 = Aq q(pμ_p~~

conductivity, $\sigma_i = n_i(\mu_p + \mu_n)q$

$$\Rightarrow n_i = \frac{\sigma_i}{q(\mu_p + \mu_n)} = \frac{\sigma_i}{\mu_p q\left(1 + \frac{\mu_n}{\mu_p}\right)}$$

$$n_i = \frac{\sigma_i}{\mu_p q (1+b)}$$

also, Einstein relation.

$$D_p = V_T \mu_p \quad , D_n = V_T \mu_n$$

$$\Rightarrow I_0 = Aq \left[\frac{\sigma_i}{\mu_p q (1+b)}\right]^2 \left[\frac{V_T \mu_p}{L_p N_D} + \frac{V_T \mu_n}{L_n N_A} \right]$$

$$= Aq \frac{\sigma_i^2}{\mu_p^2 q^2 (1+b)^2} \cdot V_T \mu_p \mu_n \left[\frac{q}{L_p \mu_n N_D \cdot q} + \frac{q}{L_n \mu_p N_A \cdot q} \right]$$

$$= A \frac{\sigma_i^2 V_T}{(1+b)^2} \cdot \frac{\mu_n}{\mu_p} \left[\frac{1}{L_p \sigma_n} + \frac{1}{L_n \sigma_p} \right]$$

$$\boxed{I_0 = \frac{A V_T \sigma_i^2 b}{(1+b)^2} \left[\frac{1}{L_p \sigma_n} + \frac{1}{L_n \sigma_p} \right]}$$

$(\because \sigma_n = \mu_n N_D q \quad , \sigma_p = \mu_p q N_A)$

OOOO

Space for Rough Work

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