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ESE 2019 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electronics & Telecommunication Engineering

Test-5: Analog Circuits + Materials Science
Electronic Devices & Circuits-1 + Advanced Electronics Topics-1
Analog and Digital Communication Systems-2

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Roll No : E C 1 9 M B D L A 0 5 3

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Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. Answer must be written in English only.
3. Use only black/blue pen.
4. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
5. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
6. Last two pages of this booklet are provided for rough work. Strike off these two pages after completion of the examination.

FOR OFFICE USE

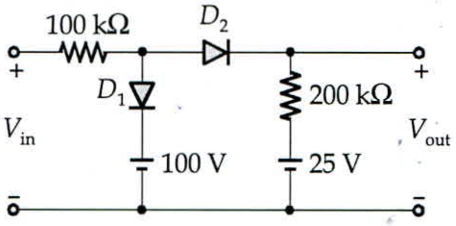
Question No.	Marks Obtained
Section-A	
Q.1	42
Q.2	44
Q.3	36
Q.4	–
Section-B	
Q.5	31
Q.6	–
Q.7	36
Q.8	
Total Marks Obtained	189

Signature of Evaluator

Cross Checked by

Section A : Analog Circuits + Materials Science

(a) Consider the circuit shown in the figure below:



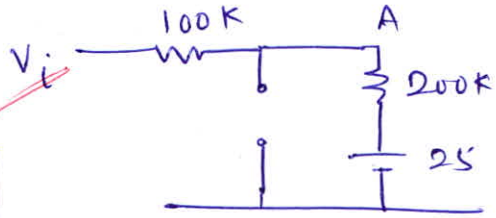
By assuming that the diodes are ideal, develop the transfer characteristic curve of the above circuit.

[12 marks]

उ. let D_1, D_2 off

for $V_{in} > 25$, D_2 ~~off~~ On D_1 OFF

$$V_A = \frac{V_i(200) + 25\times 100}{300}$$



$$\boxed{V_A = \frac{2V_i}{3} + \frac{25}{3} = V_0}$$

for $V_A > 100$ D_1 also on

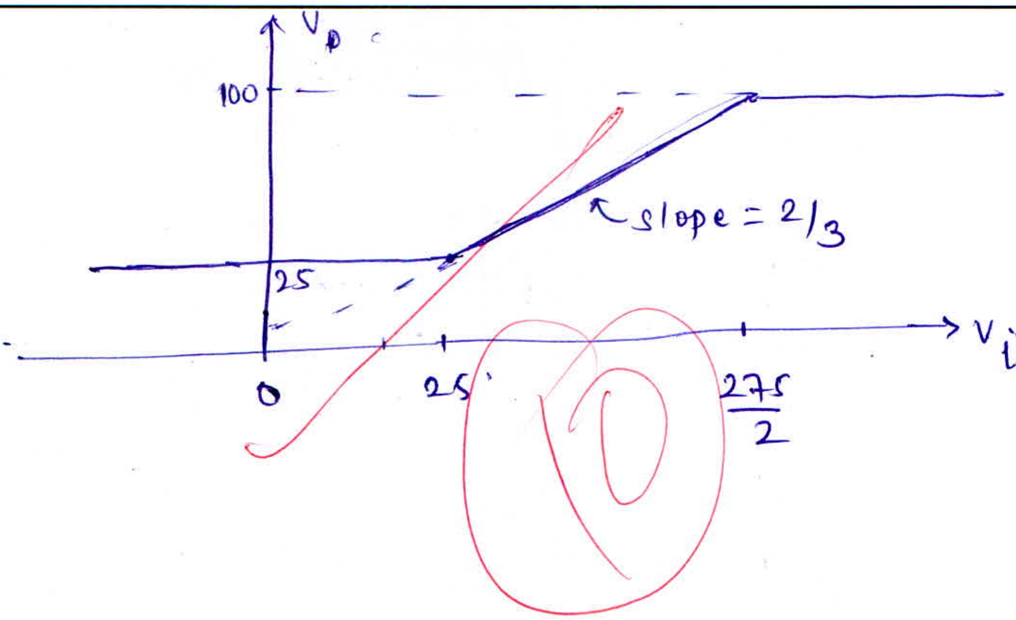
$2V_i + 25 > 300$

$V_i > \frac{275}{2}$

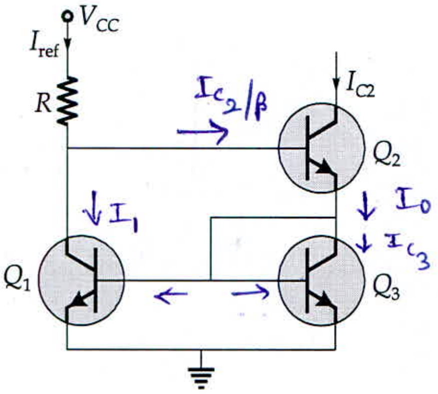
$$\boxed{V_0 = 100 \text{ V}}$$

for $V_i < 25$, both D_1, D_2 off

$$\boxed{V_0 = 25 \text{ V}}$$



Q.1 (b) Consider the Wilson current mirror circuit as shown in the figure below:



Assume that the three transistors to be matched with $V_{BE1} = V_{BE3}$ and $\beta_1 = \beta_2 = \beta_3 = \beta$. Derive an expression for I_{C2} in terms of I_{ref}.

[12 marks]

Sol.

from KVL ⇒

$$V_{cc} = I_{ref} \cdot R + V_{BE_2} + V_{BE_1}$$

$$V_{BE_2} = V_{BE_1} = V_{BE}$$

$$V_{cc} = I_{ref} \cdot R + 2V_{BE} \quad \text{—— ①}$$

Since $I_{C_1} = I_{C_3} = I_1$

$$I_0 = I_1 + \frac{2I_1}{\beta}$$

$$I_1 = \frac{I_0}{1 + 2/\beta} \quad \text{—— ②}$$

since $I_0 = I_{c_2} + \frac{I_{c_2}}{\beta} = I_{c_2}\left(1+\frac{1}{\beta}\right)$

$$I_1 = \frac{I_{c_2}\left(1+\frac{1}{\beta}\right)}{\left(1+\frac{2}{\beta}\right)}$$

Now $I_{ref} = I_1 + \frac{I_{c_2}}{\beta}$

$$I_{ref} = \frac{I_{c_2}\left(1+\frac{1}{\beta}\right)}{\left(1+\frac{2}{\beta}\right)} + \frac{I_{c_2}}{\beta}$$

$$I_{ref} = I_{c_2}\left[\frac{\beta+1}{\beta+2} + \frac{1}{\beta}\right]$$

$$I_{ref} = I_{c_2}\left[\frac{\beta^2+2\beta+2}{\beta(\beta+2)}\right]$$

$$I_{c_2} = I_{ref} \cdot \frac{\beta(\beta+2)}{\beta^2+2\beta+2}$$

10

$$I_{c_2} = \frac{I_{ref}}{1+\frac{2}{\beta(\beta+2)}}$$

Q.1 (c) A long narrow rod (having cubic structure) has an atomic density of 5×10^{28} atoms/m^3. Each atom has a polarizability of 10^{-40} F-m^2. Calculate the internal electric field in the rod when an external axial field of 1 V/m is applied.

[12 marks]

Sol.

atomic density $N = 5 \times 10^{28} \ \frac{\text{atoms}}{m^3}$

$\alpha = 10^{-40}$ F-m^2

$E = 1$ V/m

since $E_i = E + \frac{\gamma P}{\epsilon_0}$

$\gamma = 1/3$ for cubic

$P = N \alpha E_i$

$\Rightarrow \quad E_i = E + \frac{N \alpha E_i}{3\epsilon_0}$

$$E_i = \frac{E}{1 - \frac{N\alpha}{3\epsilon_0}}$$

$$= \frac{1}{1 - \frac{5 \times 10^{28} \times 10^{-40}}{3 \times \epsilon_0}}$$

$$= \frac{1}{0.811} = 1.231 \ \frac{V}{m}.$$

$E_i = 1.231$ V/m

(d) Explain Silsbee's rule for superconductors. Also give some applications of superconductors.

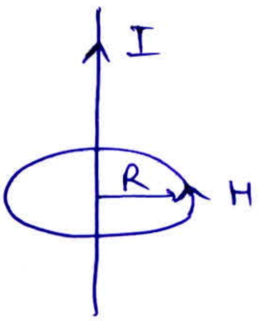
[12 marks]

Silsbee's Rule –

In a current carrying superconducting material, if magnetic field produced by that current is greater than critical magnetic field then superconductivity of the material destroys and it transitions into normal state.

The current density at ~~with~~ which material transitions into normal state from superconducting state, is called critical current density.

from Ampere's law.



$$\oint H\,dl = I$$

$$\boxed{H_c = \frac{I}{2\pi R}}$$

↑

critical Magnetic field.

Critical current

$$I_c = H_c \cdot 2\pi R .$$

$$J_c \cdot \pi R^2 = H_c \cdot 2\pi R$$

$$\boxed{J_c = \frac{2H_c}{R}}$$

Applications of super conductor

① used in MRI.

② Magnetic leviatation transportation

③ in microwave oscillater like cryoton.

④ In transformers and inductors.

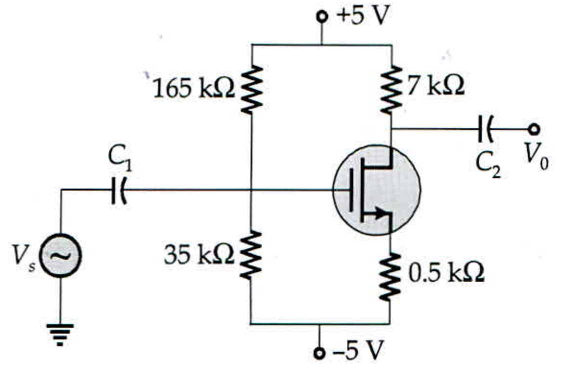
1 (e) Write short notes on the following nanomaterials:

(i) Quantum dots

(ii) Carbon nanotubes

[6 + 6 marks]

(a) **(i)** Consider the common source transistor circuit shown in the figure below:



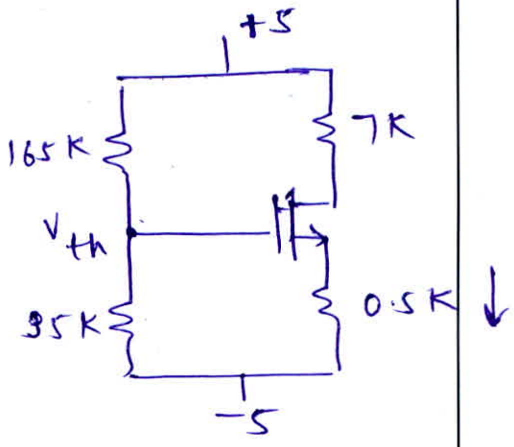
The transistor parameters are $V_{TN} = 0.8$ V, $K_n = \frac{\mu_n C_{ox} W}{2L} = 1\ \text{mA/V}^2$ and $\lambda = 0$.

Calculate the value of small signal voltage gain V_0/V_s of the circuit.

(ii) A differential amplifier has input voltages $V_1 = 1$ mV and $V_2 = 3$ mV. The amplifier has differential gain $A_d = 5 \times 10^3$ and CMRR = 1000. Calculate the output voltage of the amplifier.

[15 + 5 marks]

from DC analysis.



$$V_{th} = \frac{5\times35 - 5\times165}{35+165}$$

$$= -3.25\ \text{V}.$$

$$V_{th} = V_{GS} + I_D \times 0.5 - 5$$

$$-3.25 = V_{GS} + 0.5 I_D - 5$$

$$I_D = \frac{1.75 - V_{GS}}{0.5}\ \text{mA}.$$

since $I_D = \frac{K_n}{2}(V_{GS} - V_T)^2$

$$\frac{1.75 - V_{GS}}{0.5} = \frac{1}{2}(V_{GS} - 0.8)^2$$

$$3.5 - 2V_{GS} = \frac{1}{2}(V_{GS}^2 - 1.6V_{GS} + 0.64)$$

$$V_{GS}^2 + 2.4V_{GS} - 6.36 = 0$$

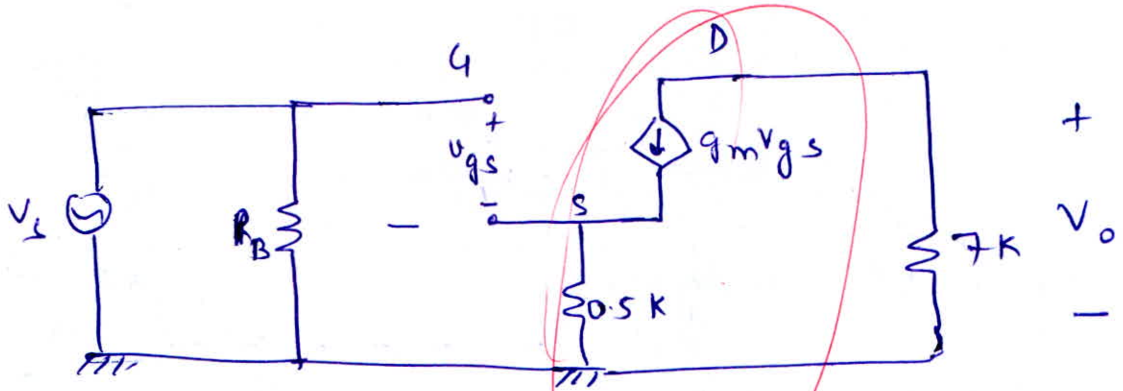
$V_{GS} = 1.59$ V , -3.99 V. X

$g_m = K_n (V_{GS} - V_T)$

$= 1(1.59 - 0.8) = 0.79$ m℧

AC Equi ckt

$R_B = 35 || 165 = 28.875$ KΩ



$$\frac{V_o}{V_s} = \frac{-g_m v_{gs}(7\times10^3)}{v_{gs} + g_m v_{gs}(0.5\times10^3)}$$

$$= \frac{-g_m \times 7\times10^3}{1 + g_m \times 0.5\times10^3}$$

$$= \frac{-0.79\times7}{1 + 0.79\times0.5}$$

$$\frac{V_o}{V_s} = -3.96$$

wrong ckt

(iii)

o/p voltage

$$V_o = A_d(V_2 - V_1) + A_{cm} V_{cm}.$$

$$CMRR = \frac{A_d}{A_{cm}}$$

$$A_{cm} = \frac{A_d}{CMRR} = \frac{5\times10^3}{1000} = 5$$

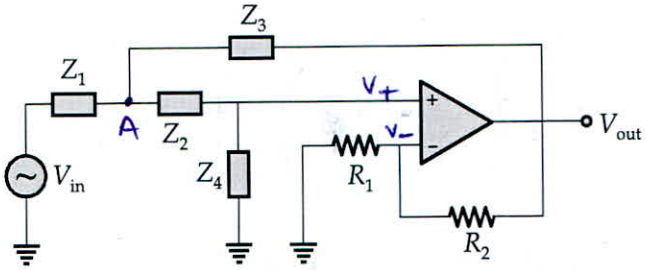
$$V_0 = 5\times10^3 (3-1)\times10^{-3} + 5\times\left(\frac{3+1}{2}\right)\times10^{-3}$$

$$= 5\times2 + 5\times2\times10^{-3}$$

$$= 10 + 0.01$$

$$V_0 = 10.01 \text{ V}.$$

Q.2 (b) Consider the circuit shown in the figure below:



The figure represents a second order active filter system.

(i) Derive an expression for V_{out}/V_{in}.

(ii) If each of the impedance elements Z_1 through Z_4 are replaced by a resistor of value R, then find the value of V_{out}/V_{in}.

[20 marks]

Sol.

$$V_- = V_{out}\cdot\frac{R_1}{R_1+R_2} = V_+ \quad \text{(virtual short)} \quad \text{—①}$$

KCL at node A.

$$\frac{V_A - V_{in}}{Z_1} + \frac{V_A - V_{out}}{Z_3} + \frac{V_A - V_+}{Z_2} = 0 \quad \text{—②}$$

Also $\frac{V_A}{Z_2+Z_4} = \frac{V_+}{Z_4}$

from eq ②

$$V_A\left[\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right] = \frac{V_{in}}{Z_1}+\frac{V_{out}}{Z_3}+\frac{V_+}{Z_2}$$

$$\Rightarrow V_+\left[\frac{Z_2+Z_4}{Z_4}\right]\left[\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right] = \frac{V_{in}}{Z_1}+\frac{V_{out}}{Z_3}+\frac{V_+}{Z_2}$$

$$\Rightarrow V_+\left[\left(1+\frac{Z_2}{Z_4}\right)\left(\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}\right)-\frac{1}{Z_2}\right] = \frac{V_{in}}{Z_1}+\frac{V_{out}}{Z_3}$$

$$V_+\left[\frac{1}{Z_1}+\frac{1}{Z_3}+\frac{Z_2}{Z_4}\left(\frac{1}{Z_1}+\frac{1}{Z_3}\right)+\frac{1}{Z_4}\right] = \frac{V_{in}}{Z_1}+\frac{V_{out}}{Z_3}$$

$$\left(V_{out}\frac{R_1}{R_1+R_2}\right)\left[\left(\frac{1}{Z_1}+\frac{1}{Z_3}\right)\left(1+\frac{Z_2}{Z_4}\right)+\frac{1}{Z_4}\right] - \frac{V_{out}}{Z_3} = \frac{V_{in}}{Z_1}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/Z_1}{\left(\frac{R_1}{R_1+R_2}\right)\left[\left(\frac{1}{Z_1}+\frac{1}{Z_3}\right)\left(1+\frac{Z_2}{Z_4}\right)+\frac{1}{Z_4}\right] - \frac{1}{Z_3}}$$

Now $Z_1 = Z_2 = Z_3 = Z_4 = R$

$$\frac{V_{out}}{V_{in}} = \frac{1/R}{\left(\frac{R_1}{R_1+R_2}\right)\left[\frac{2}{R}\times 2+\frac{1}{R}\right] - \frac{1}{R}}$$

10

$$= \frac{1}{\left(\frac{R_1}{R_1+R_2}\right)5 - 1}$$

$$= \frac{R_1+R_2}{5R_1 - R_1 - R_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{4R_1-R_2}$$

Q.2 (c) **(i)** For a dielectric, establish an expression for the relationship between the polarizability and permittivity. How does this relation lead to Clausius-Mossotti equation?

(ii) When an NaCl crystal is subjected to an electric field of 1000 V/m, the resulting polarization is 4.3×10^{-8} C/m^2. Calculate the relative permittivity of NaCl.

[15 + 5 marks]

Sol. (i) Since $\vec{p} = \alpha \vec{E}$

polarisation $P = N\vec{p}$

$\Rightarrow P = N\alpha\vec{E}$ —— ①

$\Rightarrow \epsilon_0(\epsilon_R - 1)\vec{E} = N\alpha\vec{E}$

$$\Rightarrow \boxed{\alpha = \frac{\epsilon_0(\epsilon_R - 1)}{N}}$$

Since Internal E field

$$E_i = E + \frac{\gamma P}{\epsilon_0}$$

$$\gamma = 1/3$$

$$p = N\alpha E_i \qquad \text{from eq ①}$$

$$p = N\alpha\left[E + \frac{p}{3\epsilon_0}\right]$$

$$p = N\alpha E + \frac{N\alpha}{3\epsilon_0}p$$

$$p\left[1 - \frac{N\alpha}{3\epsilon_0}\right] = N\alpha E$$

$$\frac{p}{E} = \frac{N\alpha}{1 - \frac{N\alpha}{3\epsilon_0}}$$

$$\epsilon_0(\epsilon_R - 1) = \frac{N\alpha}{1 - \frac{N\alpha}{3\epsilon_0}}$$

$$\epsilon_R - 1 = \frac{N\alpha/\epsilon_0}{1 - \frac{N\alpha}{3\epsilon_0}}$$

$$\epsilon_R + 2 = \frac{N\alpha/\epsilon_0}{1 - \frac{N\alpha}{3\epsilon_0}} + 3 = \frac{3}{1 - \frac{N\alpha}{3\epsilon_0}}$$

$$\frac{\epsilon_R - 1}{\epsilon_R + 2} = \frac{N\alpha/\epsilon_0}{1 - \frac{N\alpha}{3\epsilon_0}} \cdot \frac{1 - \frac{N\alpha}{3\epsilon_0}}{3}$$

$$\boxed{\frac{\epsilon_R - 1}{\epsilon_R + 2} = \frac{N\alpha}{3\epsilon_0}}$$

This is CM eq^n.

(ii)

$$E = 1000 \text{ V/m}$$

$$P = 4.3 \times 10^{-8} \text{ C/m}^2$$

$$\epsilon_R = ?$$

since $\boxed{P = \epsilon_0(\epsilon_R - 1)\, E}$

$$4.3 \times 10^{-8} = \epsilon_0(\epsilon_R - 1) \times 1000$$

$$\epsilon_0(\epsilon_R - 1) = 4.3 \times 10^{-11}$$

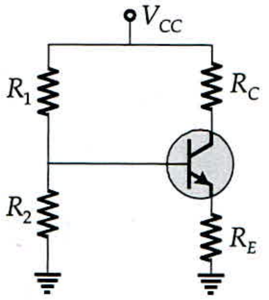
$$\epsilon_R - 1 = \frac{4.3 \times 10^{-11}}{8.85 \times 10^{-12}}$$

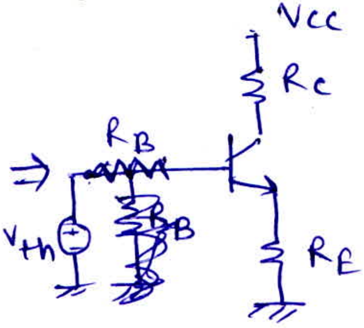
$$\epsilon_R - 1 = 4.856$$

$$\boxed{\epsilon_R = 5.856}$$

3 (a) Consider the voltage divider biasing circuit shown in the figure below:

$R_B = R_1 || R_2$





For this circuit,

(i) Derive an expression for stability factor S [i.e., the variation of I_C w.r.t. I_{CO}].

(ii) Derive an expression for stability factor S' [i.e., the variation of I_C w.r.t. V_{BE}].

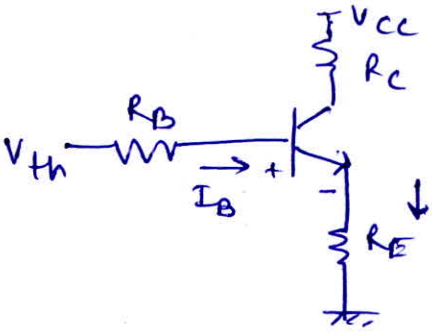
(iii) Derive a relation between S and S'.

[20 marks]

sol

$$V_{th} = R_B I_B + V_{BE} + I_E R_E$$

$$= I_B R_B + V_{BE} + (I_C + I_B) R_E$$



$$V_{th} = I_B (R_B + R_E) + I_C R_E + V_{BE}$$

2

Since $I_C = \beta I_B + I_{CO}$

$$I_B = \left(\frac{I_C - I_{CO}}{\beta}\right)$$

$$V_{th} = \left(\frac{I_C - I_{CO}}{\beta}\right)(R_B + R_E) + I_C R_E + V_{BE} \quad ①$$

diff wrt I_{CO}

$$0 = \left(\frac{R_B + R_E}{\beta}\right)\frac{\partial I_C}{\partial I_{CO}} - \left(\frac{R_B + R_E}{\beta}\right) + \frac{\partial I_C}{\partial I_{CO}} R_E$$

$$\frac{\partial I_C}{\partial I_{CO}} = \frac{\left(\frac{R_B + R_E}{\beta}\right)}{R_E + \frac{R_B + R_E}{\beta}}$$

$$s = \frac{\partial I_c}{\partial I_{co}} = \frac{\frac{R_B + R_E}{\beta}}{R_B + \frac{R_B + R_E}{\beta}}$$

$$S = \frac{1}{1 + \frac{\beta R_E}{R_B + R_E}} \quad \text{— ①}$$

diff. eq ① wrt V_{BE}

$$0 = \frac{R_B + R_E}{\beta} \cdot \frac{\partial I_c}{\partial V_{BE}} + R_E \frac{\partial I_c}{\partial V_{BE}} + 1$$

$$\frac{\partial I_c}{\partial V_{BE}} = \frac{-1}{R_E + \frac{R_B + R_E}{\beta}} = s'$$

$$S' = \frac{-\beta / R_B + R_E}{1 + \frac{\beta R_E}{R_B + R_E}} \quad \text{— ②}$$

~~$S \cdot S' = \frac{1}{1 + \frac{\beta R_E}{R_B + R_E}} \times \frac{-\beta / R_B + R_E}{1 + \frac{\beta R_E}{R_B + R_E}}$~~

$$\frac{S'}{S} = \frac{-\beta}{R_B + R_E}$$

$$\boxed{S' = \frac{-\beta}{R_B + R_B} \cdot S}$$

3 (b) What are the types of cubic crystal structure? Derive the atomic packing factor of all the cubic crystal structures.

[20 marks]

Sol.

Types. –

① simple cubic (SC)

② Body centred cubic (BCC)

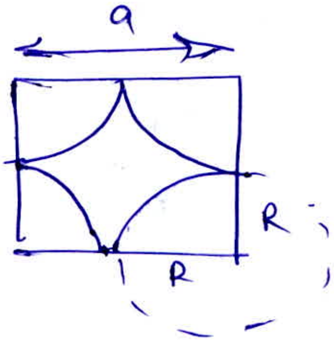
③ FCC (face centred cubic)

① SC →

3

No. of atoms / unit cell

$N = \frac{1}{8} \times 8 = 1$



lattice const

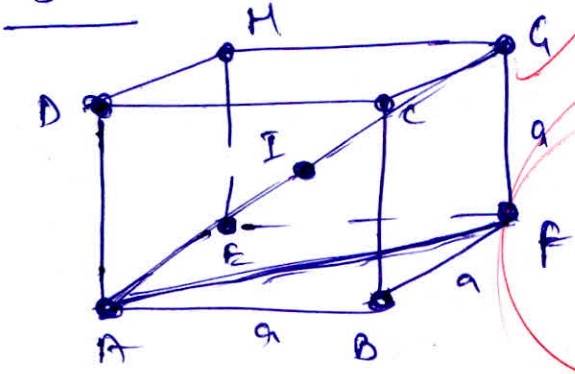
$a = 2R$ (from fig.)

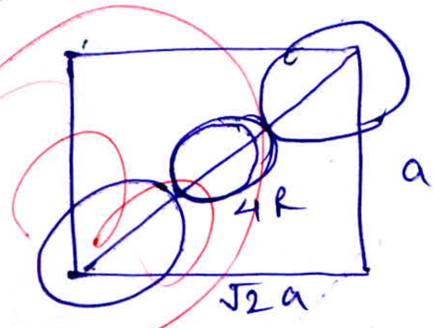
$$APF = \frac{\text{Atomic vol. per unit cell}}{\text{vol. of unit cell}}$$

$$= \frac{N \cdot \frac{4}{3}\pi R^3}{a^3}$$

$$= \frac{1 \times \frac{4}{3}\pi R^3}{(2R)^3} = \frac{\pi}{6} = \underline{0.52}$$

② $\underline{\text{BCC}}$ —





$\underline{\text{In } \Delta AFG}$ ..

$AG = 4R$

$AF = \sqrt{2}\,a$

$(4R)^2 = (\sqrt{2}a)^2 + a^2$

$4R = \sqrt{3}\,a$

$$\boxed{a = \frac{4R}{\sqrt{3}}}$$

No. of atoms / unit cel

$N = \frac{1}{8} \times 8 + 1$

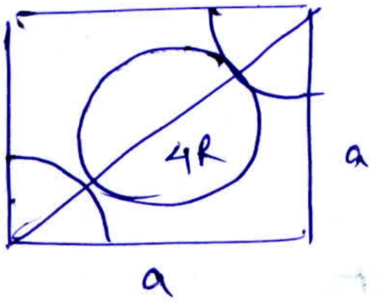
$= 2$

$$APF = \frac{N \times \frac{4}{3}\pi R^3}{a^3}$$

$$APF = \frac{2 \times \frac{4}{3}\pi R^3}{\left(\frac{4R}{\sqrt{3}}\right)^3} = \frac{2 \times \frac{4\pi}{3}}{\frac{64}{3\sqrt{3}}} = \frac{\sqrt{3}\,\pi}{8}$$

$$= \underline{0.68}$$

3) FCC:



$$(4R)^2 = a^2 + a^2$$

$$4R = \sqrt{2}\,a$$

$$a = \frac{4R}{\sqrt{2}} = 2\sqrt{2}R$$

$$N = 8 \times \frac{1}{8} + 3$$

$$= 4$$

$$APF = \frac{N \times \frac{4}{3}\pi R^3}{a^3} = \frac{4 \times \frac{4}{3}\pi R^3}{(2\sqrt{2}R)^3}$$

$$= \frac{\frac{16\pi}{3}}{16\sqrt{2}} = \frac{\pi}{3\sqrt{2}} = \underline{0.74}$$

Q.3 (c) Electron drift mobility in indium (In) has been measured to be 6 cm^2 V^{-1}s^{-1}. At room temperature (27°C), the resistivity of In is 8.37×10^{-8} Ω m and its atomic mass and density are 114.82 g mol^{-1} and 7.31 g cm^{-3} respectively.

(i) Based on the resistivity value, determine the effective number of free electrons donated by each In atom in the crystal.

(ii) If the mean speed of conduction electrons in In is 1.74×10^8 cm s^{-1}, what is the mean free path?

(iii) Calculate the thermal conductivity of In at room temperature.

[20 marks]

Sol.

$\mu = 6 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$ $\rho = 8.37 \times 10^{-8} \ \Omega \text{ m}.$

$= 8.37 \times 10^{-6} \ \Omega\text{-cm}$

density $d = 7.31 \text{ gm cm}^{-3}$

$A = 114.82 \text{ gm mol}^{-1}$

$$d = N \cdot \frac{A}{N_A}$$

$$7.31 = N \times \frac{114.82}{6.023 \times 10^{23}}$$

$$N = 3.83 \times 10^{22} / \text{cm}^3$$

(i)

$$\sigma = ne\mu$$

$$\frac{1}{\rho} = ne\mu$$

$$n = \frac{1}{\rho e \mu}$$

$$= \frac{1}{8.37 \times 10^{-6} \times 1.6 \times 10^{-19} \times 6}$$

$$\boxed{n = 1.24 \times 10^{23} \text{ cm}^{-3}}$$

(ii) $\lambda = v \cdot \tau$

Since $\mu = \frac{e\tau}{m}$

$\Rightarrow \tau = \frac{\mu m}{e}$

2

$\lambda = v \cdot \frac{\mu m}{e}$

$= \frac{1.74 \times 10^{8} \times 6 \times 9.1 \times 10^{-31} \times 10^{3}}{1.6 \times 10^{-19}}$

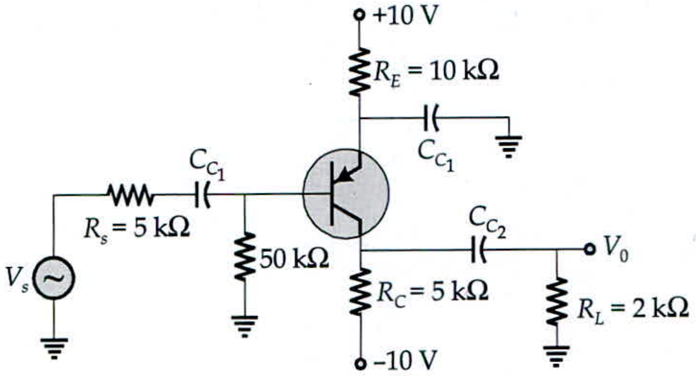
$\underline{\lambda = 5.94 \text{ cm.}}$

(iii) Thermal conductivity

$K = L\sigma T = LT/\rho$

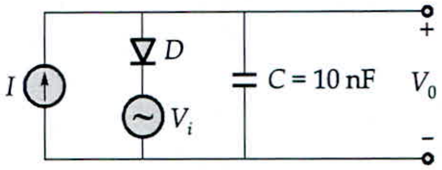
$= L \times 300 \times \frac{1}{8.37 \times 10^{-6}}$

Q.4 (a) Consider a *p-n-p* transistor shown in the figure below. The transistor has $V_{EB(on)} = 0.7$ V, $\beta = 150$ and $V_A = \infty$. Draw a neat and labelled graph for DC and AC load line. Mark the *Q*-point on the graph.



[20 marks]

Q.4 (b) Consider the circuit shown in the figure below:



I is DC current and V_i is a sinusoidal signal with small amplitude and frequency of 100 kHz. Thus for small signal input and output voltages V_i and V_0, calculate:

(i) Phase angle difference between V_i and V_0.

(ii) The value of DC current I for which the phase shift between V_i and V_0 is –45°. (Assume $V_T = 25$ mV)

(iii) The range of phase shift that is achieved as I is varied over the range of 0.1 to 10 times of the value obtained in part (ii).

[20 marks]

.4 (c) **(i)** What do you understand by magnetic hysteresis? Differentiate between hard and soft magnetic materials?

(ii) In a magnetic material, the field strength is found to be 10^6 A/m. If the magnetic susceptibility of the material is 0.5×10^{-5}, calculate the intensity of the magnetization and the magnetic flux density in the material.

[12 + 8 marks]

Section B : Electronic Devices & Circuits-1 + Advanced Electronics Topics-1 + Analog and Digital Communication Systems-2

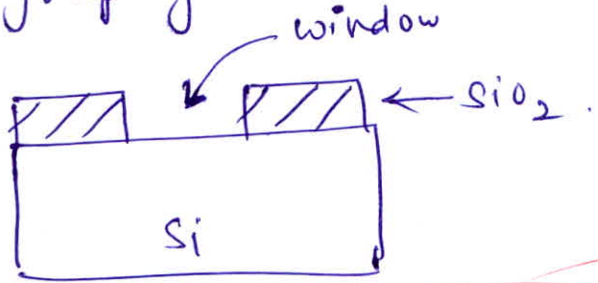
(a) With neat diagrams, explain the Local Oxidation of Silicon (LOCOS) isolation technique used in IC fabrication.

[12 marks]

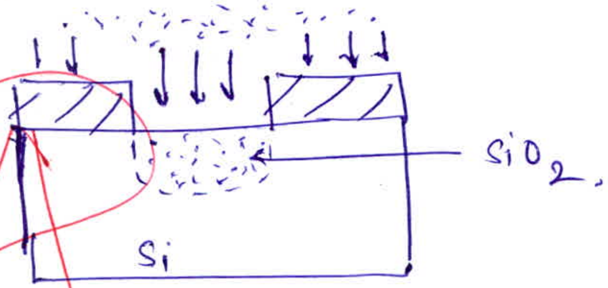
St. LOCOS – Growing local oxide (SiO_2) in a Si wafer using diffusion.

Step 1 – Take a Si wafer.

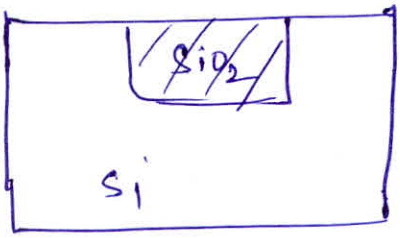
Step 2 Open a window using photolithography.



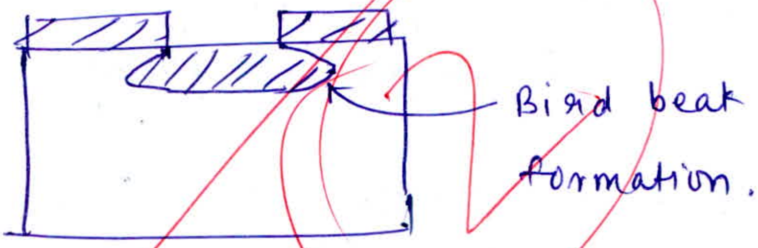
Step 3 – Diffusion of oxygen in the wafer.



Why to diffuse oxygen?



Practically Bird beak formation occurs during locos technique Because durin diffusion oxygen atoms diffuse in all directions in the Wafer.



Q.5 (b) **(i)** The oxide removal rate and the removal rate of a layer underneath the oxide (called a stop layer) are *r* and 0.1*r* respectively. To remove 1 μm of oxide and a 0.01 μm stop layer, the total removal time is 5.5 minutes. Find the oxide removal rate (*r*).

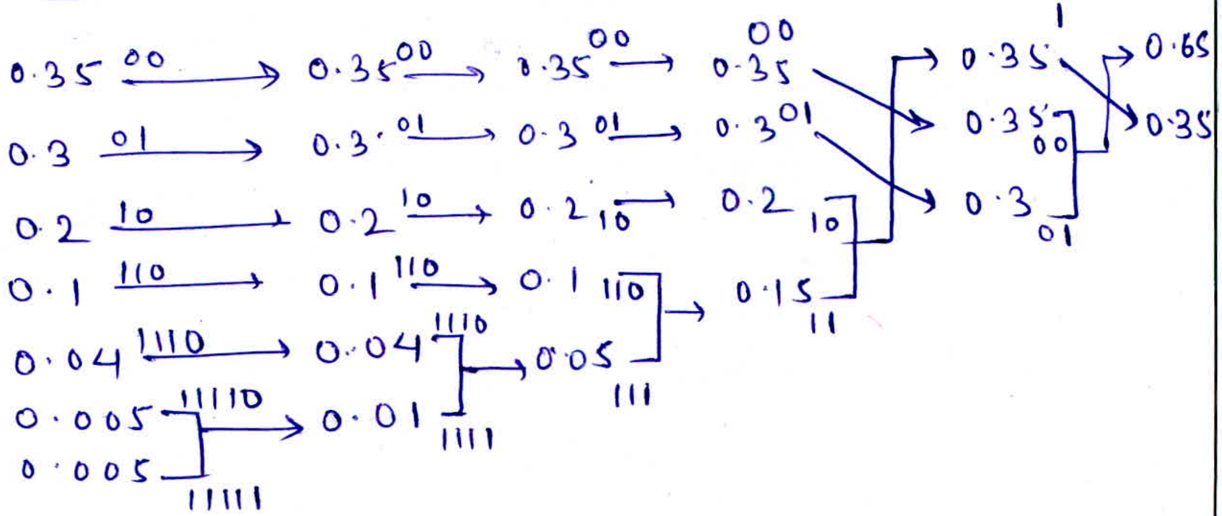
(ii) Calculate the Al average etch rate and etch rate uniformity on a 200 mm diameter silicon wafer, assuming the etch rates at the center, left, right, top and bottom of the wafer are 750, 812, 765, 743 and 798 nm/min respectively.

[6 + 6 marks]

Q.5 (c) A source emits seven symbols with probabilities 0.35, 0.3, 0.2, 0.1, 0.04, 0.005, 0.005. Give Huffman coding for these symbols and calculate average bits of information and average binary digits of information per symbol.

[12 marks]

Sol: Huffman coding.



	~~Code~~ code	code length
0·35 →	00	2
0·3 →	01	2
0·2 →	10	2
0·1 →	110	3
0·04 →	1110	4
0·005 →	11110	5
0·005 →	11111	5

Avg code length = ~~2+2+2+3+4~~

$$L = \sum P_i \cdot L_i$$

$$= 2.21 \text{ bits / symbol.}$$

~~Entor~~ Entropy ⇒

$$H(x) = -\sum p(x_i) \cdot \log_2 p(x_i)$$

$$= -[0.35 \log_2 0.35 + 0.3 \log_2 0.3 +$$

$$0.2 \log_2 0.2 + 0.1 \log_2 0.1 + 2 \times 0.05$$

$$= 2.109 \text{ bits / symbol.}$$

Avg bits of information

$H(X) = 2.109$ bits/symbol.

Avg. binary digits of info per symbol

$L_{avg} = 2.21$ bits/symbol.

(d) The distribution (with respect to energy) of electron concentration in the conduction band is given by density of allowed quantum states times the probability that state being occupied by an electron. i.e., $n(E) = g_C(E)\,f(E)$
where, $g_C(E)$ = Density of allowed states, $f(E)$ = probability of state being occupied.
Assuming that Boltzmann approximation in a semiconductor is valid, calculate the ratio of $n(E)$ at $E = E_C + 4kT$ to that at $E = E_C + (kT/2)$. Here, k = Boltzmann constant, E_C = edge of the conduction band and T = temperature in °K.

[12 marks]

Sol

$n(E) = g_c(E) \cdot f(E)$

where $g_c(E) = g_c \sqrt{E - E_c}$ ~~E−E_c~~

$= g_c (E - E_c)^{1/2}$

$f(E) = \dfrac{1}{1 + e^{(E - E_F)/kT}}$

$n(E)$ at $E = E_c + 4kT$

$n_1(E) = g_c [4kT]^{1/2} \cdot \dfrac{1}{1 + e^{\frac{(E_c + 4kT - E_F)}{kT}}}$

$$n_2(E) = g_c \left(\frac{KT}{2}\right)^{1/2} \frac{1}{1+e^{\left(E_c+\frac{KT}{2}-E_f\right)/KT}}$$

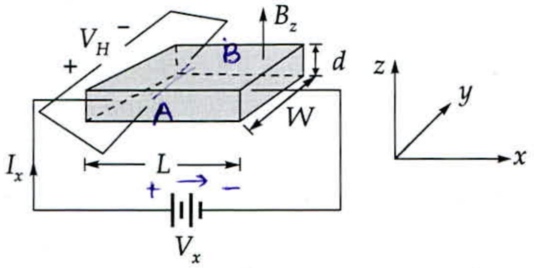
$$\frac{n_1(E)}{n_2(E)} = \sqrt{\frac{4KT}{KT/2}} \cdot \frac{1+e^{\frac{(E_c-E_f)+KT/2}{KT}}}{1+e^{\frac{(E_c-E_f)+4KT}{KT}}}$$

$$= \sqrt{8} \cdot \frac{1+e^{\frac{E_c-E_f}{KT}} \cdot e^{1/2}}{1+e^{\frac{E_c-E_f}{KT}} \cdot e^{4}}$$

Since $E_c - E_{fn} = KT \cdot \ln \frac{N_c}{N_D}$

$$\frac{n_1(E)}{n_2(E)} = 2\sqrt{2} \cdot \frac{1+\frac{N_c}{N_D} e^{1/2}}{1+\frac{N_c}{N_D} e^{4}}$$

(e) Consider a silicon Hall effect device which is used for the experiment as shown below:



The device has dimensions $d = 5 \times 10^{-3}$ cm, $W = 5 \times 10^{-2}$ cm and $L = 0.5$ cm. The electrical parameters measured as the result of the experiment are $I_x = 0.5$ mA, $V_x = 1.25$ V and $B_z = 6.5 \times 10^{-2}$ T. If the induced Hall electric field is $E_{Hy} = -16.5$ mV/cm, then determine:

- **(i)** Hall voltage (V_H)
- **(ii)** The type of semiconductor
- **(iii)** The majority carrier concentration

[12 marks]

Sol.

$$V_H = \frac{BI}{ne \cdot d} \qquad \rho = ne$$

$$V_H = \frac{6.5\times10^{-2} \times 0.5\times10^{-3}}{\rho \times 5\times10^{-3}\times10^{-2}} \quad — ①$$

since $\boxed{V_H = E_{Hy} \times W}$,

$$V_H = -16.5\times10^{-3} \times 5\times10^{-2}$$

$$= -82.5\times10^{-5} \text{ V.}$$

from eq ①

$$\rho = ne = 7.878\times10^{2}$$

$$n = \frac{7.878\times10^{2}}{1.6\times10^{-19}} = 4.92\times10^{21} \text{ per m}^3$$

Since $V_H = V_{AB} \Rightarrow$ Neg.

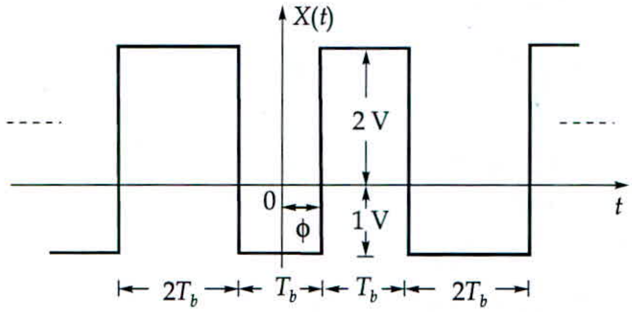
$\Rightarrow e^-$ will come to A

$$F = q(\vec{v}\times\vec{B}) = -e(-\hat{a}_x \times \hat{a}_z)$$

$$= -\hat{a}_y$$

Semiconductor is n type

Q.6 (a) Consider the random binary wave shown below:



In this binary wave, logic-1 is represented with positive rectangular pulse and logic-0 is represented with negative rectangular pulse, both with different amplitudes. ϕ is an independent random variable uniformly distributed in the range $[0, T_b]$, where T_b is the bit duration. Determine and sketch the auto-correlation function of $X(t)$. Assume that logic-1 and logic-0 are occurring with equal probability.

[20 marks]

Q.6 (b) A 1 cm long bar of *n*-type Ge has a cross section of 1 mm × 1 mm. The resistivity of material is 20 Ω-cm and the lifetime of the carriers is 100 microseconds.

(Assume $\mu_n = 3800\ \text{cm}^2/\text{V-s}$, $\mu_p = 1800\ \text{cm}^2/\text{V-s}$ and intrinsic carrier concentration $n_i = 2.5 \times 10^{13}/\text{cm}^3$).

(i) Calculate the resistance of the bar.

(ii) Calculate the donor concentration.

(iii) Calculate the resistance of the bar when it is illuminated such that excess electron-hole pairs are generated at a rate of $10^{15}\ \text{cm}^{-3}\ \text{s}^{-1}$, uniformly all over the bar.

[20 marks]

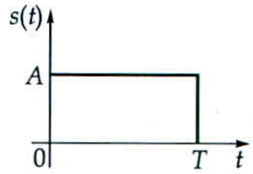
c) **(i)** Binary data (equiprobable bits) with a rate of 1 Mbps is transmitted through an AWGN channel using different modulation schemes. The two sided power spectral density of the channel noise is 0.5×10^{-11} W/Hz and the carrier signal used in the transmitters is $5\cos(2\pi f_c t)$ mV. In each case of different modulation schemes, the signals are received by their respective correlator receivers with exact phase synchronisation and with optimum threshold detection. Find the average symbol error probability for modulation schemes BASK, BFSK and BPSK.

(ii) Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t)n(t)\,dt \text{ and } n_2 = \int_0^T s_2(t)n(t)\,dt$$

Prove that n_1 and n_2 are orthogonal. **[15 + 5 marks]**

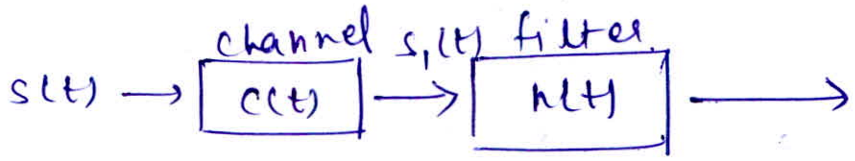
Q.7 (a) Consider the signal shown in the figure below:



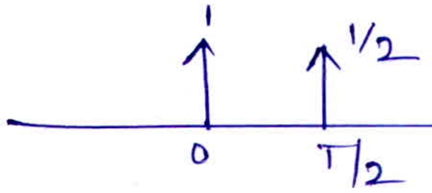
This signal is passed through a channel and applied to a filter matched to the signal $s(t)$ at the receiving end. If the channel is not ideal, but has an impulse response $c(t) = \delta(t)+\frac{1}{2}\delta\left(t-\frac{T}{2}\right)$, then determine and sketch the output of the matched filter.

[20 marks]

Sol.



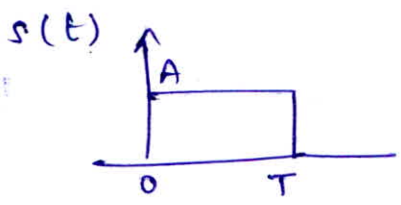
$$c(t) = \delta(t) + \frac{1}{2}\delta(t - T/2)$$

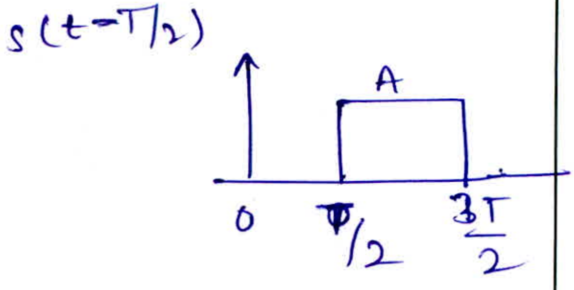


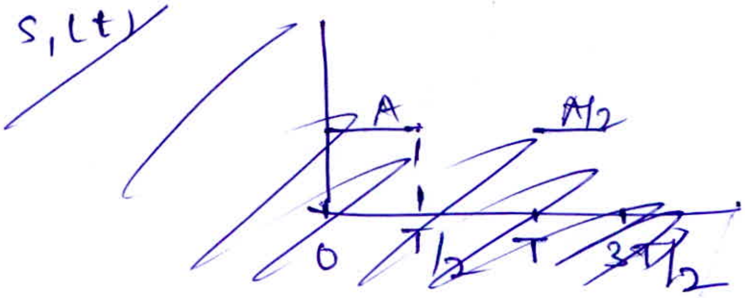
$$s_1(t) = s(t) * c(t)$$

$$= s(t) * \left[\delta(t) + \frac{1}{2}\delta(t - T/2)\right]$$

$$s_1(t) = s(t) + \frac{1}{2} s(t - T/2)$$

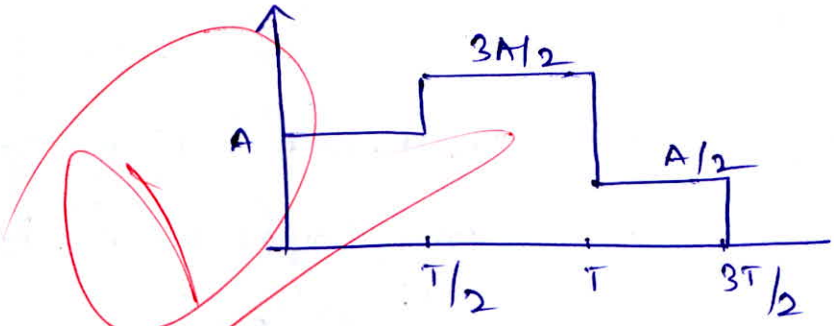




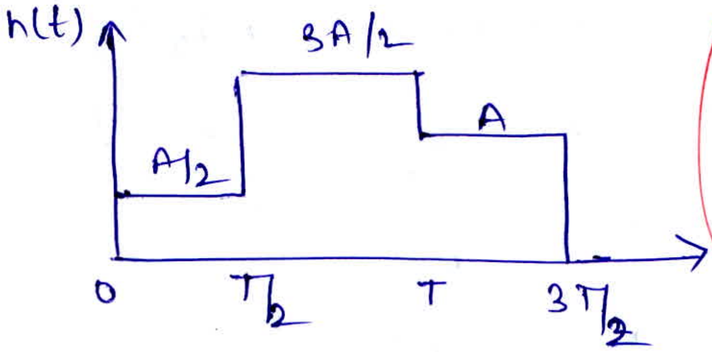


$s_1(t) \Rightarrow$

$$s_1(t) = A u(t) + \frac{A}{2} u\left(t - \frac{T}{2}\right) - A u(t-T) - \frac{A}{2} u\left(t - \frac{3T}{2}\right)$$

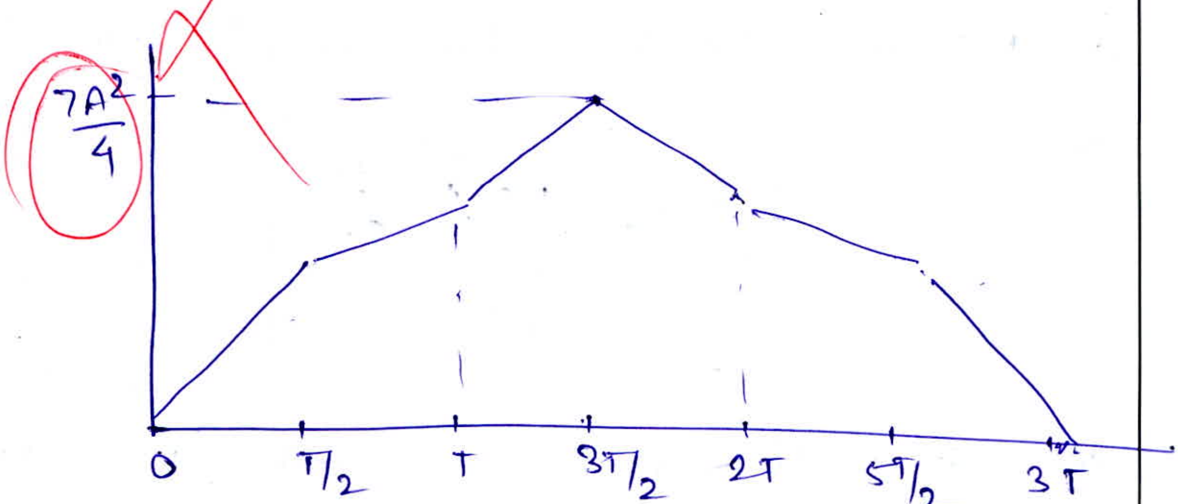


$$h(t) = s_1\left(\frac{3T}{2} - t\right)$$



2

$\therefore\ y(t) = s_1(t) * h(t)$



Q.7 (b) Explain the basic steps involved in the fabrication of a CMOS transistor using silicon on sapphire (SOS) process.

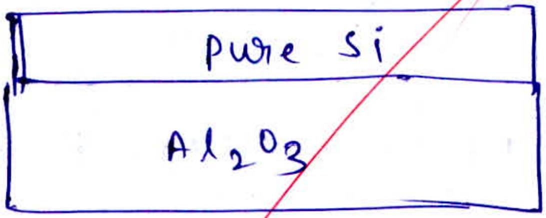
[20 marks]

SOS technique. — subtrate used is insulator Al_2O_3 (Saphhire), hence the name si on sapphire

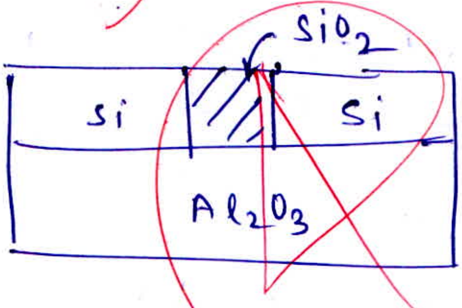
Process. —

① Take a substrate Al_2O_3

② Grow Si layer on the substrate using epitaxy.

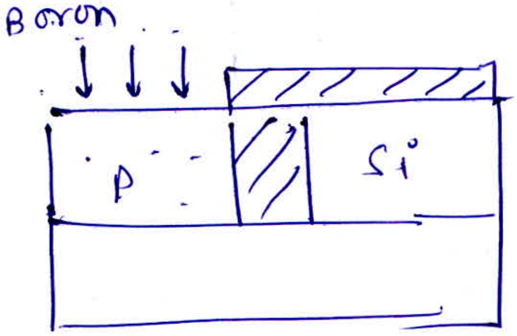


③ Perform LOCOS to grow SiO_2 in Si



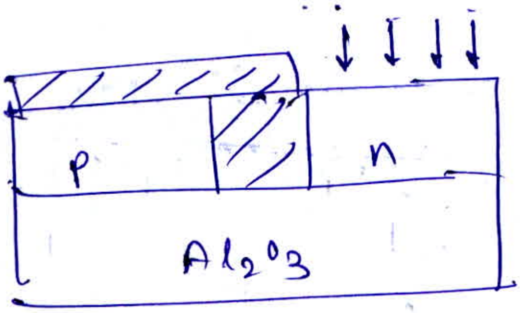
④ P-well and n-well formation

(i) P well formation. —



perform boron diffusio on one side by masking other side.

(ii) n well formation. — perform phosphorous diffusion on other side.



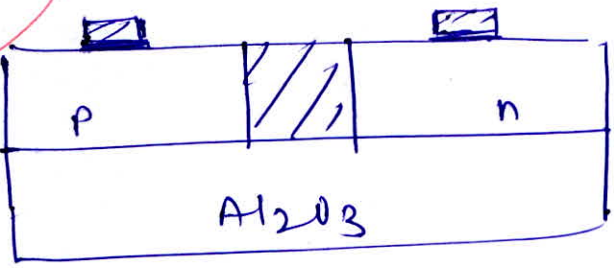
4

⑤ Gate formation for p-mos, n-mos.

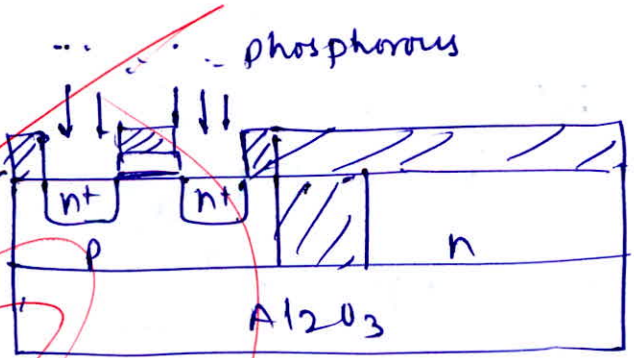
→ Gate oxide layer (wet oxidation)

→ Poly si gate.

2

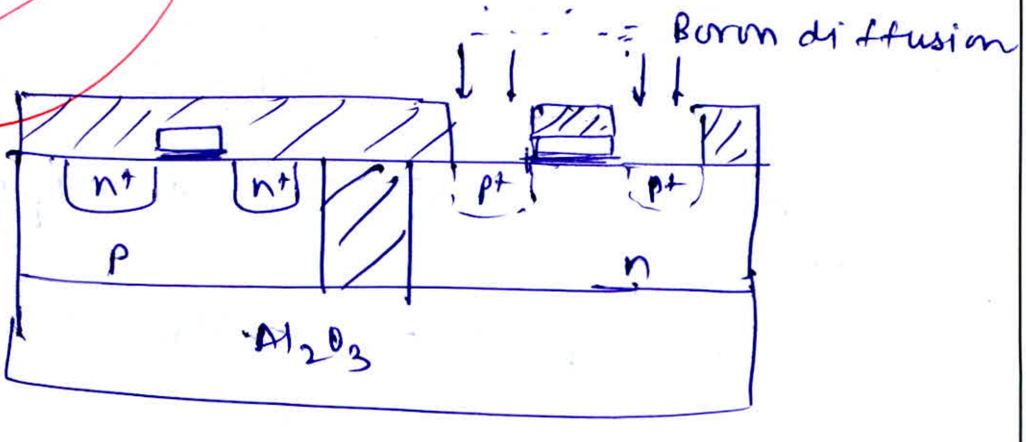


⑥ Source and drain formation of mos

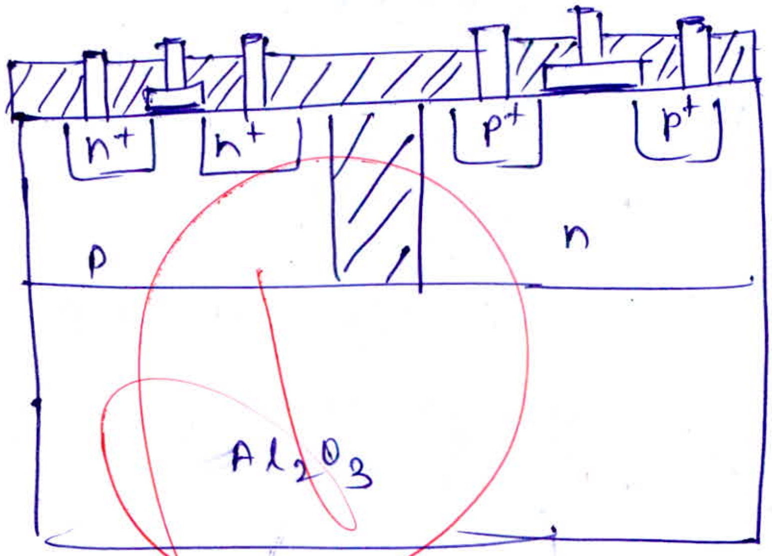


2

⑦ Source and drain formation of p mos.



(8) Metallisation –



Q.7 (c) A *p*-type lightly doped semiconductor has electron mobility μ_n, hole mobility μ_p, intrinsic carrier concentration n_i and the acceptor impurity concentration N_A.

- **(i)** Derive an expression for the hole concentration '*p*' in terms of n_i, μ_n and μ_p, such that the conductivity of the semiconductor is minimum.
- **(ii)** Derive an expression for the minimum conductivity of the semiconductor.
- **(iii)** If $n_i = 1.5 \times 10^{10}$ cm^{-3}, $\mu_n = 1300$ cm^2/V-sec and $\mu_p = 500$ cm^2/V-sec, then calculate the value of minimum conductivity.
- **(iv)** If there is 100% ionization of doping atoms, then calculate the value of acceptor impurity concentration (N_A).

[20 marks]

Sol- (i) Conductivity

$$\sigma = n \mu_n e + p \mu_p e$$

since $np = n_i^2$

$$\sigma = \frac{n_i^2}{p} \mu_n e + p \mu_p e$$

$\sigma \to \min$ $\quad \frac{d\sigma}{dp} = 0$

$$\frac{d\sigma}{dp} = n_i^2 \mu_n e \left(\frac{-1}{p^2}\right) + \mu_p e = 0$$

$$\mu_p = \frac{n_i^2 \mu_n}{p^2}$$

$$p^2 = n_i^2 \frac{\mu_n}{\mu_p}$$

$$\boxed{p = n_i \sqrt{\frac{\mu_n}{\mu_p}}}$$

$$\sigma_{min} = \frac{n_i^2}{p}\mu_n e + p\mu_p e$$

$$= \frac{n_i^2 \mu_n e}{n_i \sqrt{\frac{\mu_n}{\mu_p}}} + n_i \sqrt{\frac{\mu_n}{\mu_p}}\,\mu_p e$$

$$= n_i e\sqrt{\mu_p \mu_n} + n_i e\sqrt{\mu_n \mu_p}$$

3

$$\boxed{\sigma_{min} = 2 n_i \sqrt{\mu_p \mu_n}\, e}$$

$$\sigma_{min} = 2 \times 1.5 \times 10^{10} \sqrt{1300 \times 500} \times 1.6 \times 10^{-19}$$

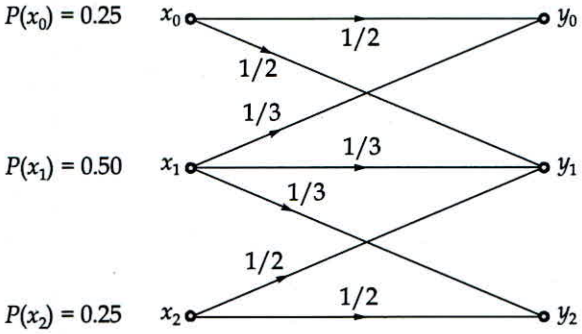
$$= 3.86 \times 10^{-6} \quad \Omega^{-1}\,cm^{-1}$$

v) 100% ionization ⇒

$$p = N_A$$

$$np. \quad n_i^2$$

) Consider the discrete memoryless channel shown below:



Determine the mutual information $I(X ; Y)$.

[20 marks]

Q.8 (b) For a boron diffusion in silicon at 1000°C, the surface concentration is maintained at $10^{19}\ \text{cm}^{-3}$ and the diffusion time is 1 hour. Assume that the diffusivity (D) of Boron in Silicon at 1000°C is $2 \times 10^{-14}\ \text{cm}^2/\text{s}$. Determine:

(i) The total number of dopant atoms per unit area of semiconductor.

(ii) The distance of the location from the surface where the dopant concentration reaches $10^{15}\ \text{cm}^{-3}$. Assume that $\text{erfc}^{-1}(10^{-4}) = 2.75$.

(iii) The gradient of the diffusion profile at the surface.

(iv) The gradient of the diffusion profile at the distance from the surface obtained in part (ii).

[20 marks]

Do not write in this margin

Q.8 (c) **(i)** Find the expression for reverse saturation current I_0 in a *p*-*n* junction diode in terms of intrinsic carrier concentration n_i.

(ii) Find an expression for the reverse saturation current in terms of the conductivity of the device and prove that, $I_0 = AV_T \dfrac{b\sigma_i^2}{(1+b)^2}\left[\dfrac{1}{L_p\sigma_n} + \dfrac{1}{L_n\sigma_p}\right]$ where, $b = \dfrac{\mu_n}{\mu_p}$

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work